Rendering: Light

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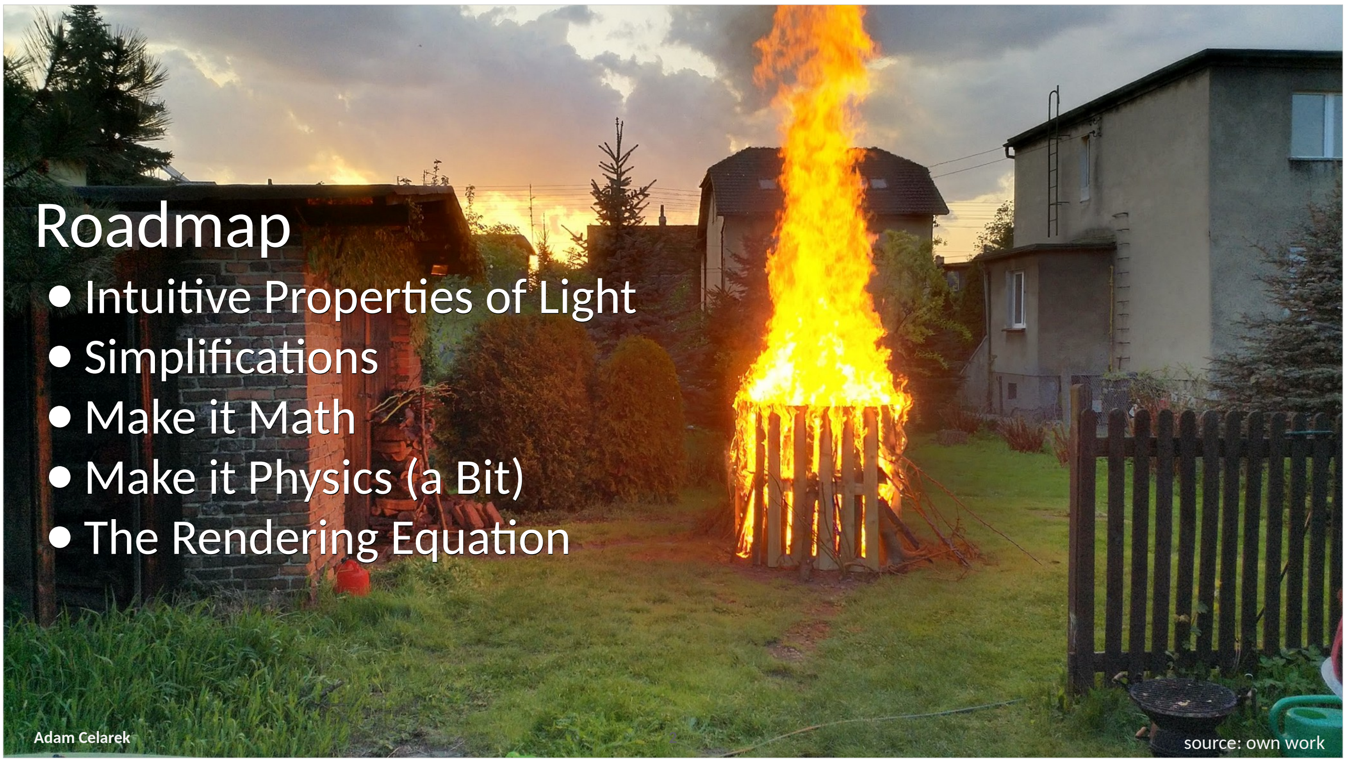
Hi, and welcome to our second rendering lecture. My name is Adam Celarek, and I will talk about light

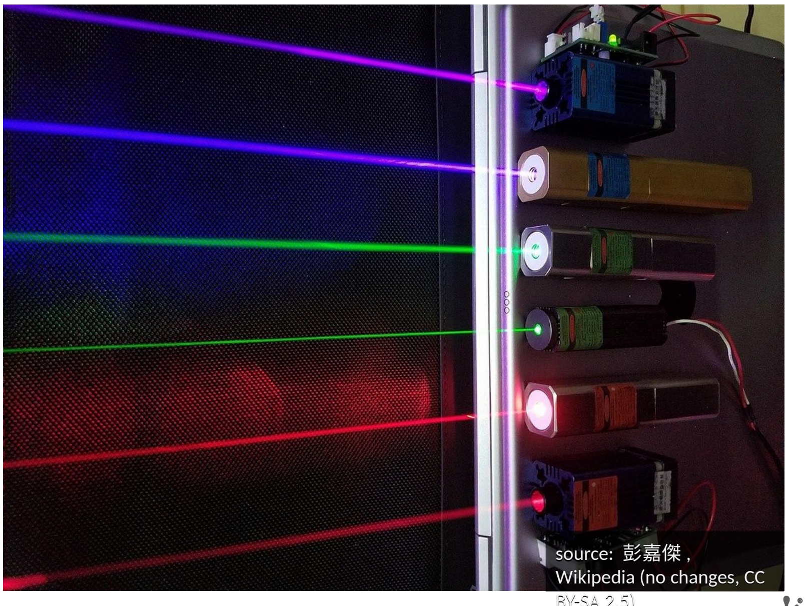


I'll first try to give you intuition, explain some basics, then make some simplifications, because we don't need to compute everything, then go a bit into math and physics and finally tell you how to apply what you learned to compute direct light, or in other words -- soft shadows.

Let's begin..

Intuitive Properties of Light

- It travels in straight lines



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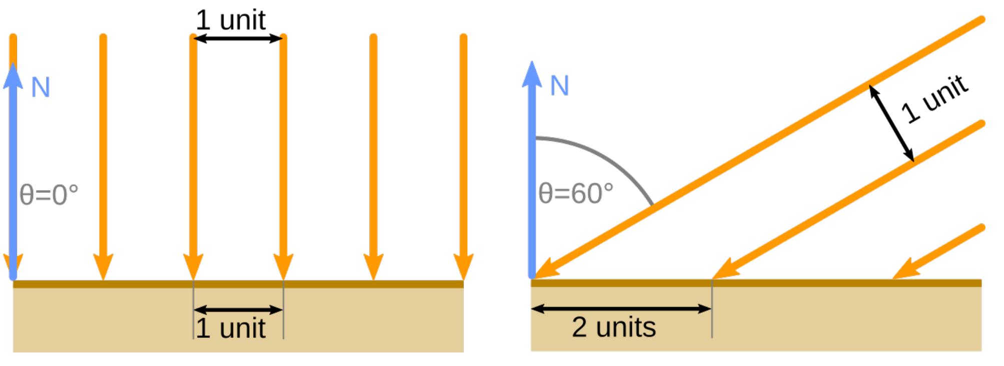
.. to talk about stuff that you probably know about light.

It travels in straight lines -- that you certainly know because cats don’t know it, and hence you were able to tease them with a laser pointer.

Intuitive Properties of Light

- It travels in straight lines
- Angle θ plays a role (cos(θ) rule)



what you should know as well, is, that the incident angle theta, plays a role. You might have heard about it in a previous computer graphics course, but we'll repeat for the youtube audience and for completeness.

when the sun is shining right from above, one packet of light hits one unit on the surface. but when you tilt the sun or your surface, then one packet of light is distributed on a larger and larger area.

Intuitive Properties of Light

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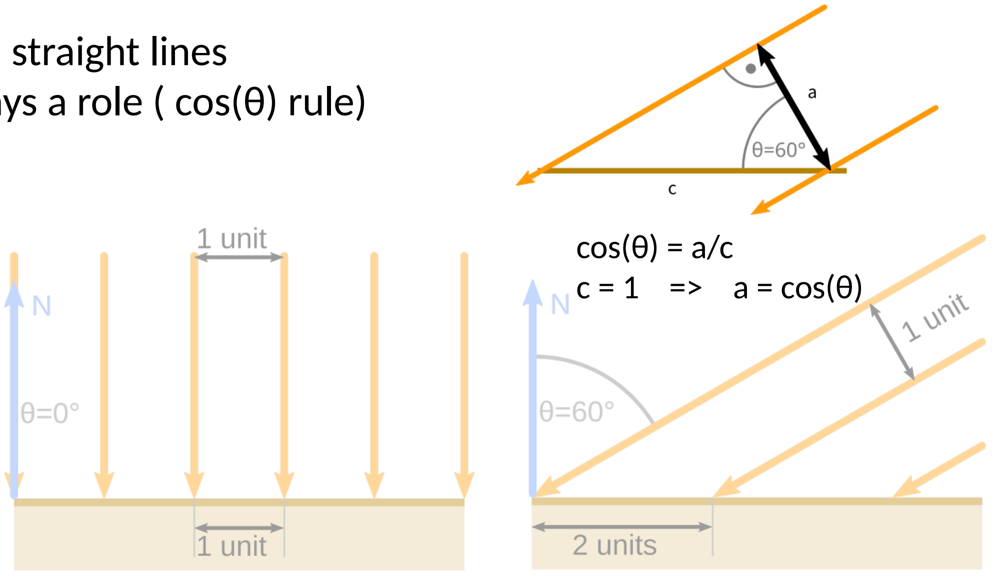
- Angle θ plays a role (cos(θ) rule)

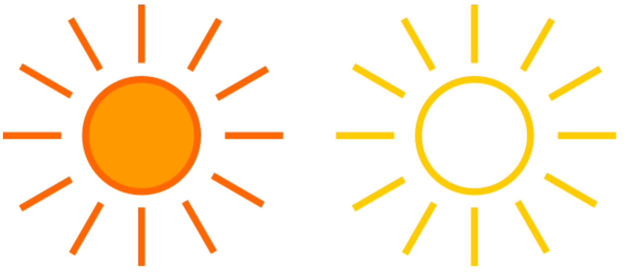


and the percentage of light is precisely the cosinus of the angle between the normal and the incident light direction, which can be calculated using simple trigonometry. right here the cosinus is the adjacent leg divided by the hypotenuse. if we want to compute how much light arrives at a unit length, we set c to one and get cos(θ) -- done.

and yes, obviously you will compute that in practice by taking the dot product between the normal and the light vector.

Intuitive Properties of Light

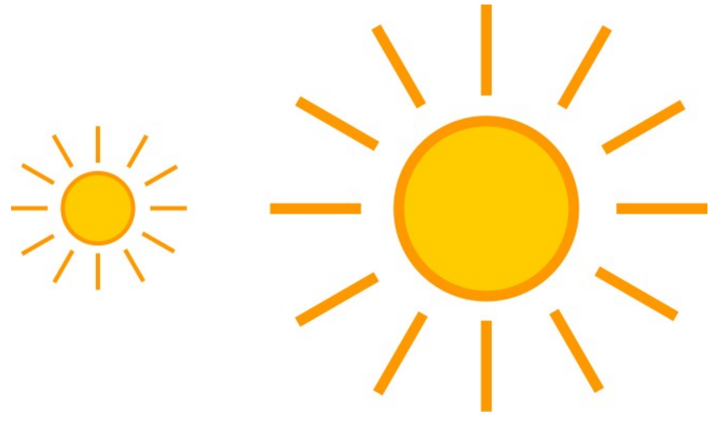
- It travels in straight lines
- Angle θ plays a role (cos(θ) rule)
- Intensity is linear (believe me)



Next, intensity, obviously that plays a role. a brighter light gives a brighter surface. this relation is linear, which also shouldn't be surprising..

