

# IMAGE ENHANCEMENT IN PROJECTORS VIA OPTICAL PIXEL SHIFT AND OVERLAY

SAJADI Behzad et al.

Julian Pegoraro, I225472, TU Wien

# SETUP

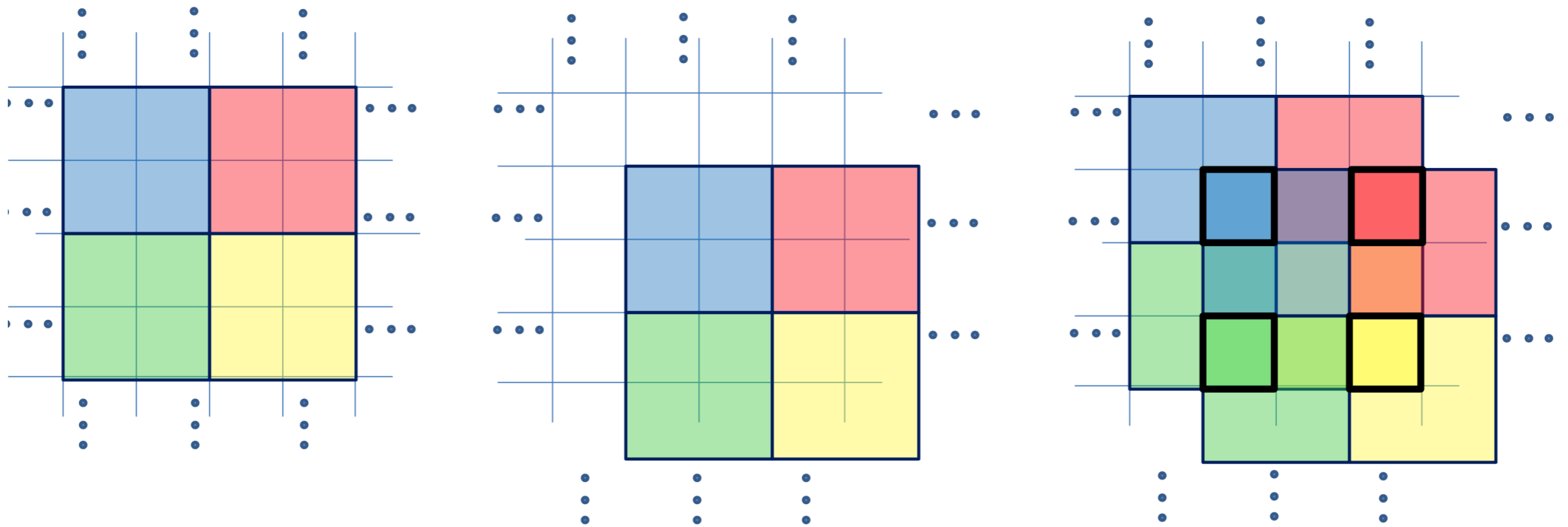
- Known target image  $I_T$  with size  $2n \times 2n$
- Pixel in  $I_T$  is denoted by  $I_T(i,j)$  where  $1 \leq i,j \leq 2n$
- $I_T$  is normalised:  $0 \leq I_T(i,j) \leq 1$
- Projector with resolution  $n \times n$

# GOAL

- Find image  $I$  with size  $n \times n$
- $I$  is shifted and overlay with itself to enhance  $I_R$
- $I_R$  is perceptual close to  $I_T$
- $I$  is shifted by  $\mathbf{s}_x, \mathbf{s}_y : \{-0.5, 0.5\}$

$$I_T(i, j) \approx \frac{1}{2} I\left(\left\lceil \frac{i}{2} \right\rceil, \left\lceil \frac{j}{2} \right\rceil\right) + \frac{1}{2} I\left(\left\lceil \frac{i}{2} + s_y \right\rceil, \left\lceil \frac{j}{2} + s_x \right\rceil\right)$$

SAJADI Behzad et al. 2013  
Equation 1



# SHIFT AND OVERLAY OF $S_x = S_y = 0.5$

SAJADI Behzad et al. 2013  
Figure 1

# EQUATION

- $\mathbf{AI} \approx \mathbf{I}_T$
- $\mathbf{I}_T$  is a known column vector of size  $4n^2 \times 1$
- $\mathbf{A}$  is a known matrix of size  $4n^2 \times n^2$
- $\mathbf{I}$  can be found by solve the constrained **linear least square problem**

# SOLVING EQUATION

- $\mathbf{A}\mathbf{I} \approx \mathbf{I}_T$  (2) can be re-expressed as  $\min_{\mathbf{I}} \frac{1}{2} \mathbf{I}^T \mathbf{A}^T \mathbf{A} \mathbf{I} - \mathbf{I}^T \mathbf{A}^T \mathbf{I}_T$  *s.t.*  $0 \leq \mathbf{I} \leq 1$   
(SAJADI Behzad et al. 2013 Equation 3)
- defining  $\mathbf{J} = \mathbf{A}^T \mathbf{A}$
- $\mathbf{h} = \mathbf{A}^T \mathbf{I}_T$
- This equation is solved with **Gaussian Belief Propagation** which is faster as **Jacobi** or **Gauss-Seidel** methods.  
Although it does **not always return a solution**.

