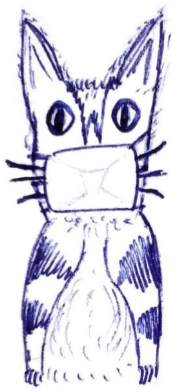
Rendering: Advanced Sampling

Adam Celarek



Research Division of Computer Graphics

Institute of Visual Computing & Human-Centered Technology

TU Wien, Austria

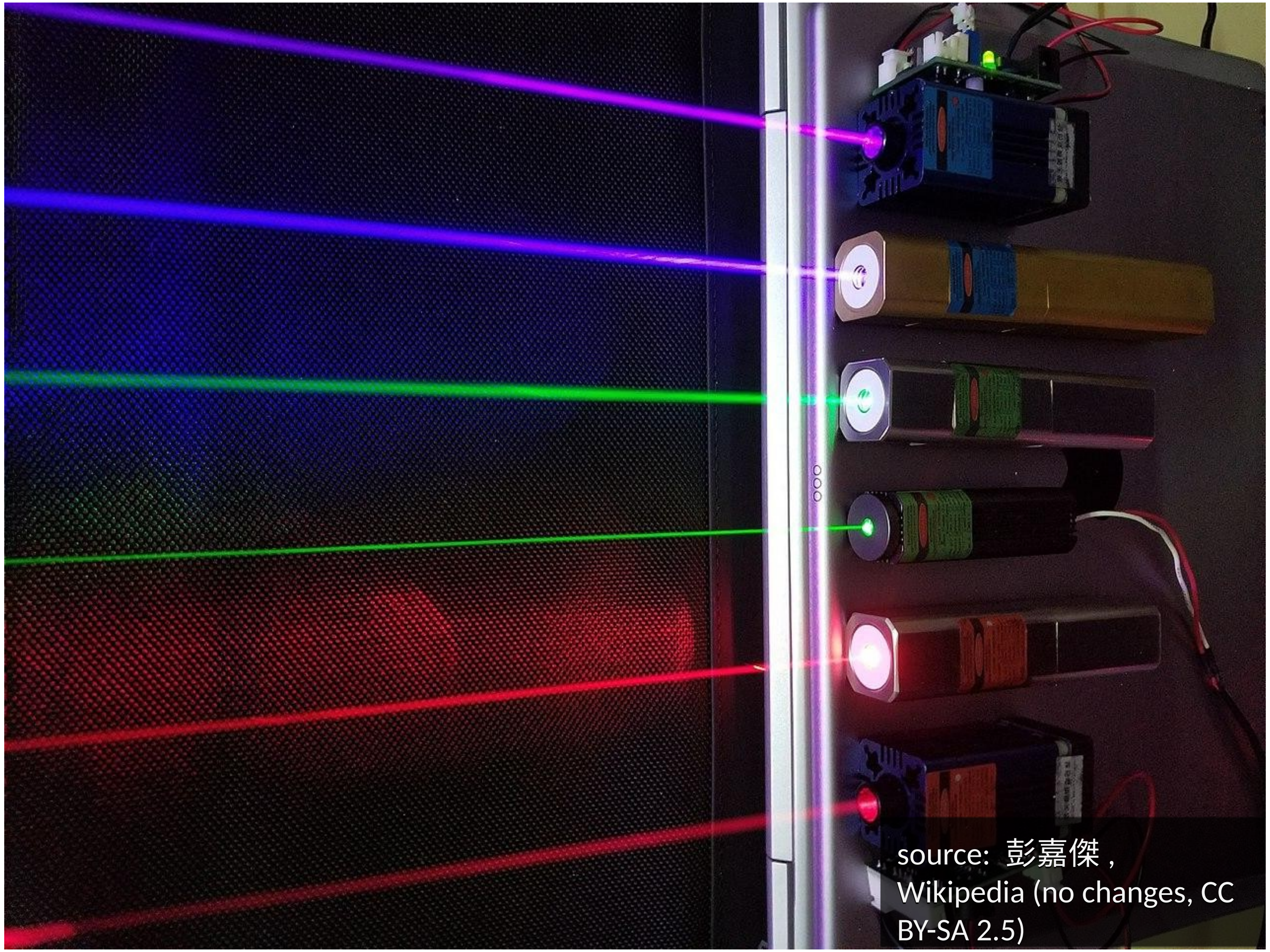
Roadmap

- Intuitive Properties of Light (recap)
- Light Surface Sampling
- Multiple Importance Sampling
- Next Event Estimation

source: own work

Intuitive Properties of Light

- It travels in straight lines



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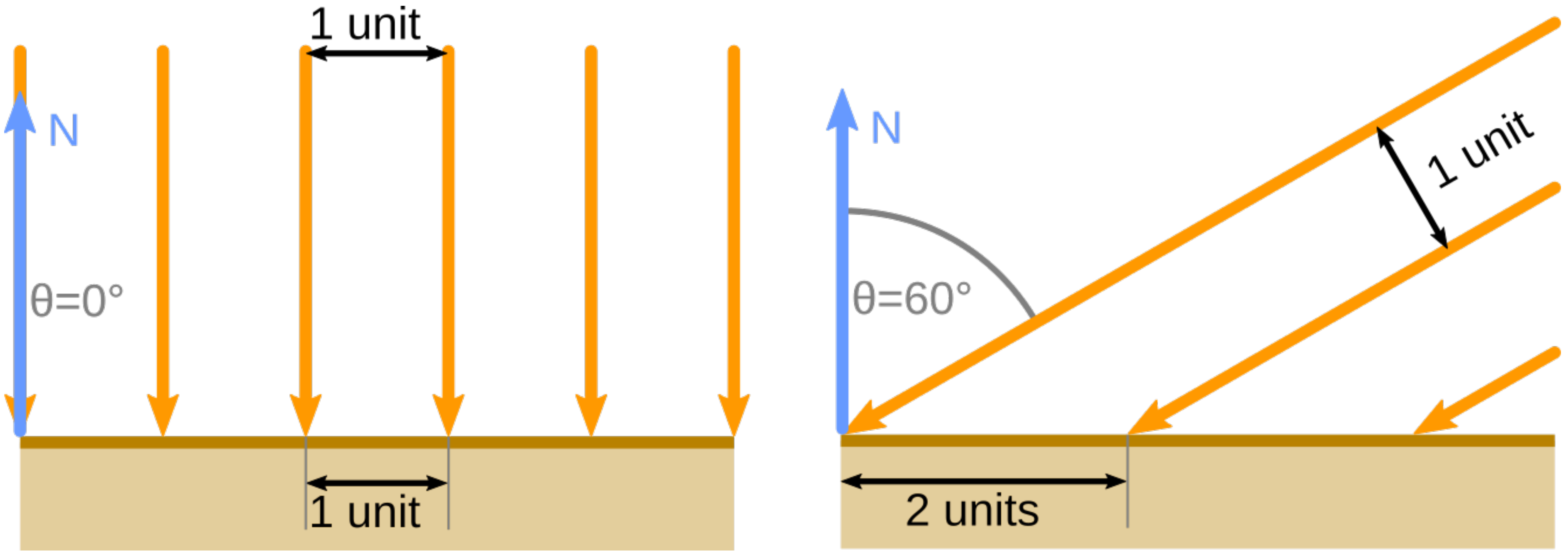
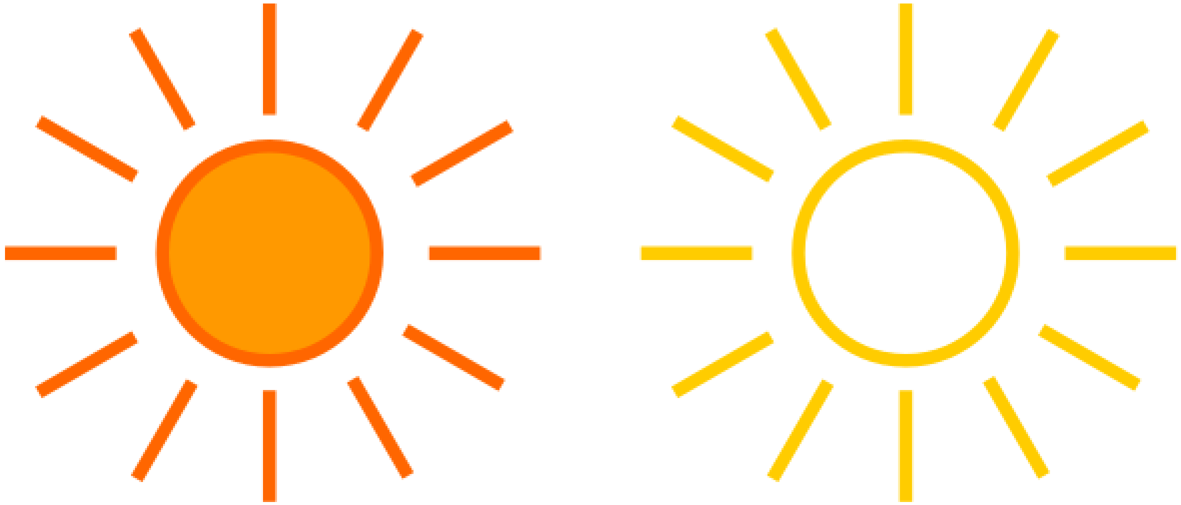
Intuitive Properties of Light

- It travels in straight lines
- Angle θ plays a role (cos(θ) rule)



Intuitive Properties of Light

- It travels in straight lines
- Angle θ plays a role (cos(θ) rule)
- Intensity is linear (believe me)



