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Mental Map Preservation for Progressively Labeling Railway Networks

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ABSTRACT

Schematizing railway networks for better readability is often achieved by aligning railway lines along the octilinear directions. However, such railway map layouts require further adjustment when placing station name labels. In this article, the authors present a novel approach to automating the placement of station names around the railway network while maximally respecting its original layout as the mental map. The key idea is to progressively annotate stations from congested central downtown areas to sparse rural areas. This is accomplished by introducing the sum of geodesic distances over the railway network to properly order the stations to be annotated first, and then elongating the line segments of the railway network while retaining their directions to spare enough labeling space around each station. Additional constraints are also introduced to restrict the aspect ratios of the region confined by the railway network for better preservation of the mental map.

KEYWORDS

Geodesic Distances, Mental Maps, Mixed-Integer Programming, Progressive Annotation, Railway Maps, Schematic Layouts

INTRODUCTION

This article presents a progressive approach for automatically annotating stations with their names while maximally respecting the original layout of the schematic railway map. This is accomplished by extending the authors' previous work (Yoshida et al., 2018) for placing station names progressively as annotation labels from crowded downtown areas to sparsely-populated rural areas. The authors implemented this labeling scheme by computing the geodesic (i.e., shortest topological) distances of each station from the other stations through the railway network, and sorting the stations in terms of

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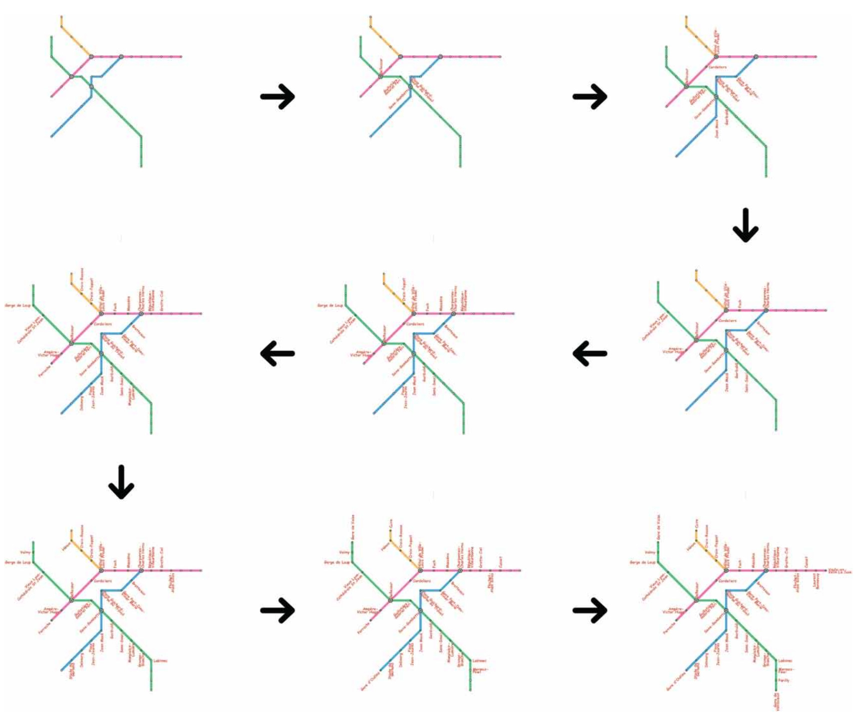
the sum of such distances for the progressive annotation. To faithfully aligning station names along one of the octilinear directions, the proposed approach elongates railway line segments of the original schematic network while retaining their original directions so as to spare enough labeling space around the stations in the schematic map. Furthermore, this article specifically differs from the previous work (Yoshida et al., 2018) in that the approach imposes additional constraints that restrict the variation in the aspect ratio of the regions confined by the railway network, which allows maximal retention of the mental map from the original schematic map. The feasibility of this work will be demonstrated through side-by-side comparison between the results of previous and new approaches, together with evaluation through an informal user study.

Figure 1 shows how name labels are progressively placed around the corresponding stations in a schematic railway map. Suppose that we take an octilinear layout of a railway network as input as shown on the top-left of the figure. The annotation process begins with labeling stations in the congested downtown area around the center of the map, including interchange stations. The station name labels are placed one by one while stretching the railway line segments to spare more labeling space when necessary. This process can successfully label all of the stations while retaining the overall layout of the schematic railway network given as input, as shown on the right-bottom of Figure 1.