# SOLVING EQUATION

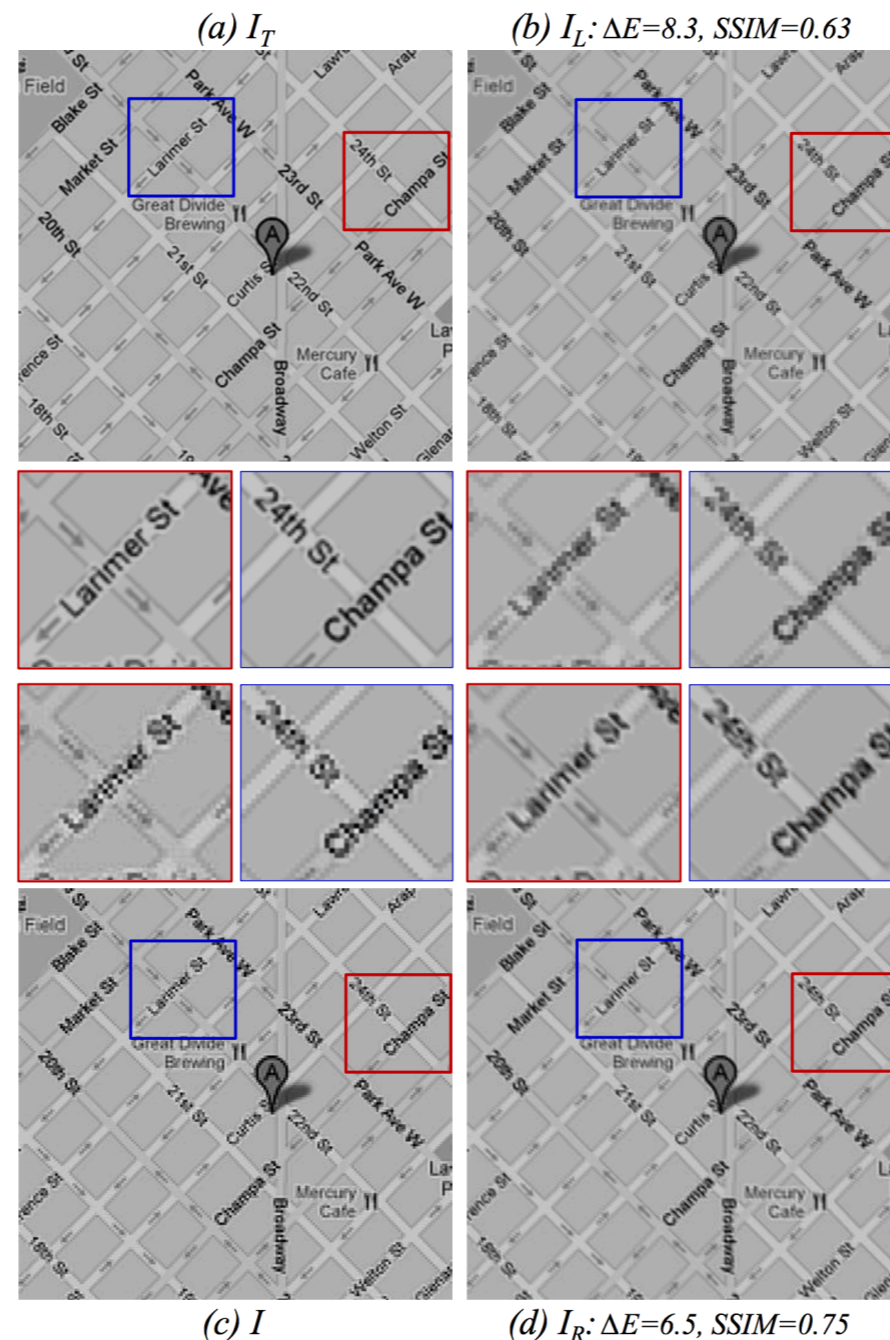
$$\max_{\lambda, \gamma \geq 0} \min_{\mathbf{I}} \frac{1}{2} \mathbf{I}^T \mathbf{J} \mathbf{I} - \mathbf{I}^T \mathbf{h} - \mathbf{I}^T \boldsymbol{\gamma} + (\mathbf{I} - \mathbf{1})^T \boldsymbol{\lambda}$$

SAJADI Behzad et al. 2013  
Equation 4



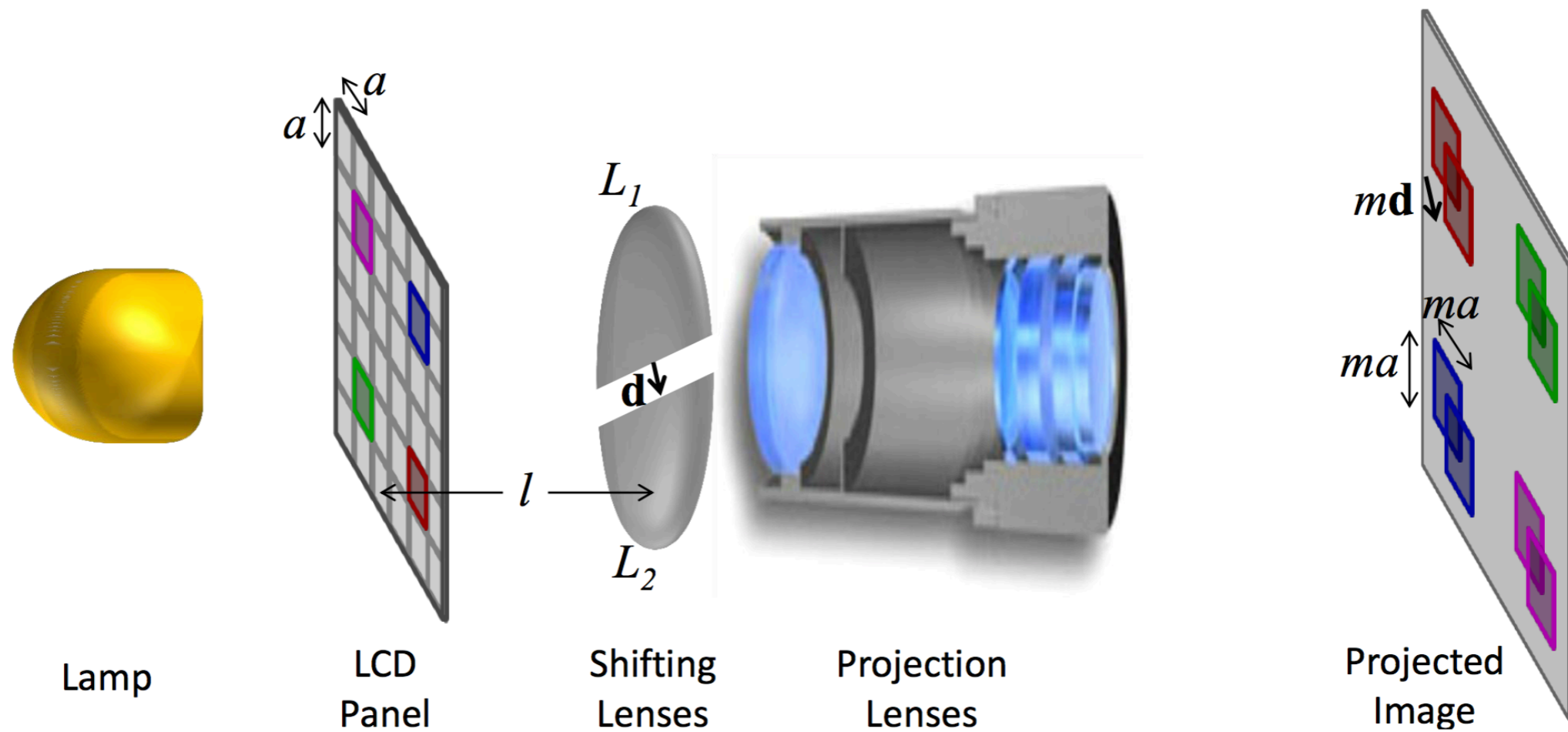
# EVALUATION

- Mean CIELAB distance value
- SSIM index [Wang et al. 2004, Image quality assessment: From error measurement to structural similarity]