Intuitive Properties of Light

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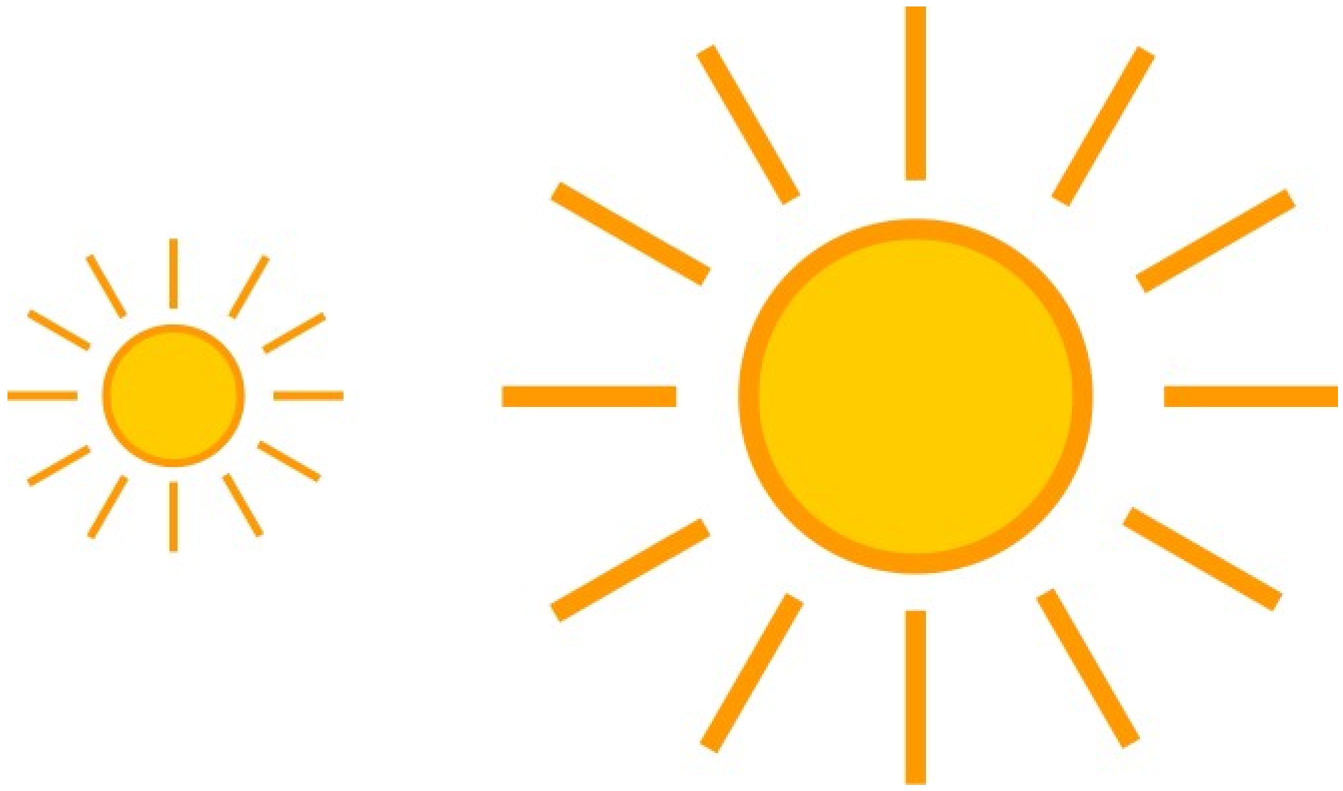
- Size of the light source



Intuitive Properties of Light

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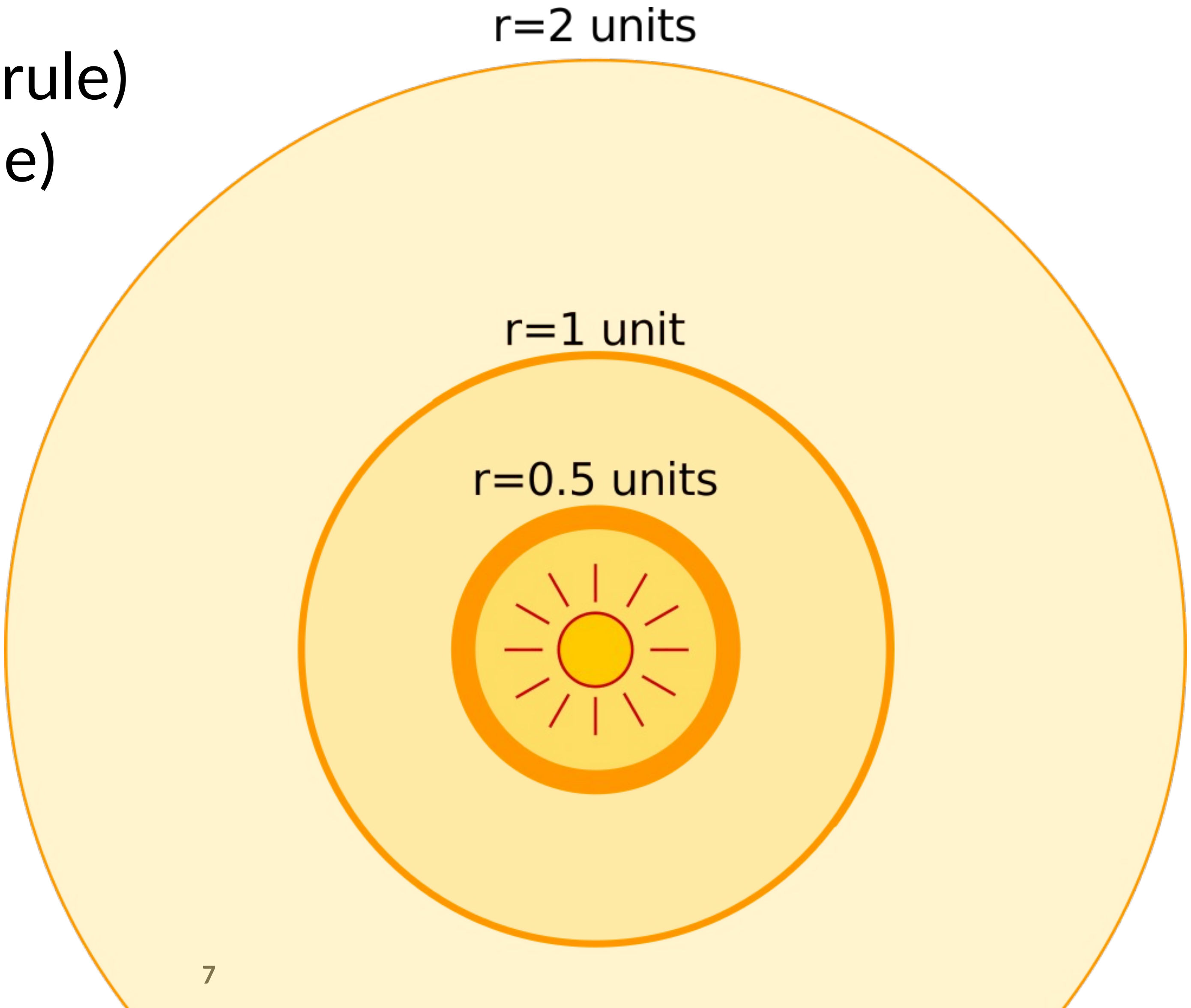
- Size of the light source
- Distance to light source



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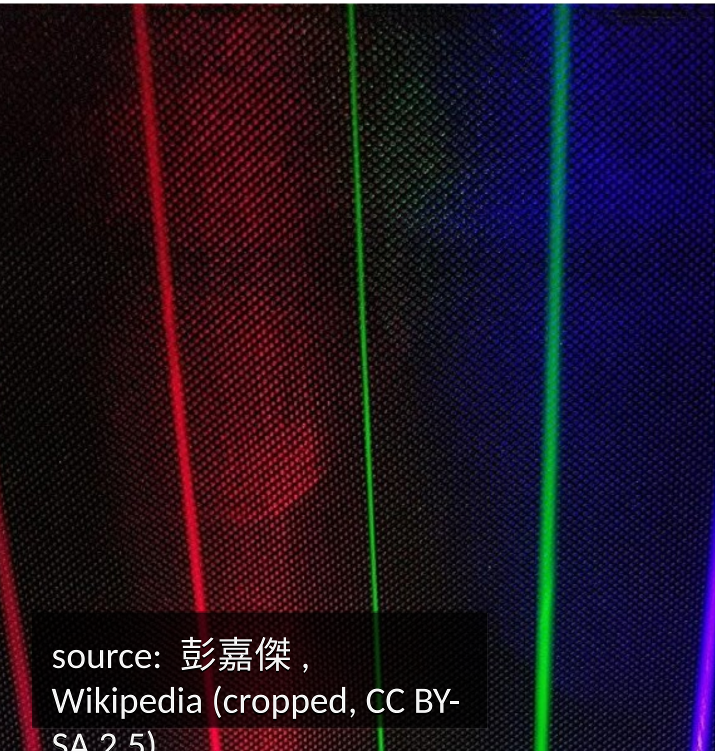

