Rendering: Path Tracing Basics

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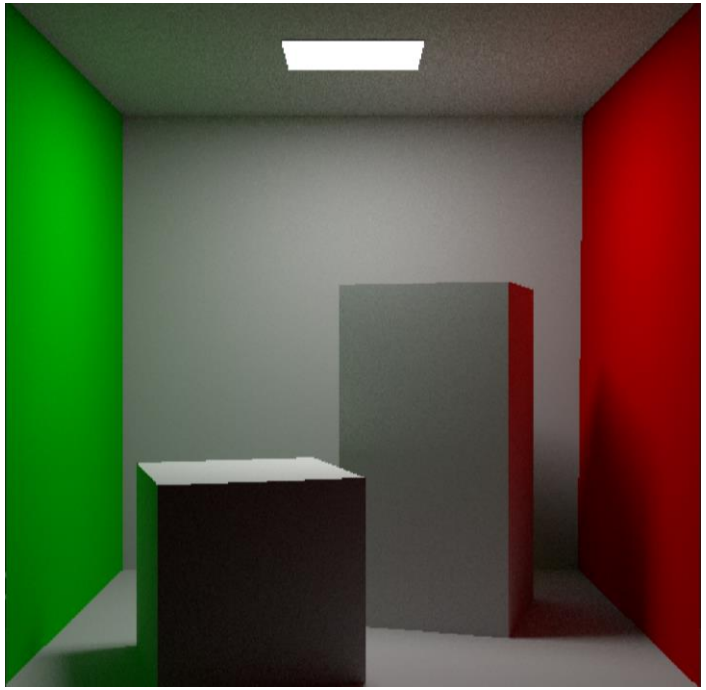
Disclaimer and Errata

- The following slides make heavy use of mathematical manipulations and pseudo code snippets for demonstration purposes
- It is quite easy to make mistakes when setting them up as slides
- If you find any issues, please feel free to notify us!
- The recorded version of this lecture will include an Errata section

Today's Goal

- We combine the things we learned so far to make our first unbiased path tracer for diffuse materials:
  - Light Physics
  - Monte Carlo Integration
  - The Rendering Equation
  - The Path Tracing Algorithm



- Different iterations and considerations for performance
- Introduce a dedicated BSDF/BRDF material data structure

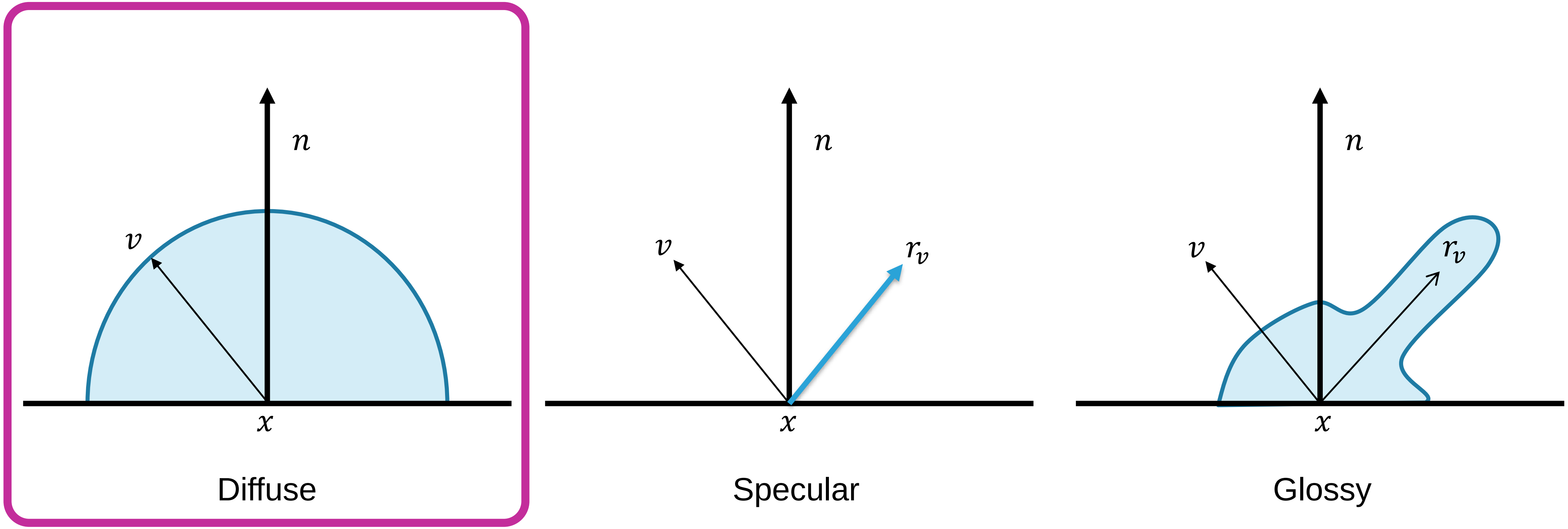
BSDF/BRDF..?

$$L_e(x, v) = E(x, v) + \int_\Omega \boxed{f_r(x, \omega \rightarrow v)} L_i(x, \omega) \cos(\theta_x) \, \mathrm{d}\omega$$

- Bidirectional Scattering Distribution Function (BSDF) accounts for the light transport properties of the hit material
- Bidirectional Reflectance Distribution Function (BRDF) considers only the **reflection** of incoming light onto a surface
- Only diffuse materials for now, more in Materials lecture

Material Types

- We usually distinguish three basic material types
  - Perfectly diffuse (light is scattered equally in/from all directions)
  - Perfectly specular (light is reflected in/from exactly one direction)
  - Glossy (mixture of the other two, specular highlights)



Material Types

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  - Perfectly diffuse (light is scattered equally in/from all directions)
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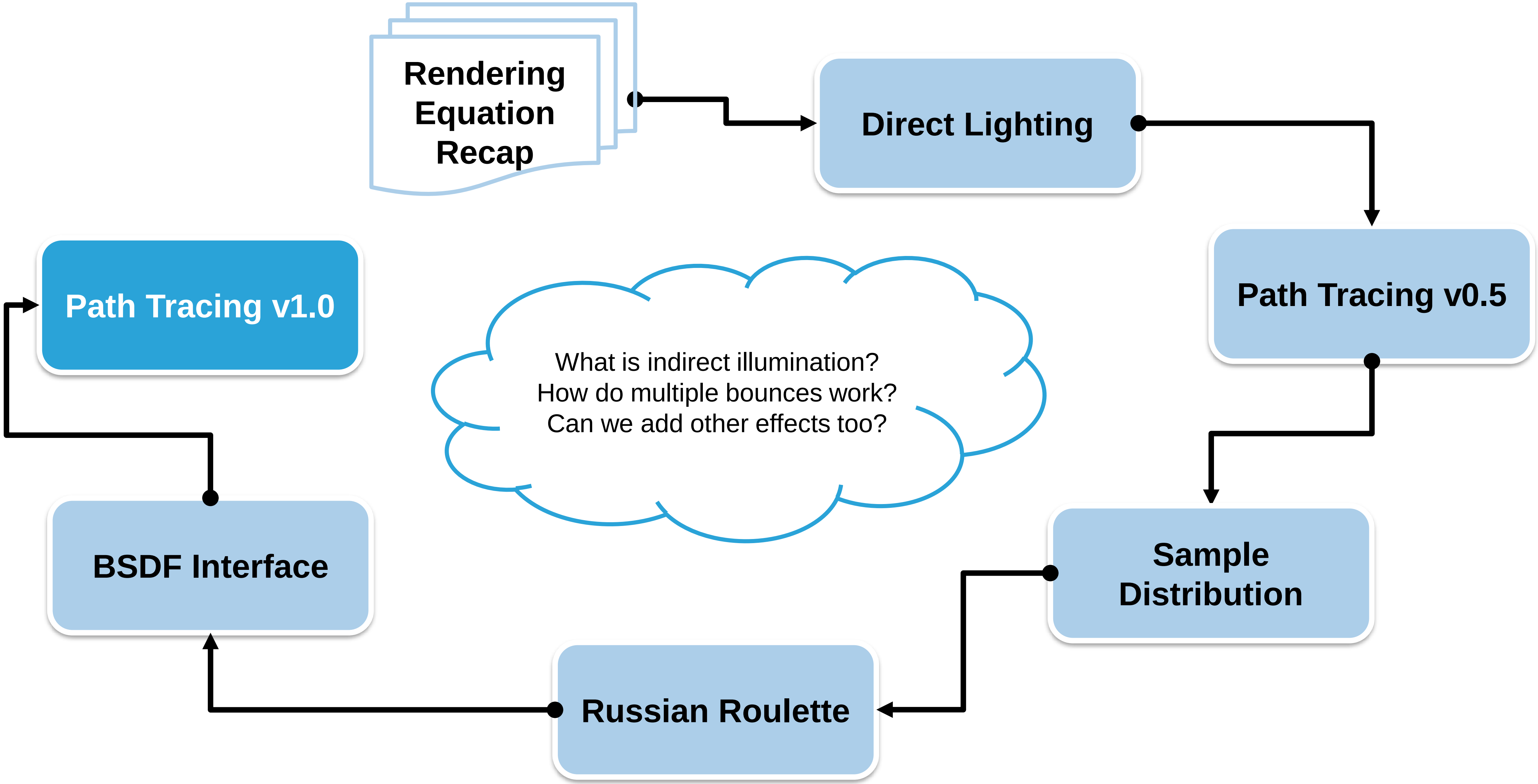
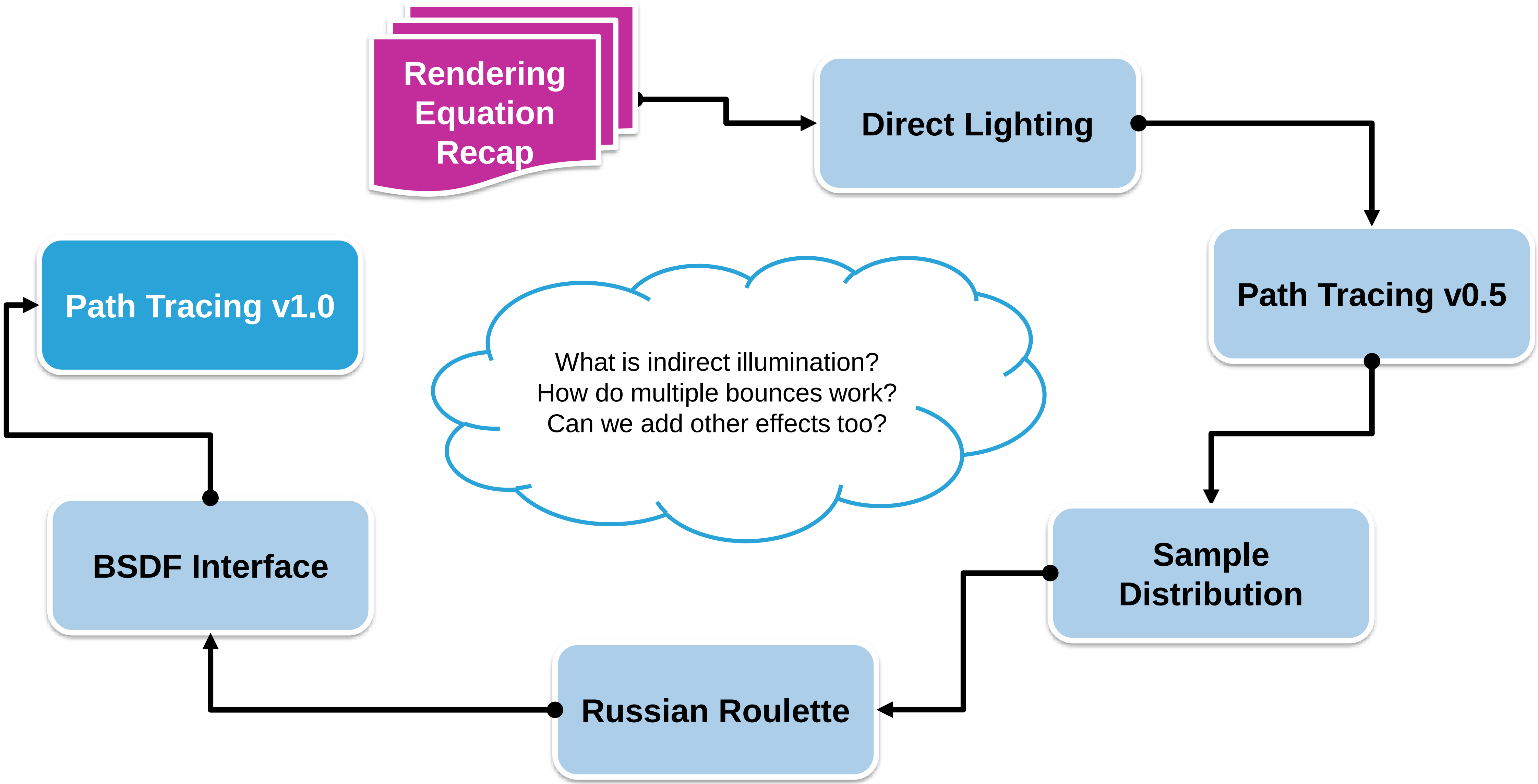
What you will need

- A basic scene with objects, some of which emit light
- An output image to store color information in
- A camera model in the scene for which you can make rays
  - E.g., with this detailed description by Scratchapixel
- A function to trace rays through your scene and find the closest hit
  - E.g., using the famous Möller-Trumbore algorithm
- Additional information for each hit point (normal, material, object)
  - So you can detect if an object was a light source, or
  - Use surface normals for computations
  - …

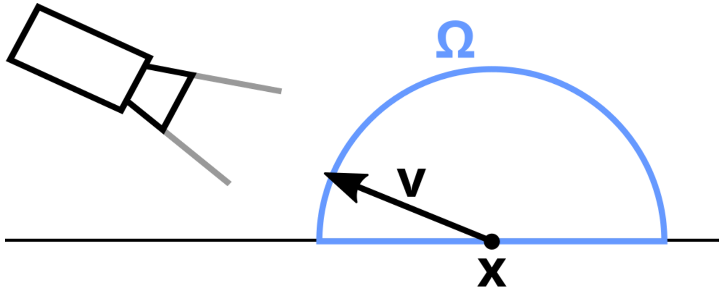
Today's Roadmap



Today's Roadmap



Recursive Rendering Equation, Recap



Material, modelled by the BRDF

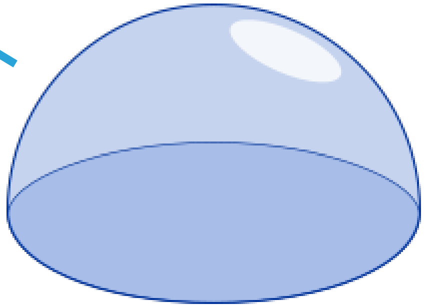
Light from direction ω

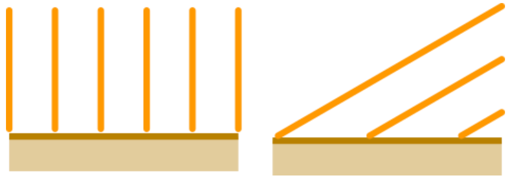
Solid angle

$$L_e(x, v) = E(x, v) + \int_{\Omega} f_r(x, \omega \to v) L_i(x, \omega) \cos(\theta_x) \, \mathrm{d}\omega$$

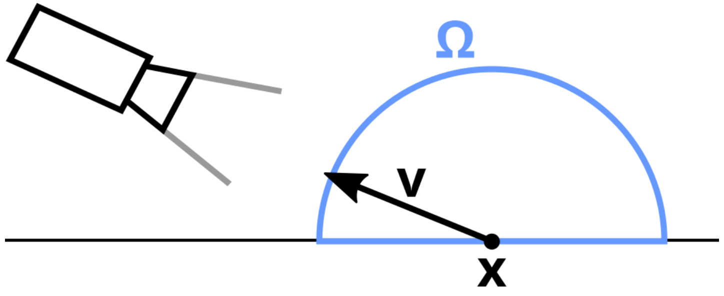
Light going in direction v

Light emitted from x in direction v





Recursive Rendering Equation, Recap



Material, modelled by the BRDF

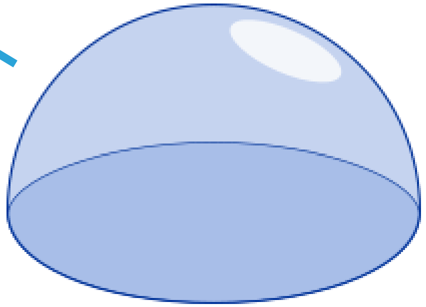
Evaluate light from direction ω **recursively**

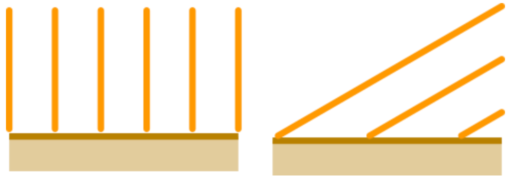
Solid angle

$$L_e(x,v) = E(x,v) + \int_\Omega f_r(x,\omega \to v) L_i(x,\omega) \cos(\theta_x)\, \mathrm{d}\omega$$

Light going in direction v

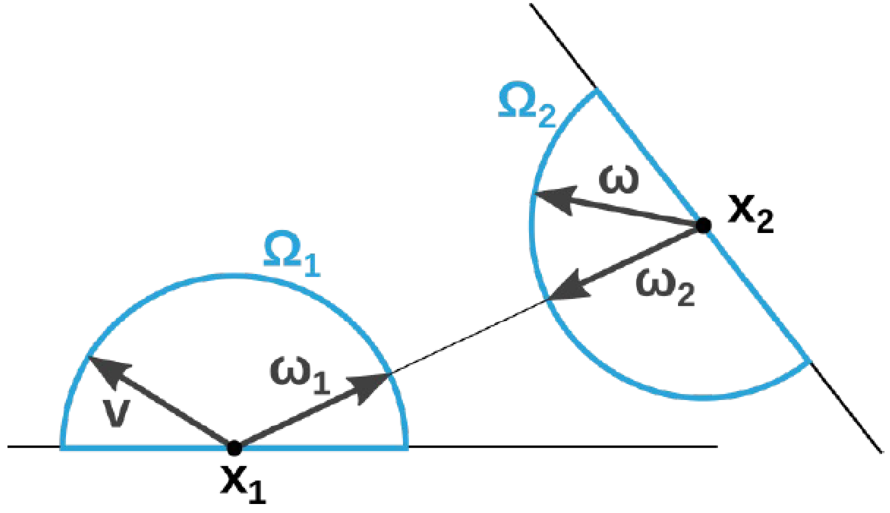
Light emitted from x in direction v

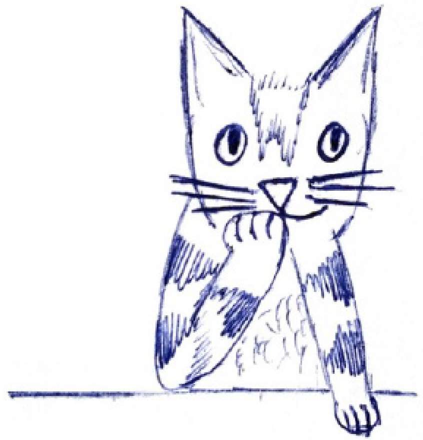




Recursive Rendering Equation, Recap

- To get the next bounce, we just evaluate this function recursively



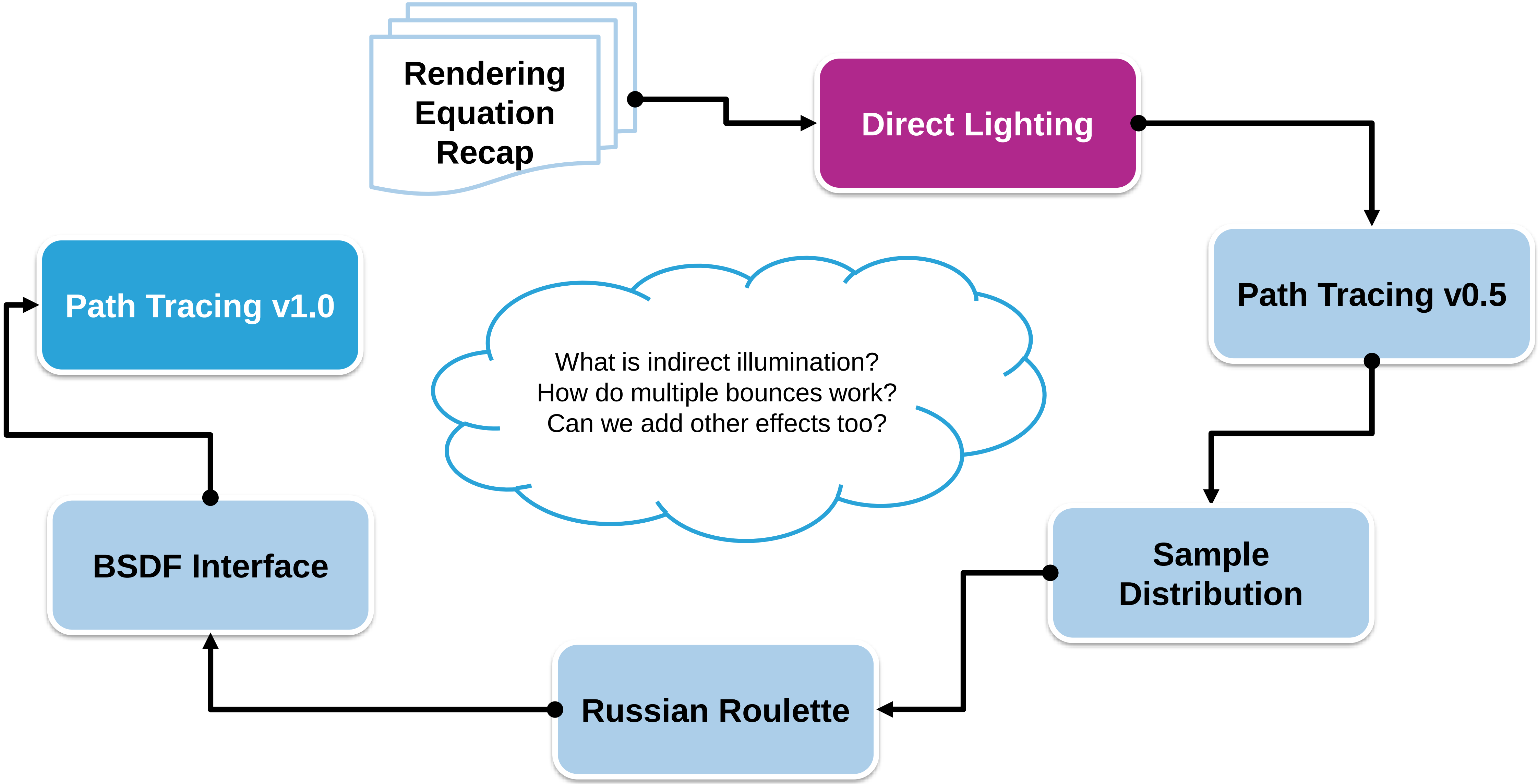


$$L(x_1 \to v) = E(x_1 \to v) + \int_{\Omega_1} f_r(x_1, \omega_1 \to v)\, L(x_1 \leftarrow \omega_1) \cos(\theta_x)\, d\omega_1$$

$$L(x_1 \leftarrow \omega_1) = L(x_2 \to \omega_2) \quad !$$

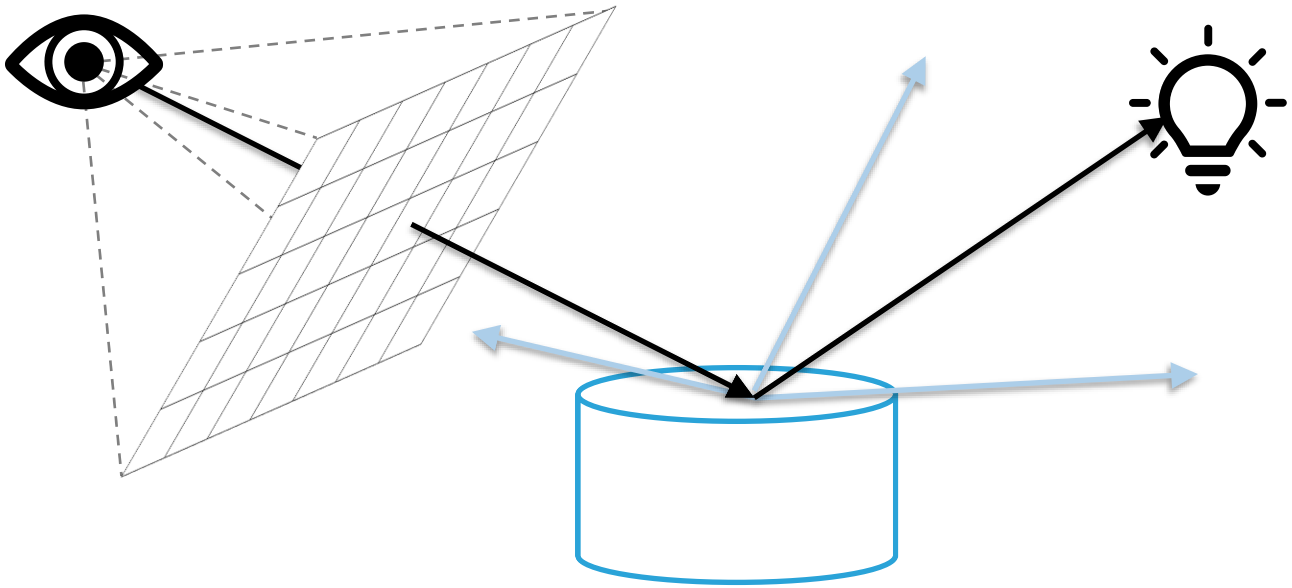
$$L(x_2 \to \omega_2) = E(x_2 \to \omega_2) + \int_{\Omega_2} f_r(x_2, \omega \to \omega_2)\, L(x_2 \leftarrow \omega) \cos(\theta_x)\, d\omega$$

Today's Roadmap



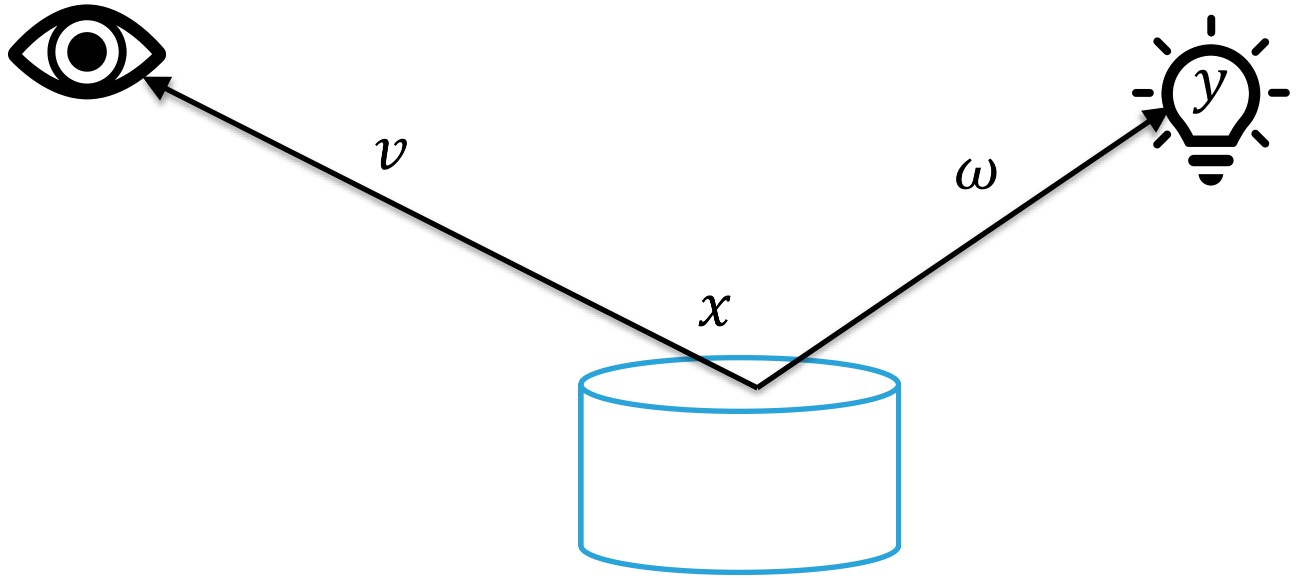
Direct Lighting with the Rendering Equation

- We trace rays through a pixel that randomly bounce into the scene
  - Some will get lucky and hit light sources at first or second hit point!



