Visualizing Poincaré maps together with the underlying flow

Helwig Löffelmann, Thomas Kučera, and Eduard Gröller

visualization to (better) communicate Poincaré maps!

Institute of Computer Graphics, Vienna University of Technology

Overview and Outline

- **Introduction, Context –**
  - ○ dynamical systems: special scope
  - ○ Poincaré section & Poincaré map
  - ○ scientific context: flow visualization

- **Visualization** to communicate . . .
  - . . . the instantaneous dynamics of a Poincaré map
  - . . . the long-term evolution induced by a Poincaré map
  - . . . the continuous context of a Poincaré map
  - . . . the suitability of the Poincaré map

- **Video, Conclusions**

Dynamical Systems – Terms and Scope

$$\begin{aligned} \dot{x} &= a \cdot x - y \\ \dot{y} &= a \cdot y + x \\ \dot{z} &= C \cdot z \end{aligned}$$

$$\begin{aligned} a &= A \cdot (r - 1/r) \\ r &= \sqrt{x^2 + y^2} \end{aligned}$$

x, y, z ... system variables

A, C ... system parameters

Sample system characteristics:

- **3D system (2+1), 2 parameters**
- **continuous = flow** (vs. discrete = map)
- **analytically specified** (vs. set of samples)
- **steady = time-independend**
- **periodic**

Dynamical Systems & Flow Visualization

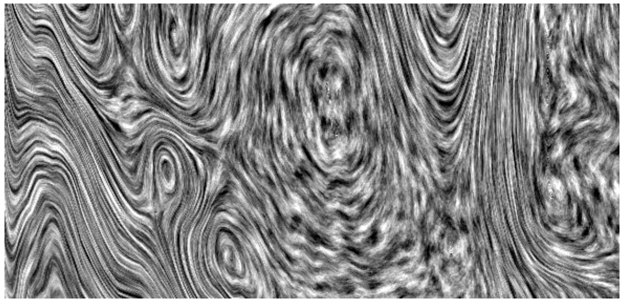
2D techniques

Hedgehog plots

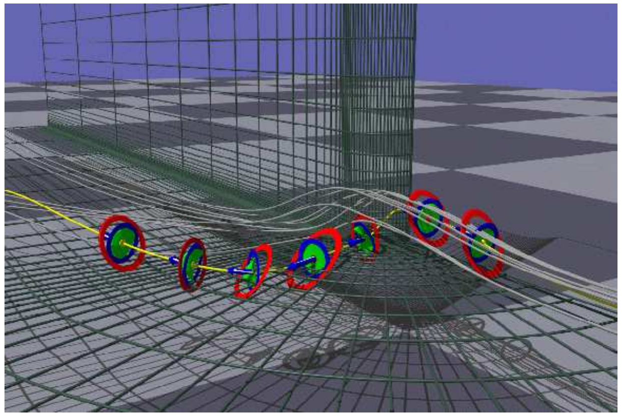
Streamlines, streaklines, etc.

Spot noise, line integral convolution

[image by W. C. de Leeuw]



[image by F. J. Post]



3D techniques

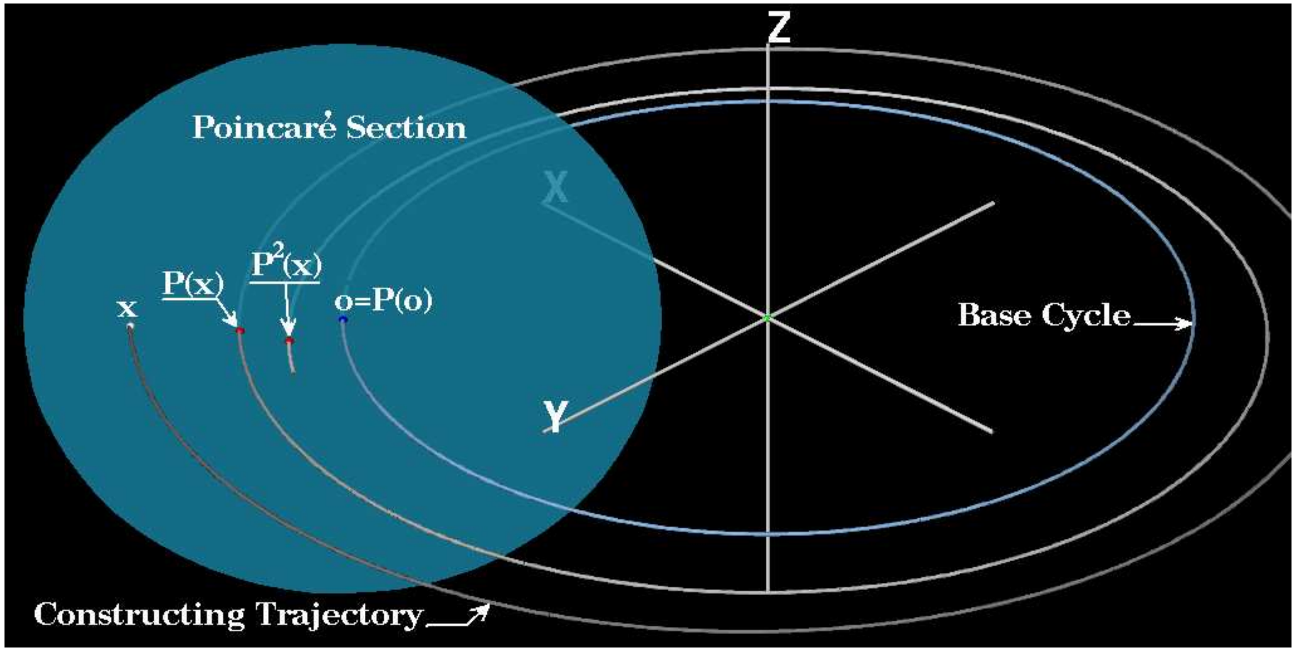
Streamlines, streamribbons, streamtubes, etc.