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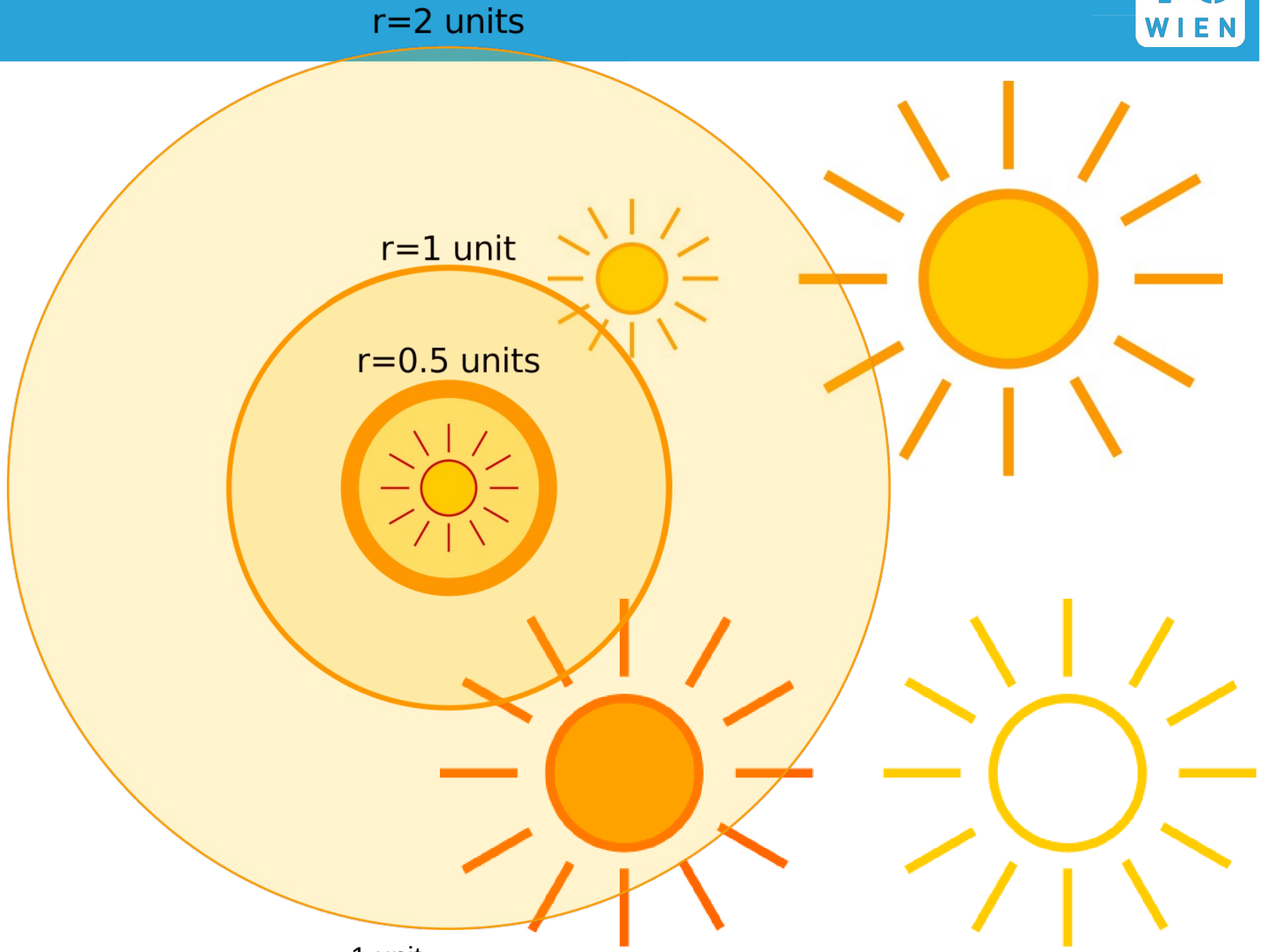

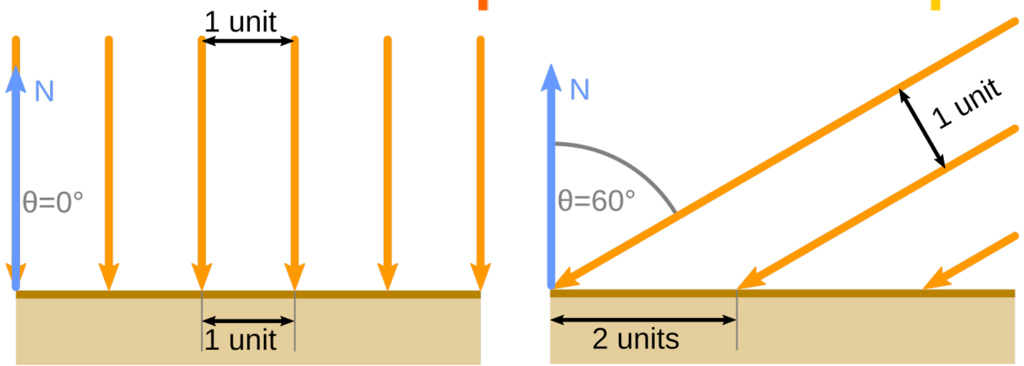

Intuitive Properties of Light

- How "bright" something is doesn't directly tell you how brightly it *illuminates* something
  - The lamp appears just as bright from across the room and when you stick your nose to it ("intensity does not attenuate")
  - Also, the lamp's apparent brightness does not change much with the angle of exitance

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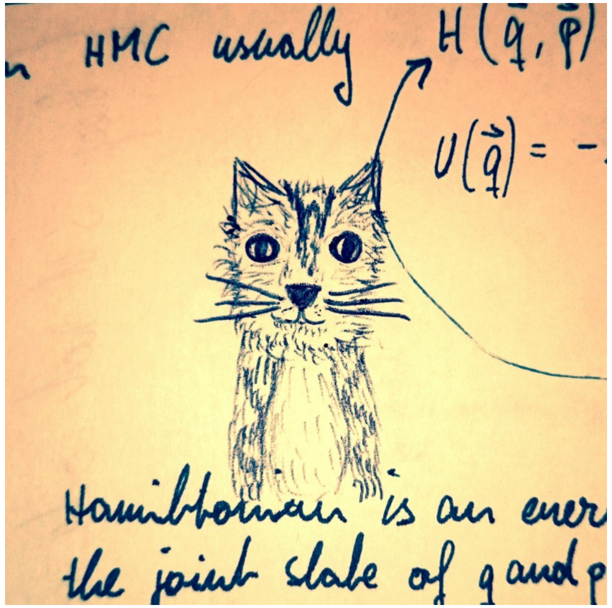
Intuitive Properties of Light

- How "bright" something is doesn't directly tell you how brightly it *illuminates* something
  - The lamp appears just as bright from across the room and when you stick your nose to it ("intensity does not attenuate")
  - Also, the lamp's apparent brightness does not change much with the angle of exitance
- **However**:
  - If you take the receiving surface further away, it will reflect less light and appear darker
  - If you tilt the receiving surface,it will reflect less light and appear darker



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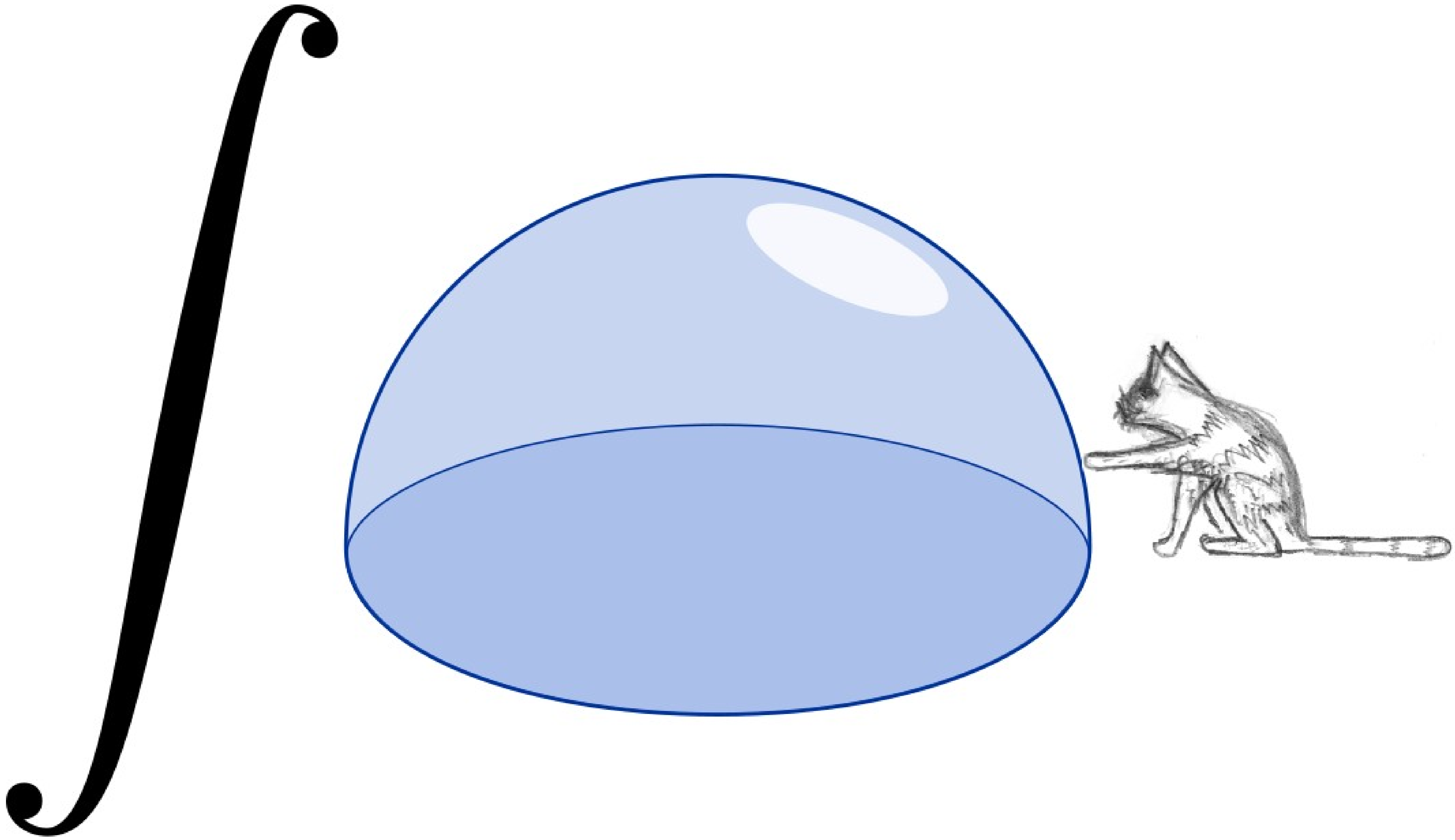
Make it math