Direct Lighting with the Rendering Equation

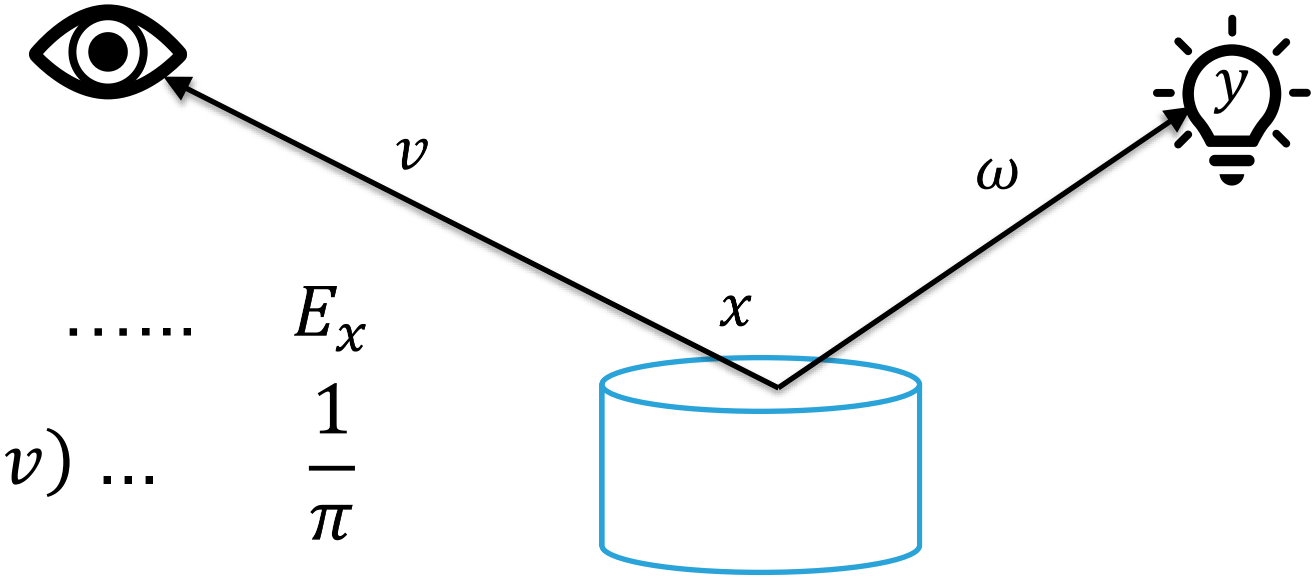
- Let's use the rendering equation to resolve direct light



$$L(x \rightarrow v) = E(x \rightarrow v) + \int_{\Omega} f(x, \omega \rightarrow v) L(x \leftarrow \omega) \cos(\theta_{\omega})\, d\omega$$

Direct Lighting with the Rendering Equation

- Let's simplify our notation a bit for our current scenario



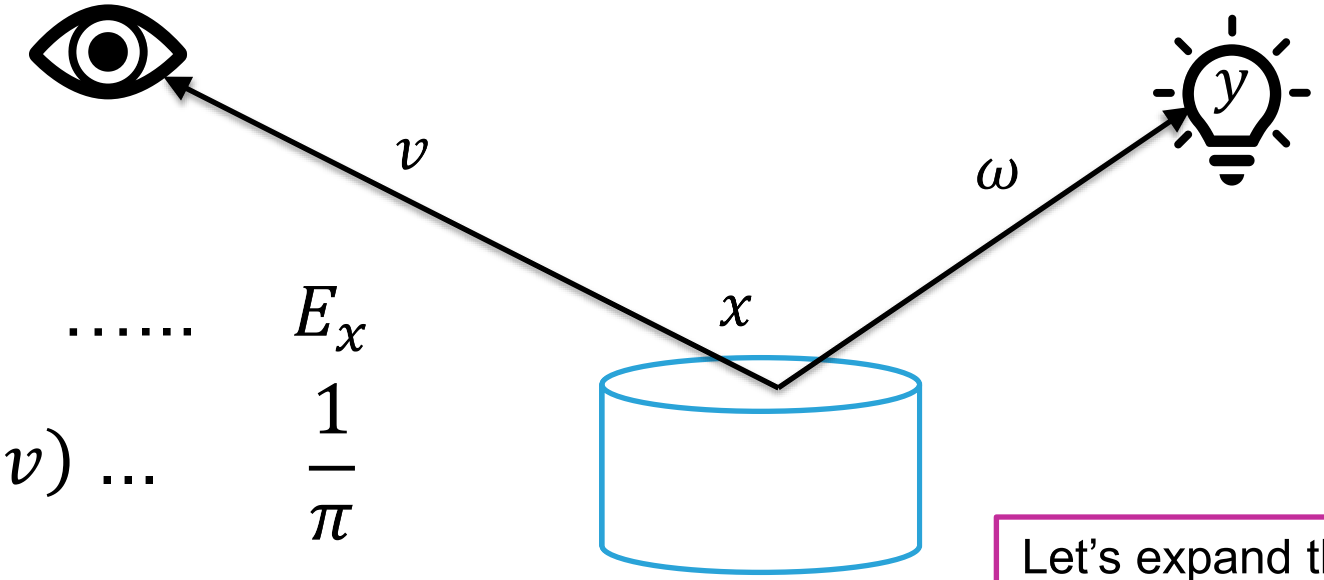
$E(x \to v)$ …… E_x

$f(x, \omega \to v)$ … $\frac{1}{\pi}$

$$L(x \to v) = E_x + \int_{\Omega} \frac{1}{\pi} L(x \leftarrow \omega) \cos(\theta_\omega)\, d\omega$$

Direct Lighting with the Rendering Equation

- Let's simplify our notation a bit for our current scenario



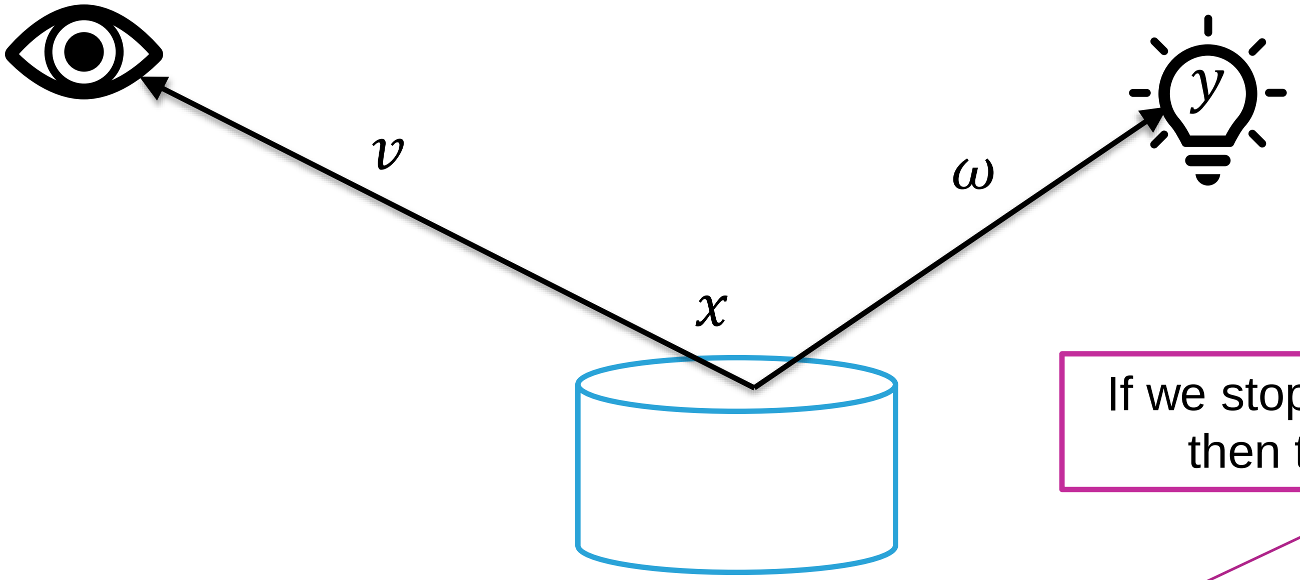
$E(x \to v)$ …… E_x

$f(x, \omega \to v)$ … $\frac{1}{\pi}$

Let's expand this!

$$L(x \to v) = E_x + \int_\Omega \frac{1}{\pi} \boxed{L(x \leftarrow \omega)} \cos(\theta_\omega)\, d\omega$$

- If all we look for is direct lighting, then we stop after the first bounce

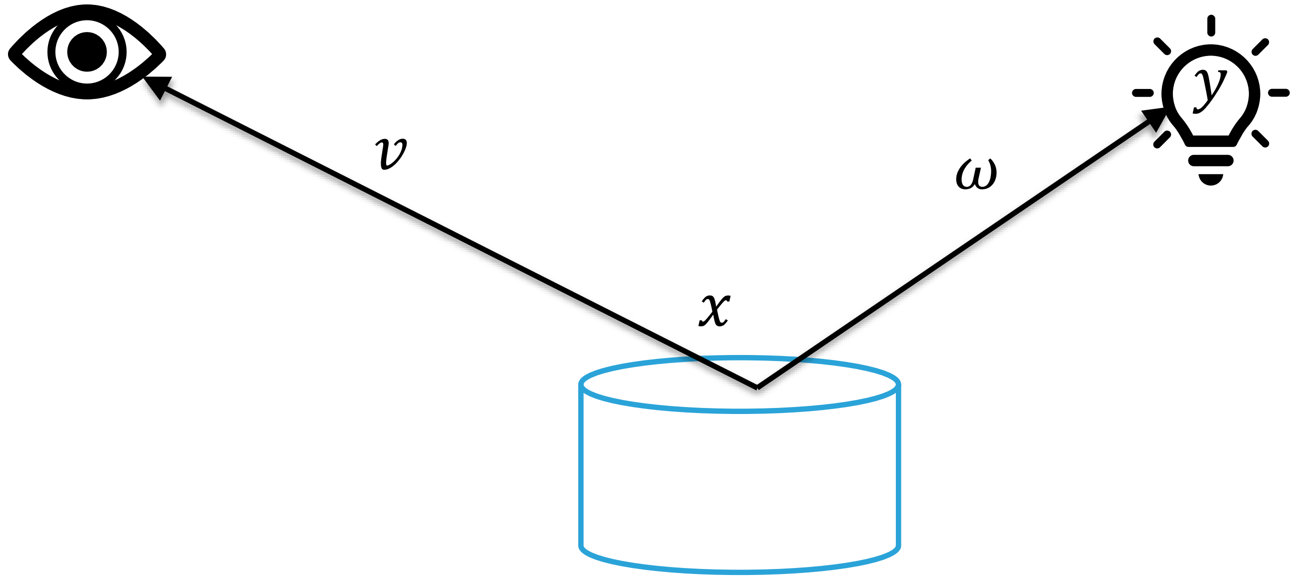


If we stop after one bounce, then this becomes 0

$$L(x \to v) = E_x + \int_{\Omega} \frac{1}{\pi} \left(E_y + \int_{\Omega'} \ldots\right) \cos(\theta_\omega)\, d\omega$$

Direct Lighting with the Rendering Equation

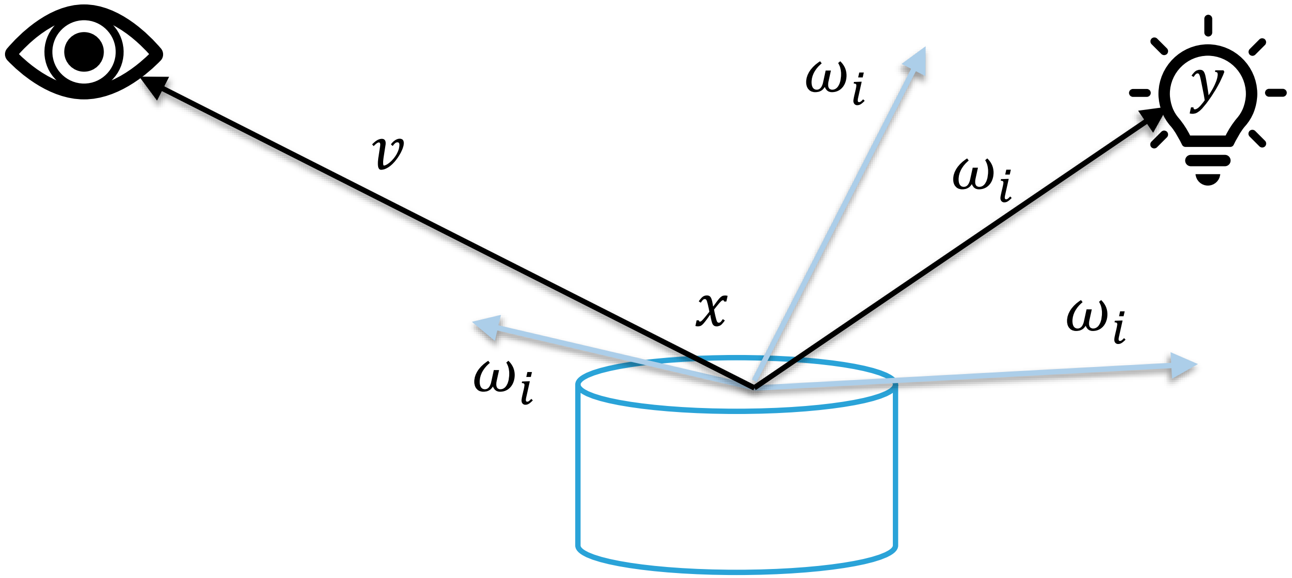
- If all we look for is direct lighting, then we stop after the first bounce



$$L(x \to v) = E_x + \int_{\Omega} \frac{1}{\pi} E_y \cos(\theta_\omega)\, d\omega$$

Direct Lighting with the Rendering Equation

- Replace indefinite integral with Monte Carlo integral



$$L(x \rightarrow v) = E_x + \frac{1}{N}\sum_{i=1}^{N}\left(\frac{1}{\pi}E_y \cos(\theta_{\omega_i})\frac{1}{p(\omega_i)}\right)$$

A quick word on ω and $p(\omega)$

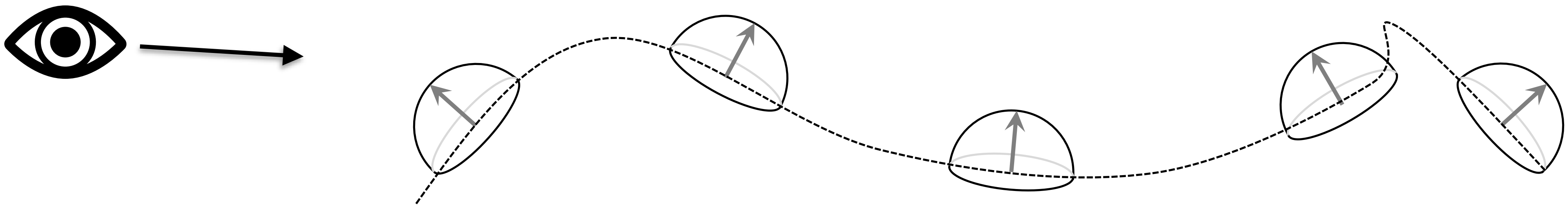
- Ensure for Monte Carlo integration that ω and $p(\omega)$ match
  - This can actually be quite tricky!

- You can use **uniform** hemisphere sampling (we will derive it later)
  - For each ω, draw two uniform random numbers x_1, x_2 in range [0, 1)
  - Calculate $\cos(\theta) = x_1, \sin(\theta) = \sqrt{1 - \cos^2(\theta)}$
  - Calculate $\cos(\phi) = \cos(2\pi x_2)\,, \sin(\phi) = \sin(2\pi x_2)$
  - $\omega = Vector3(\cos(\phi)\sin(\theta)\,, \sin(\phi)\sin(\theta), \cos(\theta))$
  - $p(\omega) = \frac{1}{2\pi}$ (yes, always!)

Bring ω into "world space"

- Resulting ω is in local coordinate frame: `z` axis is normal to surface

- To intersect scene, rays will usually need to be in *world space*



- Use coordinate transform between local and world[1]. Nori users:
  - From world space to local → `Intersection::toLocal()`
  - From local to world space → `Intersection::shFrame::toWorld()`

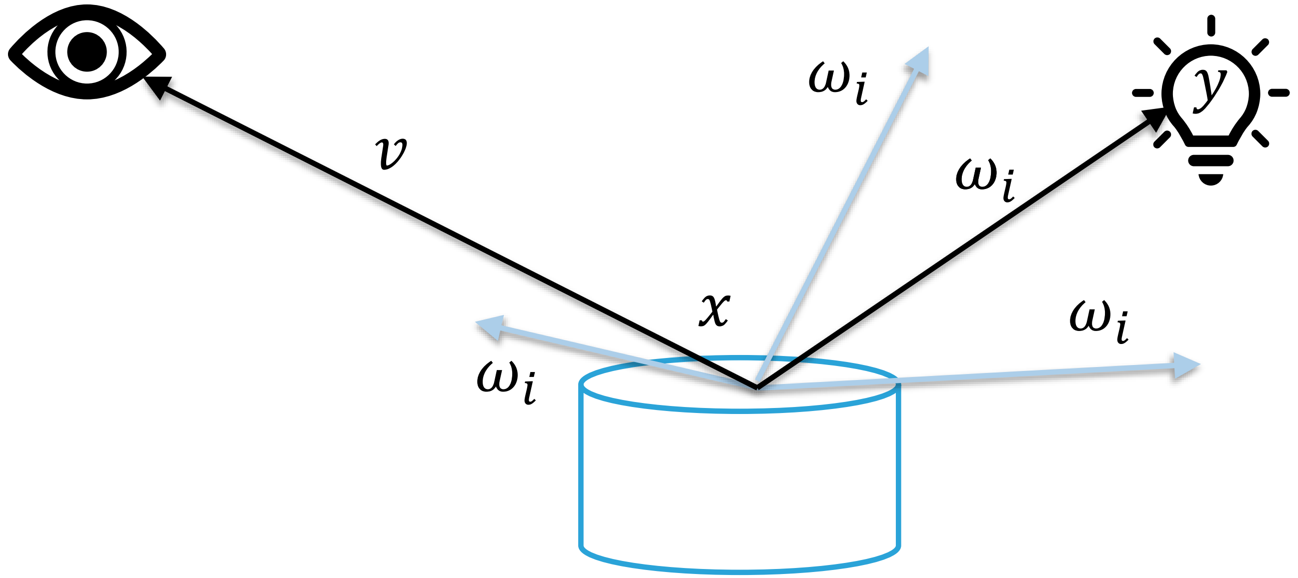
Pseudo Code for Direct Lighting of each Pixel `(px, py)`

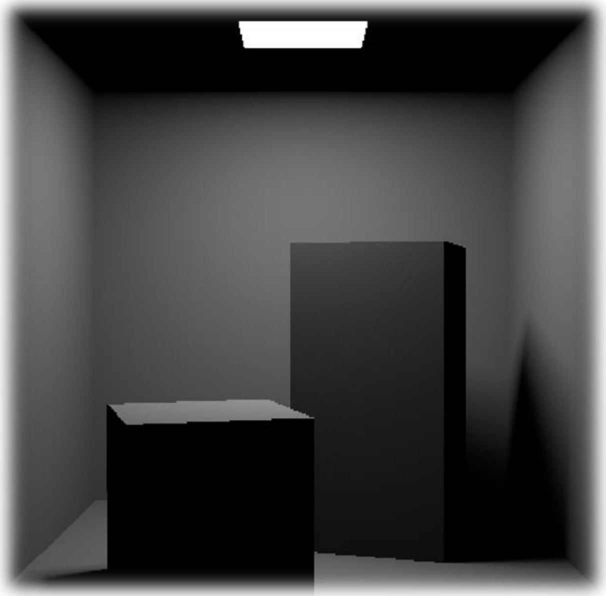
```
v_inv = camera.gen_ray(px, py)
x = scene.trace(v_inv)
pixel_color = x.emit
f = 0
for (i = 0; i < N; i++)
    omega_i, prob_i = hemisphere_uniform_world(x)
    r = make_ray(x, omega_i)
    y = scene.trace(r)
    f += 1/pi * y.emit * dot(x.normal, omega_i) / prob_i
pixel_color += f/N
```

$$L(x \to v) = E_x + \frac{1}{N}\sum_{i=1}^{N}\left(\frac{1}{\pi} E_y \cos\left(\theta_{\omega_i}\right) \frac{1}{p(\omega_i)}\right)$$

Direct Lighting with the Rendering Equation

- Success!





$$L(x \to v) = E_x + \frac{1}{N} \sum_{i=1}^{N} \left(\frac{1}{\pi} E_y \cos(\theta_{\omega_i}) \frac{1}{p(\omega_i)} \right)$$

We can move the sum out…

```
for (i = 0; i < N; i++)
      v_inv = camera.gen_ray(px, py)
      x = scene.trace(v_inv)
      f = x.emit
      omega_i, prob_i = hemisphere_uniform_world(x)
      r = make_ray(x, omega_i)
      y = scene.trace(r)
      f = 1/pi * y.emit * dot(x.normal, omega_i) / prob_i
      pixel_color += f
pixel_color /= N
```

$$L(x \to v) = \frac{1}{N}\sum_{i=1}^{N}\left(E_x + \frac{1}{\pi}E_y \cos(\theta_{\omega_i})\frac{1}{p(\omega_i)}\right)$$

...and call a function to compute color from the view ray

```
for (i = 0; i < N; i++)
     v_inv = camera.gen_ray(px, py)
     pixel_color += Li(v_inv)
pixel_color /= N
```

```
function Li(v_inv)      // can be anything! Direct light, AO…
     …                  // Must return a Monte Carlo f(x)/p(x)
```

Note: In our test framework, Nori, the different *integrators* will implement different behaviors for *Li*. The main loop over N takes care of the **sum** and the **division by N**, which are part of a Monte Carlo integral:

$$\frac{1}{N}\sum^{N}\frac{f(x)}{p(x)}$$

It is our job to write integrator functions that return $\frac{f(x)}{p(x)}$ so the result will be a proper Monte Carlo integration.

What if i don‘t hit anything in the scene?

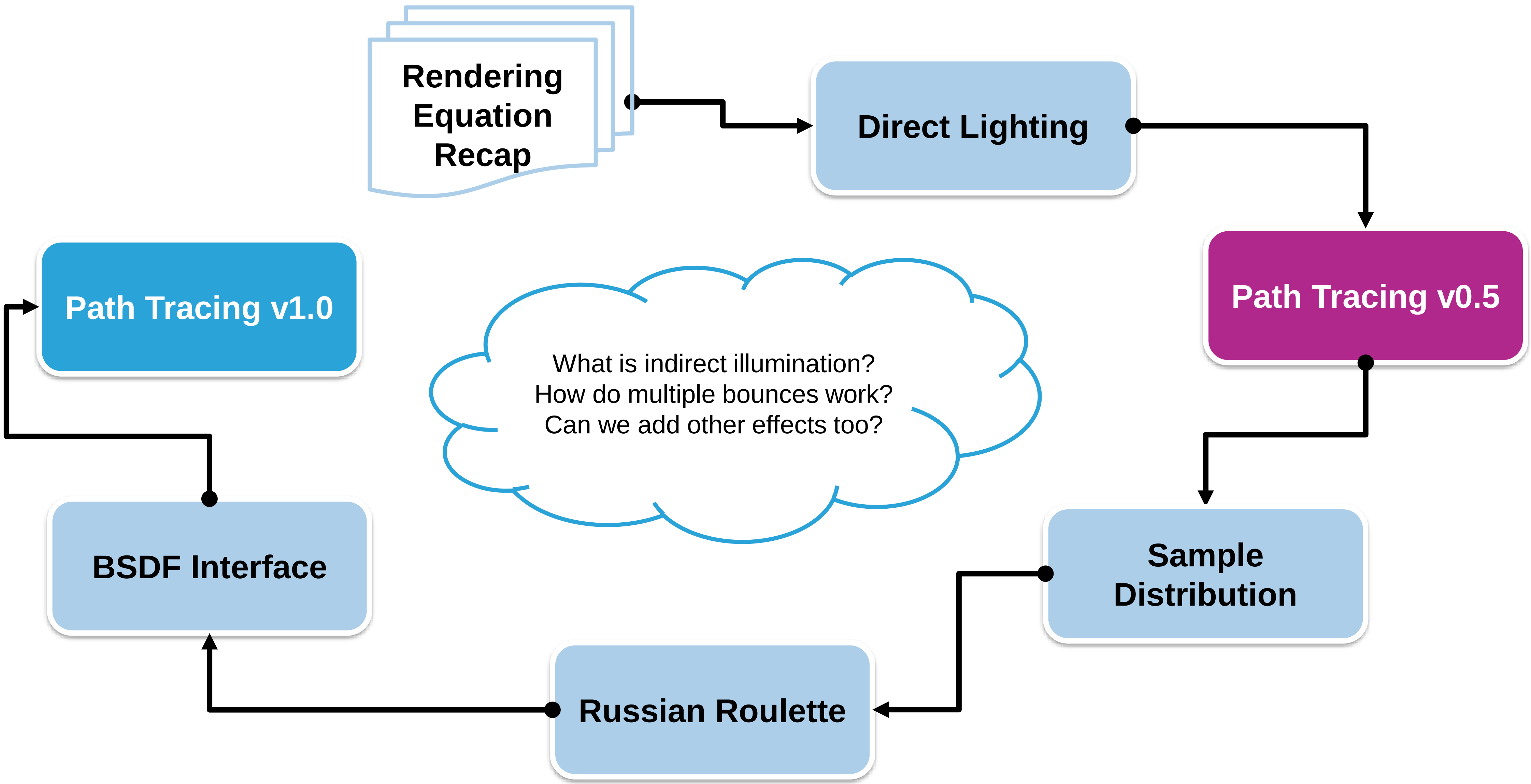
- Importance rays go on forever
- In reality, sooner or later you would hit some sort of medium
- In practice, your digital scene might just „end“
- In that case, there are no follow-up bounces
- Just return 0, or „black“

Path Tracing

Just keep bouncing

…this one weird trick instantly makes your renderings prettier!

