Rendering: Spatial Acceleration Structures

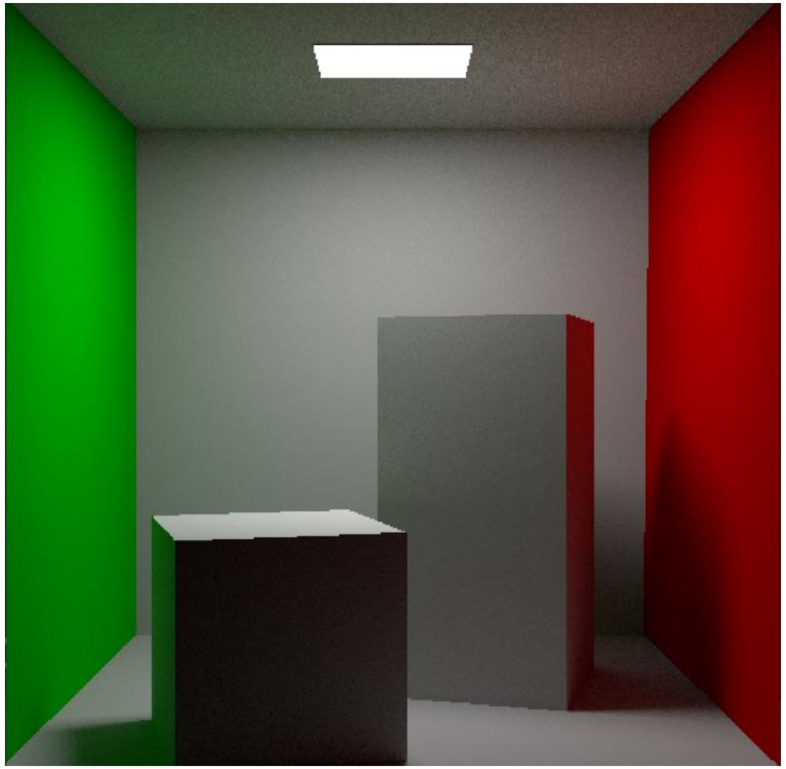
Bernhard Kerbl

Research Division of Computer Graphics
Institute of Visual Computing & Human-Centered Technology
TU Wien, Austria

With slides based on material by Jaakko Lehtinen, used with permission

What we have and what we want

- Larger images, more geometry!



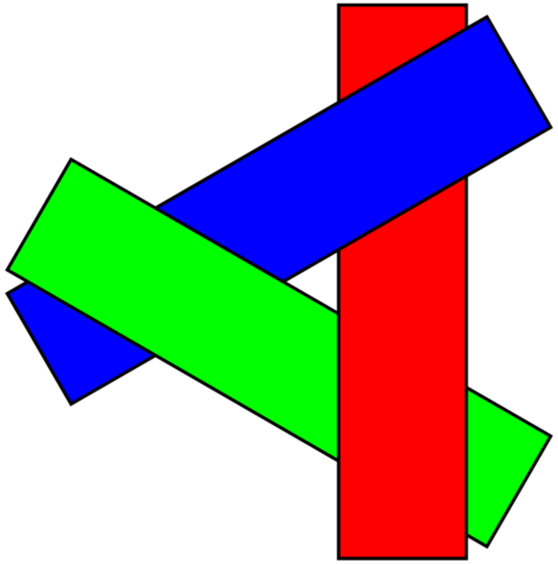
32 triangles

500k+ triangles

How do we resolve visibility right now?

- A good image needs both realistic *intensity* and *visibility*
  - **Intensity** taken care of by simulating correct light transport
  - **Visibility** makes sure that objects adhere to depth

    How would you process the scene on the right to make sure the rendered output image is correct?



Source: Wojciech Mula, Wikipedia “Painter's algorithm”

- (Naïve) Ray Casting-based Visibility
  - Shoot a ray through **each** pixel into the scene
  - Test against **all** objects for intersection
  - Record the **closest** intersection, use for intensity computations

Render Loop with Intersection Method

```
for (i = 0; i < N; i++)
        v_inv = camera.gen_ray(px, py)
        pixel_color += Li(v_inv, 0)
pixel_color /= N

function Li(v_inv, D)
        x = scene.trace(v_inv)
        …

method trace(Ray ray)
        x_min(t = INF);
        for (i = 0; i < scene.num_triangles; i++)
                x = ray.intersect(scene.triangles[i])
                if (x.t < x_min.t)
                        x_min = x
        return x_min
```

Render Loop with Intersection Method Run Time

```
for (i = 0; i < N; i++)
        v_inv = camera.gen_ray(px, py)
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```
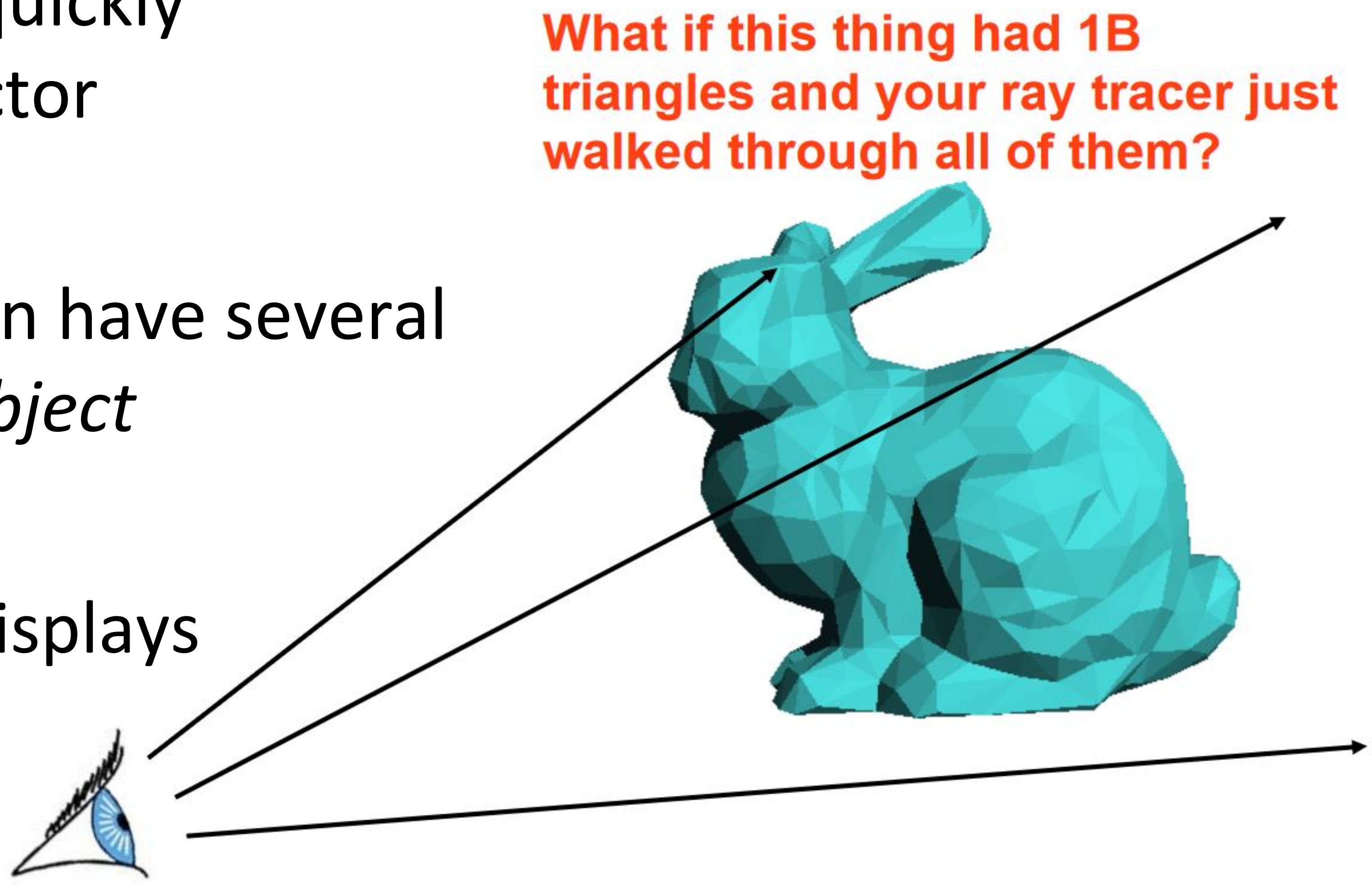

- This is $\mathcal{O}(N \cdot \#\Delta)$, but even worse, it's $\Omega(N \cdot \#\Delta)$!

Is that actually a problem?

- Run time complexity quickly becomes a limiting factor
- High-quality scenes can have several million triangles *per object*
- Current screens and displays are moving towards 4k resolution



Amazon Lumberyard “Bistro”
3,780,244 triangles
1200x675 pixels
32 samples p.p.?

100 trillion ray/triangle intersection tests?

At 10M per second, one frame will take ~120 days.

Good luck with your movie!

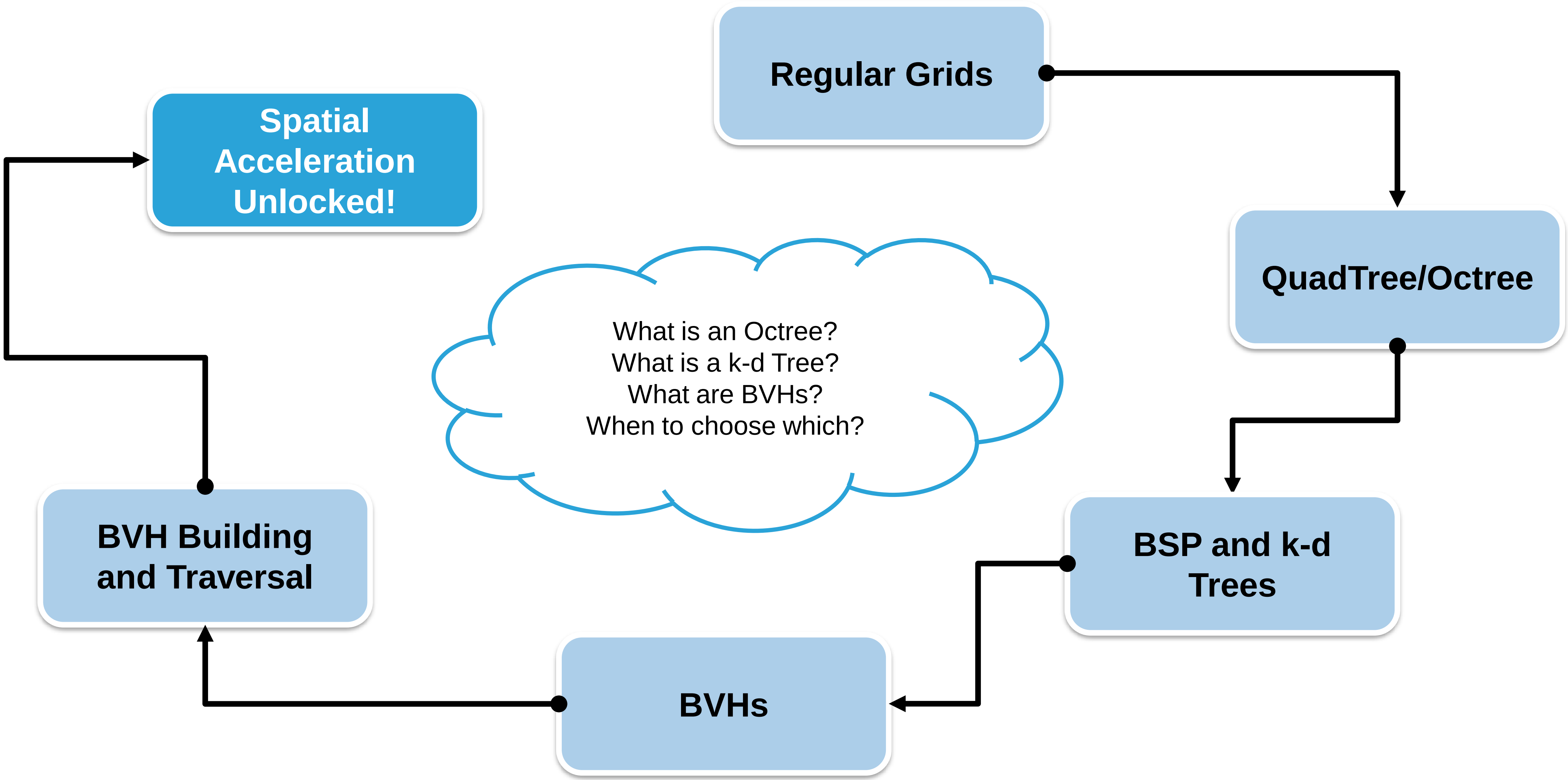
What can we do about it?

- Find ways to speed up the basic loop for visibility resolution
- Enter “spatial acceleration structures”
- Essentially, pre-process the scene geometry into a structure that reduces expected traversal time to something more reasonable
- Pick smart traversal strategies to further raise performance

Spatial Acceleration Structures

Structure	Additional Memory	Building Time	Traversal Time
none	none	none	abysmal

Today's Roadmap



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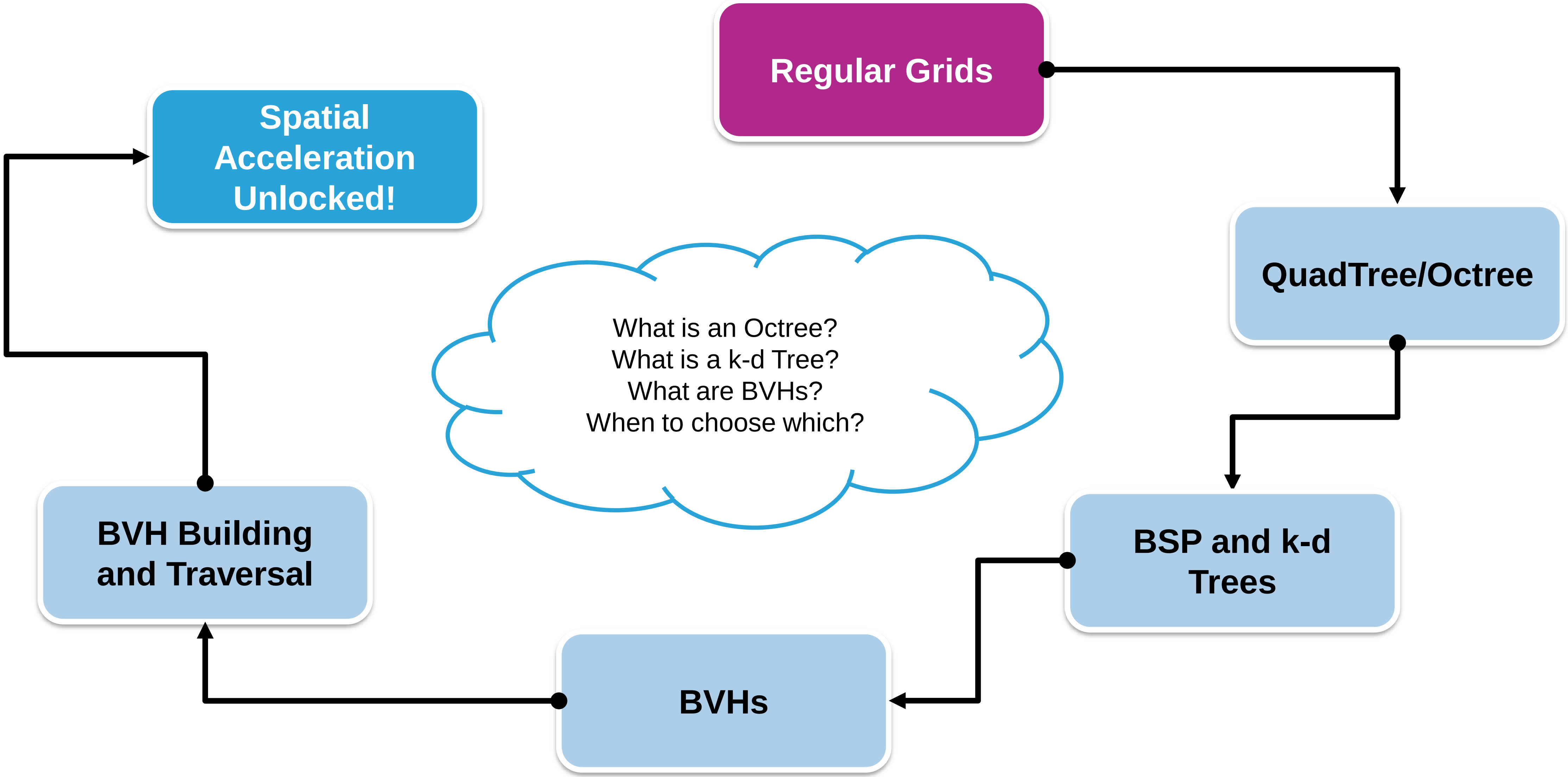
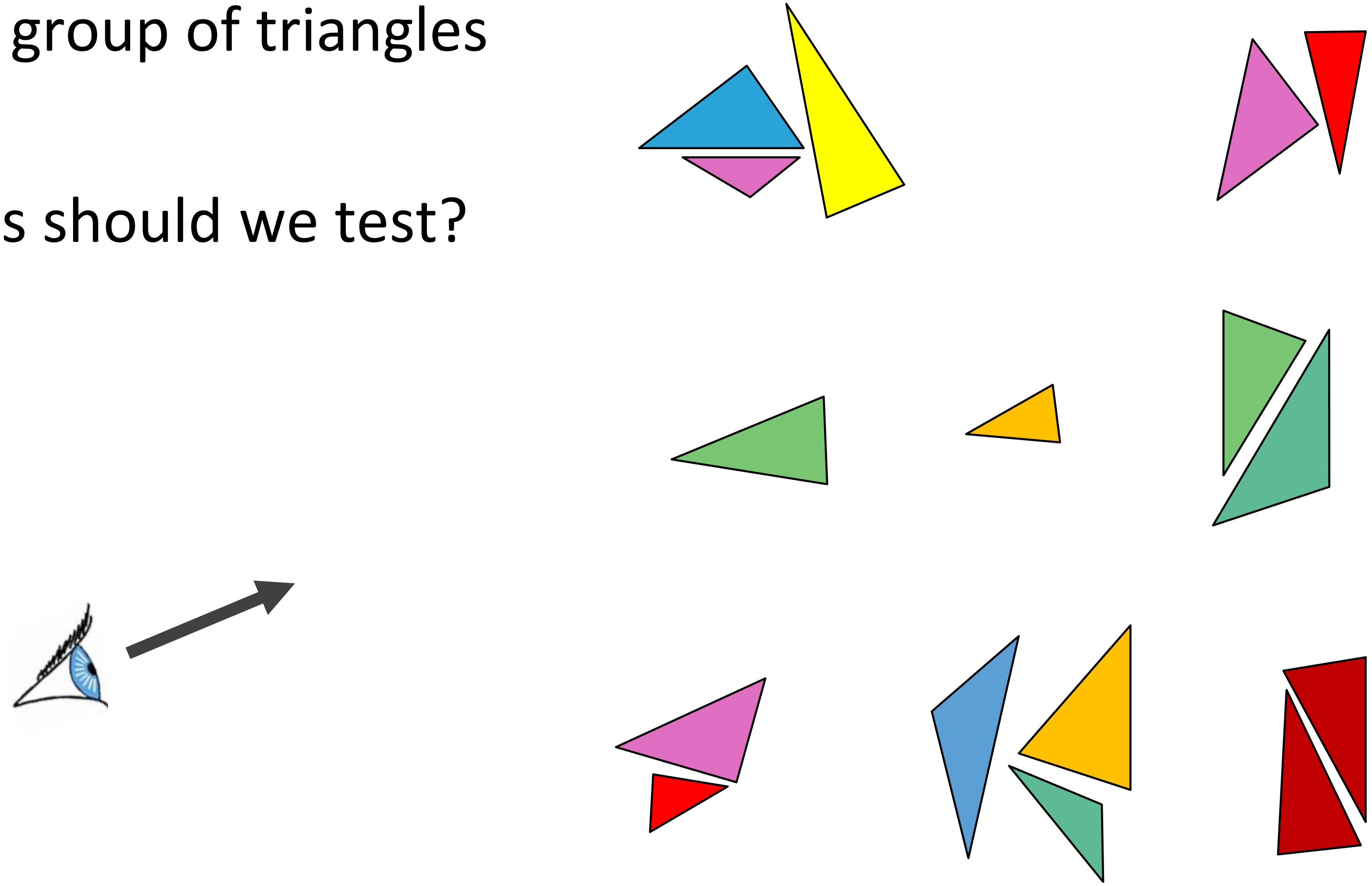


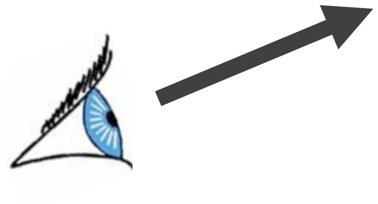
Speeding Up Intersection Tests

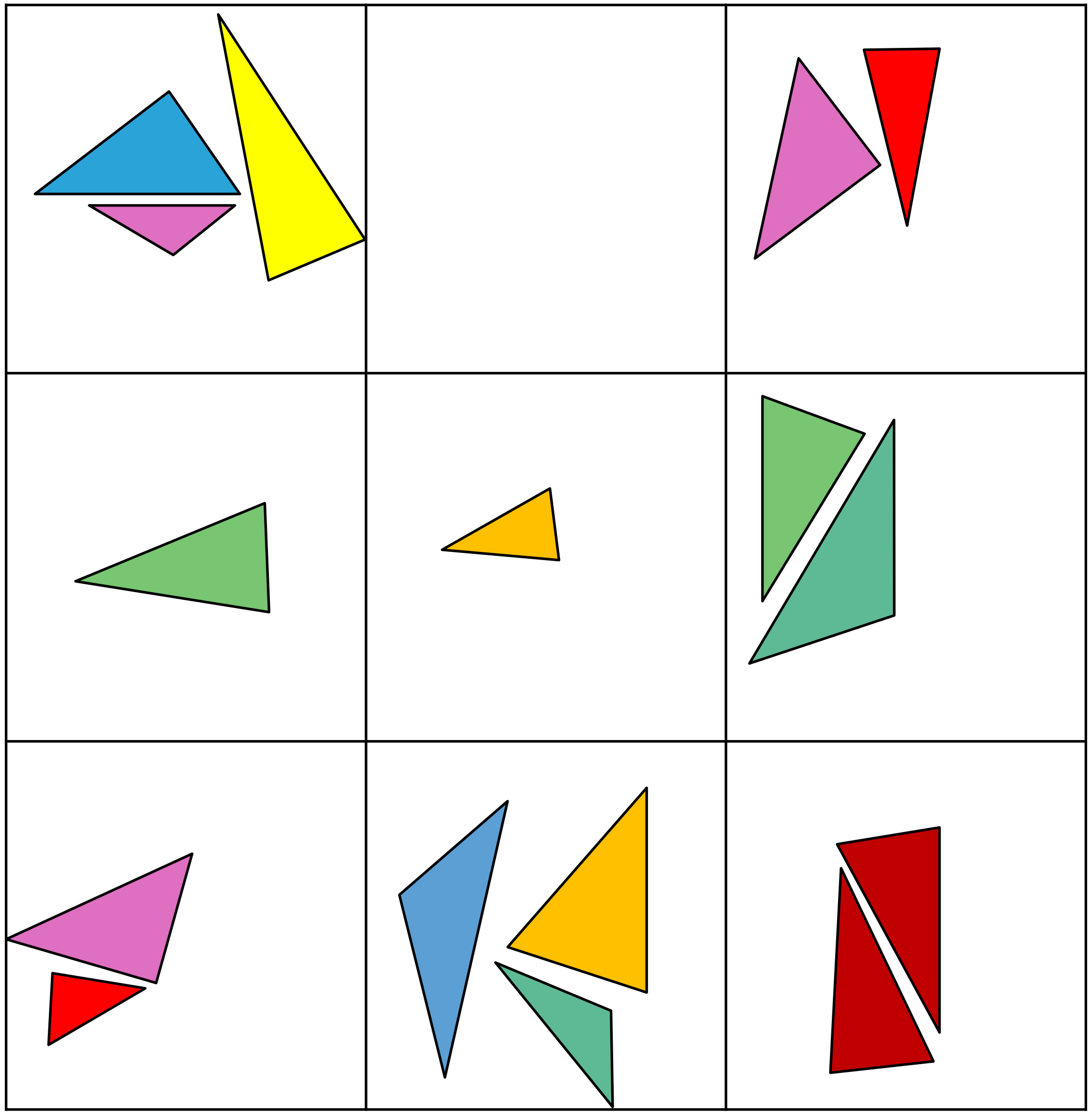
- Consider a group of triangles
- Which ones should we test?



Regular Grids

- Overlay scene with regular grid
- Sort triangles into cells
- Traverse cells and test against their contents





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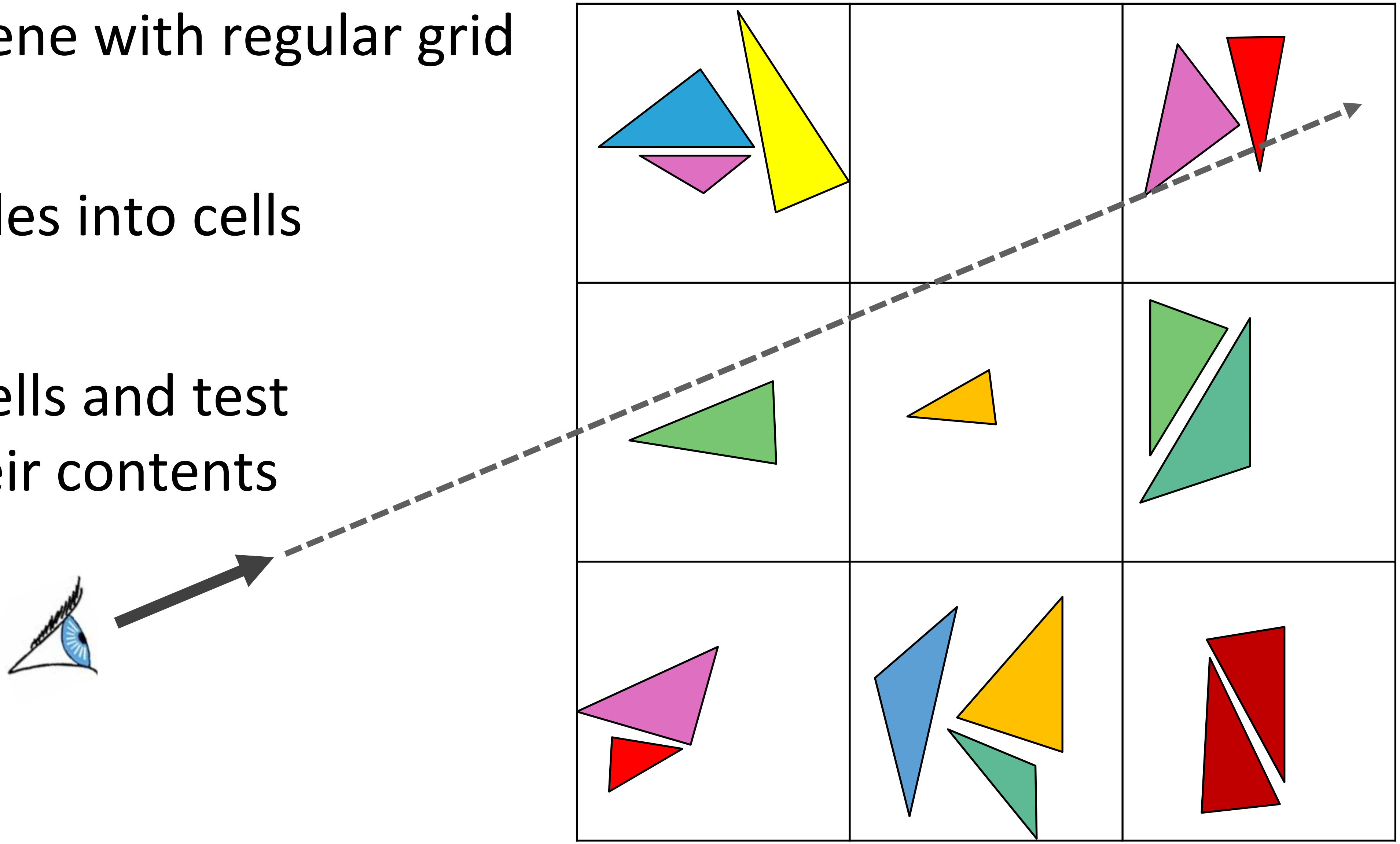

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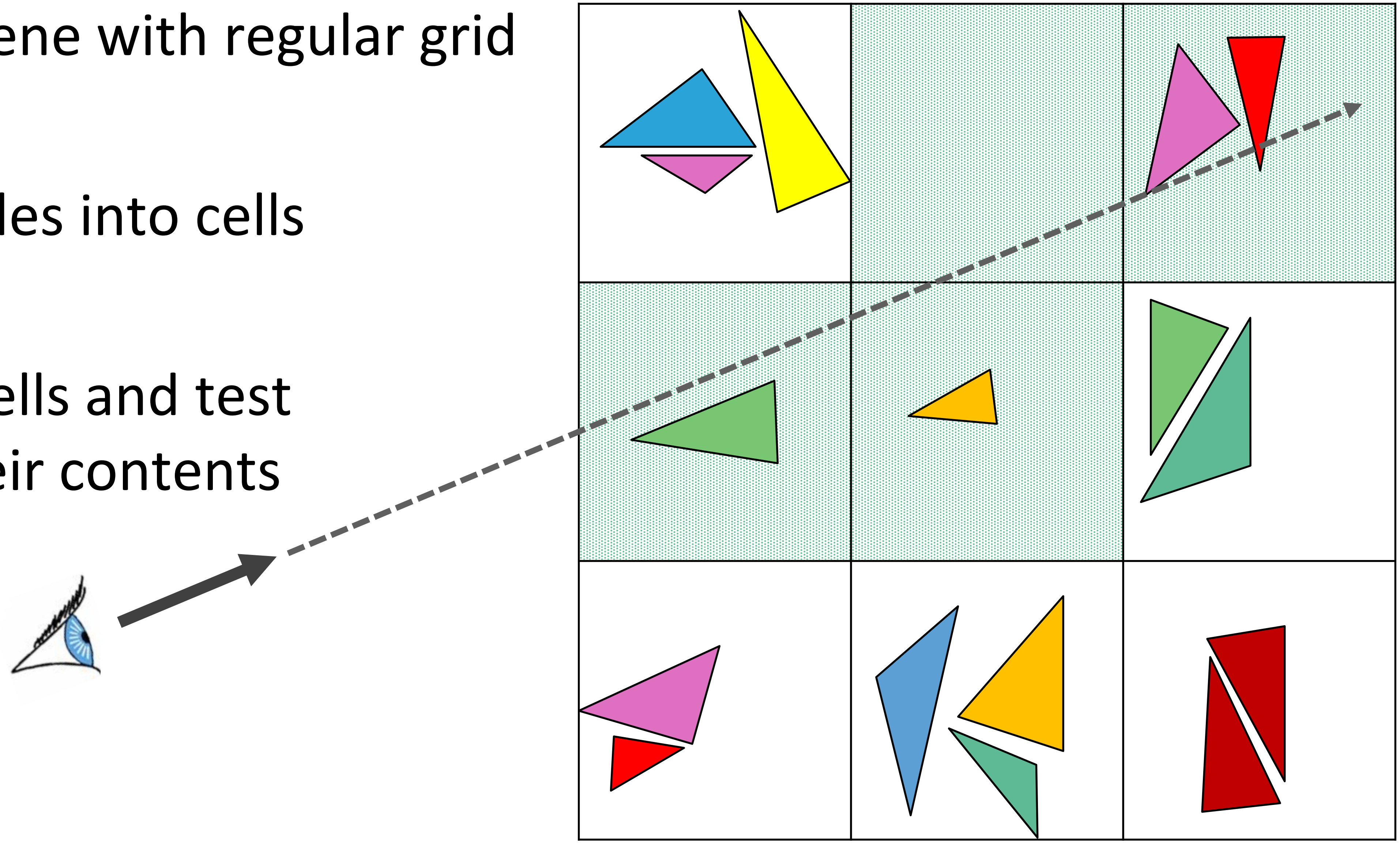
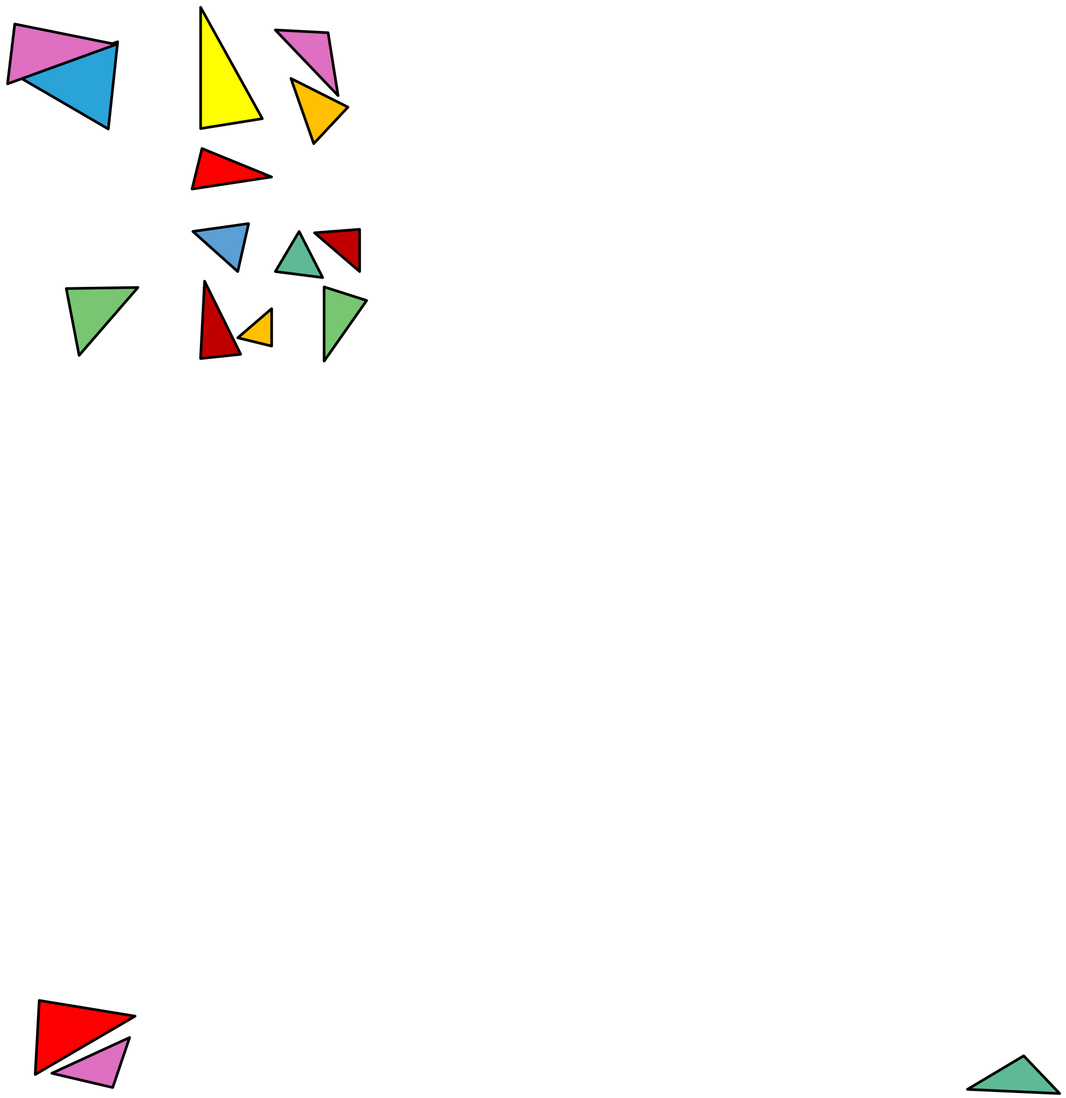


Regular Grids

- Geometry is usually not uniform
- Comes in clusters (buildings, characters, vegetation…)



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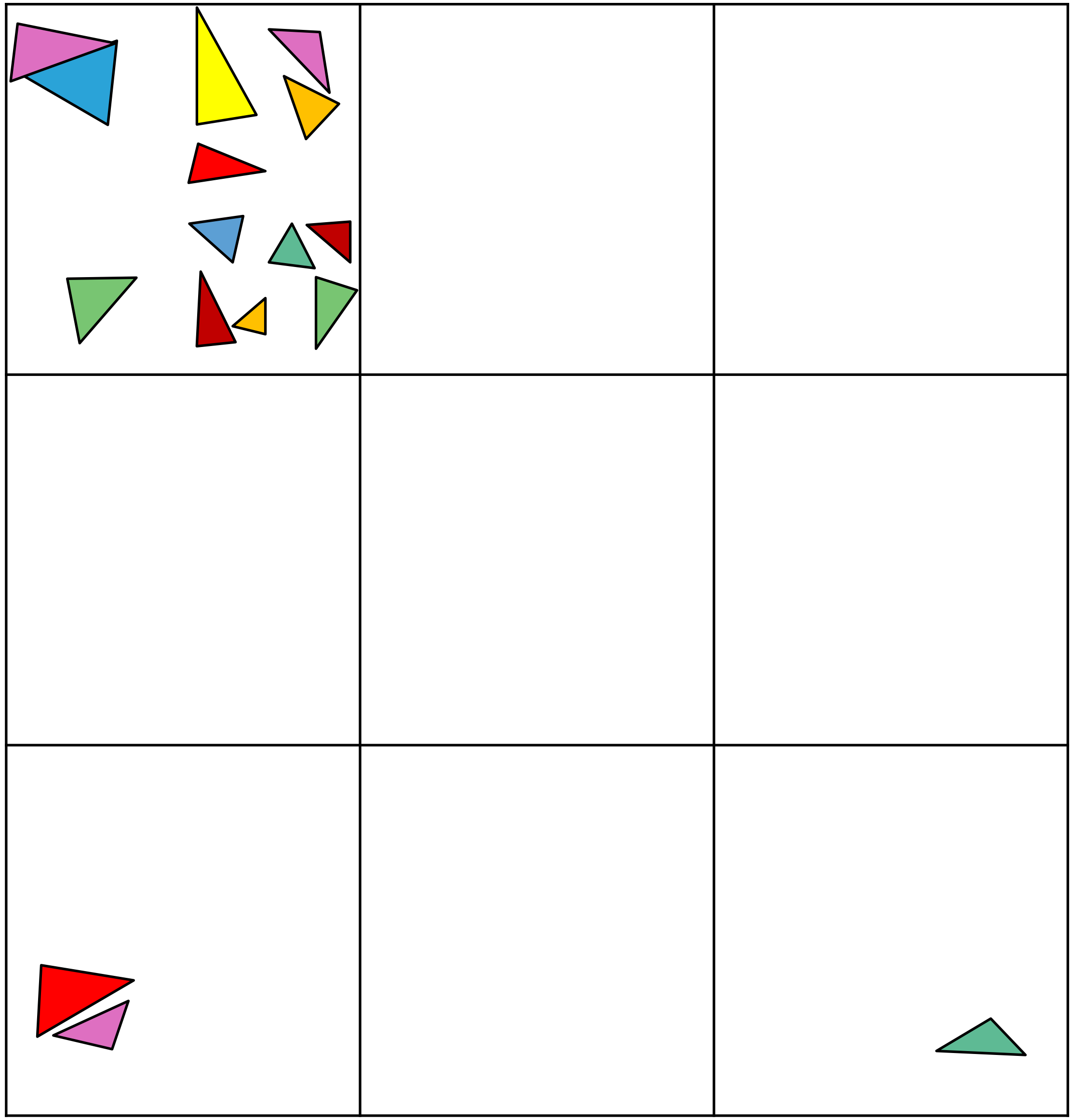
- Almost all triangles in one cell! Hitting this cell will be costly!