COMPARISON OF TARGET IMAGE (UP, LEFT), DOWNSAMPLED IMAGE (UP, RIGHT), OPTIMAL INPUT OF IMAGE (DOWN, LEFT) AND RESULTING IMAGE WITH OPTIMAL INPUT (DOWN, RIGHT)

SAJADI Behzad et al. 2013  
Figure 2

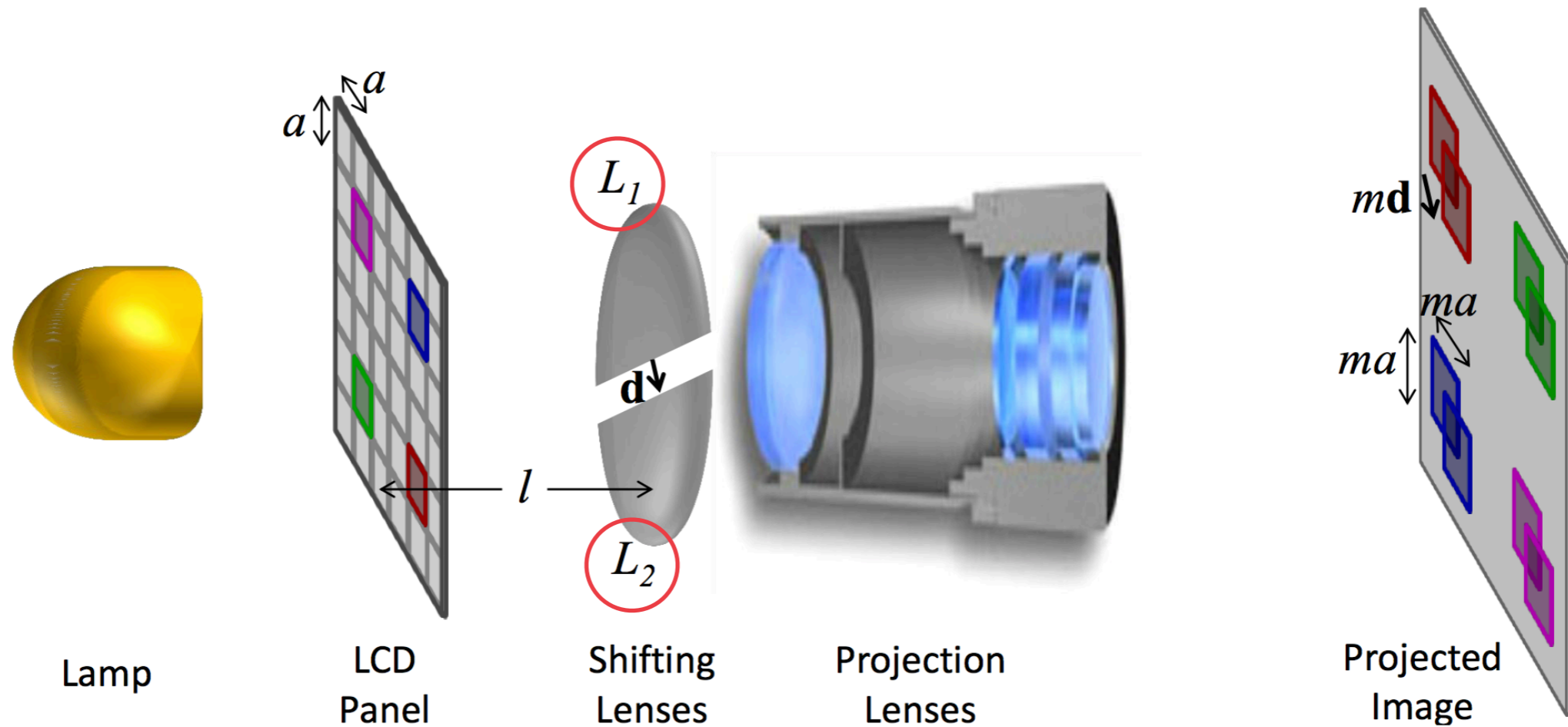