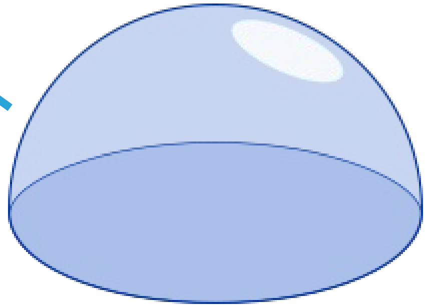
Make it math

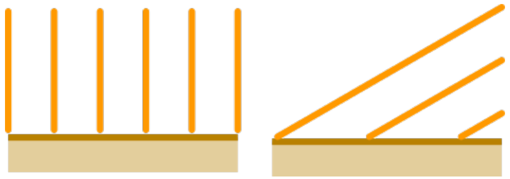
Light arriving at point x

Light from direction ω

Solid angle (next)

$$L_i(x) = \int_\Omega L_i(x,\omega)\cos(\theta_x)\,\mathrm{d}\omega$$





(not useful for rendering)

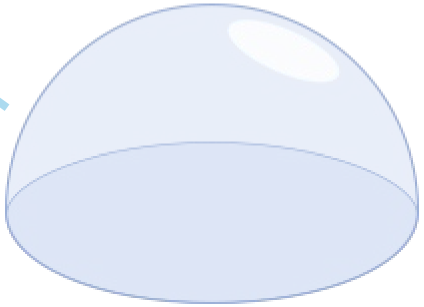
Light Surface Sampling

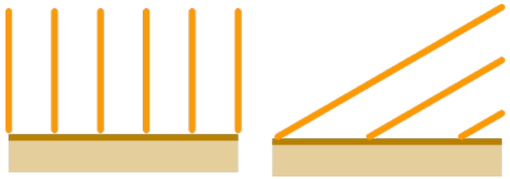
Light arriving at point x

Light from direction ω

Solid angle (next)

$$L_i(x) = \int_\Omega L_i(x, \omega)(\omega \cdot n)\,\mathrm{d}\omega$$





(not useful for rendering yet)

Light Surface Sampling

- What's going on with that object size, distance etc?

- "Illumination power" is determined by the solid angle subtended by the light source (simple, how big something looks).

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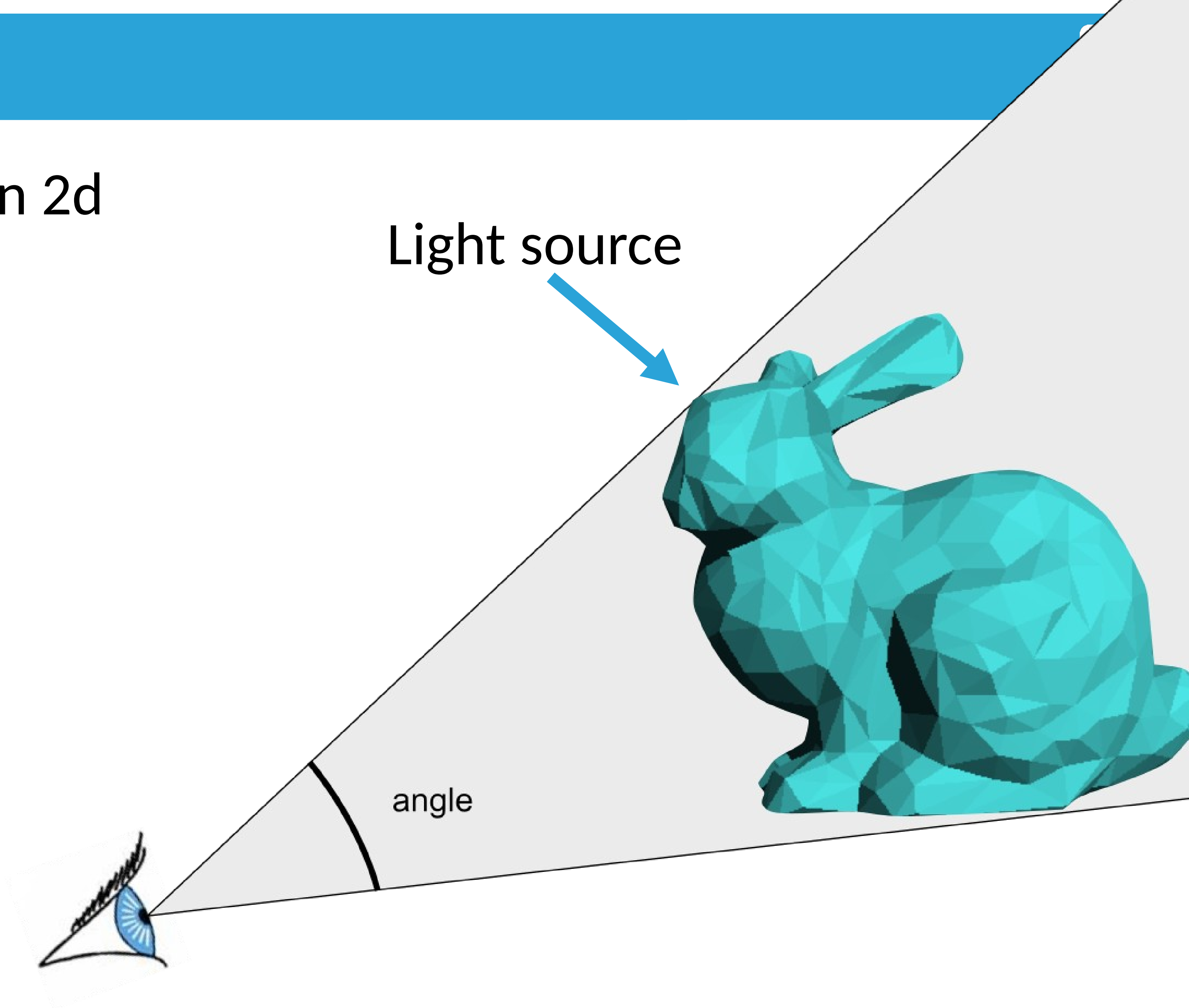
Light Surface Sampling

- How big something looks in 2d



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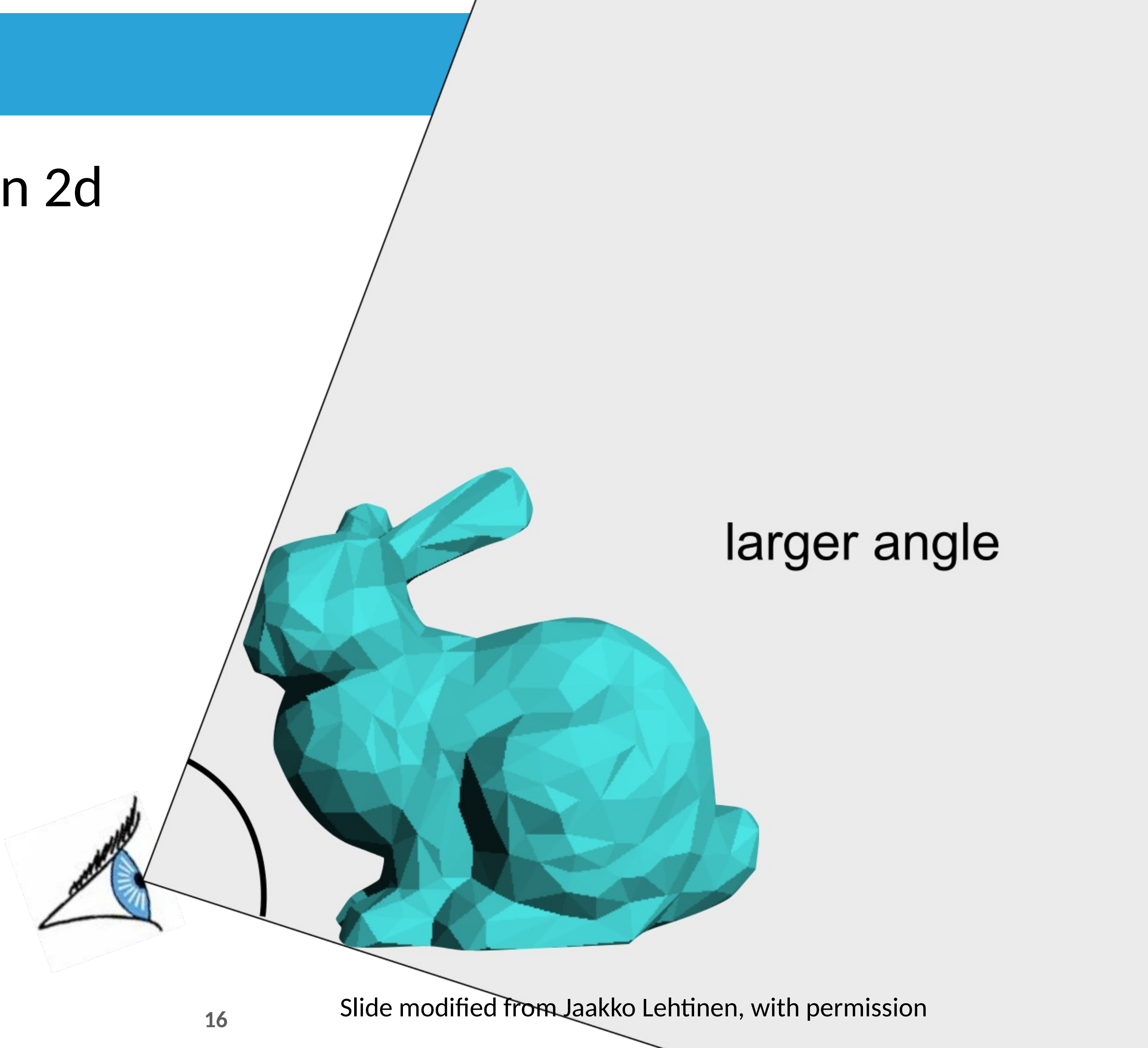
Light Surface Sampling