Regular Grids

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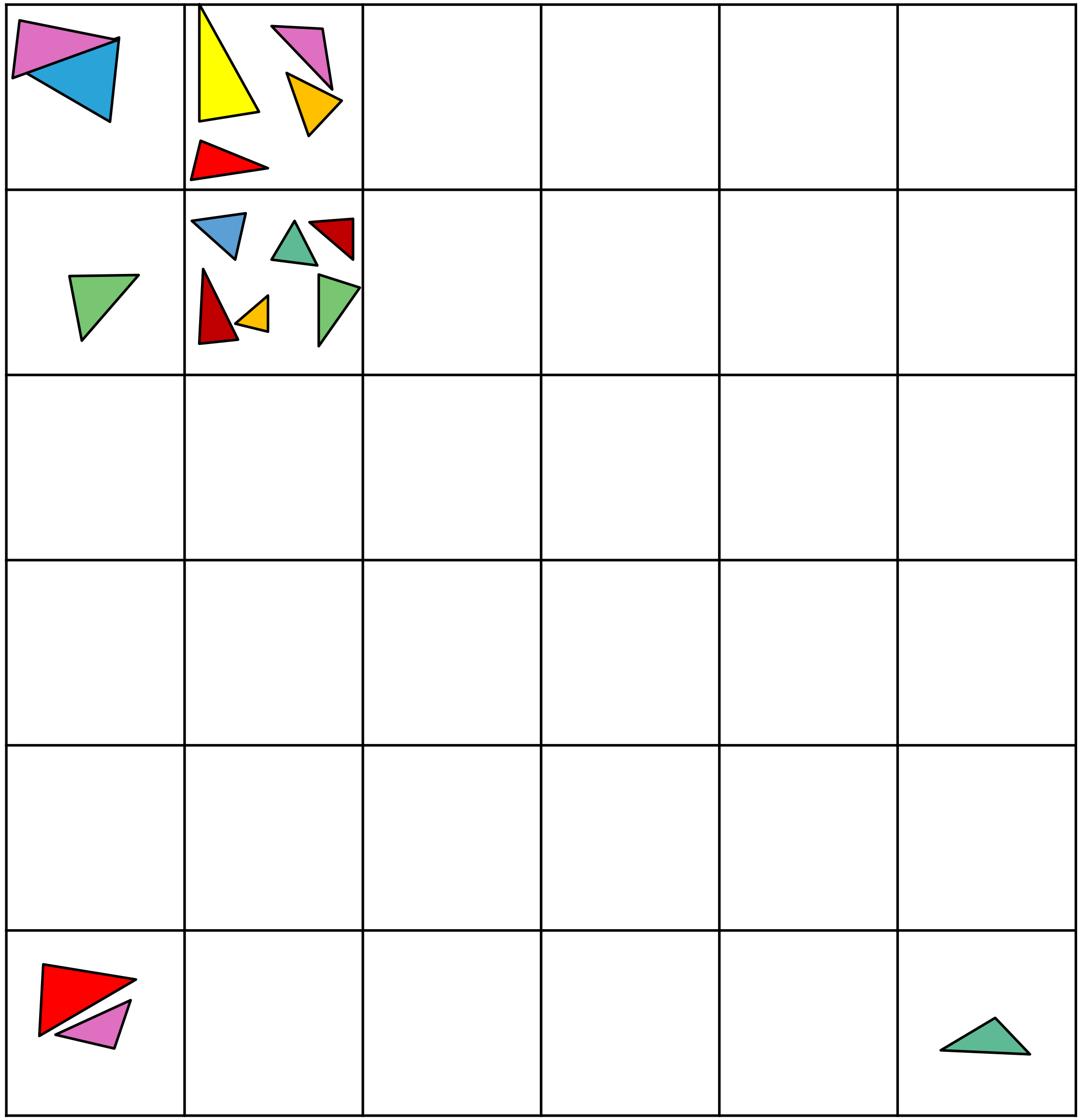
- Comes in clusters (buildings, characters, vegetation…)
- ~~Almost all triangles in one cell! Hitting this cell will be costly!~~
- Using a finer grid works



Regular Grids

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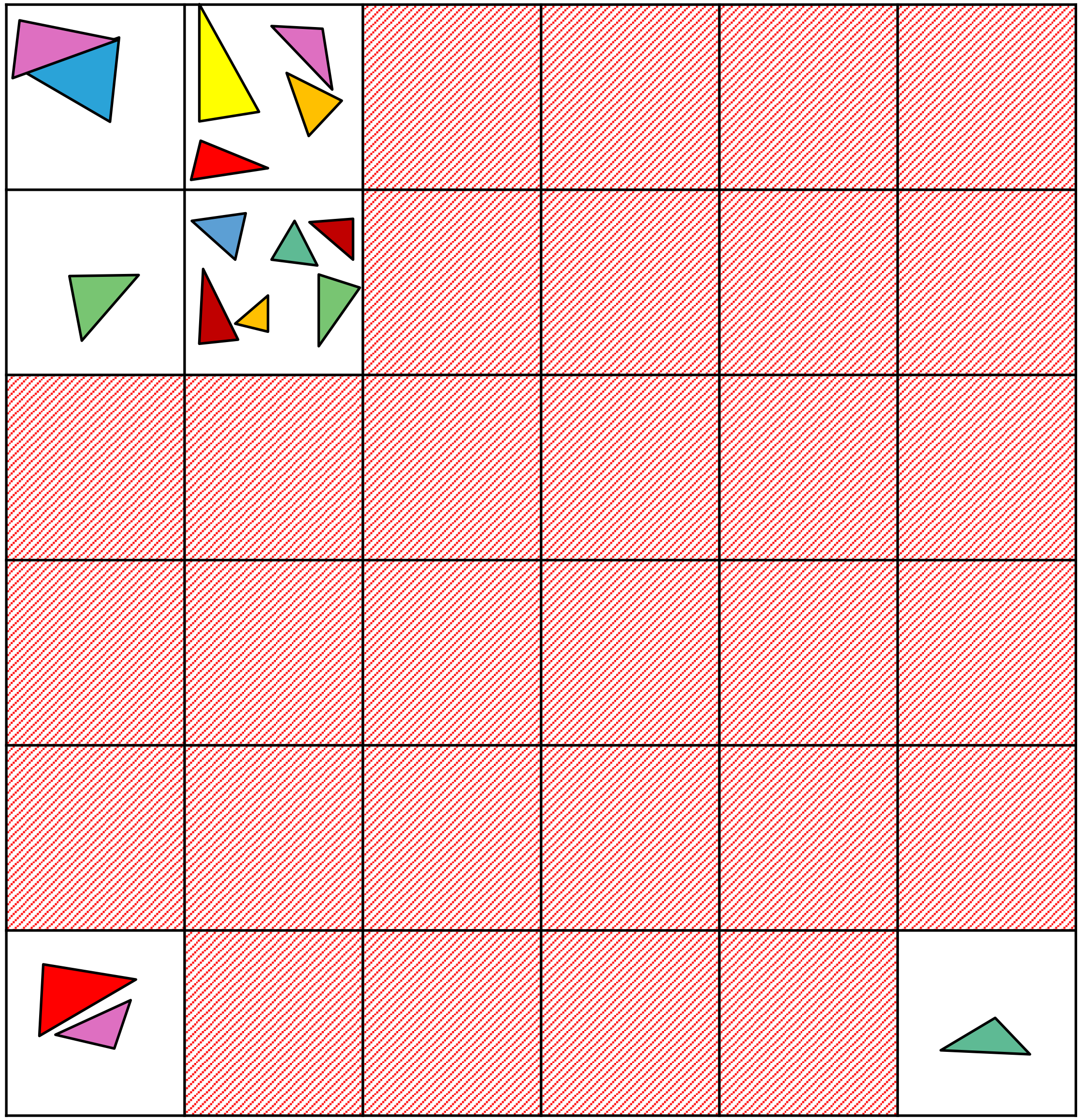
- Comes in clusters (buildings, characters, vegetation…)
- ~~Almost all triangles in one cell! Hitting this cell will be costly!~~
- Using a finer grid works, but most of its cells are unused!



Spatial Acceleration Structures

Structure	Memory Consumption	Building Time	(Expected) Traversal Time
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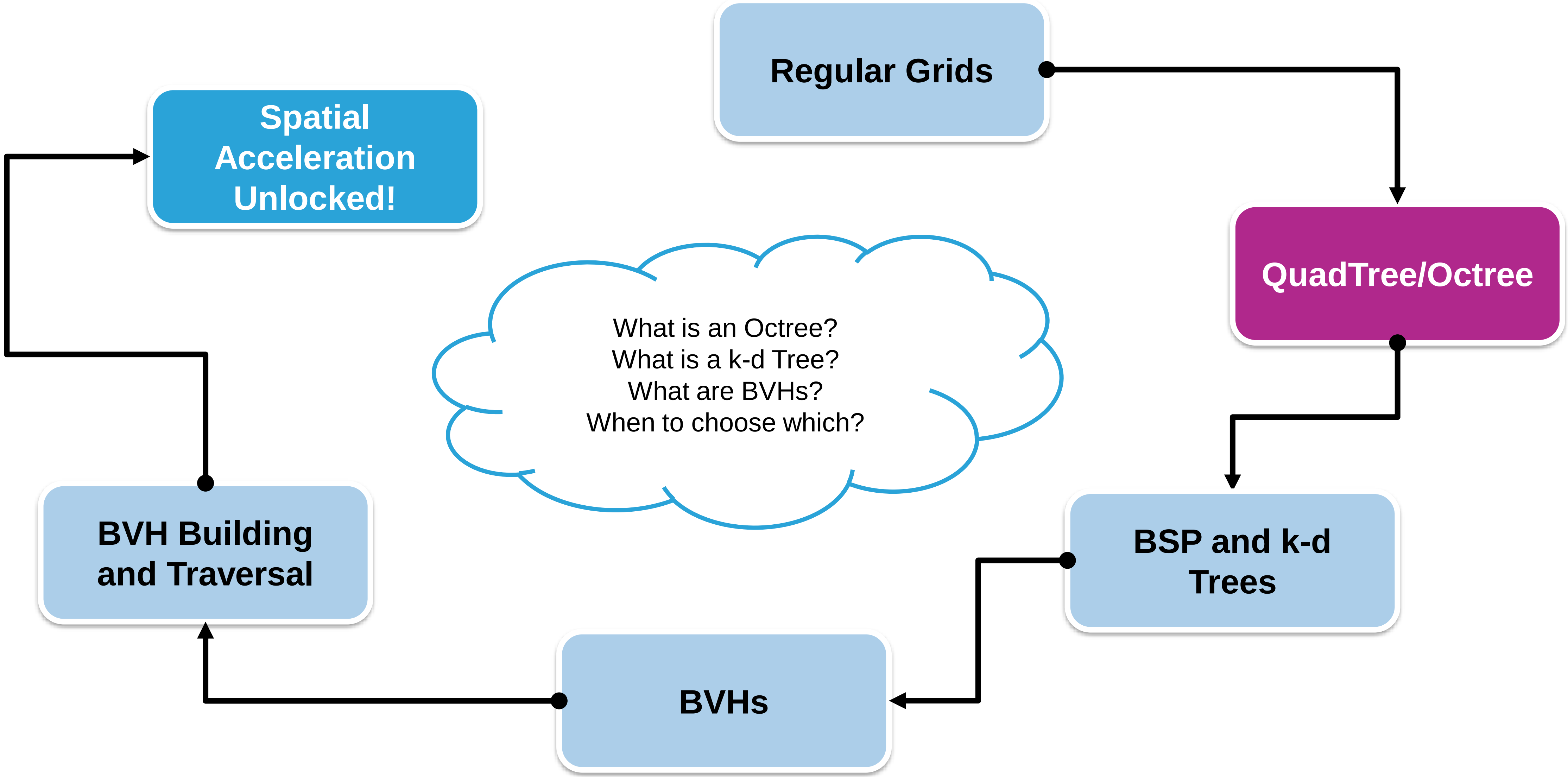
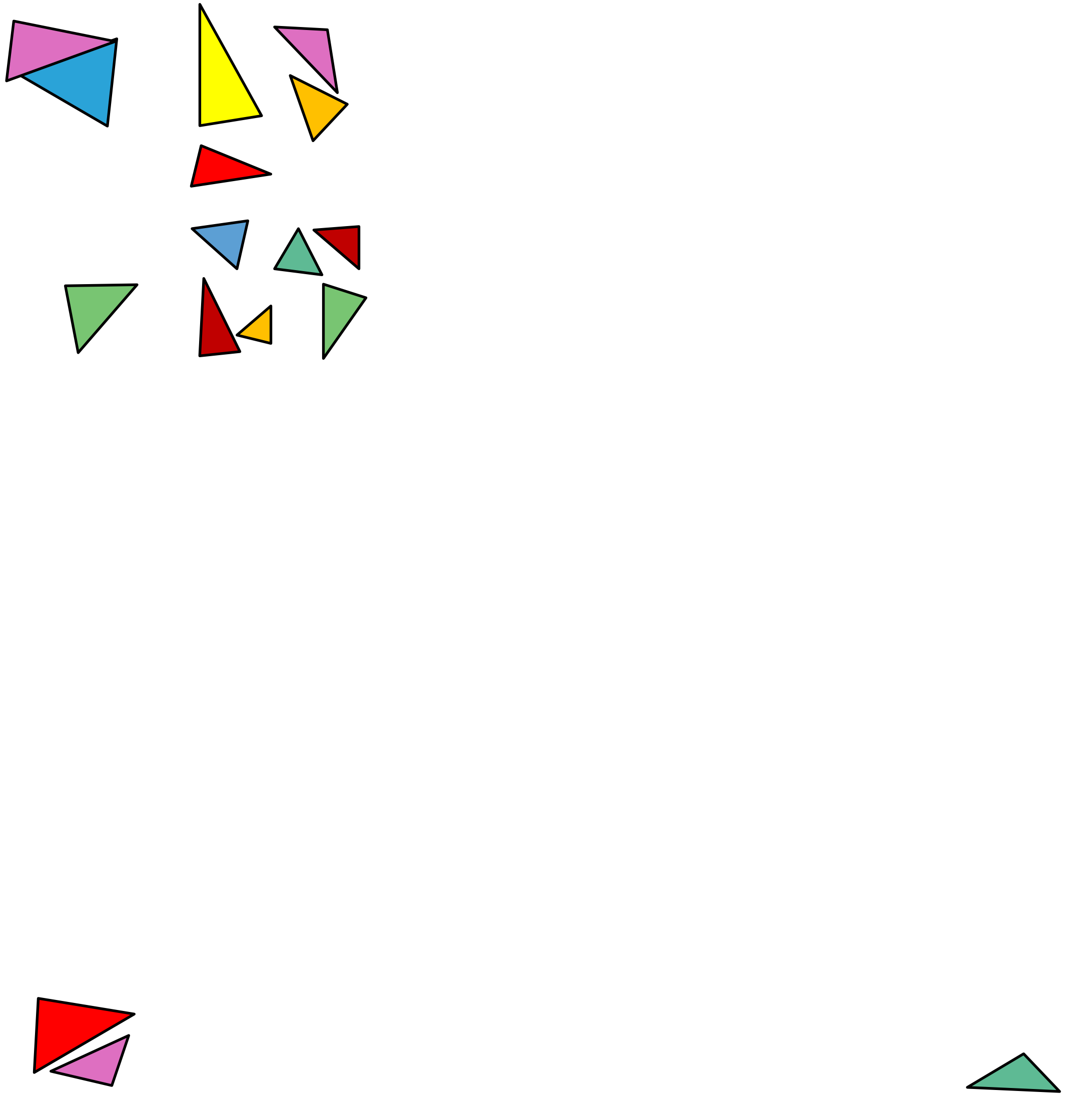


Quadtrees and Octrees

- Start with scene bounds, do finer subdivisions only if needed
- Define parameters S_{max}, N_{leaf}
- Recursively split bounds into *quadrants* (2D) or *octants* (3D)
- Stop after S_{max} subdivisions or if no cell has $> N_{leaf}$ triangles



Quad and Octrees: $N_{leaf} = 4$

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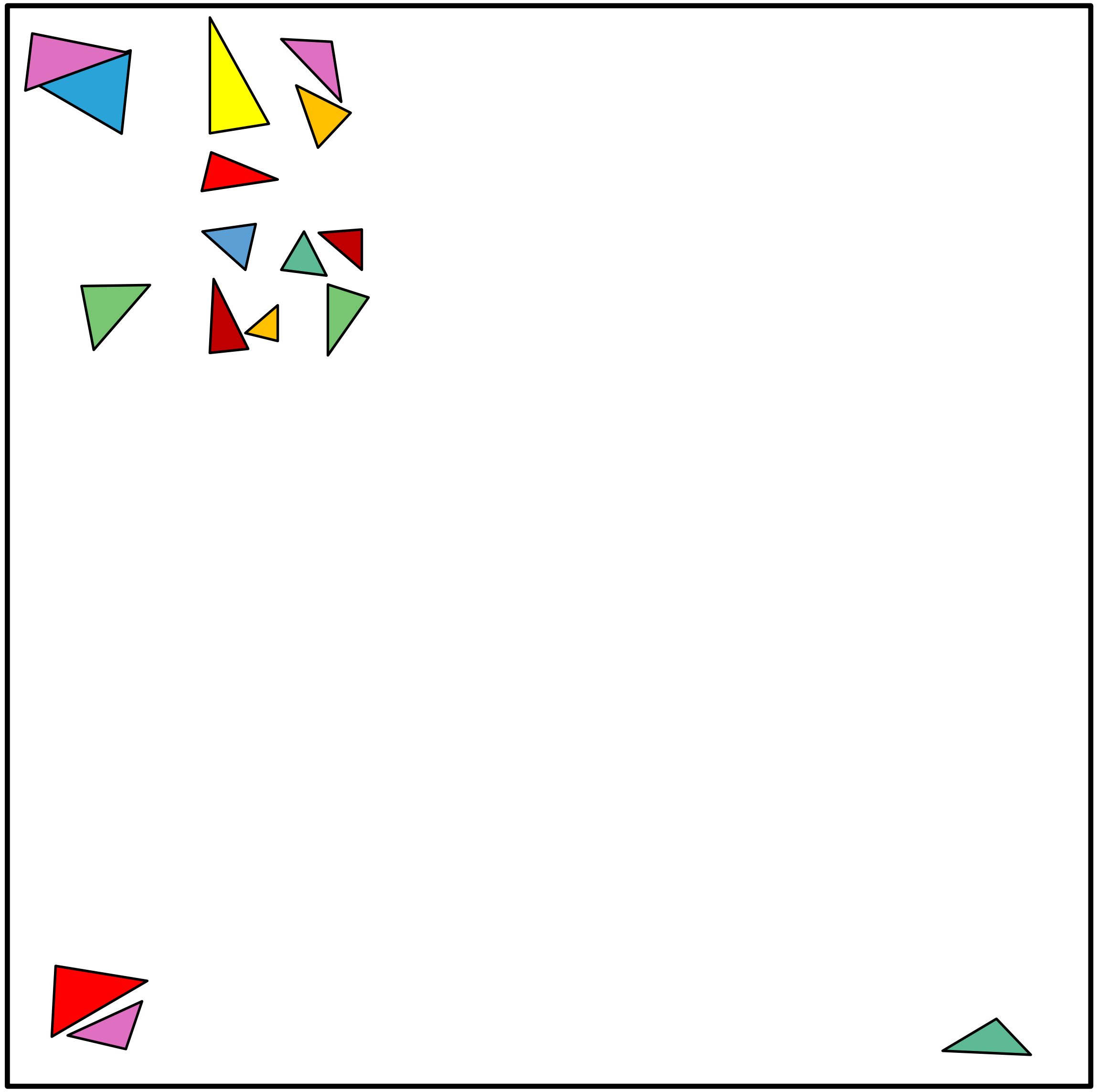

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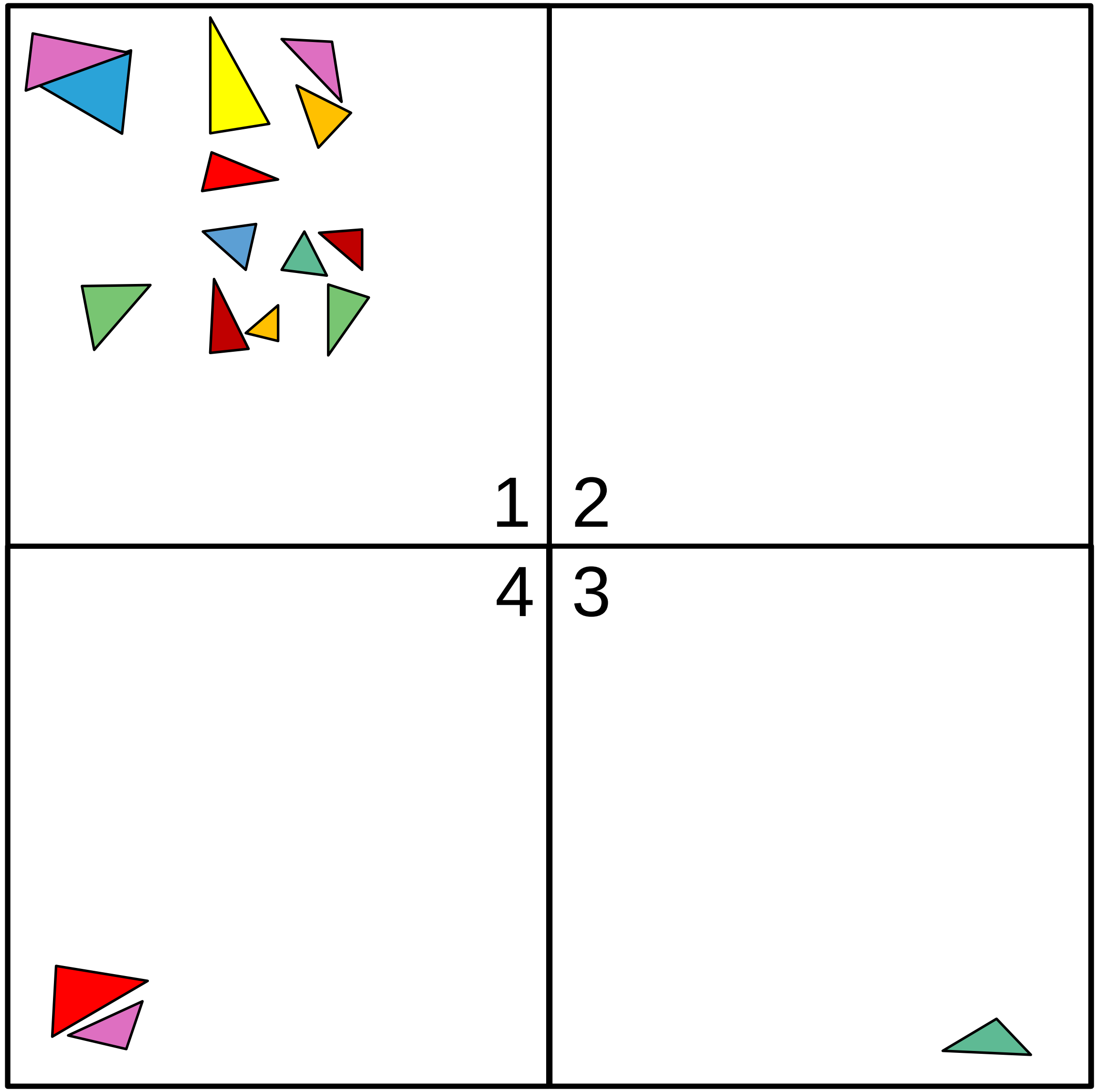

