Rendering: Advanced Sampling

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Roadmap

- Intuitive Properties of Light (recap)
- Light Surface Sampling
- Multiple Importance Sampling
- Next Event Estimation
Intuitive Properties of Light

- It travels in straight lines
Intuitive Properties of Light

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• Angle $\theta$ plays a role (cos($\theta$) rule)
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• Intensity is linear (believe me)
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- Size of the light source
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- Distance to light source
Intuitive Properties of Light

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Intuitive Properties of Light

- How “bright” something is doesn’t directly tell you how brightly it *illuminates* something
  - The lamp appears just as bright from across the room and when you stick your nose to it (“intensity does not attenuate”)
  - Also, the lamp’s apparent brightness does not change much with the angle of exitance
Intuitive Properties of Light

• How “bright” something is doesn’t directly tell you how brightly it *illuminates* something
  • The lamp appears just as bright from across the room and when you stick your nose to it (“intensity does not attenuate”)
  • Also, the lamp’s apparent brightness does not change much with the angle of exitance

• However:
  • If you take the receiving surface further away, it will reflect less light and appear darker
  • If you tilt the receiving surface, it will reflect less light and appear darker
Light arriving at point $x$:

$$L_i(x) = \int L_i(x, \omega) \cos(\theta_x) \, d\omega$$

Light from direction $\omega$:

Solid angle (next): (not useful for rendering)
Light Surface Sampling

\[ L_i(x) = \int_{\Omega} L_i(x, \omega)(\omega \cdot n) \, d\omega \]

Light arriving at point \( x \)

Light from direction \( \omega \)

Solid angle (next)

(not useful for rendering yet)
Light Surface Sampling

• What’s going on with that object size, distance etc?

• “Illumination power” is determined by the solid angle subtended by the light source (simple, how big something looks).
Light Surface Sampling

• How big something looks in 2d

Light source

angle
Light Surface Sampling

- How big something looks in 2d

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Light Surface Sampling

- How big something looks in 2d
- Angle $\alpha$ in radians $\iff$ length on unit circle
- Full circle is $2\pi$
Light Surface Sampling

- How big something looks in 3d
- replace unit circle with unit sphere
- Same thing: projected area on unit sphere $\Leftrightarrow$ solid angle
- Unit: steradian (sr)
- Full solid angle is $4\pi$ (unit sphere surface)
Light Surface Sampling

Relationship between a surface patch and the solid angle
=> what determines the area of the projected patch (solid angle)

The tiny solid angle $dw$ that corresponds to $dA = \text{projection of } dA \text{ onto the surface of the sphere}$

Some small surface patch $dA$ in the scene

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Relationship between a surface patch and the solid angle

=> what determines the area of the projected patch (solid angle)

\[ A = \int_{a}^{b} f(x) \, dx \]

- The tiny solid angle \( d\omega \) that corresponds to \( dA = \) projection of \( dA \) onto the surface of the sphere.
- \( d\omega \) also encodes direction.
- \( dA \) also encodes position in the world.
- Some small surface patch \( dA \) in the scene.

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Light Surface Sampling

Relationship between a surface patch and the solid angle
It holds for infinitesimally small surface patches $dA$ and the corresponding differential solid angles $d\omega$

$$d\omega = \frac{dA \cos \theta}{r^2}$$

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Light Surface Sampling

Larger Surfaces
Actual surfaces consist of infinitely many tiny patches $dA$ -- do you see where we are going?

$$d\omega = \frac{dA \cos \theta}{r^2}$$
Larger Surfaces
Actual surfaces consist of infinitely many tiny patches $dA$
------ do you see where we are going?

Change of variables $dA \leftrightarrow d\omega$
We can integrate over the surface $S$

$$d\omega = \frac{dA \cos \theta}{r^2}$$
We have seen this before, but now we want to integrate over a single light surface. How do we need to change the formula?

\[ L_i(x) = \int_{\Omega} L_i(x, \omega) \cos(\theta_x) \, d\omega \]

Light arriving at point \( x \)

Light from direction \( \omega \)

Solid angle (just before)

(not useful for rendering)
Light Surface Sampling

Light arriving at point $x$

$$L_i(x) = \int_{\Omega} L_i(x, \omega) \cos(\theta_x) \, d\omega$$

Light from source $[l]$ arriving at point $x$

$$L_i^{[l]}(x) = \int_{S_l} L_e^{[l]}(y) \cos(\theta_x) \cos(\theta_y) \frac{1}{r^2} \, dA_y$$

Light from direction $\omega$

Solid angle (just before)

Light intensity at position $y$ on the surface
Light Surface Sampling

situation

\[ L_{i}^{[l]}(x) = \int_{S_{l}} L_{e}^{[l]}(y) \left( \frac{y - x}{|y - x|} \cdot n_{x} \right) \left( \frac{x - y}{|x - y|} \cdot n_{y} \right) dA_{y} \]

(light intensity at position \( y \) on the surface)

(not useful for rendering)
Light Surface Sampling

situation

\[ L_i^l(x) = \int_{S_l} L_e^l(y) \left( \frac{y - x}{|y - x|} \cdot n_x \right) \left( \frac{x - y}{|x - y|} \cdot n_y \right) dA_y \]

(light intensity at position \( y \) on the surface)

(y=A)

emitter \( \cos(\theta) \)

receiver \( \cos(\theta) \)

(distance)

(not useful for rendering)
Light Surface Sampling

\[ L_e(x, v) = \int_{\Omega} f_r(x, \omega \rightarrow v) L_i(x, \omega) \cos(\theta_x) \, d\omega \]

- **Light going in direction** \( v \)
- **Material**, modelled by the BRDF
- **Light from direction** \( \omega \)
- **Solid angle**

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Light Surface Sampling
Change of variables

Next:
Multiple Importance Sampling
Useful reading (links)

• Change of variables
• Jaakko Lehtinen's slides
  (I borrowed a lot from lecture 2, but there is more on point lights, intuition, links..)
• Károly Zsolnai-Fehér's slides, previously lecturing at TUW
  (more on history, physics, different approach on solid angle etc.)
• Károly Zsolnai-Fehér's lecture on YouTube