Intuitive Properties of Light

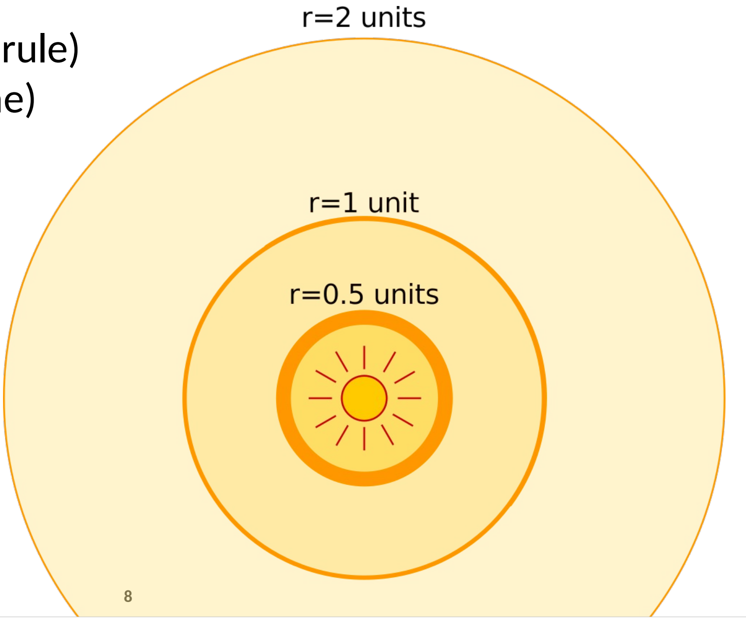
- It travels in straight lines
- Angle θ plays a role (cos(θ) rule)
- Intensity is linear (believe me)
- Size of the light source



size of the light source: Now that relationship is a bit trickier, it’s not linear. we will see the math later. for now just imagine you are standing one meter in front of a flat rectangular light source, that has the same brightness everywhere. you would look brighter if you increased the size from 10 centimetres to one metre, but the change would be minimal if you increased the size from 1 kilometer to 100. the angle theta comes in -- for starters, but also the distance to the far away points.

Intuitive Properties of Light

- It travels in straight lines
- Angle θ plays a role (cos(θ) rule)
- Intensity is linear (believe me)
- Size of the light source
- Distance to light source



which leads us to the last property -- distance.

you probably know from previous courses that a point light attenuates with 1 over distance squared.

this is simple to explain, imagine light to be the skin of a balloon growing around the source. the balloon grows because light travels away from the source. the number of light particles doesn’t increase when the balloon grows, so the density is inversely proportional to the surface area of the ballon, and the surface area of a sphere grows by the square of the radius.

good.

Intuitive Properties of Light

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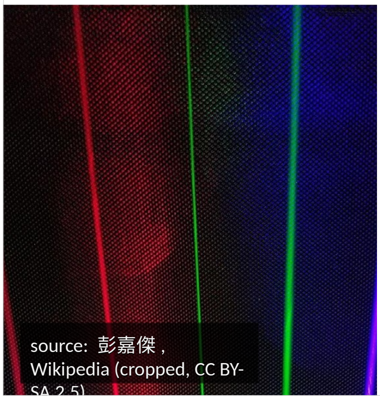

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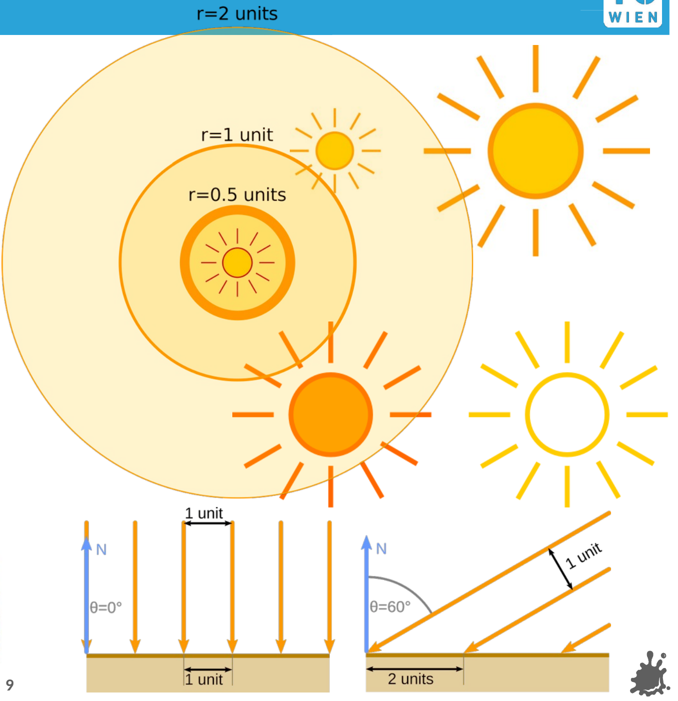

ok, but how should we put all that into a coherent framework?

we have to focus onto what's important: the brightness of a certain point on my surface.

Intuitive Properties of Light

- How "bright" something is doesn't directly tell you how brightly it *illuminates* something
  - The lamp appears just as bright from across the room and when you stick your nose to it ("intensity does not attenuate")
  - Also, the lamp's apparent brightness does not change much with the angle of exitance

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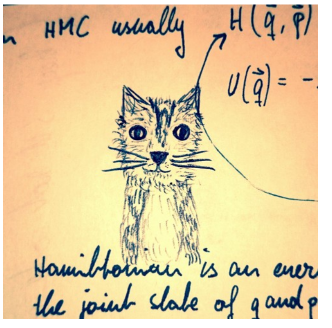
Intuitive Properties of Light

- How “bright” something is doesn’t directly tell you how brightly it *illuminates* something
  - The lamp appears just as bright from across the room and when you stick your nose to it (“intensity does not attenuate”)
  - Also, the lamp’s apparent brightness does not change much with the angle of exitance
- **However**:
  - If you take the receiving surface further away, it will reflect less light and appear darker
  - If you tilt the receiving surface,it will reflect less light and appear darker



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Light
travels in straight lines
cos rule
distance
intensity
size
Next: Less intuitive effects
Adam Celarek
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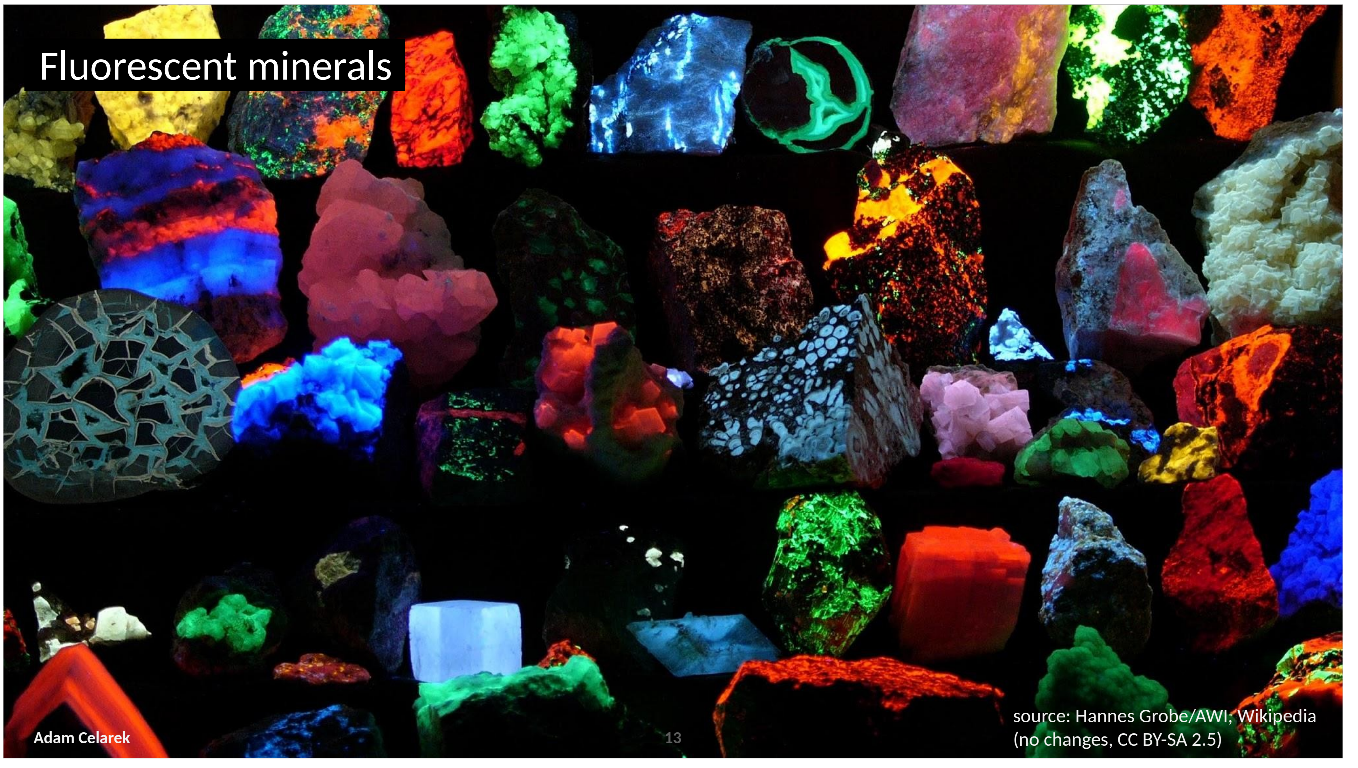
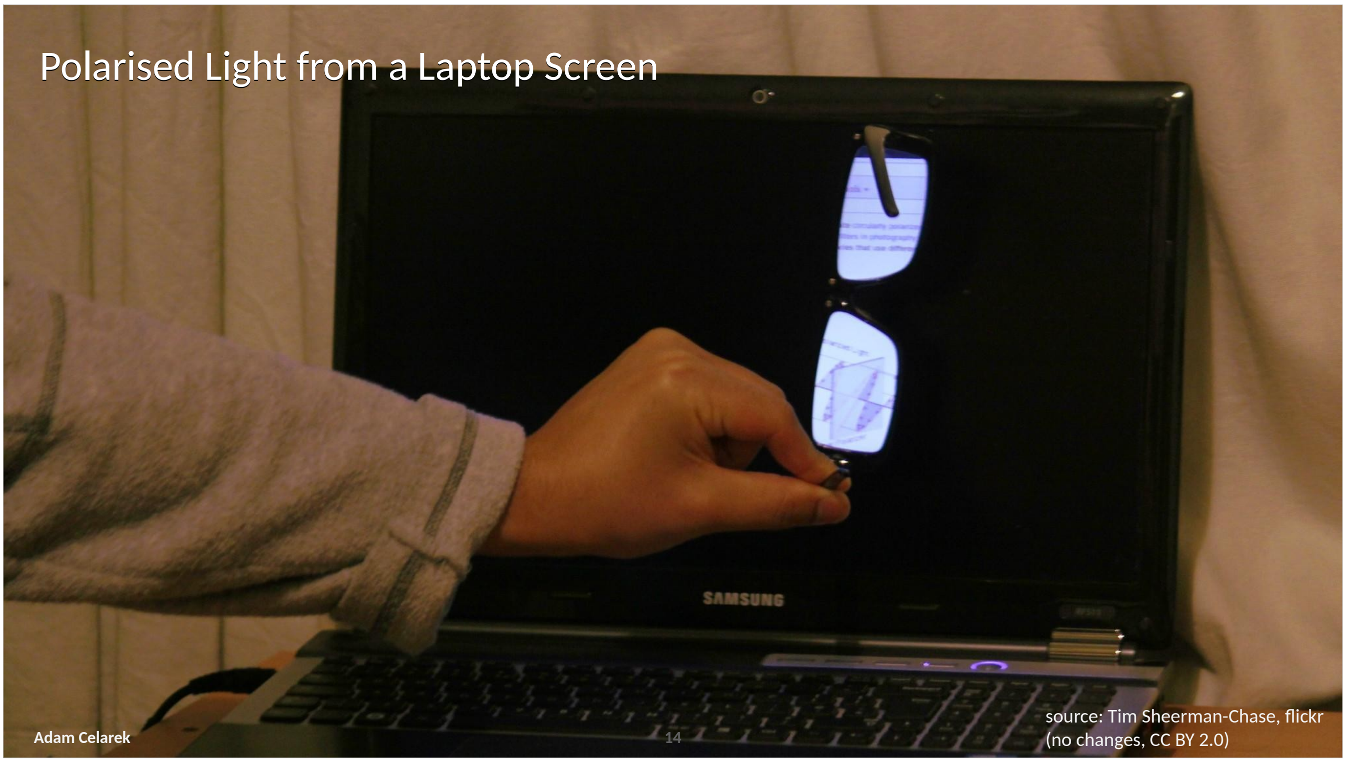
source: own work