Streamsurfaces, particle systems

Flow volumes, virtual smoke

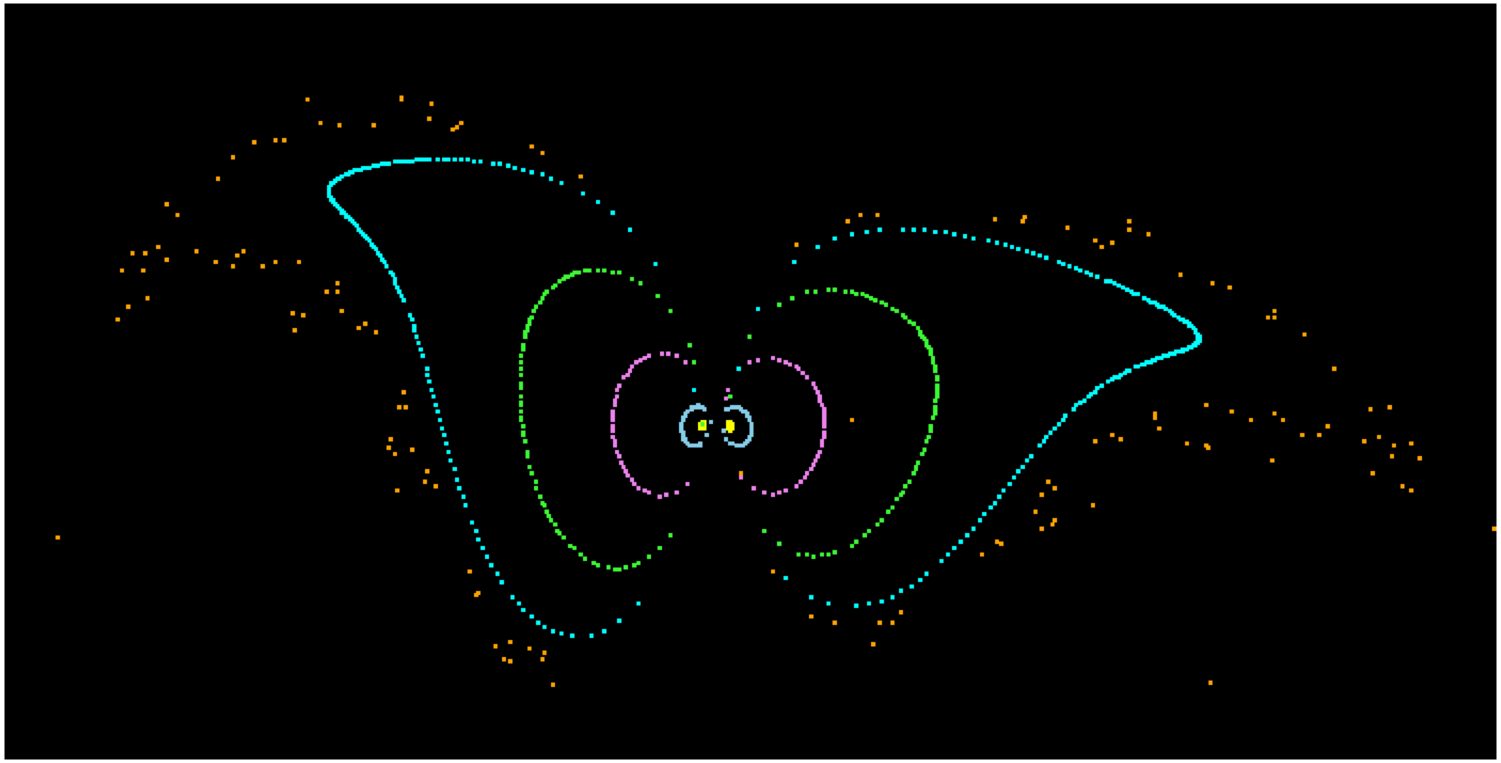
Topology visualization, skin-friction lines, etc.

Poincaré Section & Poincaré Map



continuous system (flow) in 3D $\leftrightarrow$ discrete system in 2D (Poincaré map)

2D Poincaré Map – Traditional Example



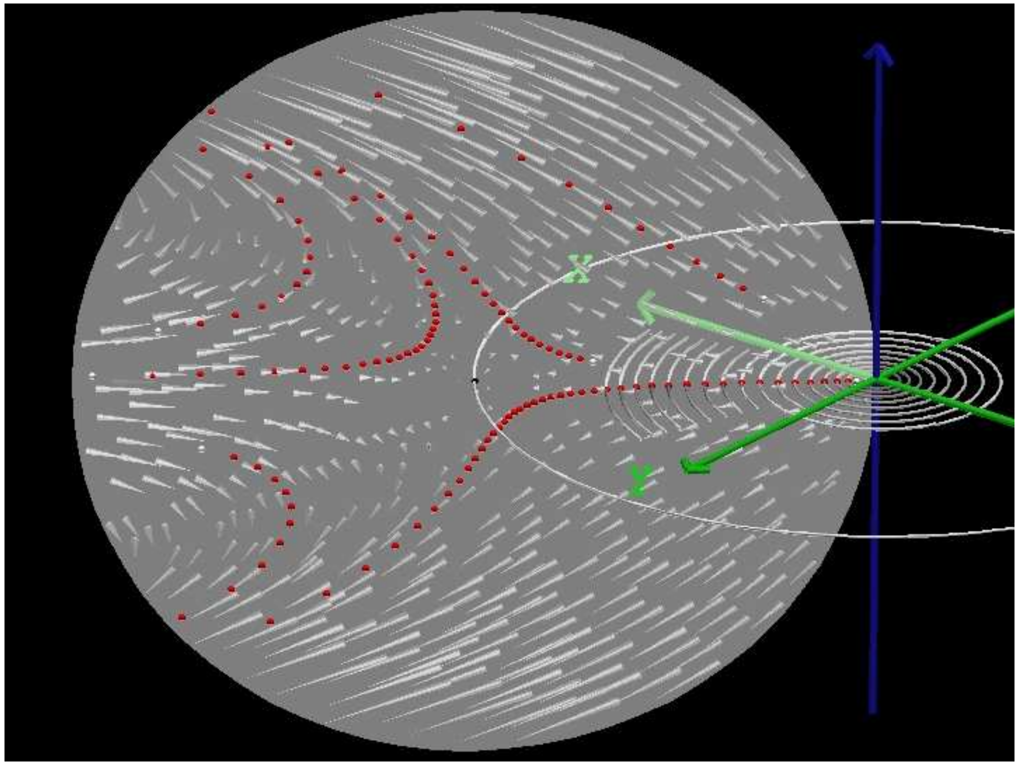
Visualizing sets $\{P(x_i) - x_i\}$ and/or $\{P^i(x_0)\}$