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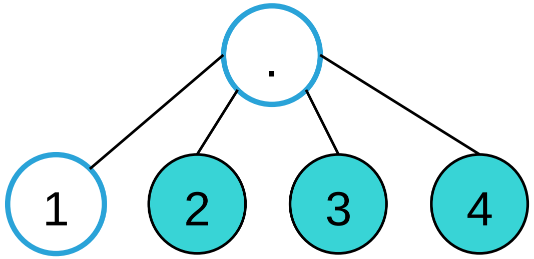

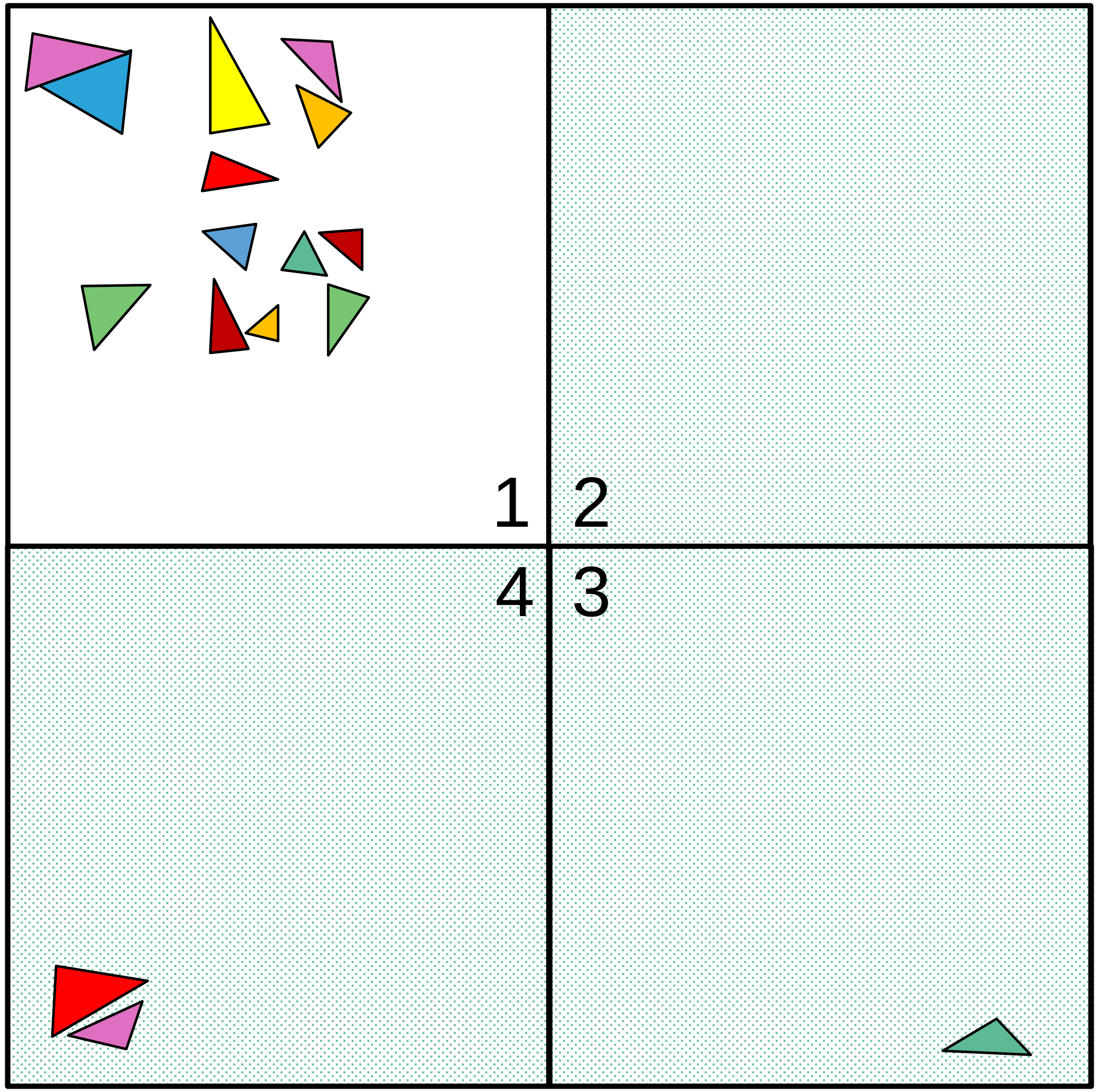

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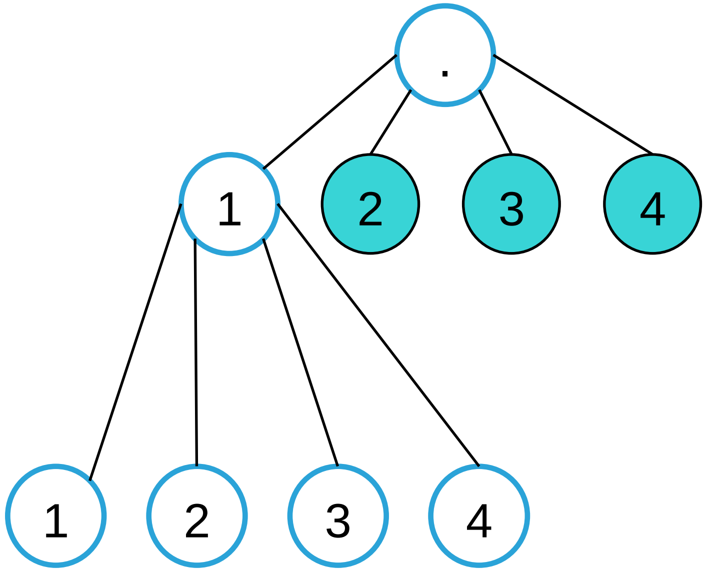

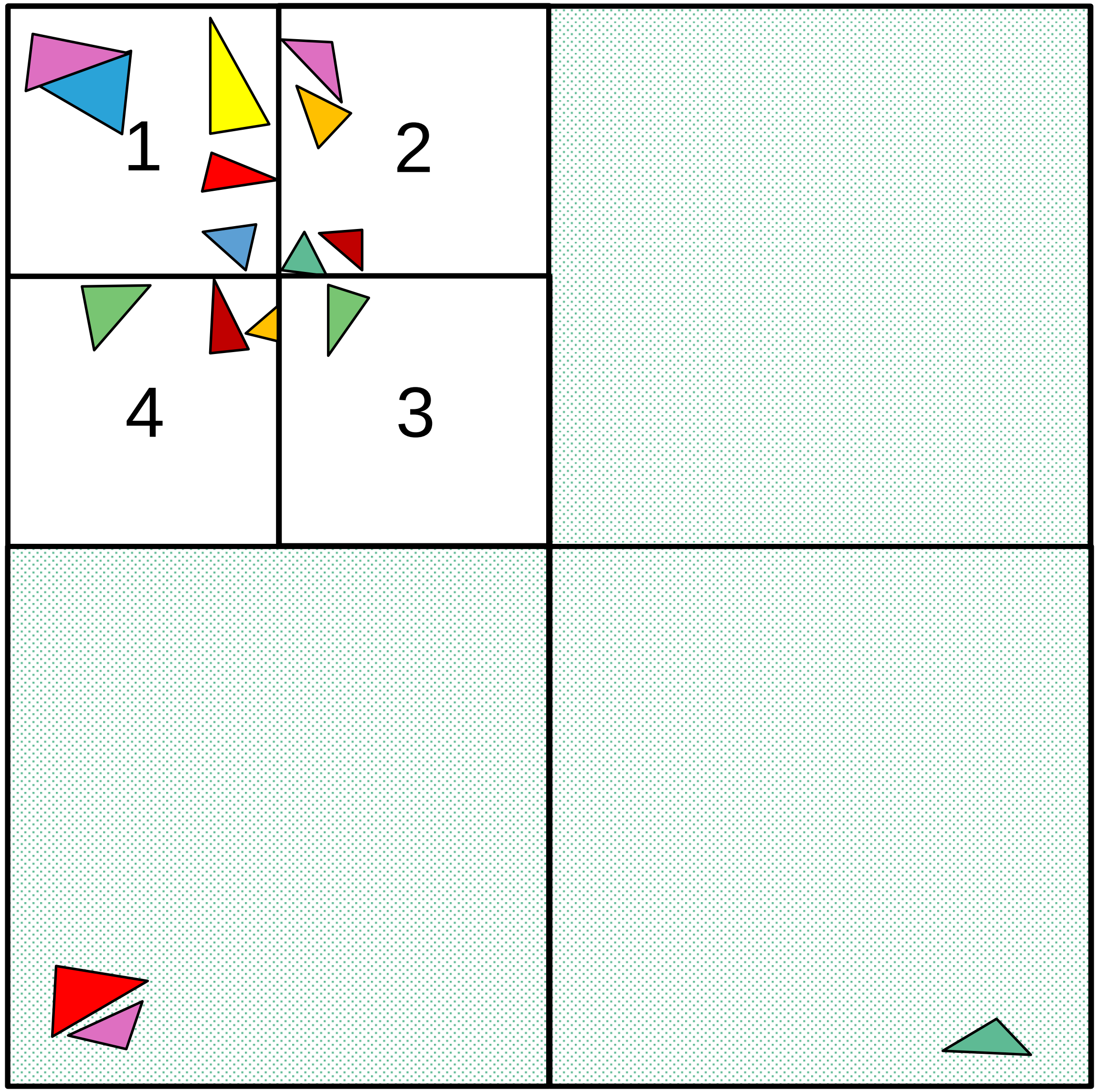

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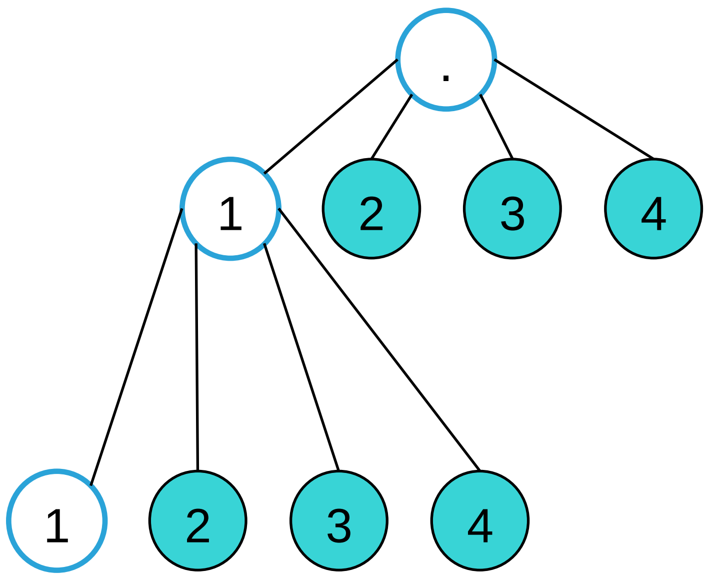

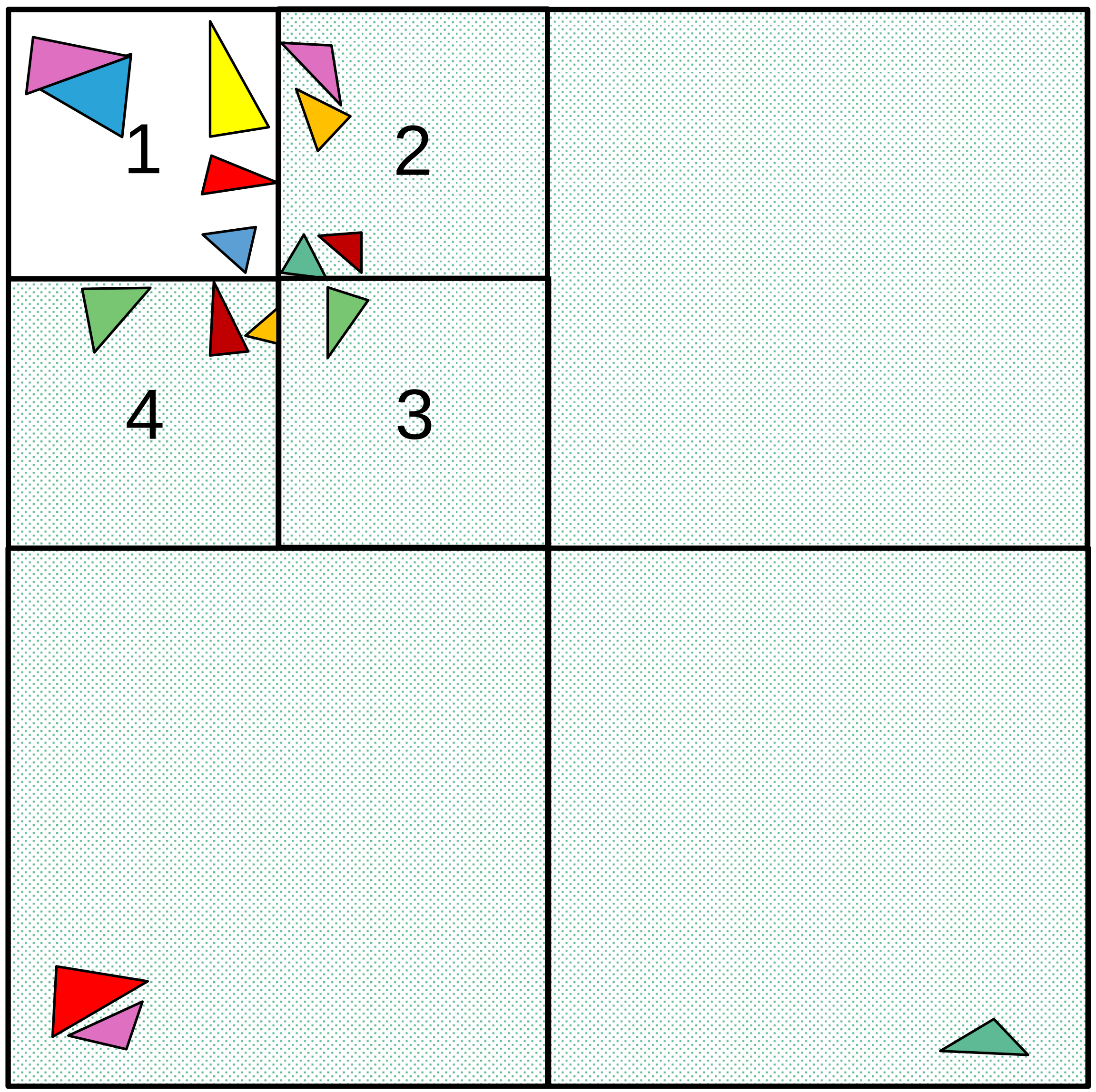

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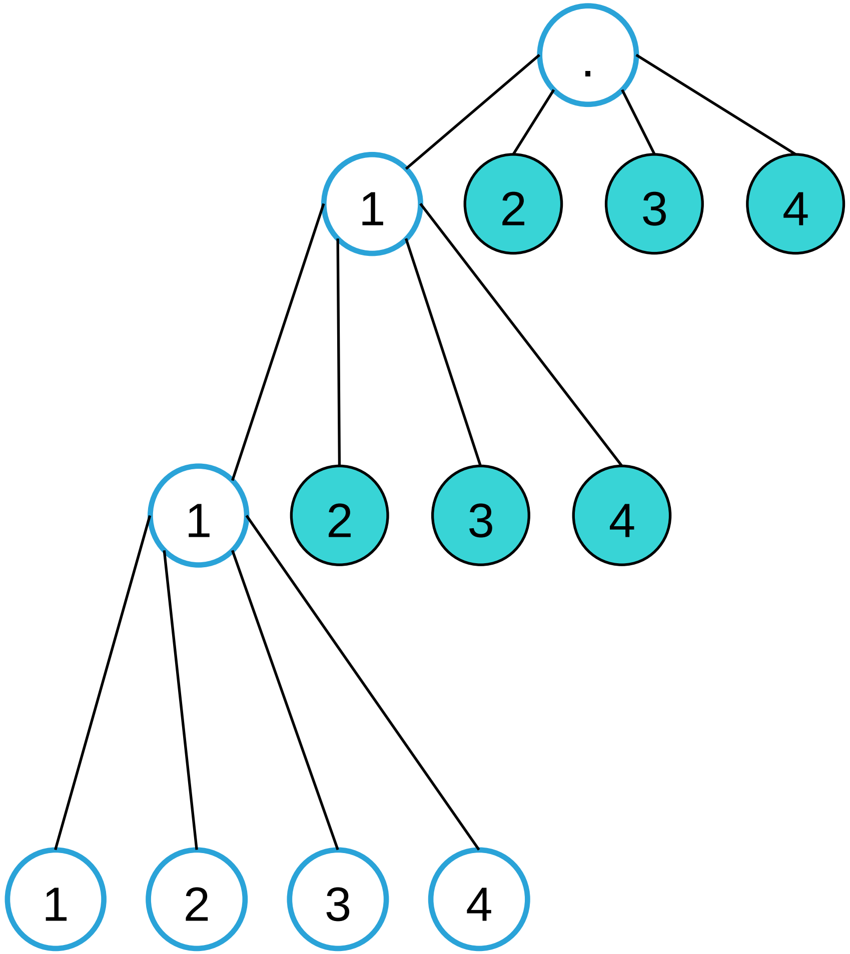

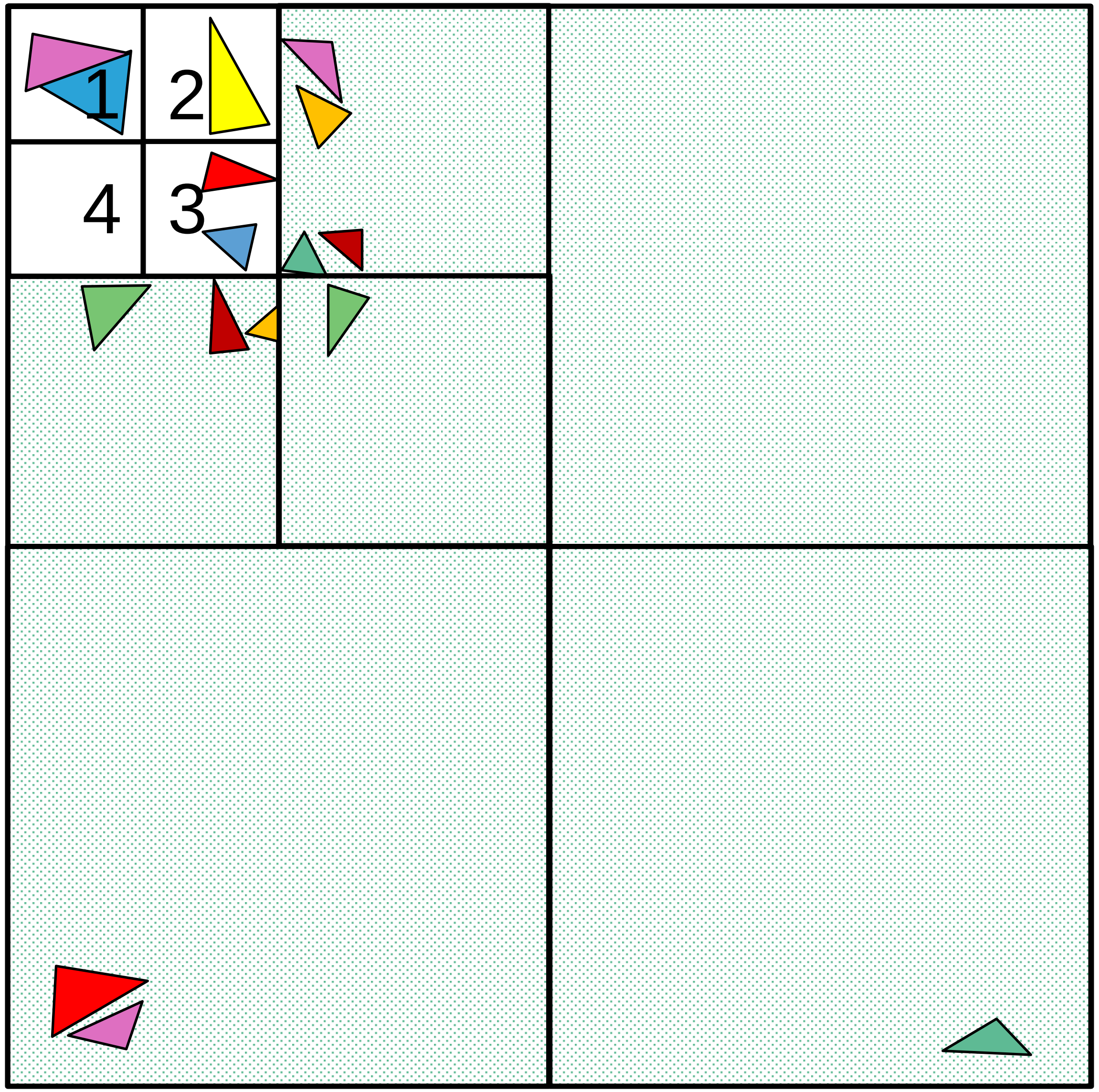

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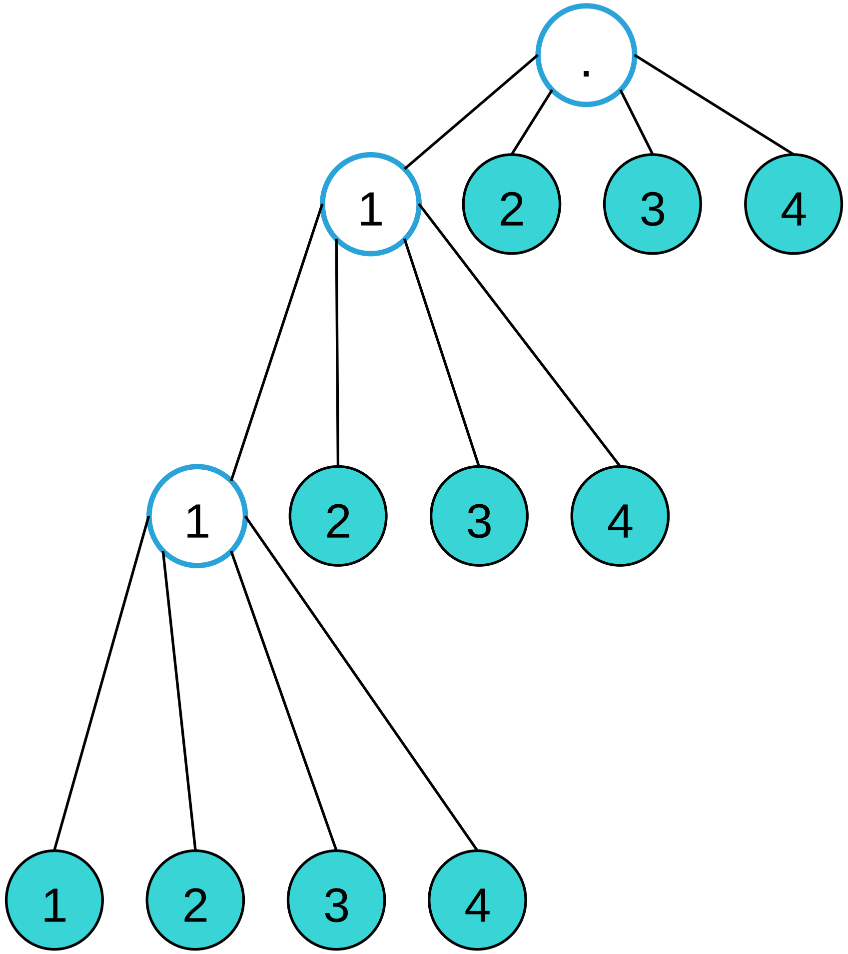

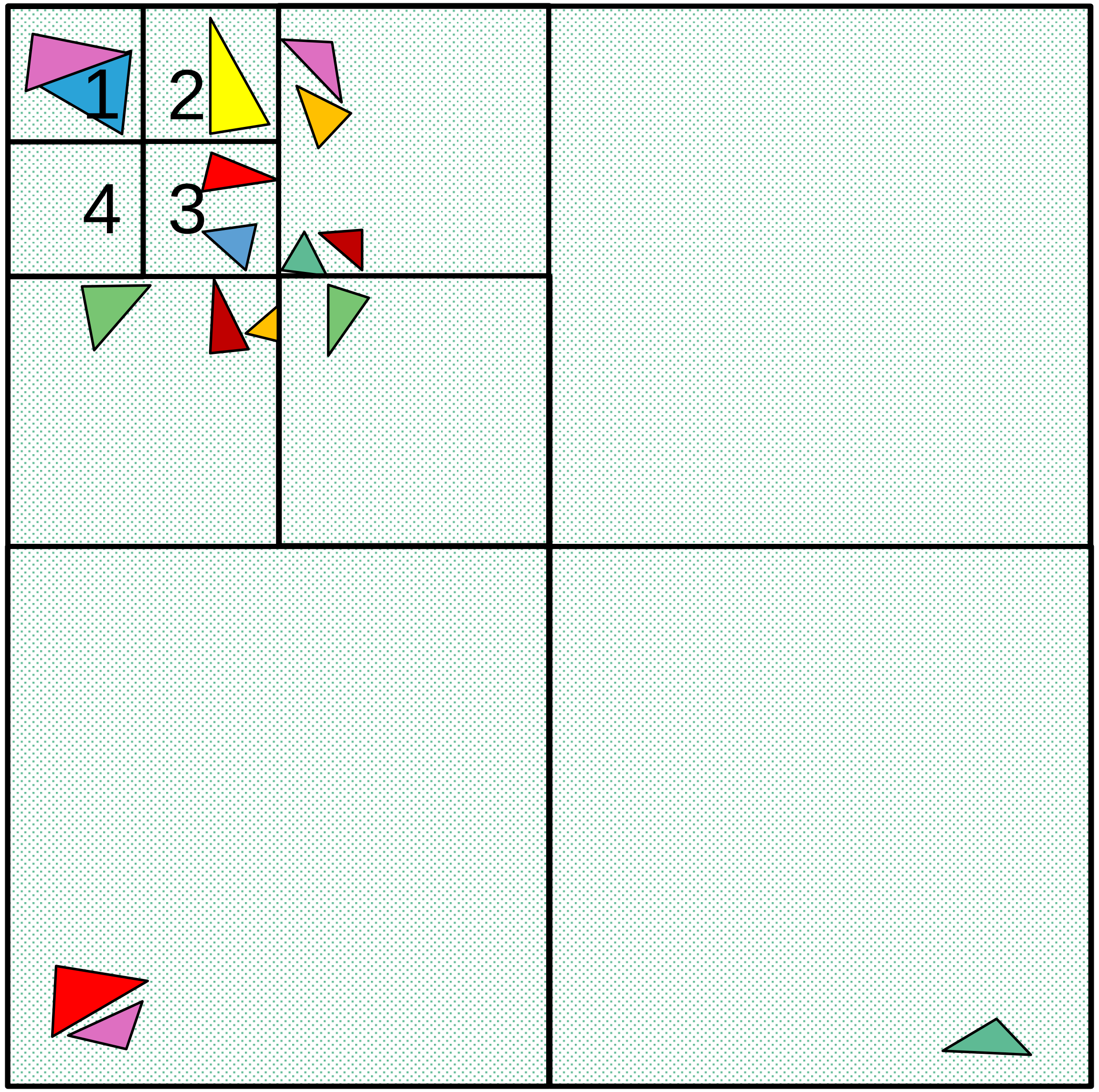

Quad and Octrees

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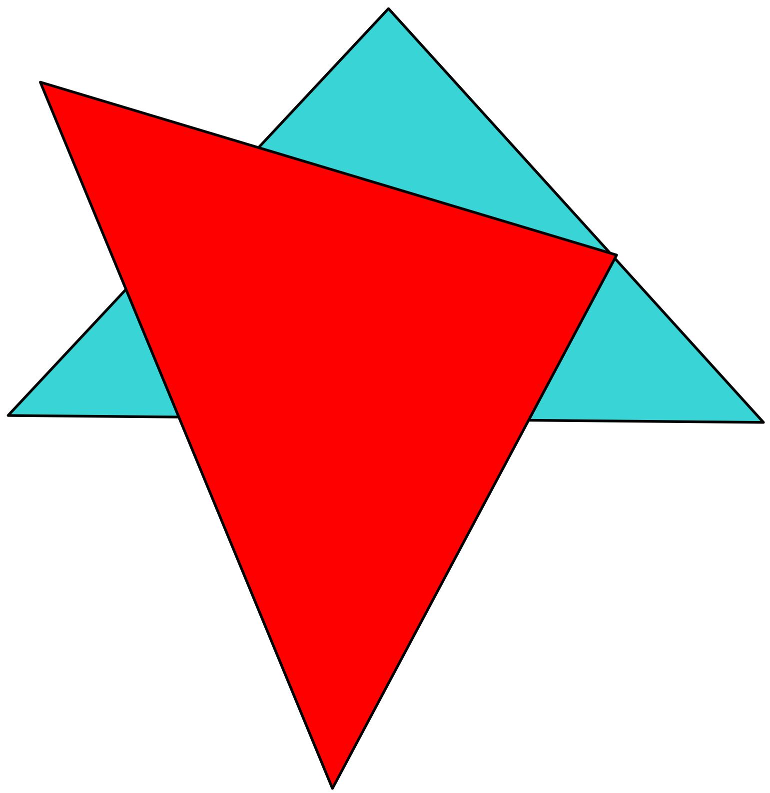
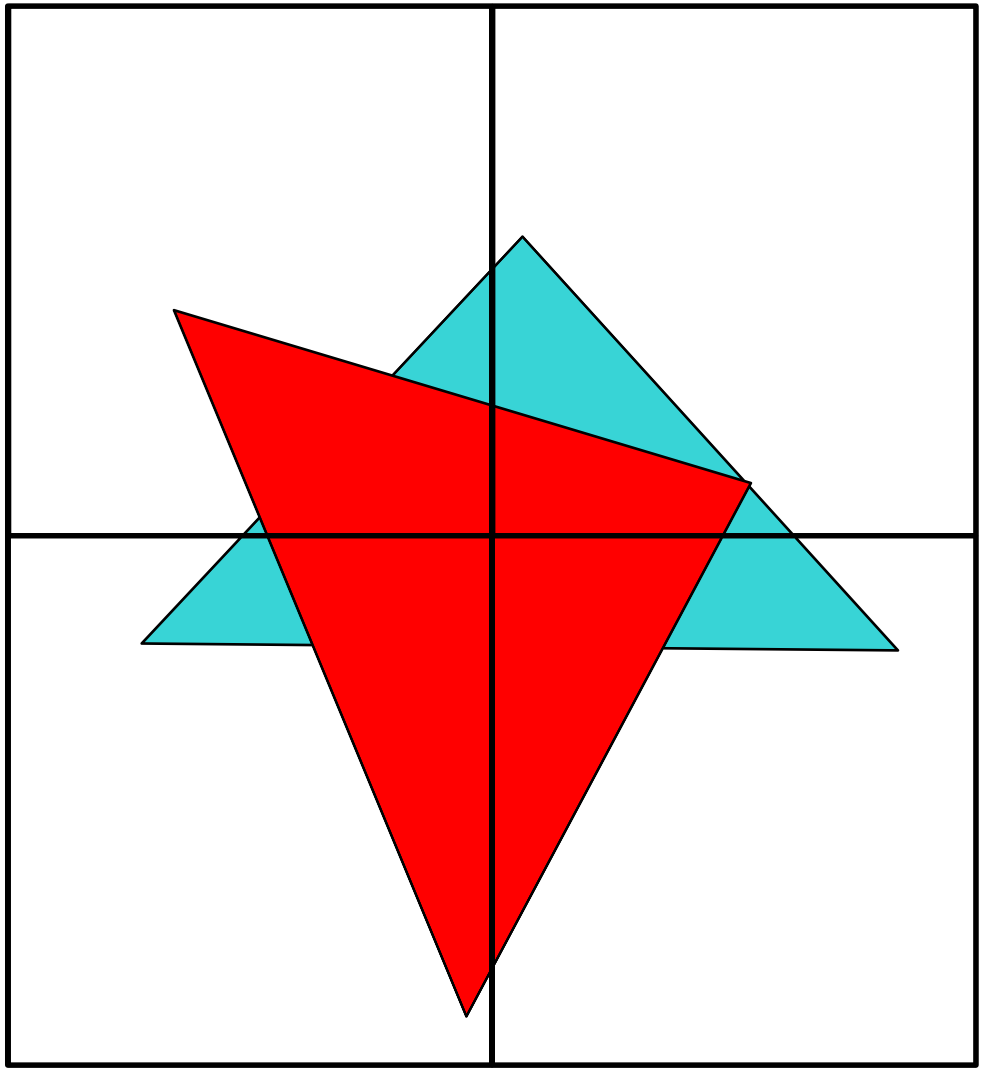


Quad and Octrees

- Triangles may not be contained within a quadrant or octant
- Triangles must be referenced in all overlapping cells or *split* at the border into smaller ones
- Can drastically increase memory consumption!



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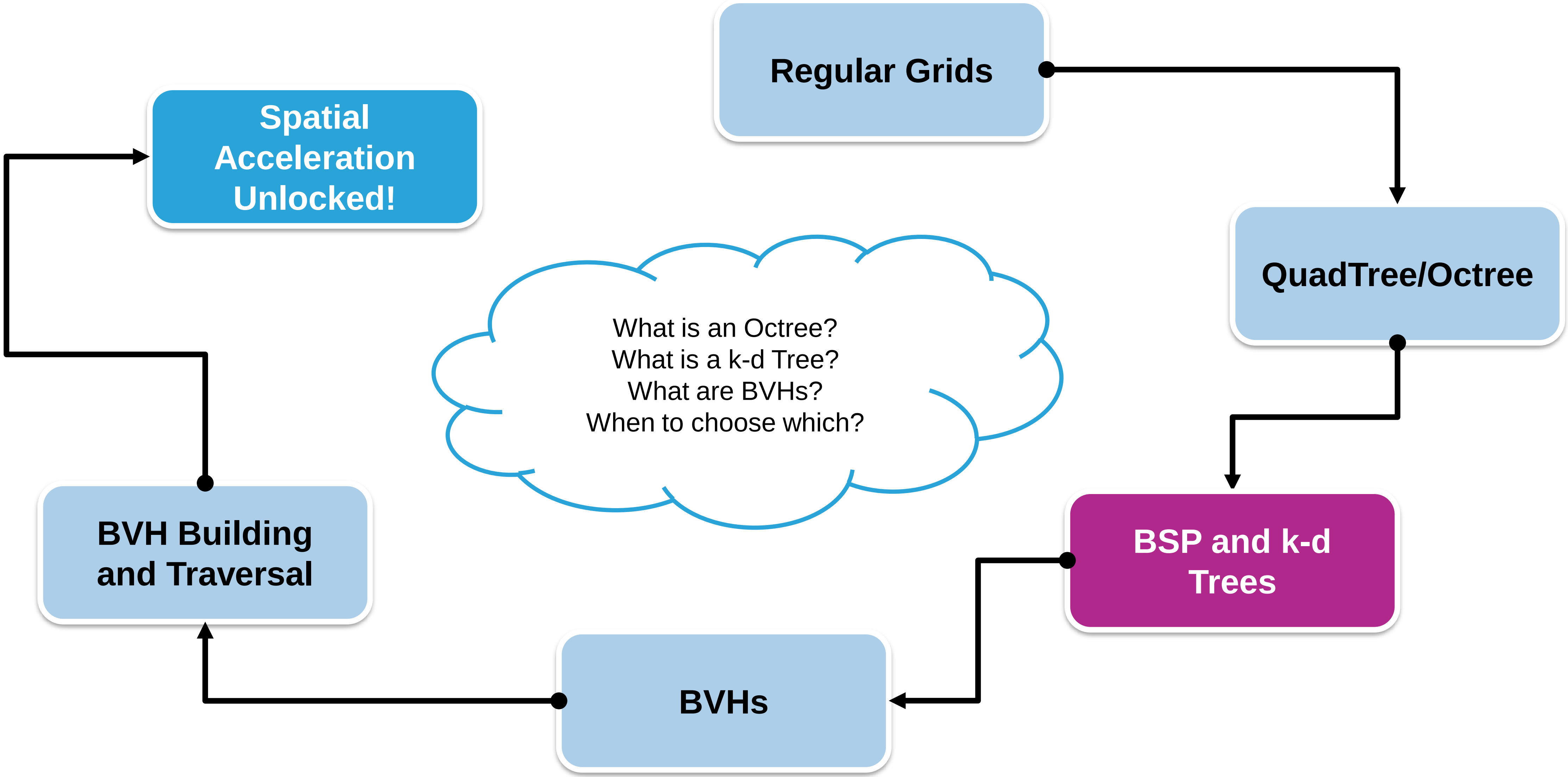
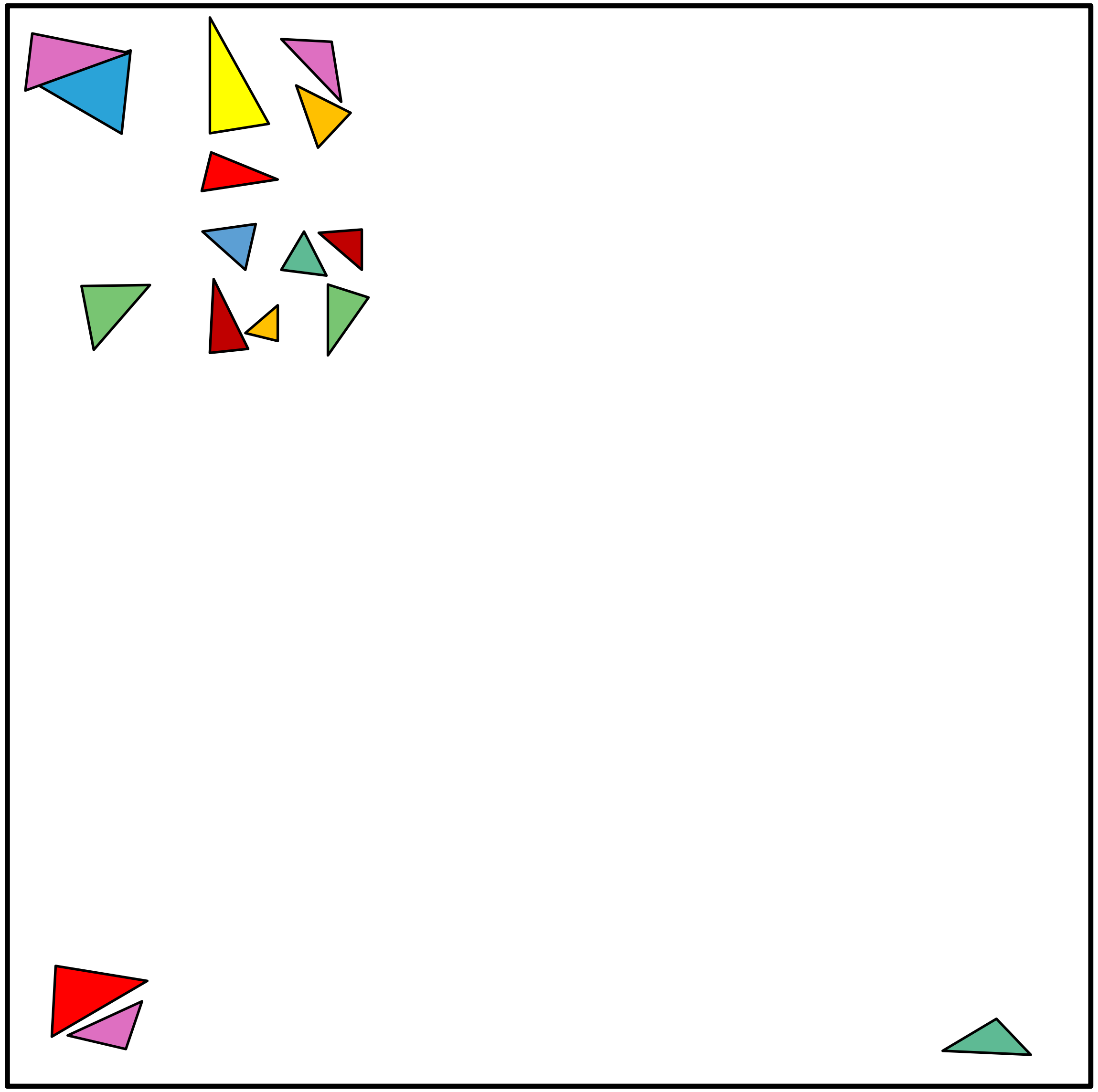


BSP Trees & k-d Trees

- Binary Space Partition Tree
  - Recursive split via *hyperplanes*
  - Left/right child nodes treat objects in each *half-space*
  - Splits can be arbitrary!