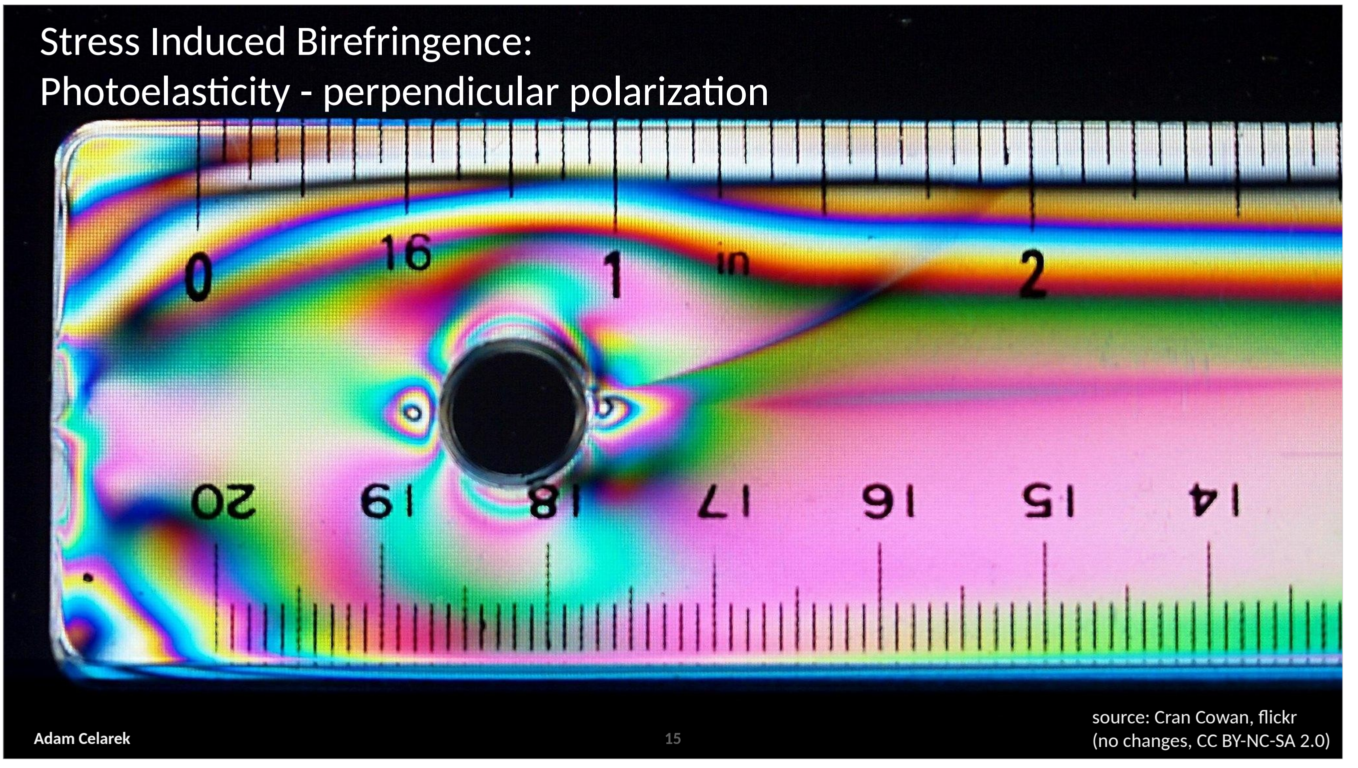
materials that change the wavelength. for instance uv -> visible light.

good example are stripes on ambulances that appear brighter than they should



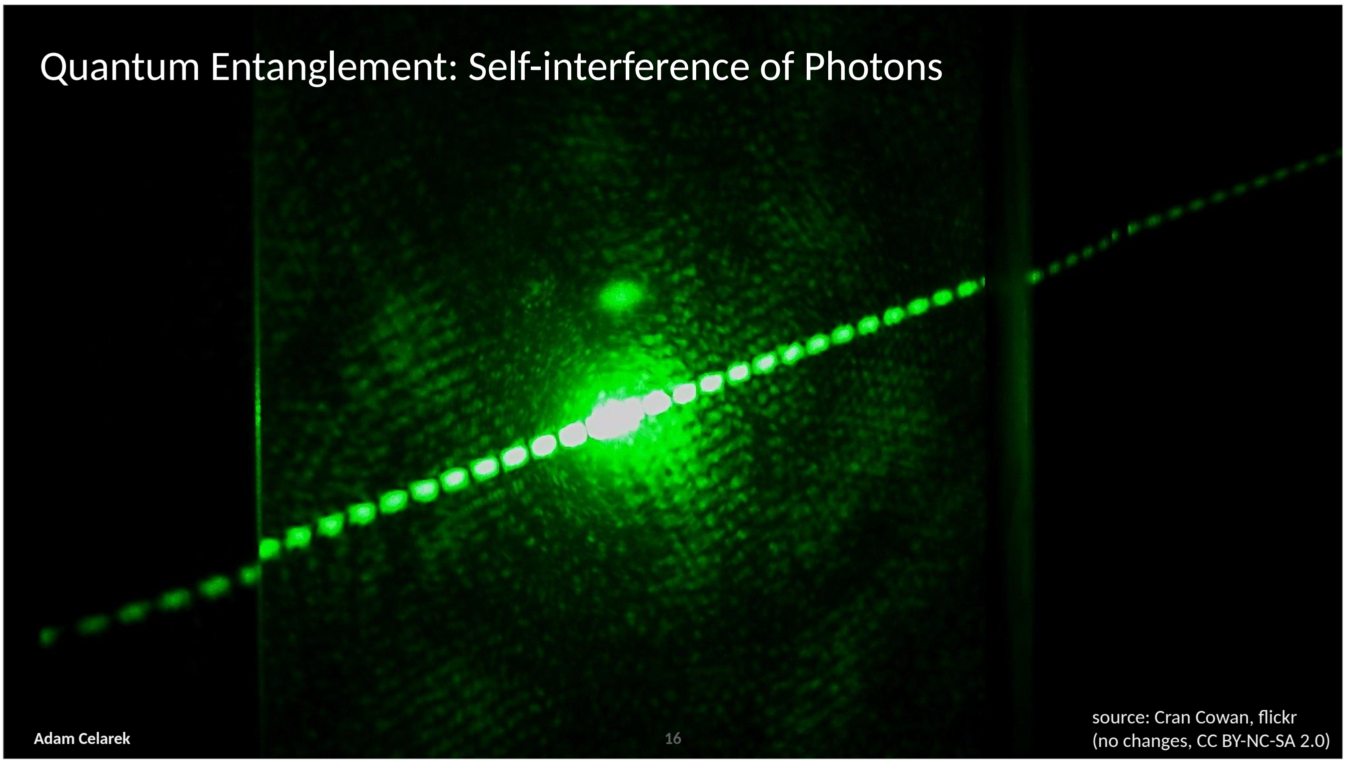
The next special effect is polarised light. linear and circular, circular can be cw and ccw, while linear can have different angles.

I don’t know how this was made exactly, but the laptop screen emits polarised light, the glasses are a polarisation filter and there is another on the camera.



Using my laptop's display as a source for polarized light, a polyvinyl chloride ruler was photographed using an analyzing polarizer in front of the camera lens.

The color patterns are due to interference caused by phase retardation of the light going through the plastic. Internal stresses were frozen when the plastic cooled creating a stress tensor field that resulted in a varying birefringence which is seen by a spectral color pattern.



Shown here is the interference pattern produced by a green 532nm laser beam passing around a wire (0.254mm in diameter) at a distance of about 4.5m from the wall acting as a screen.

The interference pattern is created by individual photons interfering with themselves. The interference pattern occurs even when the intensity of the light is so small that only one photon leaves the laser at a time

Simplifications (things that we will not do)

- We use ray optics (also called geometrical optics)
  - Doesn’t account for phenomena like diffraction or interference (rendering optical discs is hard)
- No energy transfer between frequencies (fluorescence)
- In this course we disregard the spectrum and just compute RGB separately (though production renderers often simulate a spectrum)
- And we will ignore polarisation.

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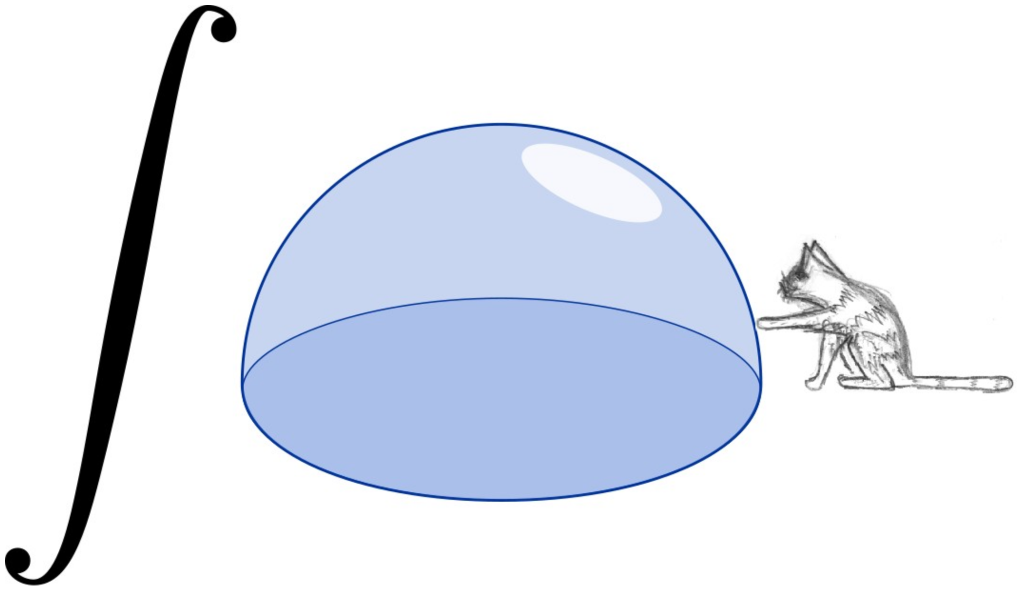
Make it math



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how to compute the amount of light that reaches a certain point on a certain surface.

ok.

we have to sum up all the light. yes that is an integral.

we have to sum up from all direction, that is a hemisphere (we ignore light coming from inside the material, like glass, for now)

Make it math

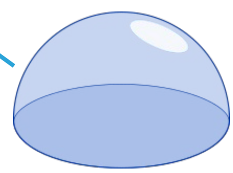
- Don’t be afraid of integrals, we’ll learn how to compute them later
- Basically, look into all directions and sum up all incoming light

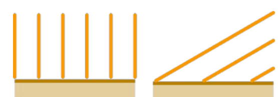
Light arriving at point x

Light from direction ω

Solid angle (next)

$$L_i(x) = \int_\Omega L_i(x, \omega) \cos(\theta_x)\, \mathrm{d}\omega$$





(not useful for rendering yet)

light from direction ω: by ray tracing.