- How big something looks in 2d



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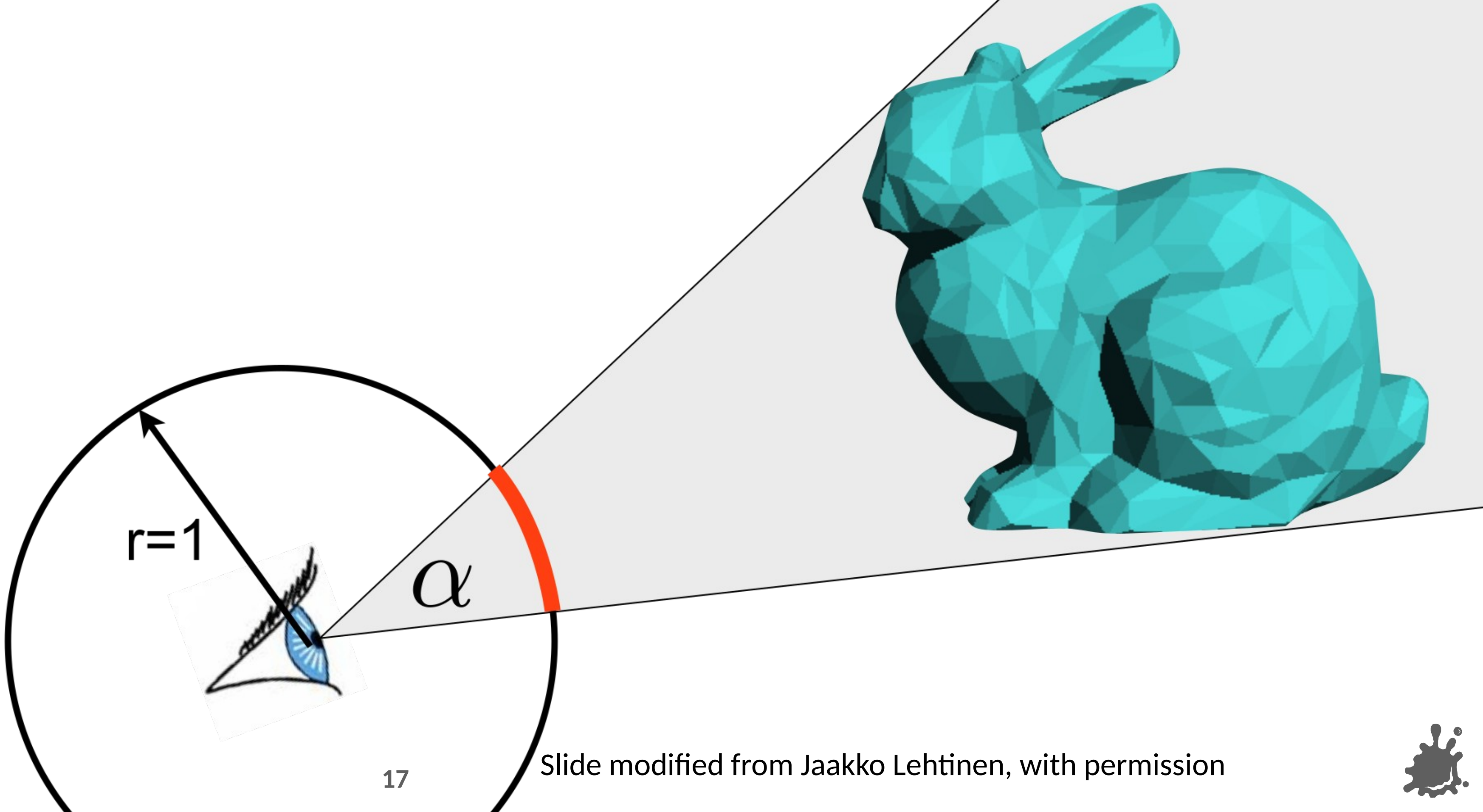
Light Surface Sampling

- How big something looks in 2d
- Angle α in radians ⇔ length on unit circle
- Full circle is 2π



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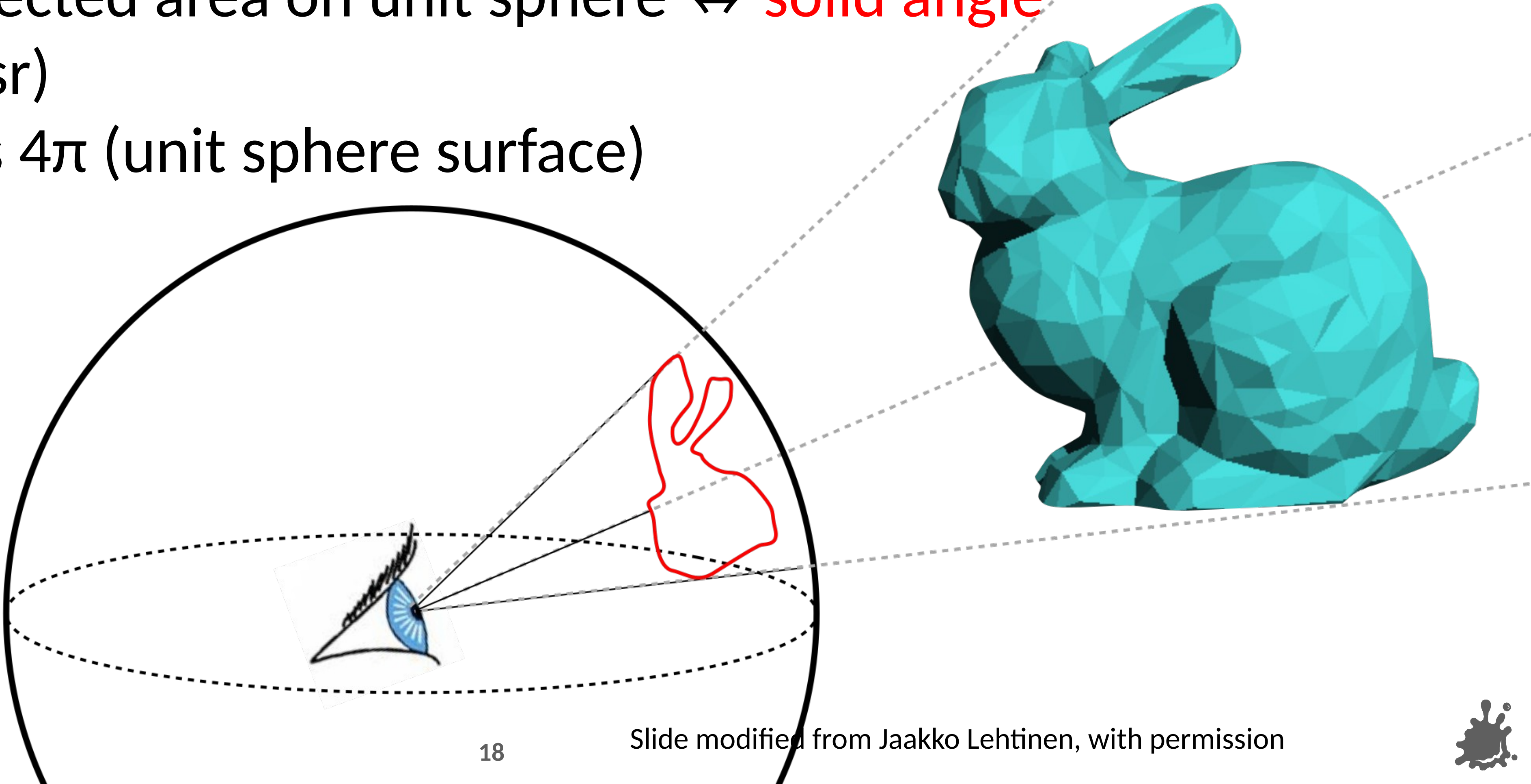
Light Surface Sampling

- How big something looks in **3d**
- replace unit circle with unit sphere
- Same thing: projected area on unit sphere ⇔ solid angle
- Unit: steradian (sr)
- Full solid angle is 4π (unit sphere surface)