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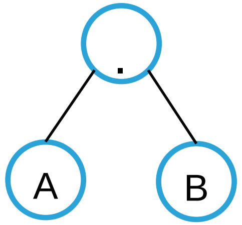

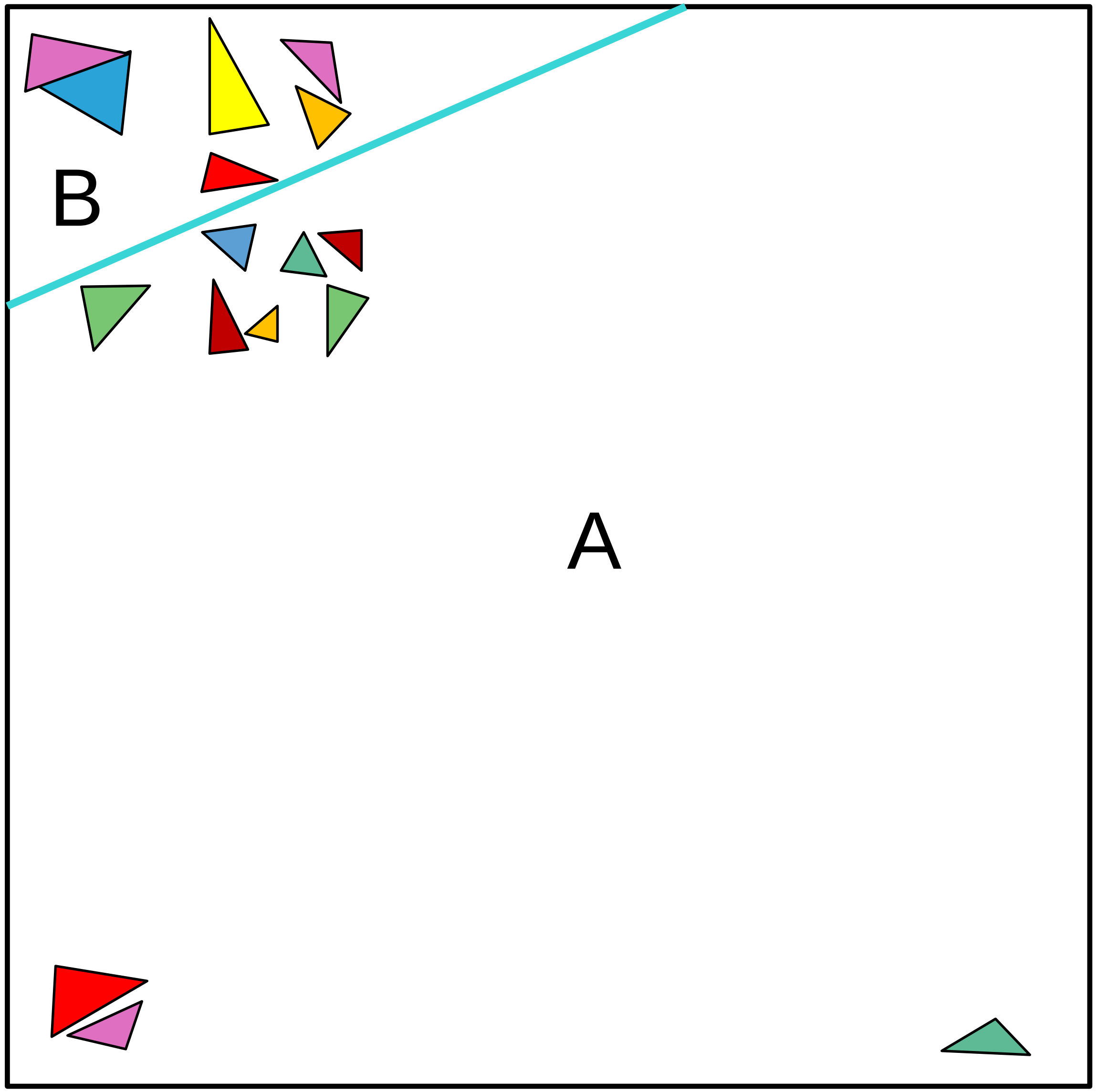

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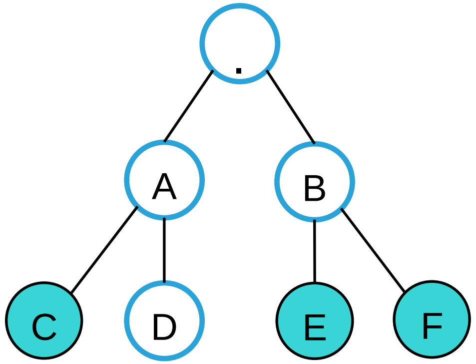

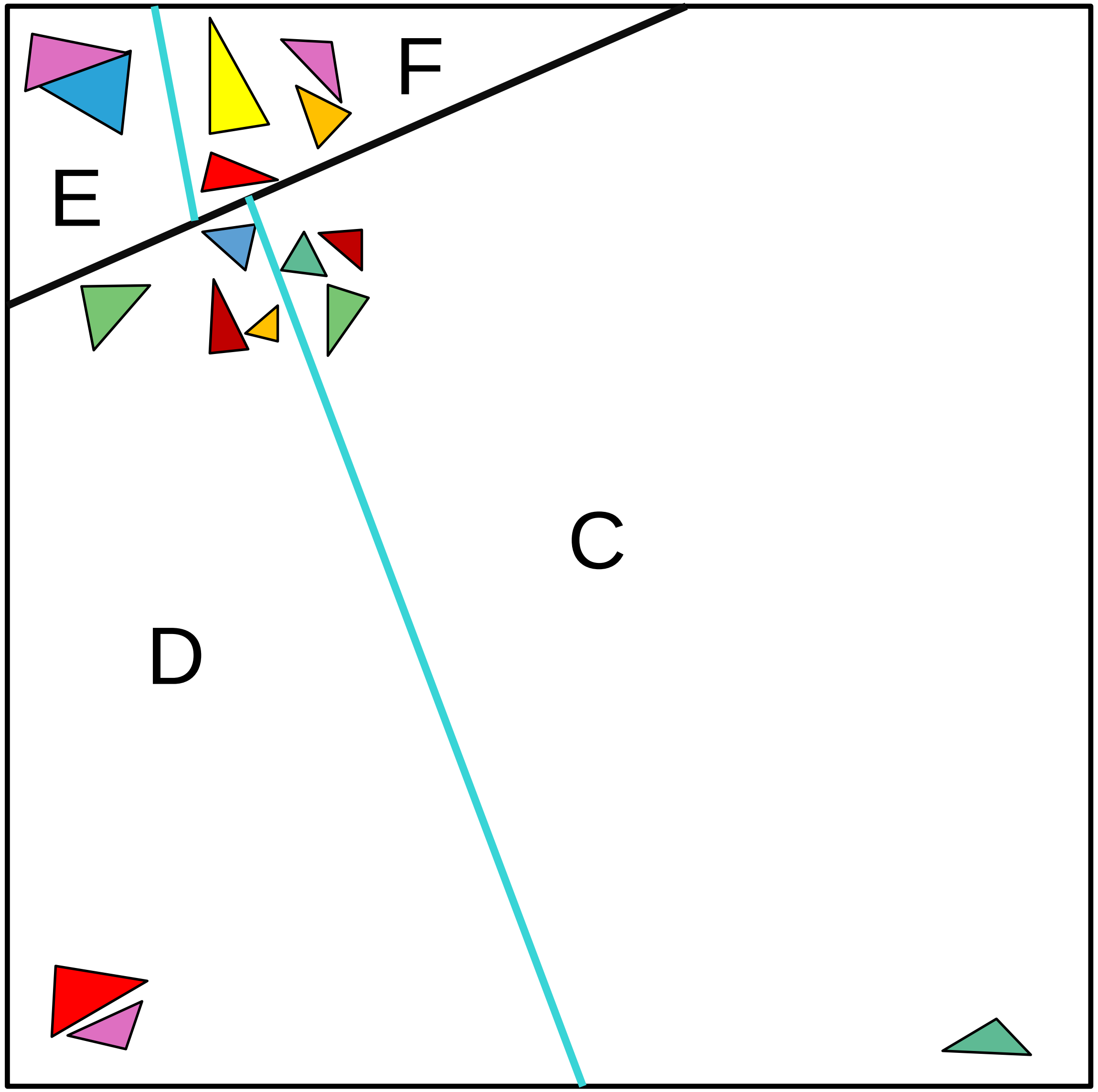

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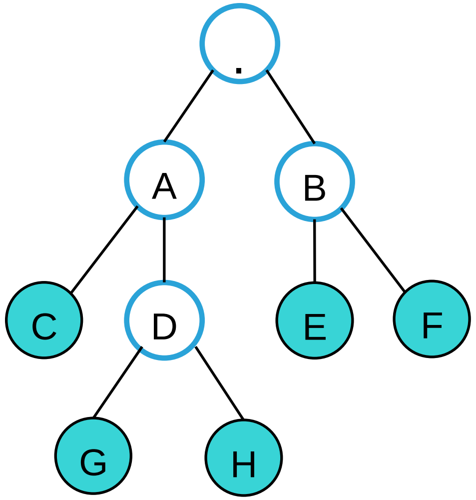

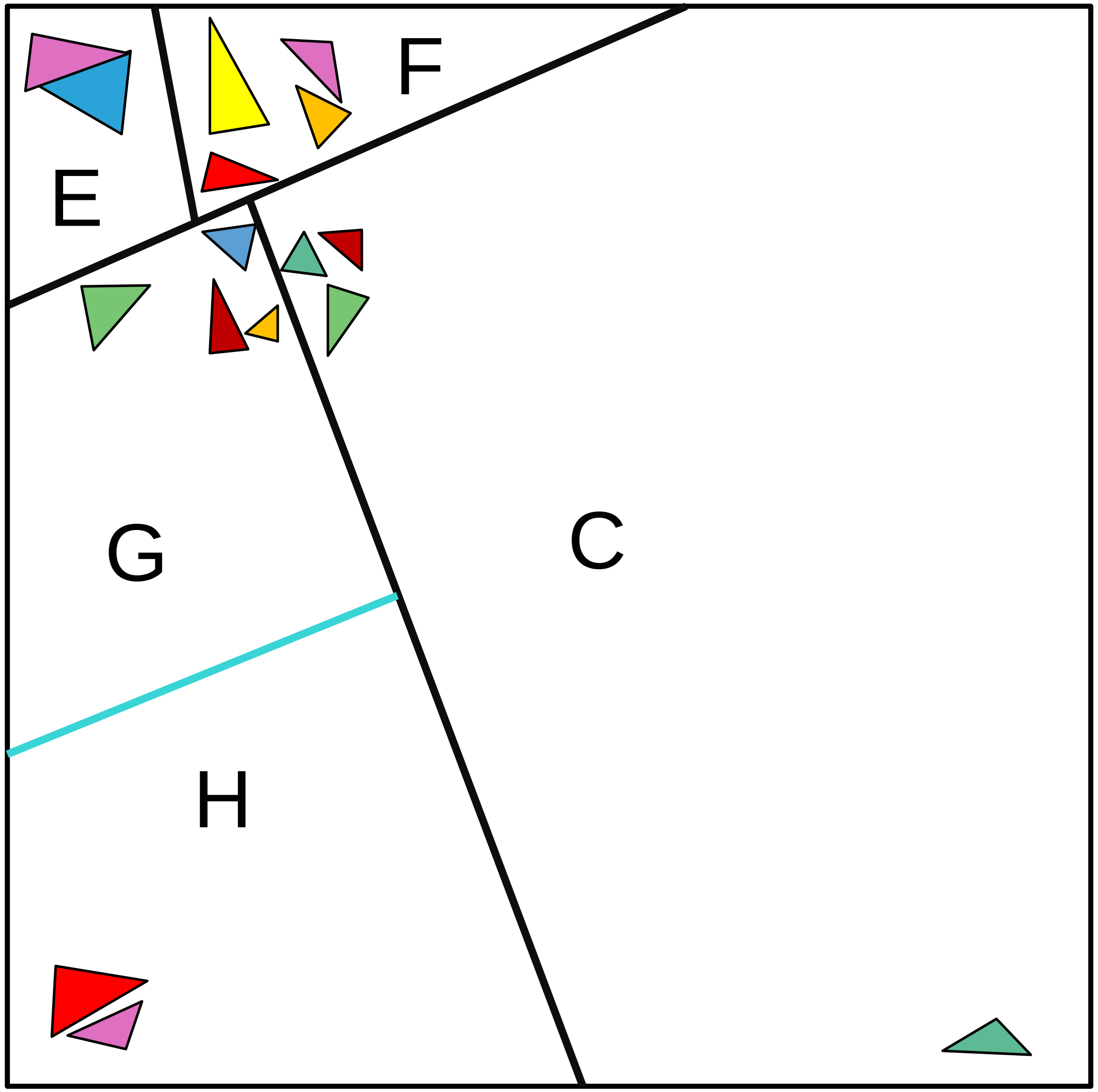

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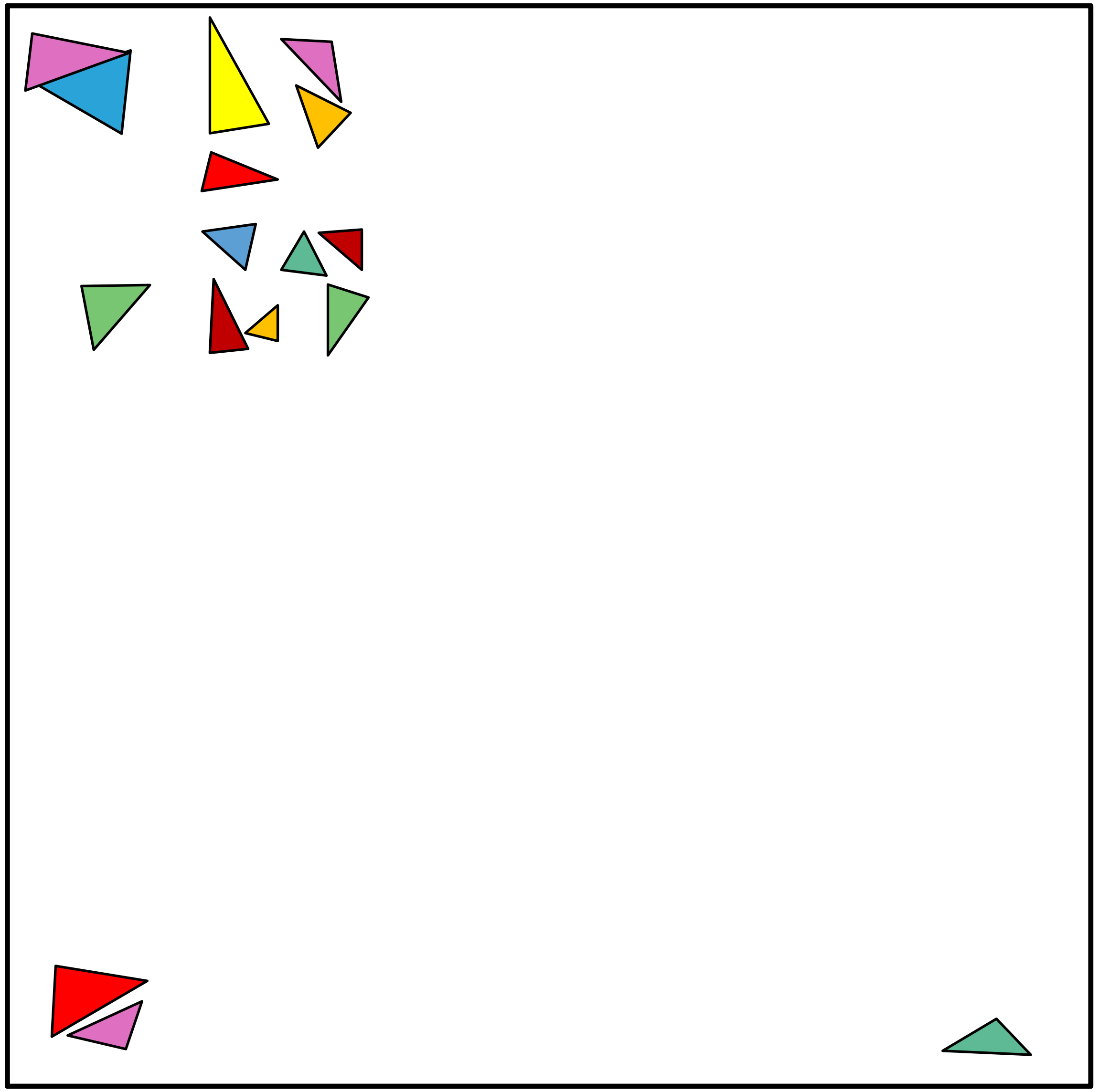
  - Every hyperplane must be perpendicular to a base axis
  - Limits search space for splits



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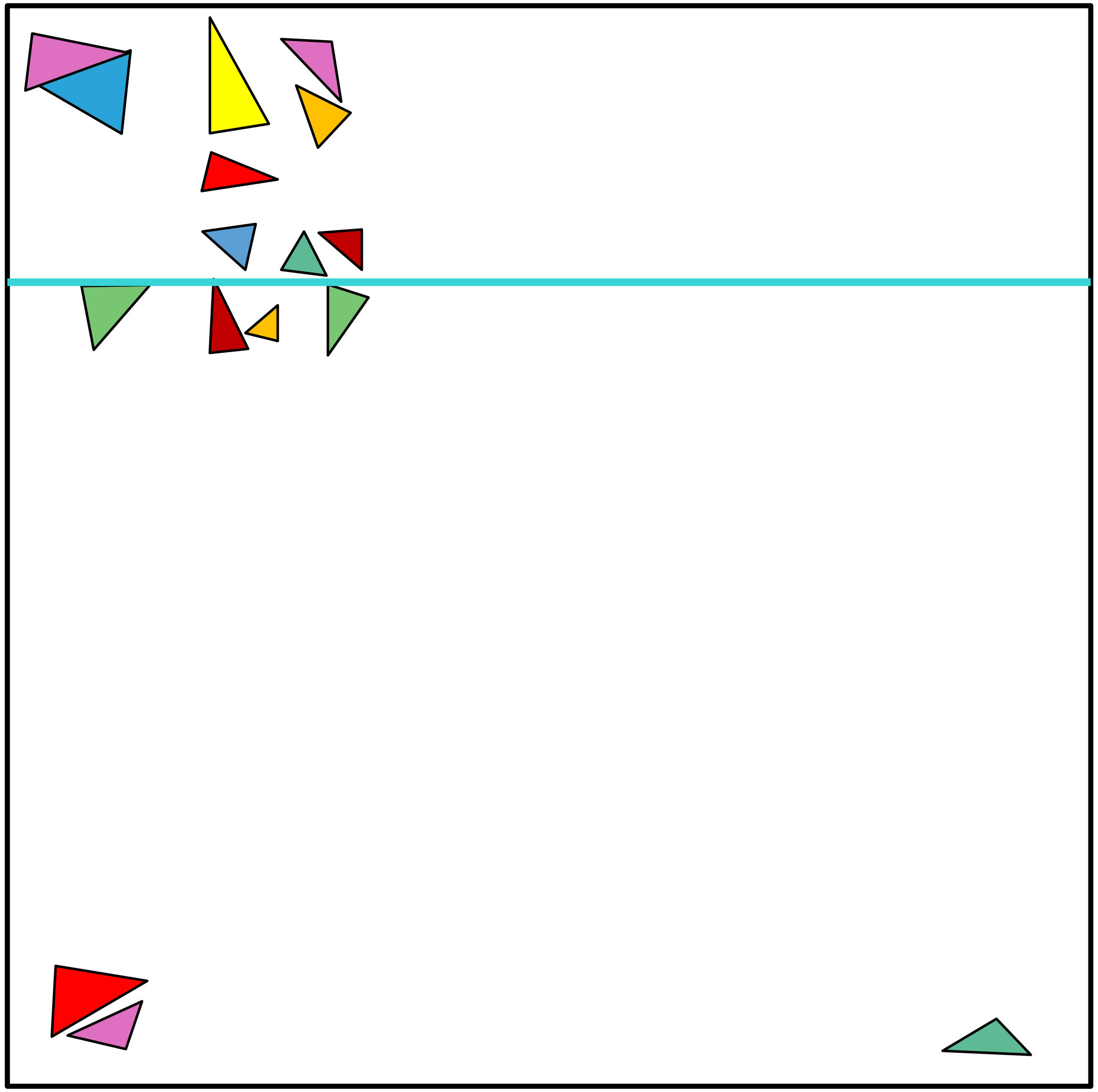

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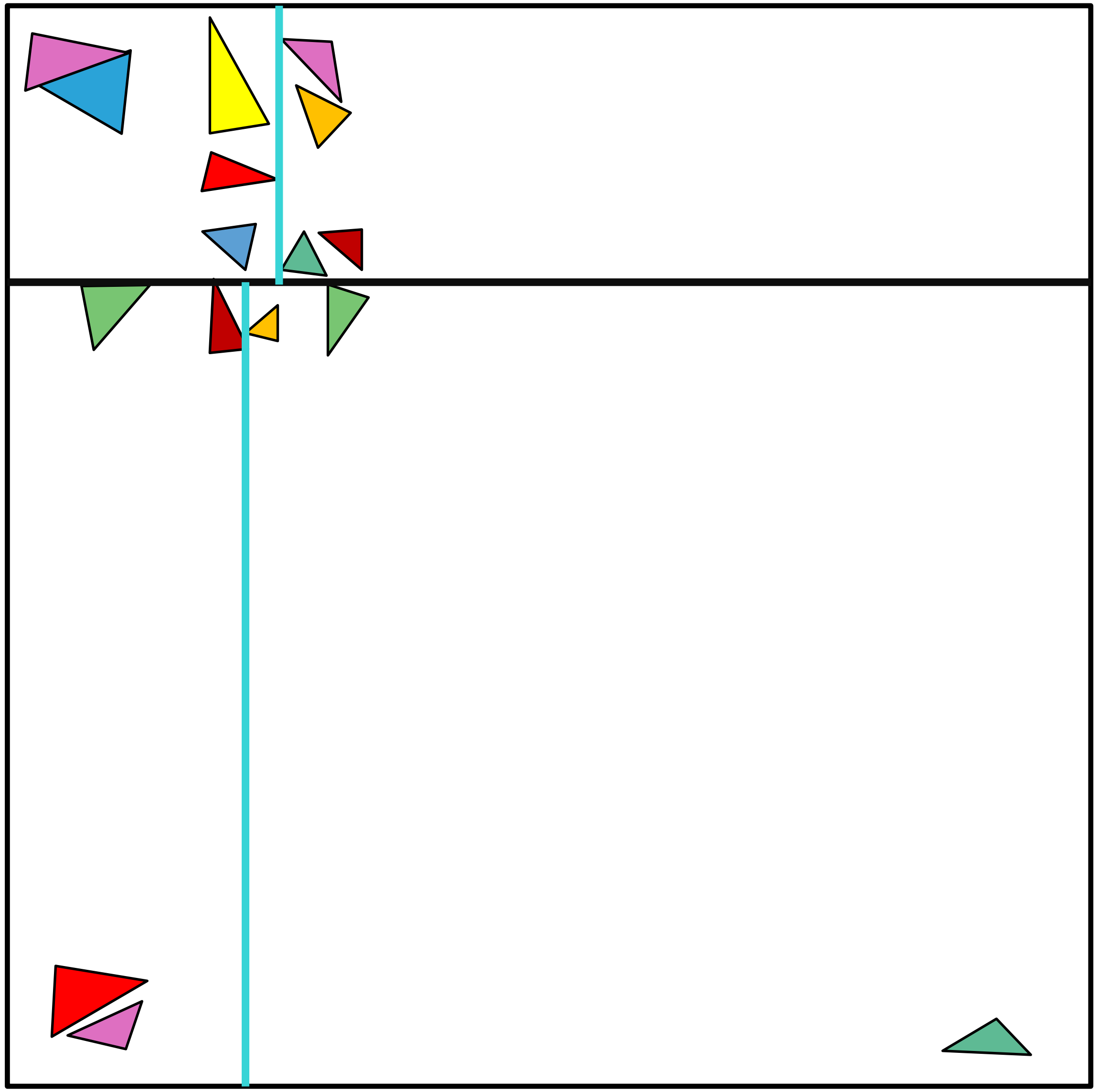

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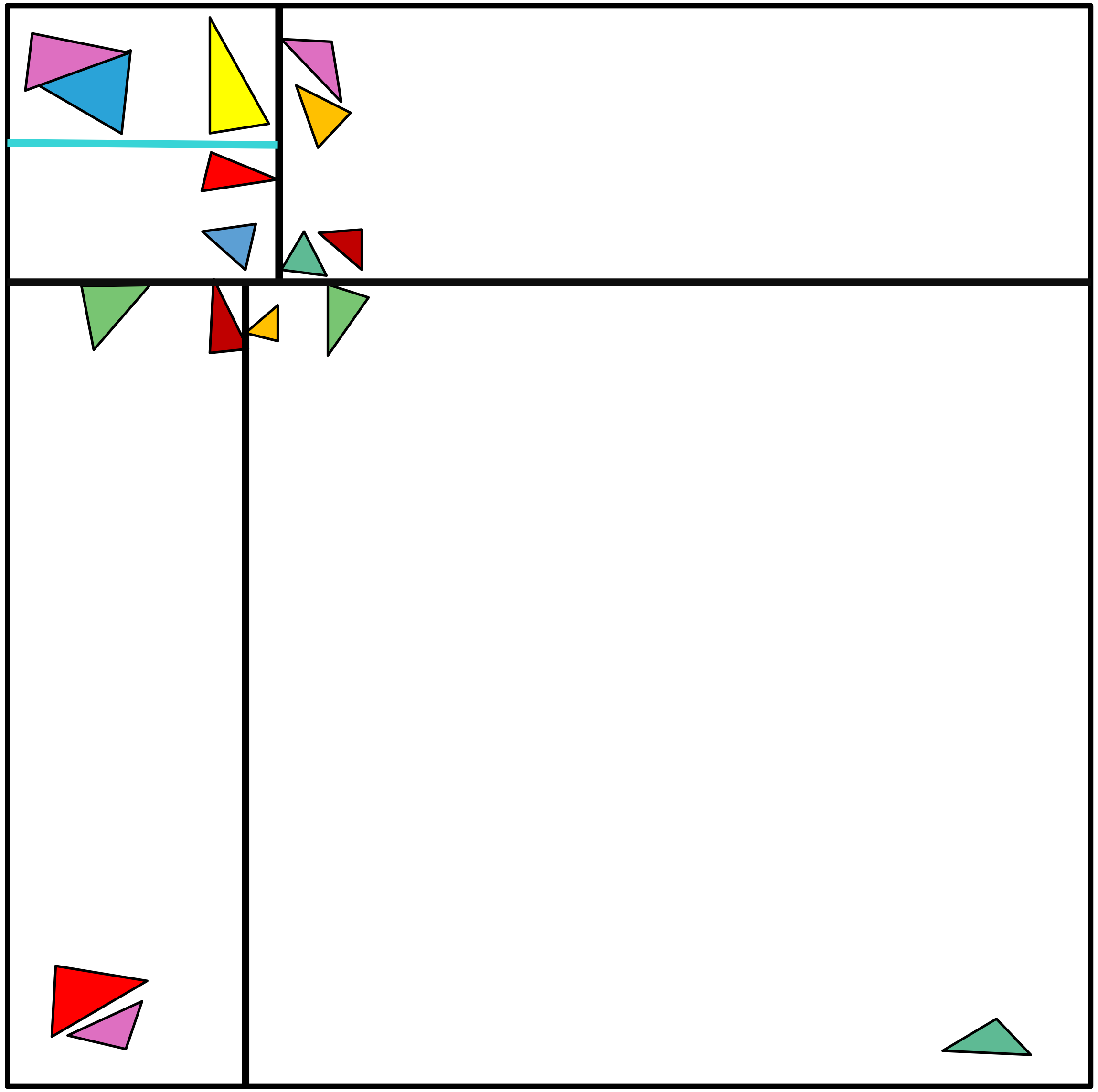

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k-d Tree	low – high (overlap)	low – high	good – excellent

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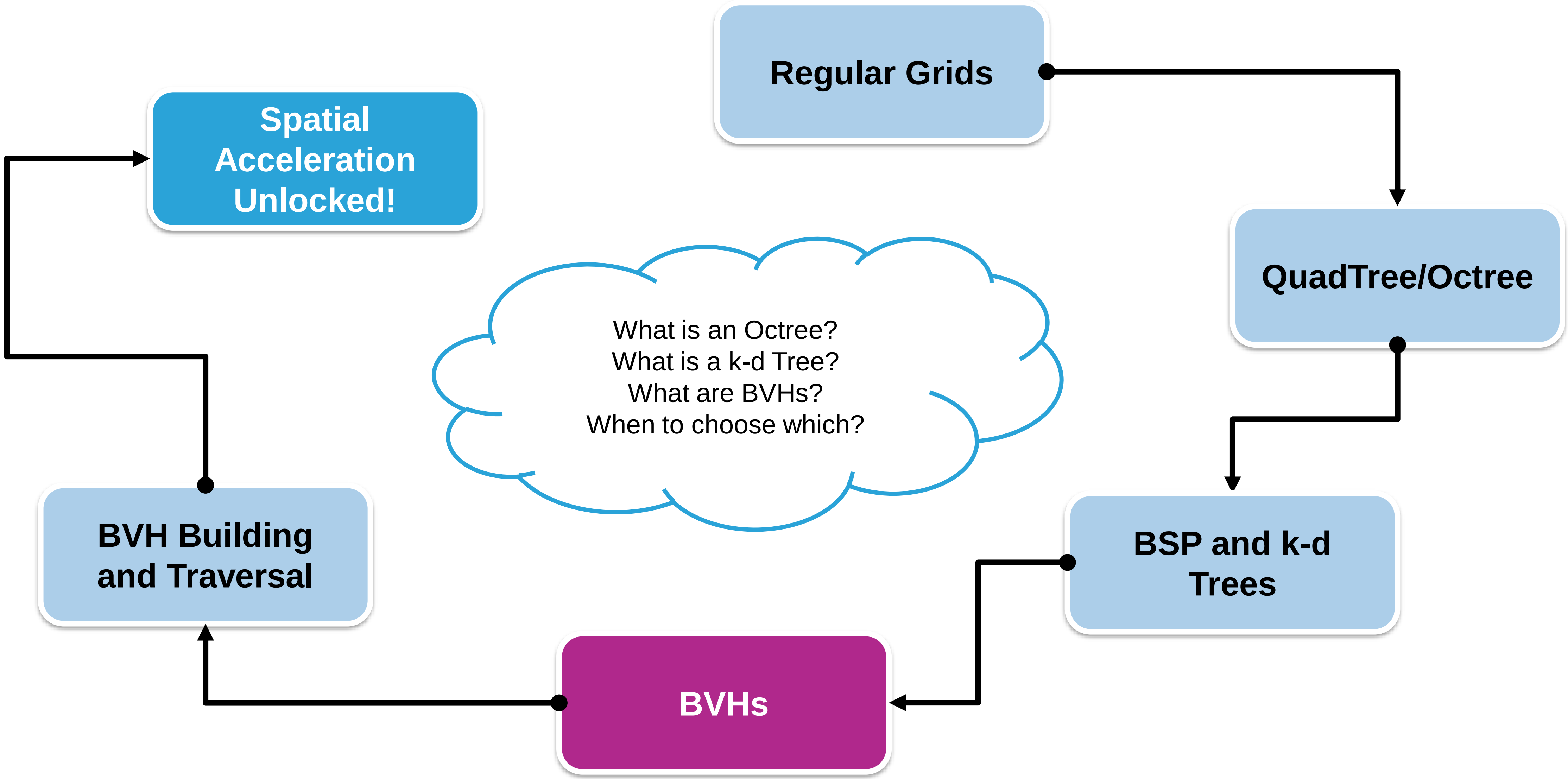
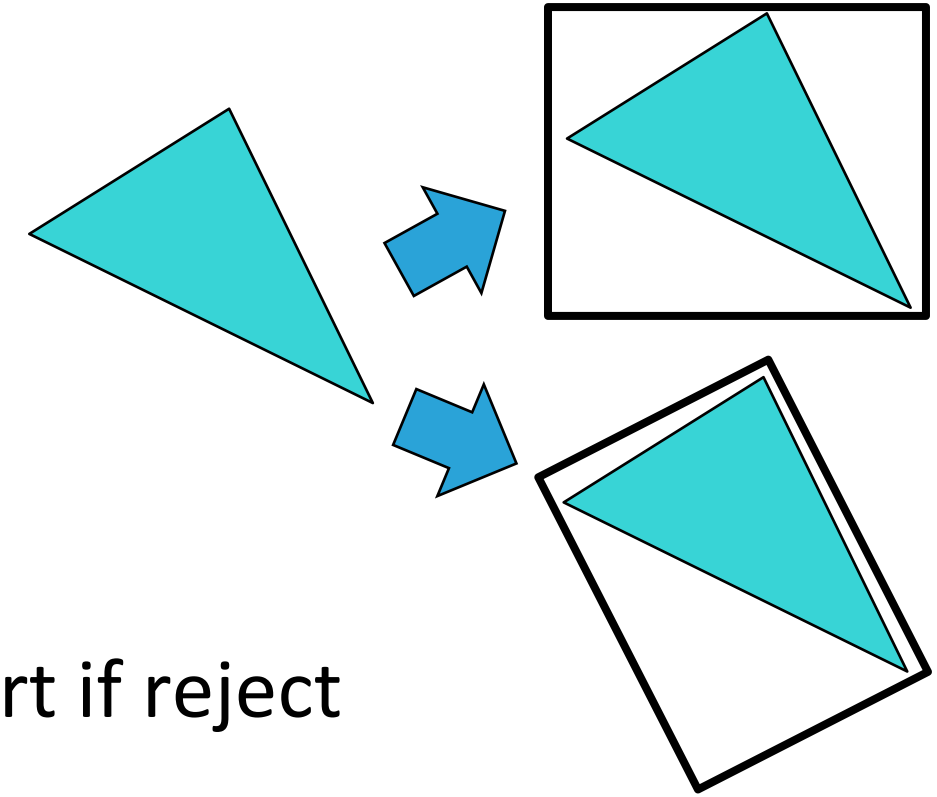


Bounding Volumes

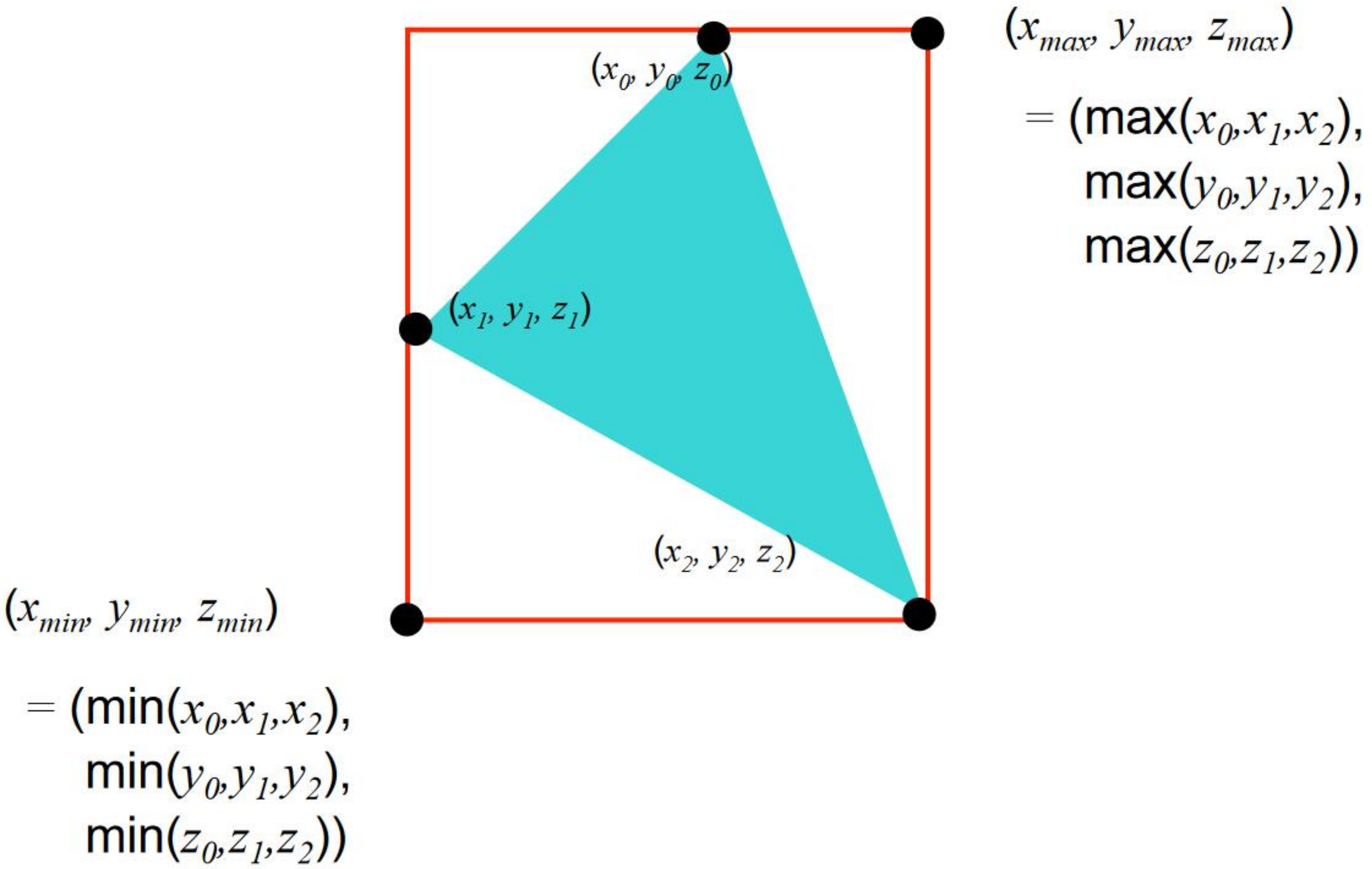
- Find enclosing ("conservative") volumes that are easier to test
- Ideally: tight, but easy to check for intersection with ray
- Common choices:
  - Bounding Spheres
  - Bounding Boxes
    - Axis-aligned (AABB)
    - Oriented (OBB)
- Saves on computational effort if reject



Axis-Aligned Bounding Boxes (AABBs)

- AABBs are defined by their two extrema (min/max)

- Linear run time to compute
  - Iterate over all vertices
  - Keep min/max values for each dimension
  - Done!