Make it math

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- Basically, look into all directions and sum up all incoming light

Light arriving at point x

Light from direction ω

Solid angle (next)

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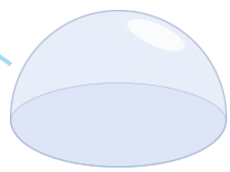


compare to a 1d integral from basic calculus

$$A = \int_a^b f(x) \, \mathrm{d}x$$

dx and dω are differentials

Make it math

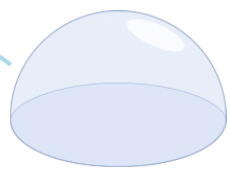
- Don't be afraid of integrals, we'll learn how to compute them later
- Basically, look into all directions and sum up all incoming light

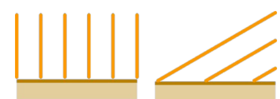
Light arriving at point x

Light from direction ω

Solid angle (next)

$$L_i(x) = \int_\Omega L_i(x, \omega)(\omega \cdot n)\, \mathrm{d}\omega$$





(not useful for rendering yet)

Make it math

- What's going on with that object size, distance etc?

- "Illumination power" is determined by the solid angle subtended by the light source (simple, how big something looks).

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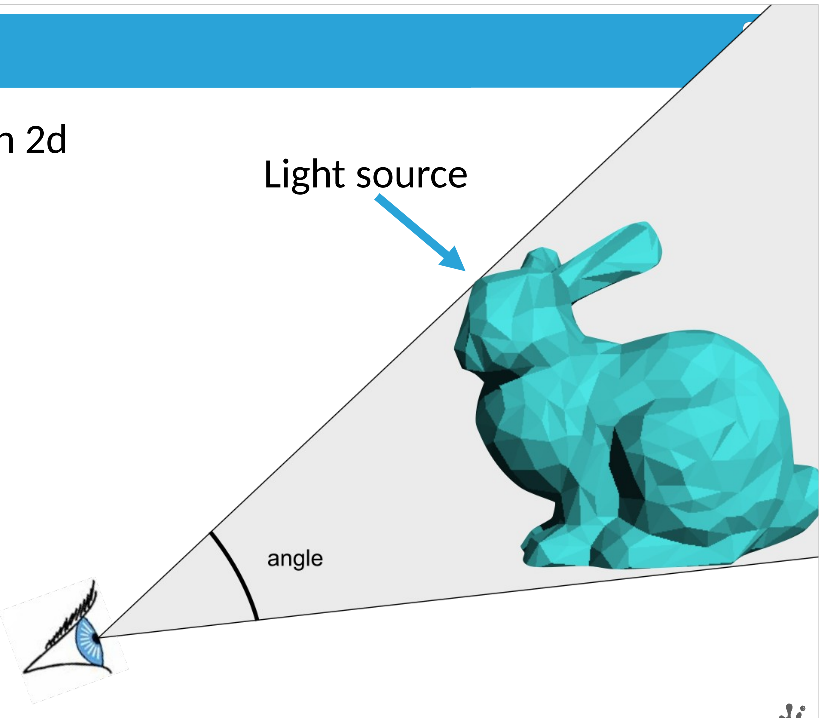
Make it math

- How big something looks in 2d



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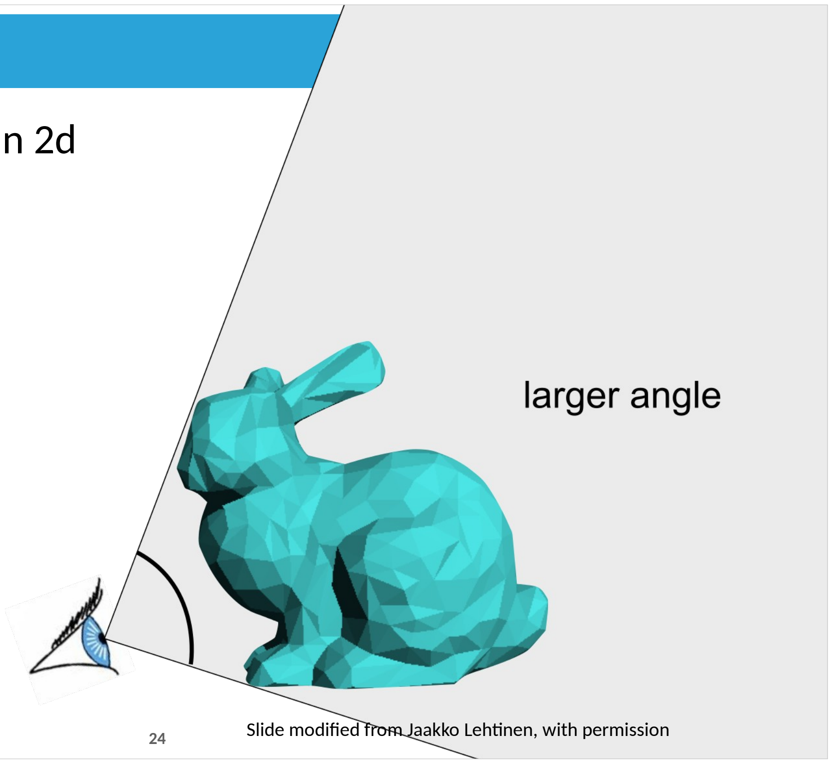
Make it math

- How big something looks in 2d



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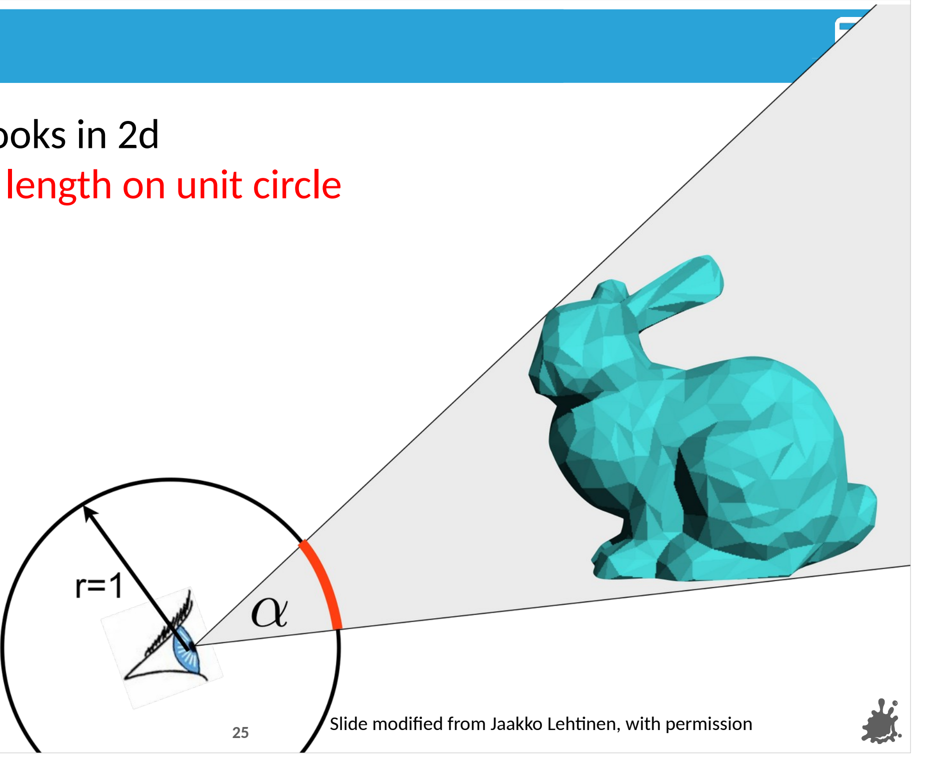
Make it math

- How big something looks in 2d
- Angle α in radians ⇔ length on unit circle
- Full circle is 2π



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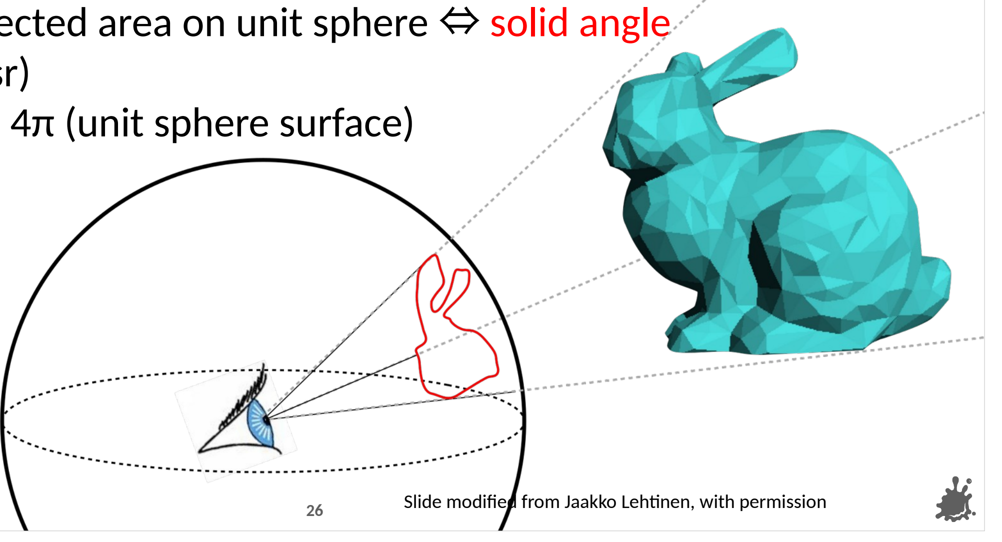
Make it math

- How big something looks in **3d**
- replace unit circle with unit sphere
- Same thing: projected area on unit sphere ⇔ solid angle
- Unit: steradian (sr)
- Full solid angle is 4π (unit sphere surface)



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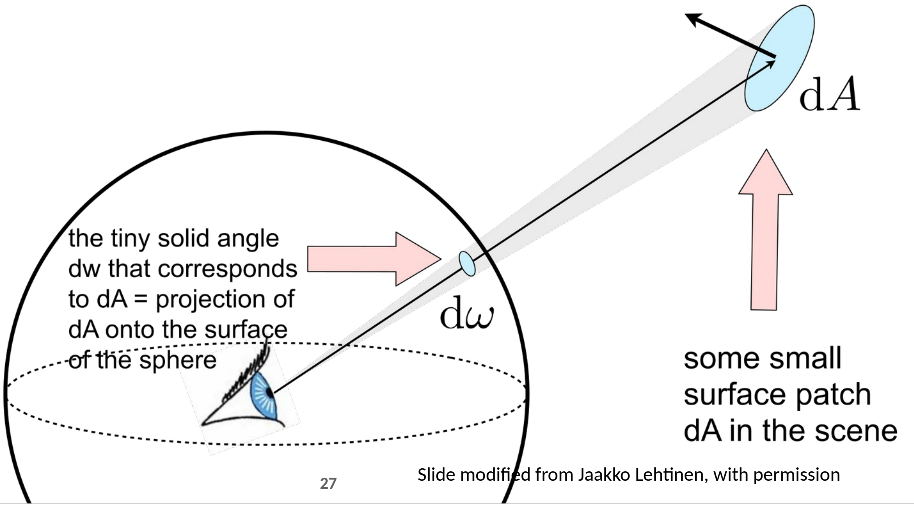
Make it math

Relationship between a surface patch and the solid angle

=> what determines the area of the projected patch (solid angle)



