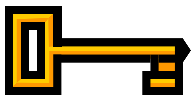
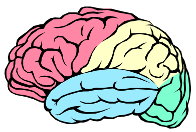
Participating Media

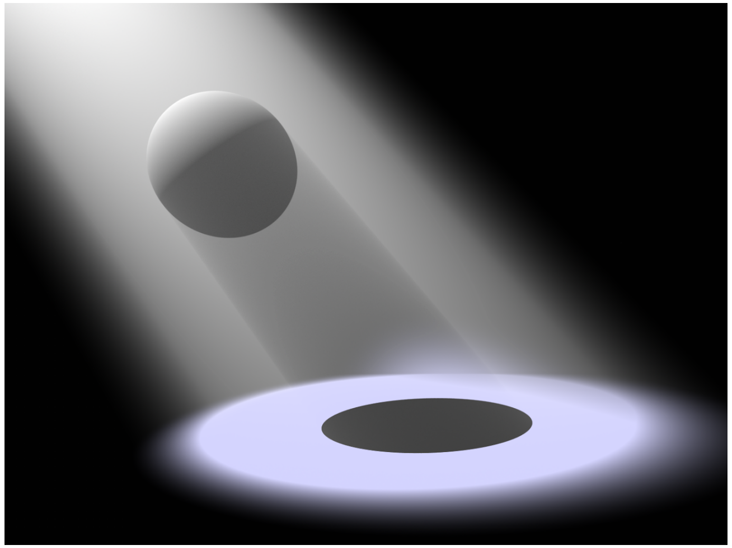
Image Synthesis
Torsten Möller

Reading

-  Chapter 11, 15 of "Physically Based Rendering" by Pharr&Humphreys
-  Chapter 19, 20 in "Principles of Digital Image Synthesis," by A. Glassner
-  "Radiative Transfer," by S. Chandrasekhar (1960)

Participating Media

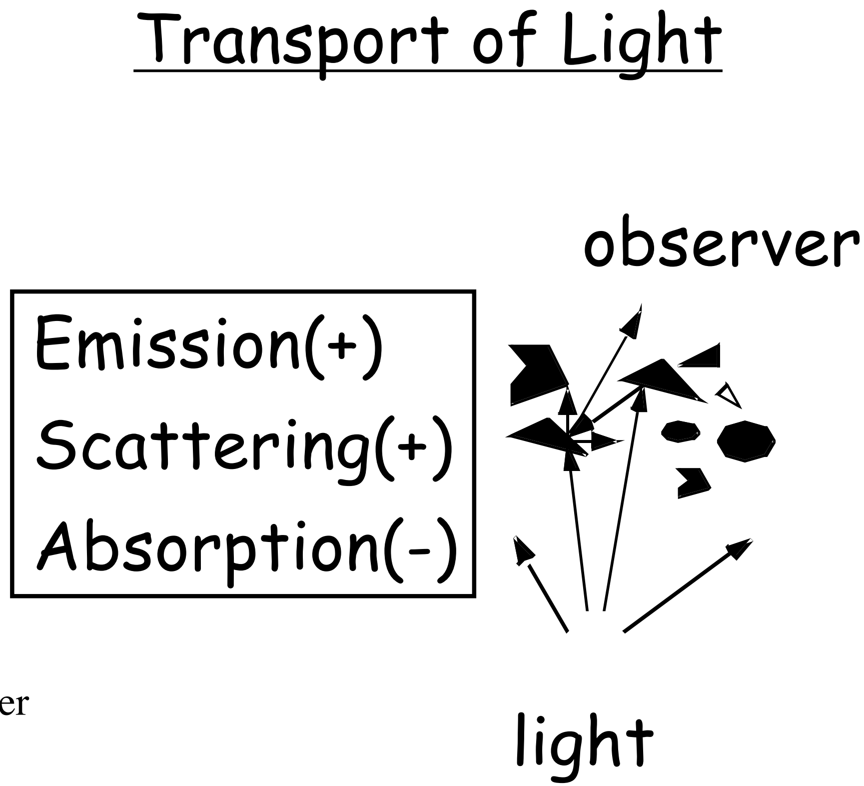
- Natural phenomena
  - Fog, smoke, fire, water, …
  - Atmospheric haze
  - beams of light through clouds
  - subsurface scattering



- Can't really be described through 'surface' models
- Volumetric effects, rendering and modeling

Participating Media

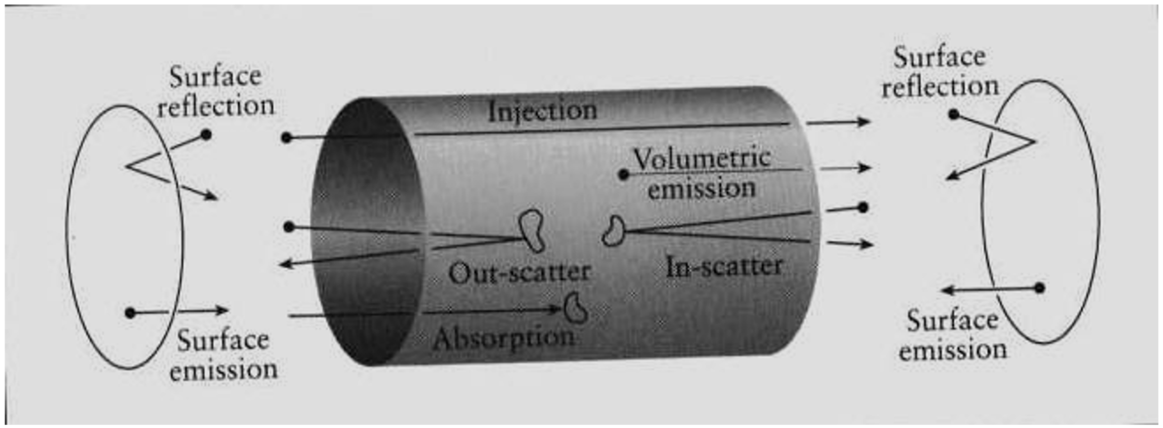
- radiative transport theory
- light: particles of energy or photons moving in straight lines in a vacuum
- no interaction among particles
- infinite number of particles moving at same speed
- interact with surfaces in a closed environment
- energy is conserved
- a steady state of energy transfer between all surfaces



Steady State

- Accumulation =
  flow through boundaries
  - flow out of boundaries
  + generation within system
  - absorption within system

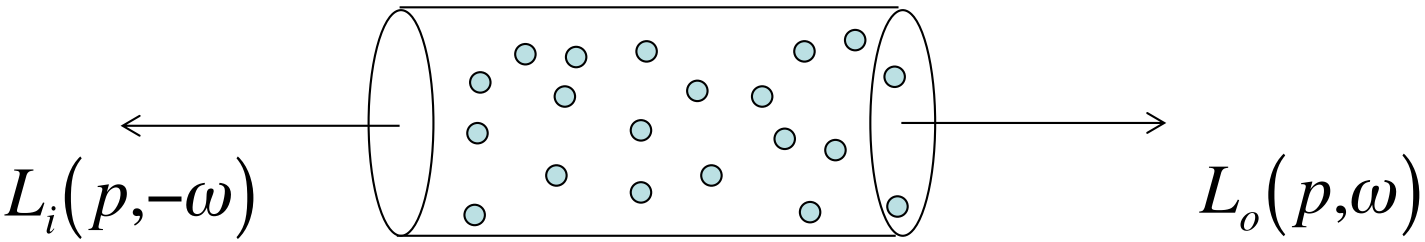
Streaming + *Absorbance* + *Outscattering* = *Emission* + *Inscattering*