Background

Railway maps serve as the common media for travelers to explore the transportation networks of the railway lines available in major cities. Such maps are often transformed into schematic diagrams for better readability of the network topology. In particular, octilinear layouts are the most representative form, which is obtained by aligning railway line segments to horizontal, vertical, and 45-degree slanting directions. This representation originates from the design criteria invented by Henry Beck (Garland, 1994), an English engineering draftsman famous for his London underground tube map

Figure 1. Metro network in Lyon. Station names are placed from the map center while adaptively extending railway line segments for sparing enough labeling space.



created in 1931. Recent advancement of practical techniques (Nöllenburg & Wolff, 2011; Stott, Rodgers, Martinez-Ovando, & Walker, 2011; Wang & Chi, 2011) enables automatic schematization of geographical layouts of railway networks in a visually coherent fashion.

The Problem

Properly labeling station names on schematic railway maps is also important, while usually incurring additional technical challenges. This is primarily because the central downtown area of such a schematic map is usually congested with multiple railway lines around the interchange stations and, thus, space is unavailable for labeling station names, when compared with areas around the sparsely populated rural terminals of the railway lines. This labeling problem becomes further complexified if the alignment of the name labels around the corresponding station is restricted to one of the octilinear directions. The problem is often alleviated by manually creating more labeling space around the stations in the congested downtown areas, which usually involves a lengthy trial and error process. It is also unfeasible to optimize the layout of the railway network and placement of station names simultaneously, because an excessive number of possible solutions for this combinatorial problem must be enumerated in such a case.

Contribution

The technical contributions of this article can be summarized as follows:

- A novel progressive approach is developed for automatically placing station names in schematic railway maps;
- Pairwise geodesic distances between stations are accumulated to properly label stations from the central downtown area to the exterior rural areas;
- A new set of constraints is introduced to maximally preserve the mental map of the original layout of the schematic railway network.

Organization of the Article

This article is structured as follows: The authors first perform a brief survey of previous studies related to this work, and then provide a review of fundamental techniques of the proposed approach, including railway network schematization based on mixed-integer programming (MIP). This is followed by the technical details of the proposed approach for progressively annotating stations with their names on schematic railway maps, and a new set of constraints that maximally retain the overall layout of the schematic railway map for better preservation of the mental map. After experimental results and evaluation through an informal user study are presented, the authors conclude this article and refer to possible future extensions.

LITERATURE REVIEW / RELATED WORK

In this chapter, the authors conduct a brief survey on related topics including schematic railway map visualization, map labeling, and mental map preservation.

Schematic Railway Map Visualization

Recent advancements in graph drawing techniques successfully permit us to transform geographic layouts of railway network maps into their schematic versions. Among them, constrained optimization approaches successfully align railway line segments to octilinear directions, i.e., horizontal, vertical, and 45-degree diagonal directions. Stott, Rodgers, Martinez-Ovando, and Walker (2011) formulated a set of criteria for schematic railway maps and optimized them through a hill climbing method. Nöllenburg and Wolff (2011) introduced a mixed-integer programming framework for designing octilinear railway maps and placing station name labels. Wu et al. extended the work of (Nöllenburg & Wolff, 2011) to elongate a specific route as a central line (Wu, Takahashi, Lin, & Yen, 2012) and a loop line (Wu et al., 2015) in the schematic maps to effectively direct visual attention on it from map users. Another sophistication was achieved by Wang and Chi (2011), who successfully accelerated the computation of octilinear railway layouts by formulating the optimization as a quadratic programming problem. Claudio and Yoon (2014) developed a method of placing thumbnail images in areas of interest on railway maps, where the schematic representation is created by applying proper distortion to the entire map domain. The proposed approach employs the technique of (Nöllenburg & Wolff, 2011) as a basis for schematizing the geographic layout of the target railway map.

Map Labeling

Placing annotation labels aesthetically on maps is another important research subject for cartographic and geospatial visualization. Labeling techniques can be classified into internal labeling (Christensen, Marks, & Shieber, 1995) for placing an annotation label in the neighborhood of each point feature on the map, and external labeling (Bekos, Kaufmann, Symvonis, & Wolff, 2007; Lin, 2010) for placing the label away from the corresponding point feature while they are connected by a leader line to clarify the correspondence. The combination of internal and external labeling techniques (Wu, Takahashi, Lin, & Yen, 2011) together with several enhanced variants (Fekete & Plaisant, 1999; Fink, Haunert, Schulz, Spoerhase, & Wolff, 2012) have also been pursued. In addition, scale-dependent consistent labeling techniques have been investigated both in internal (Been, Daiches, & Yap, 2006; Wu, Takahashi, Poon, & Arikawa, 2017a, 2017b) and external (Wu, Takahashi, Poon, & Arikawa, 2017a, 2017b) labeling techniques, due to the demand for the use in digital maps available on mobile displays. Niedermann and Haunertand (2018) developed a heuristic algorithm, which requires users to carefully choose the scale of labels to find their proper placement. When one applies the external labeling technique for annotating point features, designers usually prefer to align leader lines along the octilinear directions (Bekos, Kaufmann, Nöllenburg & Symvonis, 2010; Wu et al., 2013). The proposed work, which is inspired by the idea presented in (Wu et al., 2013), employs an internal labeling technique to place station names in the octilinear layout of a railway network.