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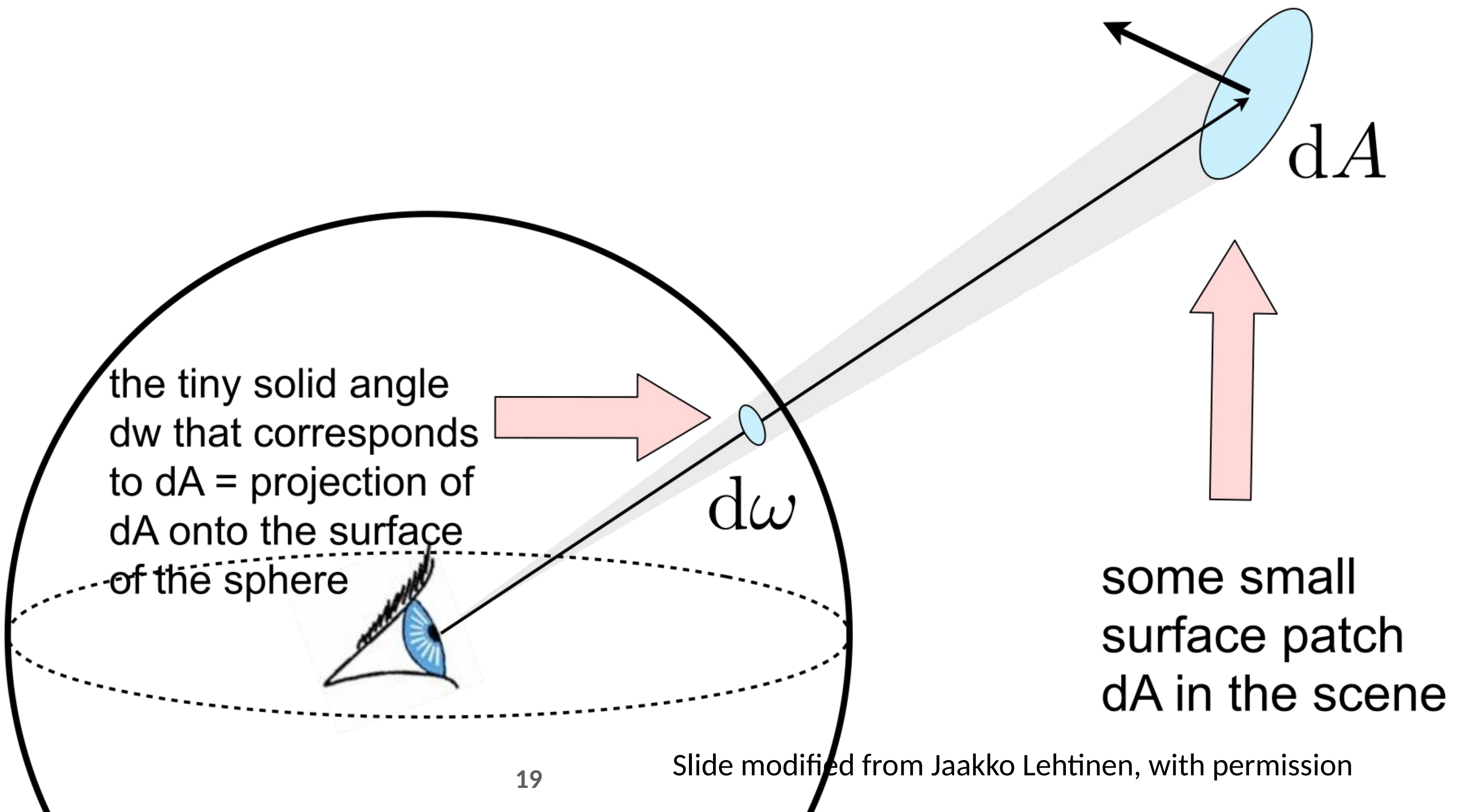
Relationship between a surface patch and the solid angle

=> what determines the area of the projected patch (solid angle)



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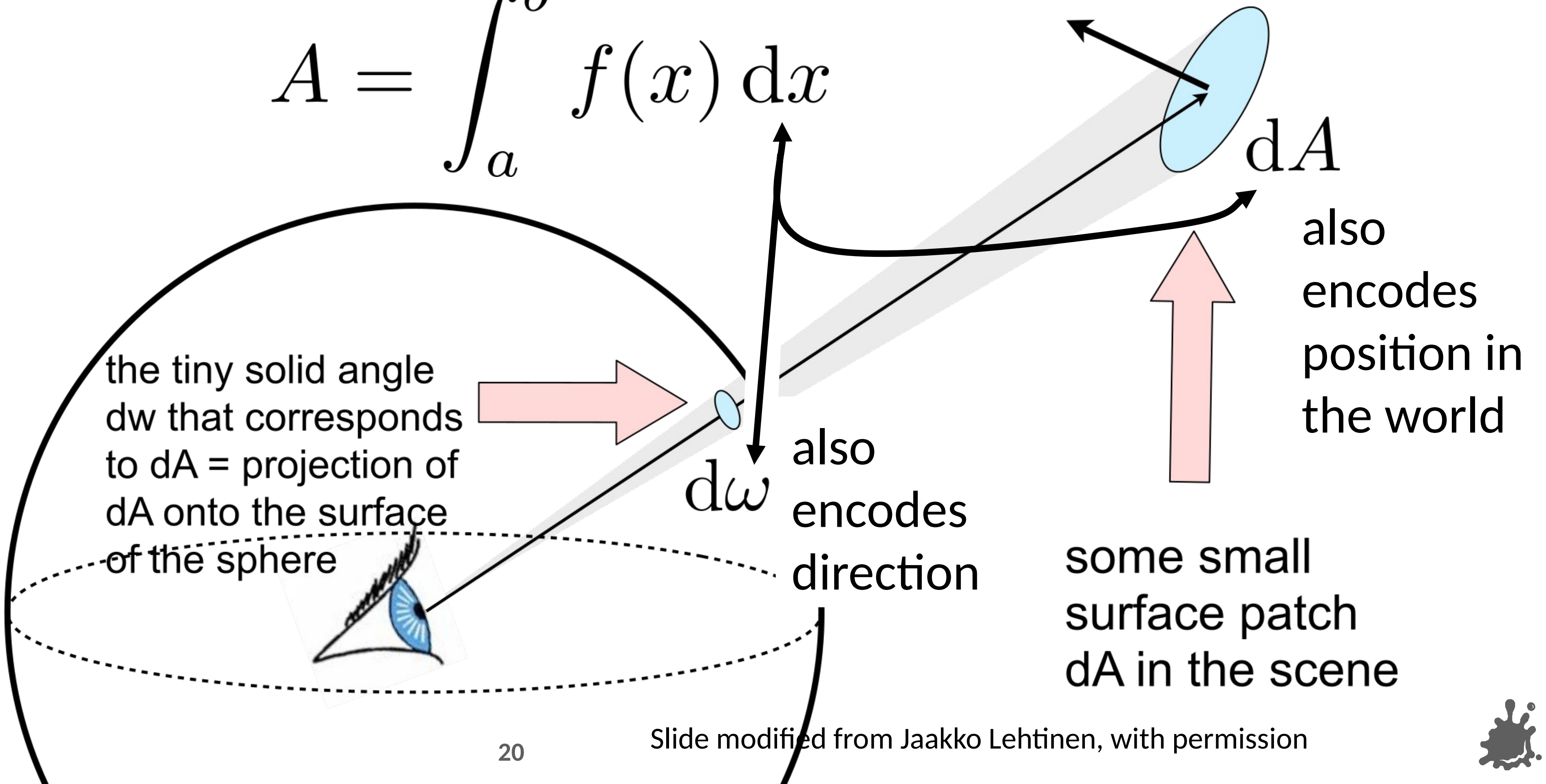
Relationship between a surface patch and the solid angle

=> what determines the area of the projected patch (solid angle)

$$A = \int_a^b f(x)\,\mathrm{d}x$$



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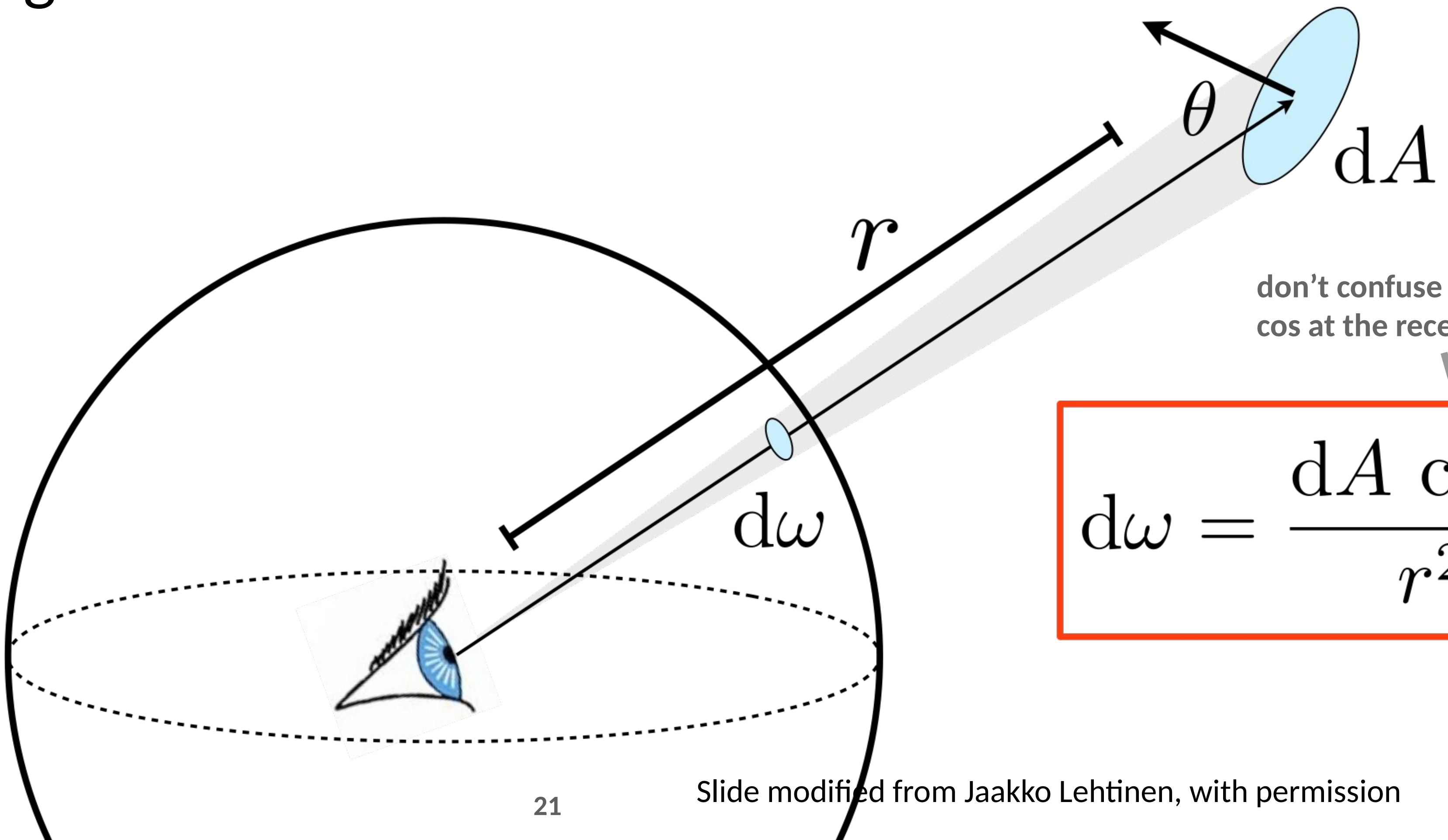
Relationship between a surface patch and the solid angle

