

# Introduction to Parametric interpolation for computer animation

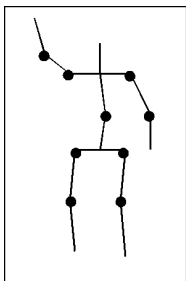
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EVASION-LJK

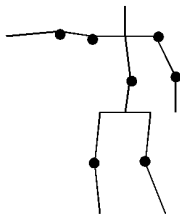
- 1 Parametric curves
- 2 Cubic bases
- 3 Arclength parameterization
- 4 Velocity control

# Main idea

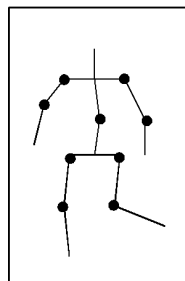
- Series of pairs (time, parameter values)
- interpolate inbetween



key  $T_0, q_0$



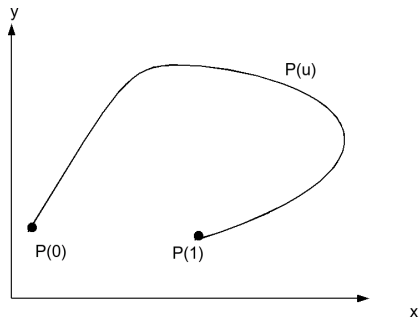
$t$ , interpolated  $q$



key  $T_1, q_1$

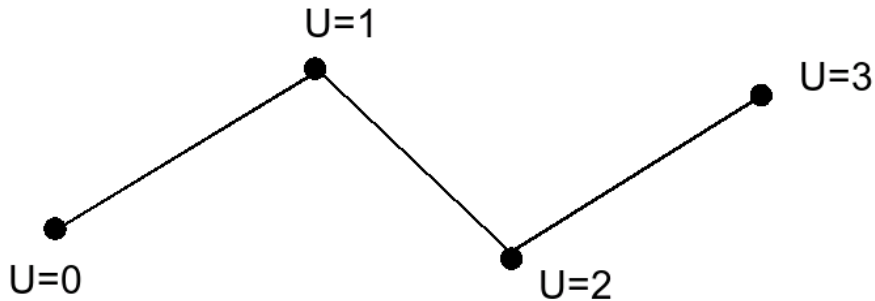
# Parametric curves

- General form :  $\mathbf{P}(u) = (x(u), y(u), z(u)) \quad 0 \leq u \leq 1$



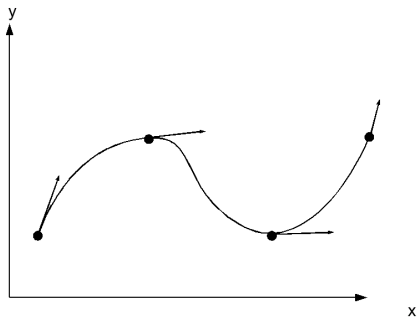
# Linear curves

- General form :  $\mathbf{P}(u) = (1 - u)\mathbf{P}_0 + u\mathbf{P}_1$
- Use fractional value of global parameter  $U$  for piecewise curves
- Sharp discontinuities



# Cubic curves

- Patches are defined using endpoints and tangents (*Hermite splines*)
- $x(u)$  and  $y(u)$  are cubic functions
- Smooth continuity