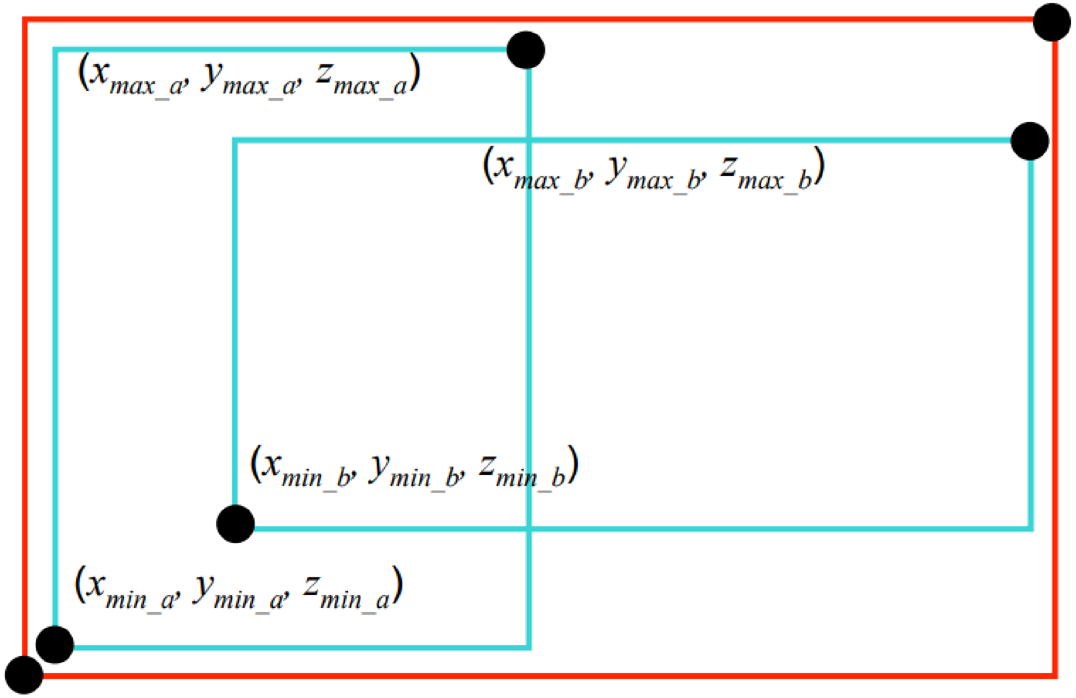


Merging AABBs

- Find the AABB that encloses multiple, smaller AABBs

- Operates only on extrema of each smaller AABB

- Merging process is commutative

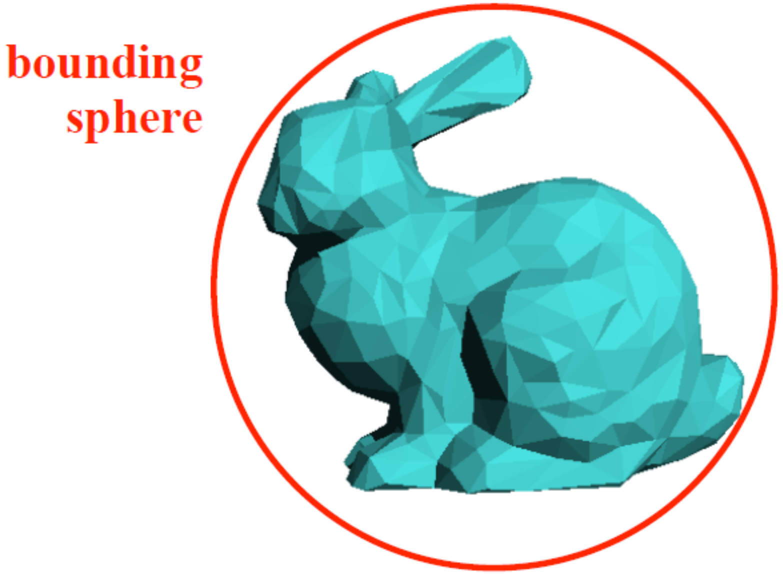


$$(x_{max}, y_{max}, z_{max}) = (max(x_{max\_a}, x_{max\_b}), max(y_{max\_a}, y_{max\_b}), max(z_{max\_a}, z_{max\_b}))$$

$$(x_{min}, y_{min}, z_{min}) = (min(x_{min\_a}, x_{min\_b}), min(y_{min\_a}, y_{min\_b}), min(z_{min\_a}, z_{min\_b}))$$

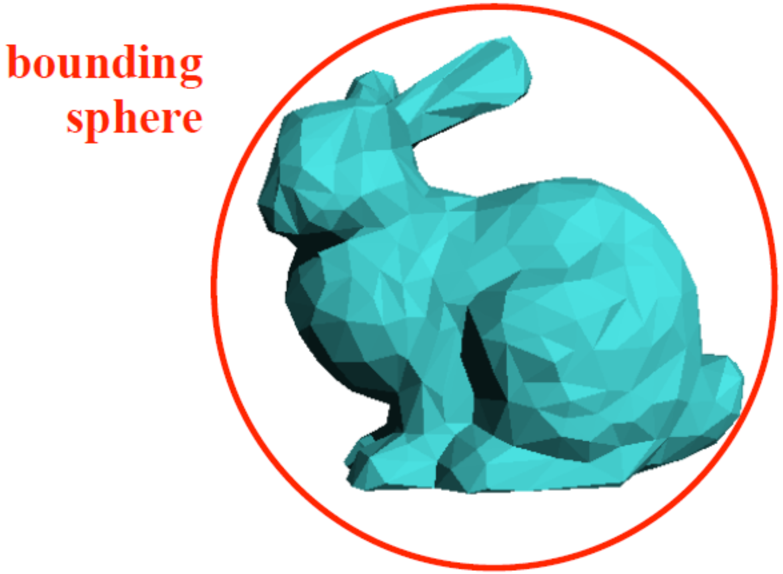
Bounding Spheres

- Bounding spheres need a center $\vec{c}$ and a radius r
- For $\vec{c}$, can pick the mean vertex position or center of AABB
- Once center is chosen, find vertex position $\vec{v}_{max}$ farthest from it
- $r = |\vec{c} - \vec{v}_{max}|$



How to Use Bounding Volumes

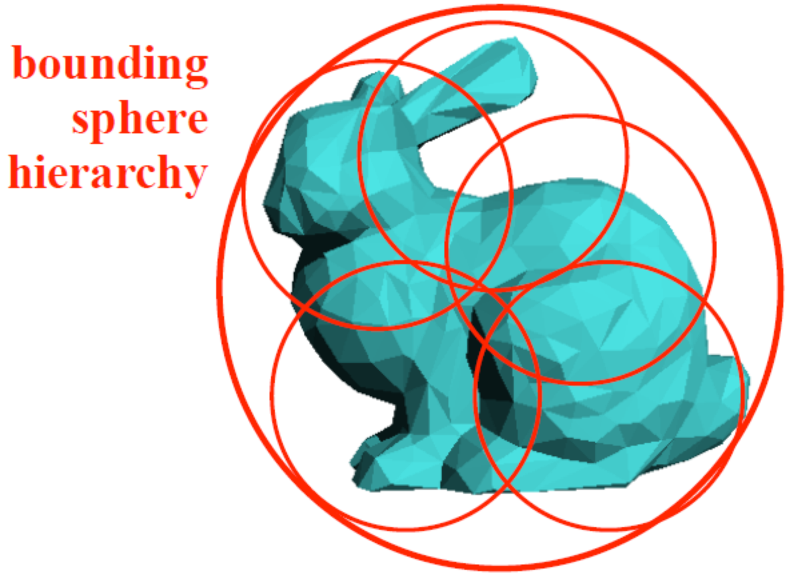
- Can also be applied to entire objects
- Reject entire object if volume is not hit
- Good start, but what if…
  - …scene is not partitioned into objects?
  - …objects are extremely large (terrain)?
  - …objects are extremely detailed (characters)?
  - …there are millions of objects with ~ 2 triangles each (leaves)?



Bounding Volume Hierarchy (BVH)

- Each node of the hierarchy has its own bounding volume

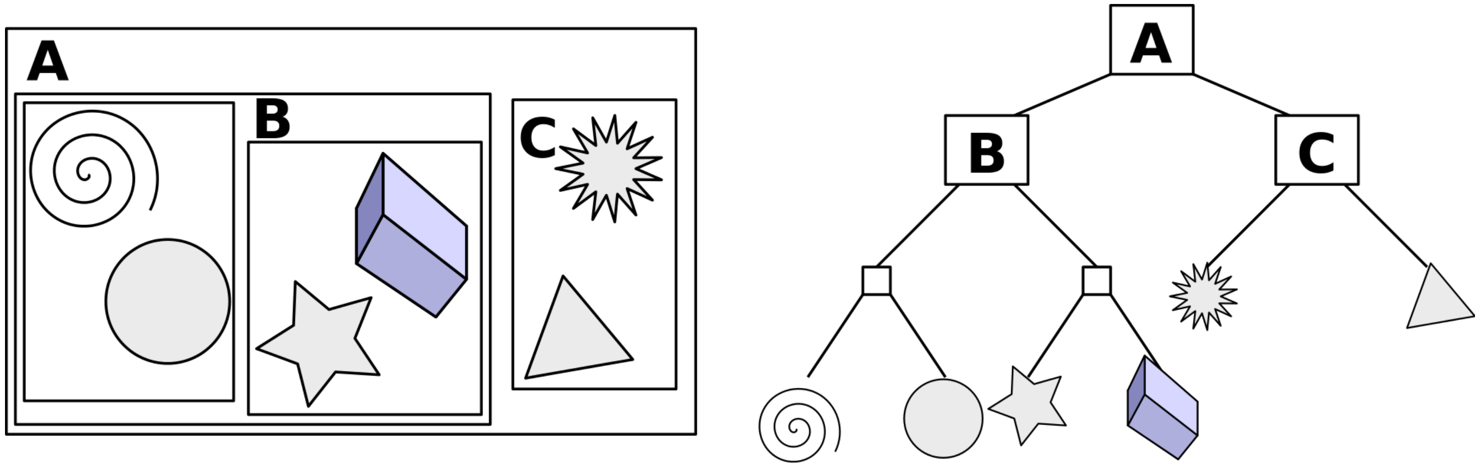
- Every node can be
  - An inner node: references child nodes
  - A leaf node: references triangles



- Each node's bounding volume is a subset of its parent's bounding volume (i.e., child nodes are spatially contained by their parents)

Bounding Volume Hierarchy (BVH)

- The final hierarchy is (again) a tree structure with N leaf nodes
- Leaf nodes can be
  - Individual triangles
  - Clusters (e.g., $\leq 10\Delta$)

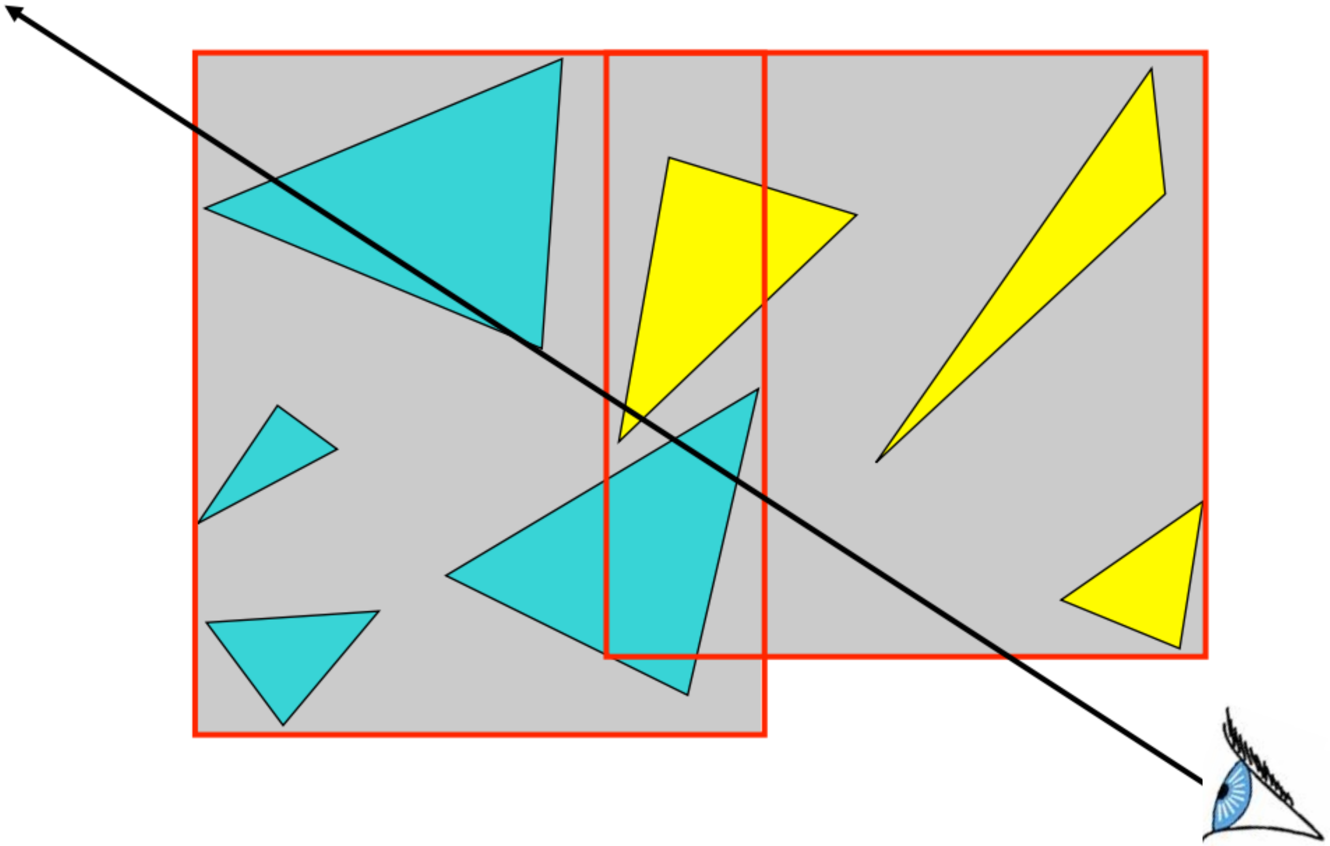


Source: Schreiberx, Wikipedia "Bounding Volume Hierarchy"

- Total number of nodes for a binary tree: $2N - 1$
  - If balanced, it takes $\sim \log N$ steps to reach a leaf from the root
  - If trees have more than 2 branches, they require fewer nodes

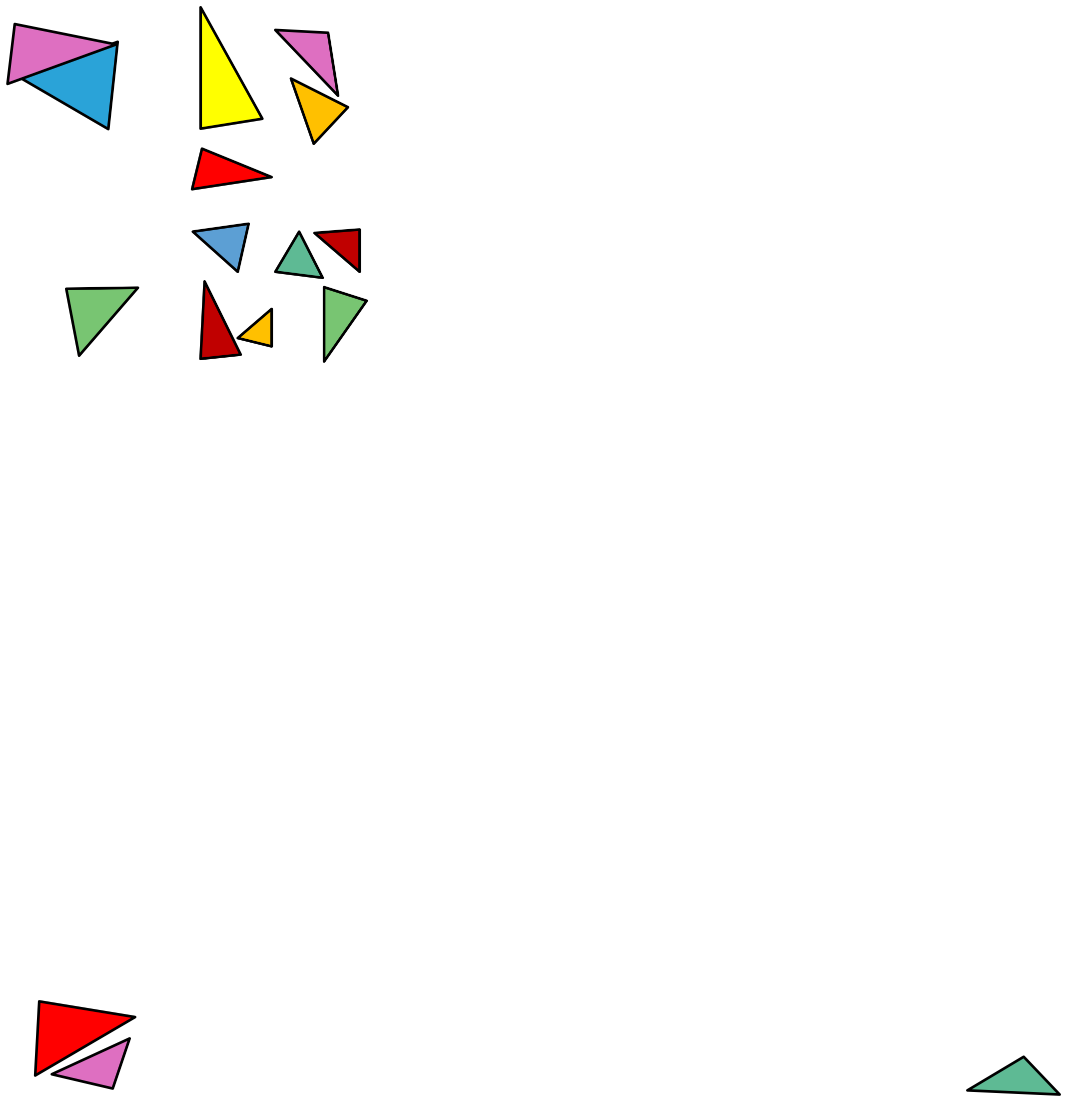
What makes BVHs special?

- Important feature: bounding volumes can ***overlap!***
- No duplicate references or split triangles necessary!
- Implicitly limits the amount of memory required



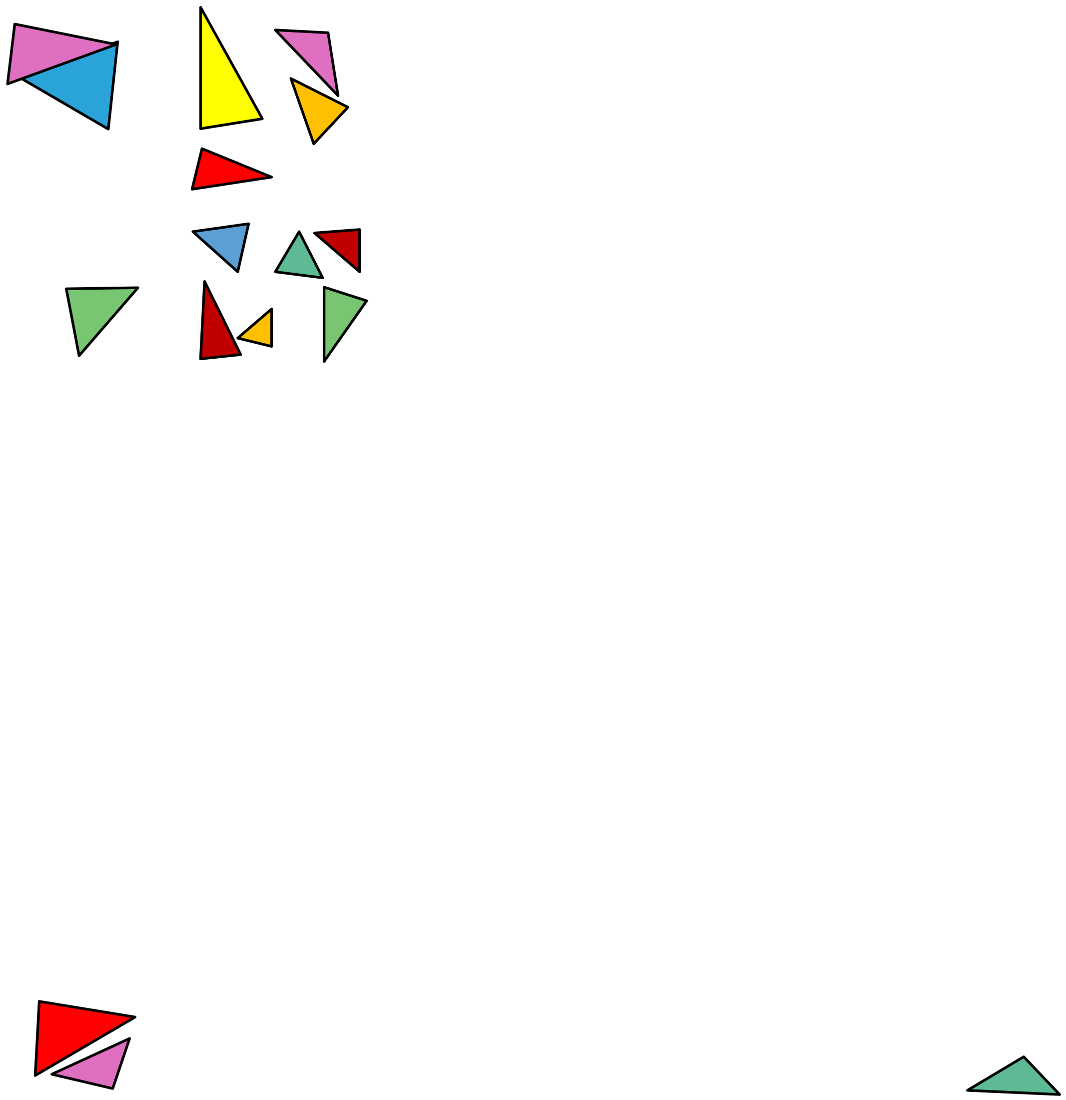
BVH Building

- Generating BVH and tree for input triangle geometry
- CPU: usually top-down
  GPU: usually bottom-up
- From here on out, we will consider box BVHs only



BVH Building, Top-Down

- Define N_{leaf} for leaves
- For each node, do the following:
  - Compute bounding box that fully encloses triangles & store
  - Holds $\leq N_{leaf}$ triangles? Stop.
  - Else, split into child groups
  - Make one new node per group
  - Set them as children of current
  - Repeat with child nodes



BVH Building, Top-Down, $N_{leaf} = 4$

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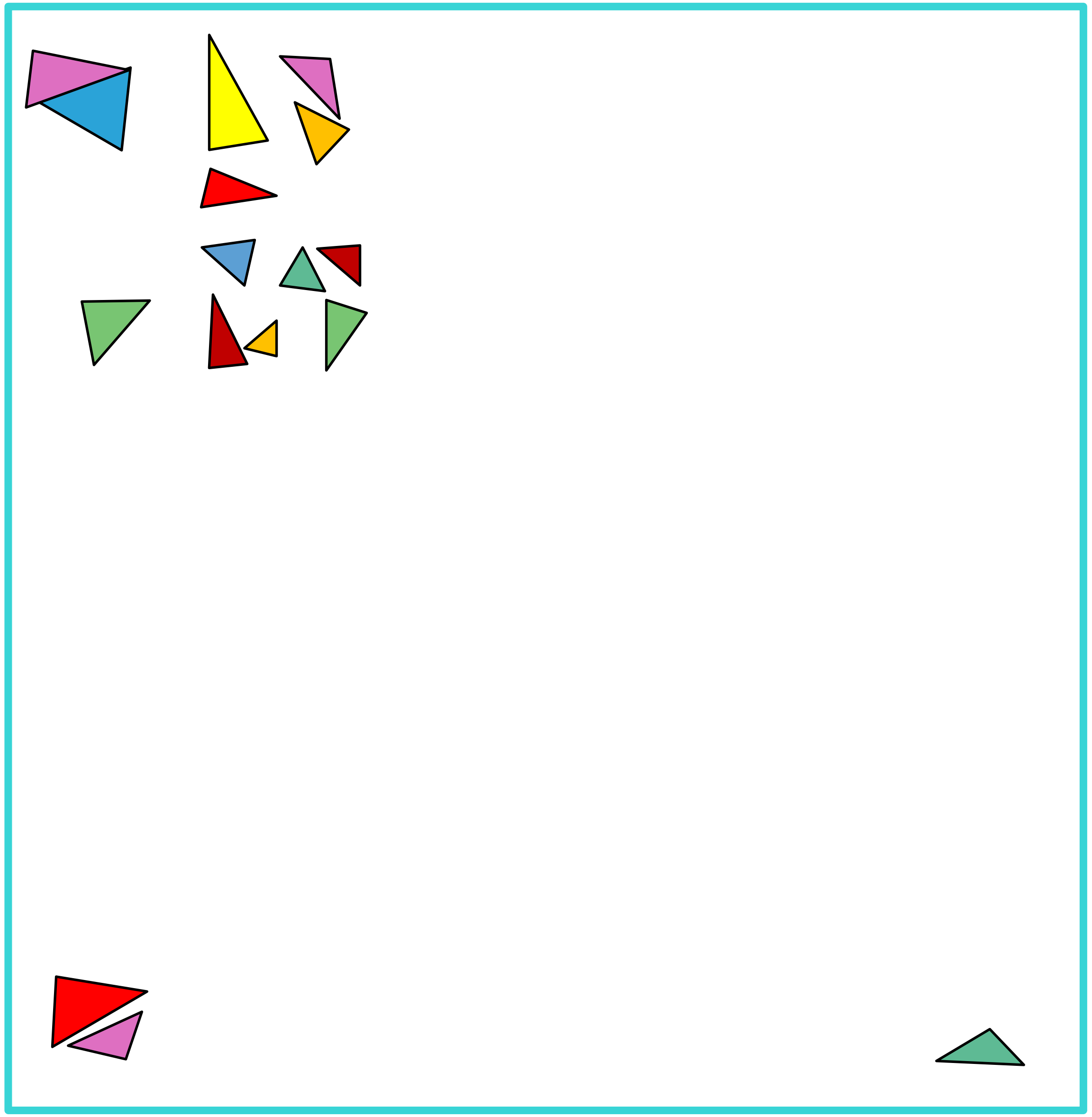

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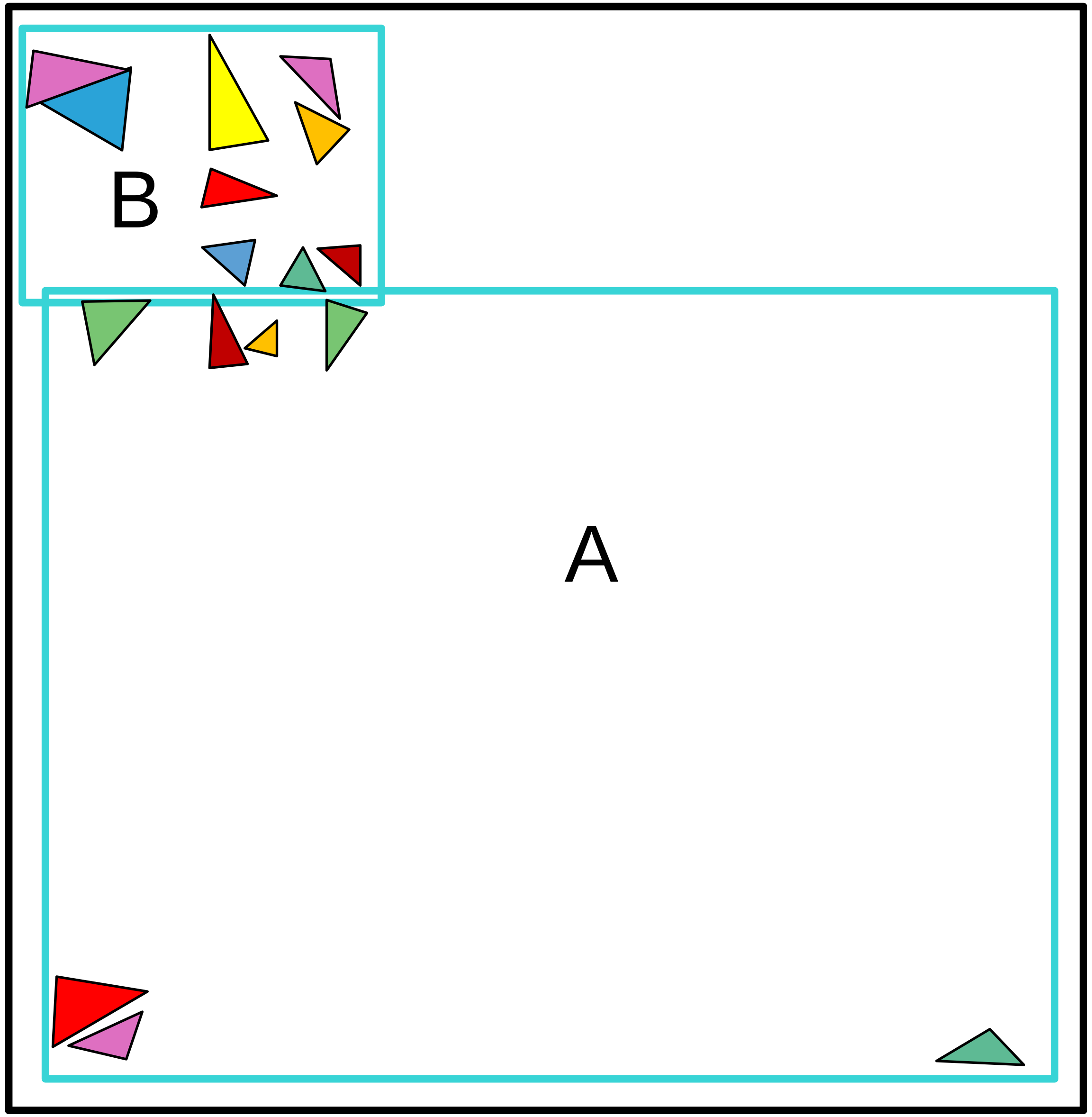

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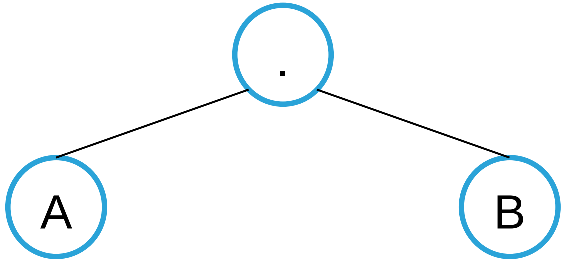

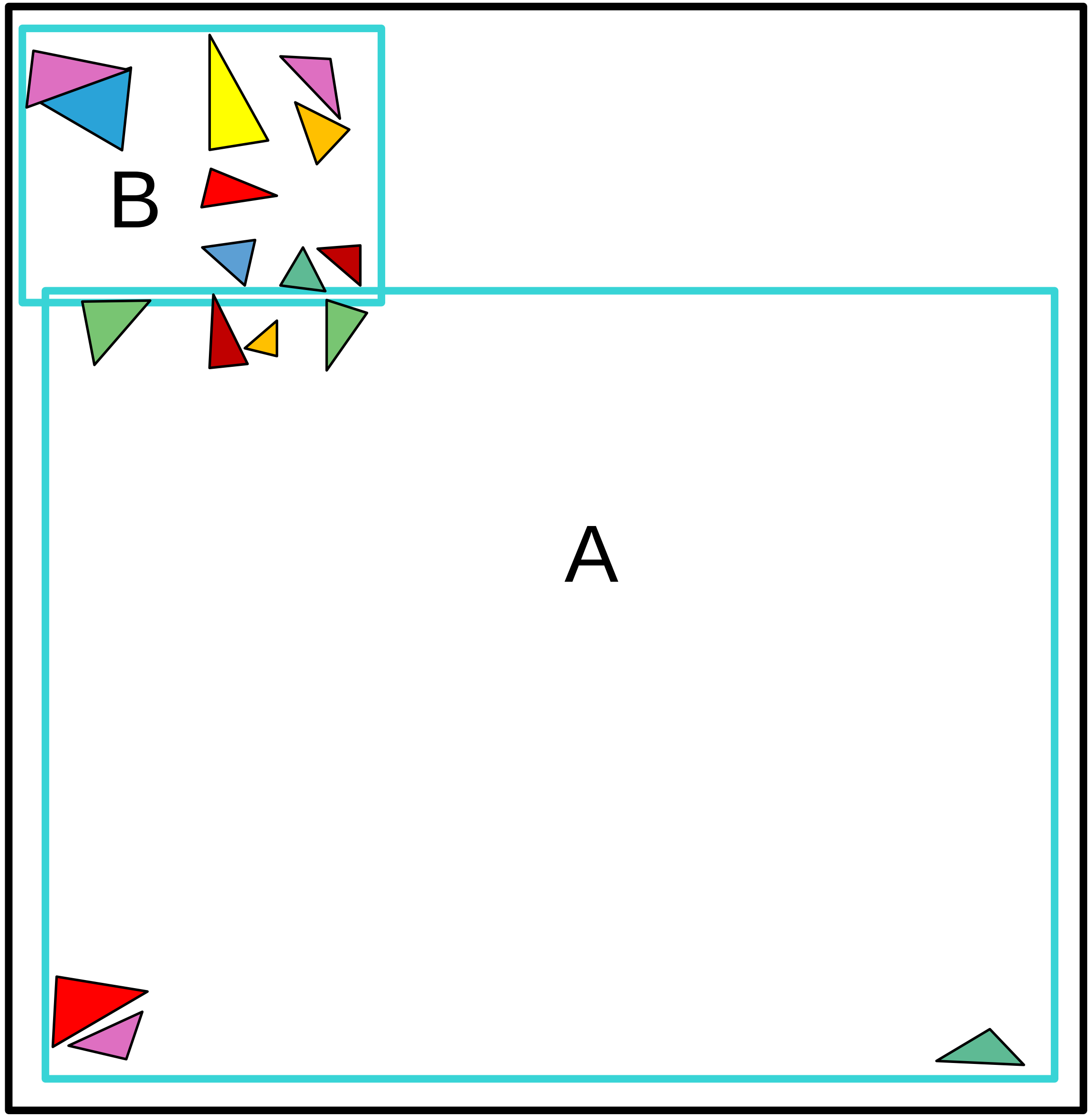

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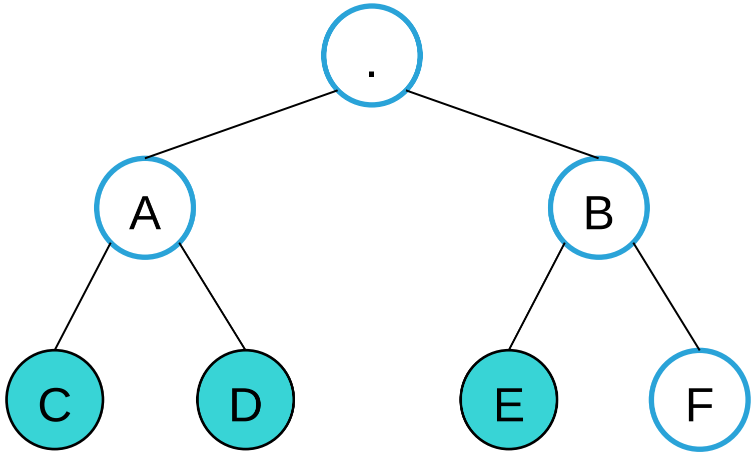

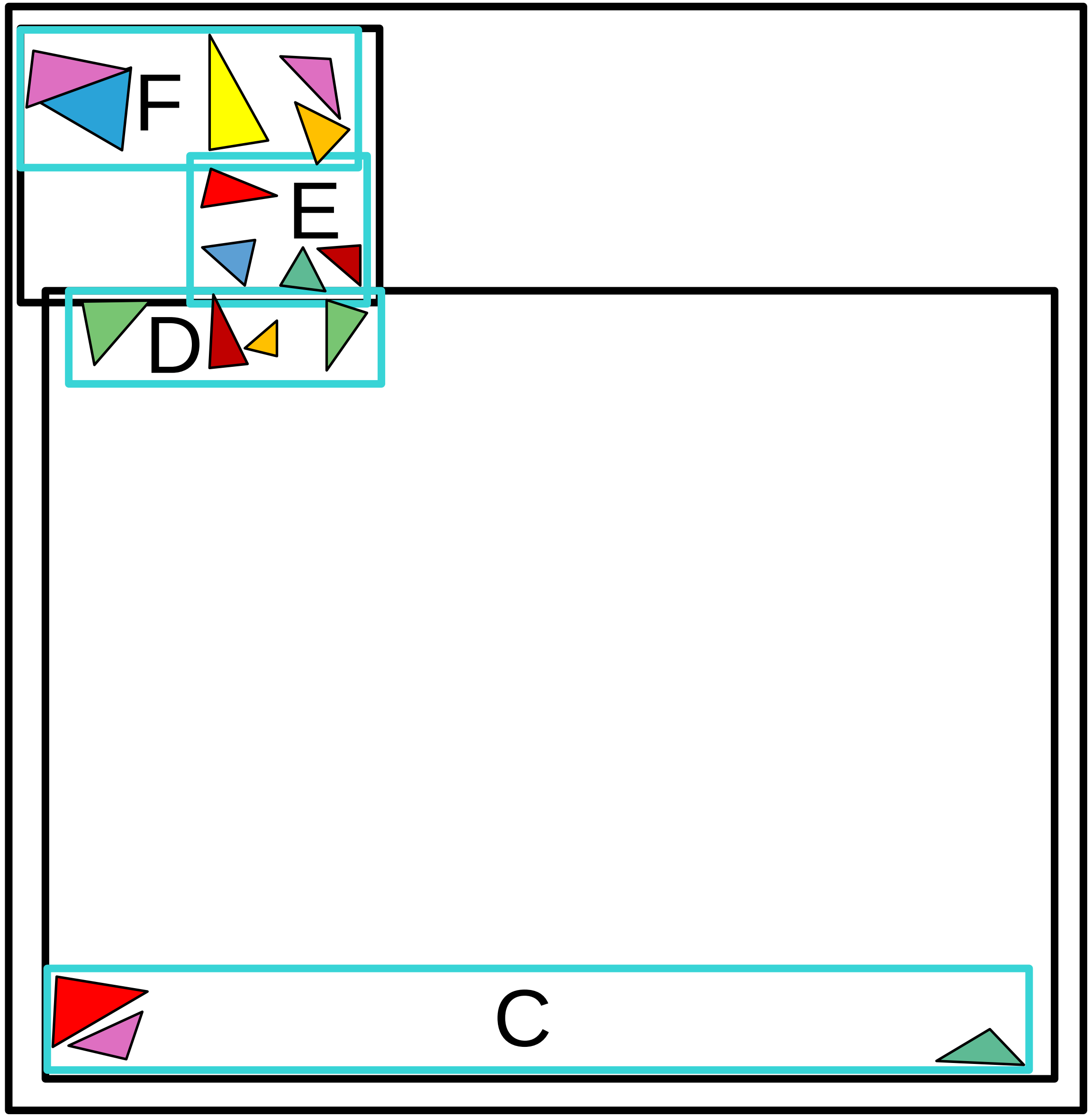

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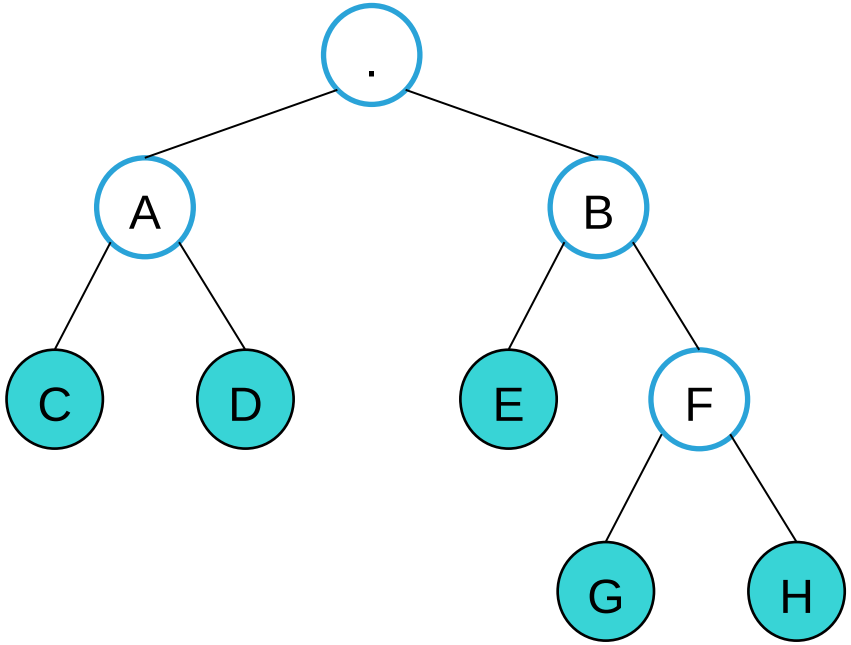
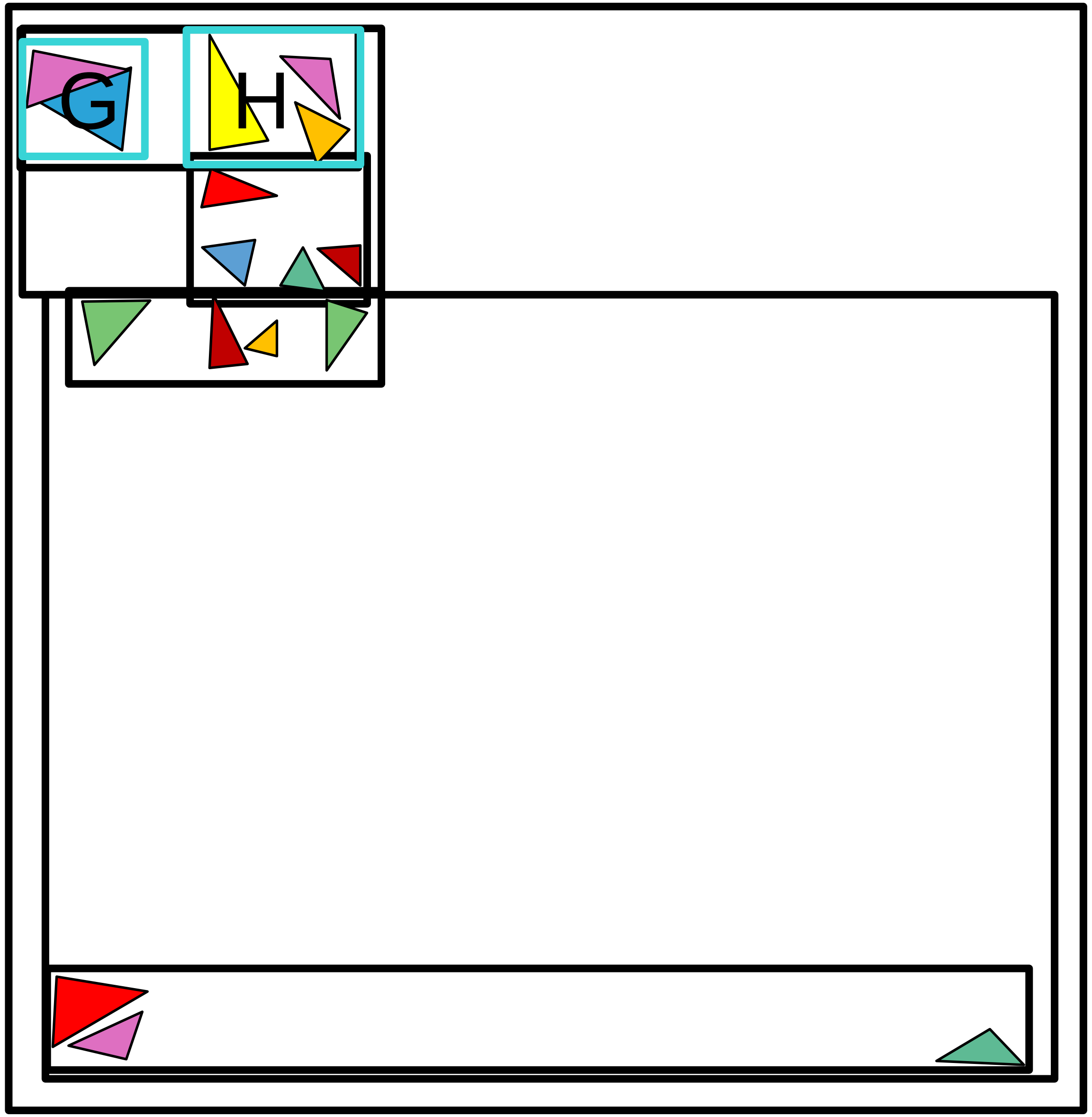
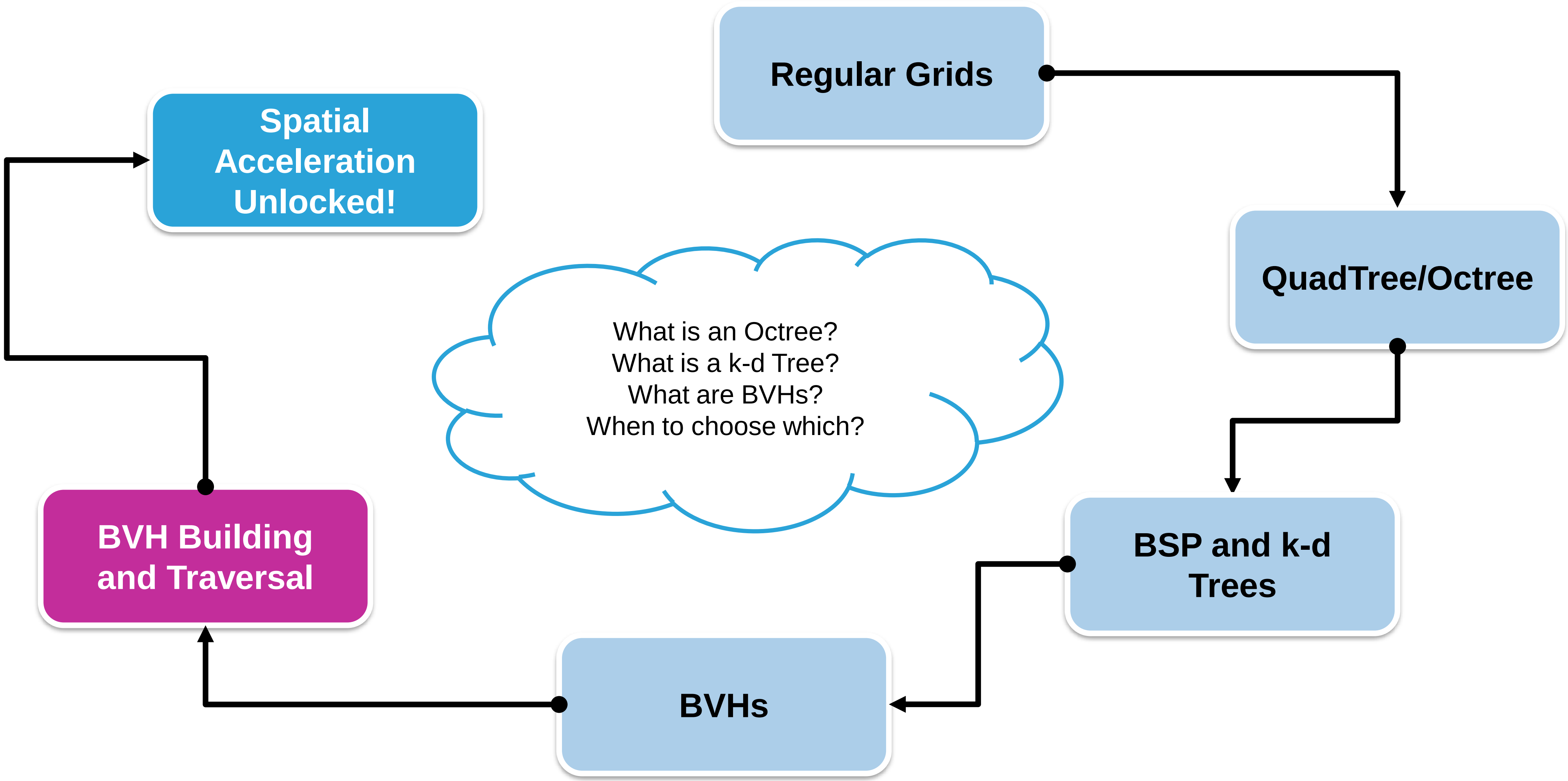




Today's Roadmap



How to split a node?

- Which axes to consider for building bounding boxes/splitting?
  - Basis vectors $(1,0,0), (0,1,0), (0,0,1)$ only
  - Oriented basis vectors only
  - Arbitrary

- Where to split?
  - Spatial median
  - Object median
  - Something more elaborate...

How to split a node?

- Which axes to consider for building bounding boxes/splitting?
  - Basis vectors (1,0,0), (0,1,0), (0,0,1) only
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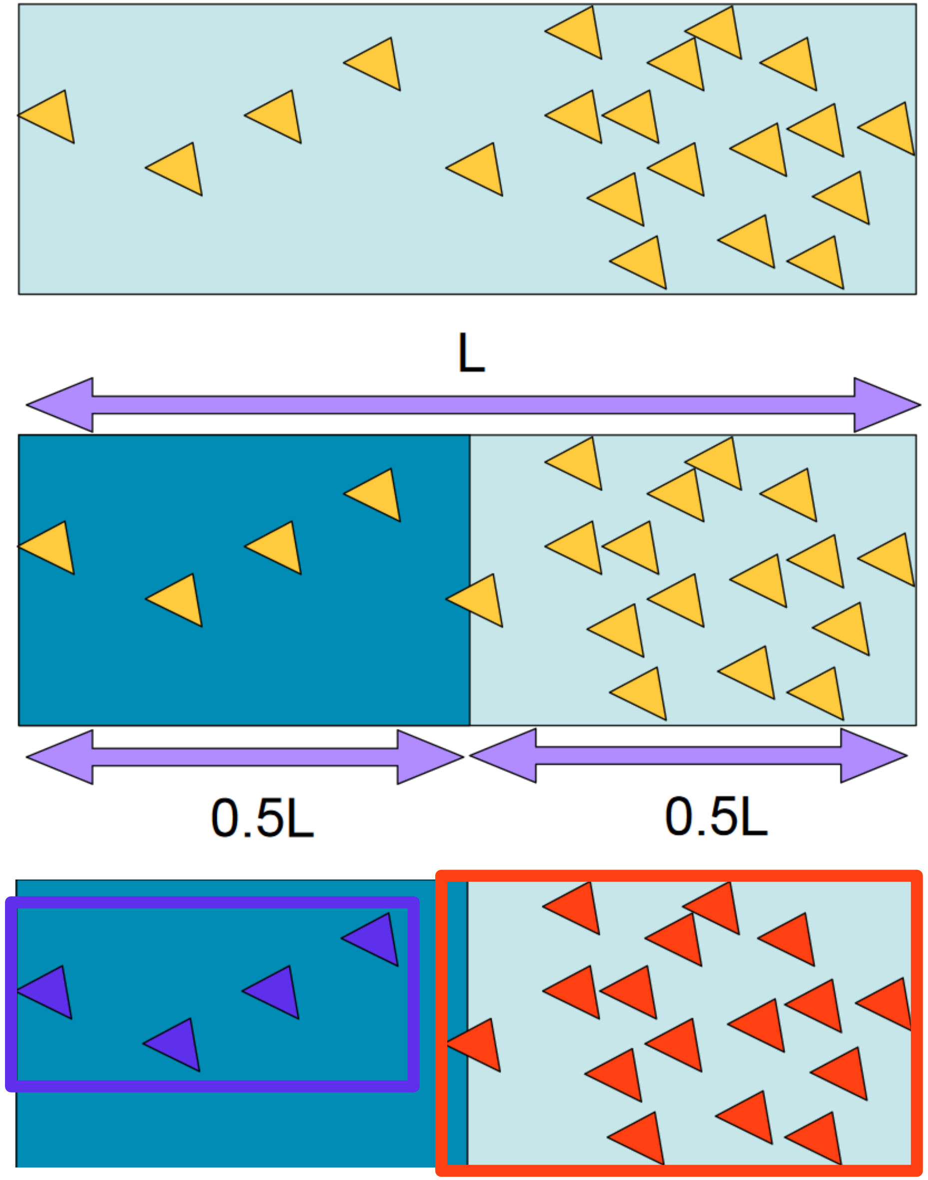
  - Arbitrary

Algorithms exist (e.g. "separating axis theorem"), but usually very slow!

- Where to split?
  - Spatial median
  - Object median
  - Something more elaborate...

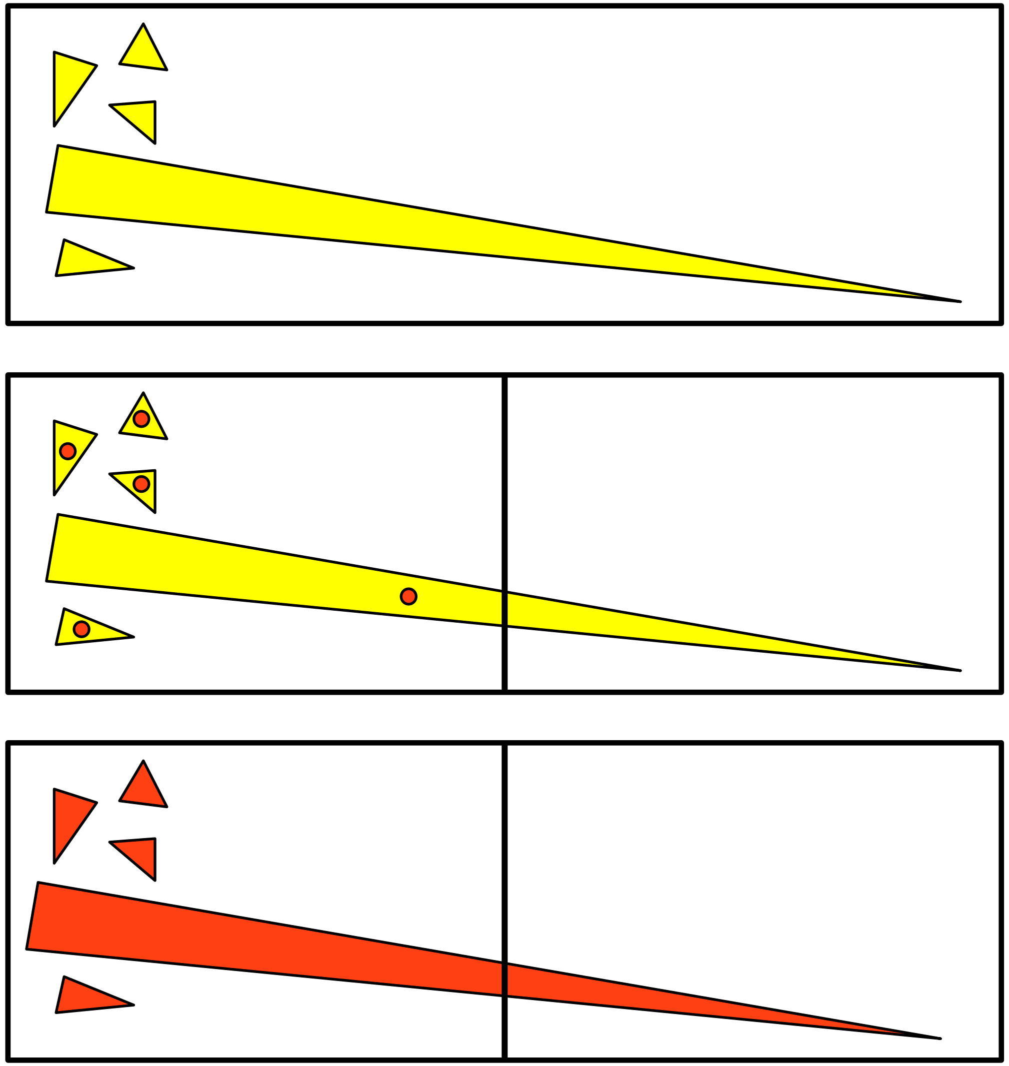
Splitting at spatial median

- Pick the longest axis (X/Y/Z) of current node bounds
- Find the midpoint on that axis
- Assign triangles to A/B based on which side of the midpoint each triangle's *centroid* lies on
- Continue recursion with A/B



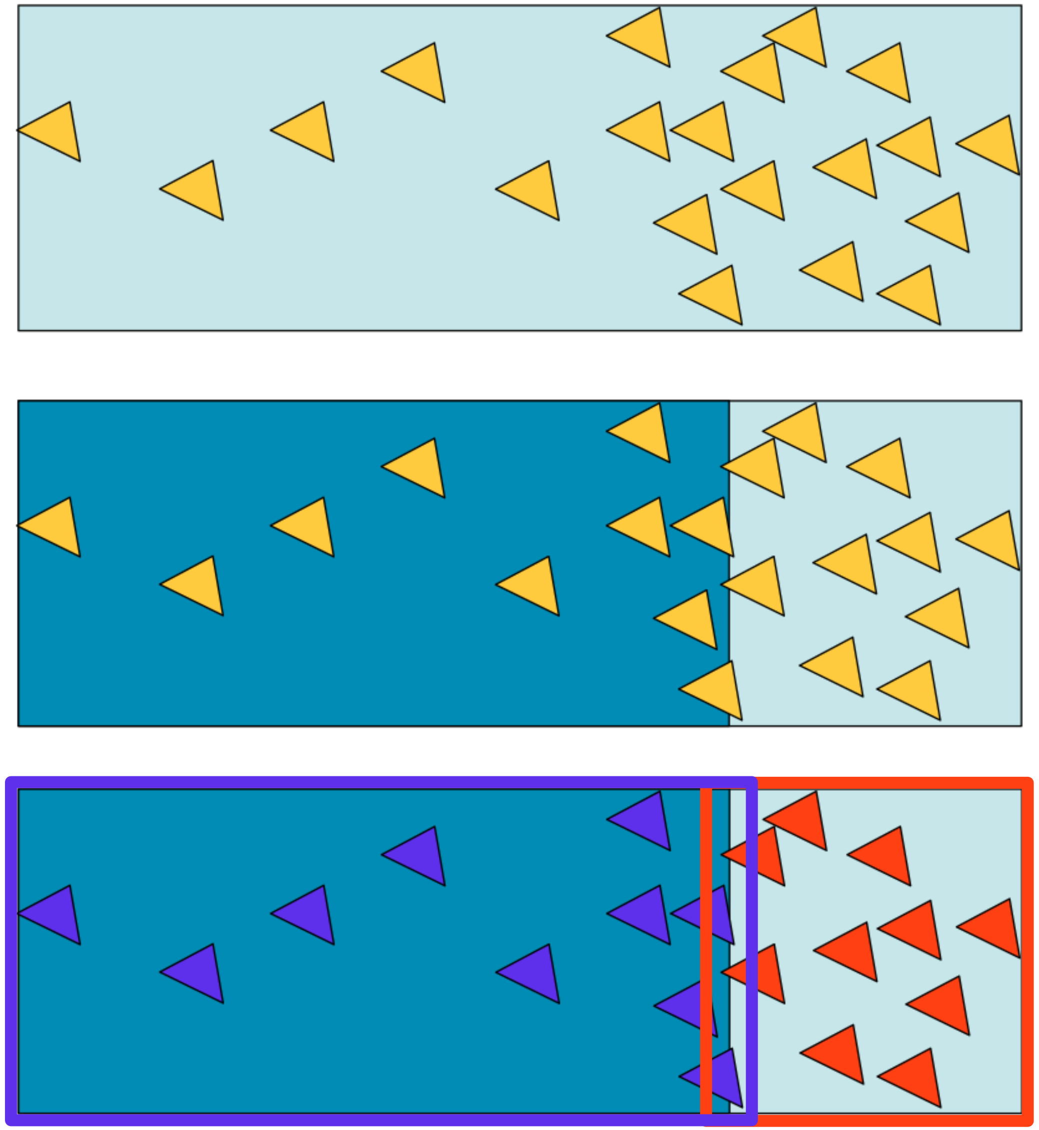
Splitting at spatial median

- Careful: can result in infinite recursion!
- All triangles are assigned again to one node, none in the other
- Can guard against it in several ways
  - Limit max. number of split attempts
  - Try other axes if one node is empty
  - Compute box over triangle centroids and split that on longest axis instead



Splitting at object median

- Pick an axis. Can try them all, don't pick the same every time
- Sort triangles according to their centroid's position on that axis
- Assign first half of the sorted triangles to A, the second to B
- Continue recursion with A/B



BVH Traversal