It holds for infinitesimally small surface patches dA and the corresponding differential solid angles dω



don't confuse this with the cos at the receiving surface

$$\boxed{\mathrm{d}\omega = \frac{\mathrm{d}A \cos\theta}{r^2}}$$

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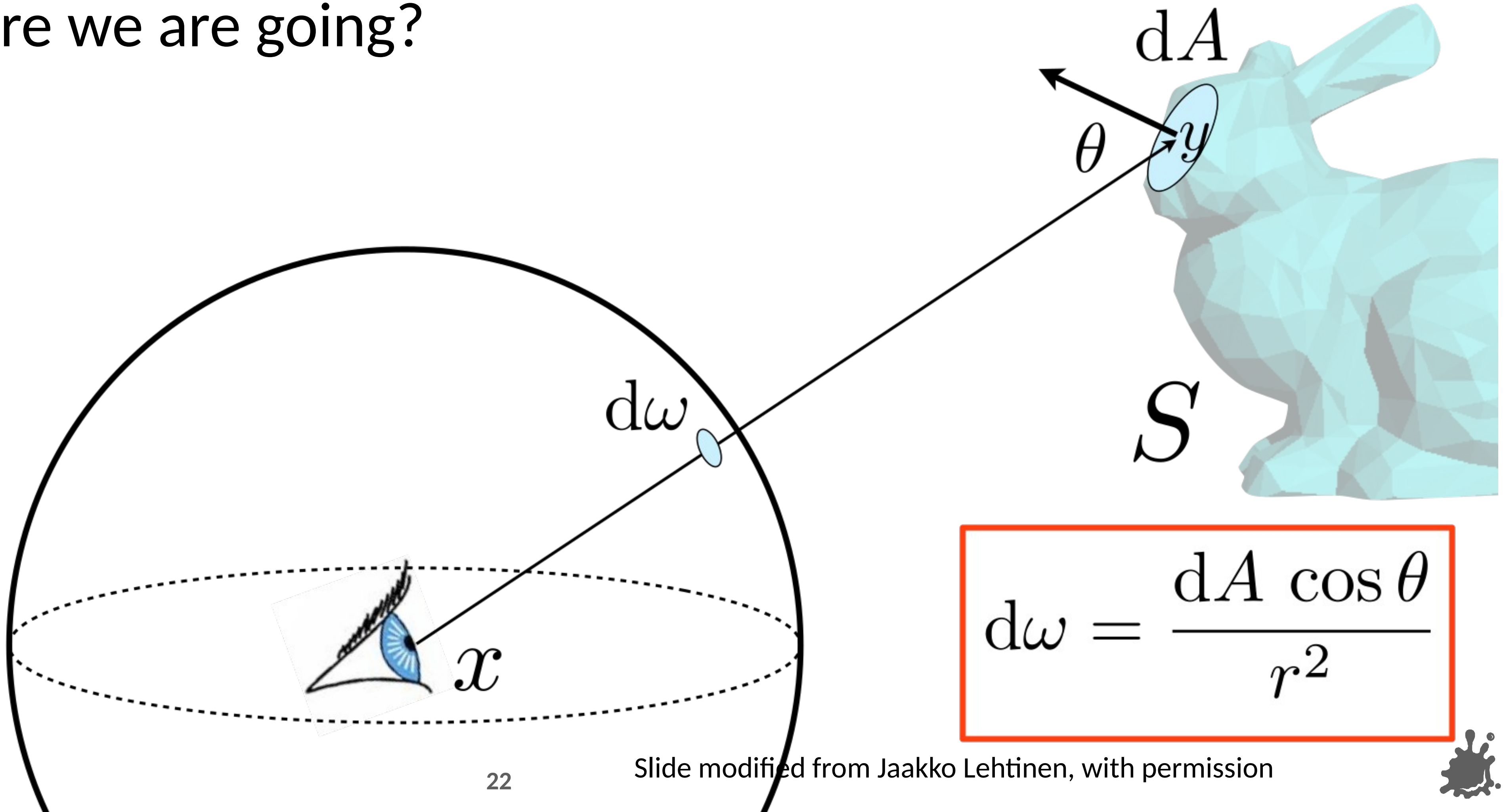
Larger Surfaces

Actual surfaces consist of infinitely many tiny patches dA

-- do you see where we are going?



$$d\omega = \frac{dA \cos\theta}{r^2}$$

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Light Surface Sampling

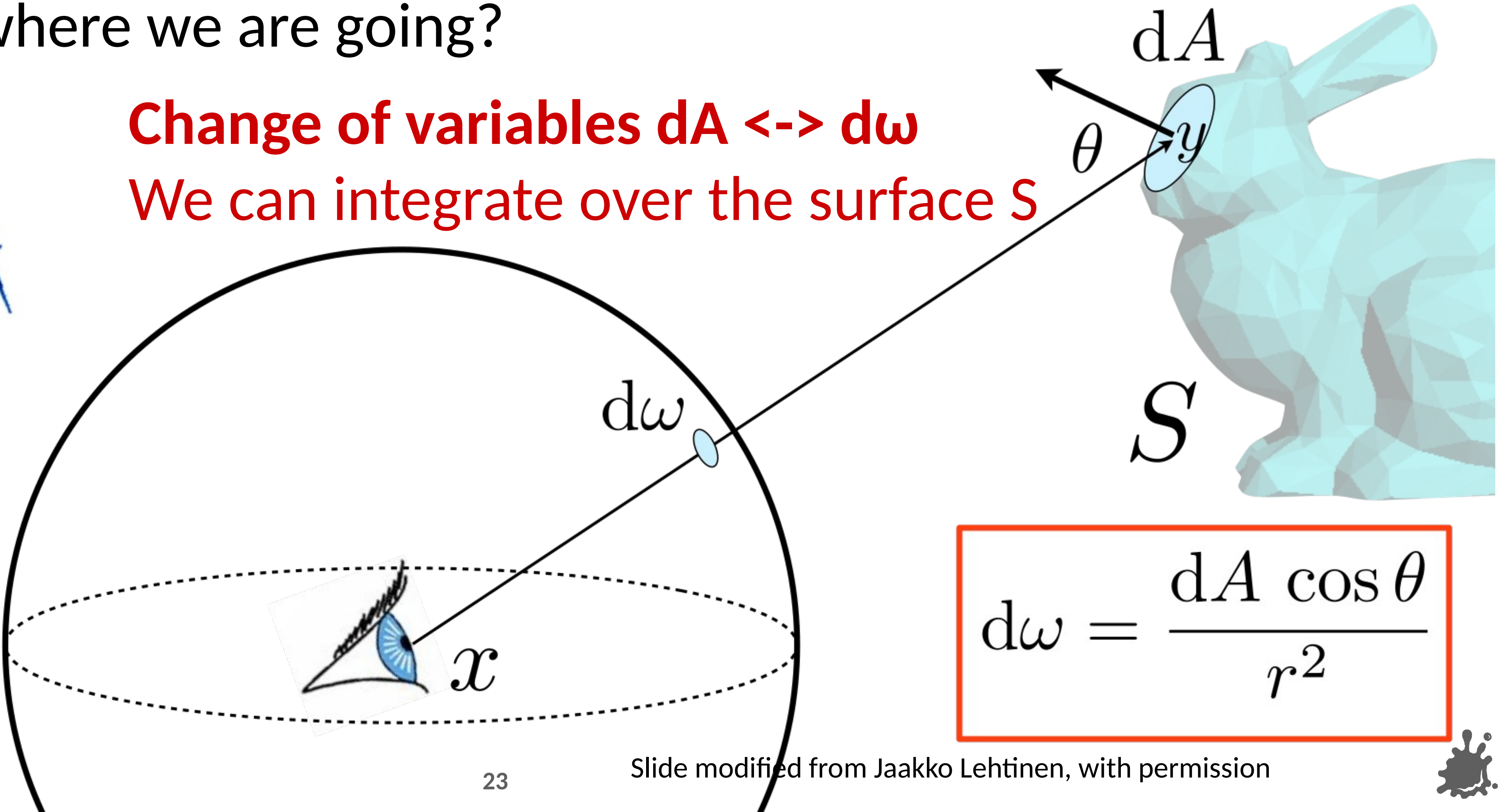
Larger Surfaces

Actual surfaces consist of infinitely many tiny patches dA

------- do you see where we are going?