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Make it math

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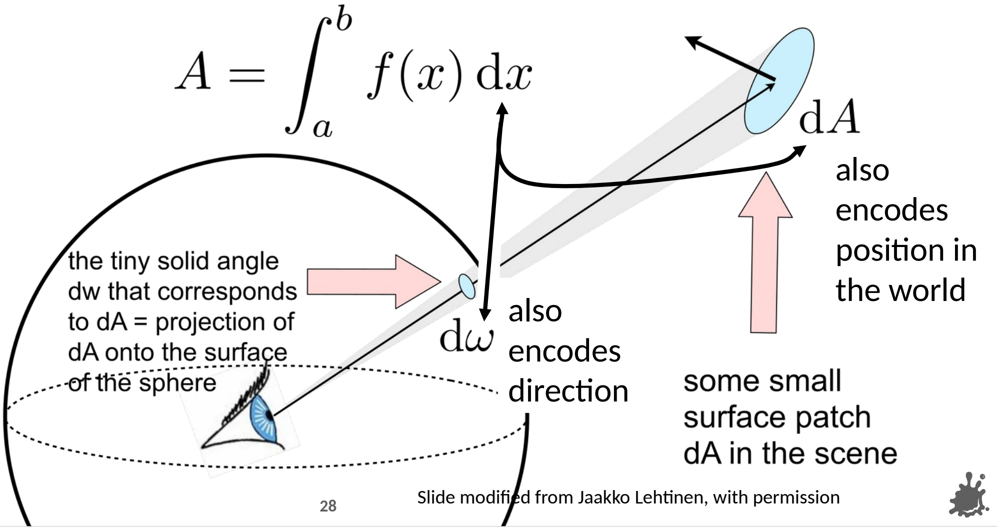

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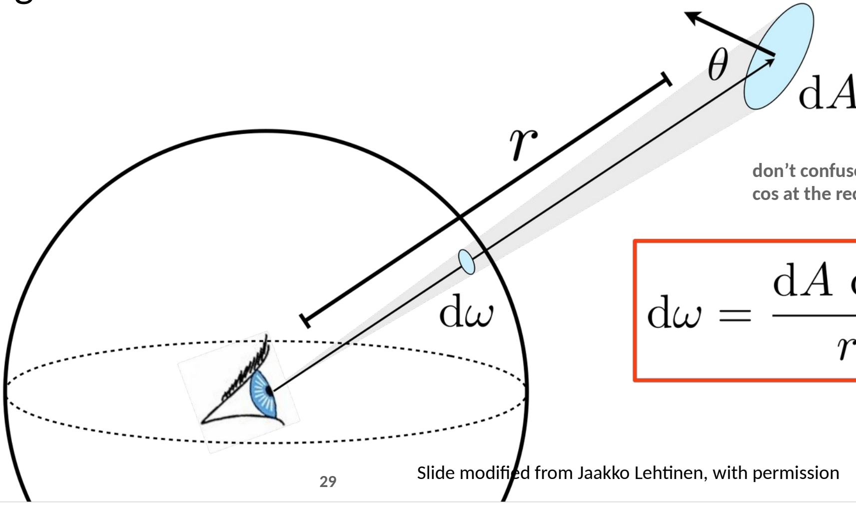
dx is called differential.

Make it math

Relationship between a surface patch and the solid angle

It holds for infinitesimally small surface patches dA and the corresponding differential solid angles dω



don't confuse this with the cos at the receiving surface

$$\boxed{\mathrm{d}\omega = \frac{\mathrm{d}A\,\cos\theta}{r^2}}$$

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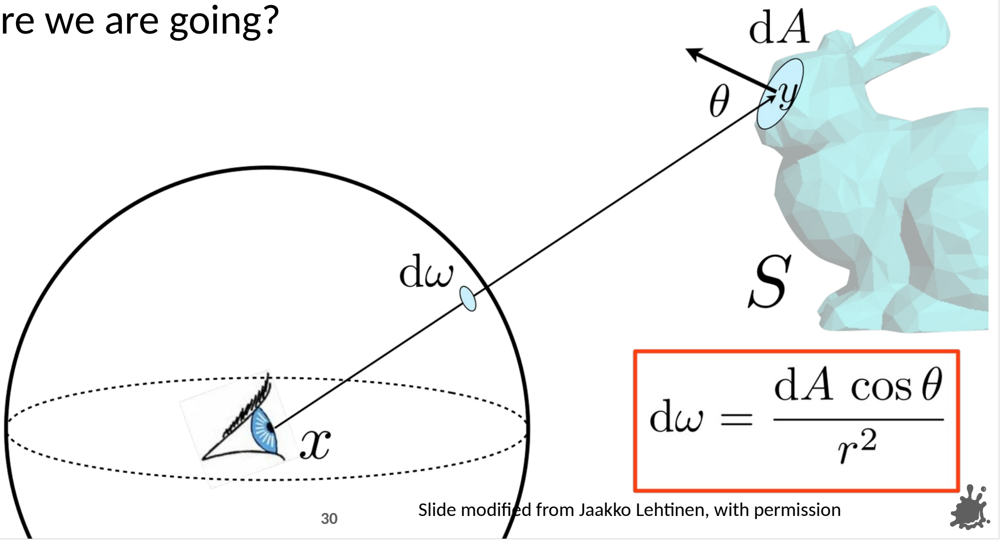
Make it math

Larger Surfaces

Actual surfaces consist of infinitely many tiny patches dA
-- do you see where we are going?



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Make it math

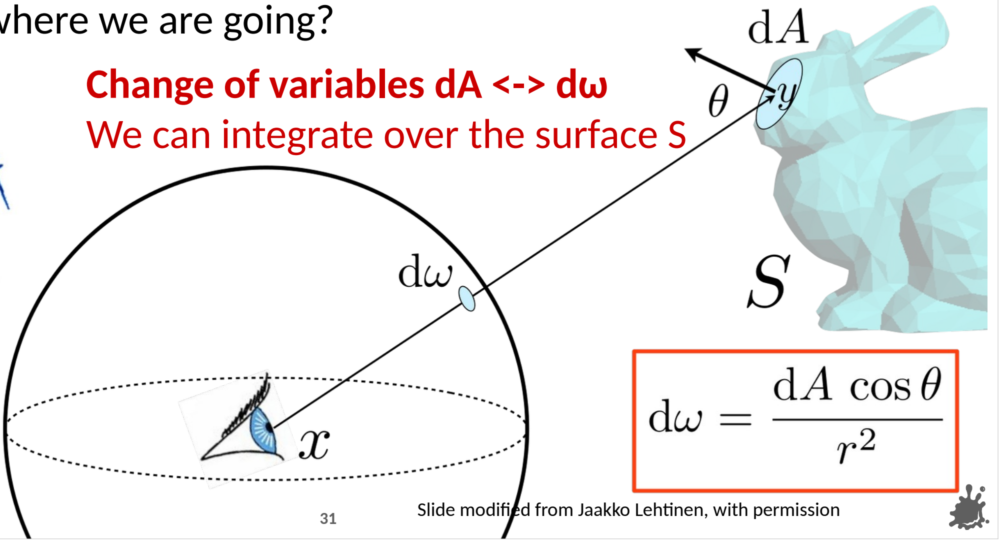
Larger Surfaces

Actual surfaces consist of infinitely many tiny patches dA

------- do you see where we are going?

Change of variables dA <-> dω

We can integrate over the surface S



$$\boxed{\mathrm{d}\omega = \frac{\mathrm{d}A\ \cos\theta}{r^2}}$$

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Make it math

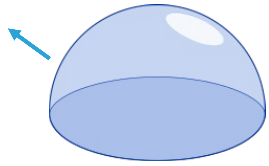
We have seen this before, but now we want to integrate over a single light surface. How do we need to change the formula?

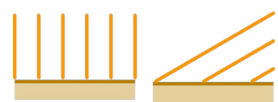
Light arriving at point x

Light from direction ω

Solid angle (just before)

$$L_i(x) = \int_\Omega L_i(x, \omega) \cos(\theta_x) \, \mathrm{d}\omega$$





(not useful for rendering yet)

Make it math

Light arriving at point x

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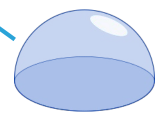
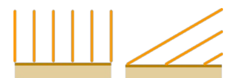
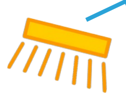




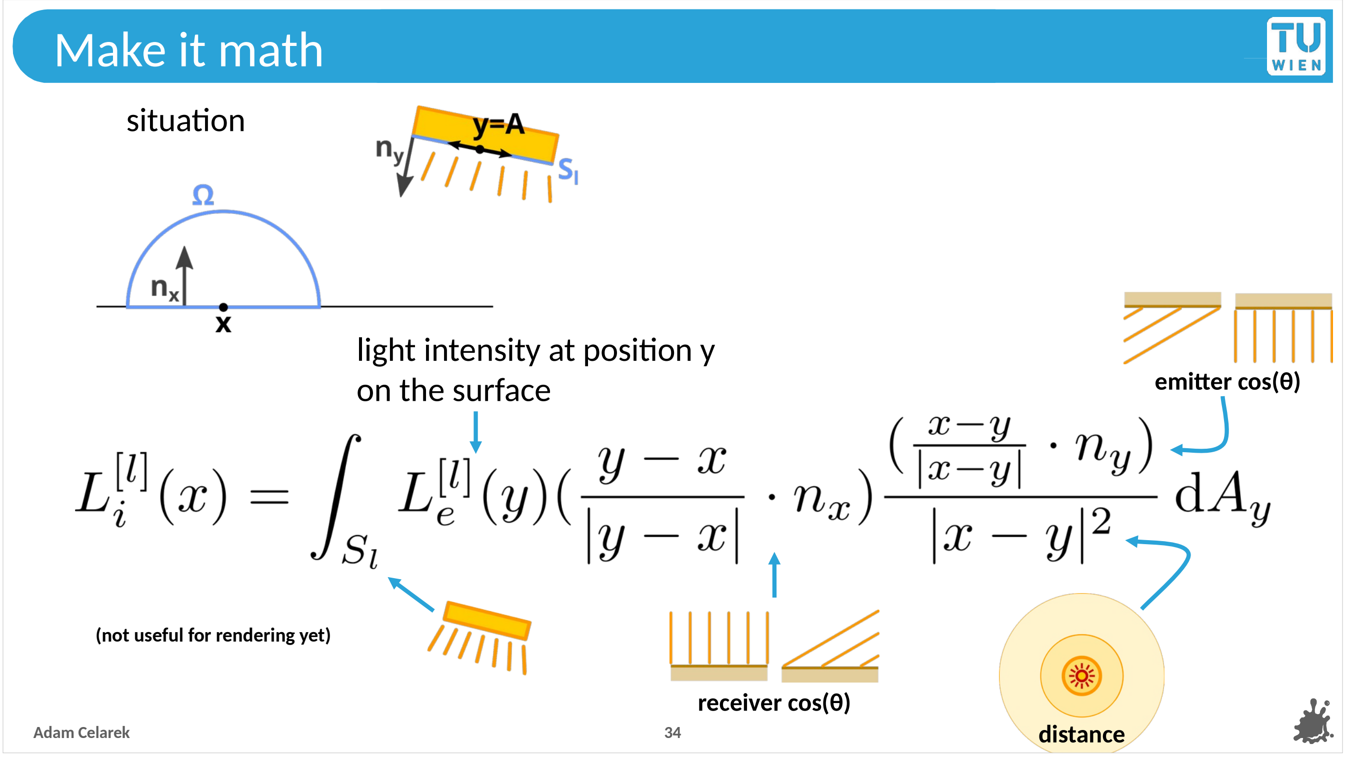
Solid angle (just before)

Light from source [l] arriving at point x

$$L_i^{[l]}(x) = \int_{S_l} L_e^{[l]}(y) \cos(\theta_x) \boxed{\frac{\cos(\theta_y)}{r^2} \, \mathrm{d}A_y}$$



light intensity at position y on the surface



ok, and since it could be hard to imagine how that works in practise, here i've expanded all the variables.

we employ numerical integration, that means we have to evaluate the integral at certain points. these points are y, and they are used together with the point x to compute

Light integral

How to compute the amount
of light that reaches a certain point?