# OPTICAL SETUP OF THE PROJECTOR WITH SHIFTING LENSES

SAJADI Behzad et al. 2013

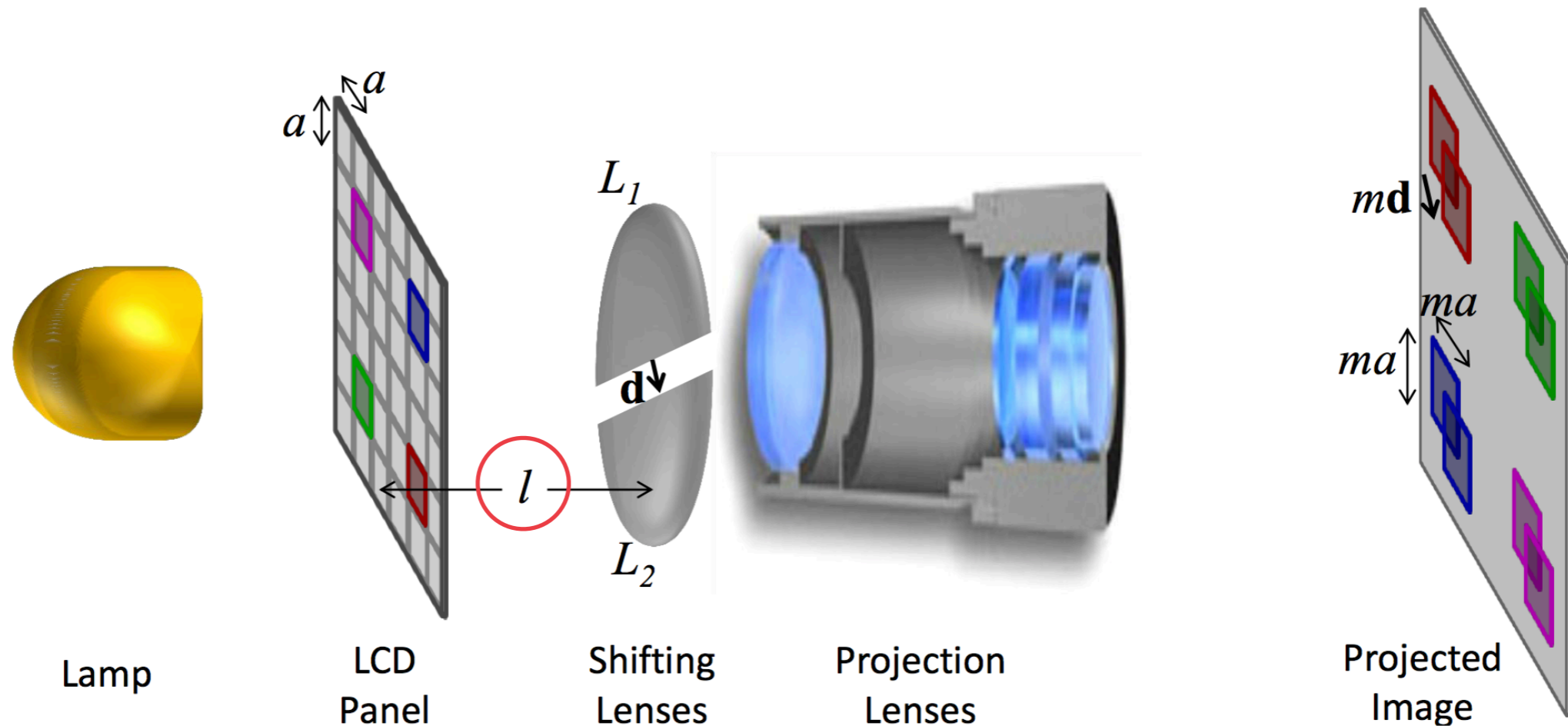
Figure 3

# SHIFTING LENSES



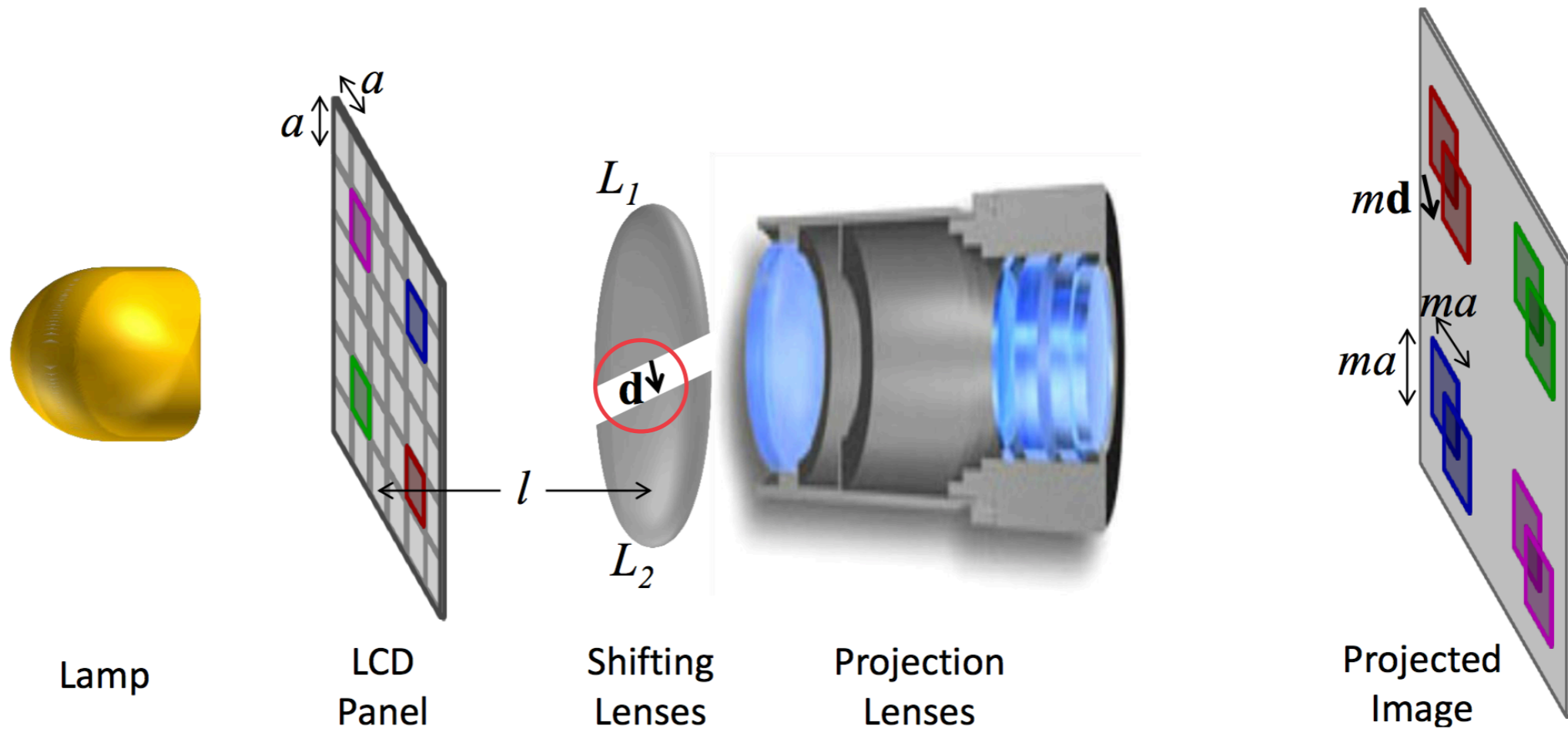
The lenses have the same focal length  $f$