Visualizing $\{P(x_i) - x_i\}$:

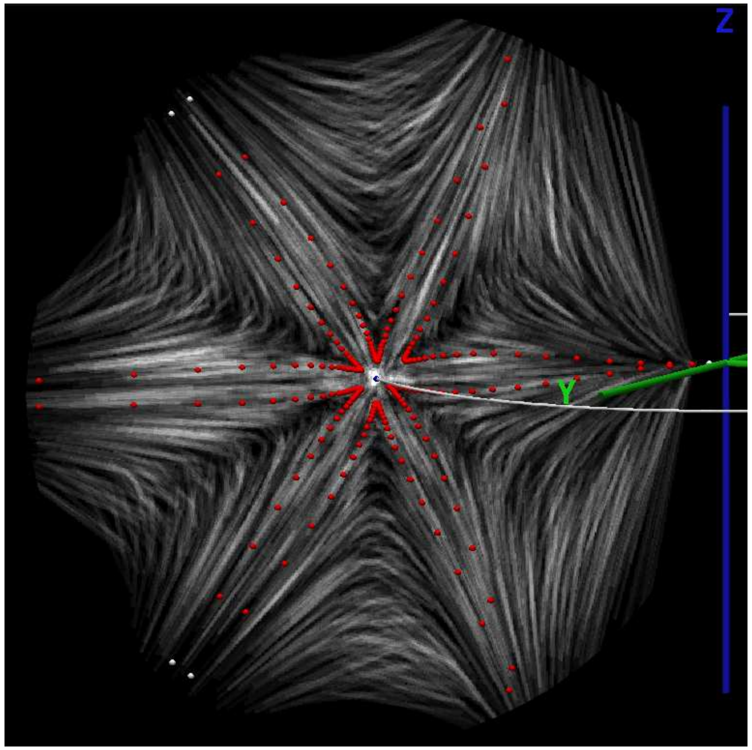
- as set of tiny arrows

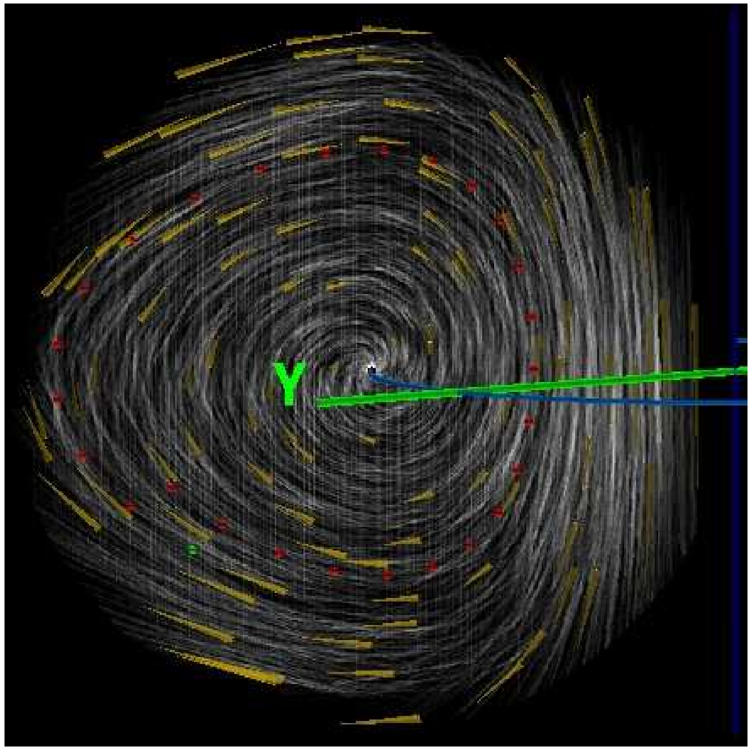
Visualizing $\{P^i(x_0)\}$:

- with constructing trajectory
- with short streamlets
- just $\{P^i(x_0)\}$

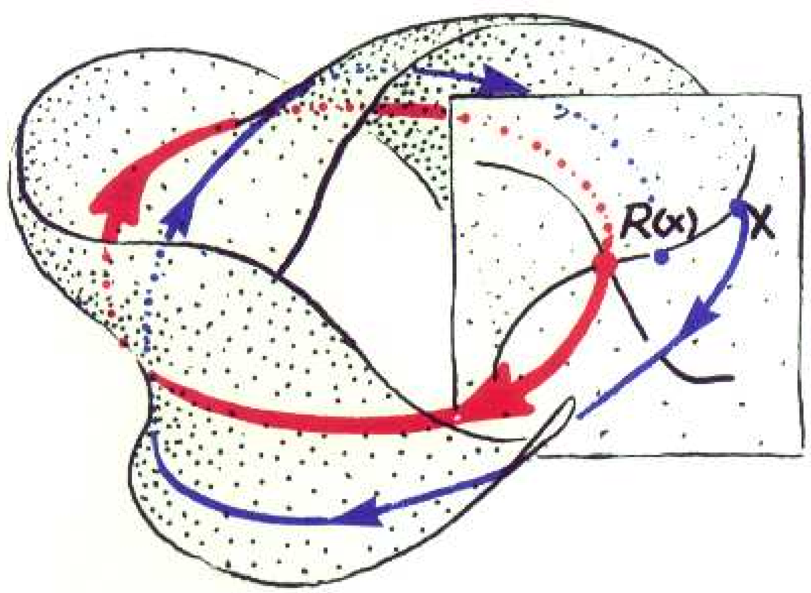


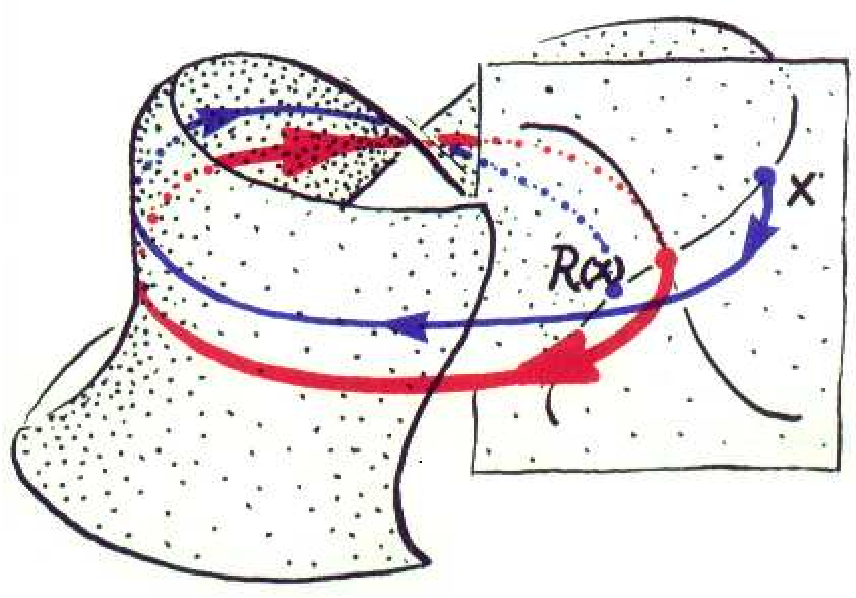
Visualizing $P(x) - x$ Using Spot Noise



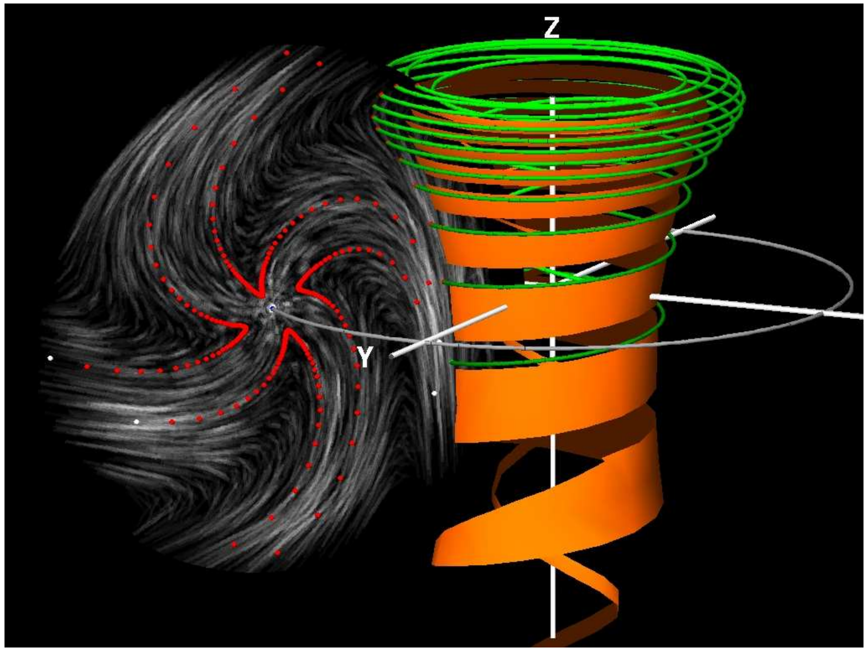


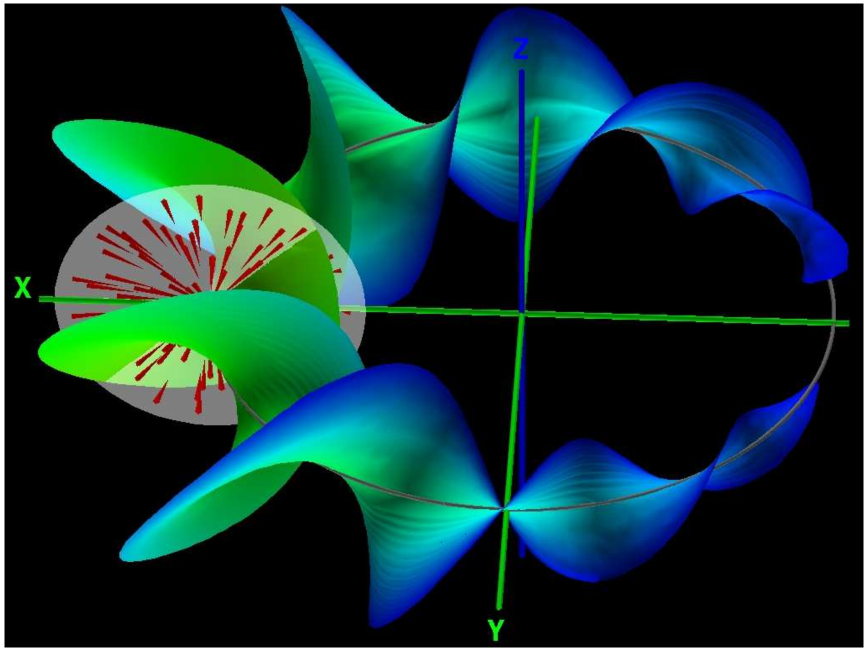
Poincaré Map + Flow Viz [Abraham/Shaw ’92]