Change of variables dA <-> dω

We can integrate over the surface S



$$\boxed{\mathrm{d}\omega = \frac{\mathrm{d}A \cos\theta}{r^2}}$$

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Light Surface Sampling

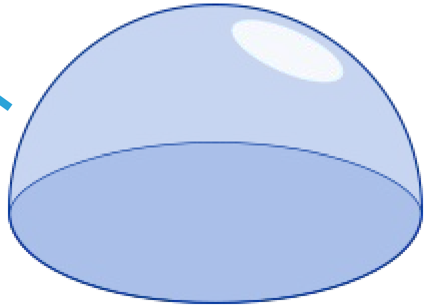
We have seen this before, but now we want to integrate over a single light surface. How do we need to change the formula?

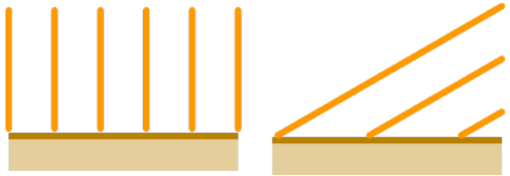
Light arriving at point x

Light from direction ω

Solid angle (just before)

$$L_i(x) = \int_\Omega L_i(x, \omega) \cos(\theta_x)\, \mathrm{d}\omega$$





(not useful for rendering)

Light Surface Sampling

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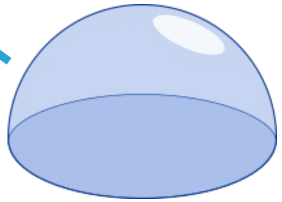
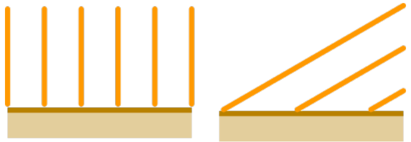




Solid angle (just before)

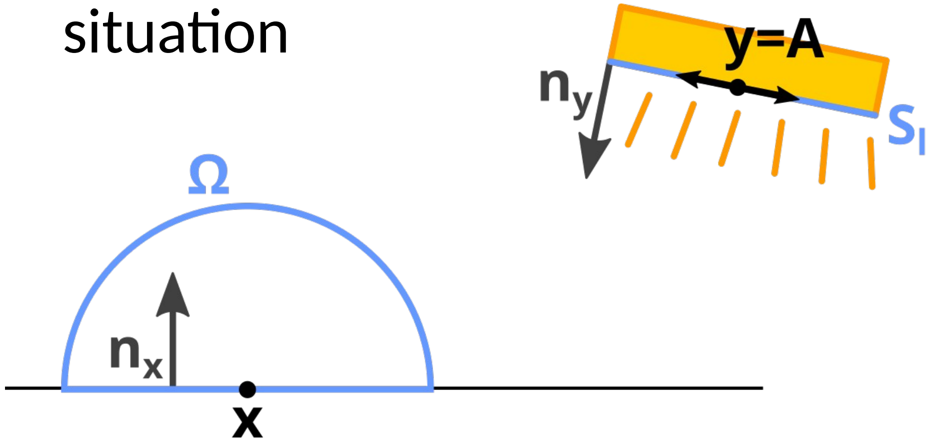
Light from source [l] arriving at point x

$$L_i^{[l]}(x) = \int_{S_l} L_e^{[l]}(y) \cos(\theta_x) \boxed{\frac{\cos(\theta_y)}{r^2} \, \mathrm{d}A_y}$$

light intensity at position y on the surface

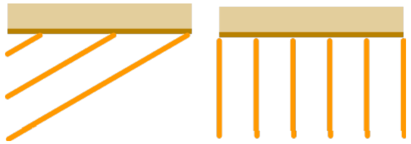
Light Surface Sampling

situation



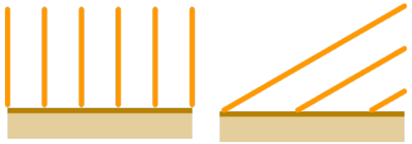
light intensity at position y on the surface

$$L_i^{[l]}(x) = \int_{S_l} L_e^{[l]}(y)\left(\frac{y-x}{|y-x|}\cdot n_x\right)\frac{\left(\frac{x-y}{|x-y|}\cdot n_y\right)}{|x-y|^2}\,\mathrm{d}A_y$$

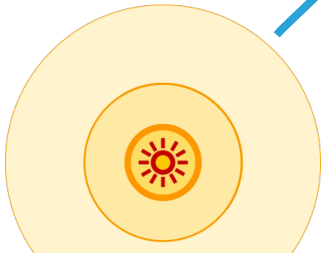


emitter cos(θ)

(not useful for rendering)

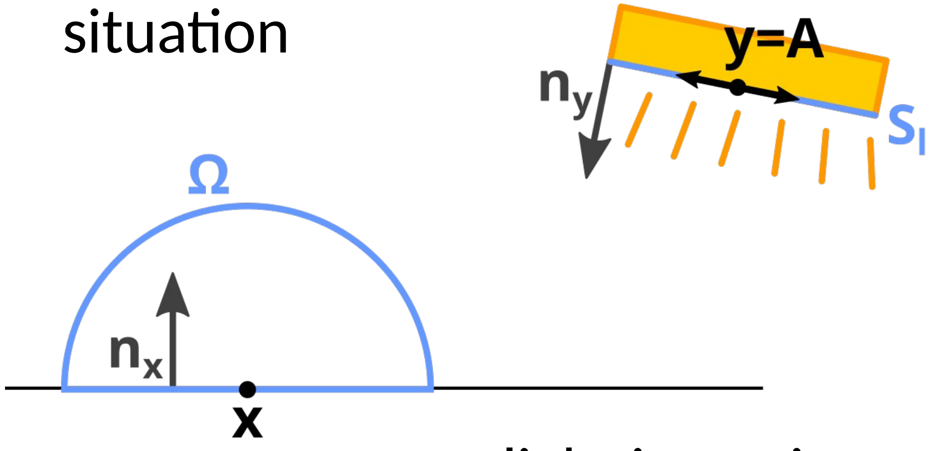


receiver cos(θ)



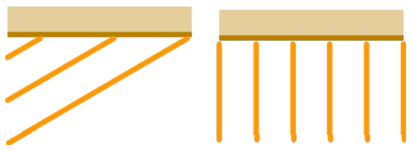
distance

situation



light intensity at position y on the surface

$$L_i^{[l]}(x) = \int_{S_l} L_e^{[l]}(y)\left(\frac{y-x}{|y-x|}\cdot n_x\right)\frac{\left(\frac{x-y}{|x-y|}\cdot n_y\right)}{|x-y|^2}\,\mathrm{d}A_y$$



emitter cos(θ)

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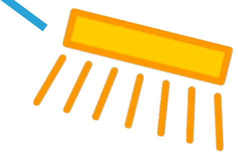

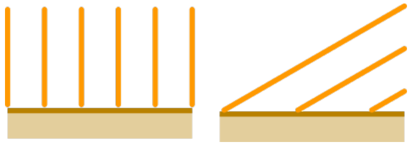

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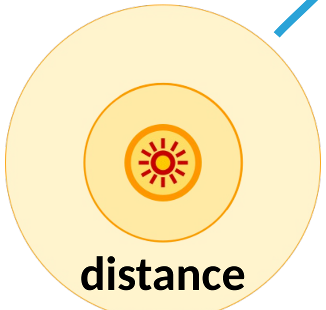
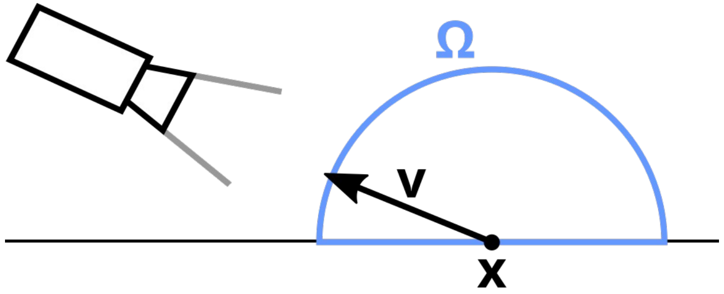


Light Surface Sampling



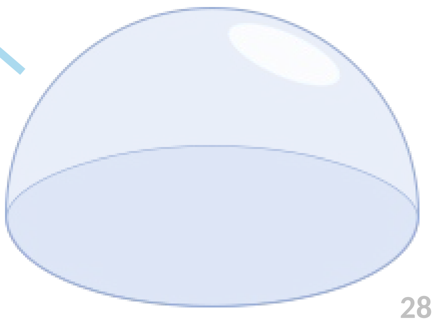
Material, modelled by the BRDF

Light from direction ω

Solid angle

$$L_e(x,v) = \int_\Omega \boxed{f_r(x,\omega \to v)} L_i(x,\omega)\cos(\theta_x)\,\mathrm{d}\omega$$

Light going in direction v



Light Surface Sampling

Change of variables

Next:
Multiple Importance Sampling

source: own work

Useful reading (links)

- Change of variables
- Jaakko Lehtinen's slides
  (I borrowed a lot from lecture 2, but there is more on point lights, intuition, links..)
- Károly Zsolnai-Fehér's slides, previously lecturing at TUW
  (more on history, physics, different approach on solid angle etc.)
- Károly Zsolnai-Fehér's lecture on YouTube