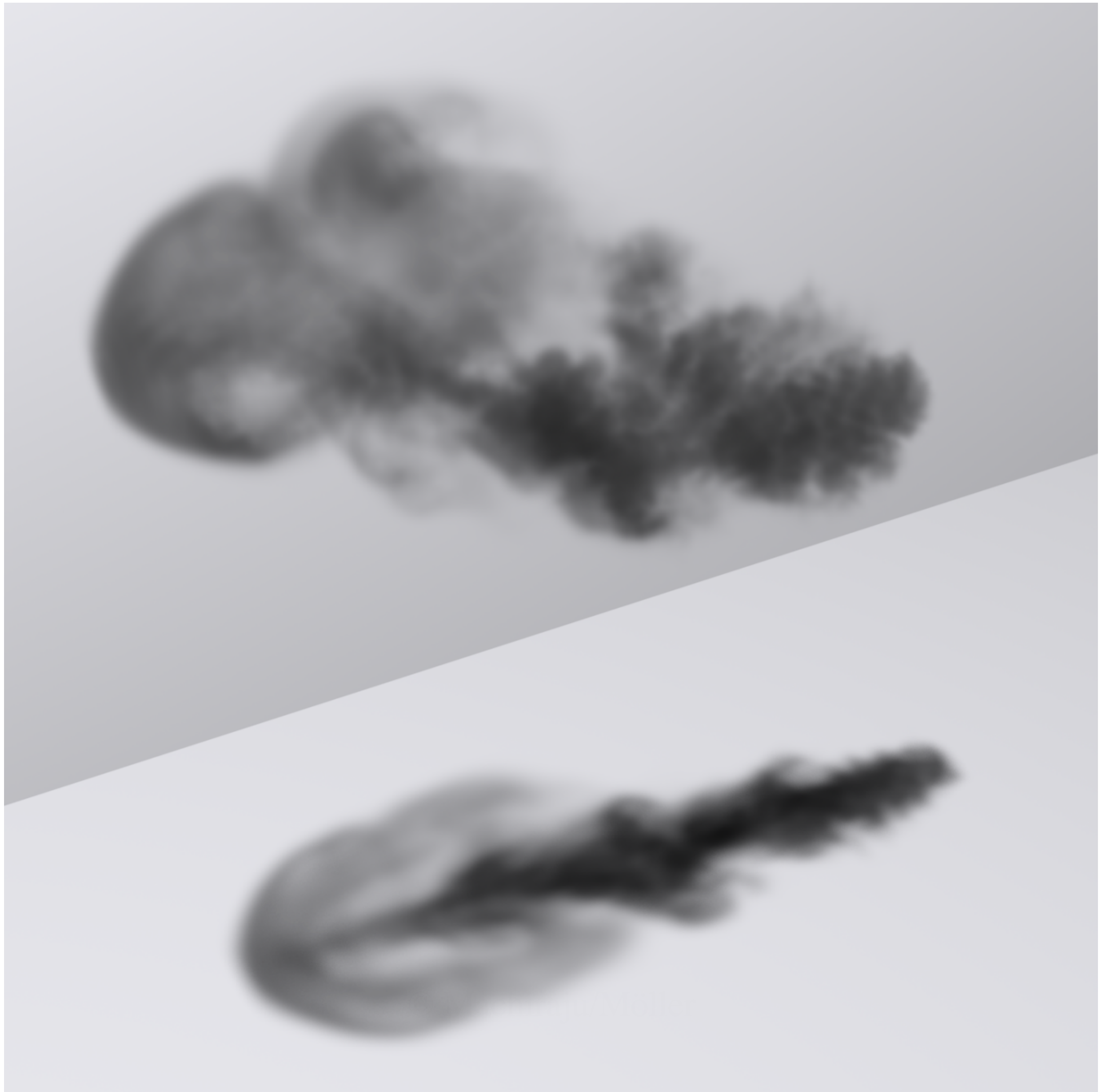
Absorption

- The reduction of radiance due to conversion of light to another form of energy (e.g. heat)
- σ_a: *absorption cross section* - probability density that light is absorbed per unit distance traveled

$$L_o(p,\omega) - L_i(p,-\omega) = dL_o(p,\omega) = -\sigma_a L_i(p,-\omega)dt$$



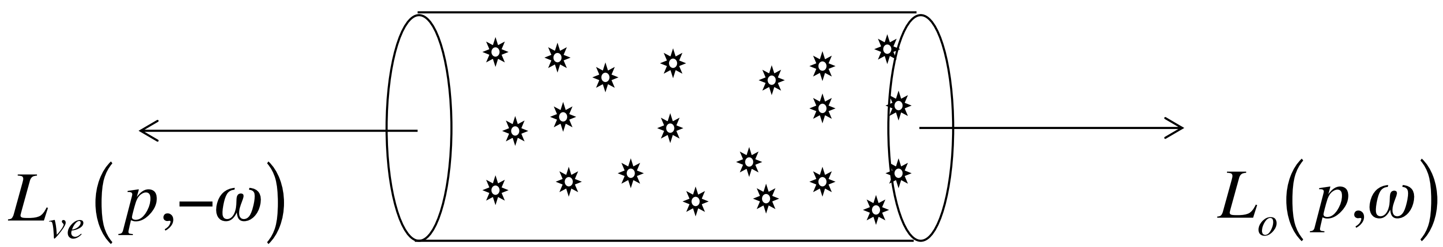
Absorption



Emission

- Energy that is added to the environment from luminous particles
- L_{ve}: *emitted light* - not depending on incoming light!

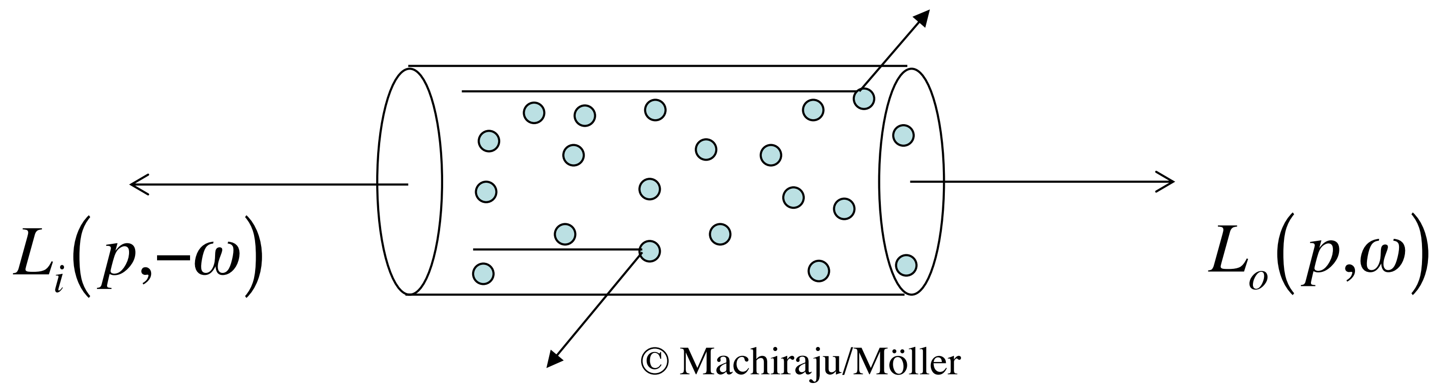
$$dL_o(p,\omega) = L_{ve}(p,-\omega)dt$$



Emission

Out-scattering + extinction

- Light heading in one direction is scattered to other directions due to collisions with particles
- σ_s: *scattering coefficient* - probability of an out-scattering event to happen per unit distance $dL_o(p,\omega) = -\sigma_s(p,\omega)L_i(p,-\omega)dt$



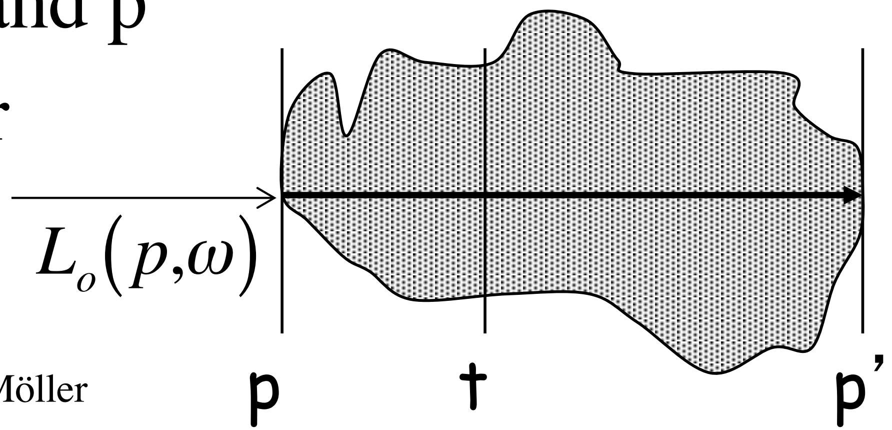
Out-scattering + extinction

- Combining absorption and out-scattering:

$$\sigma_t(p,\omega) = \sigma_s(p,\omega) + \sigma_a(p,\omega)$$

$$\frac{dL_o(p,\omega)}{dt} = -\sigma_t(p,\omega)L_i(p,-\omega)$$

- It's solution: $T_r(p \rightarrow p') = e^{-\int_0^d \sigma_t(p+t\omega,\omega)\,dt}$
  - T_r - beam transmittance
  - d - distance between p and p'
  - ω - unit direction vector



Out-scattering + extinction

- Properties of T_r:
  - In vaccum $\quad T_r(p \to p') = 1$
  - Multiplicative $\quad T_r(p \to p'') = T_r(p \to p') \cdot T_r(p' \to p'')$
  - Beer's law (in homogeneous medium)

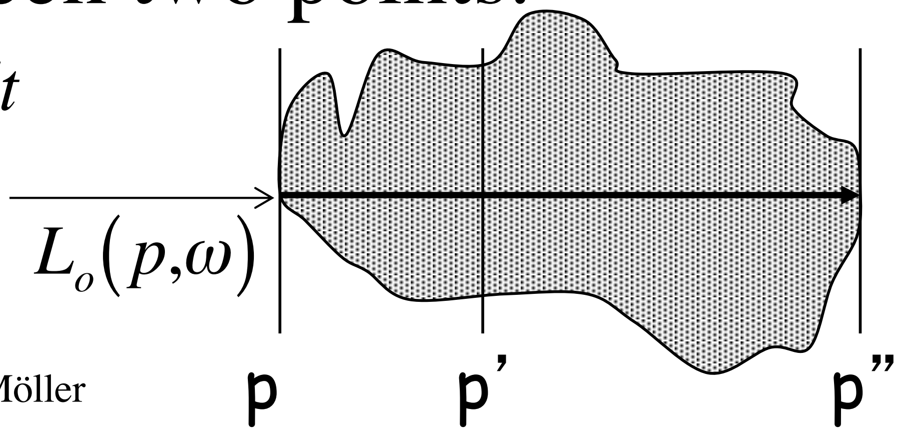
$$T_r(p \to p') = e^{-\sigma_t d}$$

- Optical thickness between two points:

$$\tau(p \to p') = \int_0^d \sigma_t(p + t\omega, \omega)\,dt$$

- Often used:

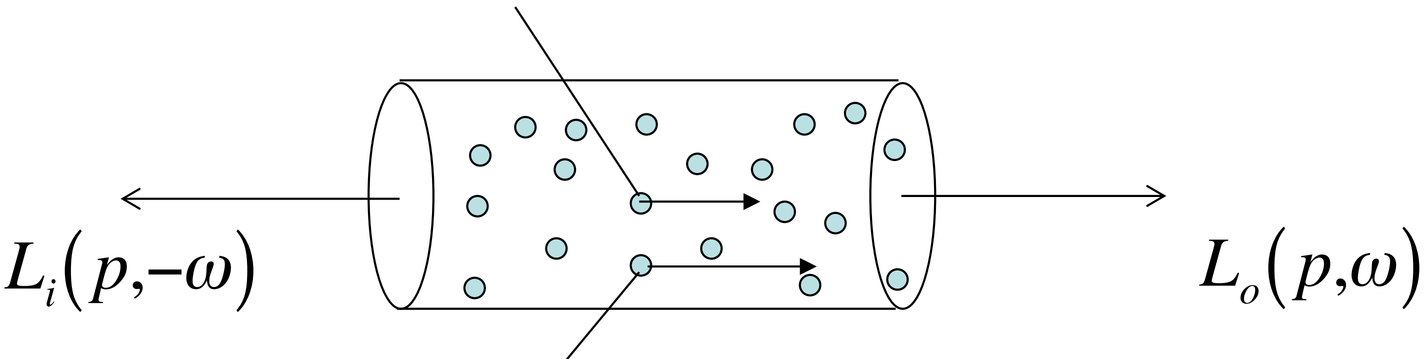
$$T_r(p \to p') \approx 1 - \tau(p \to p')$$



In-scattering

- Increased radiance due to scattering from other directions
  - Ignore inter-particle reactions
  - S - *source term*: total added radiance per unit distance

$$dL_o(p,\omega) = S(p,\omega)dt$$

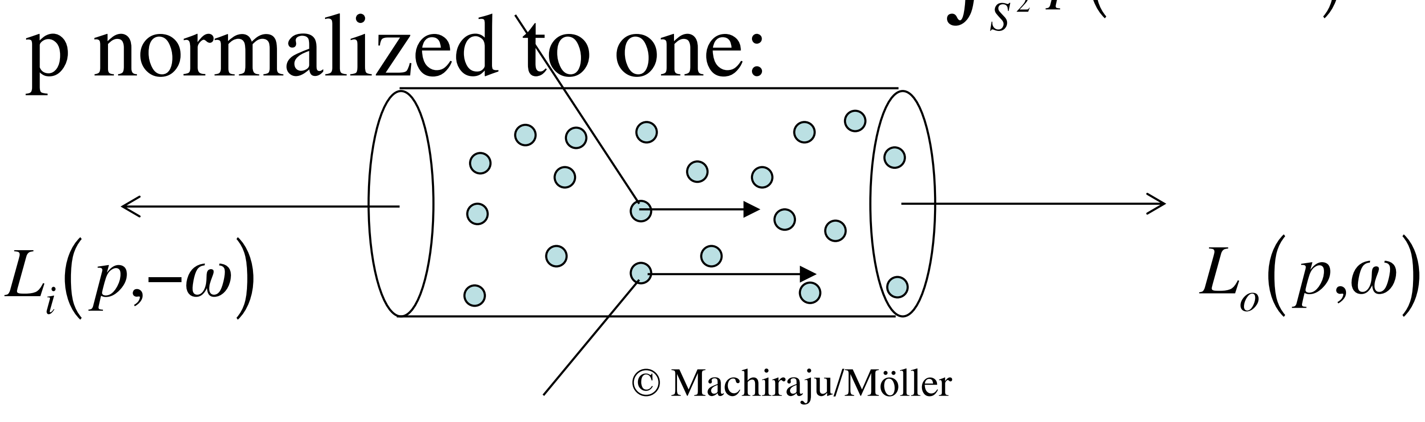


In-scattering

$$S(p,\omega) = L_{ve}(p,\omega) + \sigma_s(p,\omega)\int_{S^2} p(p,-\omega' \rightarrow \omega)L_i(p,\omega')d\omega'$$

- S determined by
  - Volume emission
  - p - *phase function*: describes angular distribution of scattered radiation (volume analog of BSDF)
- p normalized to one:

$$\int_{S^2} p(\omega \rightarrow \omega')d\omega' = 1$$



In-scattering

Phase Functions

- *Isotropic media* - phase function only depends on angle θ between the two directions ω and ω' $p(\omega \to \omega') = p(\cos\theta)$
- Energy preserving $\int_{S^2} p(\omega \to \omega')\,d\omega' = 1$
- Reciprocal (simple for isotropic media)

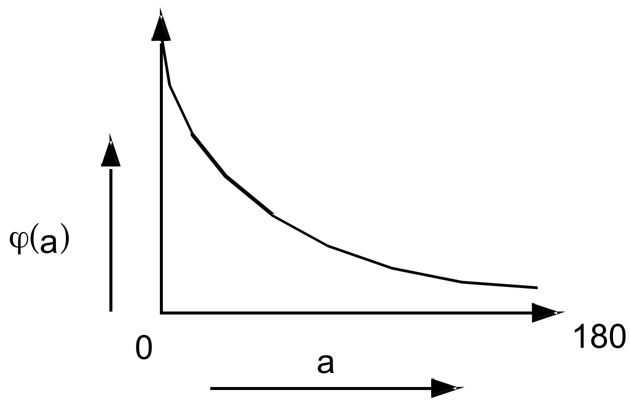
$$\cos(-\theta) = \cos\theta$$

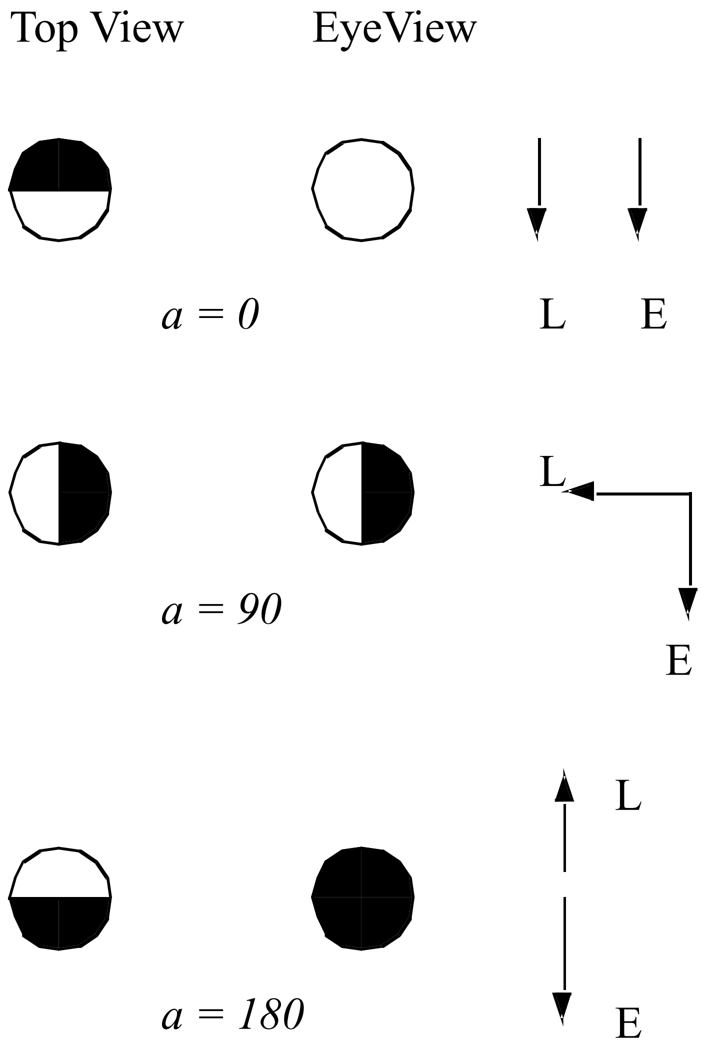
- *Isotropic phase function* - equal scattering in all directions; independent on any angle:

$$p_{\text{isotropic}}(\omega \to \omega') = \frac{1}{4\pi}$$

Phase Function

- “how” we see the particles
- depends on the angle of eye E and light vector L
- Smooth drop off …





Phase Function

- Many different models possible
- (isotropic) Constant function
  - size of particles much less then wavelength of the light

$$p(\cos\theta) = \frac{1}{4\pi}$$

- (Anisotropic) basic

$$p(\cos\theta) = 1 + x\cos\theta$$

  - more light forward then backward - essentially our diffuse shading
- Lambert surfaces

$$p(\cos\theta) = {}^{8\pi}\!/_{3}\left(\sin\theta + (\pi - \theta)\cos\theta\right)$$

  - spheres reflect according to Lamberts law
  - physically based

Phase Function

- Rayleigh Scattering $$p(\cos\theta) = \tfrac{3}{4}\left(1 + \cos^2\theta\right)$$
  - diffraction effects dominate
  - Particles have radii smaller than the wavelength λ (r/ λ < 0.05)
  - Causes sky to be blue and sunset to be red
- Mie scattering
  - Based on Maxwells equations
- Empirical Measurments
  - tabulated phase function
- sums of functions $$p(\omega \to \omega') = \sum_{i=1}^{n} w_i p_i(\omega \to \omega')$$
  - weighted sum of functions - model different effects in parallel

Phase Function

- Henyey-Greenstein
  - general model with good fit to empirical data
  - g in range [-1,+1]
  - Negative g - back scattering

$$p(\cos\theta) = \frac{1}{4\pi}\left(1-g^2\right)\Big/\left(1+g^2-2g\cos\theta\right)^{3/2}$$