Combining Poincaré Map & Flow Visualization

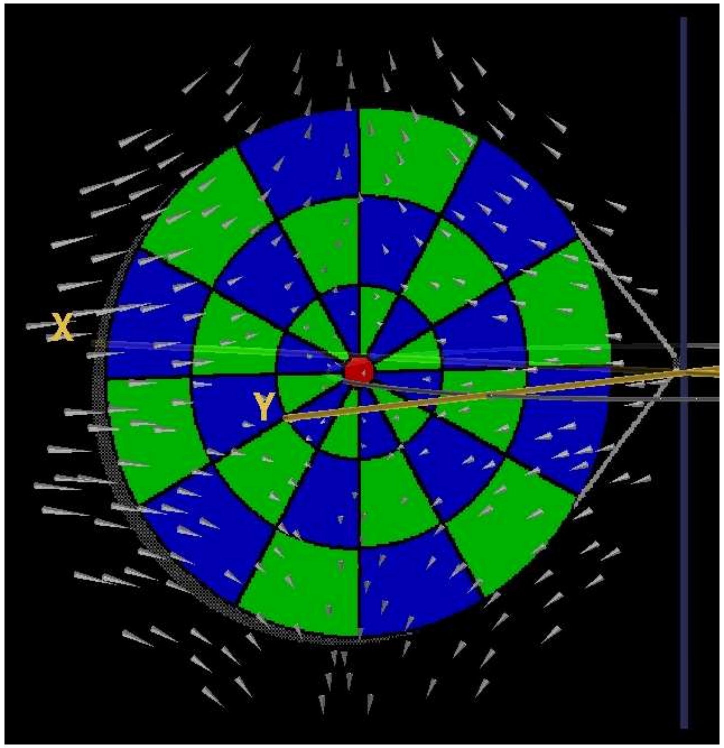


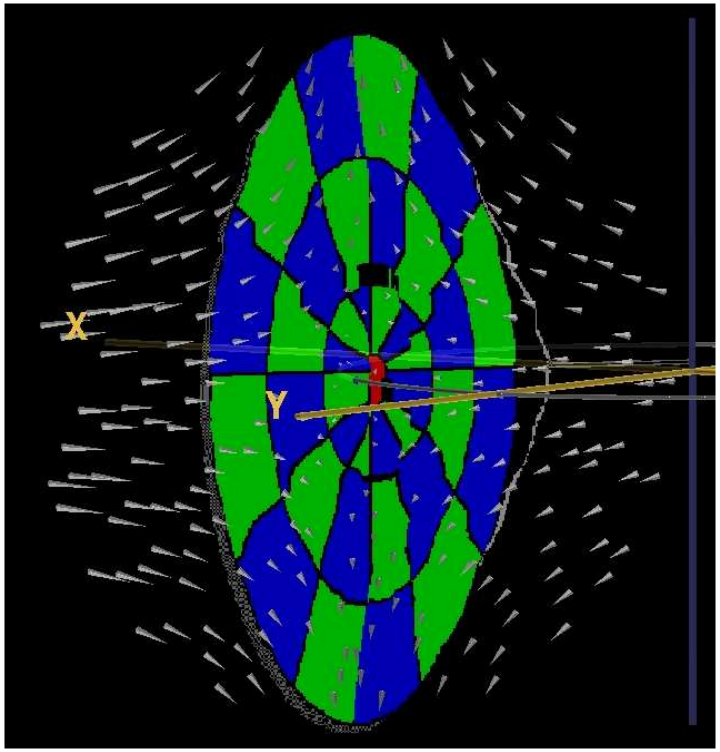


Visualizing Poincaré maps together with the underlying flow

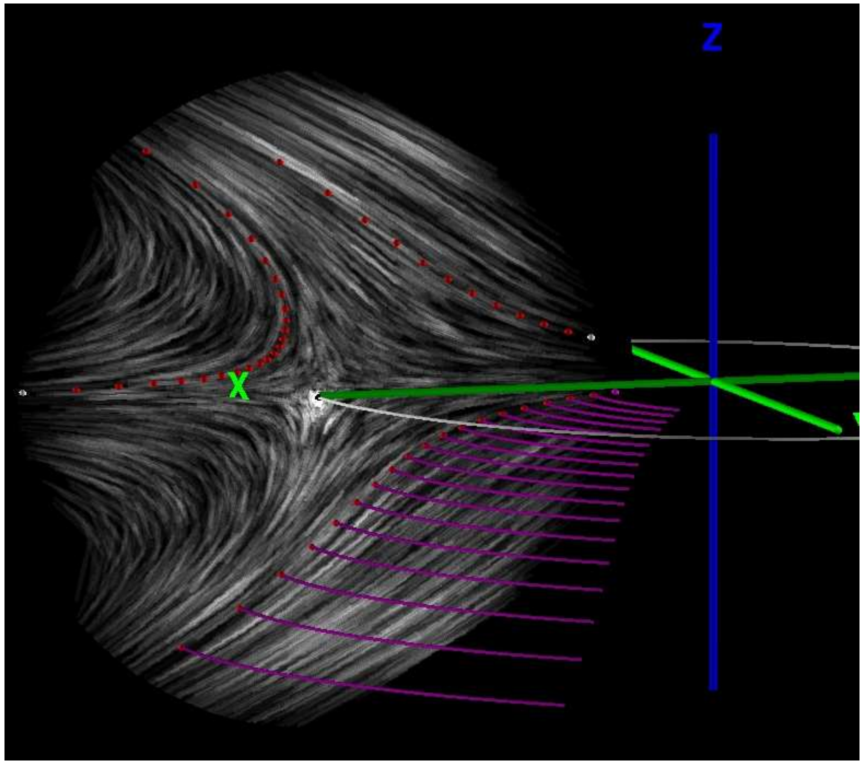
Helwig Löffelmann, Vienna University of Technology