0. Set $t_{max} = \infty$. Start at root node, return if it doesn't intersect ray.

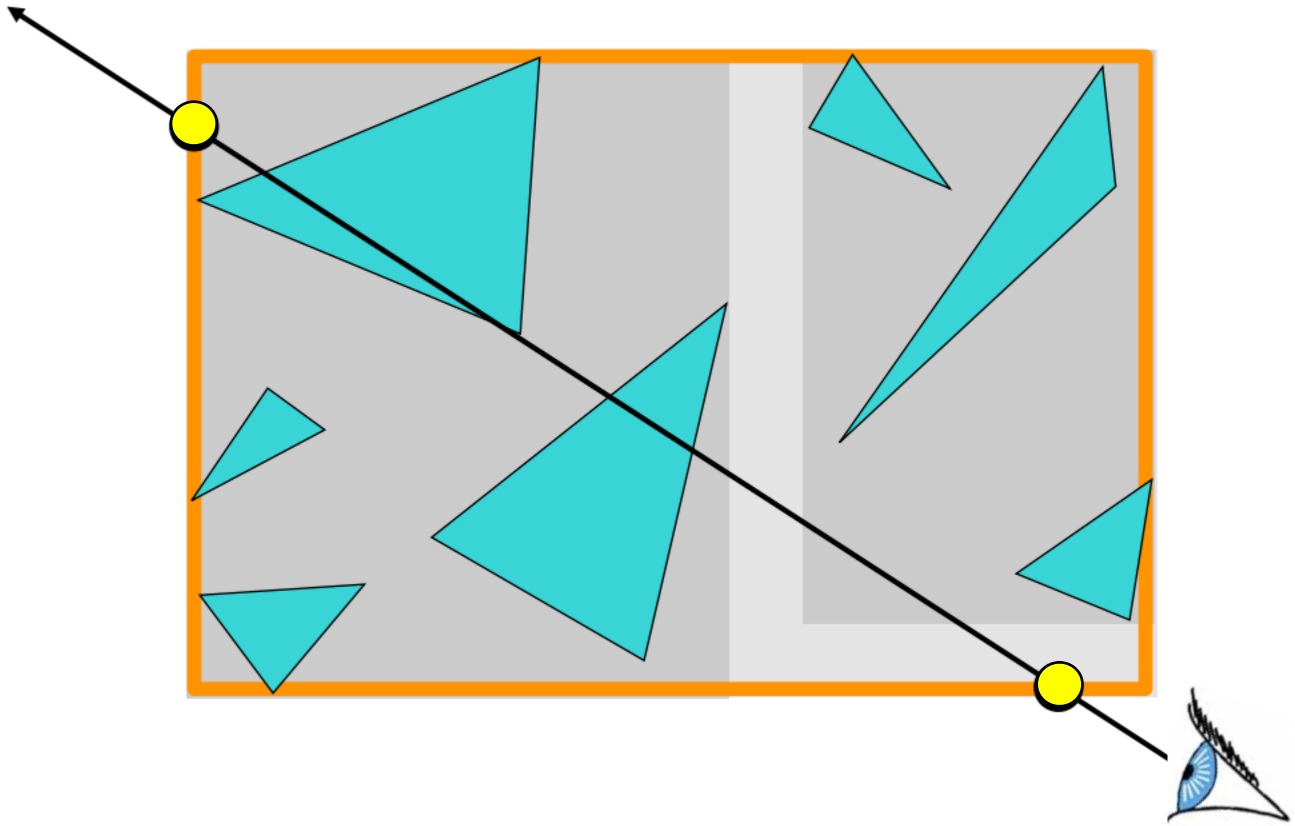
1. Process node if its closest intersection with ray is closer than t_{max}

2. If it's an inner node, run from 1. for child nodes that intersect ray
   - Process the closest node first
   - Keep others on stack to process further ones later (recursion works)

3. If it's a leaf, check triangles and update t_{max} in case of closer hit

BVH Traversal Example

1. Process node if its closest intersection with ray is closer than t_{max}



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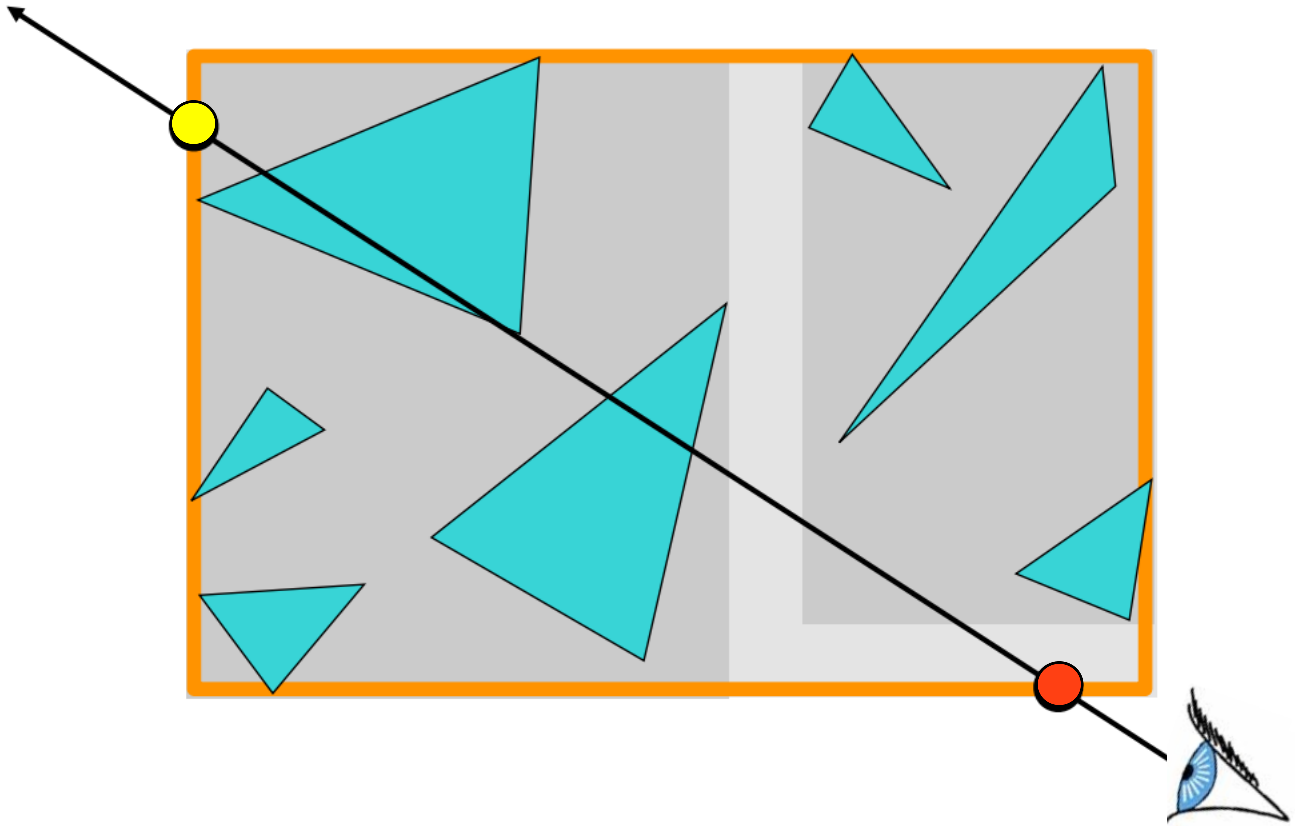
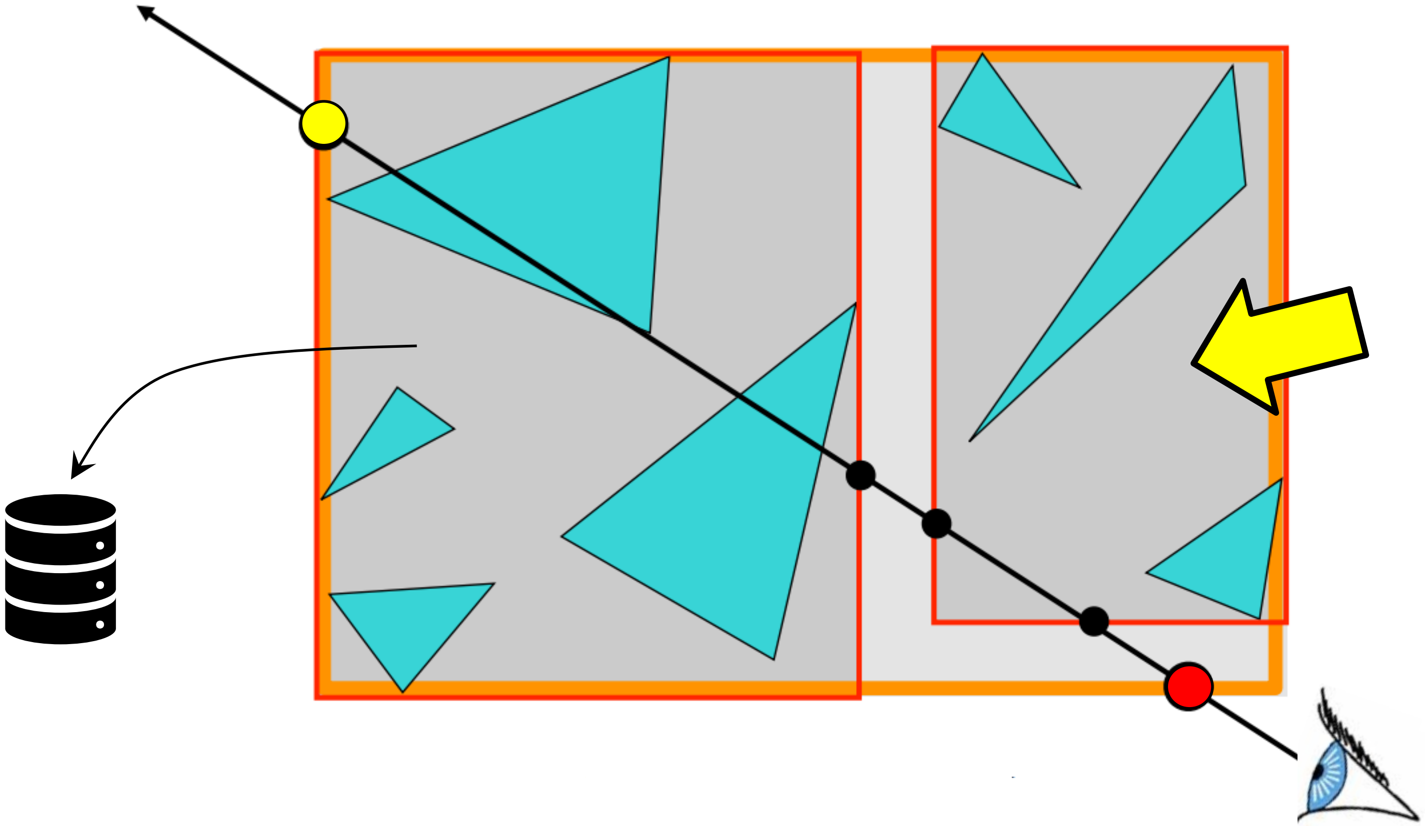


BVH Traversal Example

2. If it's an inner node, run from 1. for child nodes that intersect ray
   - Process the closest node first
   - Keep others on stack to process further ones later



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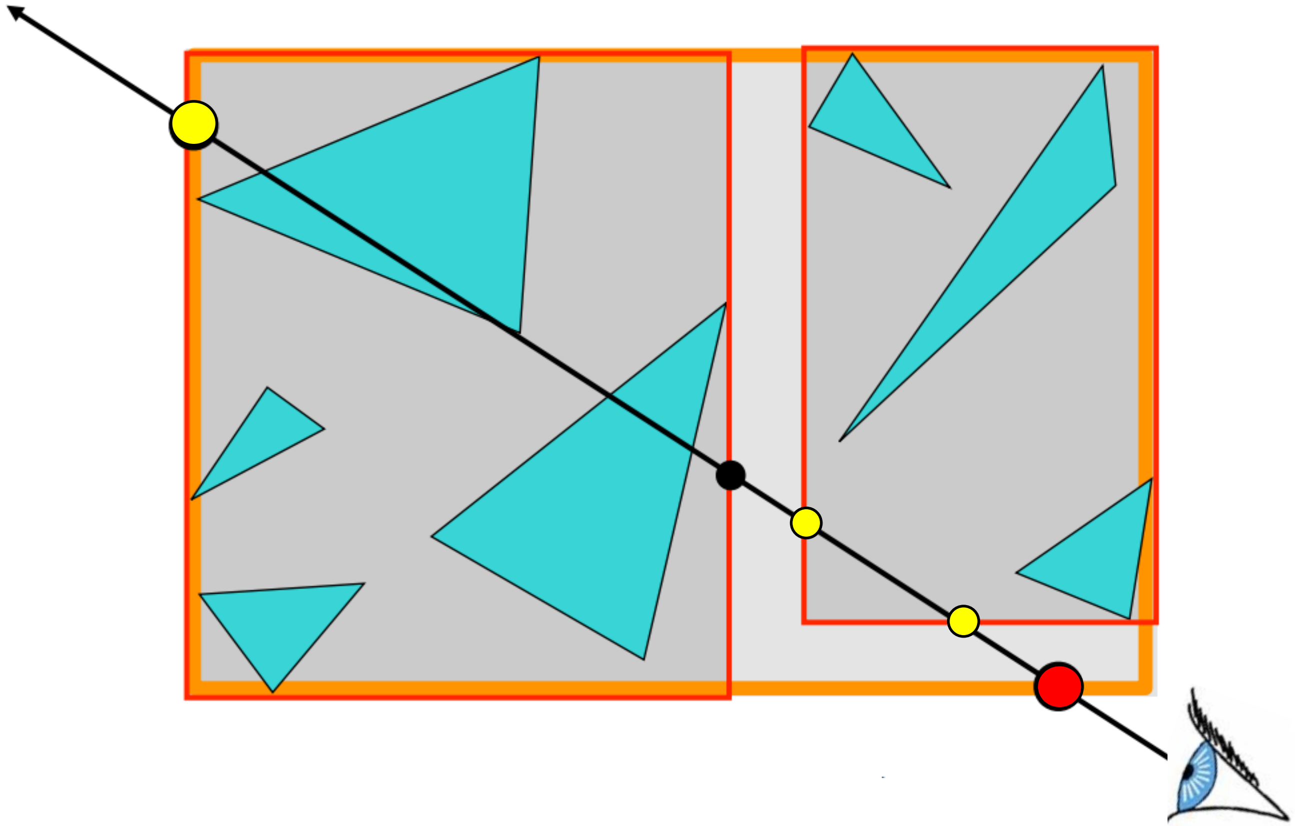

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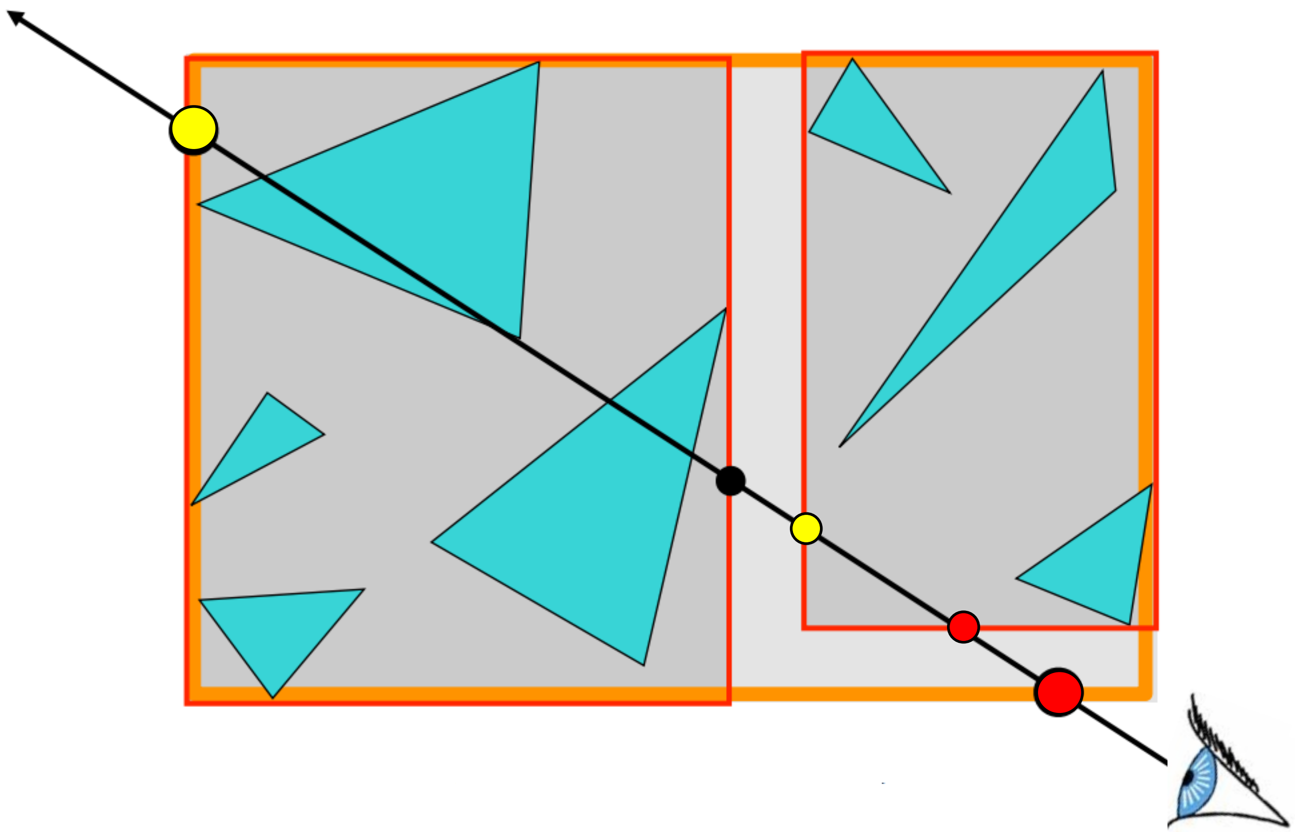
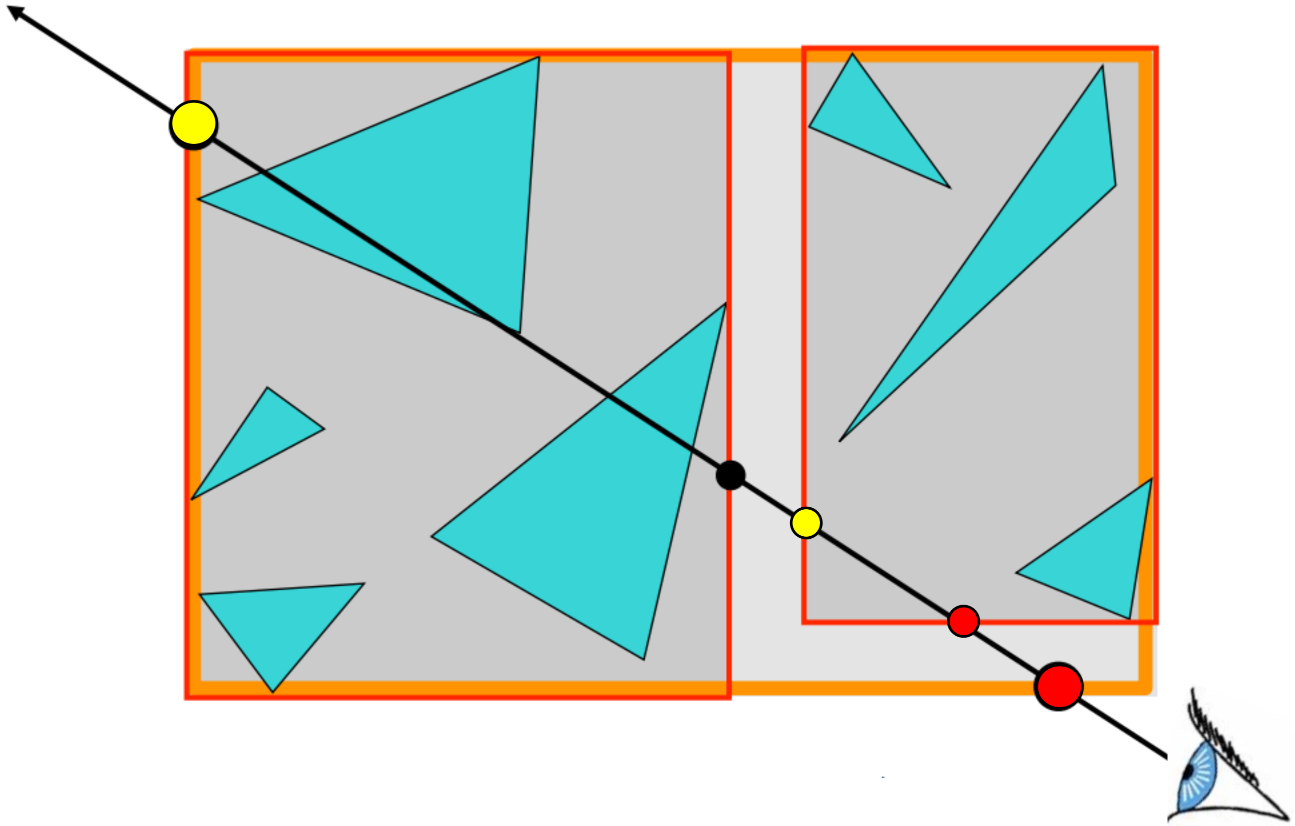


BVH Traversal Example

3. If it's a leaf, check triangles and update t_{max} in case of closer hit



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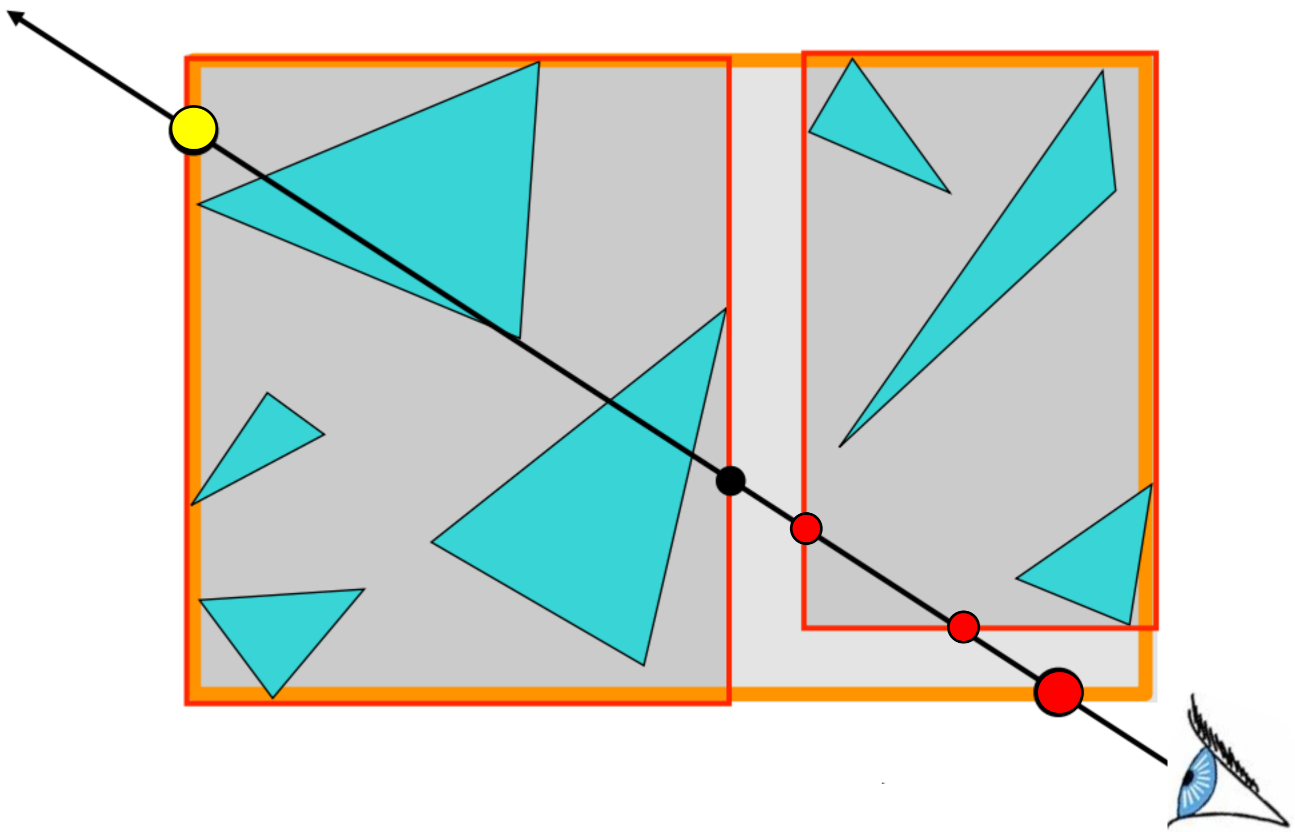

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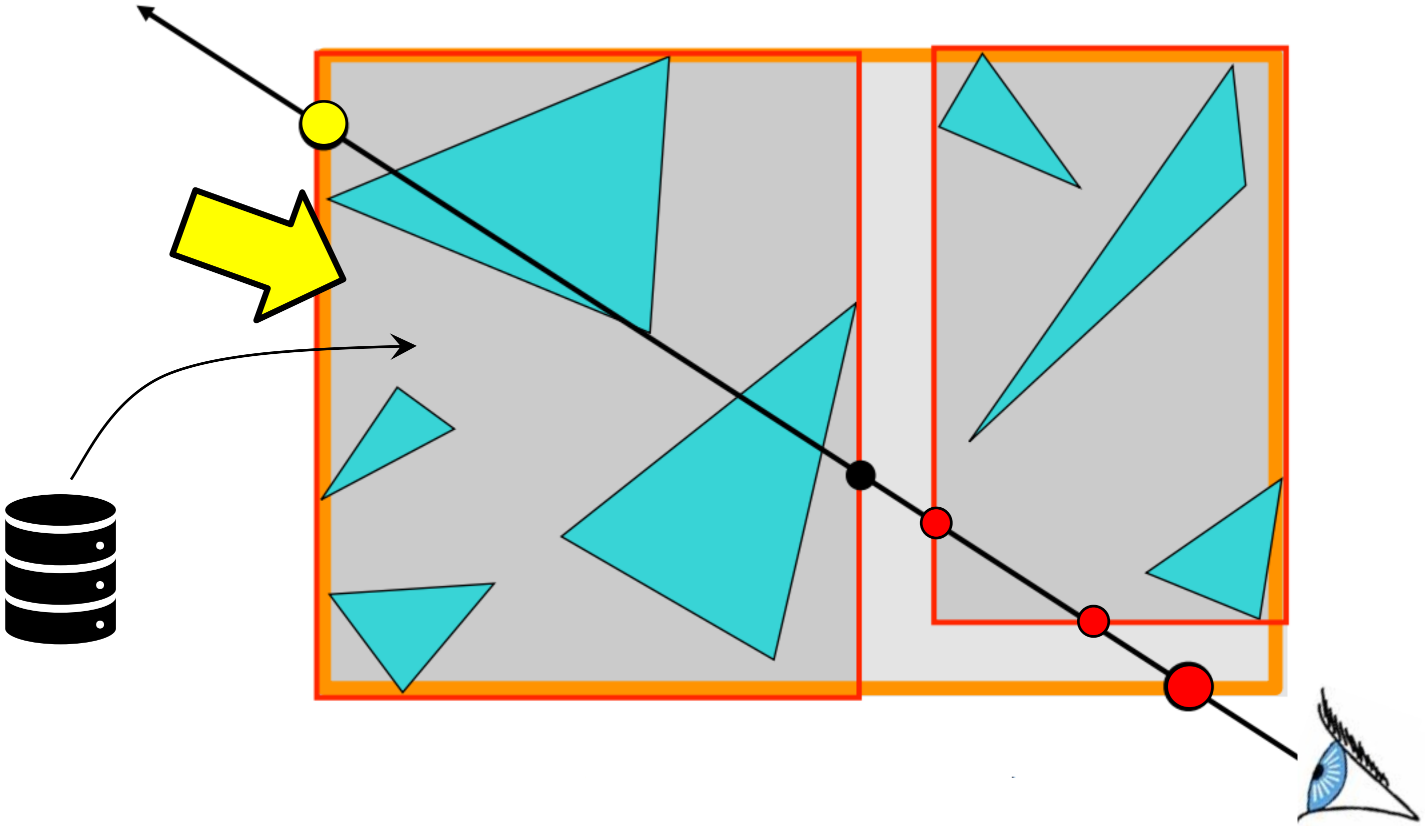

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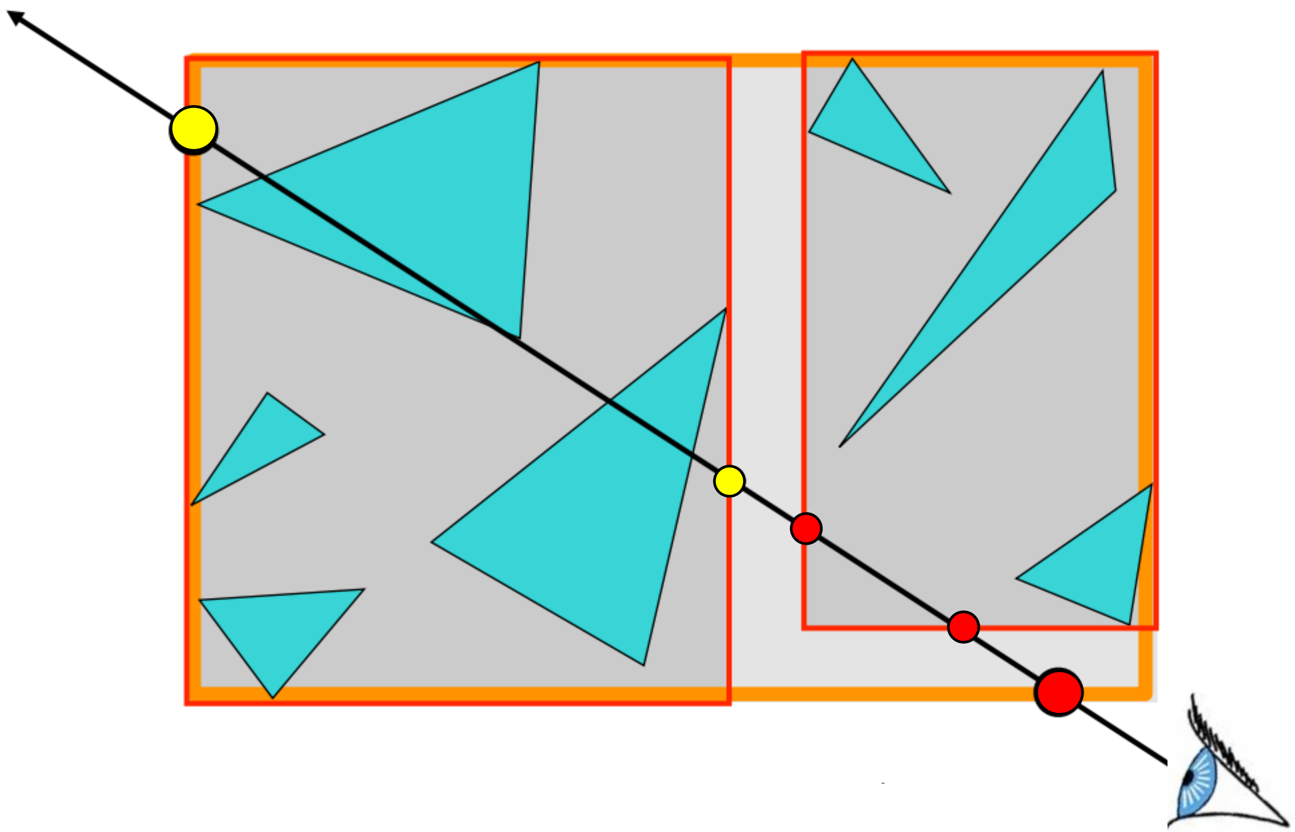

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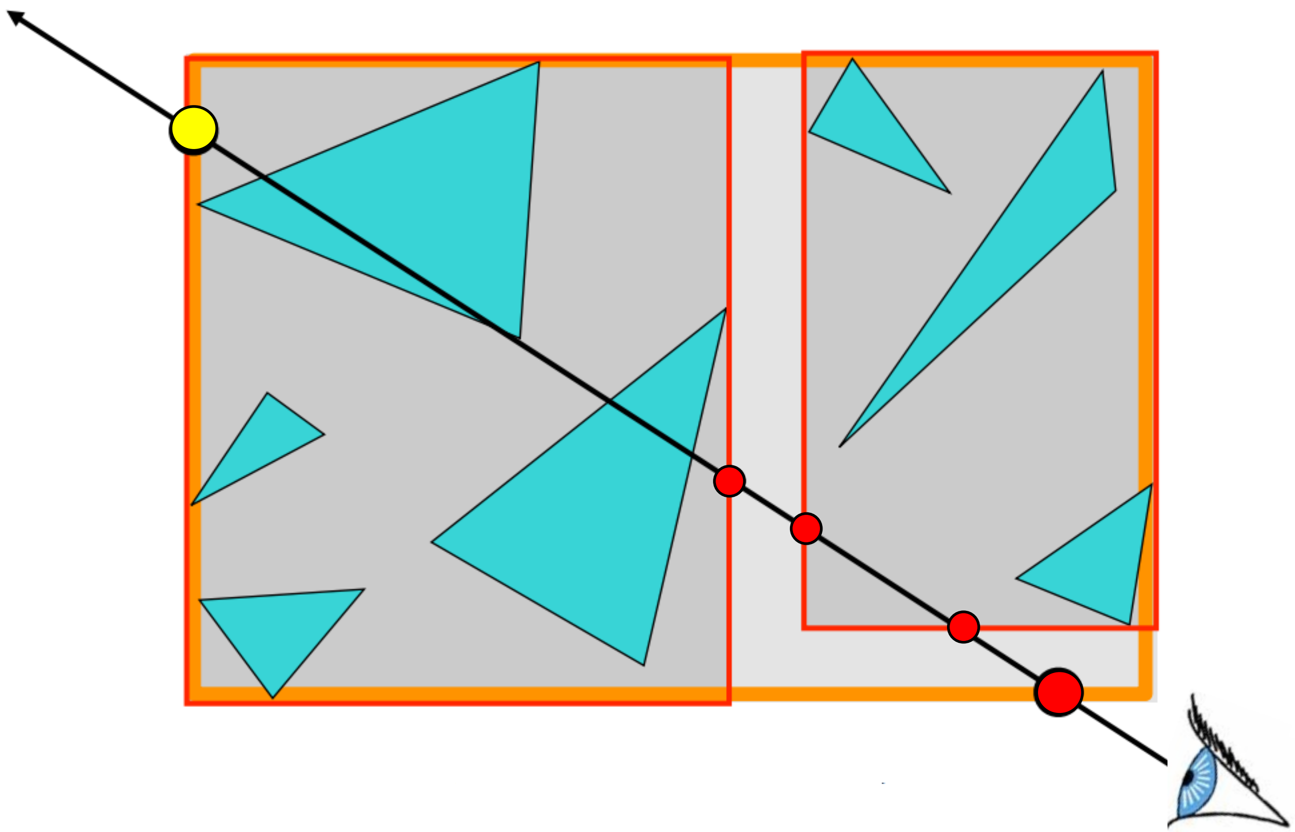
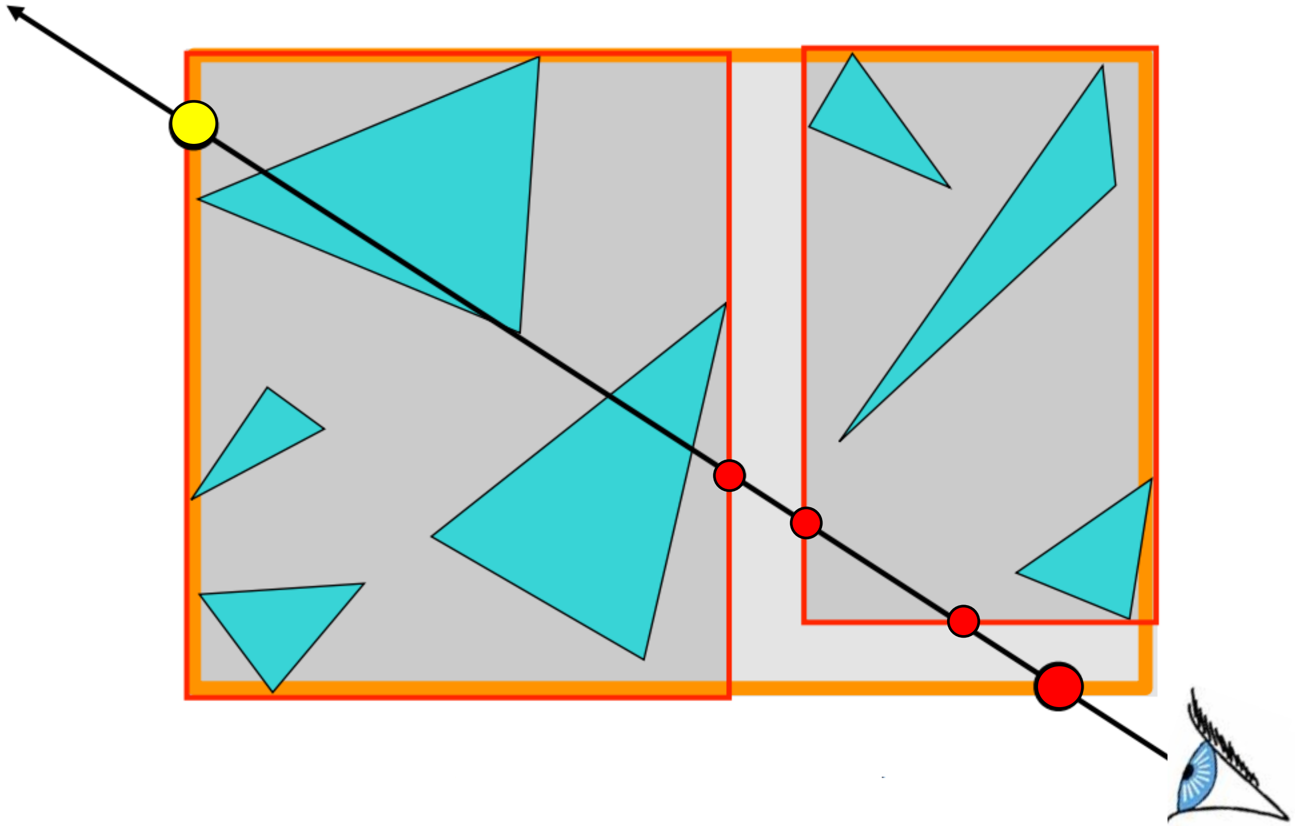


BVH Traversal Example

3. If it's a leaf, check triangles and update t_{max} in case of closer hit (●)



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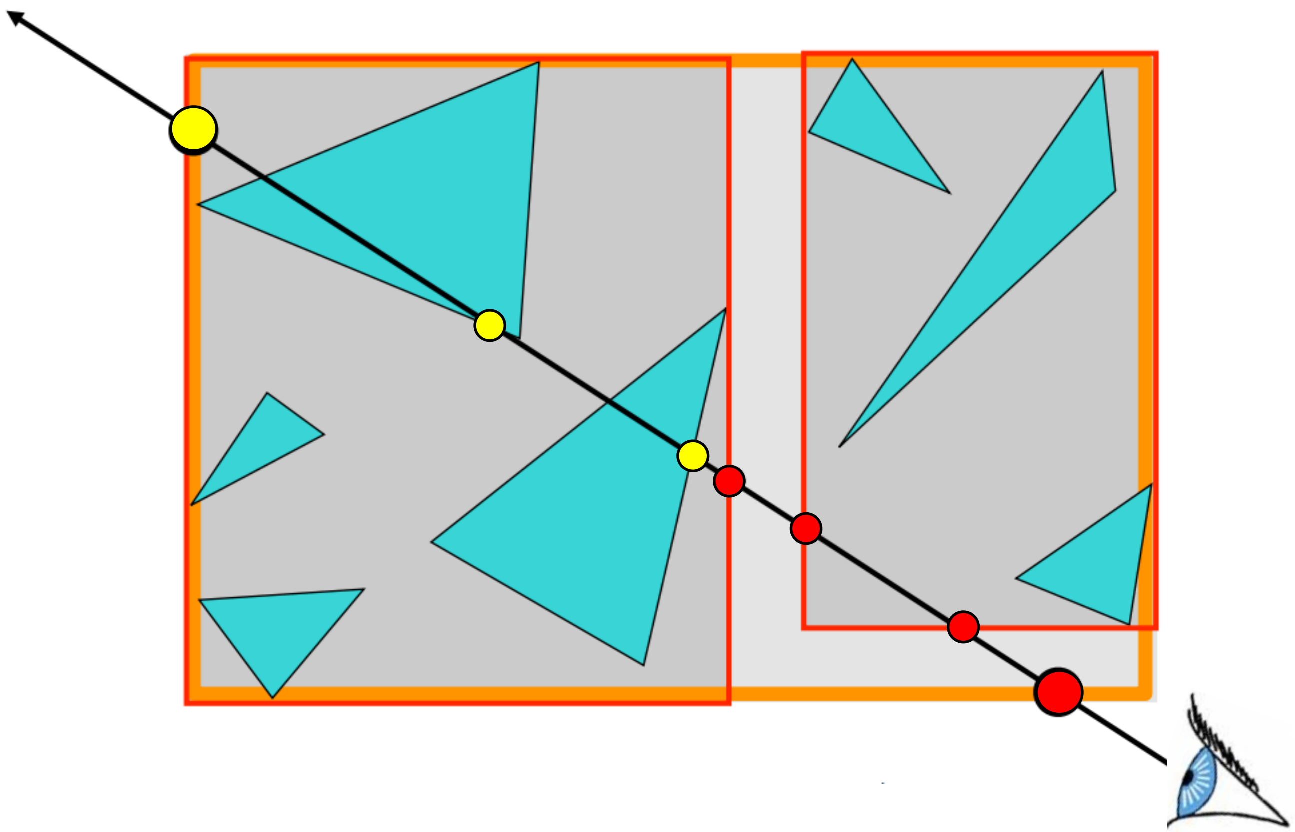

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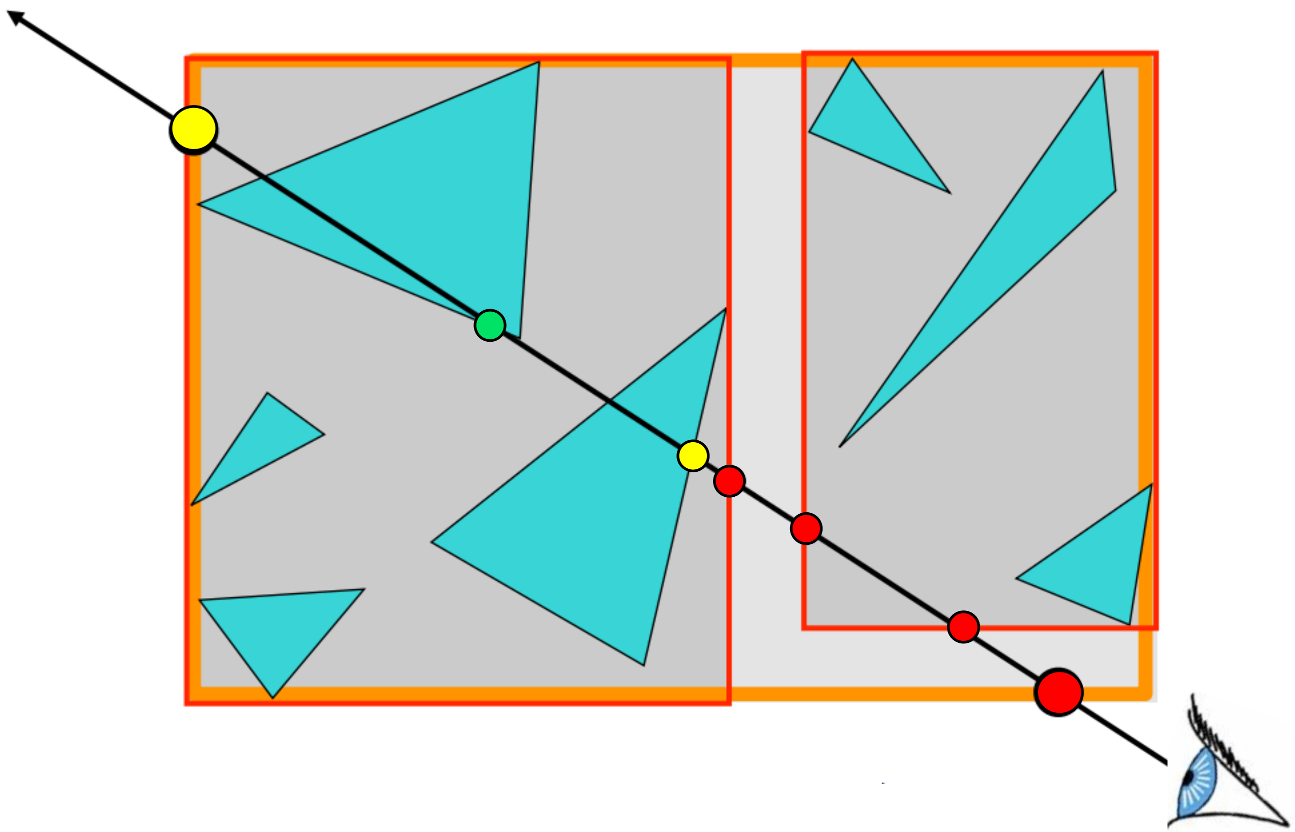

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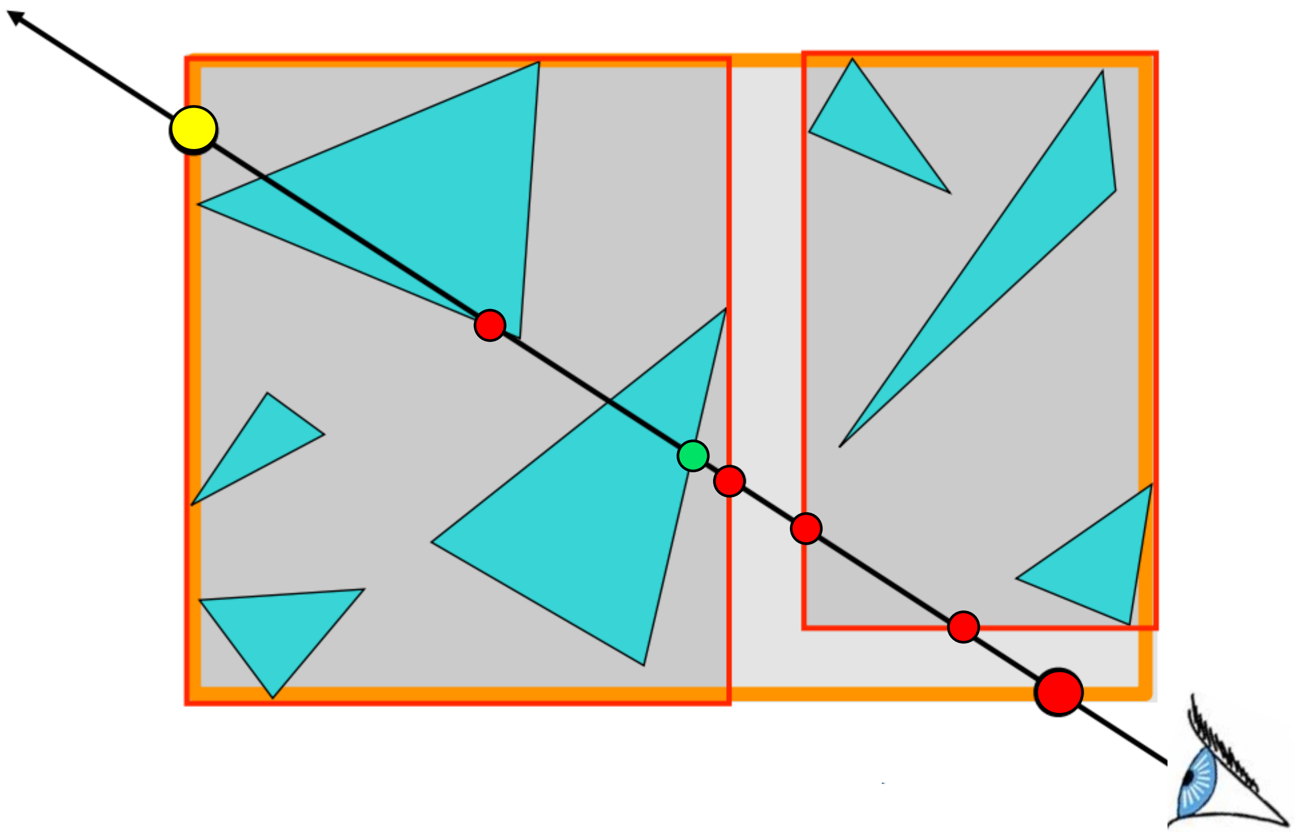

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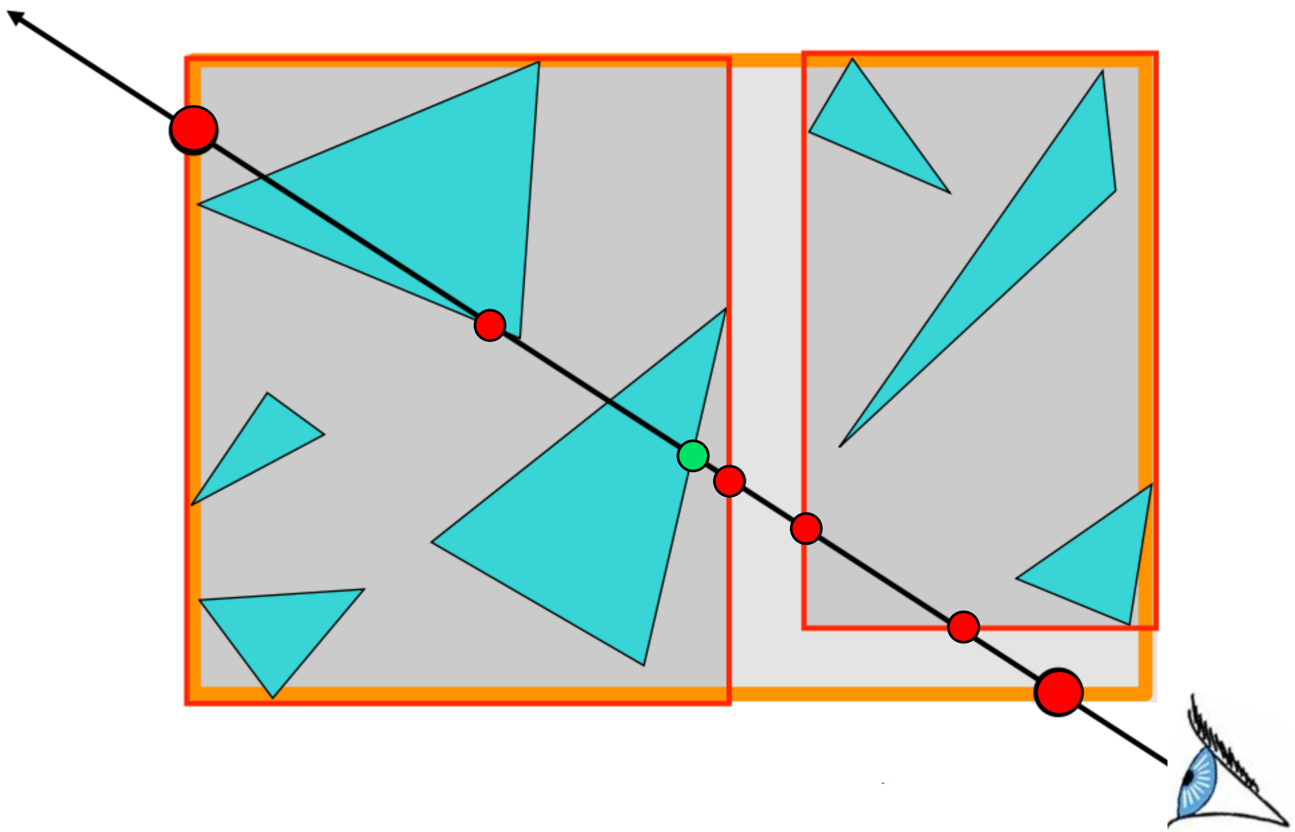


The Surface Area Heuristic [1]

- Simple, but powerful heuristic for choosing splits

- Created with traversal in mind, based on the following ideas:
  - Assume rays are uniformly distributed in space
  - Probability of a ray hitting a node is proportional to its **surface area**
  - Cost of traversing it depends on the **number of triangles** in its leaves
  - Hence, **avoid large nodes with many triangles**, because:
    - They have a tendency to get checked often
    - Getting a definite result (reject or closest hit) is likely to be expensive

Applying the Surface Area Heuristic

Goal: To split a node, find the hyperplane b that minimizes

$$f(b) = LSA(b) \cdot L(b) + RSA(b) \cdot (N - L(b)),$$ where

- $LSA(b)/RSA(b)$ are the **surface area** of the nodes that enclose the triangles whose centroid is on the “left”/“right” of the split plane b
- $L(b)$ is the **number of primitives on the “left”** of b
- N is the **total number of primitives** in the node

The Sweep SAH BVH

- We want to constrain the search space for a good split
- Pick a set of axes to test (e.g., 3D basis vectors X/Y/Z)
- When splitting a node with N triangles, for each axis
  - Sort all triangles by their centroid's position on that axis
  - Find the index i that minimizes

$f(i) = LSA(i) \cdot i + RSA(i) \cdot (N - i)$, where

  - $LSA(i)$ is the surface area of the AABB over sorted triangles $[0, i)$
  - $RSA(i)$ is the surface area of the AABB over sorted triangles $[i, N)$

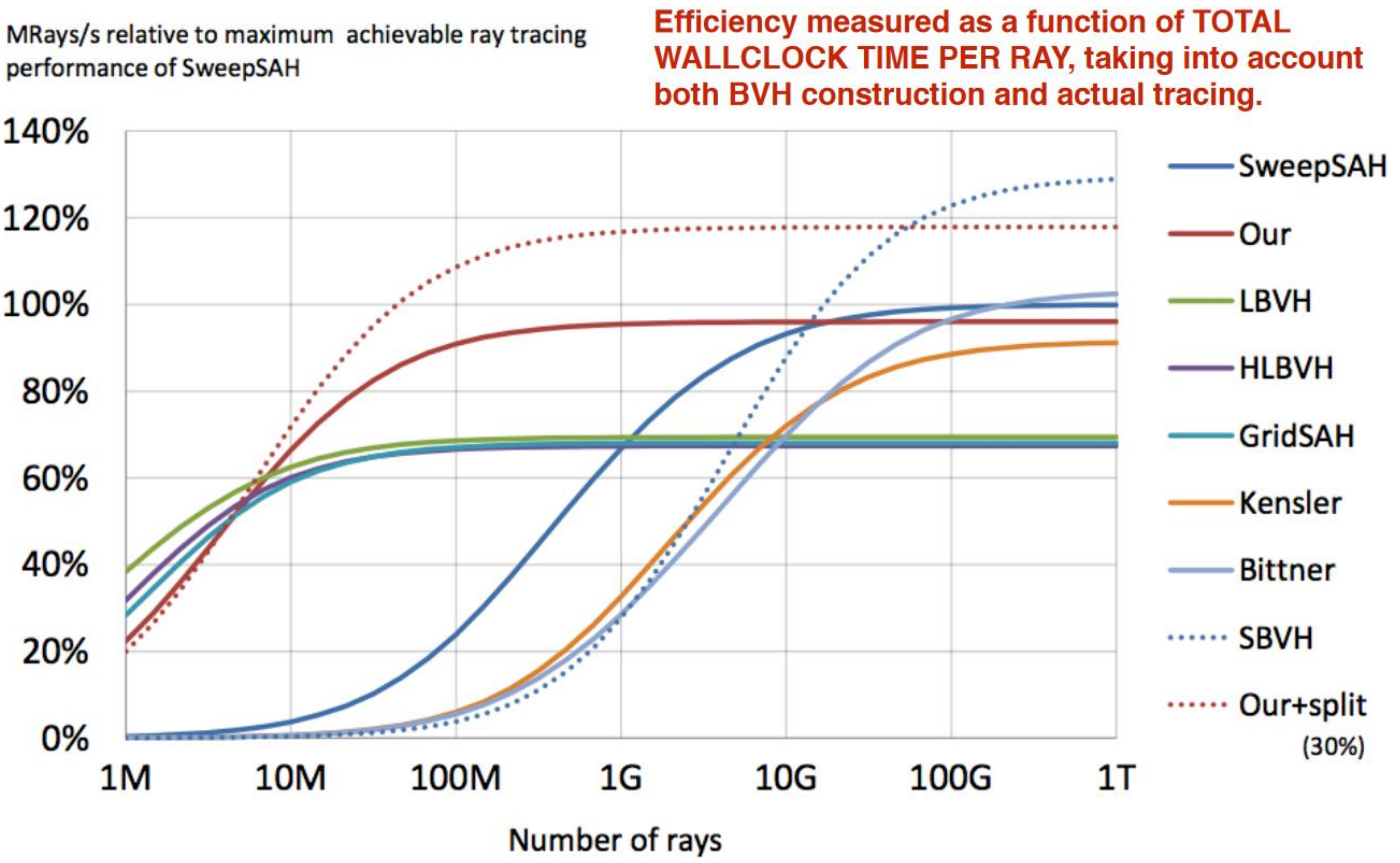
- Select the axis and index i with the best $f(i)$ for the split overall!

Importance of Optimizing Splits

- Important trade-off: building time vs. traversal time
  - Given the same tracing/traversal code, the quality of a BVH tree may have a big impact on performance!
  - Can be as high as 2x compared to naïve splitting

- Benefits depend on the parameters of your rendering scenario
  - How big is your scene and how are triangles distributed?
  - How long will your BVH be valid?
  - What are the quality requirements for your images?

Evaluation of Combined Building + Traversal [2]

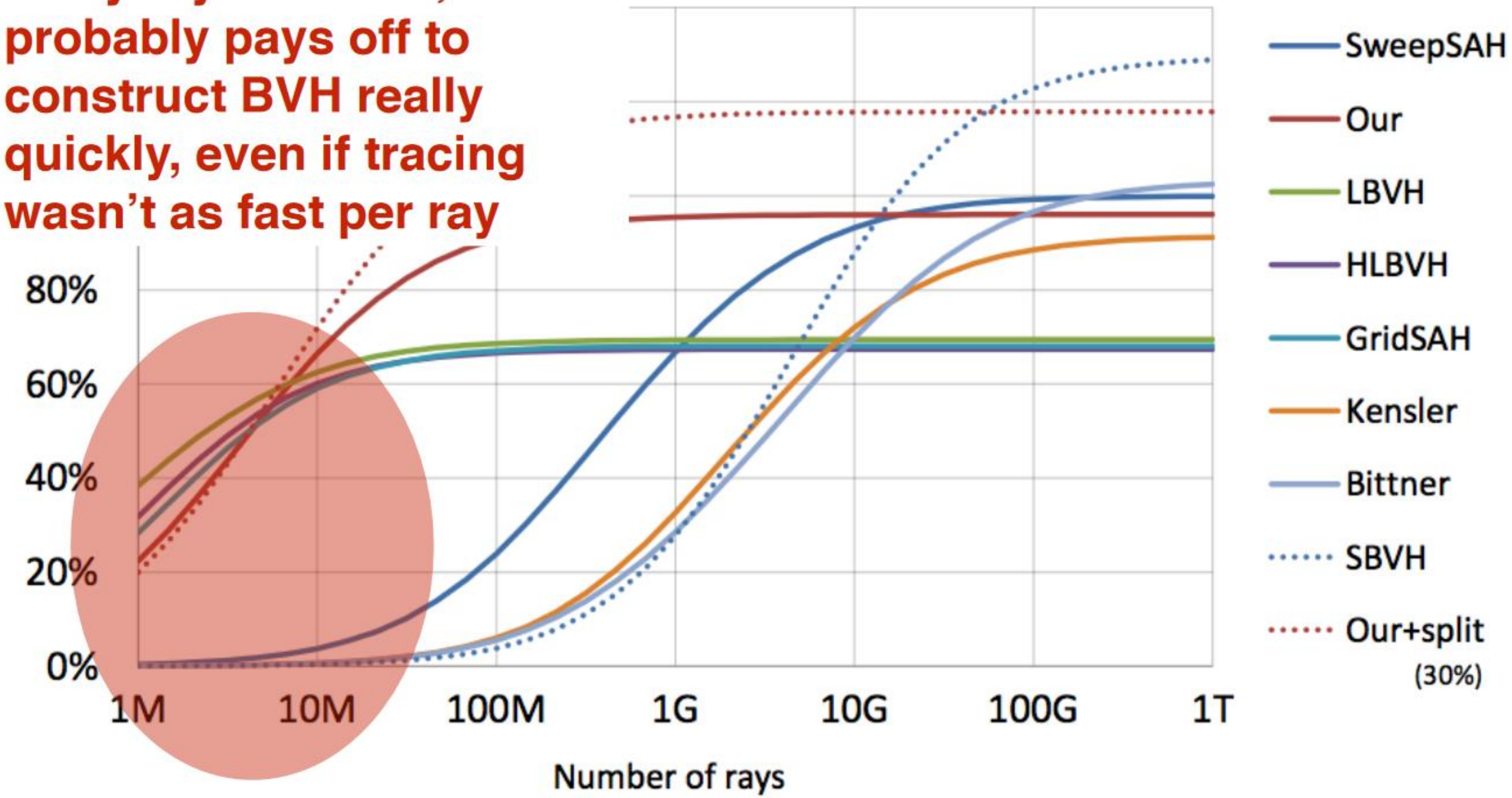