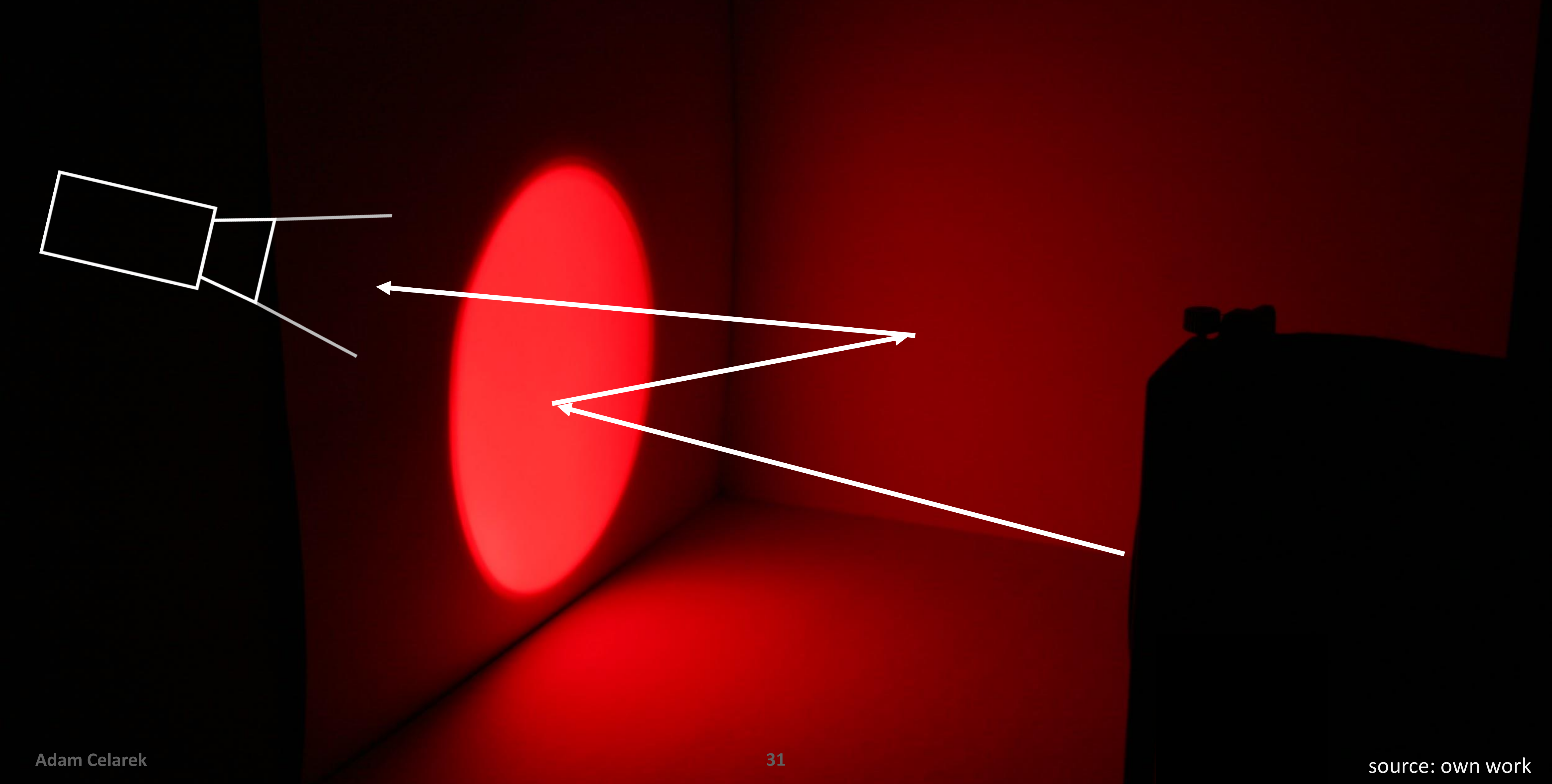
Today's Roadmap



Updating our Material Term

- Before, we assumed diffuse, white material term $\frac{1}{\pi}$
- If you have material information for your hit points, use that instead
- Produce completely diffuse materials, but now with color
- Assumed material information: albedo (diffuse RGB color) in [0, 1)
  - For each hit point, read out the albedo ρ
  - Now we will be using $f_r = \frac{\rho}{\pi}$

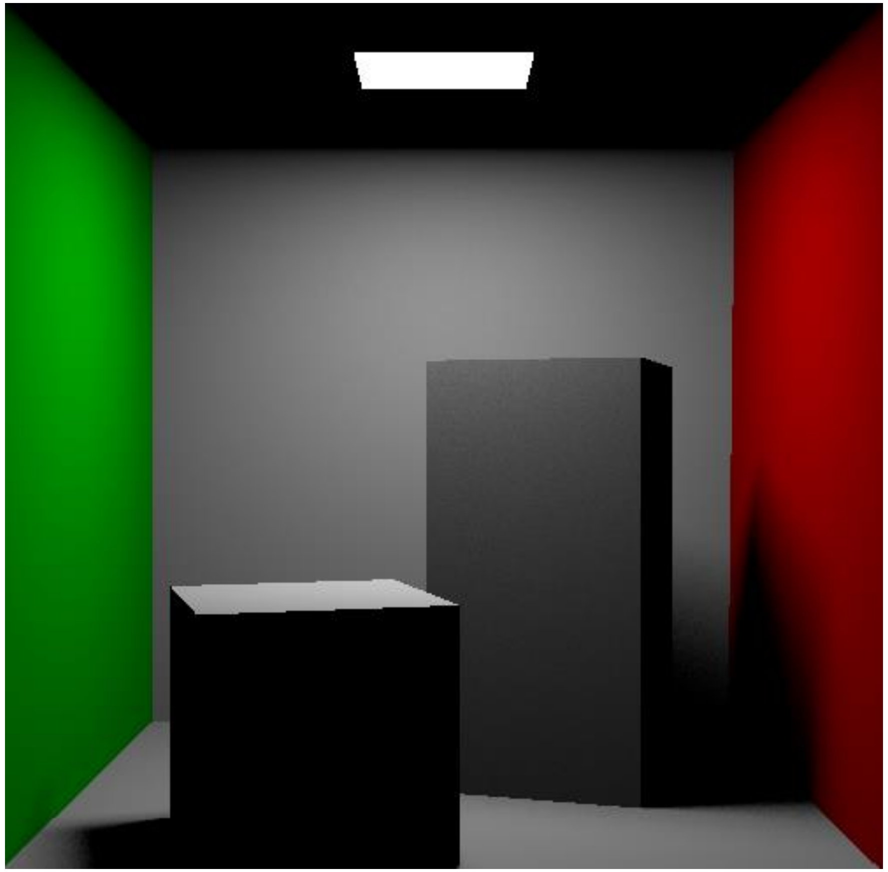
Results from following view path over multiple bounces

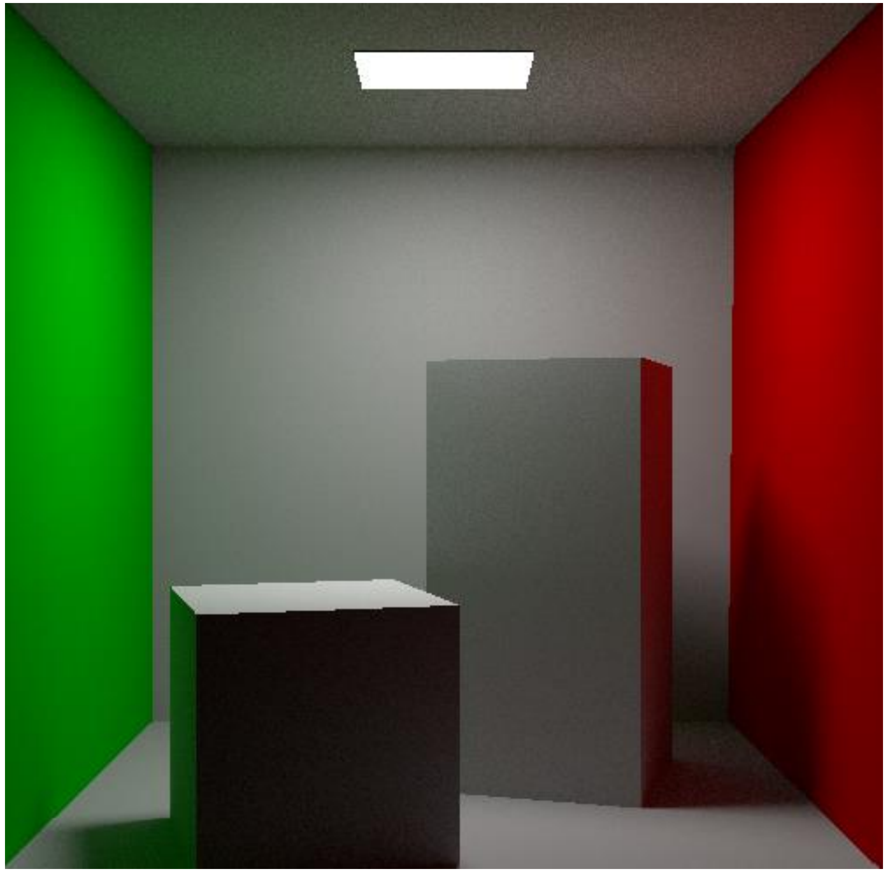


source: own work

Indirect Illumination

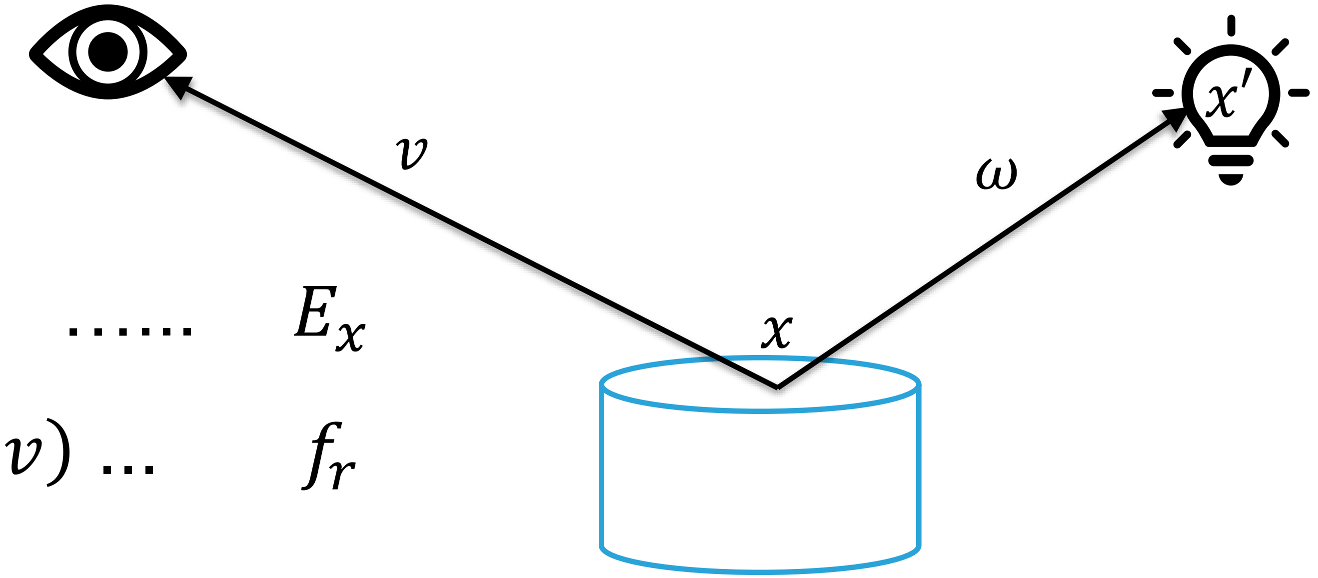
- Difficult in real-time graphics – comes naturally in path tracing!





Indirect Lighting with the Rendering Equation

- Let's go one step further!



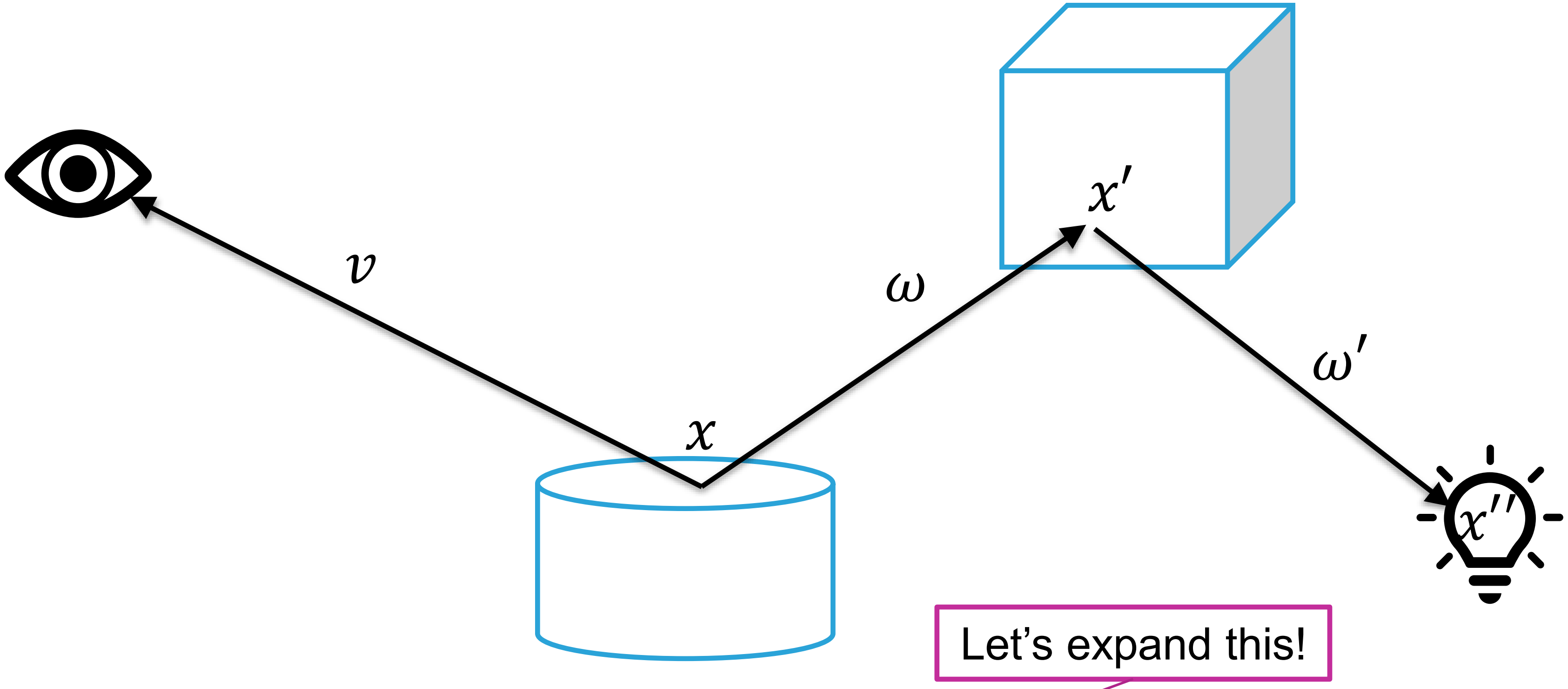
$E(x \rightarrow v)$ …… E_x

$f(x, \omega \rightarrow v)$ … f_r

$$L(x \rightarrow v) = E_x + \int_\Omega f_r \, L(x \leftarrow \omega) \cos(\theta_\omega) \, d\omega$$

Indirect Lighting with the Rendering Equation

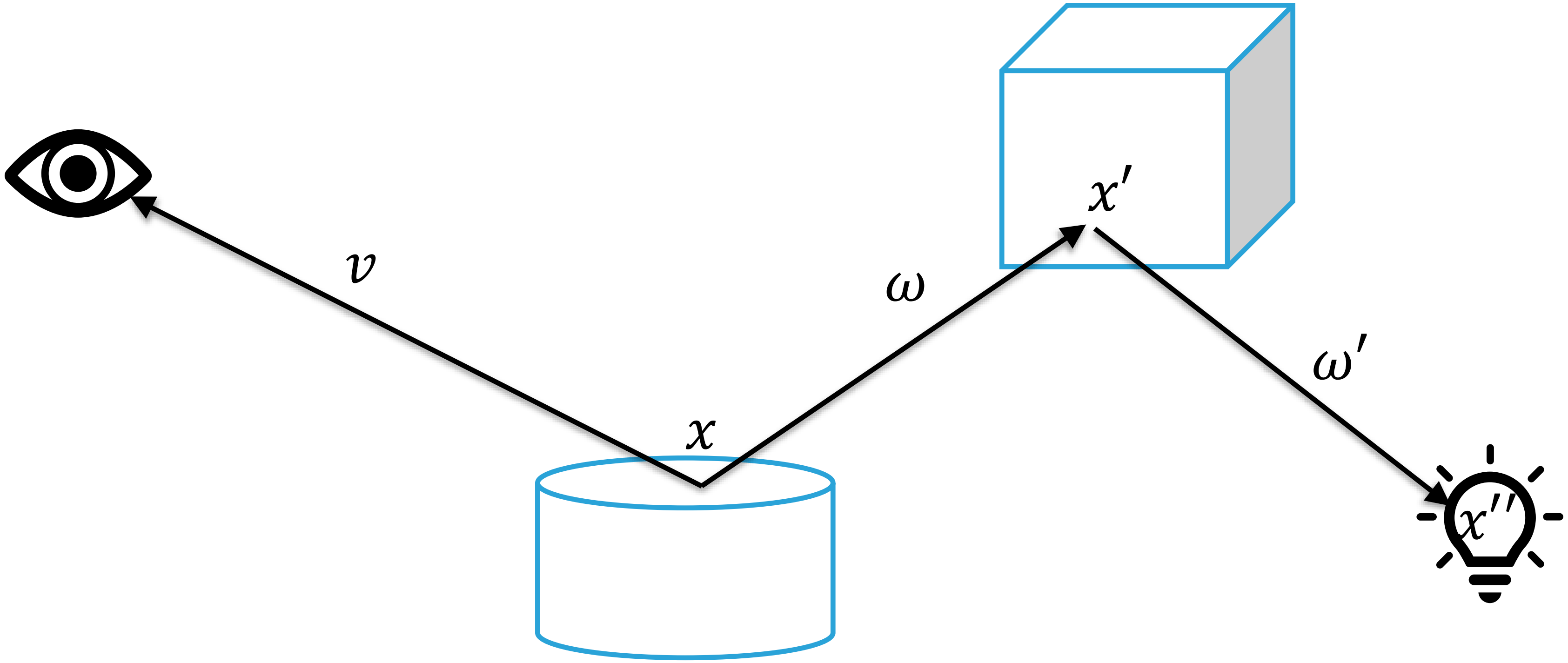
- Let's go one step further!



$$L(x \to v) = E_x + \int_\Omega f_r\, L(x \leftarrow \omega) \cos(\theta_\omega)\, d\omega$$

Indirect Lighting with the Rendering Equation

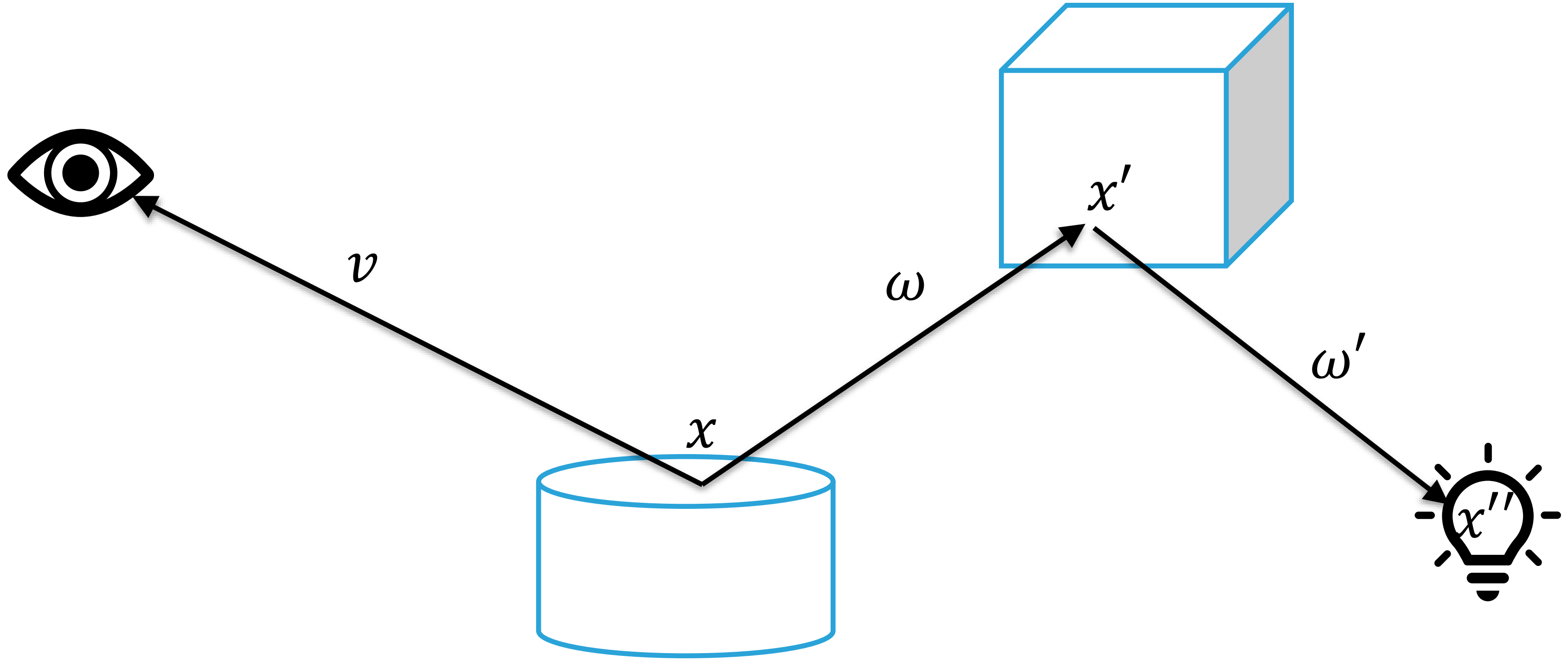
- Let's look at this new equation in detail…



$$L(x \to v) = E_x + \int_{\Omega} f_r \left(E_{x'} + \int_{\Omega'} f_r' \, L(x' \leftarrow \omega') \cos(\theta_{\omega'}) \, d\omega' \right) \cos(\theta_\omega) \, d\omega$$

Indirect Lighting with the Rendering Equation

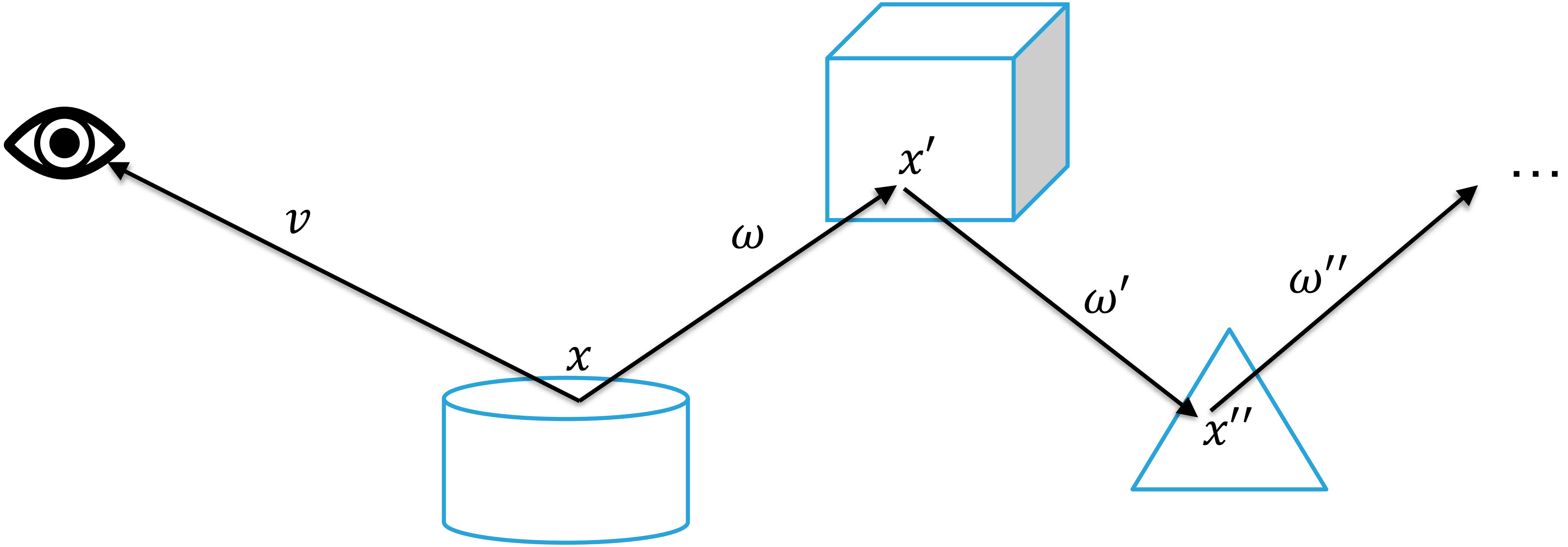
- A pattern emerges!



$$L(x \to v) = E_x + \int_\Omega f_r \left(E_{x'} + \int_{\Omega'} f_r' \, L(x' \leftarrow \omega') \cos(\theta_{\omega'}) \, d\omega' \right) \cos(\theta_\omega) \, d\omega$$

Indirect Lighting with the Rendering Equation

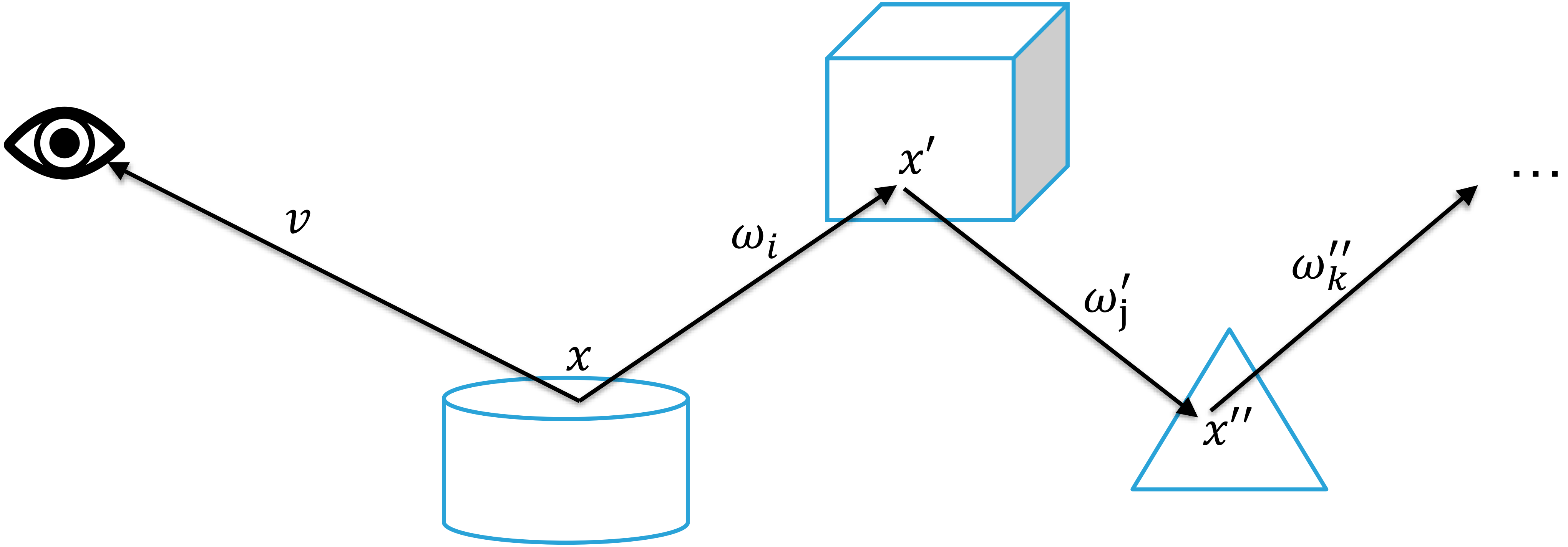
- A real solution could go on much longer (ideally indefinitely)



$$L(x \to v) = E_x + \int_{\Omega} f_r \left(E_{x'} + \int_{\Omega'} f_r' \quad \ldots \quad \cos(\theta_{\omega'})\, d\omega' \right) \cos(\theta_{\omega})\, d\omega$$

Indirect Lighting with the Rendering Equation

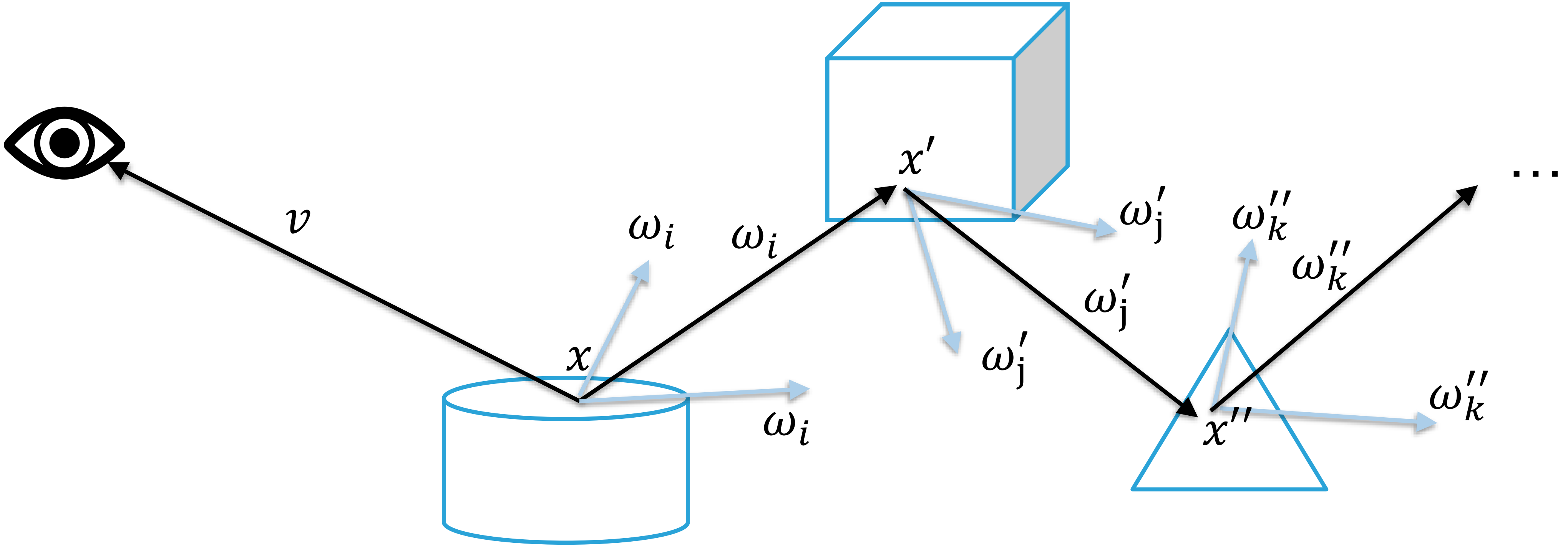
- With Monte Carlo integration, we might replace all $\int$ with $\sum$



$$L(x \to v) = E_x + \frac{1}{N}\sum_{i=1}^{N} f_r \left(E_{x'} + \frac{1}{N}\sum_{j=1}^{N} f'_r \ \ldots \ \cos\left(\theta_{\omega'_j}\right)\frac{1}{p(\omega'_j)} \right) \cos\left(\theta_{\omega_i}\right)\frac{1}{p(\omega_i)}$$

Indirect Lighting with the Rendering Equation

- With Monte Carlo integration, we might replace all $\int$ with $\sum$



$$L(x \to v) = E_x + \frac{1}{N}\sum_{i=1}^{N} f_r \left(E_{x'} + \frac{1}{N}\sum_{j=1}^{N} f_r' \; \dots \; \cos\left(\theta_{\omega_j'}\right) \frac{1}{p(\omega_j')} \right) \cos\left(\theta_{\omega_i}\right) \frac{1}{p(\omega_i)}$$

Let's implement this with recursion!

```
v_inv = camera.gen_ray(px, py)
pixel_color = Li(v_inv)

function Li(v_inv)
      x = scene.trace(v_inv)
      f = 0
      for (i = 0; i < N; i++)
            omega_i, prob_i = hemisphere_uniform_world(x)
            ray = make_ray(x, omega_i)
            f += x.alb/pi * Li(ray) * dot(x.normal, omega_i) / prob_i
      return x.emit + f/N
```

$$L(x \rightarrow v) = E_x + \frac{1}{N} \sum_{i=1}^{N} f_r \; L(x \leftarrow \omega_i) \; \cos(\theta_{\omega_i}) \frac{1}{p(\omega_i)}$$

Let‘s run it!