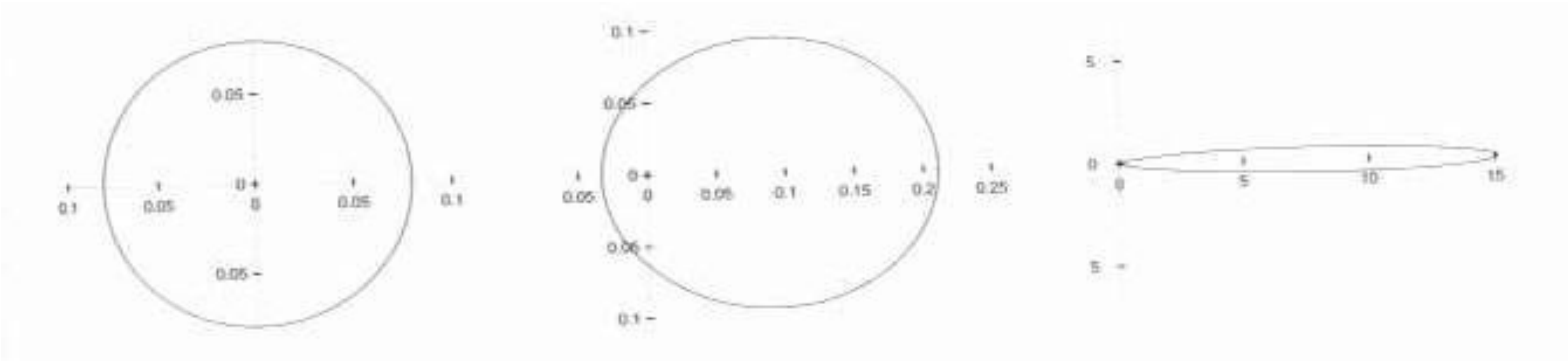
  - g chosen such that:

$$g = \frac{1}{4\pi}\int_{S^2} p(\omega \to \omega')(\omega\cdot\omega')\,d\omega' = 2\pi\int_0^{\pi} p(\cos\theta)\cos\theta\,d\theta$$

  - Isotropic - g=0

Phase Function

- Henyey-Greenstein
  - Ellipse-like shape dependent on g



Phase Function

- Approx. to Henyey-Greenstein

$$p_{\text{Schlick}}(\cos\theta) = \frac{1}{4\pi}\left(1-k^2\right)\Big/\left(1-k\cos\theta\right)^2$$

- k similar role like g
  - -1 - back scattering
  - 0 - isotropic
  - Could use: $k = 1.55g - 0.55g^2$

Importance Sampling of HG

- Can importance sample –
  new direction (at least θ) is:

$$g \neq 0 \quad \cos\theta = -\frac{1}{2g}\left(1 + g^2 - \left(\frac{1-g^2}{1-g+2g\xi}\right)^2\right)$$

$$g = 0 \qquad \theta = 1 - 2\xi$$

- Do same for ϕ

$$p(\phi) = \frac{1}{2\pi}$$

Results - Skin

© Machiraju/Möller

Results - Skin

- Henyey-Greenstein phase function (g=-.25)

Results - Skin

- Large forward scattering

albedo

- *albedo* - proportion of light reflected from a particle: in the range of 0..1
- $\sigma_s/(\sigma_s+\sigma_a)$ - fraction of scattered radiation

Homogeneous Volumes

- Determined by (constant)
  - σ_s and σ_a
  - phase functions g value
  - Emission L_{ve}
  - Plus spatial extend

Homogeneous Volumes

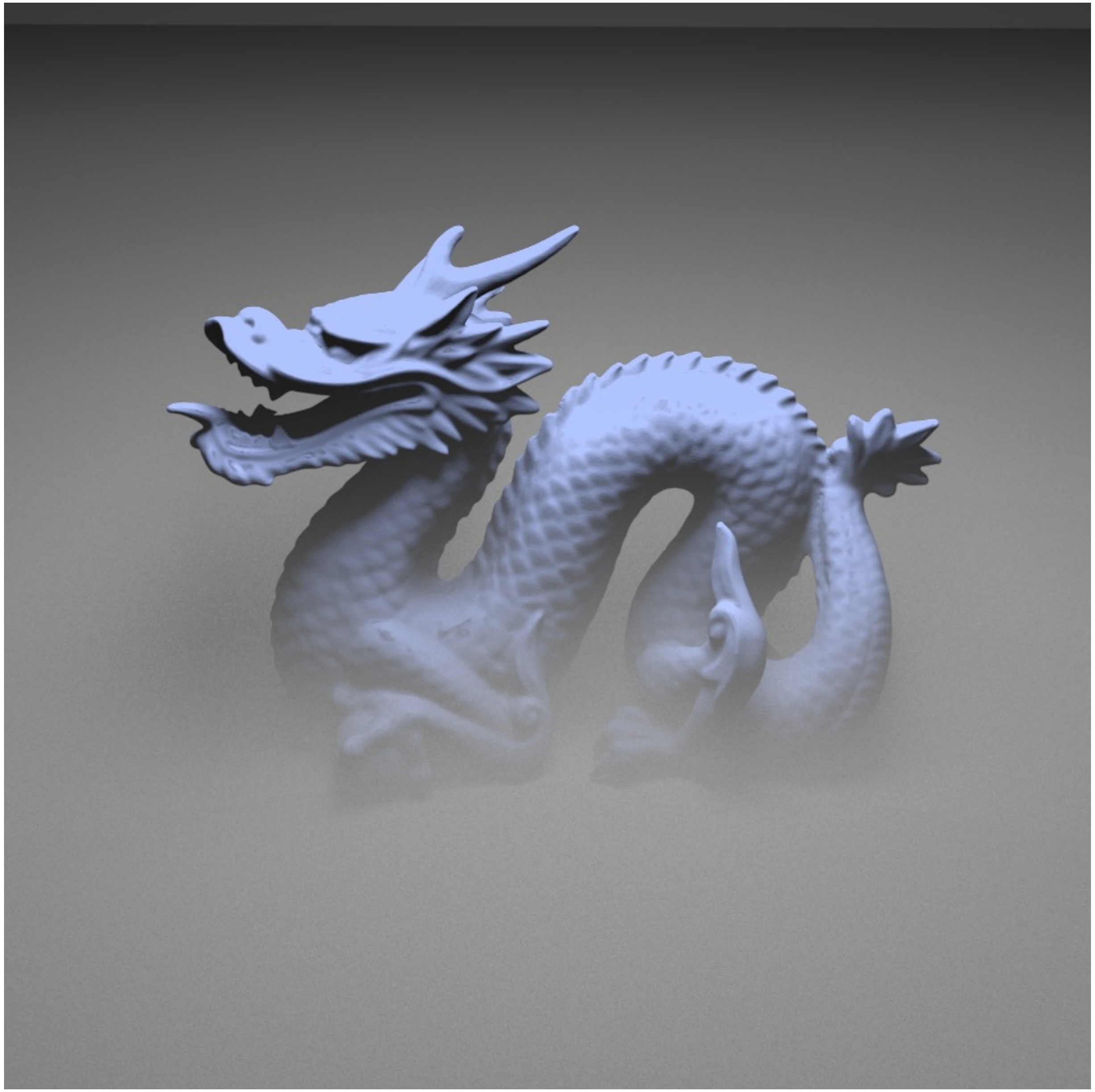
3D lattices

- Standard form of given data
- Tri-linear interpolation of data in order to get continuous volume is typically done
- Field of volume rendering / volume graphics

Exponential density

- Given by: $$d(h) = ae^{-bh}$$
- Where h is the height in the direction of the up-vector

Exponential density



Volume Aggregates

- Dealing with multiple volumes at once
- Typically just cycle through the volumes, one at a time and add up their contribution

Light Transport In Total

- Summary:
  - Emission + in-scatter (source term)

$$S(p,\omega) = L_{ve}(p,\omega) + \sigma_s(p,\omega) \int_{S^2} p(p,-\omega' \to \omega) L_i(p,\omega') d\omega'$$

  - Absorption + out-scatter

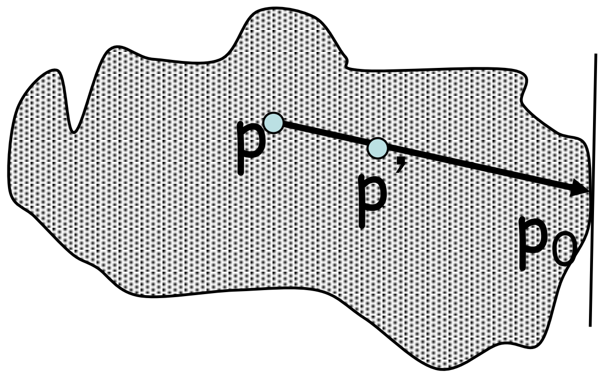
$$\frac{dL_o(p,\omega)}{dt} = -\sigma_t(p,\omega) L_i(p,-\omega)$$

- Combined:

$$\frac{dL_o(p,\omega)}{dt} = -\sigma_t(p,\omega) L_i(p,-\omega) + S(p,\omega)$$

Light Transport In Total

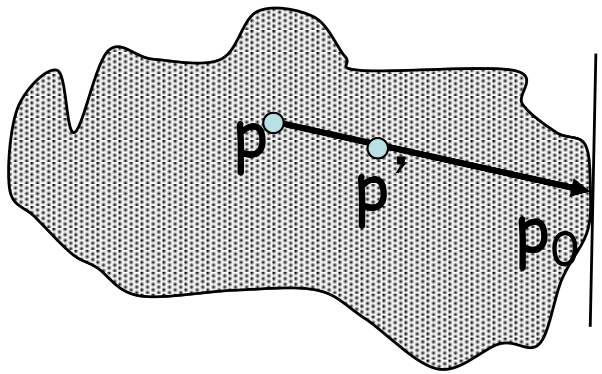
- In order to solve this PDE we need to know the boundary conditions:
  a) No surfaces (boundary is inf.)
  b) Surface at p_0 (distance d away)



a) No boundaries

- Solution of $\dfrac{dL_o(p,\omega)}{dt} = -\sigma_t(p,\omega)L_i(p,-\omega) + S(p,\omega)$
- Is: $L_i(p,\omega) = \int\limits_0^\infty T_r(p' \to p)S(p',-\omega)dt$
- where $p' = p + t\omega$

$$T_r(p' \to p) = e^{-\int_0^d \sigma_t(p+t\omega,\omega)dt}$$

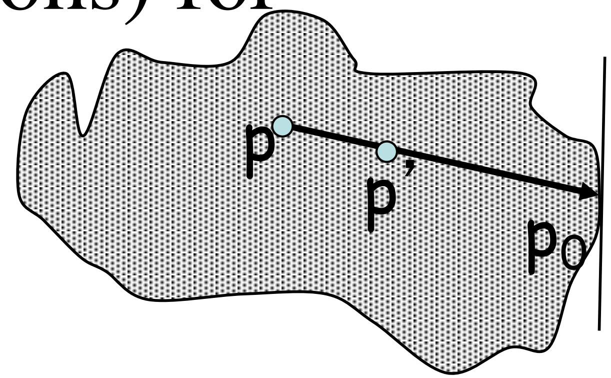


a) No boundaries

- Essentially, this can be written as:

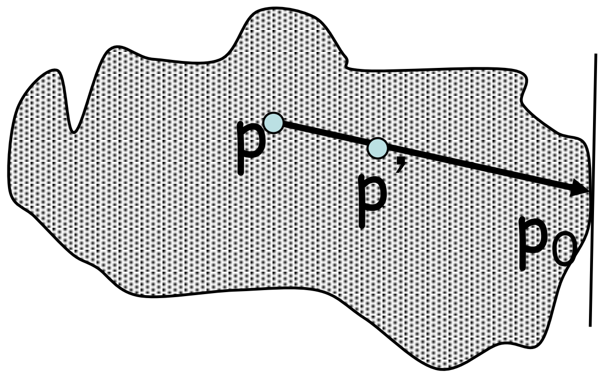
$$L_i(p,\omega) = \int_0^{\infty} e^{-g(t)dt} f(t)\,dt$$

- Not possible to distribute samples according to $e^{-g(t)t}$ for general g(t)
- Typically we know solutions (either analytical or suitable distributions) for simple versions of the integral



a) No boundaries

- If g(t)=const: $$MC = \frac{1}{N}\sum_{i}^{N}\frac{e^{-cT_i}f(T_i)}{ce^{-cT_i}} = \frac{1}{Nc}\sum_{i}^{N}f(T_i)$$
- If g(t) approx. const: $$MC = \frac{1}{N}\sum_{i}^{N}\frac{e^{-g(T_i)T_i}f(T_i)}{ce^{-cT_i}}$$
- Interval is usually not infinite; then we can also approx. using piecewise linear functions



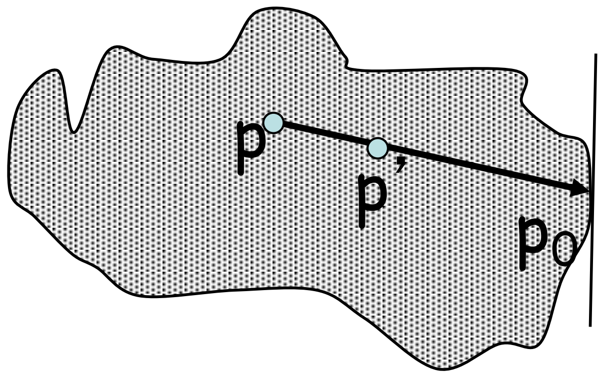
a) No boundaries

- g(t) is actually the optical thickness:

$$\tau(p \rightarrow p') = \int_0^d \sigma_t(p + t\omega, \omega)\,dt$$

- Using MC to solve: $MC = \frac{1}{N}\sum_i^N \frac{\sigma_t(p + T_i\omega, -\omega)}{p(T_i)}$
- Natural choice would be stratified sampling
- Since 1D integral MC not very efficient
- Instead use Riemann sums:

$$T_i = \frac{\xi + i}{N} d$$



b) Surface at point p_0

- To compare - the "no surface" solution:

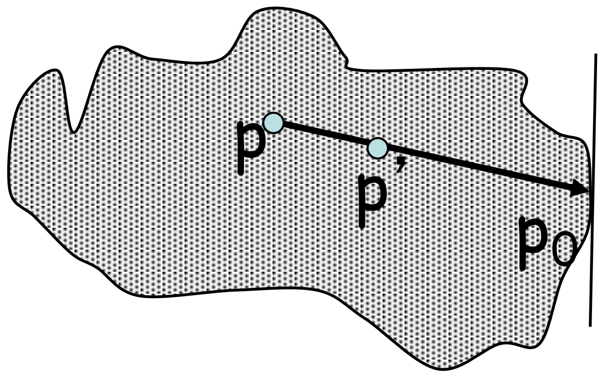
$$L_i(p,\omega) = \int_0^\infty T_r(p' \to p) S(p', -\omega) dt$$

- With surface:

$$L_i(p,\omega) = \boxed{T_r(p_0 \to p) L_o(p_o, -\omega)} + \boxed{\int_0^d T_r(p' \to p) S(p', -\omega) dt}$$

From the surface point p_0 From the participating media

- Rather complicated to solve!



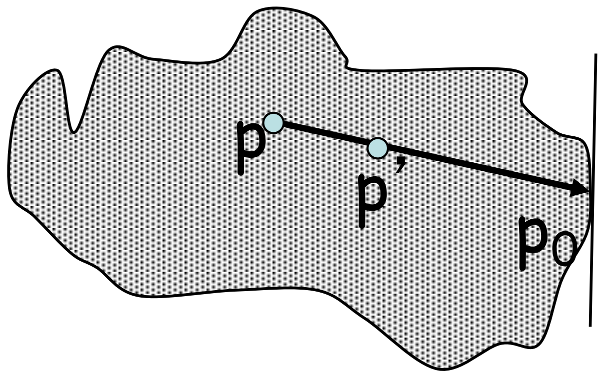
b) Surface at point p_0

- Find solutions for simplifications
  - 1. Emission only, i.e. $S(p',-\omega) = L_{ev}(p',-\omega)$
  - Hence:

$$L_i(p,\omega) = T_r(p_0 \to p)L_o(p_o,-\omega) + \int_0^d T_r(p' \to p)L_{ev}(p',-\omega)dt$$

  - And:

$$MC = \frac{1}{N}\sum_i^N \frac{T_r(p_i \to p)L_{ev}(p_i,-\omega)dt}{p(p_i)}$$

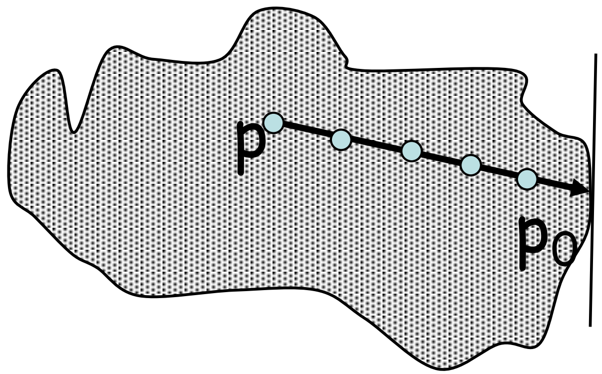


b) Surface at point p_0

- Use multiplicativity of T_r

$$T_r(p_i \to p) = T_r(p_i \to p_{i-1}) \cdot T_r(p_{i-1} \to p)$$

- Can break up integral and compute it incrementally => ray marching
- T_r can get small in a long ray
  - "early ray termination"
  - Either using Russian Roulette or deterministically

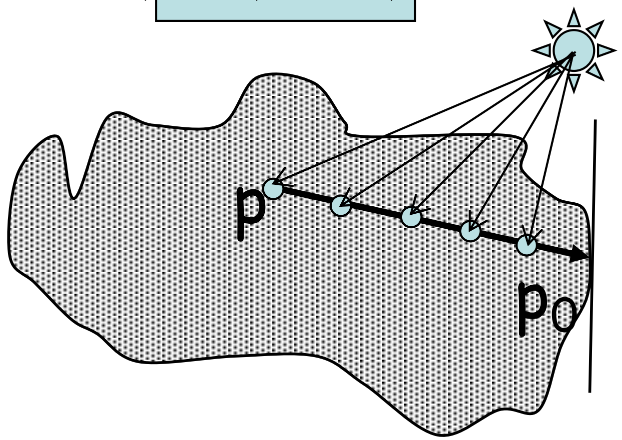


b) Surface at point p_0

- 2. Single Scattering
  - Consider incidence radiance due to direct illumination

$$L_i(p,\omega) = T_r(p_0 \rightarrow p) L_o(p_o,-\omega) + \int_0^d T_r(p' \rightarrow p) S(p',-\omega)\, dt$$

$$S(p,\omega) \approx L_{ve}(p,\omega) + \sigma_s(p,\omega) \int_{S^2} p(p,-\omega' \rightarrow \omega) \boxed{L_d(p,\omega')}\, d\omega'$$

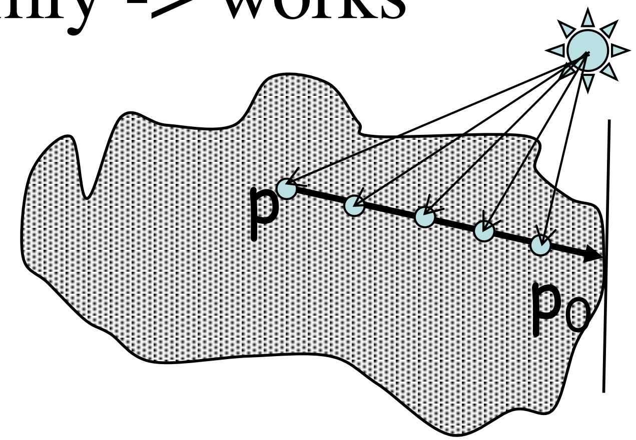


b) Surface at point p_0

- 2. Single Scattering
  - L_d may be blocked by participating media
  - At each point of the integral - could use multiple importance sampling in order to get

$$\sigma_s(p,\omega)\int_{S^2} p(p,-\omega' \to \omega) L_d(p,\omega')\,d\omega'$$