Check out the paper this comparison came from https://users.aalto.fi/~ailat1/publications/karras2013hpg_paper.pdf

Evaluation of Combined Building + Traversal [2]

If you don't have too many rays to trace, it probably pays off to construct BVH really quickly, even if tracing wasn't as fast per ray

Efficiency measured as a function of TOTAL WALLCLOCK TIME PER RAY, taking into account both BVH construction and actual tracing.



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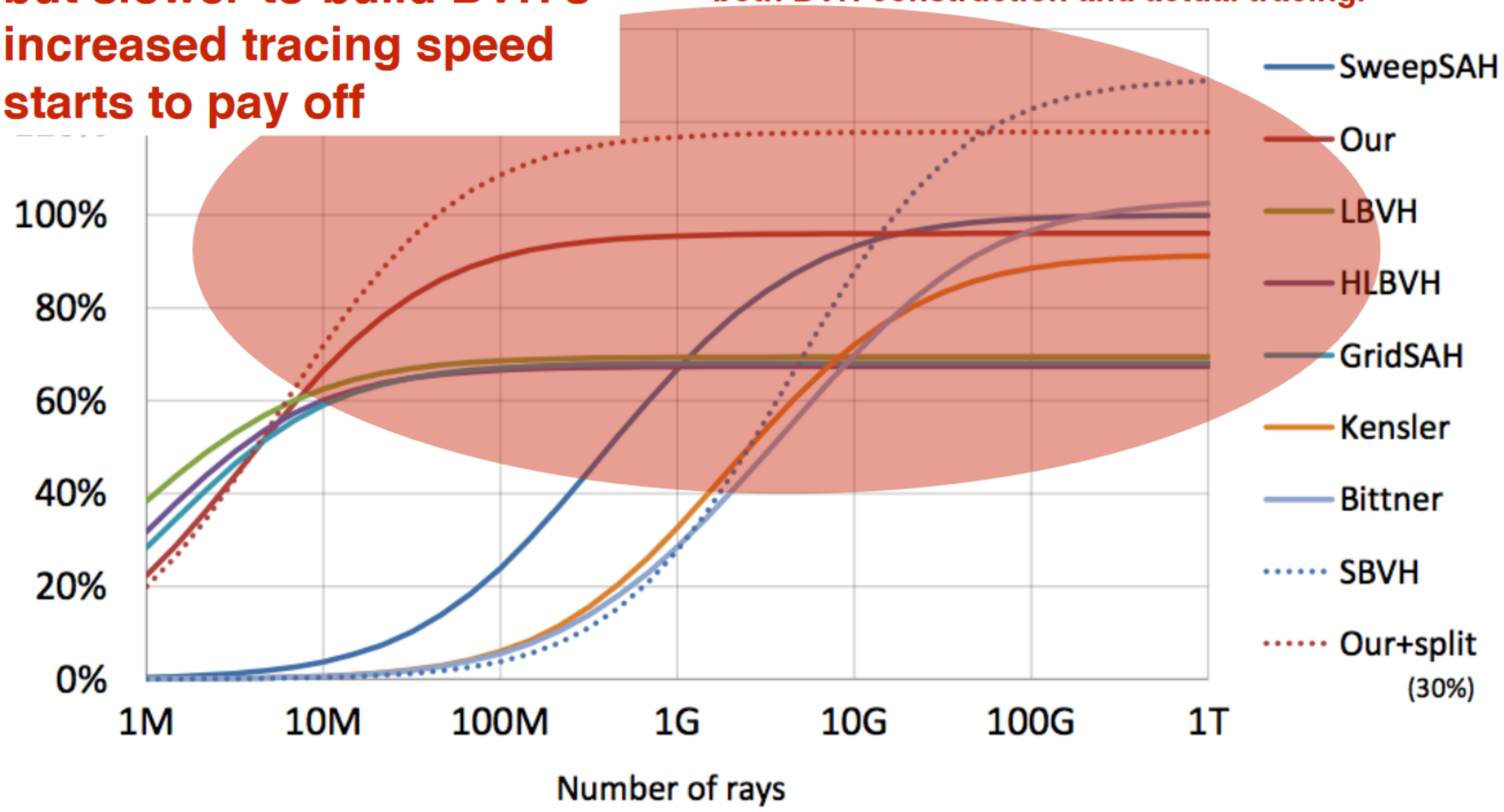
Evaluation of Combined Building + Traversal [2]

After some point a faster but slower-to-build BVH's increased tracing speed starts to pay off

Efficiency measured as a function of TOTAL WALLCLOCK TIME PER RAY, taking into account both BVH construction and actual tracing.



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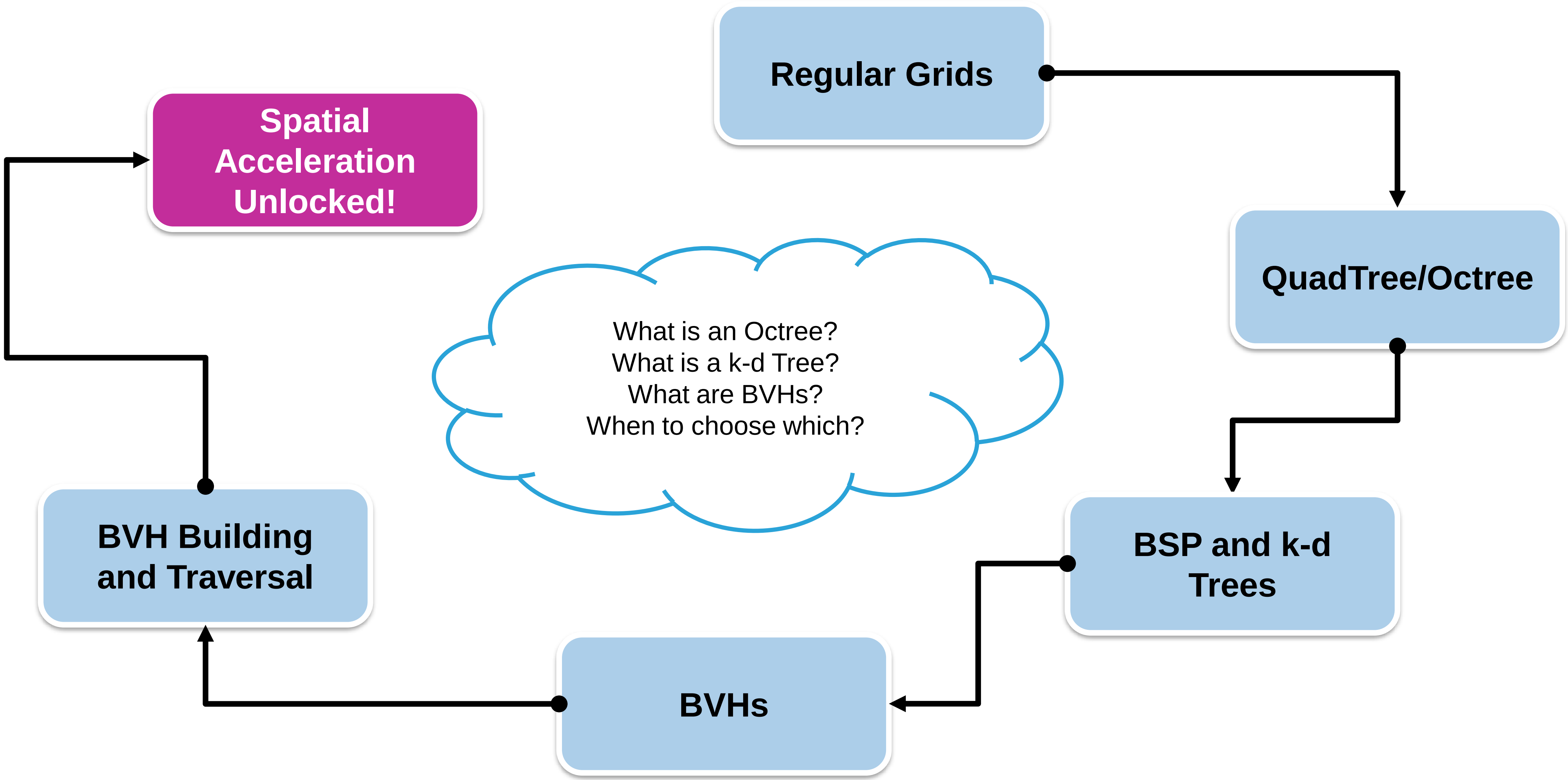
Spatial Acceleration Structures

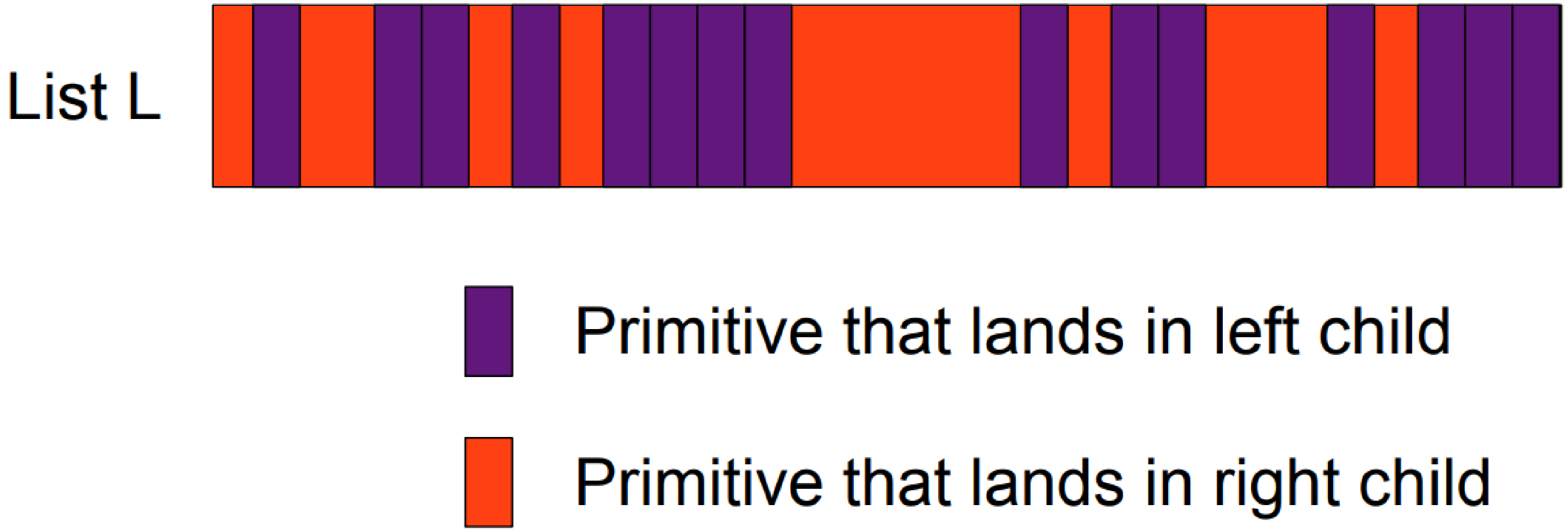
Structure	Memory Consumption	Building Time	(Expected) Traversal Time
none	none	none	abysmal
Regular Grid	low – high (resolution)	low	uniform scene: ok otherwise: poor
Quadtree/Octree	low – high (overlap/uniformity)	low	good
k-d Tree	low – high (overlap)	low – high	good – excellent
BVH	**low**	**low – high**	**good – excellent**

Today's Roadmap



BVH Building Hints

- For each split, sort the node's portion of the triangle list *L* in-place
- When constructing child nodes, pass them *L* and *start/end* indices



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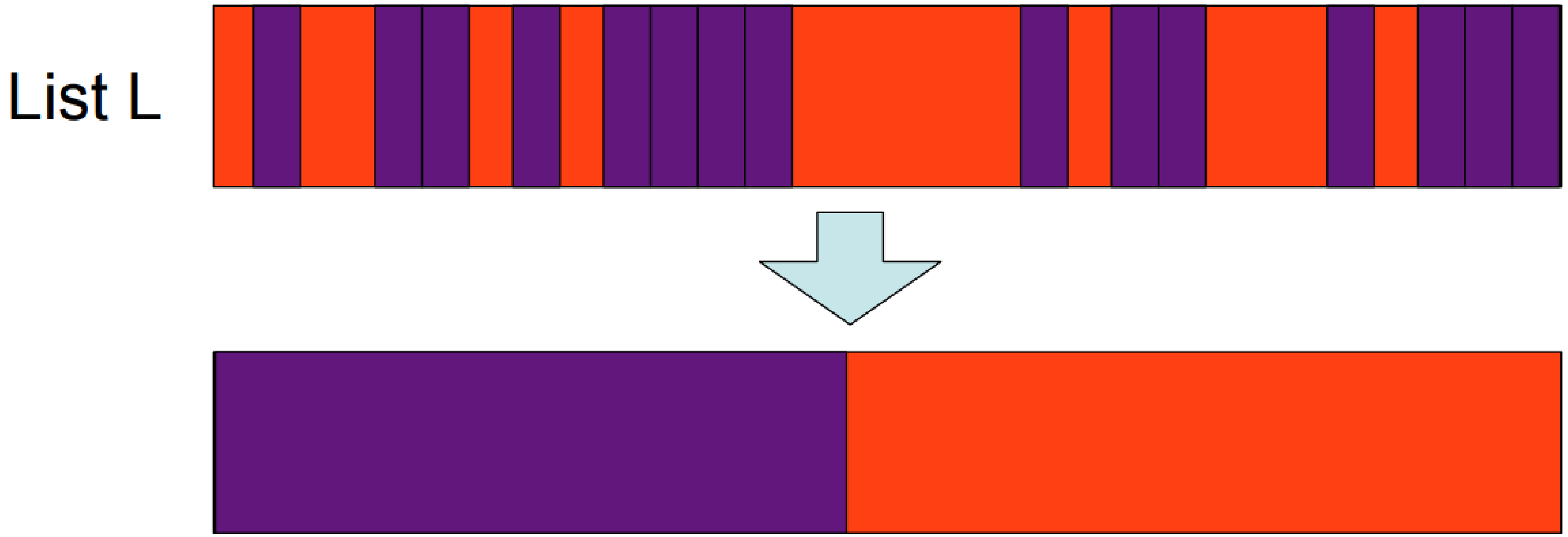

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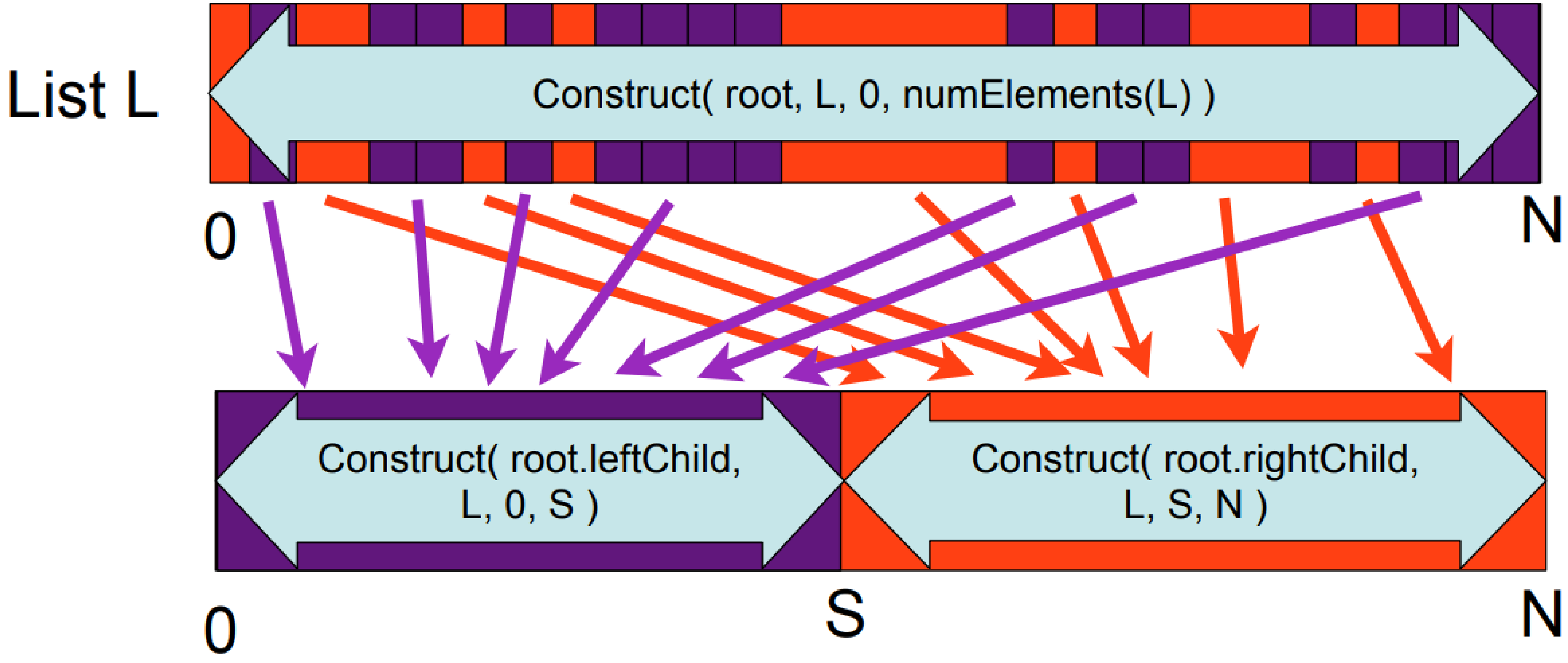

SAH Coding Hints

- Don't loop over triangles at each i to get $LSA(i)$ and $RSA(i)$!

- Precompute them once per node and axis instead
  - Create two 0-volume bounding boxes BB_L, BB_R
  - Allocate N+1 entries for LSA/RSA, set $LSA(0) = RSA(N) = 0$
  - Iterate i over range $[1, N]$, for each i:
    - Merge BB_L with the AABB of sorted triangle with index $(i-1)$
    - Store surface area of BB_L as value for $LSA(i)$
    - Merge BB_R with the AABB of sorted triangle with index $(N-i)$
    - Store surface area of BB_R as value for $RSA(N-i)$

BVH Building Hints (C++)

- Consider using *stdlib* container (e.g., vector)
- Try to avoid dynamic memory allocation
- $2N - 1$ is an upper bound for the total number of nodes you need
- *std::sort*(<first>, <last>, <predicate>)
- *std::nth_element*(<first>, <nth>, <last>, <predicate>)
  - Can be used for splitting if you don‘t need exact sorting
  - Reorders the N-sized vector such that:
    - n smallest elements are on the left
    - $N - n$ biggest are on the right
  - Faster than sorting!

BVH vs k-d Tree vs Others

- Each have their specializations, strengths and weaknesses
- E.g., k-d Trees with ropes do not require a stack for traversal [5]
- Which acceleration structure is the **best** is contentious
- Currently, BVHs are extremely widespread and well-understood

State-of-the-Art Variants and Trends

- Higher child counts (>2) per node, mixed nodes (children + triangles)
- Actually DO split triangles sometimes to get maximal performance
- Build BVHs bottom-up in parallel on the GPU [3]
- In animated scenes, reuse BVHs, update those parts that change
- Actually use built-in traversal logic of GPU hardware (NVIDIA RTX!)

References and Further Reading

- Interesting topics: BVHs for animation, LBVH, SIMD/packet/stackless traversal, Turing RTX architecture

- [1] *Heuristics for Ray Tracing Using Space Subdivision*, J. David MacDonald and Kellogg S. Booth, 1990
- [2] *On Quality Metrics of Bounding Volume Hierarchies*, Timo Aila, Tero Karras, and Samuli Laine, 2013
- [3] *Parallel BVH generation on the GPU*, Tero Karras and Timo Aila, 2012
- [4] *Fast Parallel Construction of High-Quality Bounding Volume Hierarchies*, Tero Karras and Timo Aila, 2013
- [5] *Stackless KD-Tree Traversal for High Performance GPU Ray Tracing*, Stefan Popov, Johannes Günther, Hans-Peter Seidel and Philipp Slusallek, 2007
- [6] *Realtime Ray Tracing and Interactive Global Illumination*, Phd Thesis, Ingo Wald, 2004
- [7] *Bonsai: Rapid Bounding Volume Hierarchy Generation using Mini Trees*, P. Ganestam, R. Barringer, M. Doggett, and T. Akenine-Möller, 2015