# MAGNIFICATION



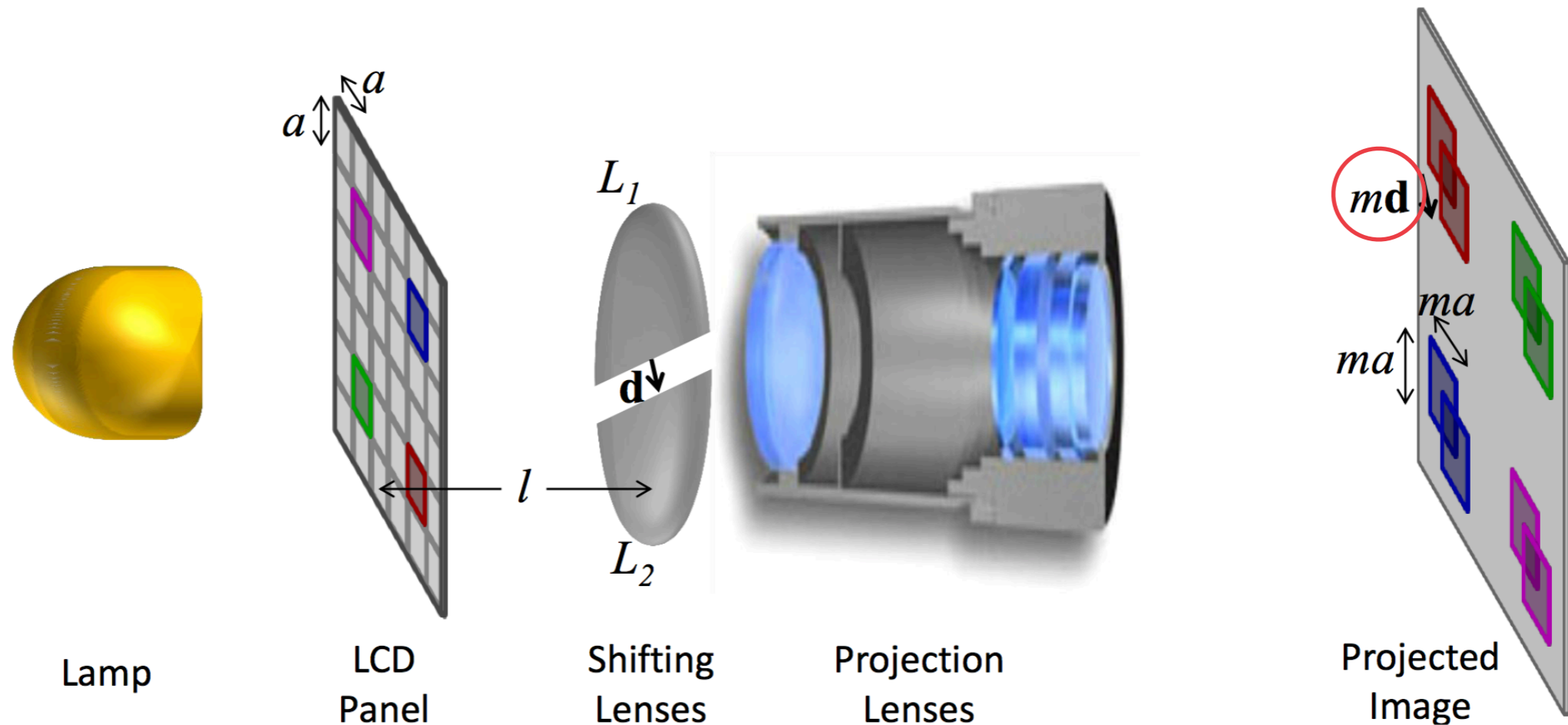
When placed at distance  $l$  from the LCD panel, we obtain a magnification  $m = f/(l-f)$

# DISTANCE BETWEEN OPTICAL AXES



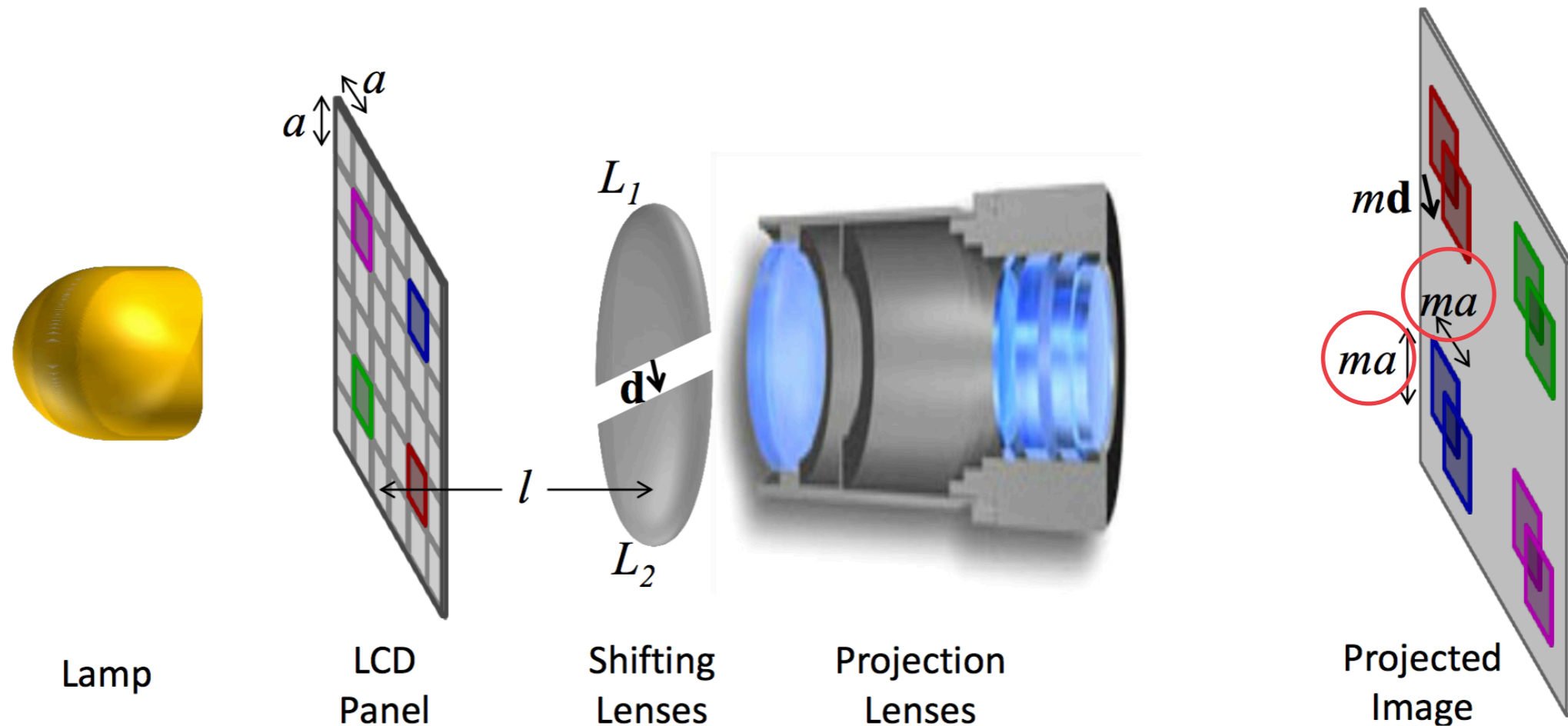
$$d = (d_x, d_y)$$

# MAGNIFICATION OF DISTANCE



Pixel  $p$  is at position  $m$  and position  $md$ , because it is shifted by  $d$