- Each hemisphere integral computed with N samples (seems fair!)
- Question: what to pick for N?
  - Let‘s start with something low
  - Like 4, 8, 16, 32
- Let‘s see what we get…
- Renderer never stops. We need a stopping criterion!

Adding hard stopping criterion

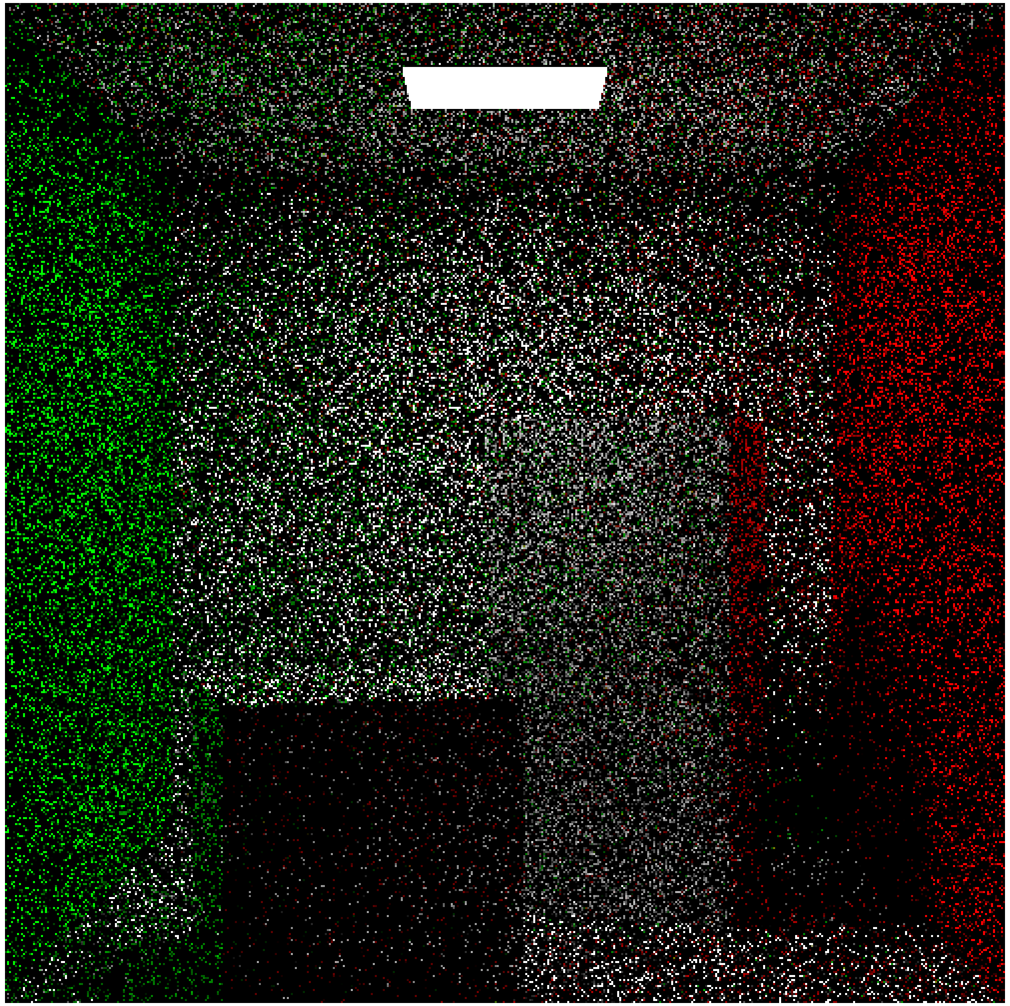
```
v_inv = camera.gen_ray(px, py)
pixel_color = Li(v_inv, 0)

function Li(v_inv, D)
      if (D >= NUM_BOUNCES)
            return 0
      x = scene.trace(v_inv)
      f = 0
      for (i = 0; i < N; i++)
            omega_i, prob_i = hemisphere_uniform_world(x)
            ray = make_ray(x, omega_i)
            f += x.alb/pi * Li(ray, D+1) * dot(x.normal, omega_i) / prob_i
      return x.emit + f/N
```

Let's run this again!

- NUM_BOUNCES = 3
- Samples for each sum and runtime

N	Time
4	3s
8	
16	
32	

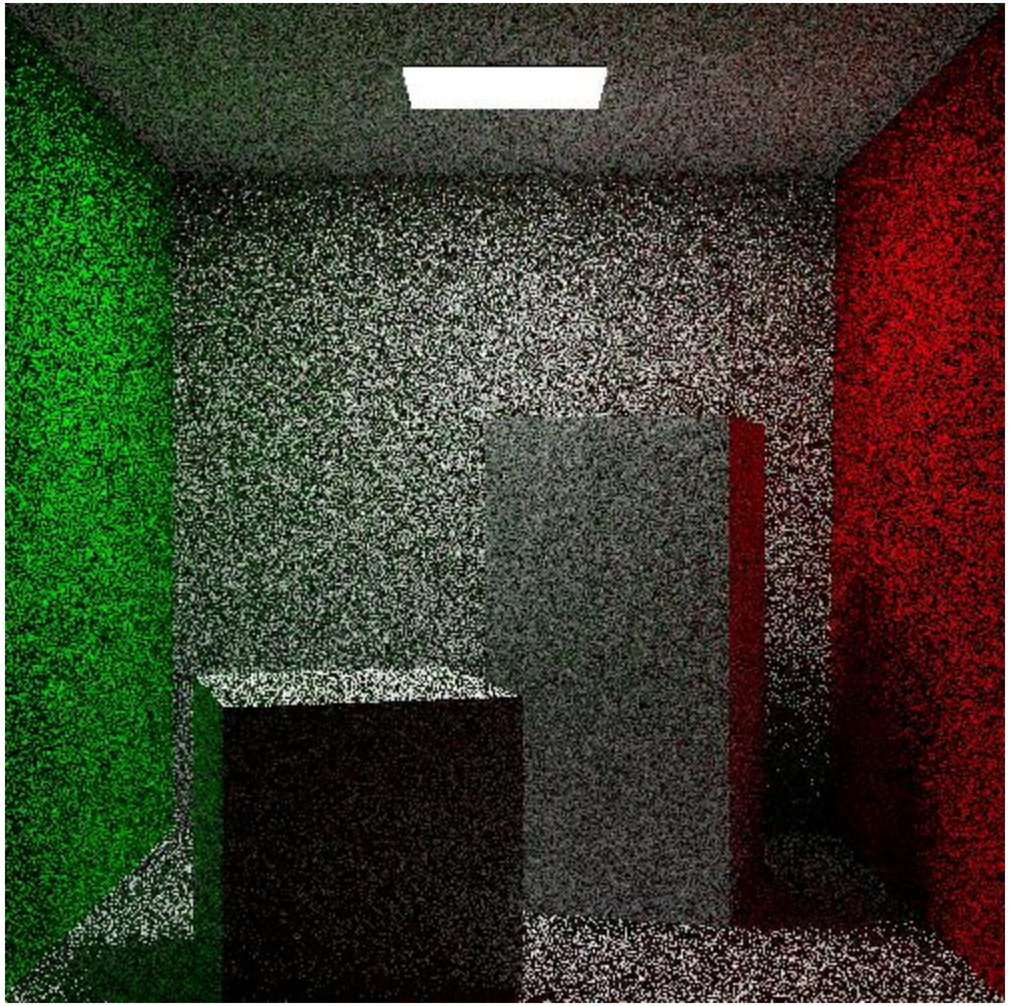


Let‘s run this again!

- NUM_BOUNCES = 3

- Samples for each sum and runtime

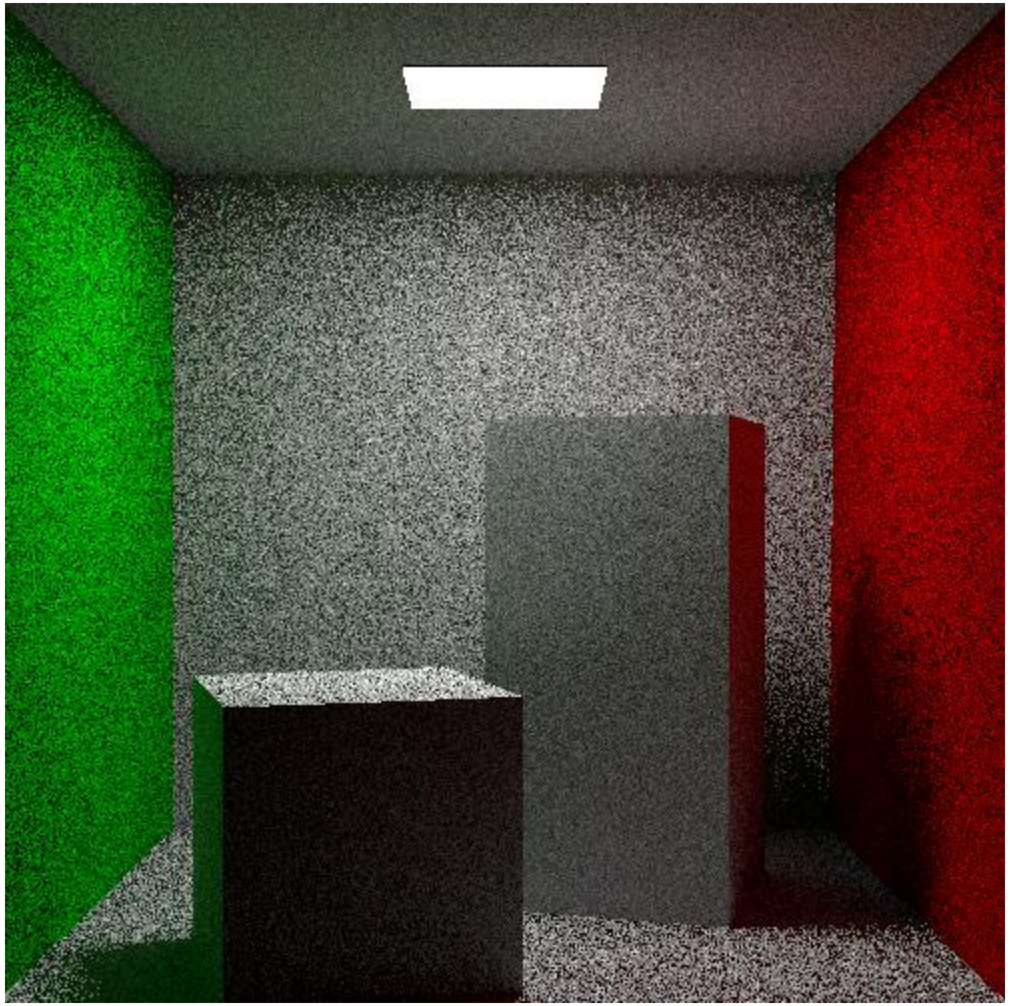
N	Time
4	3s
8	22s
16	
32	



Let‘s run this again!

- NUM_BOUNCES = 3
- Samples for each sum and runtime

N	Time
4	3s
8	22s
16	3m 33s
32	

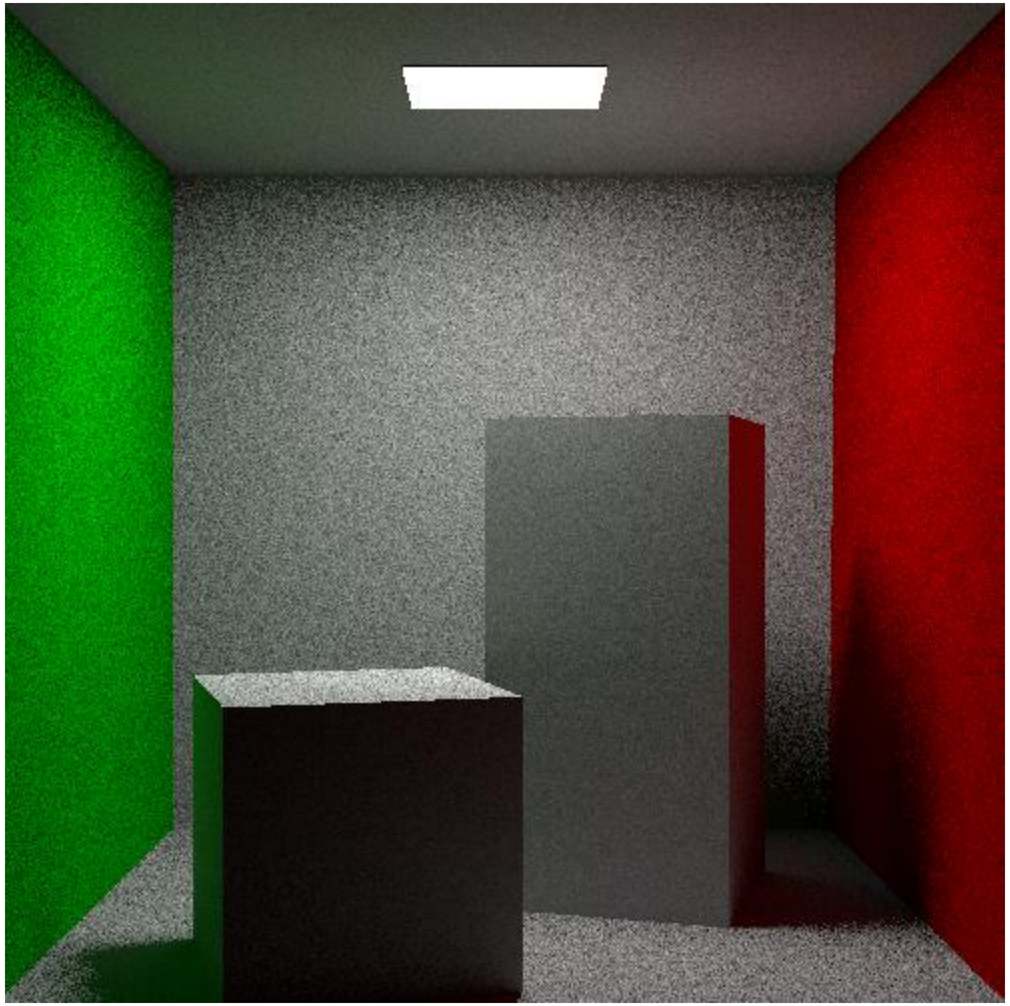


Let‘s run this again!

- NUM_BOUNCES = 3

- Samples for each sum and runtime

N	Time
4	3s
8	22s
16	3m 33s
32	43m 20s

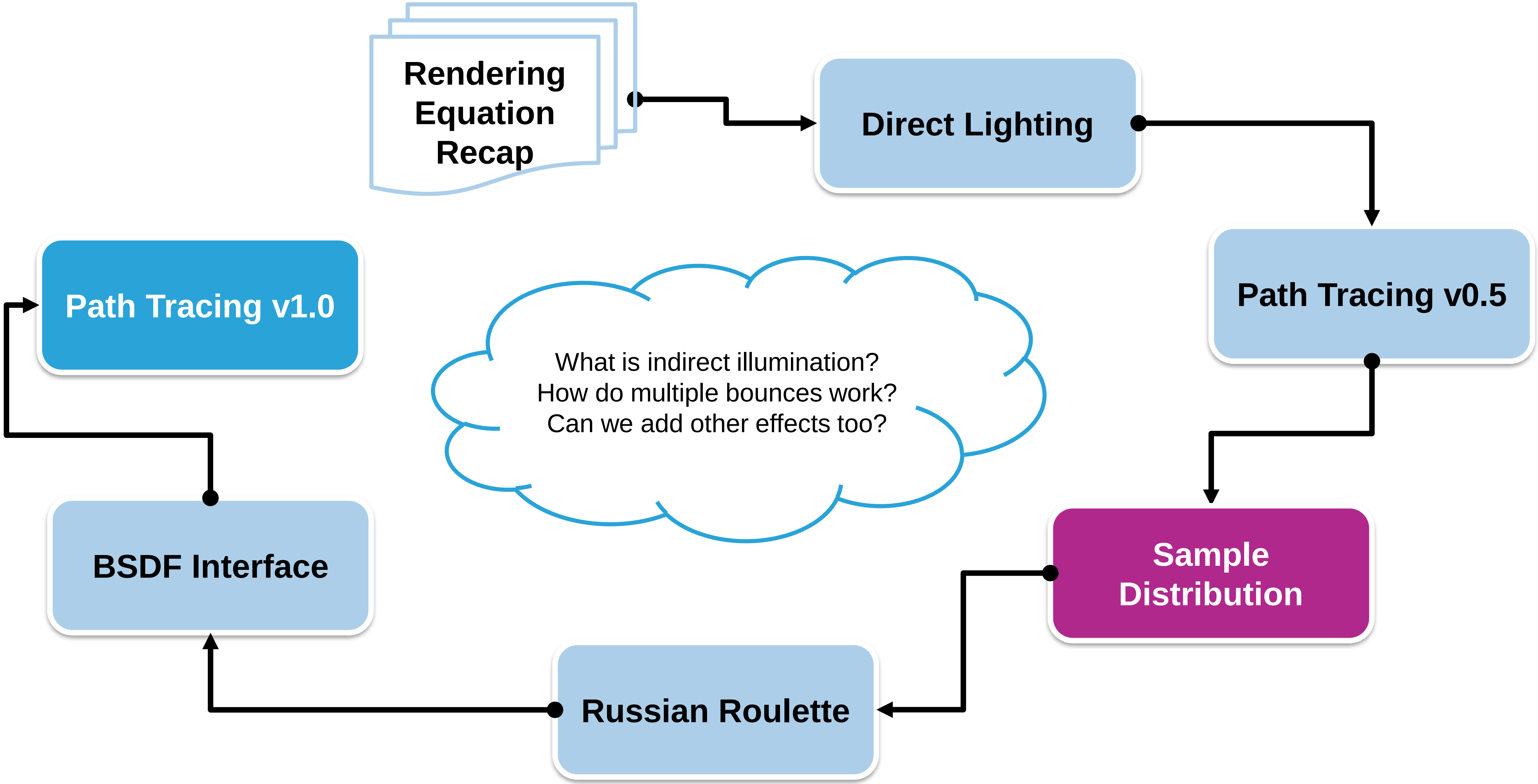


Wisdom of the Day

The more samples, the better

…and also slower

Today's Roadmap



It‘s time to reconsider our approach

- It works, but it‘s slow and the quality is not great
- Also, this was only 3 bounces
- Advanced effects can require much more than that!
- Were we too naive, is path tracing doomed?
  - Definitely no!
  - There‘s a vast range of tools we can use to raise performance

Reconsidering Sample Distribution

- We can write this one big integral slightly differently

$$L(x \to v) = E_x + \int_{\Omega} f_r \left(E_{x'} + \int_{\Omega'} f_r' \quad \dots \quad \cos(\theta_{\omega'})\, d\omega' \right) \cos(\theta_\omega)\, d\omega$$

$$\begin{aligned} L(x \to v) = E_x & \\ & + \int_{\Omega} f_r\, E_{x'} \cos(\theta_\omega)\, d\omega \\ & + \int_{\Omega} f_r \int_{\Omega'} f_r'\, E_{x''} \cos(\theta_{\omega'}) \cos(\theta_\omega)\, d\omega' d\omega \\ & + \int_{\Omega} f_r \int_{\Omega'} f_r' \int_{\Omega''} f_r''\, E_{x'''}\, cos(\theta_{\omega''}) \cos(\theta_{\omega'}) \cos(\theta_\omega)\, d\omega'' d\omega' d\omega \\ & + \dots \end{aligned}$$

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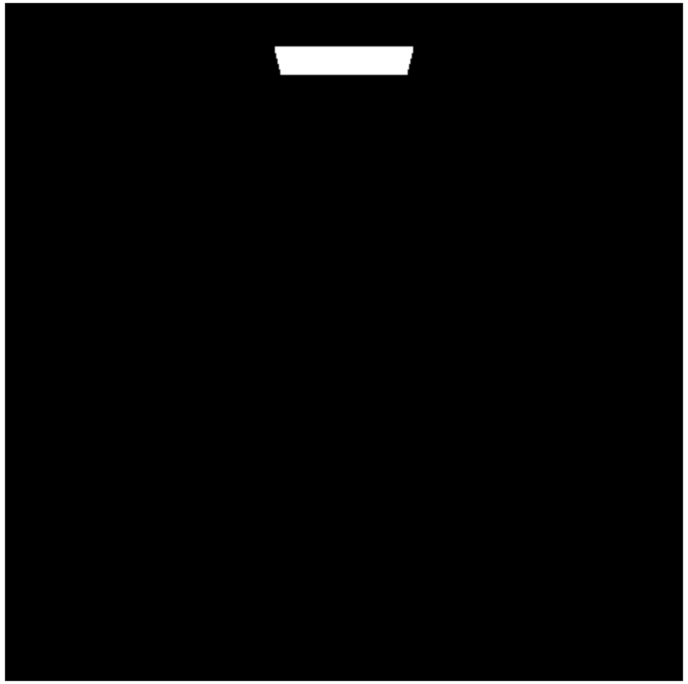
$$\begin{aligned} L(x \rightarrow v) = & \boxed{E_x} \\ & + \int_{\Omega} f_r \boxed{E_{x'}} \cos(\theta_\omega)\, d\omega \\ & + \int_{\Omega} f_r \int_{\Omega'} f_r' \boxed{E_{x''}} \cos(\theta_{\omega'}) \cos(\theta_\omega)\, d\omega' d\omega \\ & + \int_{\Omega} f_r \int_{\Omega'} f_r' \int_{\Omega''} f_r'' \boxed{E_{x'''}} \cos(\theta_{\omega''}) \cos(\theta_{\omega'}) \cos(\theta_\omega)\, d\omega'' d\omega' d\omega \\ & + \dots \end{aligned}$$

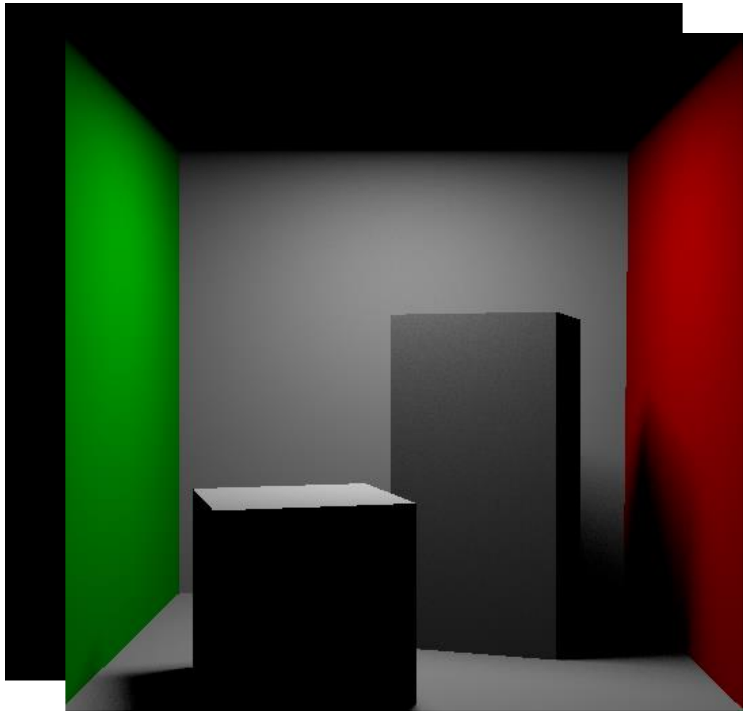
Can you guess what each of these lines does for our final image?