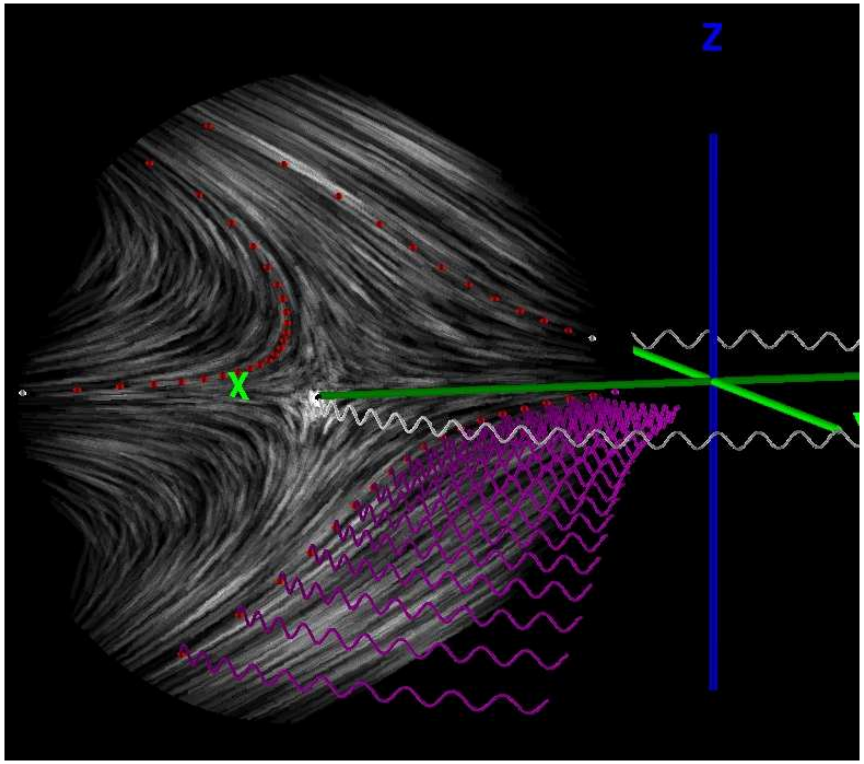
Approximating $P^i(x)$ by a Warping Function