# MAGNIFICATION OF UNITS



Pixel  $p$  is with unit size  $a$ , is magnified as  $ma$



Shift between two copies is therefore:

$$(s_x, s_y) = \frac{(1 + m)\mathbf{d}}{ma} = \frac{\left(1 + \frac{f}{l-f}\right)\mathbf{d}}{\frac{f}{l-f}a} = \frac{\frac{l}{l-f}\mathbf{d}}{\frac{f}{l-f}a} = \frac{l\mathbf{d}}{fa}$$

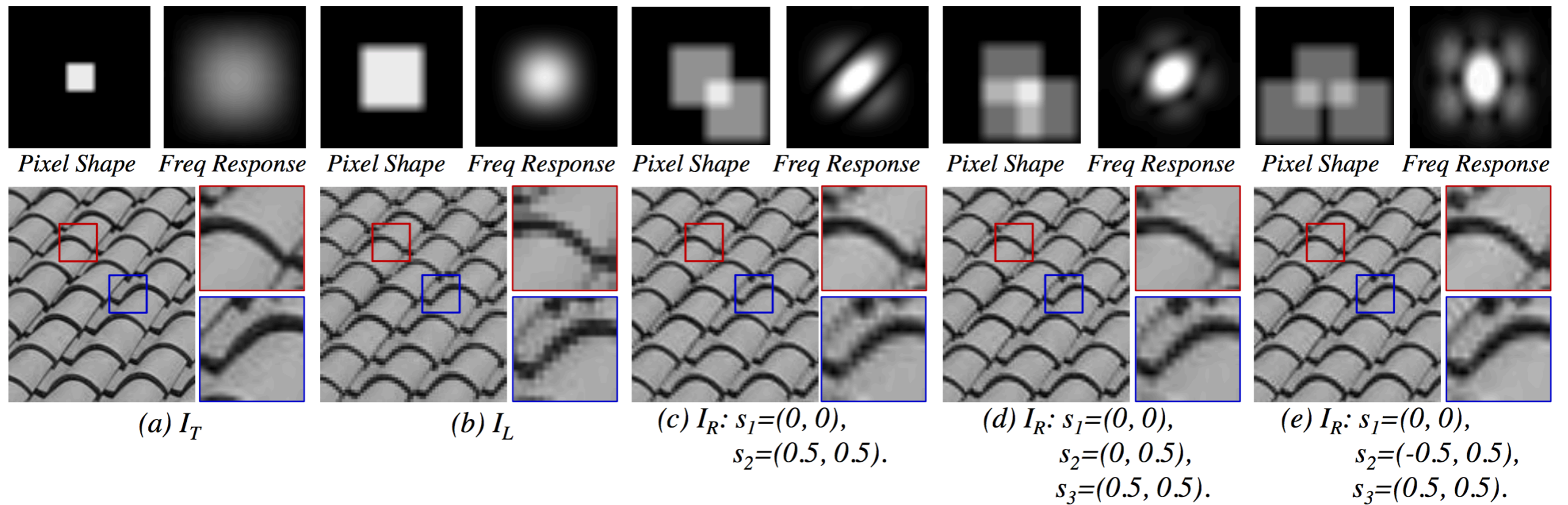
SAJADI Behzad et al. 2013  
Equation 5

# IMPROVEMENTS

- Multiple Overlays
- General Shifts
- Multiple Channels

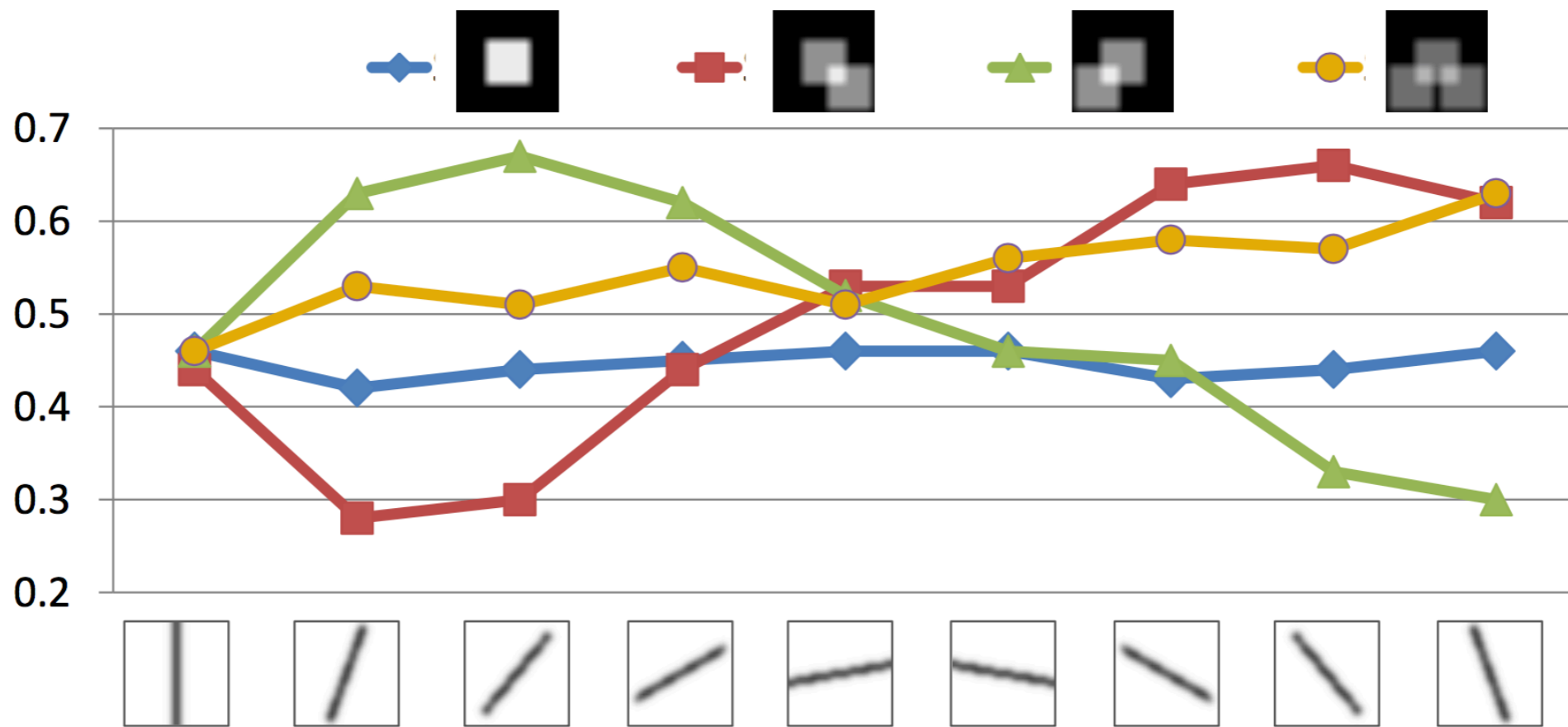
# MULTIPLE OVERLAYS

- Instead of using only two pixels, use multiple ( $m$ ) pixels
- Can be obtained by using  $k$  Lenses
- Equation 1 can be as following  $I_T(i, j) \approx \frac{1}{m} \sum_{p=1}^m I\left(\left[\frac{i}{2} + s_y(p)\right], \left[\frac{j}{2} + s_x(p)\right]\right)$