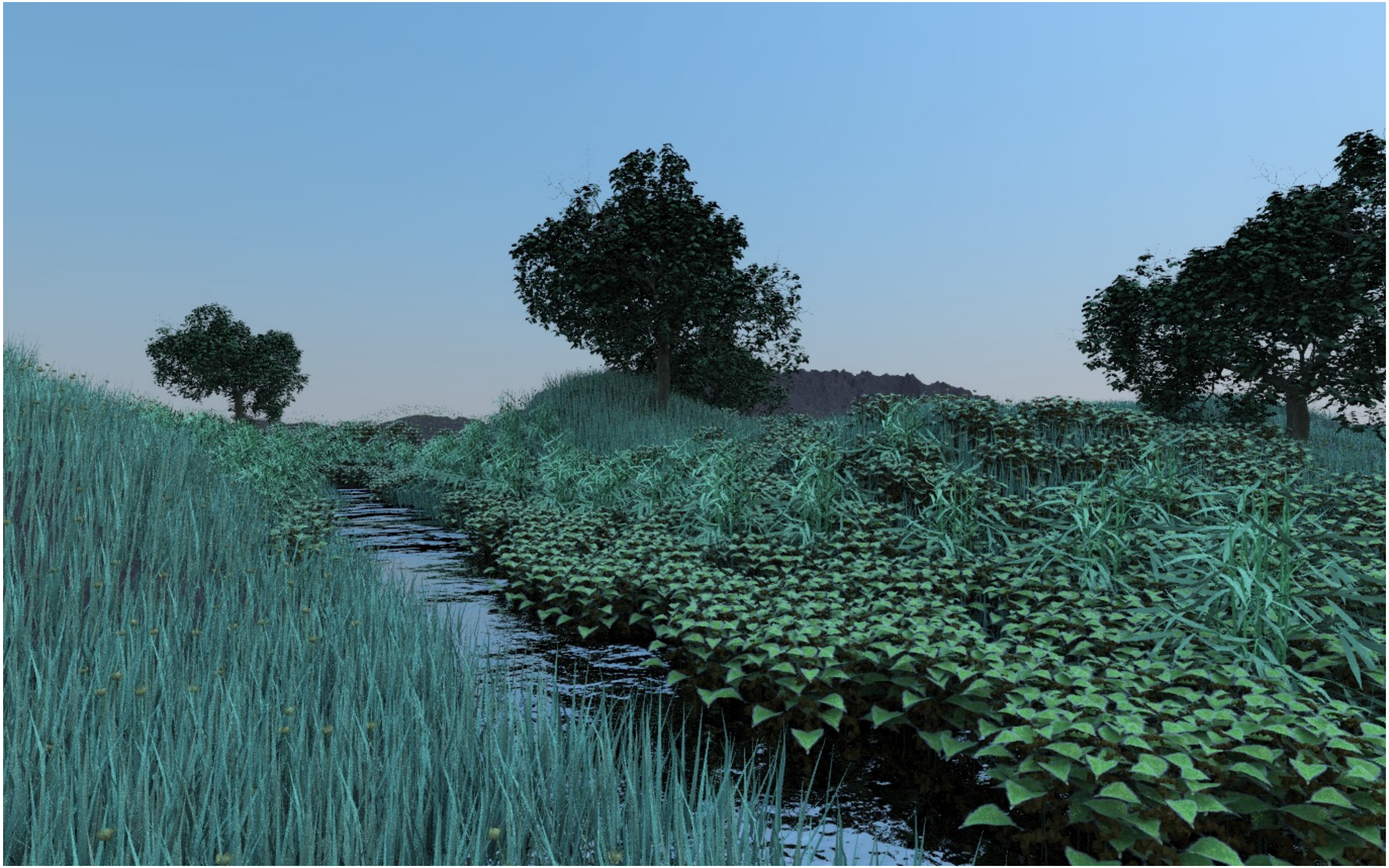
  - But just pick light source randomly -> works just as well for isotropic media



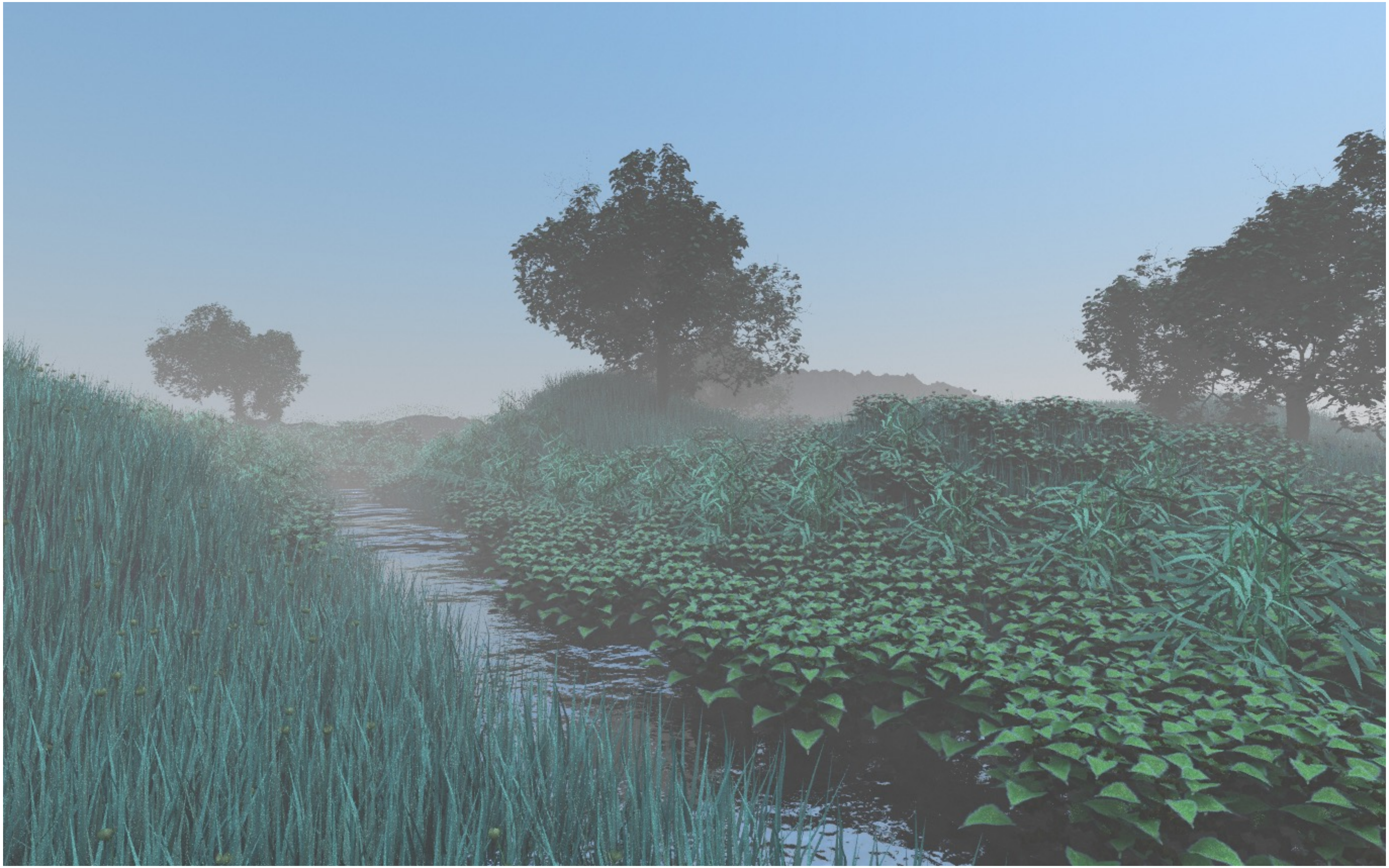
Ray Marching

- Multiple scattering - sample sphere around segment of interest
- S sample rays are used to estimate the in-scattered light
- Very expensive
- Max' 95 has a good solution

Results

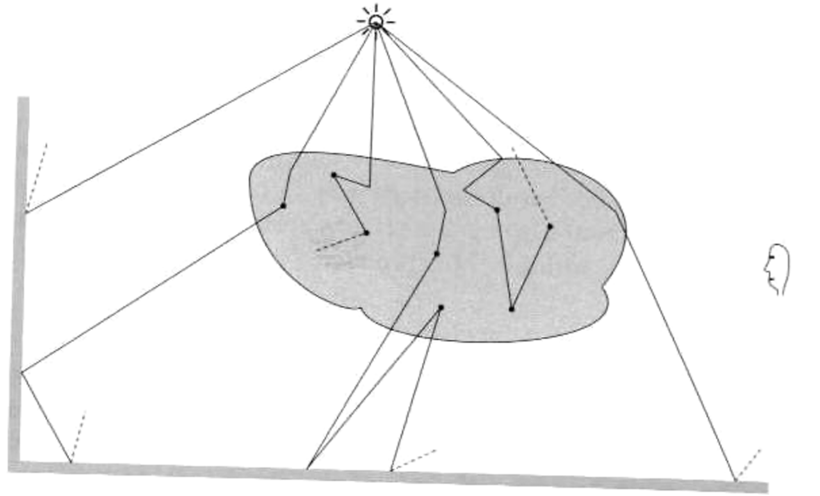


Results



Results

Photon Tracing



- Photons that enter are scattered and absorbed
- Photon maps store where events happen
- Prob. of such an event is determined by extinction coefficient
- Avg. distance a photon moves $d = \frac{1}{\sigma_t}$
- Replace σ_t by $\tau(0,d)$ for non-homogeneous media
- Hence = importance sample according to $d = -\frac{\log \xi}{\sigma_t}$

Photon Scattering

- Prob. of Scattering $\Lambda = \frac{\sigma_s}{\sigma_t}$
- Scale power of new photon --- can reduce total power (i.e. lots of low-powered photons again)
- Instead - do Russian Roulette again

$$\xi \in [0,1] \rightarrow \begin{cases} \xi \leq \Lambda & \text{Photon is scattered} \\ \xi > \Lambda & \text{Photon is absorbed} \end{cases}$$