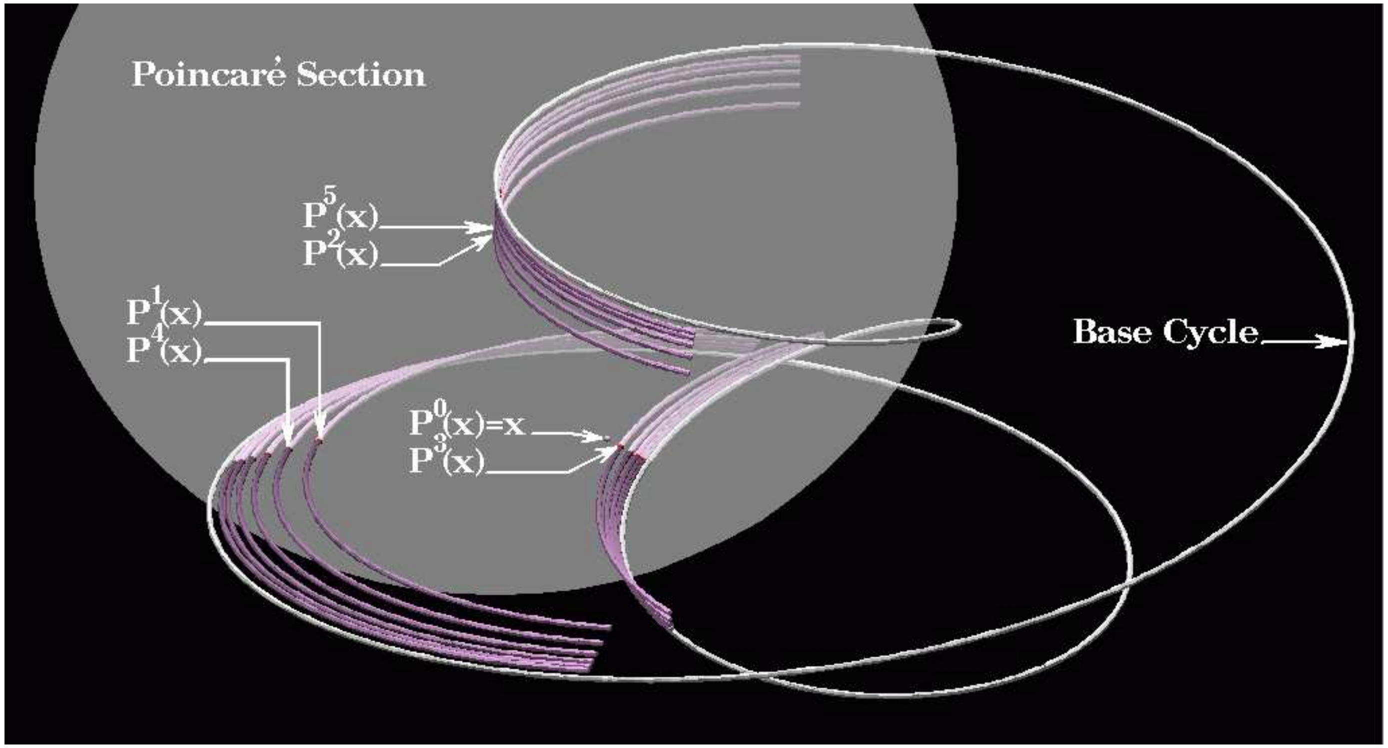


Combined Visualization – Additional Insights



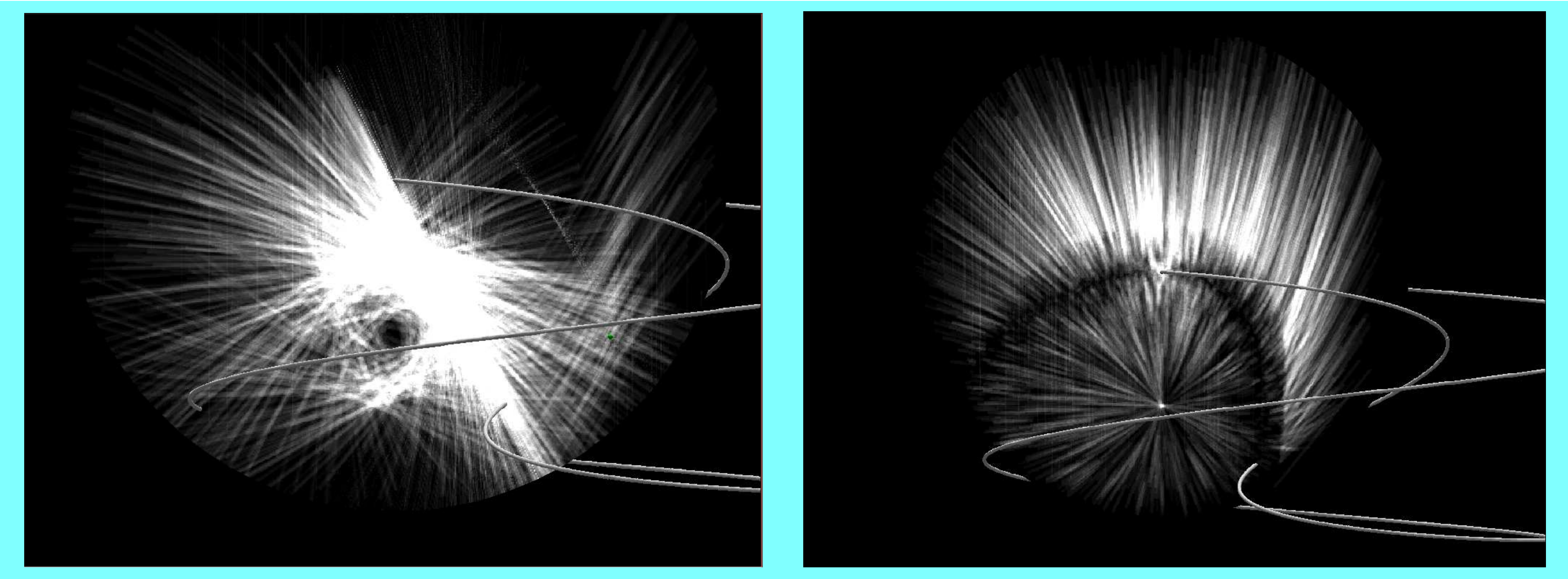


Visualizing P^q instead of P

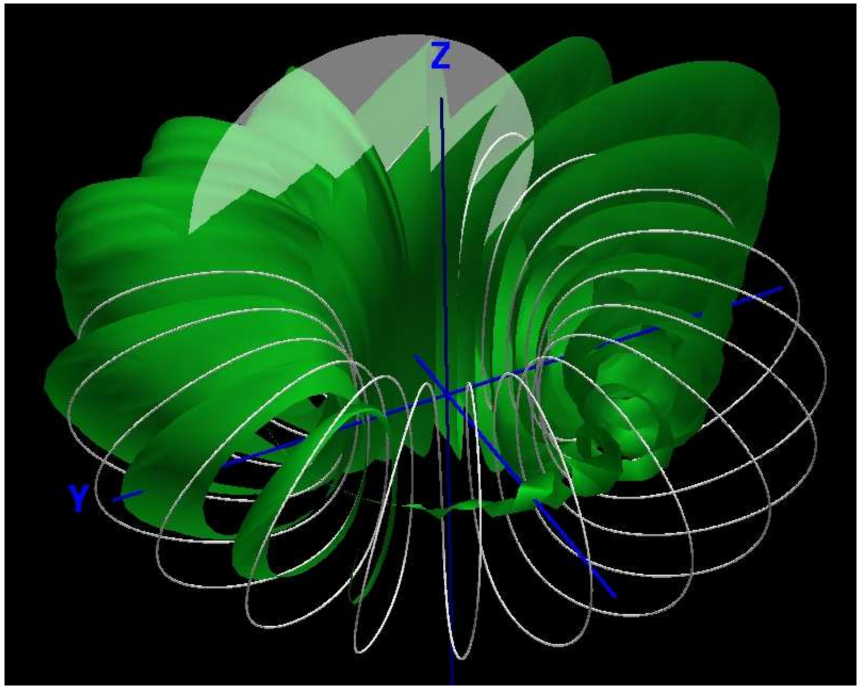


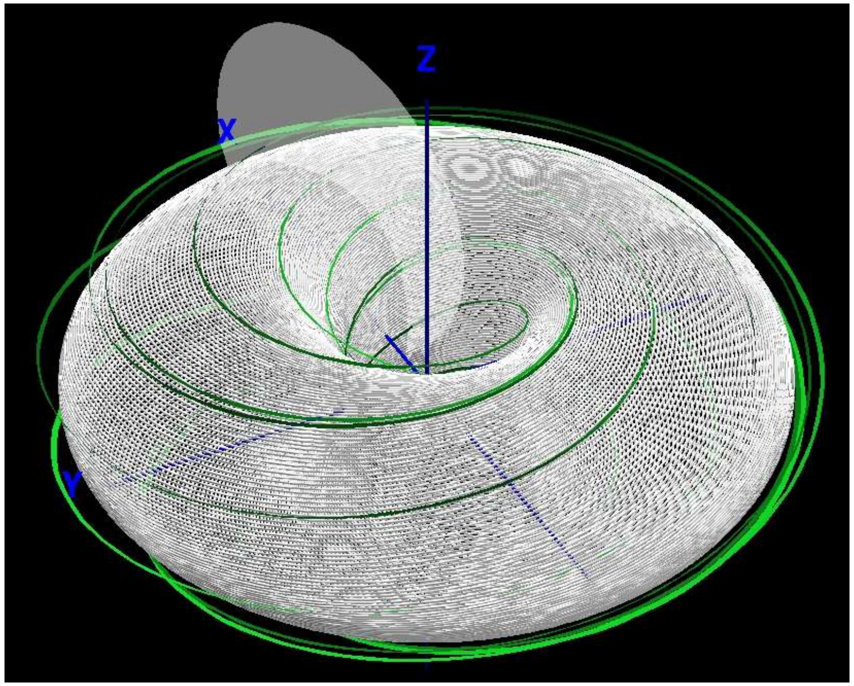
Helwig Löffelmann, Vienna University of Technology