# Hermite spline equation

$$y = b_0 + b_1 u + b_2 u^2 + b_3 u^3$$

$$y' = b_1 + 2b_2 u + 3b_3 u^2$$

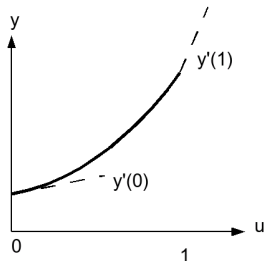
- Match values and slopes

$$b_0 = y_0$$

$$b_1 = y'_0$$

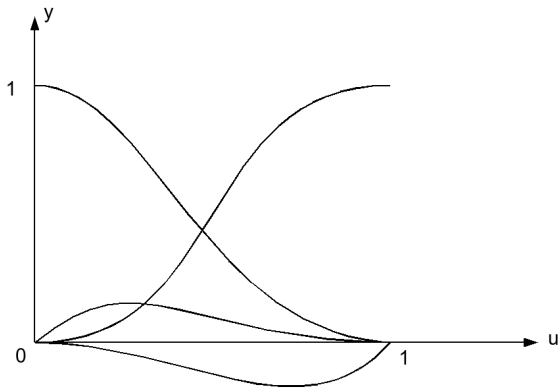
$$b_2 = 3(y_1 - y_0) - 2y'_0 - y'_1$$

$$b_3 = 2(y_0 - y_1) + y'_0 + y'_1$$



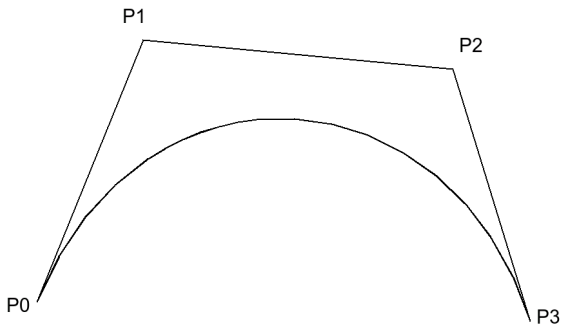
# Hermite blending functions

$$y(t) = (1-3u^2+2u^3)y_0 + (3u^2-2u^3)y_1 + (u-2u^2+u^3)y'_0 + (u^3-u^2)y'_1$$



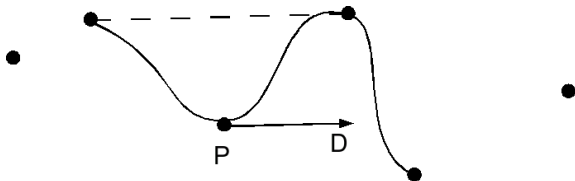
# Beziers curves

- 4 points
- tangents along the first and last line segments



# Catmull-Rom splines

- Derived from Hermite splines
- Approximate tangents using control points  
$$D_i = \frac{1}{2}(P_{i+1} - P_{i-1})$$
- Arbitrary first and last points



# Kochaneck-Bartels splines

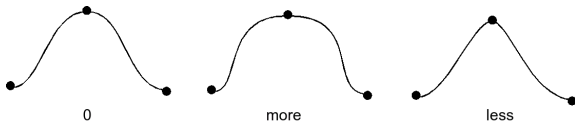
- Add intuitive control parameters to Catmull-Rom splines
  - Tension  $t$
  - Bias  $b$
  - Continuity  $c$

$$\begin{aligned}
 D^- &= \frac{(1-t)(1-b)(1+c)}{2}(P_{i+1} - P_i) \\
 &\quad + \frac{(1-t)(1+b)(1-c)}{2}(P_i - P_{i-1}) \\
 D^+ &= \frac{(1-t)(1-b)(1-c)}{2}(P_{i+1} - P_i) \\
 &\quad + \frac{(1-t)(1+b)(1+c)}{2}(P_i - P_{i-1})
 \end{aligned}$$

# Tension

- Tension  $t$  is responsible for sharpness

$$D = \frac{1-t}{2}(P_{i+1} - P_{i-1})$$



# Bias

- Bias  $b$  modifies the slope

$$D = \frac{1-b}{2}(P_{i+1} - P_i) + \frac{1+b}{2}(P_i - P_{i-1})$$

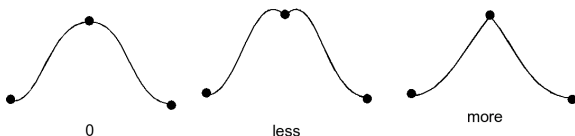


# Discontinuity

- Discontinuity  $c$  splits the tangent in two pieces

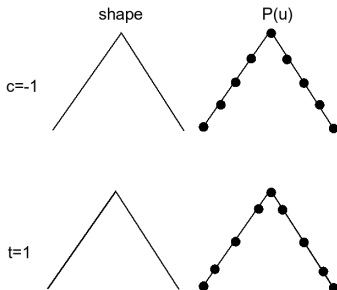
$$D^- = \frac{1+c}{2}(P_{i+1} - P_i) + \frac{1-c}{2}(P_i - P_{i-1})$$

$$D^+ = \frac{1-c}{2}(P_{i+1} - P_i) + \frac{1+c}{2}(P_i - P_{i-1})$$



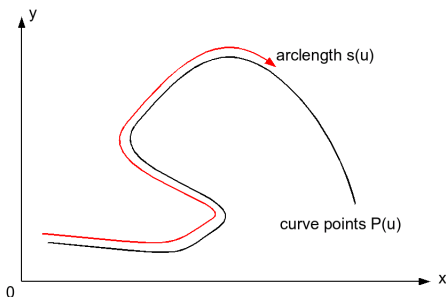
# Arclength parameterization

- It is difficult to control  $P$  and  $\frac{dP}{du}$  independently
- Example using two Kochaneck-Bartels with same shape



# Arclength parameterization

- We want to control the arclength  $s(t)$



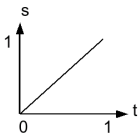
- Problem :  $s(t)$  is far from trivial

# Approximate arclength parameterization

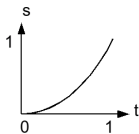
- Approximate  $s(t)$  using distances between regularly sampled points
- Model  $s(t)$  as a tabulated function
- For a given  $s$ , find the corresponding interval and apply linear approximation

# Velocity

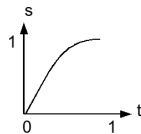
- We want to control the velocity of a moving object along a given path (spline)
- Use arclength parameterization
- Apply velocity control as  $s(t)$  with  $s = 0$  at starting point and  $s = 1$  at end point



constant  
speed



accelerated



decelerated