source: own work

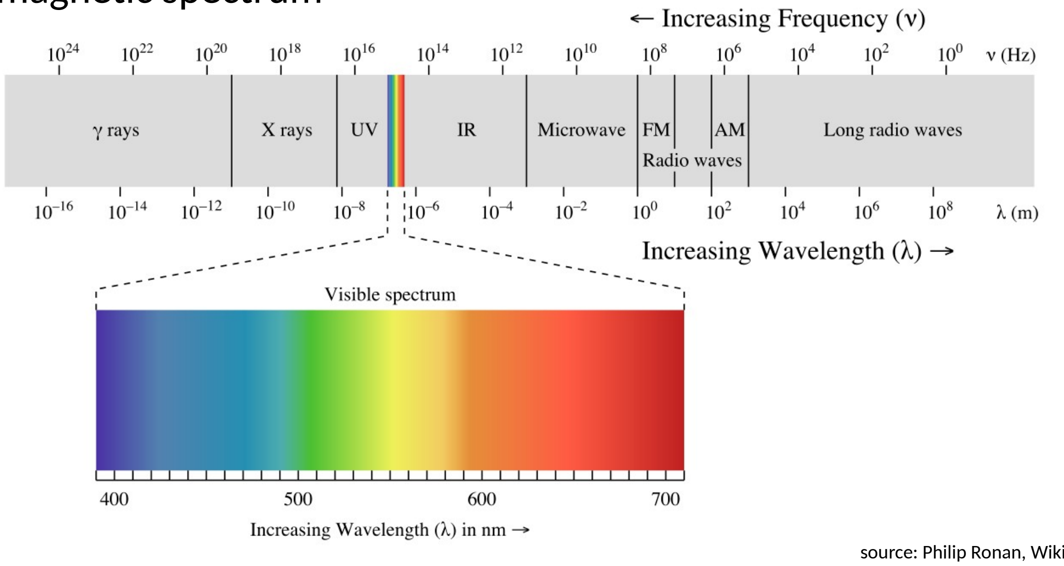
Next: Physics

Make it Physics (a Bit)

- Electromagnetic spectrum
- Radiometry and photometry
  - Units and naming
  - How is that stuff perceived in the human eye
- Radiance (constant along straight lines)
- Rendering
  - Irradiance
  - Materials
  - White furnace test (energy conservation)

Make it Physics (a Bit)

Electromagnetic spectrum



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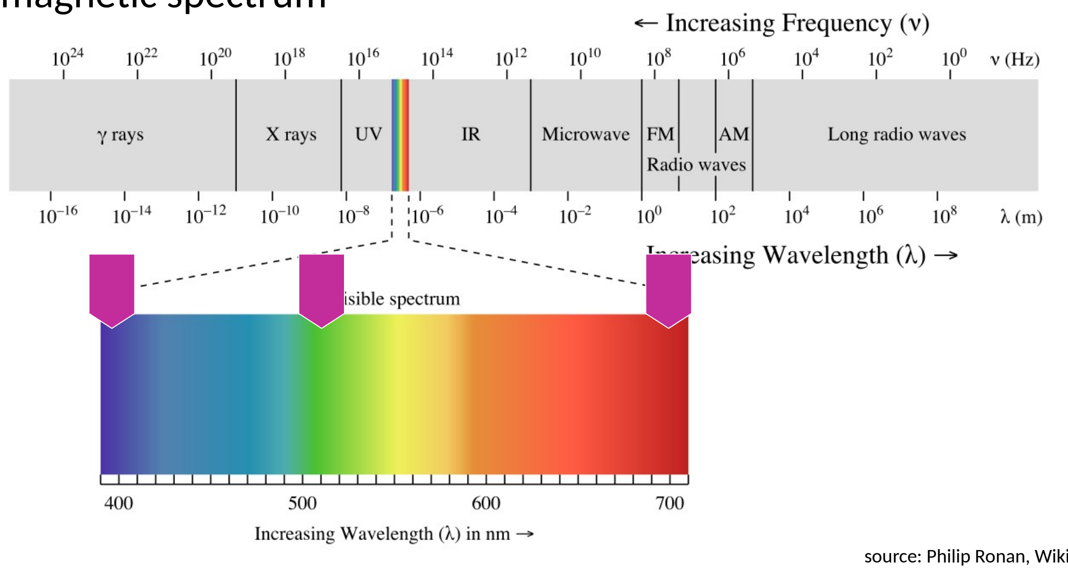
Make it Physics (a Bit)

Electromagnetic spectrum



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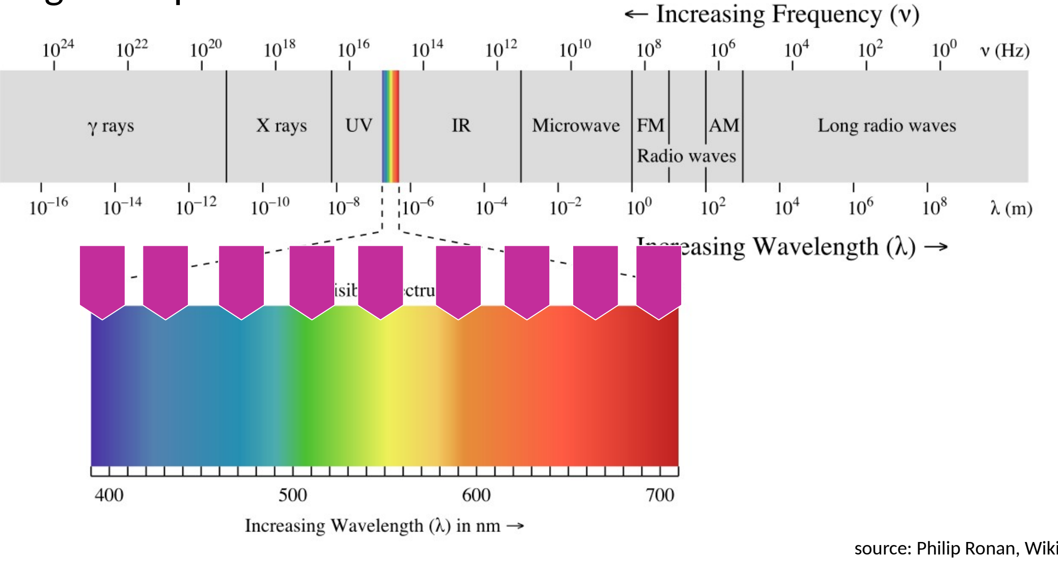
Make it Physics (a Bit)

Electromagnetic spectrum



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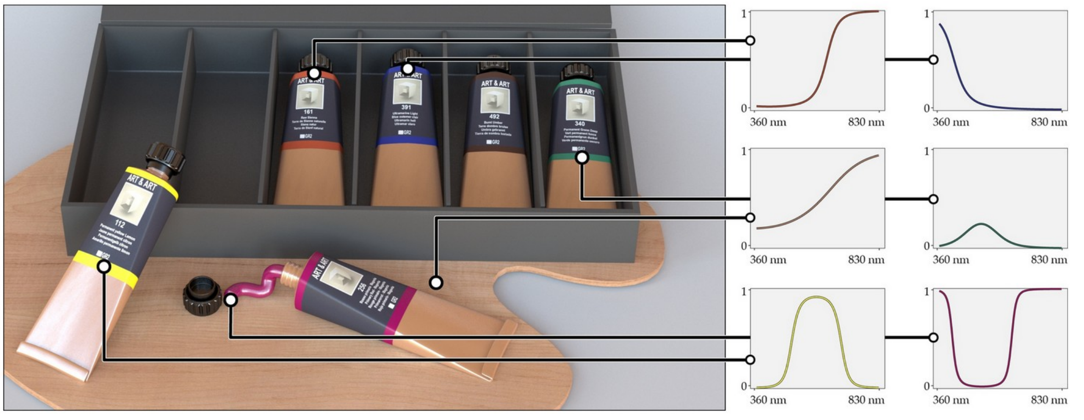
Make it Physics (a Bit)

A Low-Dimensional Function Space for Efficient Spectral Upsampling

Wenzel Jakob   Johannes Hanika

In *Computer Graphics Forum (Proceedings of Eurographics 2019)*



Left. *A spectral rendering performed using the proposed technique. This scene uses a variety of RGB textures that have been converted into reflectance spectra.* ***Right***. *Plots of highlighted surface regions over the visible range.*

Make it Physics (a Bit)

Radiometry

- Units and naming
  - Radiant energy Q_e [J] (Joule)
  - Radiant flux / power Θ_e[W=Js] (Watt = Joule seconds)
  - Radiant intensity $I_e(\omega)$ [W/sr] (Watt / steradians = solid angle)
  - Irradiance $E_e(x)$ [W/m²] (incident flux per unit area, think of photons, integral from before)
  - Radiant exitance $M_e(x)$ [W/m²] (emitted flux per unit area, i.e. light source)
  - Radiosity $J_e(x)$ [W/m²] (flux per unit area emitted + reflected)
  - Radiance $L_e(x, \omega)$ [W/(m²sr)] (flux per unit area per solid angle)
  - Radiometric quantity per wavelength $L_{e,\lambda}(x, \omega)$ [W/(m² sr nm)] (erm..)

radiant energy = compare with rain or water drops. i know might be a bit weird, but it'll work out great a bit later. so energy -- how much rain is in the air.

flux = how much water in total (no mention of area or angles)

radiant intensity= how much water drops in a certain direction

irradiance = how much water per m2 (i.e. millimetres)

exitance = how much water is coming out of the clouds per m2

radiosity = imagine the rain is flying up into the clouds, being reflected by the clouds and the clouds add some rain.

radiance = the same, but per direction and area.

rqpw = same, but per wavelength as well

Make it Physics (a Bit)

Photometry

- Measurement of perceived brightness
- The human eye has a different sensitivity to different wavelengths (colours), sometimes we have to account for that
- Radiance -> Luminance
- There are also units and names

more sensitive to yellow (which is red and green) light than to blue light

Make it Physics (a Bit)

Radiometry and Photometry

Radiometric quantity	Symbol	Unit	Photometric quantity	Symbol	Unit
Radiant energy	Q_e	[J] *joule*	Luminous energy	Q_v	[lm s] *talbot*
Radiant flux	Φ_e	[W] *watt*	Luminous flux	Φ_v	[lm] *lumen*
Radiant intensity	I_e	[W sr^{-1}]	Luminous intensity	I_v	[cd] *candela*
Radiance	L_e	[W sr^{-1} m^{-1}]	Luminance	L_v	[cd m^{-2}] *nit*
Irradiance	E_e	[W m^{-2}]	Illuminance	E_v	[lx] *lux*
Radiant exitance	M_e	[W m^{-2}]	Luminous emittance	M_v	[lx]
Radiosity	J_e	[W m^{-2}]	Luminosity	J_v	[lx]

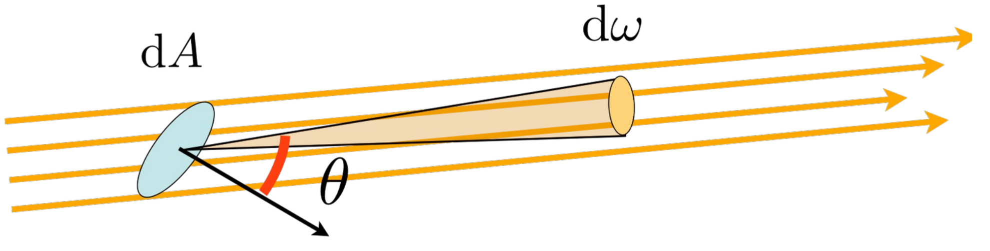
source: previous year's lecture (Auzinger and Zsolnai)

talbot, that is a unit with a nice name..