Comparing the Visualization of P vs. P^q

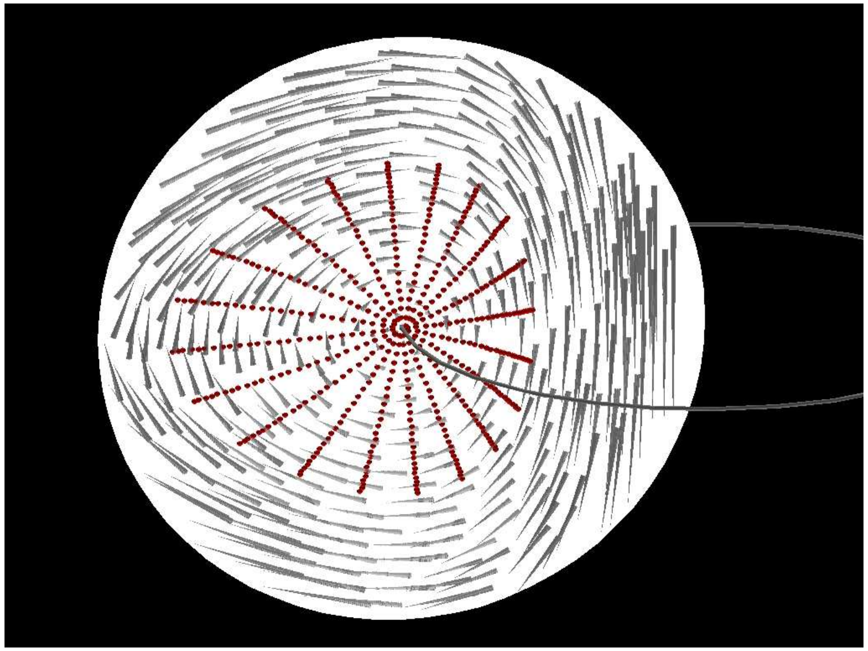


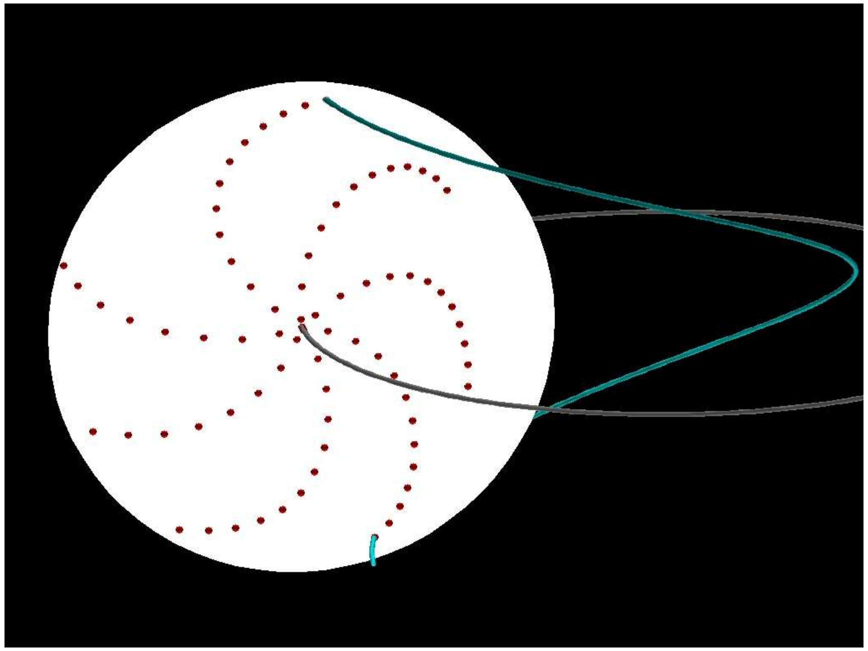
Frequency Coupling as Problematic Case





Visualization to Disambiguate Results





Animation – Video

- **Animation as another dimension for visualization –**
  - intuitively representing the temporal evolution of the Poincaré map
  - successive probing: animating the seed-point x_0 of orbit $\{P^i(x_0)\}$
  - animating parameters, i.e. comparing different dynamical systems
  - using a warping function to efficiently approximate iterated applications of P
  - animating the seed-line of a streamsurface: browsing the 3D context

Conclusions

- **Improved understanding of Poincaré maps –** visualization communicates a lot of information through an intuitive medium (images and animation sequences)
- **Supplementory help for Poincaré map investigation –** visualization can provide: a first impression, probing results, visual proofs, etc.
- **Additional insights through 3D & 2D combination –** augments the abstract representation, provides information not contained in the Poincaré map, etc.
- **Help with judging about the suitability of a Poincaré map –** be careful with using the Poincaré map abstraction!

Test Page

- **Color Test –**

white	gray02	gray05	gray10	gray25	gray50	gray75	gray90	gray95	gray98	black
white	red02	red05	red10	red25	red50	red75	red90	red95	red98	red
white	green02	green05	green10	green25	green50	green75	green90	green95	green98	green
white	blue02	blue05	blue10	blue25	blue50	blue75	blue90	blue95	blue98	blue
white	cyan02	cyan05	cyan10	cyan25	cyan50	cyan75	cyan90	cyan95	cyan98	cyan
white	mag02	mag05	mag10	mag25	mag50	mag75	mag90	mag95	mag98	magenta
white	yell02	yell05	yell10	yell25	yell50	yell75	yell90	yell95	yell98	yellow