Volume Radiance Estimate

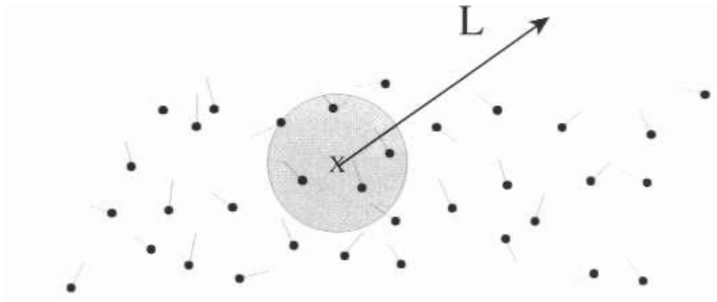
- Out-scattered Radiance

$$L_o(p,\omega) = \sigma_s(p,\omega)\int_{S^2} p(p,-\omega' \rightarrow \omega)L_i(p,\omega')d\omega'$$

- Stored photons = flux

$$L_i(p,\omega') = \frac{d^2\Phi(p,\omega')}{\sigma_s(p,\omega')d\omega' dV}$$

- Leading to:



Volume Radiance Estimate

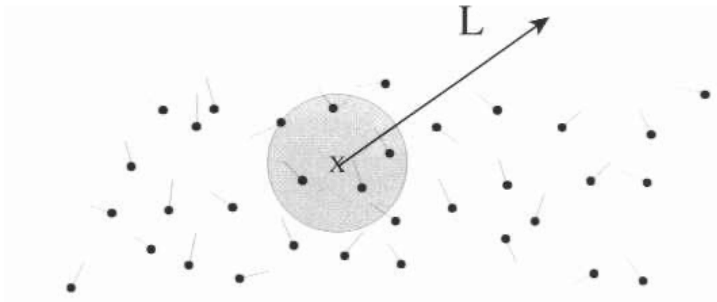
- Leading to:

$$L_o(p,\omega) = \sigma_s(p,\omega)\int_{S^2} p(p,-\omega' \to \omega)\frac{d^2\Phi(p,\omega')}{\sigma_s(p,\omega')\,d\omega' dV}d\omega'$$

$$= \int_{S^2} p(p,-\omega' \to \omega)\frac{d^2\Phi(p,\omega')}{dV}$$

$$\approx \sum_{i=1}^{N} p(p,-\omega_i' \to \omega)\frac{\Delta\Phi(p,\omega_i')}{\Delta V}$$

$$\approx \sum_{i=1}^{N} p(p,-\omega_i' \to \omega)\frac{\Delta\Phi(p,\omega_i')}{\frac{4}{3}\pi r^3}$$



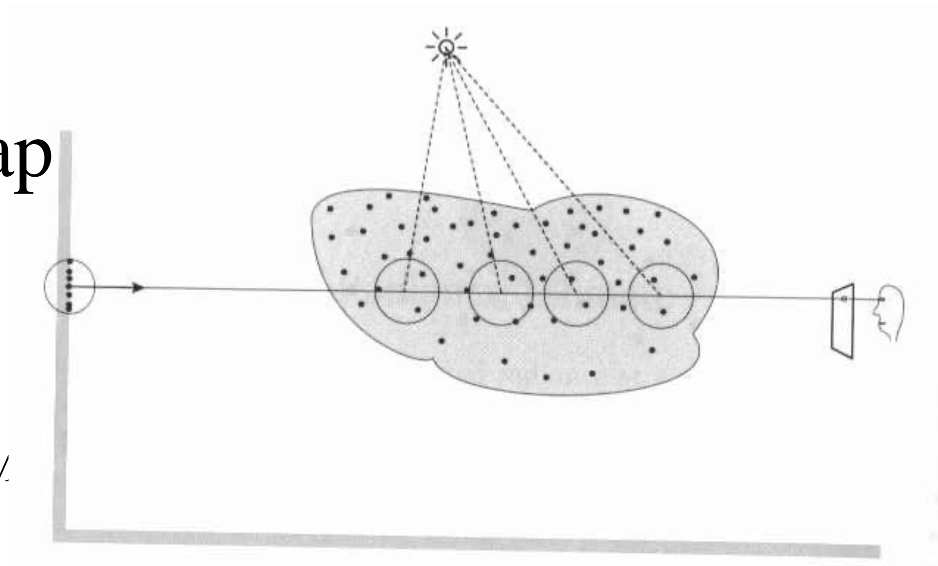
Rendering

- Surfaces - do traditional photon mapping
- Entering participating media - divide into single + multiple scattering events:

$$L(p,\omega) = L_s(p,\omega) + L_m(p,\omega)$$

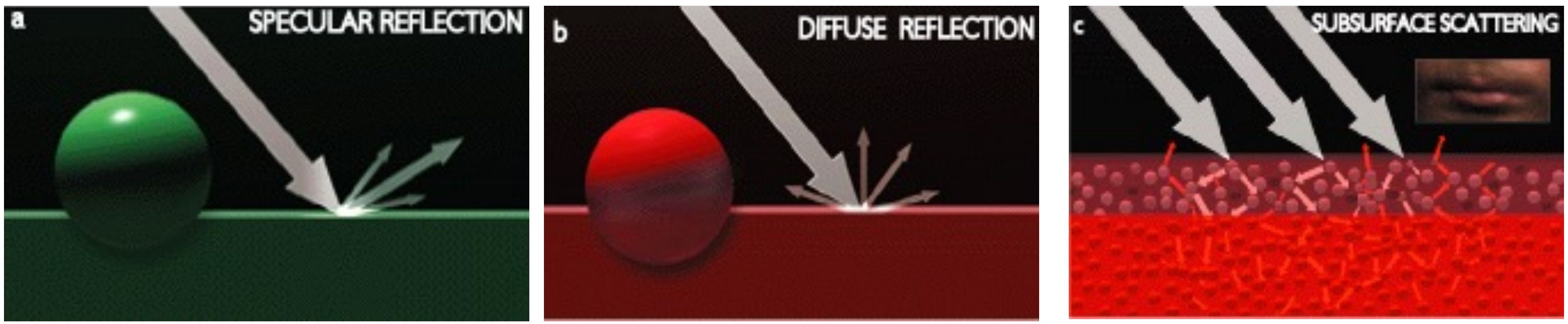
  – Single scattering - do ray tracing

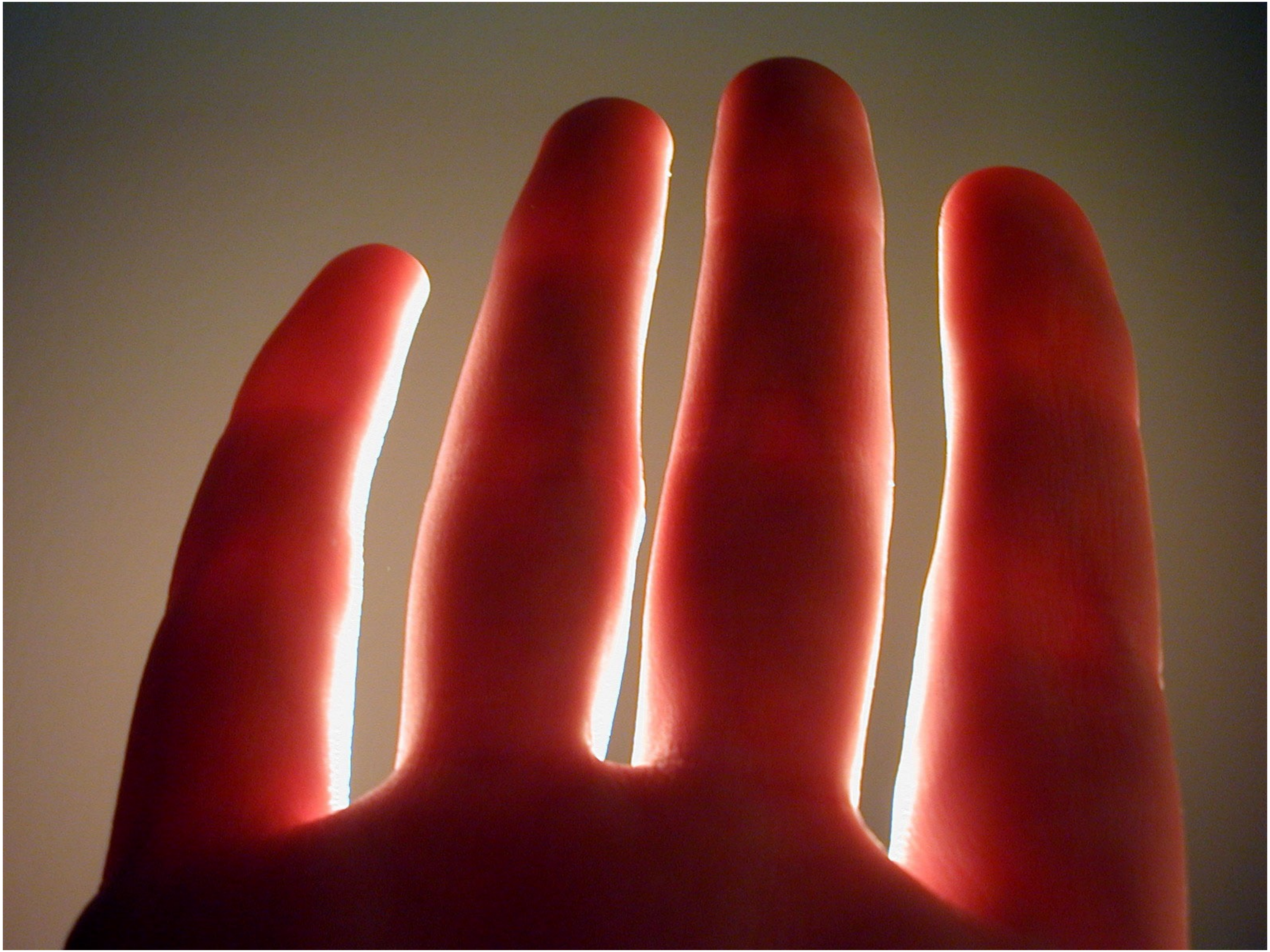
  – Multiple scattering
    - use volume photon map