Make it Physics (a Bit)

- **Radiance** is the fundamental quantity that simultaneously explains effects of both light source size and receiver orientation
- Let's consider a tiny almost-collimated beam of cross-section $dA^{\perp} = dA \cos(\theta)$ where the directions are all within a differential angle $d\omega$ of each other

dA and dω are differentials. check out 3blue1brown, if you want a really good explanation



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ok, back to serious

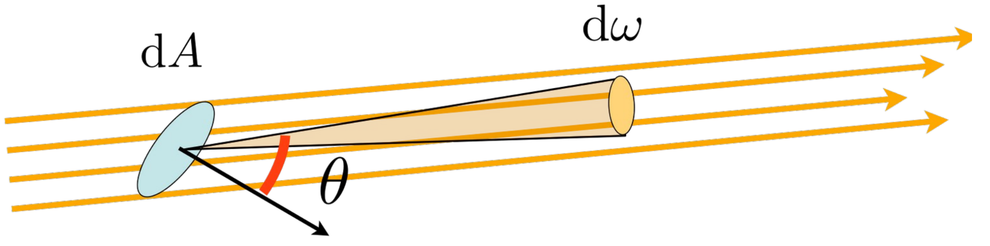
Make it Physics (a Bit)

Radiance **L =**
flux per unit projected area per unit solid angle

$$L = \frac{\mathrm{d}\Phi}{\mathrm{d}A^{\perp}\,\mathrm{d}\omega}$$

dA, dω and dΦ are differentials. check out 3blue1brown, if you want a really good explanation

$$[L] = \left[\frac{W}{m^2\ sr}\right]$$



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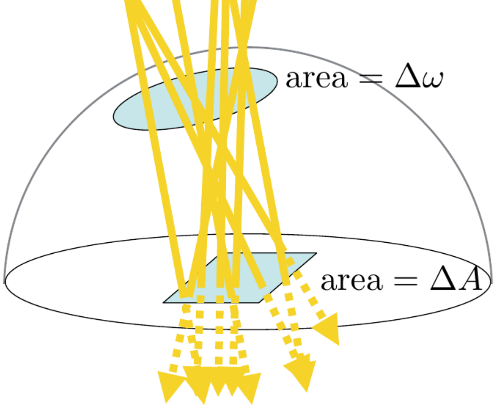
Make it Physics (a Bit)

Radiance, intuitively

Let's count energy packets, each ray carries the same ΔΦ (dΦ)

$$L = \frac{\mathrm{d}\Phi}{\mathrm{d}A^{\perp}\,\mathrm{d}\omega}$$

$$[L] = \left[\frac{W}{m^2\ sr}\right]$$



dA, dω and dΦ are differentials. check out 3blue1brown, if you want a really good explanation

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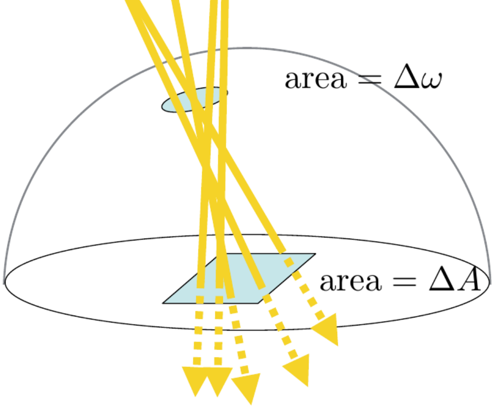
sorry for the inconsistent notation

Make it Physics (a Bit)

Radiance, intuitively

Smaller solid angle

=> fewer rays => less energy



$$L = \frac{\mathrm{d}\Phi}{\mathrm{d}A^{\perp}\,\mathrm{d}\omega}$$

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dA, dω and dΦ are differentials. check out 3blue1brown, if you want a really good explanation

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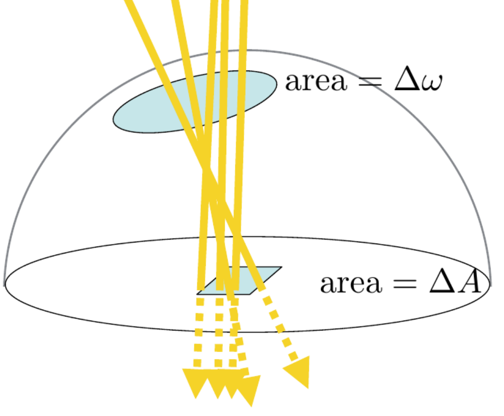
Make it Physics (a Bit)

Radiance, intuitively
Smaller projected surface area
=> fewer rays => less energy



$$L = \frac{\mathrm{d}\Phi}{\mathrm{d}A^{\perp}\,\mathrm{d}\omega}$$

$$[L] = \left[\frac{W}{m^2\ sr}\right]$$

dA, dω and dΦ are differentials. check out 3blue1brown, if you want a really good explanation

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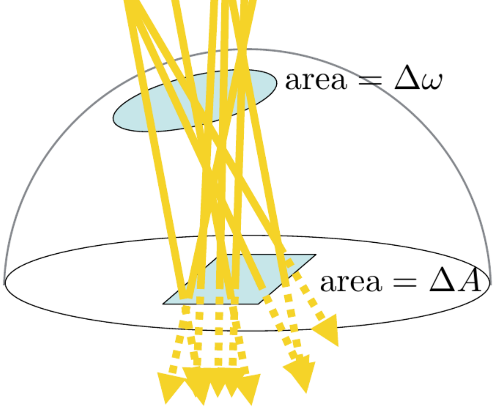
Make it Physics (a Bit)

Radiance, intuitively
I.e., radiance is a density over both space and angle



$$\boxed{L = \frac{\mathrm{d}\Phi}{\mathrm{d}A^{\perp}\,\mathrm{d}\omega}}$$

$$[L] = \left[\frac{W}{m^2\ sr}\right]$$

dA, dω and dΦ are differentials. check out 3blue1brown, if you want a really good explanation

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Make it Physics (a Bit)

Radiance

- **Sensors are sensitive to radiance**
  - It’s what you assign to pixels
  - The fundamental quantity in image synthesis
- “Intensity does not attenuate with distance”
  ⇔ **radiance stays constant along straight lines** *
- All relevant quantities (irradiance, etc.) can be derived from radiance

* unless the medium is participating, e.g. smoke, fog, wax, water, air..

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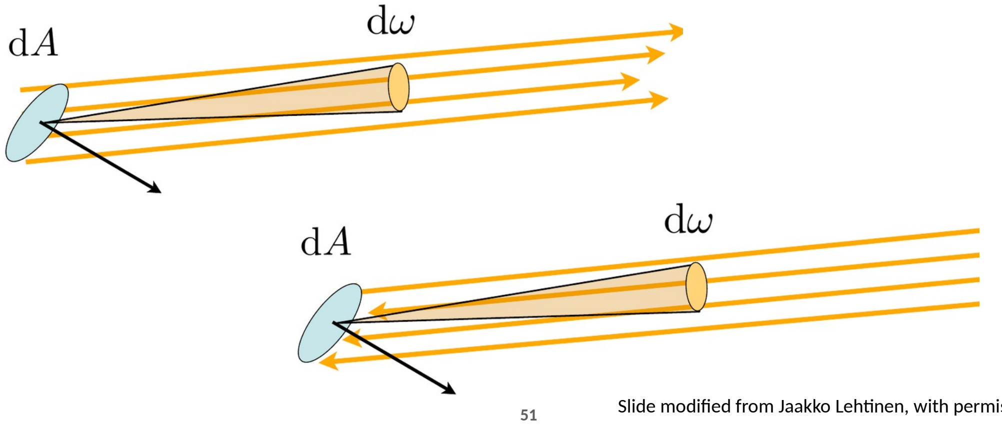
Make it Physics (a Bit)

Radiance characterises

- Light that leaves a surface patch dA to a given direction
- Light that arrives at a surface patch dA from a given direction

(just flip the direction)



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Make it Physics (a Bit)

Radiance also exists in empty space, away from surfaces

- Radiance L(x,ω), when taken as a 5d function of position (3d) and direction (2d) completely nails down the light flow in a scene
- Sometimes called the "plenoptic function"

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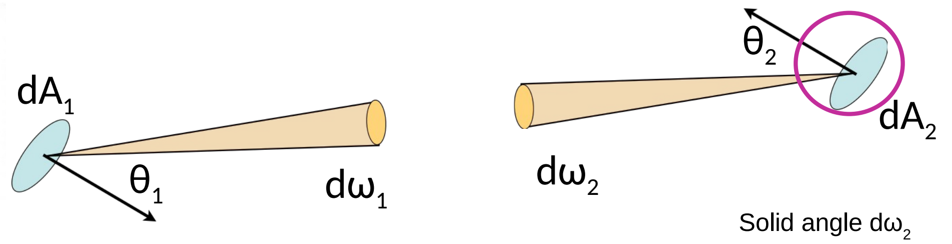
Make it Physics (a Bit)

Constancy along straight lines

Let’s look at the flux sent by dA_2 into the direction of dA_1



$$\boxed{L = \frac{d\Phi}{dA^{\perp}\, d\omega}}$$

$$d\Phi = L(x_2 \to \omega_2)\overbrace{\cos\theta_2\, dA_2}^{dA_2^{\perp}}\overbrace{\frac{dA_1 \cos\theta_1}{r^2}}^{\text{Solid angle } d\omega_2 \text{ subtended by } dA_1 \text{ as seen from } dA_2}$$

Let’s now look at the flux sent from surface patch A2 towards A1.

We have the formula at the bottom.

L is the radiance, which is constant along straight lines – and remember, those are differentials, so we the solid angle is infinitesimal, meaning just a line.

But the sending patch still obeys the cosine rule, but in reverse. Let me explain: say the surface is sending out 100 packets of light per square metre in all directions equally and the size of the surface is 1 square metre. If you are right above the surface, you can see the whole surface, you get 100 packets. If you are completely on the side, you don’t see the surface. It could send a million packets and you wouldn’t see it..

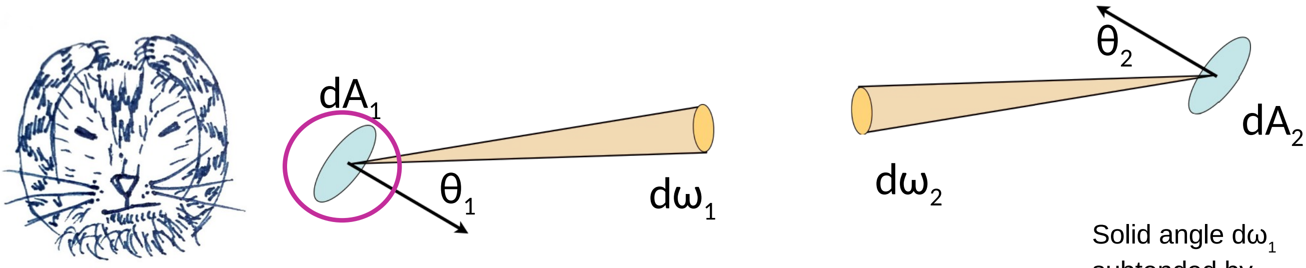
Alright, and then we have the solid angle, which answers the question of how much of the receiver is visible. That works in the same way, if the receiver is turned in a bad way, it wouldn’t receive anything, no matter how large the radiance is..

Let’s look at the reversed situation..

Make it Physics (a Bit)

Constancy along straight lines

And now the flux received by dA_1 from directions dA_2



$$\boxed{\mathrm{L} = \frac{\mathrm{d}\Phi}{\mathrm{d}A^{\perp}\,\mathrm{d}\omega}}$$

$$\mathrm{d}\Phi = \mathrm{L}(x_1 \leftarrow \omega_1)\,\overbrace{\cos\theta_1\,\mathrm{d}A_1}^{\mathrm{d}A_1^{\perp}}\,\overbrace{\frac{\mathrm{d}A_2\cos\theta_2}{r^2}}^{\text{Solid angle } d\omega_1 \text{ subtended by } dA_2 \text{ as seen from } dA_1}$$

Now this looks much more familiar:

Surface patch dA1 is receiving light from surface patch dA2. We had the exact same thing during the change of variables in the maths chapter. Radiance, times cosine rule times solid angle..

ok. compare those two friends..