Reconsidering Sample Distribution



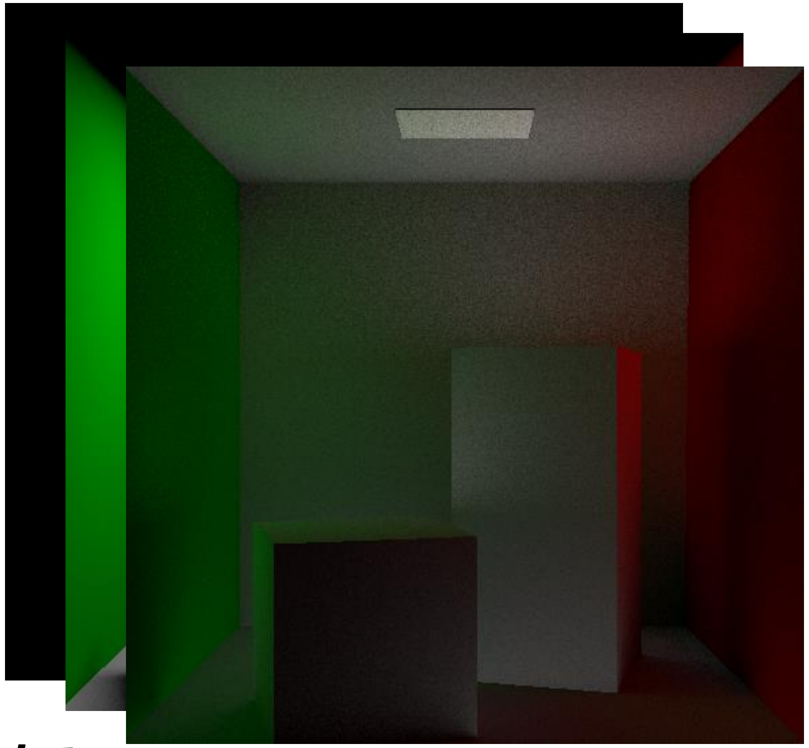
$$
\begin{aligned}
L(x \rightarrow v) = E_x \\
&+ \int_{\Omega} f_r \, E_{x'} \cos(\theta_\omega) \, d\omega \\
&+ \int_{\Omega} f_r \int_{\Omega'} f_r' \, E_{x''} \cos(\theta_{\omega'}) \cos(\theta_\omega) \, d\omega' d\omega \\
&+ \int_{\Omega} f_r \int_{\Omega'} f_r' \int_{\Omega''} f_r'' \, E_{x'''} \cos(\theta_{\omega''}) \cos(\theta_{\omega'}) \cos(\theta_\omega) \, d\omega'' d\omega' d\omega \\
&+ \ \ldots
\end{aligned}
$$

Reconsidering Sample Distribution



$$L(x \to v) = E_x$$
$$+ \int_\Omega f_r \, E_{x'} \cos(\theta_\omega) \, d\omega$$
$$+ \int_\Omega f_r \int_{\Omega'} f_r' \, E_{x''} \cos(\theta_{\omega'}) \cos(\theta_\omega) \, d\omega' d\omega$$
$$+ \int_\Omega f_r \int_{\Omega'} f_r' \int_{\Omega''} f_r'' \, E_{x'''} \cos(\theta_{\omega''}) \cos(\theta_{\omega'}) \cos(\theta_\omega) \, d\omega'' d\omega' d\omega$$
$$+ \ \ldots$$

Reconsidering Sample Distribution



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$$+ \int_\Omega f_r \, E_{x'} \cos(\theta_\omega) \, d\omega$$
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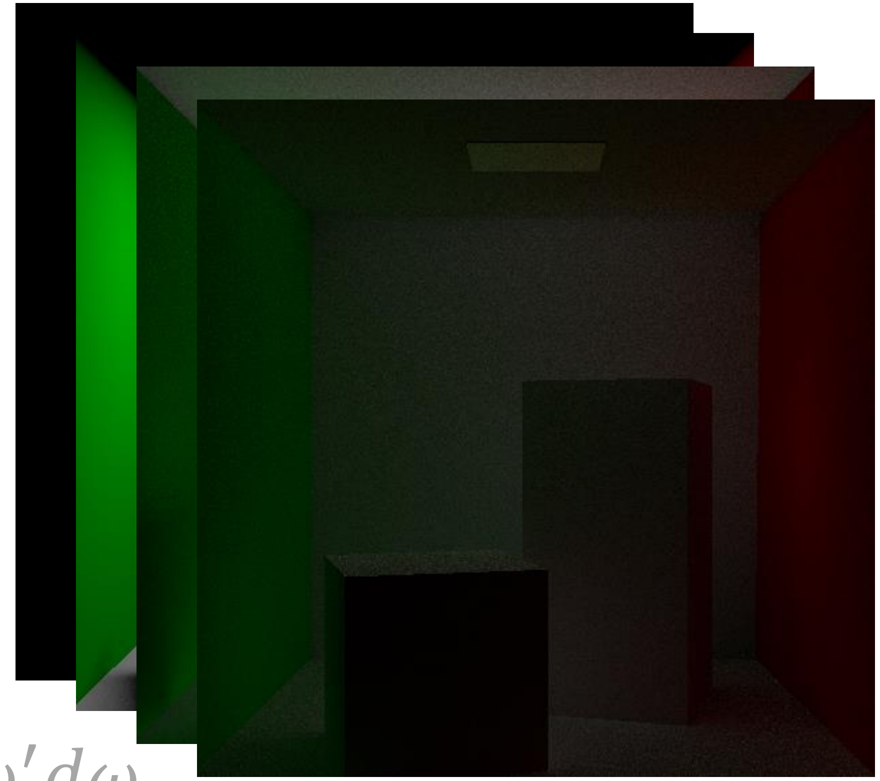
$$+ \ \ldots$$

Reconsidering Sample Distribution



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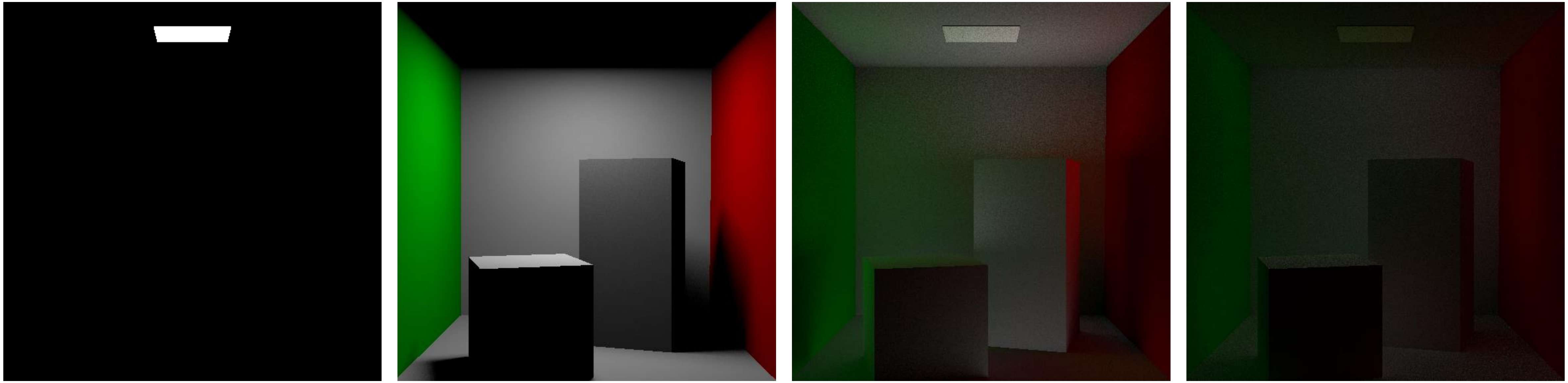
$$+ \int_\Omega f_r \, E_{x'} \cos(\theta_\omega) \, d\omega$$
$$+ \int_\Omega f_r \int_{\Omega'} f_r' \, E_{x''} \cos(\theta_{\omega'}) \cos(\theta_\omega) \, d\omega' d\omega$$
$$+ \int_\Omega f_r \int_{\Omega'} f_r' \int_{\Omega''} f_r'' \, E_{x'''} \cos(\theta_{\omega''}) \cos(\theta_{\omega'}) \cos(\theta_\omega) \, d\omega'' d\omega' d\omega$$
$$+ \ \ldots$$

Reconsidering Sample Distribution

- We used more samples for the contributions from longer light paths
- The return-on-investment is not that great!



1 sample | N samples | N^2 samples | N^3 samples

Reconsidering Sample Distribution

- We have seen a different way of writing these results last time

- The path integral form used a single integral for each bounce!

$$\int_{\Omega_0} \mathrm{f_j}(\bar{x})\, \mathrm{d}\mu(\bar{x}) + \int_{\Omega_1} \mathrm{f_j}(\bar{x})\, \mathrm{d}\mu(\bar{x}) + \ldots + \int_{\Omega_\infty} \mathrm{f_j}(\bar{x})\, \mathrm{d}\mu(\bar{x})$$

$$\begin{aligned} L(x \to v) = E_x & \\ &+ \int_\Omega f_r\, E_{x'} \cos(\theta_\omega)\, d\omega \\ &+ \int_\Omega f_r \int_{\Omega'} f_r'\, E_{x''} \cos(\theta_{\omega'}) \cos(\theta_\omega)\, d\omega' d\omega \\ &+ \int_\Omega f_r \int_{\Omega'} f_r' \int_{\Omega''} f_r''\, E_{x'''}\, cos(\theta_{\omega''}) \cos(\theta_{\omega'}) \cos(\theta_\omega)\, d\omega'' d\omega' d\omega \\ &+ \ldots \end{aligned}$$

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$$\begin{aligned} L(x \rightarrow v) = E_x & \\ &+ \int_{\Omega_1} f_r\, E_{x'} \cos(\theta_\omega)\, d\mu(\bar{x}) \\ &+ \int_{\Omega_2} f_r\, f_r' E_{x''} \cos\left(\theta_{\omega'}\right) \cos(\theta_\omega)\, d\mu(\bar{x}) \\ &+ \int_{\Omega_3} \underbrace{f_r f_r' f_r''\, E_{x'''}\, cos(\theta_{\omega''}) \cos\left(\theta_{\omega'}\right) \cos(\theta_\omega)}_{\mathrm{f_j}(\bar{x})}\, d\mu(\bar{x}) \\ &+ \ \ldots \end{aligned}$$

Reconsidering Sample Distribution

- Let's replace each integral with Monte Carlo integration again

$$L(x \to v) = E_x$$

$$+ \frac{1}{N}\sum_{i=1}^{N} f_r\, E_{x'} \cos(\theta_\omega) \frac{1}{p(\omega)}$$

$$+ \frac{1}{N}\sum_{i=1}^{N} f_r\, f_r' E_{x''} \cos(\theta_{\omega'}) \cos(\theta_\omega) \frac{1}{p(\omega)p(\omega')}$$

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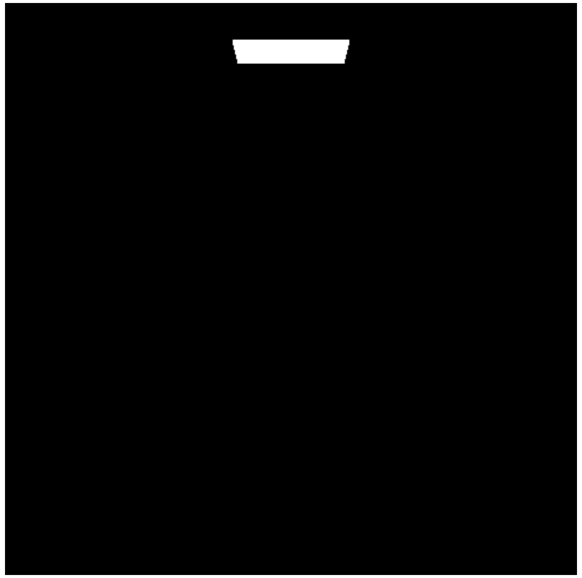
$$+ \ \ldots$$

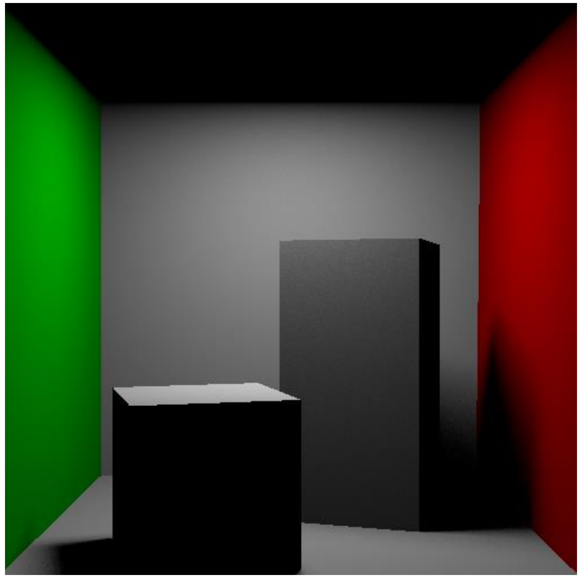
Our new sample distribution

- We use the same number of samples for all light paths

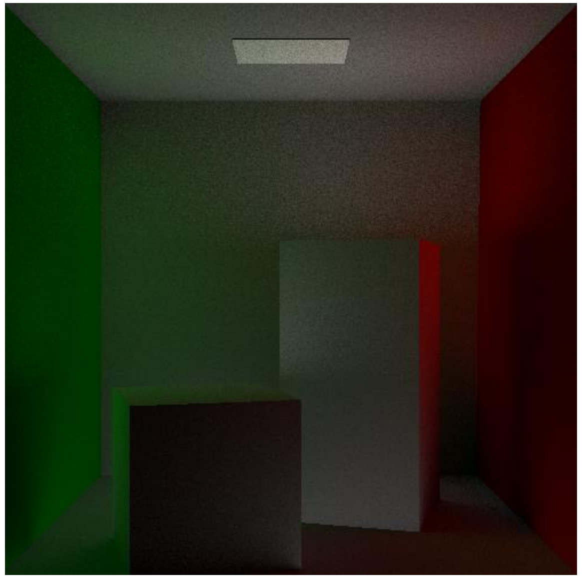
- No more exponential sample growth!



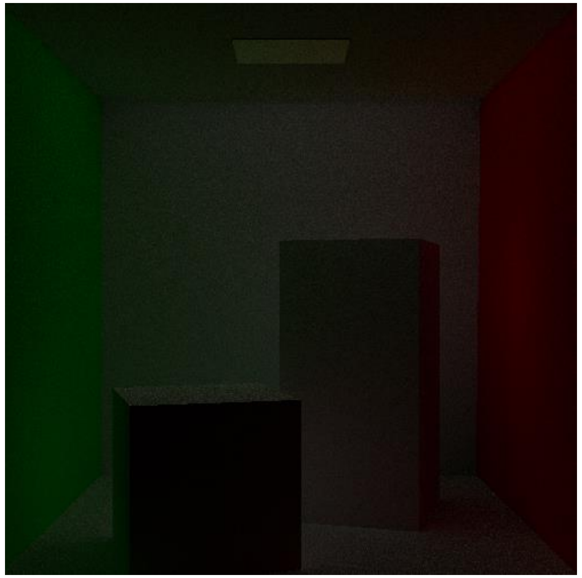
N samples



N samples



N samples



N samples

Reconsidering Sample Distribution

- Let's replace each integral with Monte Carlo integration again

$$L(x \to v) = E_x$$

$$+ \frac{1}{N}\sum_{i=1}^{N} f_r\, E_{x'} \cos(\theta_\omega) \frac{1}{p(\omega)}$$

$$+ \frac{1}{N}\sum_{i=1}^{N} f_r\, f_r' E_{x''} \cos\left(\theta_{\omega'}\right) \cos(\theta_\omega) \frac{1}{p(\omega)p(\omega')}$$

$$+ \frac{1}{N}\sum_{i=1}^{N} f_r f_r' f_r''\, E_{x'''}\, cos(\theta_{\omega''}) \cos\left(\theta_{\omega'}\right) \cos(\theta_\omega) \frac{1}{p(\omega)p(\omega')p(\omega'')}$$

$$+ \ \ldots$$

Reconsidering Sample Distribution

- We can again pull the sum to the front…

$$L(x \to v) = \frac{1}{N}\sum_{i=1}^{N}(E_x$$

$$+f_r E_{x'} \cos(\theta_\omega) \frac{1}{p(\omega)}$$

$$+f_r f_r' E_{x''} \cos\left(\theta_{\omega'}\right) \cos(\theta_\omega) \frac{1}{p(\omega)p(\omega')}$$

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$$+ \ \ldots)$$

Reconsidering Sample Distribution

- …and rewrite to highlight the original recursion

$$L(x \to v) = \frac{1}{N}\sum_{i=1}^{N} \left(E_x + f_r \left(E_{x'} + f_r'(\quad \dots \quad) \cos\theta_{\omega'} \frac{1}{p(\omega')} \right) \cos(\theta_\omega) \frac{1}{p(\omega)} \right)$$

- We are back to a single sum for integration with recursion!
- This also fits perfectly with our main loop and interface design

Path Tracing with a Single Sum

```
for (i = 0; i < N; i++)
      v_inv = camera.gen_ray(px, py)
      pixel_color += Li(v_inv, 0)
pixel_color /= N

function Li(v_inv, D)
      if (D >= NUM_BOUNCES)
            return 0
      x = scene.trace(v_inv)
      f = x.emit
      omega, prob = hemisphere_uniform_world(x)
      r = make_ray(x, omega)
      f += x.alb/pi * Li(r, D+1) * dot(x.normal, omega)/prob
      return f
```

$$L(x \to v) = \frac{1}{N}\sum_{i=1}^{N}\left(E_x + f_r\,(\ldots)\cos(\theta_\omega)\frac{1}{p(\omega)}\right)$$