COMPARISON OF (A) TARGET IMAGE, (B) LOW RESOLUTION IMAGE, (C) RESULTING IMAGE WITH SHIFT ONE SHIFT OF (0.5,0.5), (D) RESULTING IMAGE WITH TWO SHIFTS OF (0,0.5) AND (0.5,0.5) AND (E) RESULTING IMAGE WITH TWO SHIFTS OF (-0.5,0.5) AND (0.5,0.5)

SAJADI Behzad et al. 2013  
Figure 4

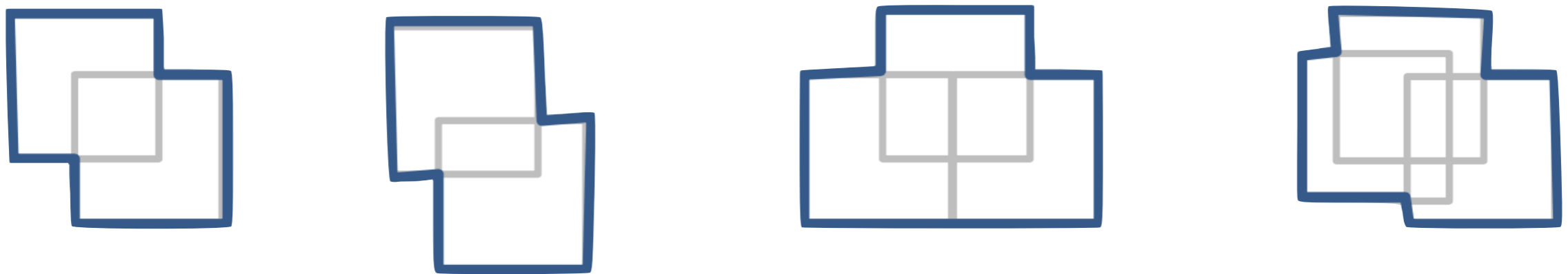


HOG COMPUTED WITH DIFFERENT PIXEL SHAPES. BLUE IS ONLY A LOWER RESOLUTION, WHEREAS GREEN AND RED ARE PIXEL SHAPES WITH ONLY ONE SHIFT AND RED IS A PIXEL SHAPE WITH TWO SHIFTS

SAJADI Behzad et al. 2013  
Figure 5

# GENERAL SHIFTS

- Instead of shifting by half a step, shift by something between 0 and 1
- Dimensions of  $\mathbf{A}$  and  $\mathbf{I}_T$  change relative to  $\mathbf{k}$
- Equation 1 change as following  $I_T(i, j) \approx \frac{1}{2}I\left(\left[\frac{i}{k}\right], \left[\frac{j}{k}\right]\right) + \frac{1}{2}I\left(\left[\frac{i}{k} + s_y\right], \left[\frac{j}{k} + s_x\right]\right)$