Make it Physics (a Bit)

Constancy along straight lines

Eureka

$$d\Phi = L(x_2 \to \omega_2) \cos\theta_2 \, dA_2 \frac{dA_1 \cos\theta_1}{r^2}$$

$$d\Phi = L(x_1 \leftarrow \omega_1) \cos\theta_1 \, dA_1 \frac{dA_2 \cos\theta_2}{r^2}$$

$$\boxed{\Rightarrow L(x_2 \to \omega_2) = L(x_1 \leftarrow \omega_1)}$$

Now look at that:

The the sent light is the same as the received one..

Make it Physics (a Bit)

- Electromagnetic spectrum
- Radiometry and photometry
  - Units and naming
  - How is that stuff perceived in the human eye
- Radiance (constant along straight lines)
- Rendering
  - Irradiance
  - Materials
  - White furnace test (energy conservation)

Make it Physics (a Bit)

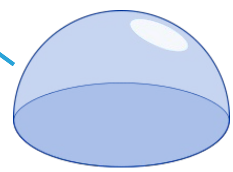
We have seen this before, this is **irradiance** (incoming light).

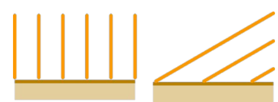
Light arriving at point x

Light from direction ω

Solid angle

$$L_i(x) = \int_\Omega L_i(x, \omega)\cos(\theta_x)\,\mathrm{d}\omega$$

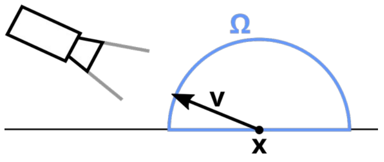




(not useful for rendering yet)

Make it Physics (a Bit)

Now we want to know how much light is going to the camera.



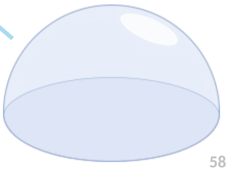
Material, modelled by the BRDF

Light from direction ω

Solid angle

$$L_e(x, v) = \int_\Omega \boxed{f_r(x, \omega \rightarrow v)} L_i(x, \omega) \cos(\theta_x) \, \mathrm{d}\omega$$

Light going in direction v



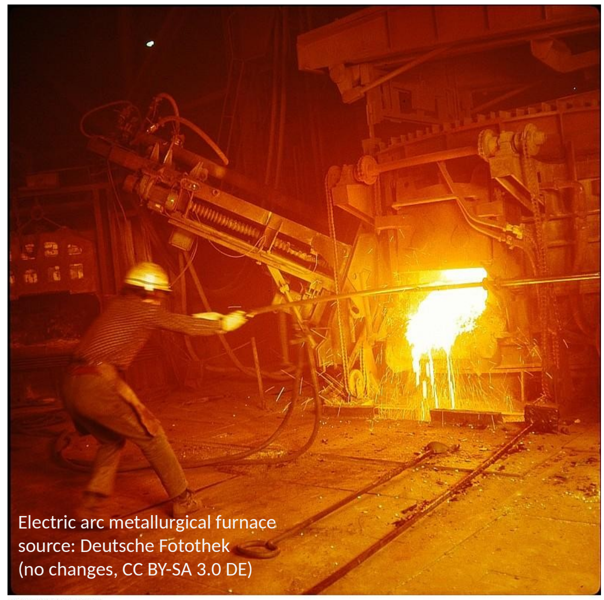
Make it Physics (a Bit)

Material BRDF = Bidirectional reflectance distribution function

- How much light is reflected from a given direction into another given direction at a given position, and in which wavelengths
- The colour
- You probably already implemented BRDFs in “*Übung Computergraphik (186.831)*”
- For now we will treat it simply as a black-box function that models the material. You will learn about the inner workings in a later lecture!

Make it Physics (a Bit)

Industrial furnace
source: Guenter Sonnenschein, Wikipedia
(no changes, CC BY 3.0)



Electric arc metallurgical furnace
source: Deutsche Fotothek
(no changes, CC BY-SA 3.0 DE)

Quelle: Deutsche Fotothek

For all non-native speakers I want to explain the word furnace – look at the pictures, it's an oven..

Why do you have to know? Well because there is the white furnace test for energy conservation. Think of an oven that is so hot it's all white..

Make it Physics (a Bit)

White furnace test (energy conservation)

- A material can not create light, otherwise it would be a light source
- It can only absorb light, turn it into another form of energy or radiation
- We can make unit tests
- Set L_i to 1 and check $L_e \leq 1$

Material, modelled by the BRDF

Light from direction ω

$$L_e(x, v) = \int_\Omega \boxed{f_r(x, \omega \rightarrow v)} L_i(x, \omega) \cos(\theta_x)\, \mathrm{d}\omega$$

Because there is the white furnace test for energy conservation.

Make it Physics (a Bit)

White furnace test (energy conservation)

- Ok cat, set L_i to 1
- Assume a white diffuse material (all light is reflected)
- And check $L_e \leq 1$

Material, modelled by the BRDF

Light from direction ω

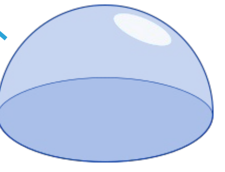
$$L_e(x, v) = \int_\Omega \boxed{1} \; 1 \; \cos(\theta_x)\, \mathrm{d}\omega$$

Make it Physics (a Bit)

White furnace test (energy conservation)

- Ok cat, how can I integrate that half sphere
- -> change of variables!

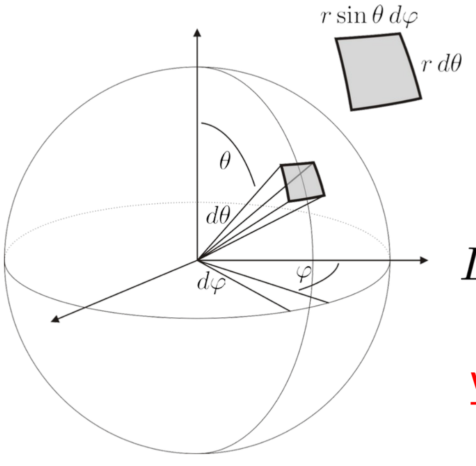
$$L_e(x, v) = \int_{\Omega} \cos(\theta)\, \mathrm{d}\omega$$



Make it Physics (a Bit)

White furnace test (energy conservation)

Change of variable



$$L_e(x, v) = \int_0^{2\pi} \int_0^{\pi/2} \cos(\theta) \sin(\theta) \, \mathrm{d}\theta \, \mathrm{d}\phi$$

WolframAlpha

source: previous year's lecture (Auzinger and Zsolnai)

Make it Physics (a Bit)

White furnace test (energy conservation)

$$L_e(x, v) = \int_0^{2\pi} \int_0^{\pi/2} \cos(\theta) \sin(\theta) \, \mathrm{d}\theta \, \mathrm{d}\phi$$

WolframAlpha: π own work: $\pi > 1$

Make it Physics (a Bit)

White furnace test (energy conservation)
Failed

$$L_e(x, v) = \int_0^{2\pi} \int_0^{\pi/2} \cos(\theta)\sin(\theta)\,\mathrm{d}\theta\,\mathrm{d}\phi$$

WolframAlpha: π own work: $\pi > 1$

Make it Physics (a Bit)

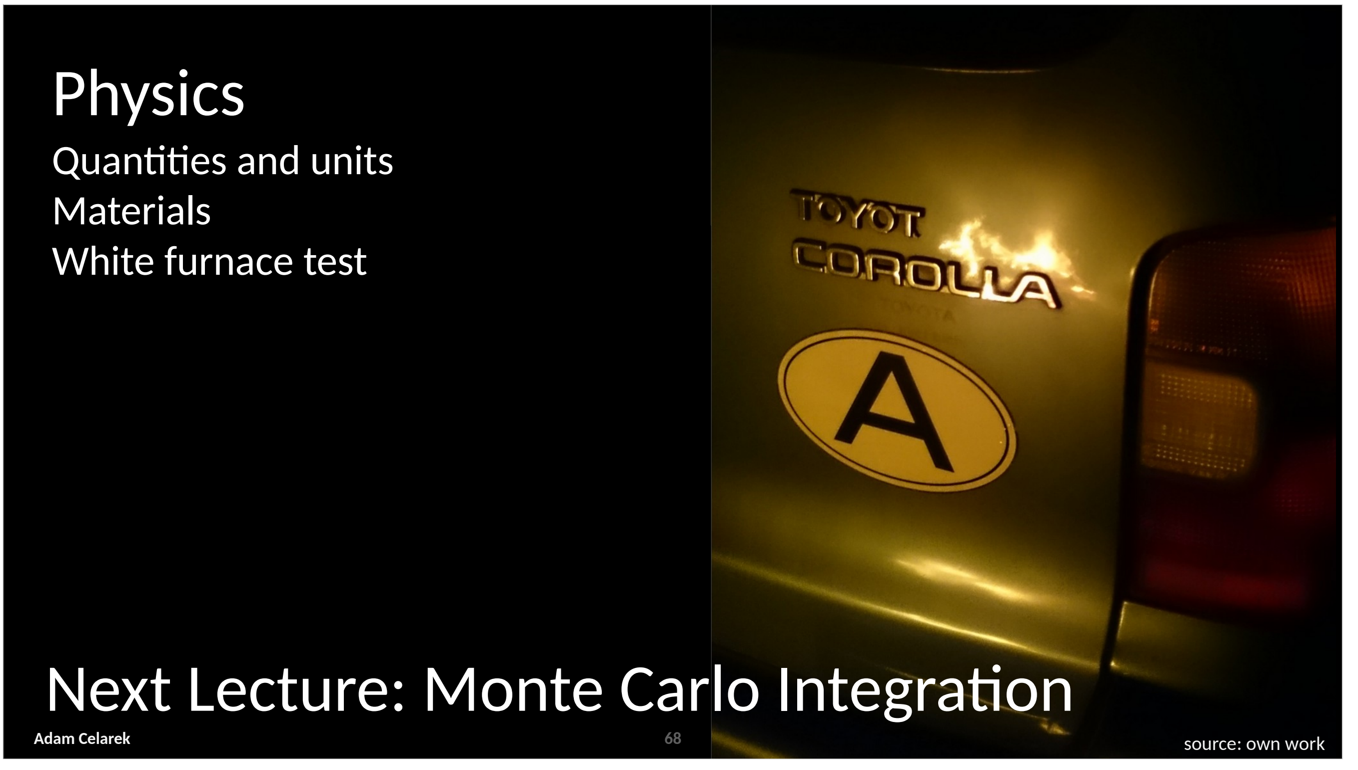
White furnace test (energy conservation)

- A material can not create light, otherwise it would be a light source
- It can only absorb light, turn it into another form of energy or radiation
- **f_r for a white diffuse material is 1/π,**
  for a general diffuse material it is ρ/π, where ρ is the colour

Material, modelled by the BRDF

Light from direction ω

$$L_e(x, v) = \int_\Omega \boxed{f_r(x, \omega \to v)} L_i(x, \omega) \cos(\theta_x) \, \mathrm{d}\omega$$



That's it for today.

In the physics chapter we skimmed over the quantities and units, we introduced the concept of material and showed how it's used in the reflected light integral, and we showed the white furnace test for materials, used to test whether a brdf is erronously producing energy.

Yes, this lecture was a bit short, we will reorder and extend it next year. This is only the second iteration of this course and we are working on improving it, but we also have other stuff to do. This year saw an overhaul of the complete schedule. We also redid the Monte Carlo integration lecture, coming up next, and, we'll probably see a new and more complete lecture about materials.

See you next time, and take care!

Useful reading (links)

- Change of variables
- Jaakko Lehtinen's slides
  (I borrowed a lot from lecture 2, but there is more on point lights, intuition, links..)
- Károly Zsolnai-Fehér's slides, previously lecturing at TUW
  (more on history, physics, different approach on solid angle etc.)
- Károly Zsolnai-Fehér's lecture on YouTube