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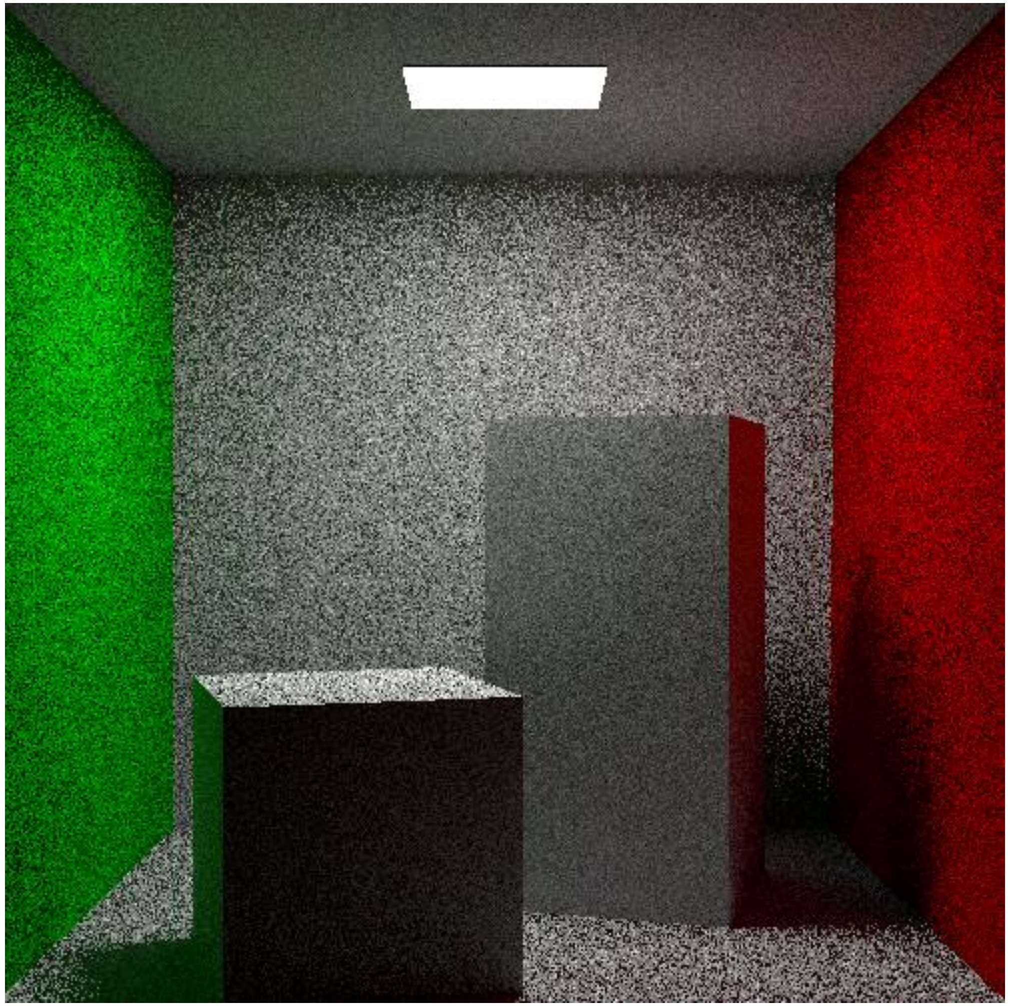
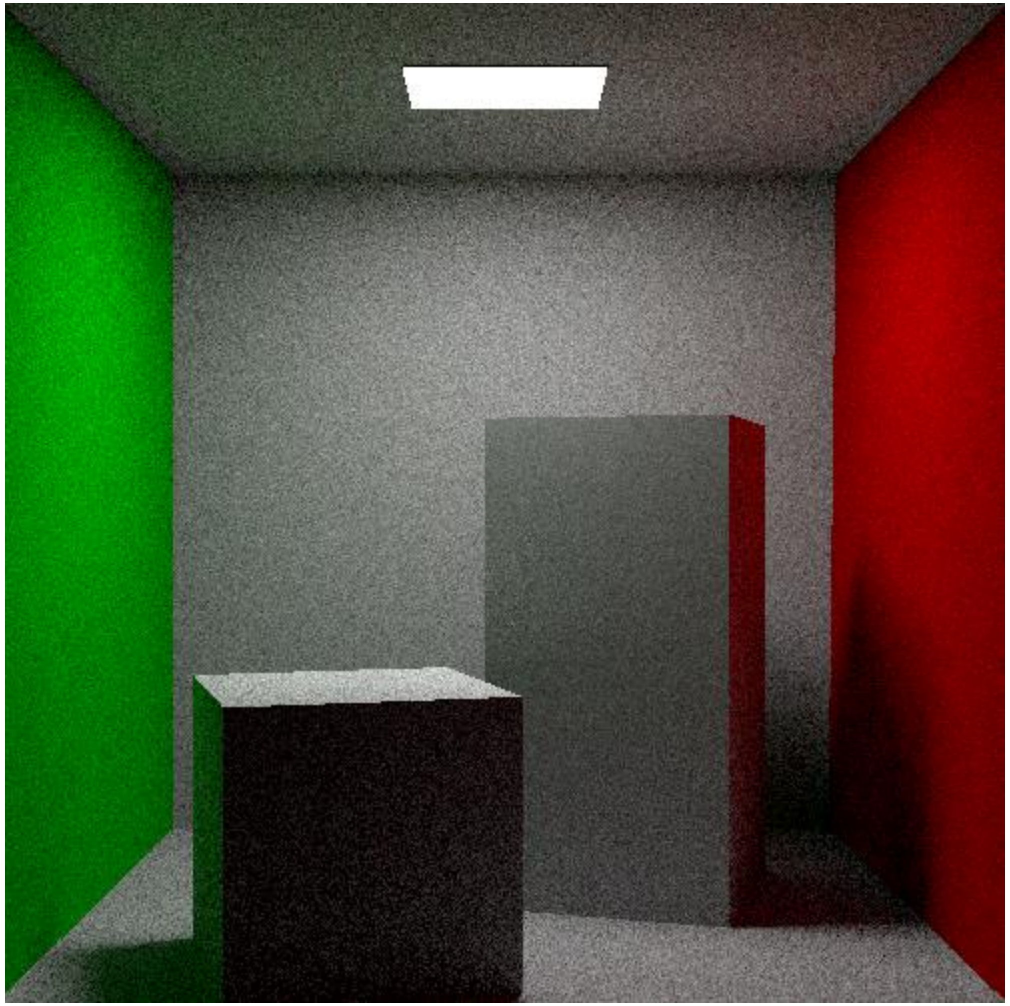
```

$$L(x \rightarrow v) = \frac{1}{N} \sum_{i=1}^{N} \left(E_x + f_r\,(\ldots)\cos(\theta_\omega)\frac{1}{p(\omega)} \right)$$

Path Tracing Implementations – Comparison



3 bounces, 3 nested sums, N = 16



3 bounces, 1 sum, N = 2048

Wisdom of the Day

Your samples are precious

…put them where they matter!

How many bounces are enough?

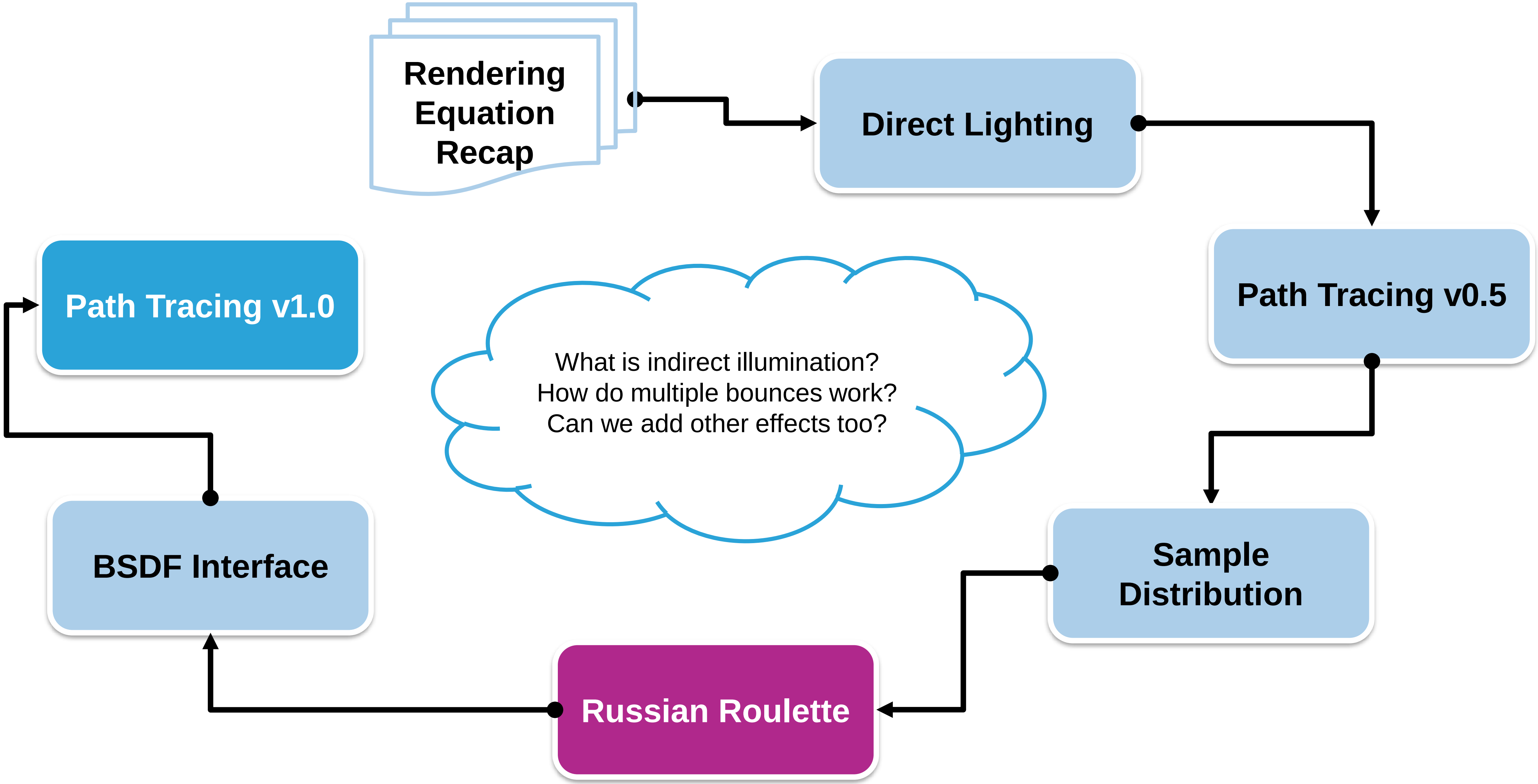
- Remember: if we want to be physically correct, then we must consider all possible light paths (i.e., journeys of photons)

- Photons stop bouncing when they have been entirely absorbed

- Problem: no real-world material absorbs 100% of incoming light

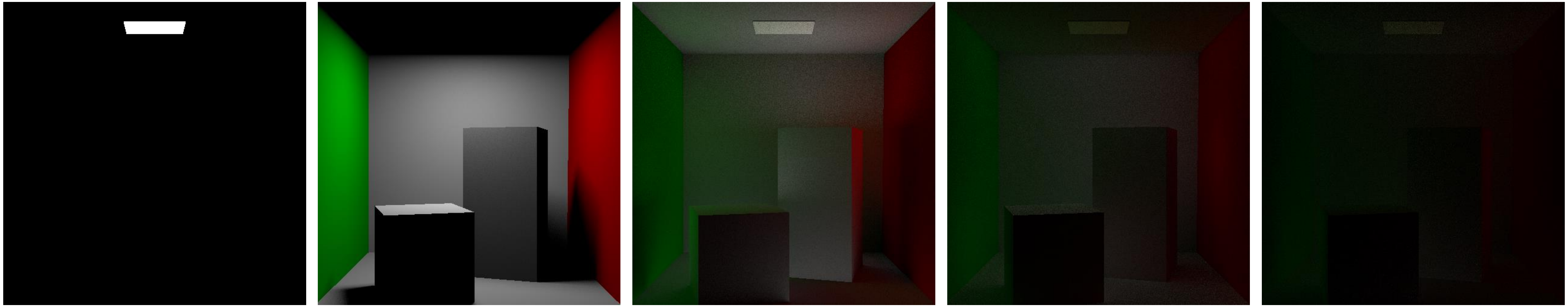
- No matter how many bounces, the probability might never go to absolute zero → so we should **never** stop?

Today's Roadmap



How do we handle infinity?

- In many cases, most contribution comes from the first few bounces



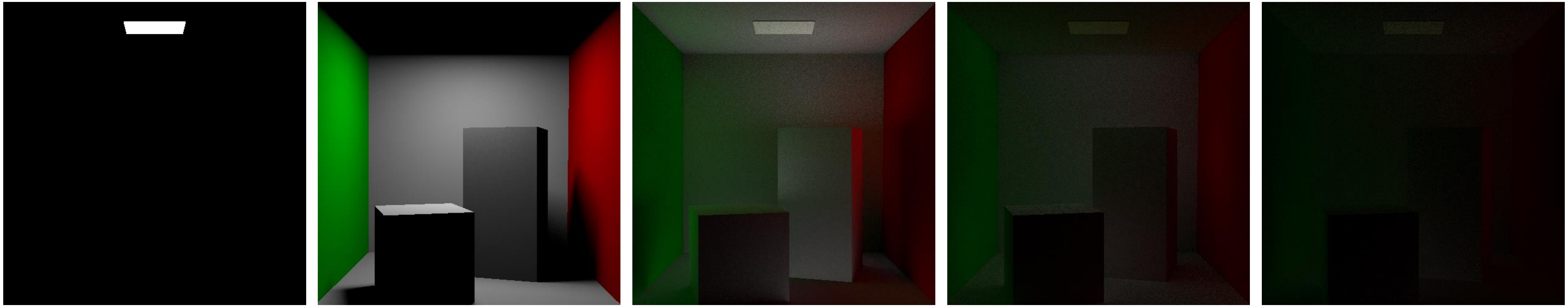
N samples | N samples | N samples | N samples | N samples

Number of bounces →

- Can we exploit this fact and make long paths possible, but unlikely?

How do we handle infinity?

- In many cases, most contribution comes from the first few bounces



N samples | N samples | N samples | **<N samples?** | **≪N samples?**

Number of bounces →

- Can we exploit this fact and make long paths possible, but unlikely?

Russian Roulette (RR)

- Pick $0 < p_{RR} < 1$. Draw uniform random value x in [0, 1) to decide
  - $x < p_{RR}$: keep going for another bounce
  - $x \geq p_{RR}$: end path

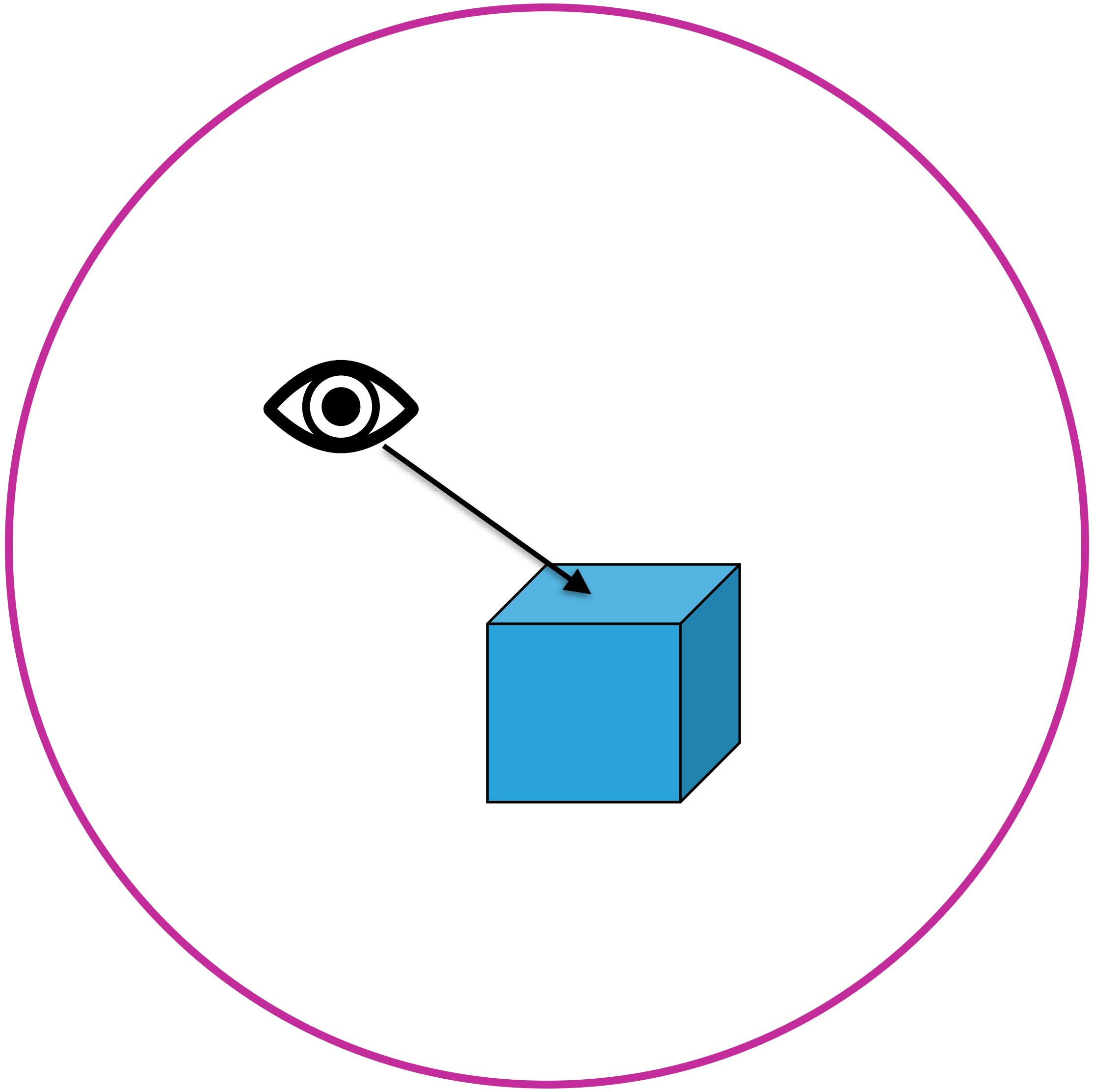
- The longer a path goes on, the more likely it is to get terminated

- The probability of a ray surviving the D^{th} bounce is p_{RR}^{D}

- Whenever a path continues with another bounce, compensate for its (un)-likeliness by weighting the returned color from L_i with $\frac{1}{p_{RR}}$

The Logic behind Russian Roulette

- Both objects emit light
- Very aggressive RR
- Assume fixed $p_{RR} = \frac{1}{5}$
- Most paths ($\frac{4}{5}$) fail to capture the pink sphere



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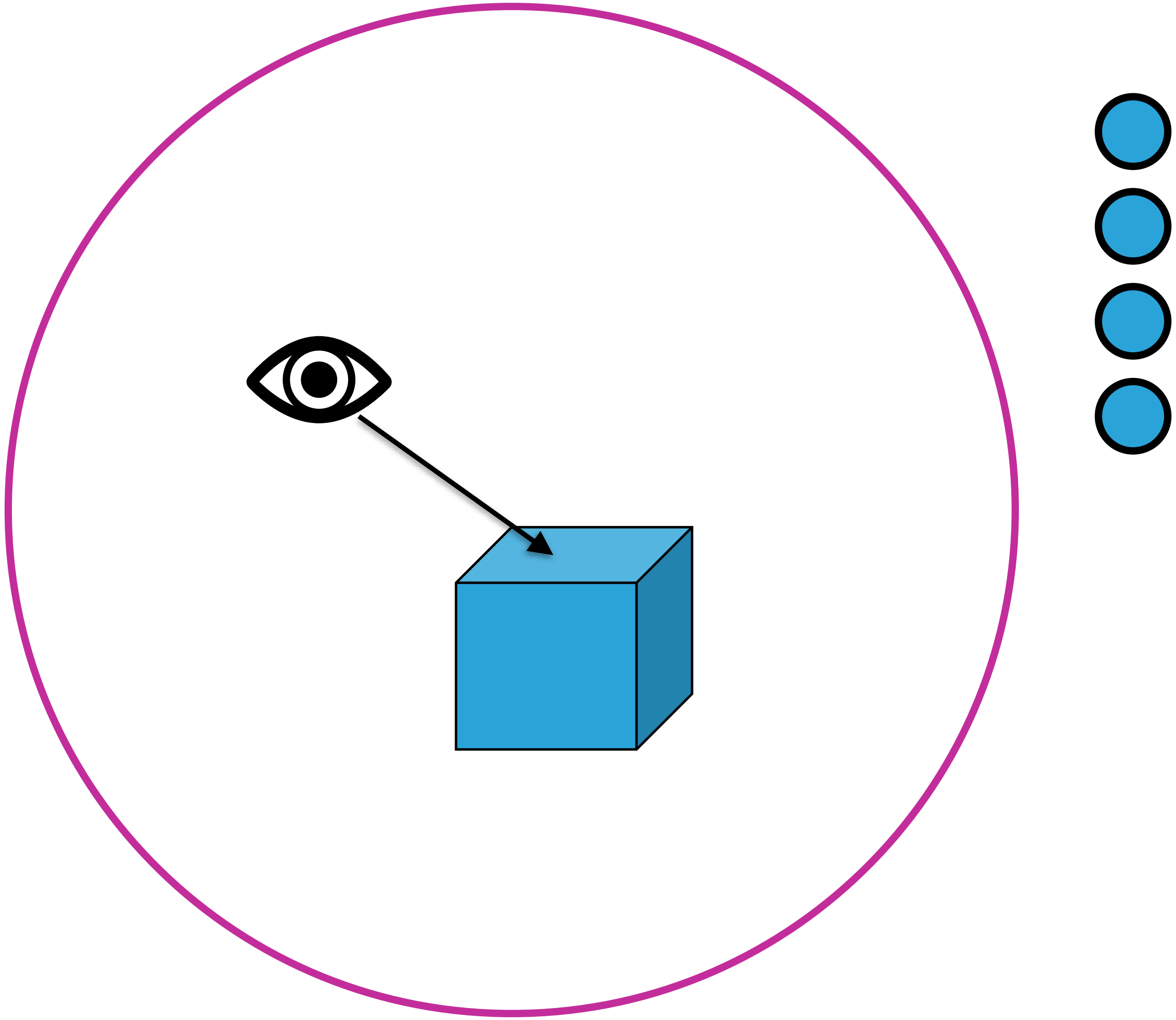
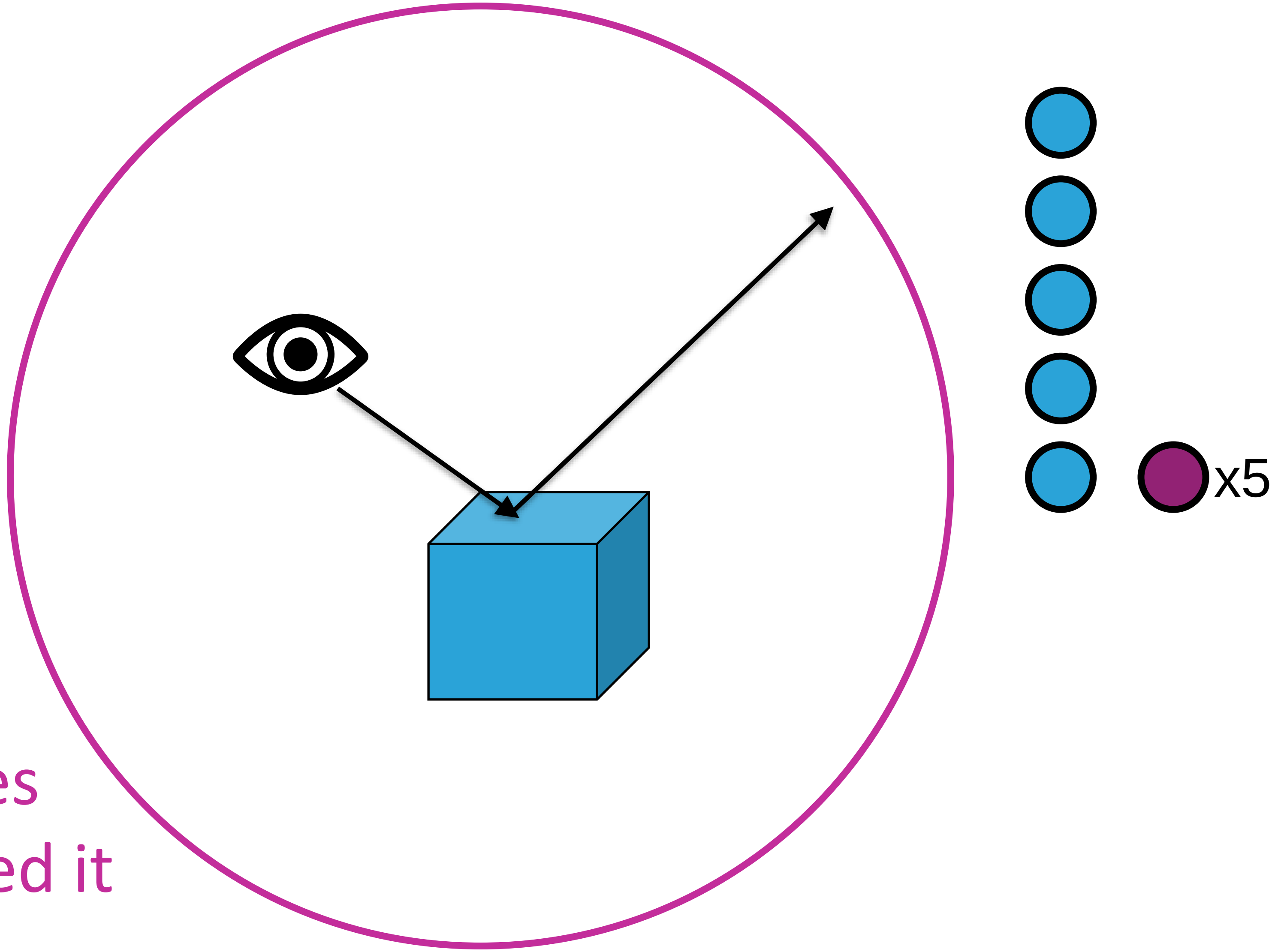


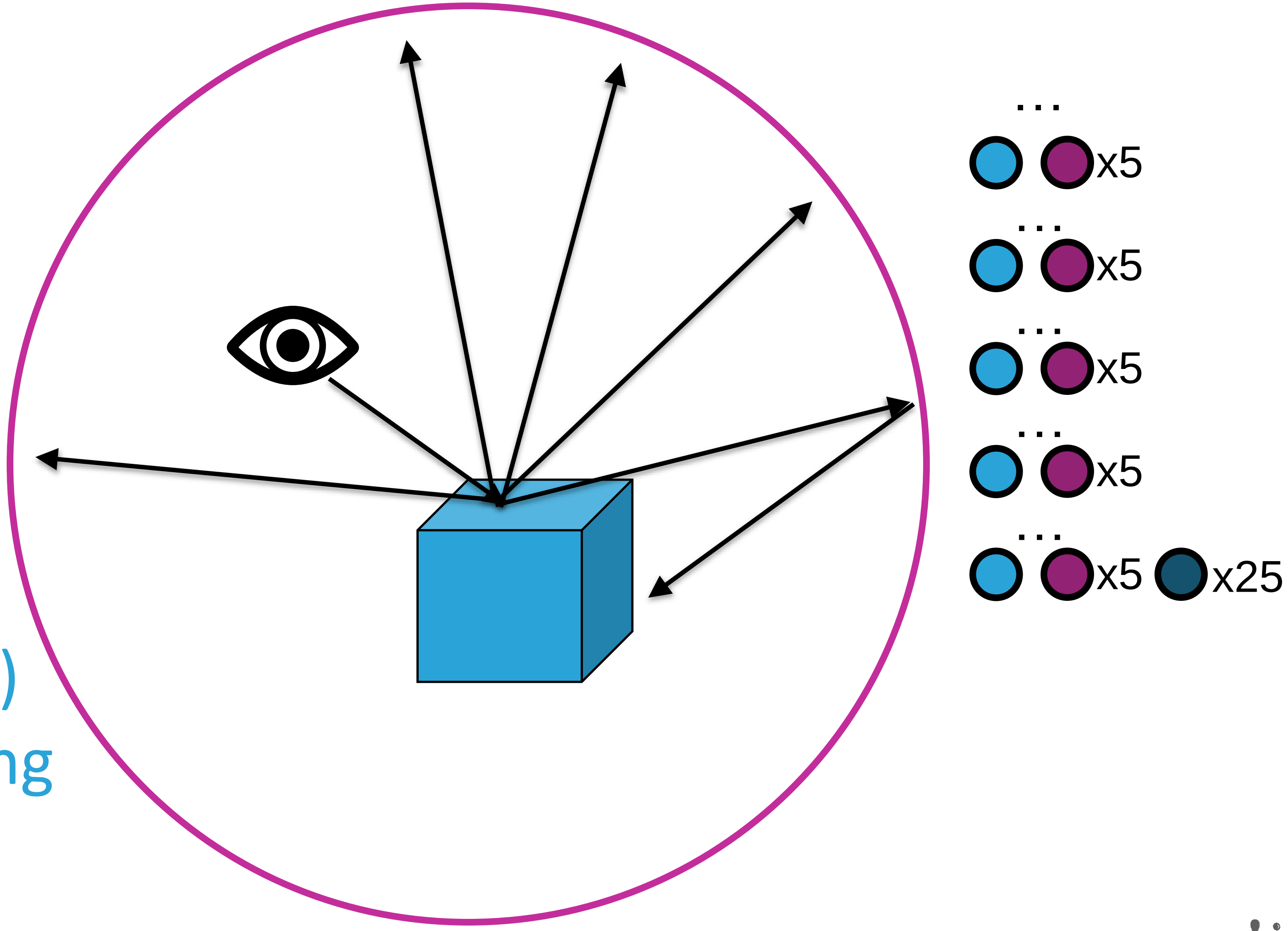
The Logic behind Russian Roulette

- Both objects emit light
- Very aggressive RR
- Assume fixed $p_{RR} = \frac{1}{5}$
- When we **do** hit it, the division by p compensates for times where we missed it



The Logic behind Russian Roulette

- Both objects emit light
- Very aggressive RR
- Assume fixed $p_{RR} = \frac{1}{5}$
- The rarity (and multiplier) of later bounces is growing continuously!



Russian Roulette Recursion Trap

- In code, you might be tempted to use $\frac{1}{p_{RR}^{D}}$ to compensate for RR
- Don‘t! $\frac{1}{p_{RR}}$ is enough for each individual bounce!
- If you use the recursive implementation, your effective RR compensation will grow with each bounce, all by itself…
- Maybe look at pseudocode and ponder the previous slide for a bit

Russian Roulette Implementation

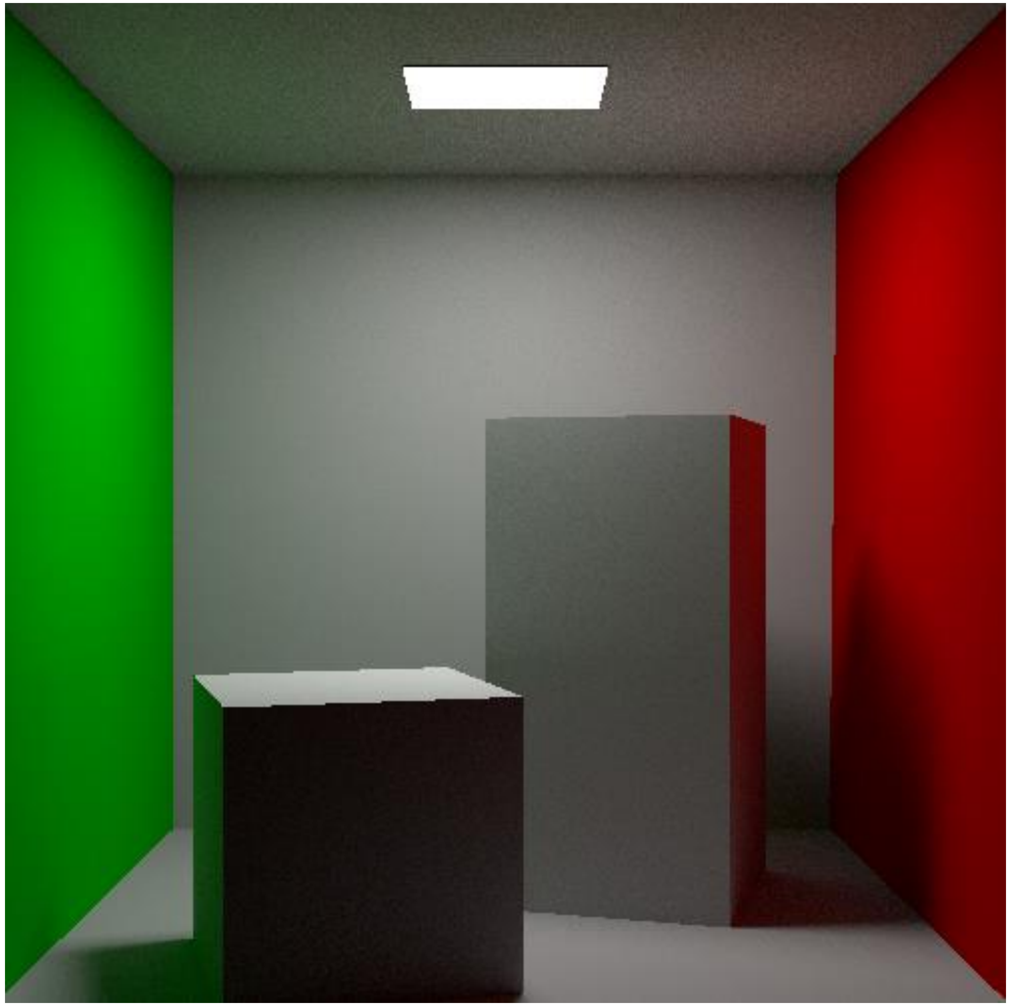
```
for (i = 0; i < N; i++)
      v_inv = camera.gen_ray(px, py)
      pixel_color += Li(v_inv, 0)
pixel_color /= N