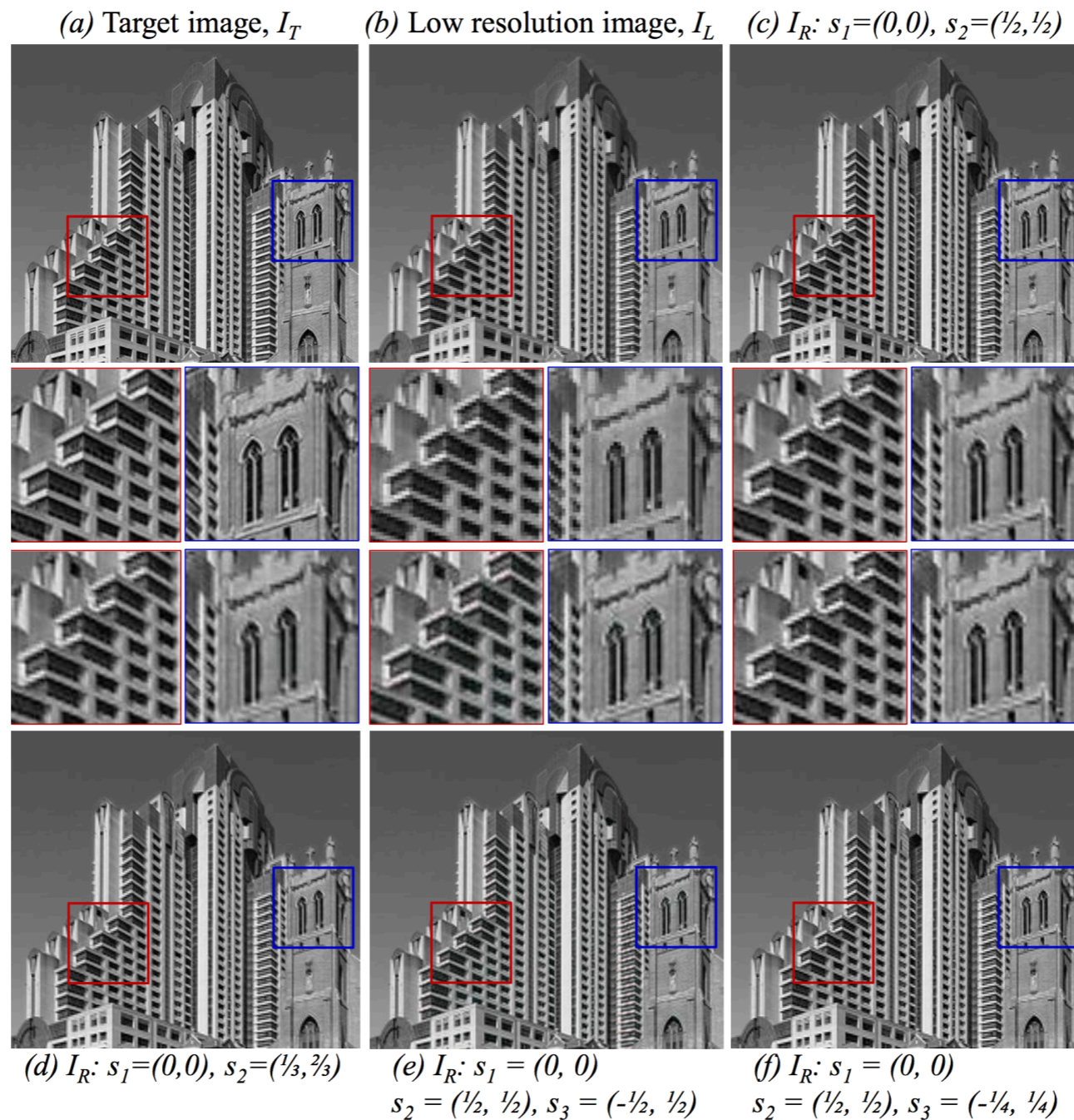


DIFFERENT PIXEL SHAPES WITH DIFFERENT K  
VALUE FOR A SHIFT

SAJADI Behzad et al. 2013

Figure 6



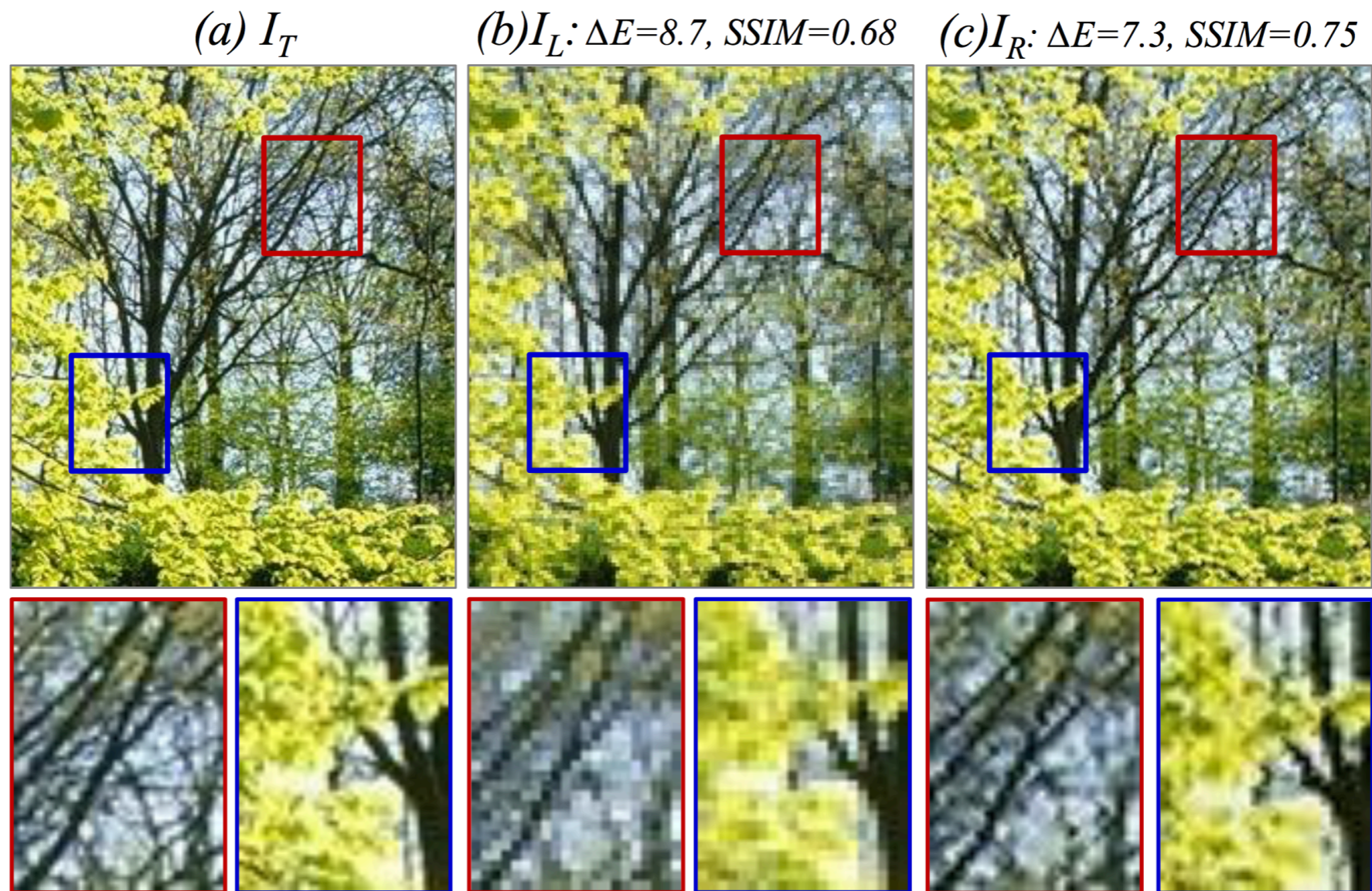


COMPARISON OF THE RESULT OF THE TARGET IMAGE, THE LOW RESOLUTION IMAGE, A SINGLE SIMPLE SHIFT OF (0.5,0.5), A SINGLE SHIFT OF (0.33..., 0.66...), TWO SHIFTS OF (0.5,0.5),(-0.5,0.5) AND TWO SHIFTS OF (0.5,0.5),(0.25,0.25)

SAJADI Behzad et al. 2013  
 Figure 7

# MULTIPLE CHANNELS

- Instead of calculation everything for one image, it has to be calculated for the r, g and b channels
- Illumination does not have to be the same
- Equation 1 change as following  $I_t^l(i, j) \approx \frac{1}{m} \sum_{p=1}^m I^l\left(\left[\frac{i}{2} + s_y(p)\right], \left[\frac{j}{2} + s_x(p)\right]\right), l \in \{r, g, b\}$



COMPARISON OF A COLOR IMAGE WITH A LOW RESOLUTION  
IMAGE AND AN RESULTING IMAGE WITH TWO HALF STEP SHIFTS

SAJADI Behzad et al. 2013  
Figure 8

# DISADVANTAGES

Thank you for your attention

Julian Pegoraro, 1225472