Subsurface Scattering

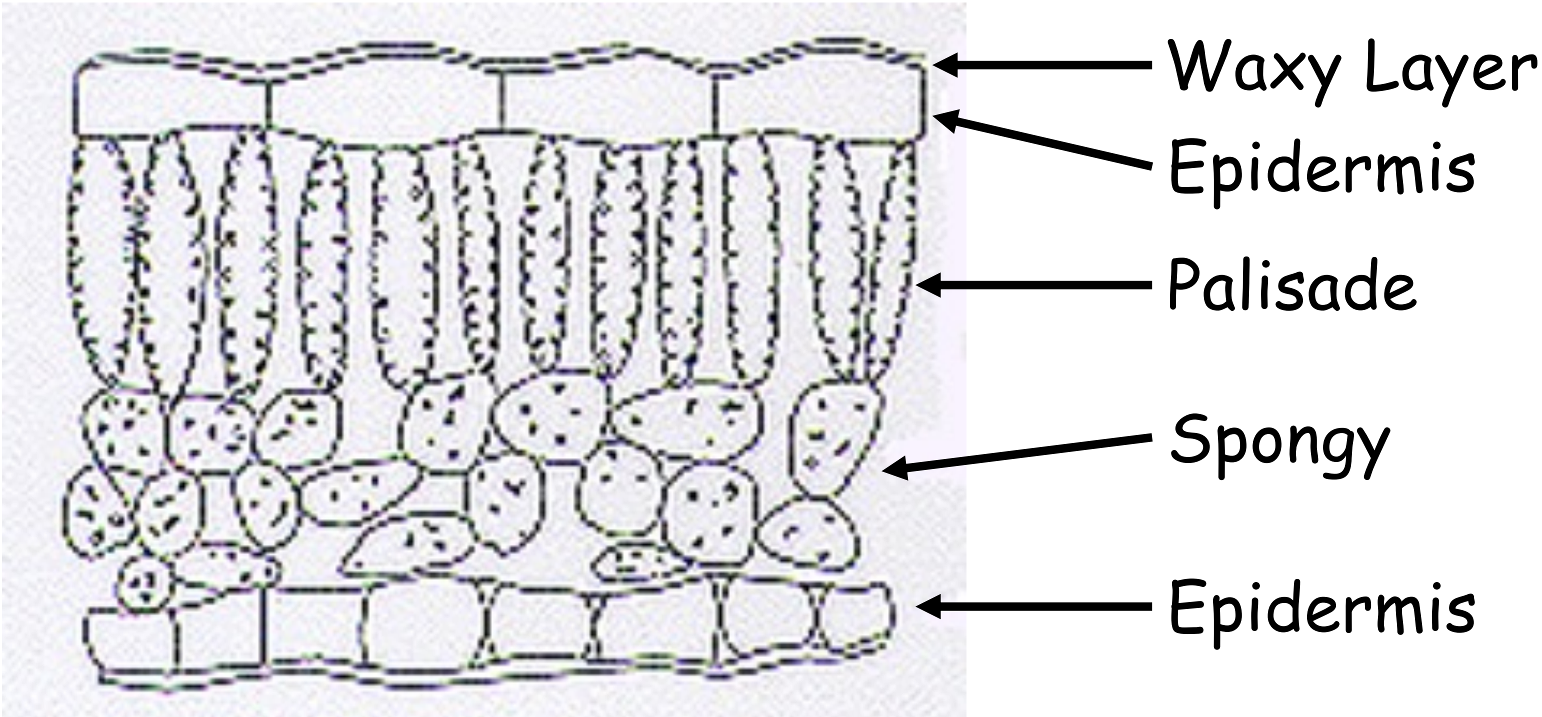
- Happens in all non-metallic materials
- Often approx. by a diffuse reflection
- Bad approx. for translucent material (marble, skin, milk)
- Light "bleeds" through thin slabs of material (material illuminated from behind)





Example: Leaves

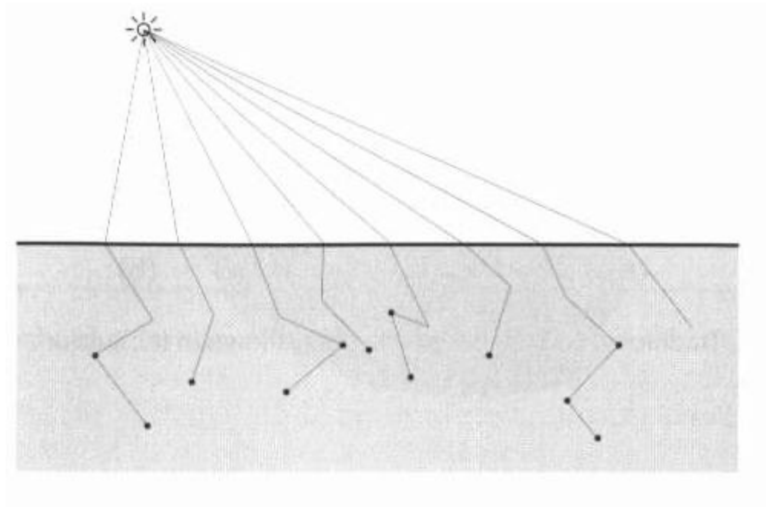
- Layers of typical leaf:



Rendered Visible Femal Foot

- Solve the actual transport equation (No Photon Mapping)

SS & PhotonMaps

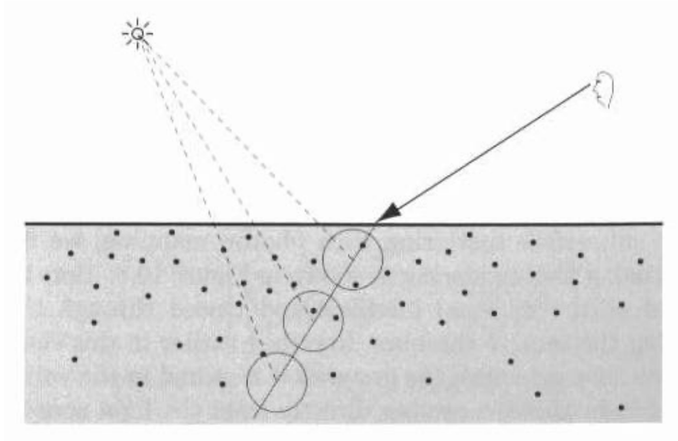


- Create Photon Map
- Photon is refracted
  (on object surface)
- Progresses through material
  – scattering and absorption
- Use photon map to record events
- No record of contribution from direct light
  (use standard ray-tracing for that)

SS & PhotonMaps

- Rendering: Use ray tracing
  - Ray is bent at interface
  - Step size important
  - Use Russian Roulette

$$d = -\frac{\log \xi}{\sigma_t}$$



  - Indirect multiple scattering – volume radiance estimate
  - Direct single scattering- ray tracing
    - distance of ray to light source (through refraction boundary)
    - d_i - Euclidean distance
    - n - surface normal
    - η - refraction coefficient

$$d_i' = d_i \frac{|\omega \cdot n|}{\sqrt{1 - \left(\frac{1}{\eta^2}\right)\left(1 - (\omega \cdot n)^2\right)}}$$

SS Examples - Utah Teapot


RENDERED USING DALI - HENRIK WANN JENSEN 2001


HENRIK WANN JENSEN 1999


HENRIK WANN JENSEN 1999

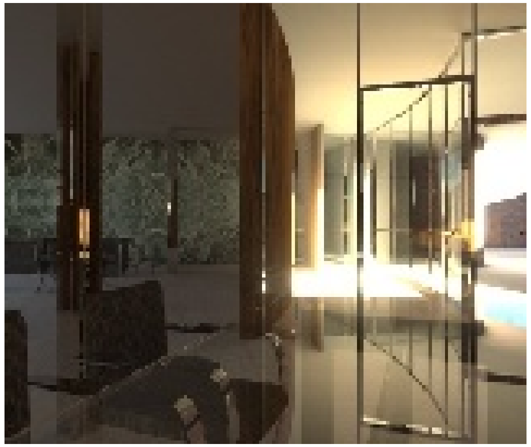
Granite + Marble Stanford Bunny

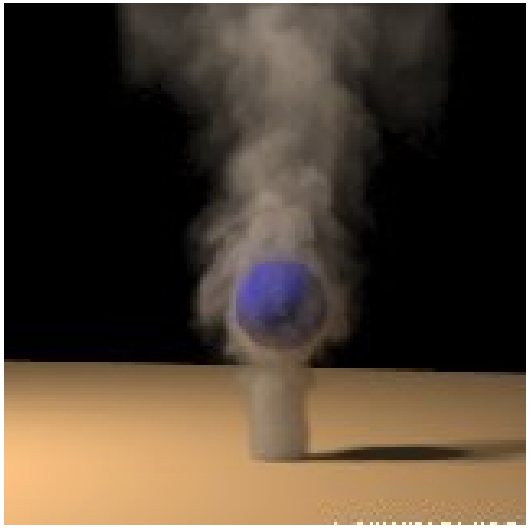

HENRIK WANN JENSEN 1999


RENDERED BY HENRIK WANN JENSEN - 2001


RENDERED BY HENRIK WANN JENSEN - 2001

Some animations





All images © Jensen