function Li(v_inv, D)
      …
      f = x.emit
      …
      rr_prob = Some value between 0 and 1
      if (uniform_random_value() >= rr_prob)     // This path ends here
            return f
      …
      f += x.alb/pi * Li(r, D+1) * dot(x.normal, omega) / (prob * rr_prob)
      return f
```

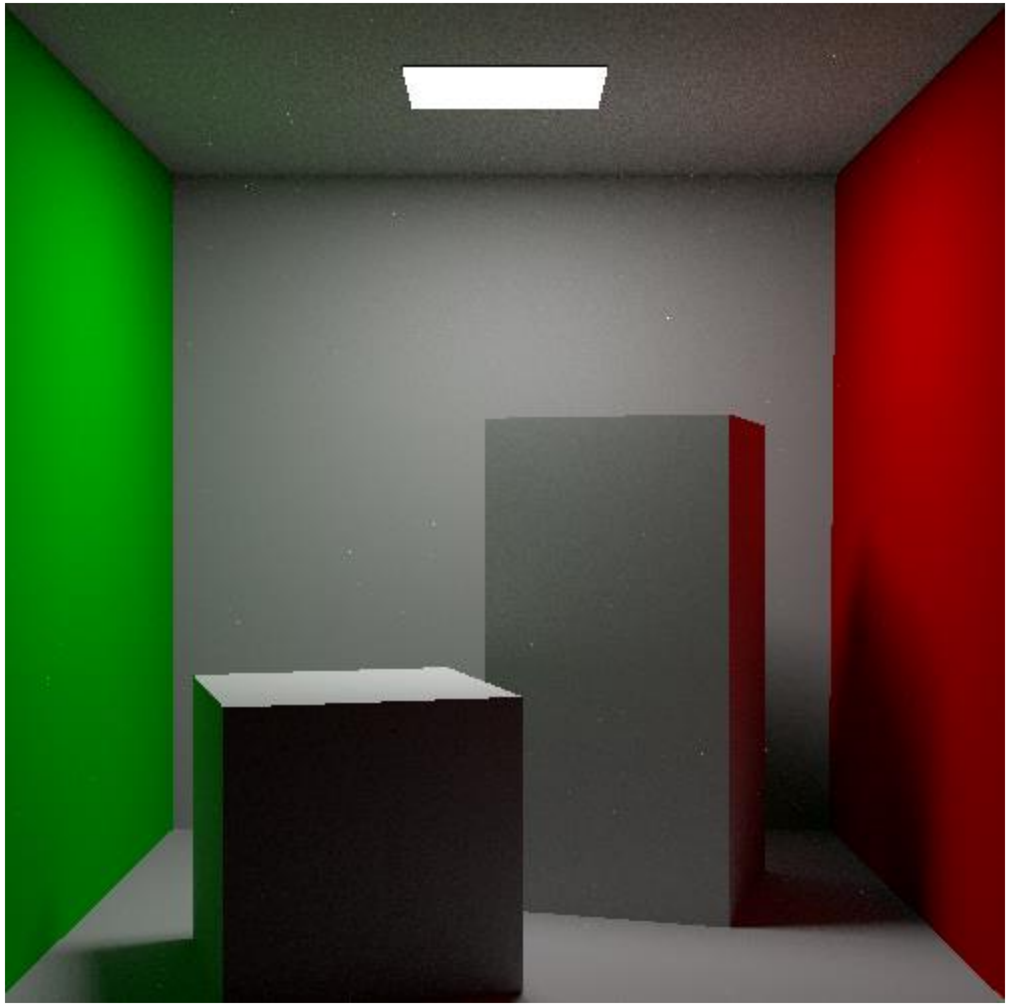
Russian Roulette..?

- *"…but if the possibility for infinitely long paths remains, doesn't that mean that my renderer may take forever to finish?"*
- Almost certainly no
- In practice, if you choose an adequate p_{RR}, you are more likely to get struck by lightning while reading this than that ever happening
- *"Ok, cool, so the lower I choose p_{RR}, the better, right? Can we just take something really small?"* Well, not exactly.

High p_{RR} vs low p_{RR}, same number of samples

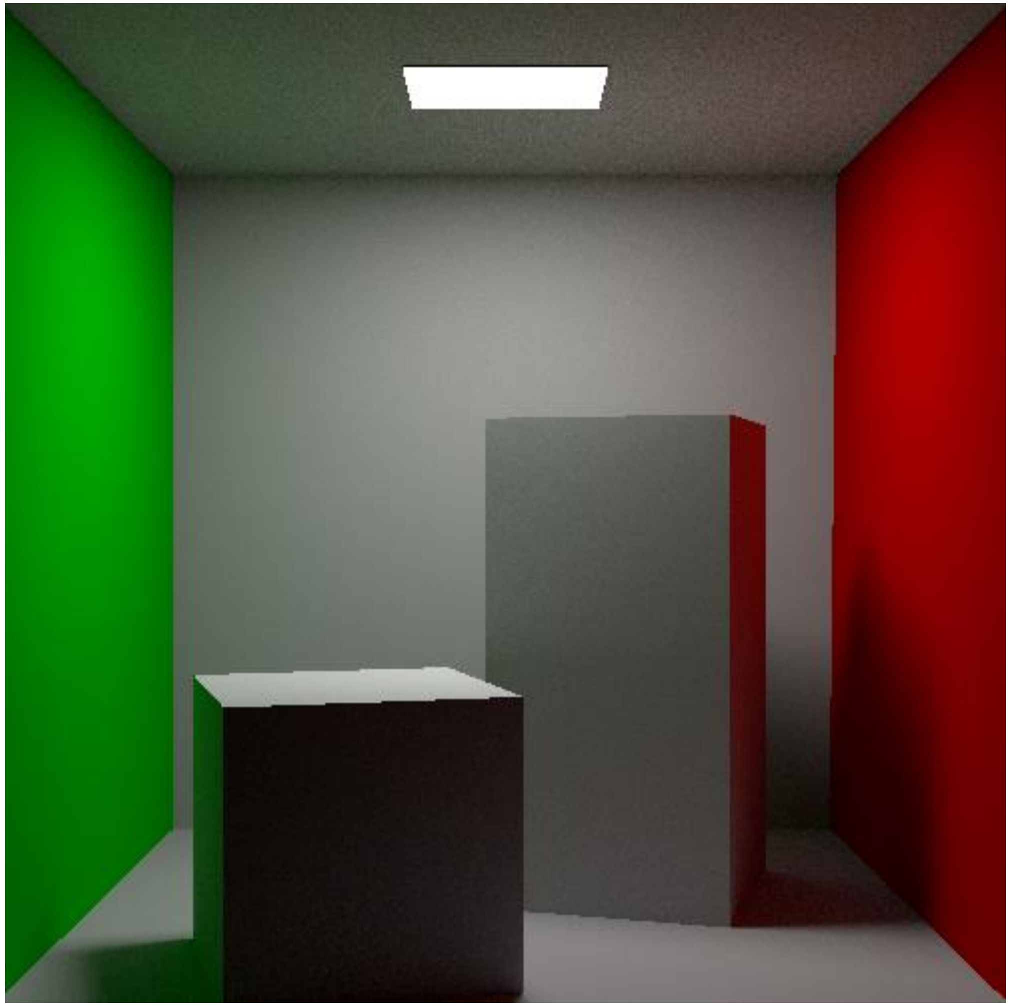


$p_{RR} = 0.7$: 50 seconds

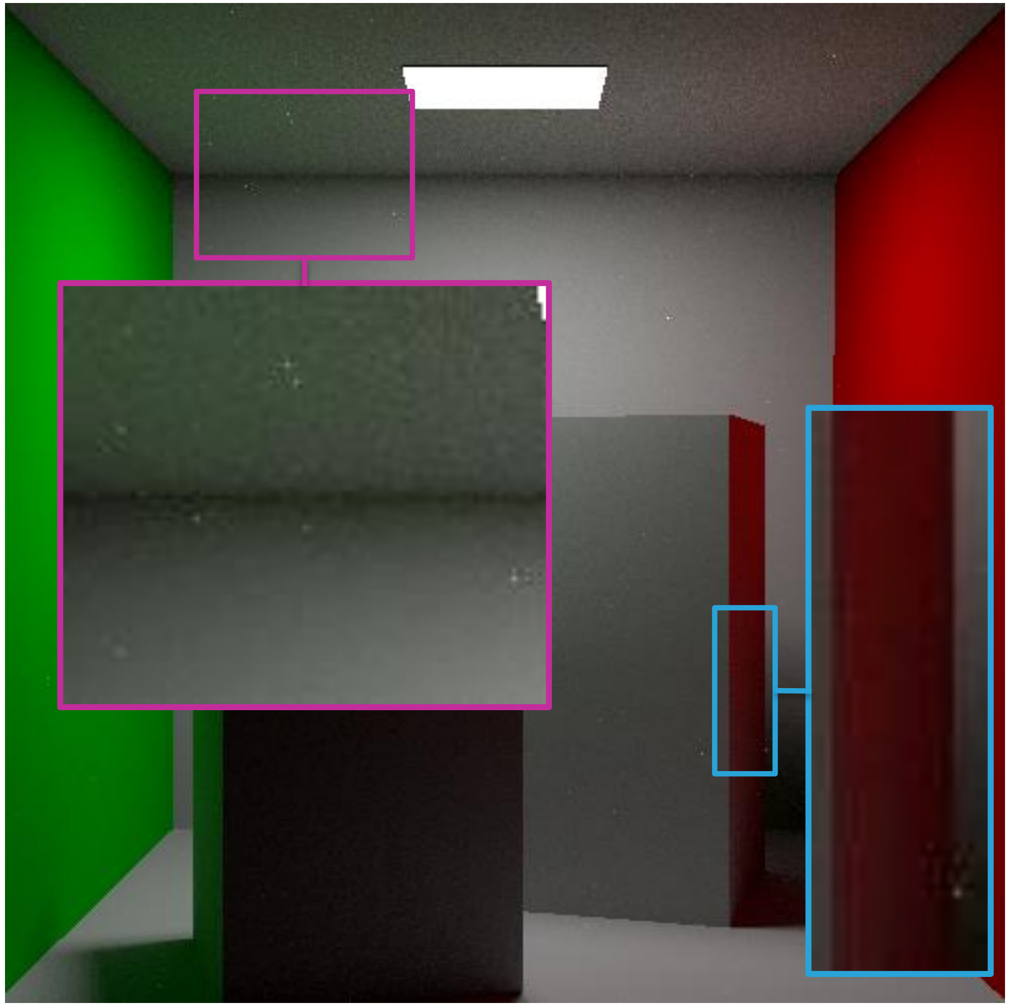


$p_{RR} = 0.1$: 35 seconds

High p_{RR} vs low p_{RR}, same number of samples

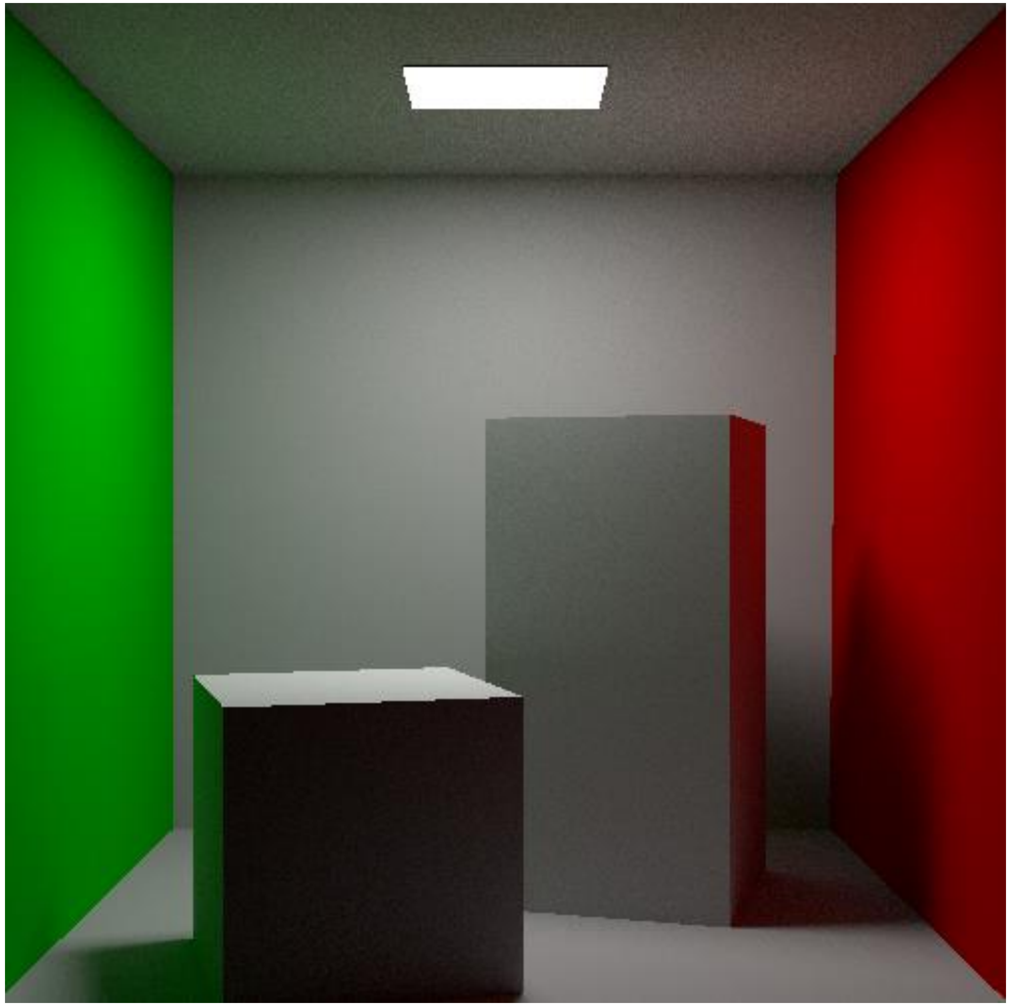


$p_{RR} = 0.7$: 50 seconds

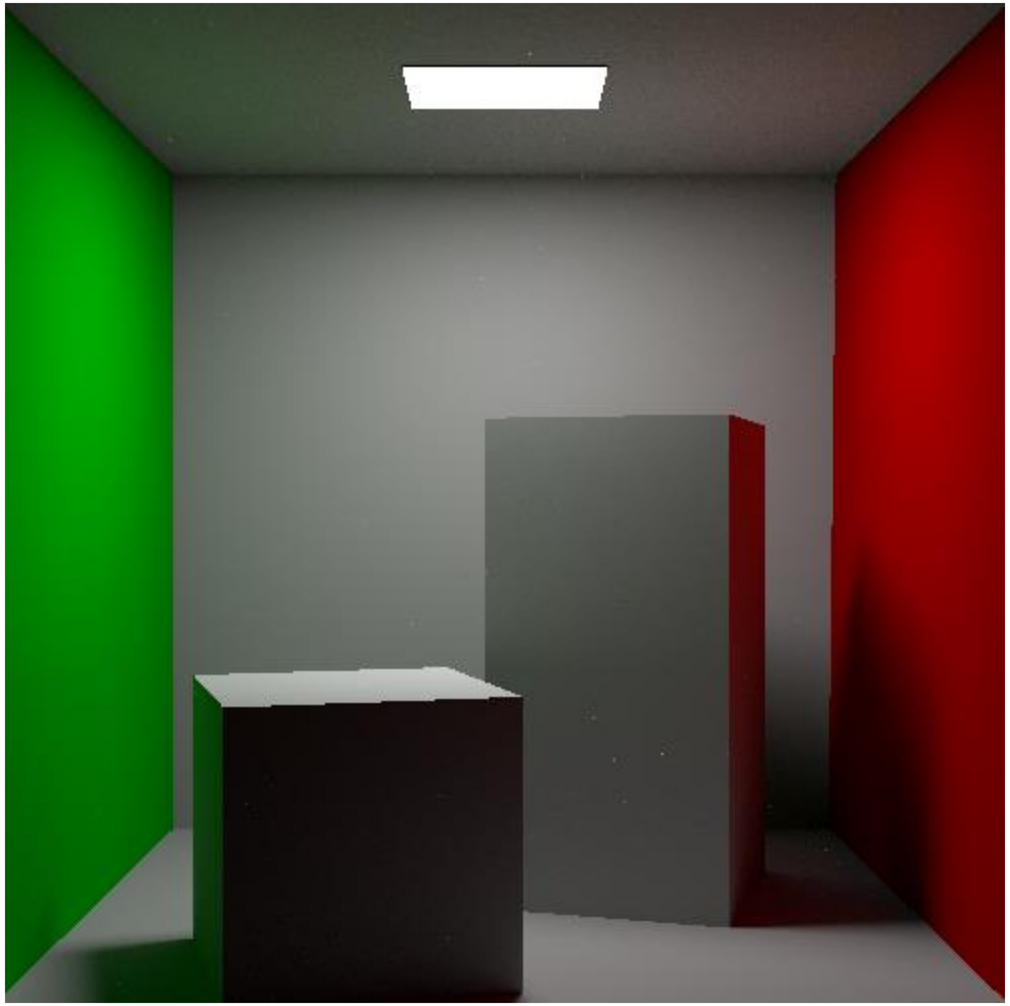


$p_{RR} = 0.1$: 35 seconds

High p_{RR} vs low p_{RR}, different number of samples

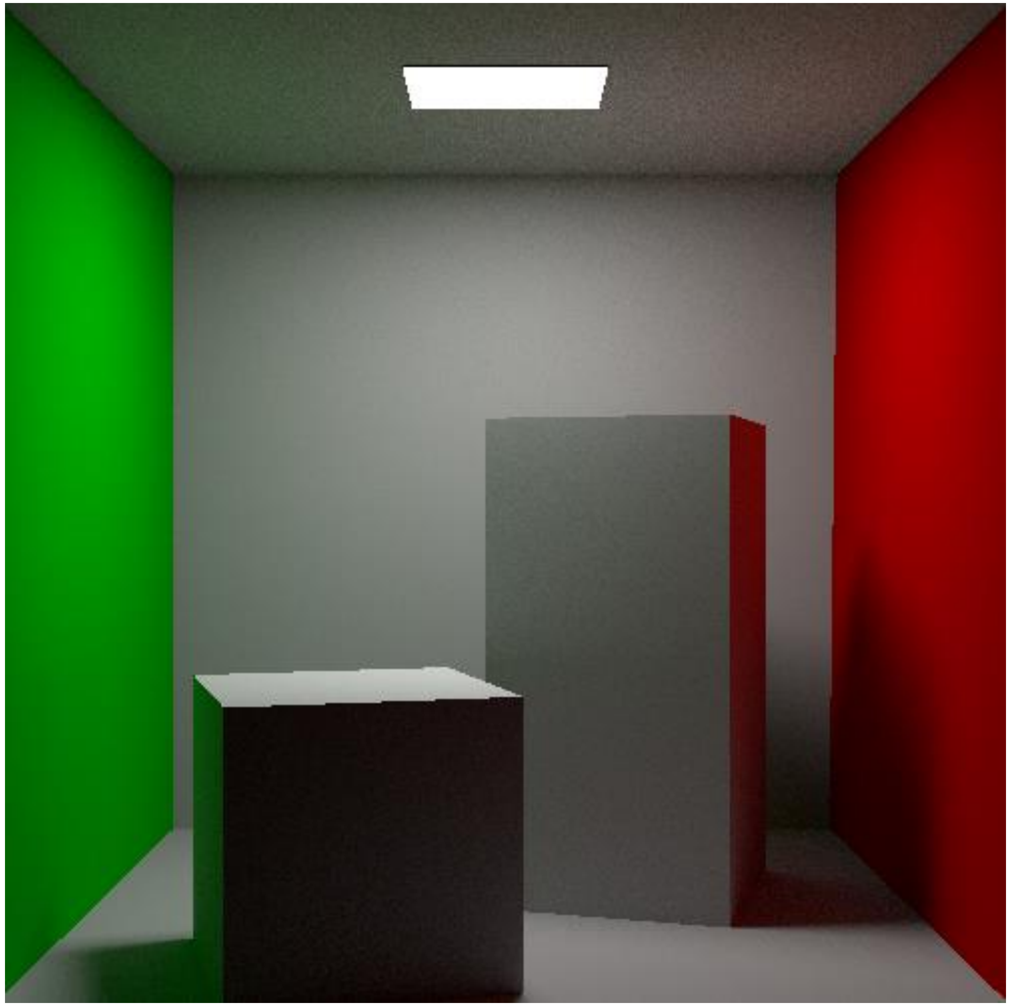


$p_{RR} = 0.7$: 50 seconds

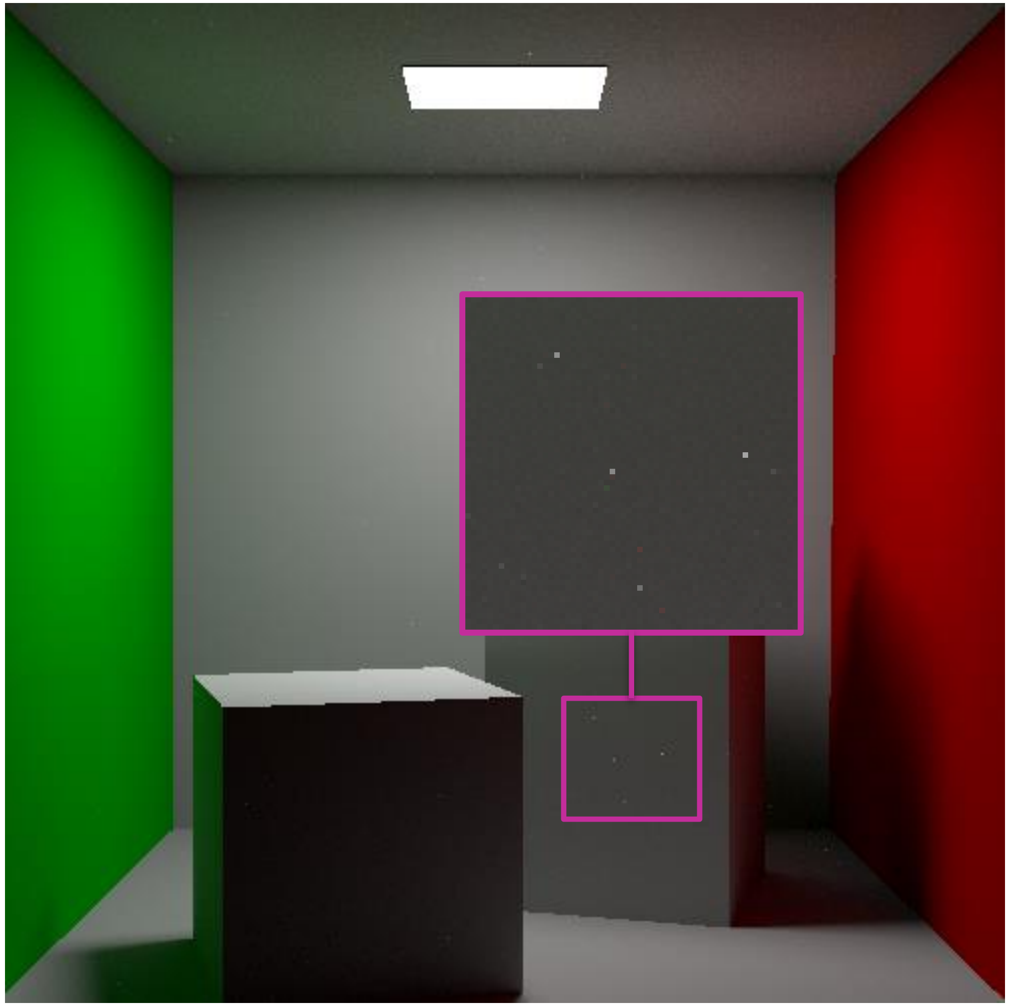


$p_{RR} = 0.1$, 4x as many samples: 150 seconds

High p_{RR} vs low p_{RR}, different number of samples



$p_{RR} = 0.7$: 50 seconds

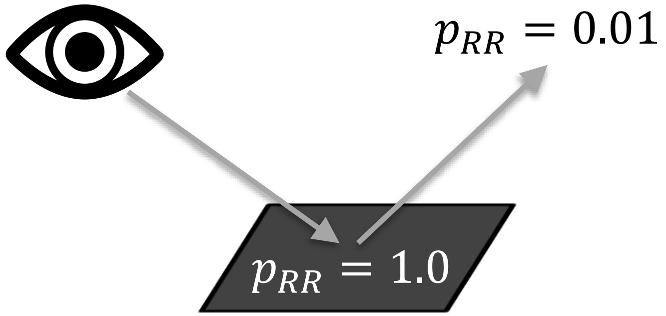


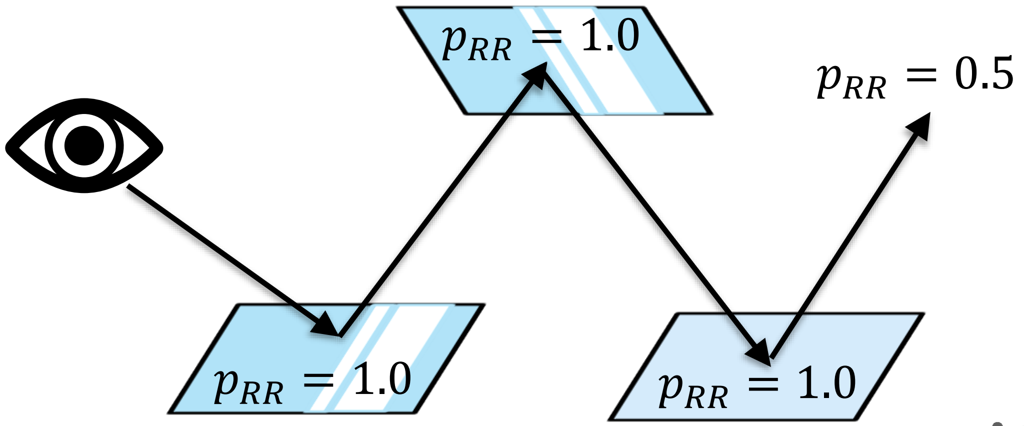
$p_{RR} = 0.1$, 4x as many samples: 150 seconds

Fireflies and Throughput

- If $p(x)$ is low, but $f(x)$ is not → high contribution of rare events!
- These "fireflies" tend to stick around!
- Choose p_{RR} dynamically: compute it at each bounce according to possible color contribution („throughput")

$$p_{RR} = \max_{\text{RGB}} \left(\prod_{d=1}^{D-1} \left(\frac{f_r(x_d, \omega_d \rightarrow v_d) \cos \theta_d}{p(\omega_d) p_{RR_d}} \right) \right)$$





Keeping track of throughput (recursive)

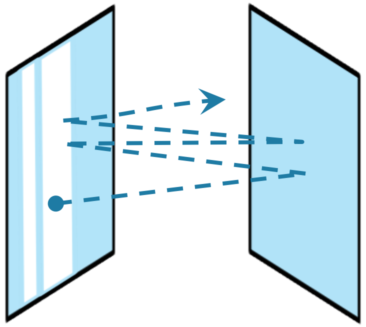
```
for (i = 0; i < N; i++)
      v_inv = camera.gen_ray(px, py)
      pixel_color += Li(v_inv, 0, 1)
pixel_color /= N

function Li(v_inv, D, throughput)
      …
      rr_prob = max_coefficient(throughput)     // Throughput is RGB
      …
      brdf = x.alb/pi
      cosTheta = dot(x.normal, omega)
      throughput *= brdf * cosTheta / (prob * rr_prob)
      …
      f += brdf * Li(r, D+1, throughput) * cosTheta / (prob * rr_prob)
      return f
```

Russian Roulette Implementation Details

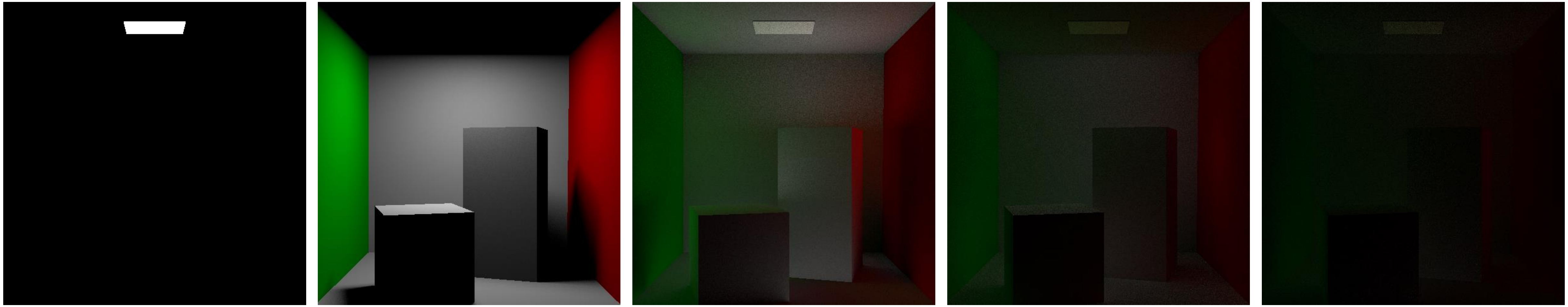
- Use guaranteed **minimal** path length before Russian Roulette starts
  - E.g., no Russian Roulette before the third bounce
  - Preserves a minimal path length for indirect illumination
  - Guaranteed bounces have $p = 1$ always

- Some materials absorb barely any incoming light (mirrors!)
  - Imagine two mirrors opposite of each other
  - Ray may bounce between them forever
  - Bad: limit bounces to a strict maximum
  - Better: clamp p_{RR} to a value < 1, e.g. 0.99



Sample Distribution in Production Renderers

- In practice, the distribution of samples is usually more dynamic:



medium | **high** | medium | **low** | **lower**

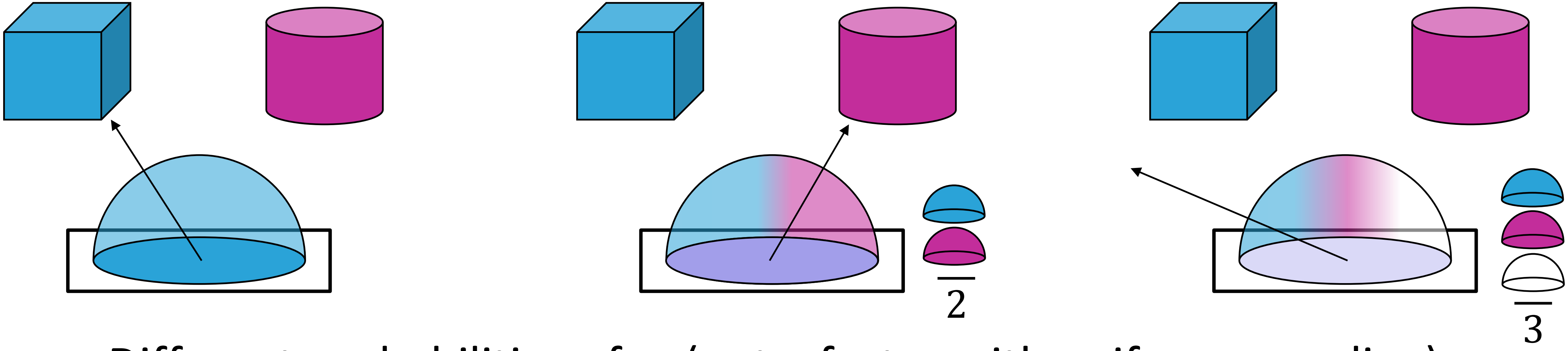
→ Number of bounces

- Especially bounce 1 (direct light/shadows) receives more attention

Choosing and Compensating

- You can interpret the p_{RR} as an additional factor to MC integral

- Originally, we divided by $p(\omega)$ to account for two things:
  - Scaling to approximate the whole domain (i.e., entire hemisphere)



  - Different probabilities of ω (not a factor with uniform sampling)

Choosing and Compensating

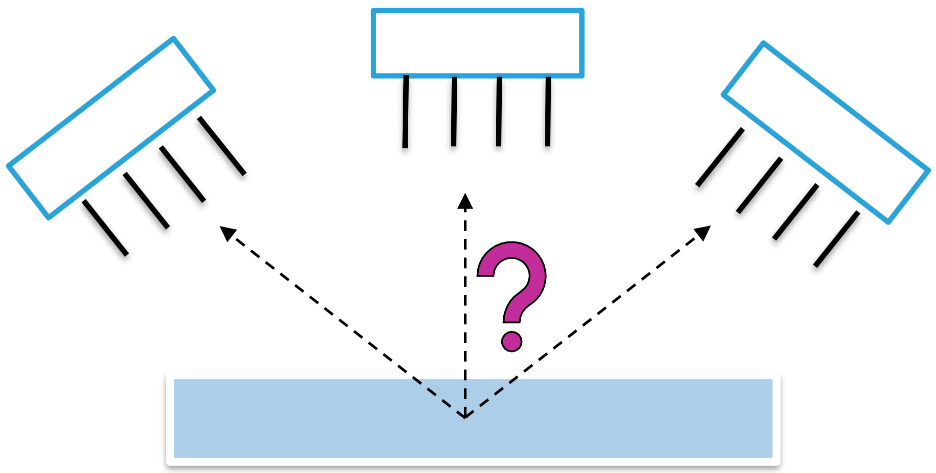
- Probability density of **following** a particular ω changes to $p(\omega)p_{RR}$
  - Compensate for the fact that sometimes we chose to stop
  - Weight rare events higher (e.g., many bounces off dark materials)

- We have to compensate whenever we pick one of multiple options

- This is usually the case when you draw a random variable to decide on choosing one option and not the others

Choosing and Compensating

- Another example: picking a light source for light source sampling
  - For light source sampling, you can sample them all or just pick one
  - If you do pick only one, must compensate for making that choice!
  - Simplest: pick uniformly, multiply result by number of lights (why?)

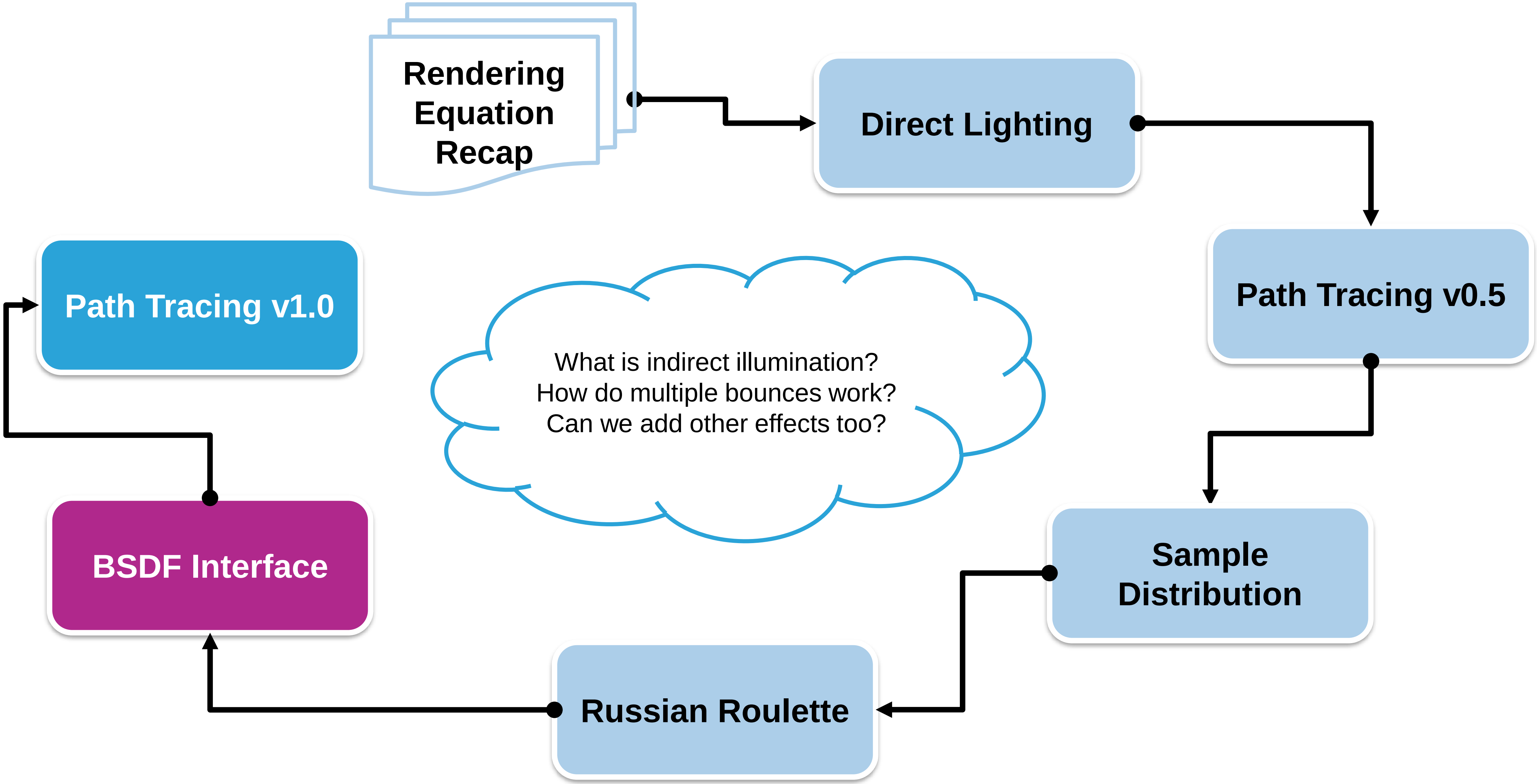


Wisdom of the Day

Monte Carlo is all about picking samples and then compensating

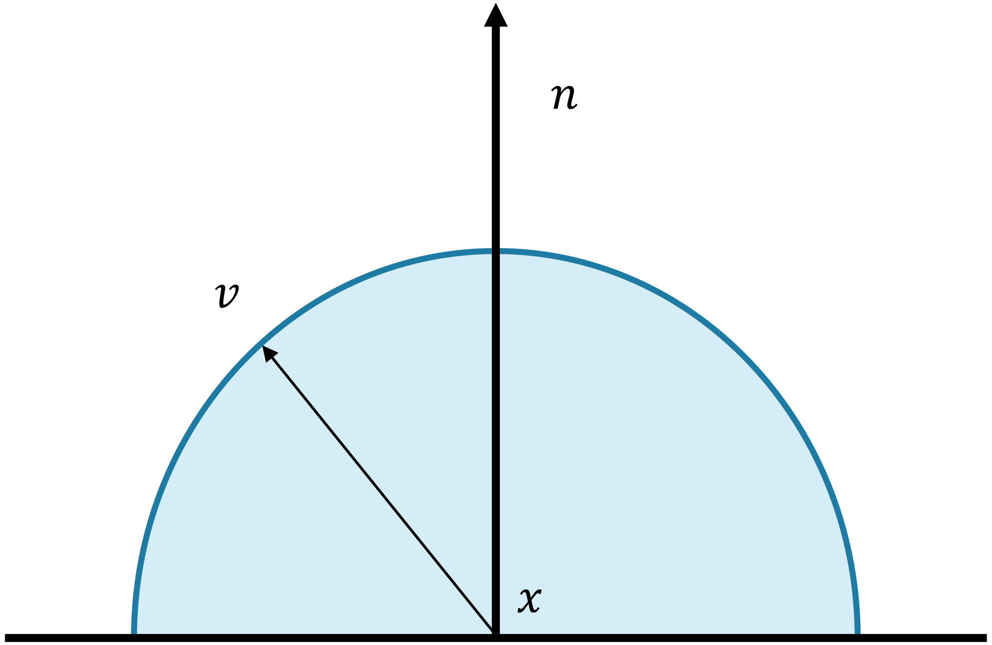
…and if you pick your samples carefully, we call it importance sampling

Today's Roadmap



Materials and the BSDF

- We made a path tracer for diffuse materials, and diffuse **only**



Reflection Behavior

Appearance

Capturing Path Behavior in a BSDF Data Structure

- We made a solution that works, but only for diffuse materials
- There are a lot of exciting other options
  - Specular
  - Glossy
  - …
- Path tracer should allow adding materials without rewriting it all
- Encapsulate material-dependent rendering factors in BSDF class

Material-Dependent Factors

```
function Li(v_inv, D, throughput)
        …
        omega, prob = hemisphere_uniform_world(x)
        …
        brdf = x.alb/pi
        throughput *= brdf * cosTheta / (prob * rr_prob)
        …
        f += brdf * Li(r, D+1, throughput) * cosTheta / (prob * rr_prob)
        …
```

Material-Dependent Factors

Some materials will reflect incoming light entirely in one single direction (mirrors). Sampling the hemisphere in this case is pointless! Also: we might be able to do something smarter than uniform sampling

```
function Li(v_inv, D, throughput)
    …
    omega, prob = hemisphere_uniform_world(x)
    …
    brdf = x.alb/pi
    throughput *= brdf * cosTheta / (prob * rr_prob)
    …
    f += brdf * Li(r, D+1, throughput) * cosTheta / (prob * rr_prob)
    …
```

Material-Dependent Factors

```
function Li(v_inv, D, throughput)
        …
        omega, prob = hemisphere_uniform_world(x)
        …
        brdf = x.alb/pi
        throughput *= brdf * cosTheta / (prob * rr_prob)
        …
        f += brdf * Li(r, D+1, throughput) * cosTheta / (prob * rr_prob)
        …
```

Super simple term that never changes. Obviously, this only makes sense if the amount of reflected light is the same in all directions, independent of v and ω. Only a fully diffuse BSDF gets away with this.

Material-Dependent Factors

```
function Li(v_inv, D, throughput)
        …
        omega, prob = hemisphere_uniform_world(x)
        …
        brdf = x.alb/pi
        throughput *= brdf * cosTheta / (prob * rr_prob)
        …
        f += brdf * Li(r, D+1, throughput) * cosTheta / (prob * rr_prob)
        …
```

For some materials, like glass, this cosine term is not needed, cancels out or has to be removed for reasons of energy conservation.

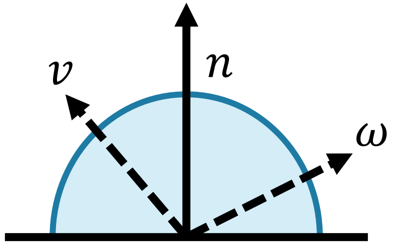
Material-Dependent Factors

```
function Li(v_inv, D, throughput)
        …
        omega, prob = hemisphere_uniform_world(x)
        …
        brdf = x.alb/pi
        throughput *= brdf * cosTheta / (prob * rr_prob)
        …
        f += brdf * Li(r, D+1, throughput) * cosTheta / (prob * rr_prob)
        …
```

Many issues! Example: we could have perfect mirrors that only reflect in a single direction. Hence the probability of other directions is 0. Danger of division by 0!

Implement Basic Diffuse BRDFs with BSDF Interface

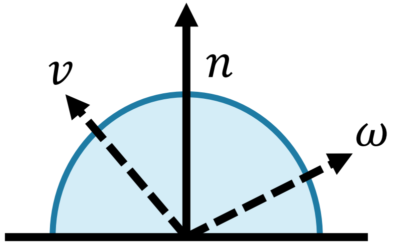
- Nori BSDF class has three methods: **eval, pdf, sample**



- Use auxiliary struct parameter `bRec` to pass v (`.wi`) and ω (`.wo`)
- **eval(**`bRec`**)**: evaluate material's ability to reflect light from ω to v
- **pdf(**`bRec`**)** : compute the relative probability of sampling direction ω
- **sample(**`bRec`**)**: make & store ω in `bRec`, compute material multiplier

Implement Basic Diffuse BRDFs with BSDF Interface

- Nori BSDF class has three methods: **eval, pdf, sample**



- Use auxiliary struct parameter `bRec` to pass v (`.wi`) and ω (`.wo`)
- **eval(**`bRec`**)**: return $\frac{\rho}{\pi}$ if v, ω lie in hemisphere around n (diffuse)
- **pdf(**`bRec`**)** : return $\frac{1}{2\pi}$ if ω lies in hemisphere around n (all ω equal)
- **sample(**`bRec`**)**: create uniform ω, return `(`$\cos\theta \cdot$ `eval())/pdf()`

Using the New BSDF Class in Path Tracing Code

- Isolates material-specific factor computations in a single function
- Simplifies code and will make extension with other materials easy

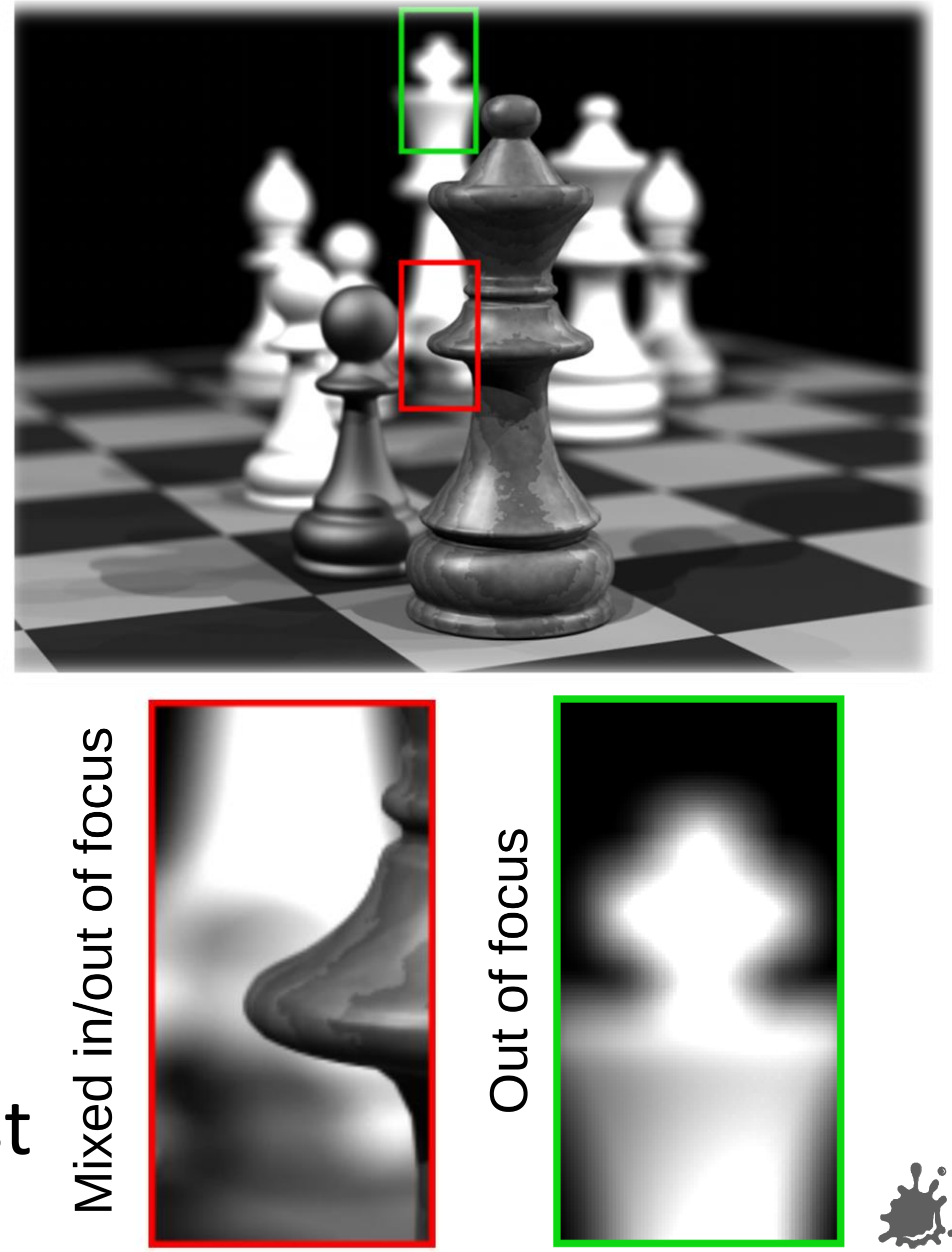
```
function Li(v_inv, D, throughput)
	…
	omega, brdf_multiplier = sample(x, v_inv)
	…
	throughput *= brdf_multiplier / rr_prob
	…
	f += brdf_multiplier * Li(r, D+1, throughput) / rr_prob
	…
```

Path-Tracing is *Multidimensional*

- We already know some of them:
  - Constructing a new ray after each bounce ($2D$)
  - Evaluating RR continuation probability (D)
  - …
- Other possible choices we have not yet considered:
  - Lens coordinates (for depth-of-field) (2)
  - Time (for motion blur) (1)
  - …

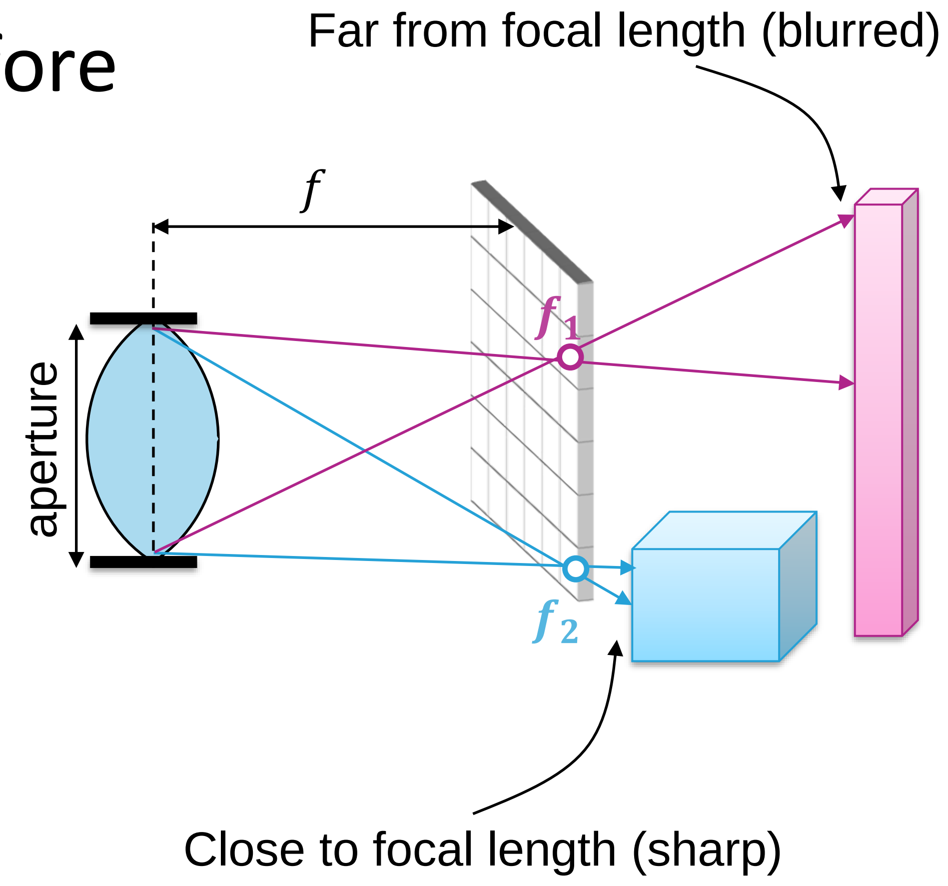
Depth-of-Field

- Simulate the behavior of camera lenses
- Depending on shape, focal length and aperture, lenses have limited distance range in which objects appear sharp
- If they are closer or farther away, they cause a blurry „circle of confusion“
- Can be used to highlight objects of interest



Depth-of-Field in Path Tracing

- Can simulate depth-of-field for a thin lens with focal length f [2]
- Create camera ray r through pixel as before
- Find focal point $\boldsymbol{f}$ along r at distance f
- Pick random location x, y on the lens (2D disk) inside the aperture
- Shoot camera ray from x, y through $\boldsymbol{f}$



Motion Blur

- Mostly an artistic effect to simulate a familiar camera phenomenon

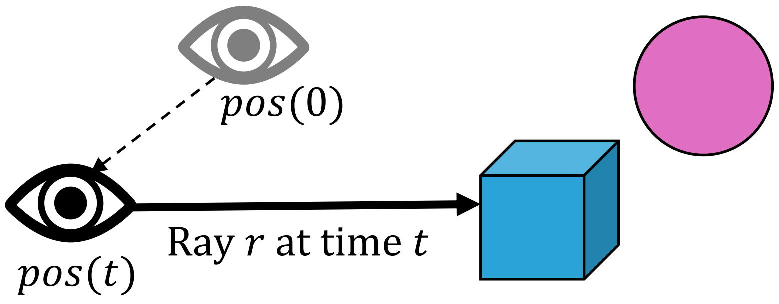
- Occurs when medium exposure is longer than rate of motion of objects in the captured scene

- Can help convey the impression of moving objects in still image

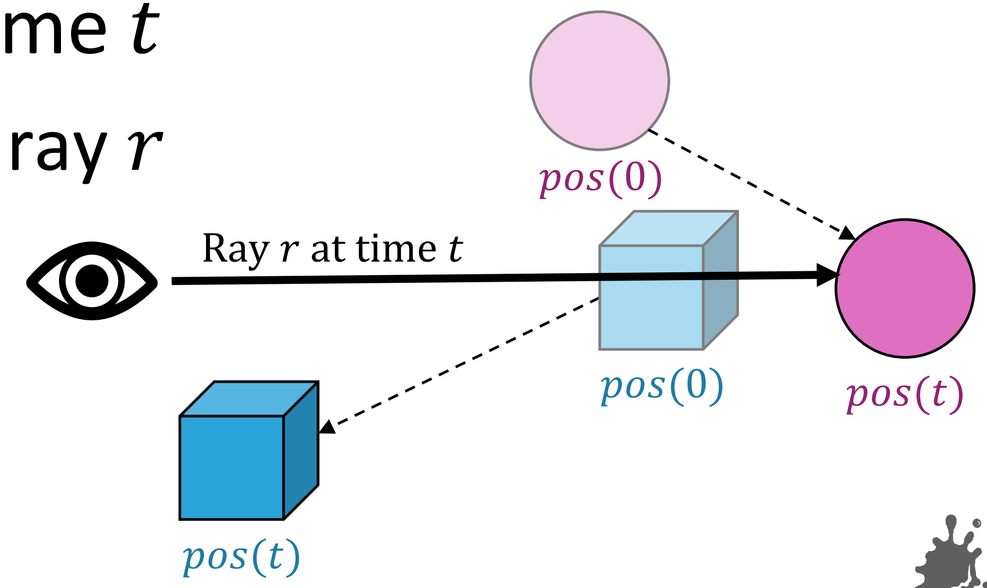
- In path tracing, all we need is an additional integrated time variable

Motion Blur in Path Tracing

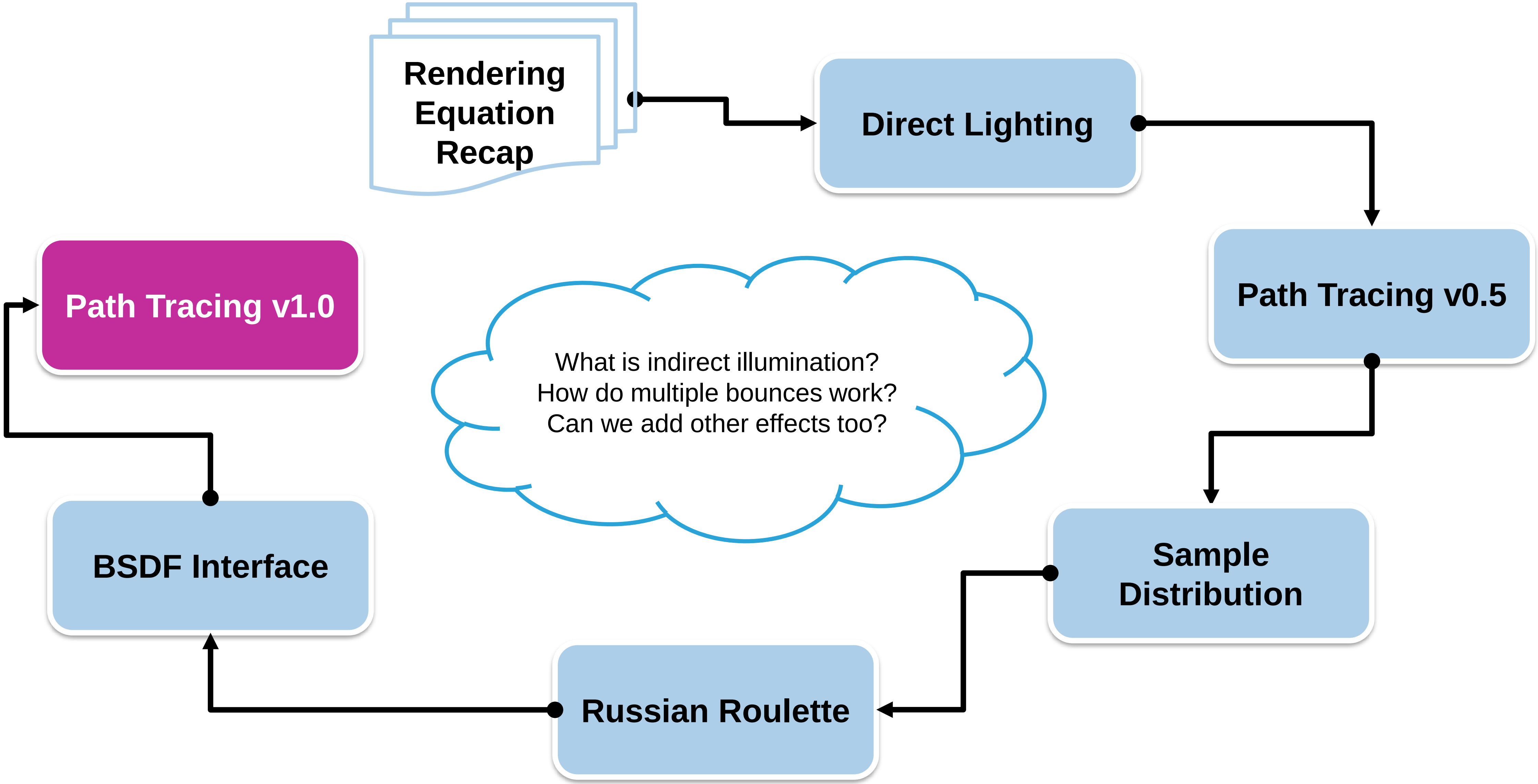
- Option 1: make camera position a function of time t
  - Draw a random t, create adapted view ray r
  - Follow path through the scene
  - Check which triangles ray intersects



- Option 2: make geometry a function of time t
  - Draw and store a random t, create view ray r
  - Follow path through the scene
  - Check which triangles ray intersects at t

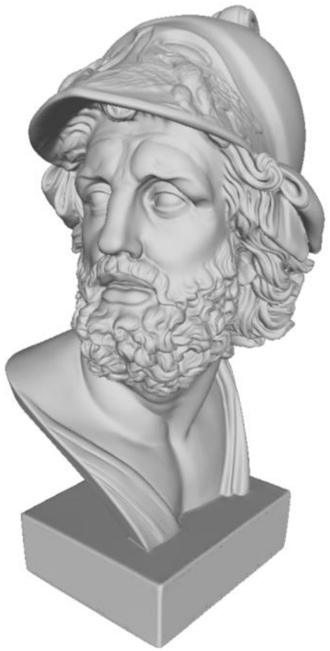


Today's Roadmap



Everything that is wrong with our path tracer right now

- So far, we only rendered very simple scenes (Cornell box)
- What happens if we run a slightly more challenging scene?
  - Ajax bust, 500k triangles
  - Takes 17 hours (!) to get a boring, noisy image…



- Is path tracing doomed (again)?
- No! We will make better images in seconds!

Room for Improvement

- Economize on samples – squeeze out whatever we can
  - Better sampling strategies (**importance sampling**)
  - Exploiting light source sampling (**next-event estimation**)
  - Combining sampling strategies (**multiple importance sampling**)

- Improving our scene intersection tests
  - Build **spatial acceleration structures**
  - Optimized traversal strategies

- Support spectacular specular, glossy and transparent **materials**

References and Further Reading

- [1] Creating an Orientation Matrix or Local Coordinate System
https://www.scratchapixel.com/lessons/mathematics-physics-for-computer-graphics/geometry/creating-an-orientation-matrix-or-local-coordinate-system

- [2] Depth-of-Field Implementation in a Path Tracer
https://medium.com/@elope139/depth-of-field-in-path-tracing-e61180417027

- [3] *Toshiya Hachisuka, Wojciech Jarosz, Richard Peter Weistroffer, Kevin Dale, Greg Humphreys, Matthias Zwicker, and Henrik Wann Jensen. 2008. Multidimensional adaptive sampling and reconstruction for ray tracing. ACM Trans. Graph. 27, 3 (August 2008)*

- [4] *Ryan Overbeck, Craig Donner, and Ravi Ramamoorthi. Adaptive Wavelet Rendering. ACM Transactions on Graphics (SIGGRAPH ASIA 09), 28(5), December 2009.*