Mental Map Preservation

The mental map is defined to be a concept that refers to the visual representation of some subject people commonly have in their mind. Preserving such mental maps is usually important, especially in interactive environments, because one can hardly make clear correspondence between the components if the entire view of the subject drastically changes in its layout. It is often the case that the mental map can be maintained if the overall shape of the subject has been kept even when it still contains minor changes in its view. The pioneering work on this topic was done by Eades, Lai, Misue, and Sugiyama (1991), which successfully investigated several mathematical models for preserving users' mental maps in the context of graph drawing issues. This was followed by the work by Misue, Eades, Lai, and Sugiyama (1995), in which a set of methods were formulated to preserve the orthogonal ordering of vertices in drawing graphs. Lin, Lee, and Yen (2011) introduced a sophisticated algorithm for preserving the mental map in drawing graphs by taking advantage of the simulated annealing technique. Several empirical studies were also conducted to understand the mental map and its impacts on dynamic graph drawing issues (Archambault, Purchase, & Pinaud, 2011; Bridgeman & Tamassia, 2001; Purchase, Hoggan, & Görg, 2007). In this article, a set of constraints is newly formulated to explicitly preserve the mental map of the schematic railway network by restricting the aspect ratios of the closed regions in the map.

METHOD

In this article, the authors describe the technical details of the proposed method, as the primary contribution.

Railway Map Schematization

Before going into the details, the authors briefly explain the previous formulation for synthesizing schematic networks developed by Nöllenburg and Wolff (2001), which is used as the basis of the proposed approach. This means that the approach takes the geographic layout of a railway network as input and transforms it to an octilinear version using their formulation, in which the railway segments are aligned to horizontal, vertical, or 45-degree diagonal directions. Here, stations and their connections are supposed to be represented as vertices and edges of the schematic network. The resultant schematic layout then serves as a base map to which station names are introduced as annotation labels. Refer to the next chapter for more details about this annotation process.

The formulation by Nöllenburg and Wolff (2001) employs a mixed-integer programming (MIP) approach to solve the constrained optimization problems for schematizing the railway networks. The MIP problem can be thought of as a discrete version of the linear programming problem and is known to be NP-hard from a technical point of view. To automatically schematizing input railway networks, the MIP-based formulation minimizes three different types of costs, which are *line bends*, *relative positions*, and *edge length*.

The first cost, *line bends*, penalizes the number of bends along the respective railway lines. The researchers define the line bends between the two edges $\overline{pq}$ and $\overline{qr}$ by referring to the angle $\angle pqr$ spanned by the two edges. The corresponding cost of the line bends $\mathrm{bd}\left(p,q,r\right)$ is set to be 0 if $\angle pqr$ equals to 180 degrees, and increment it by 1, if the angle changes by 45 degrees. This implies that $\mathrm{bd}\left(p,q,r\right)$ ranges from 0 to 3 in this setup because the angle is supposed to range from 45 to 315 degrees in general. For example, in Figure 2, $\mathrm{bd}\left(p,q,r\right)$ equals 1. Thus, the total cost of the line bends can be written as:

$$c_{(B)} = \sum_{pqr \in L} \mathrm{bd}\left(p,q,r\right)$$

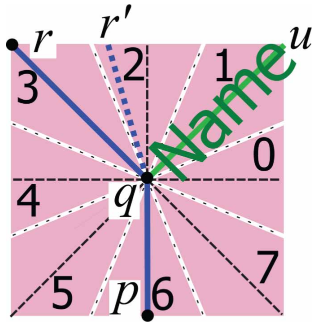
where L indicates a set of three consecutive vertices along each railway line.

The second cost, *relative positions*, evaluates the dissimilarity in the directions of the schematic railway edge and its geographic version. This cost is introduced to faithfully follow the geographic shape of the input railway network as much as possible even in its schematic layout. Suppose that the schematic railway edge $\overline{qr}$ originally runs along the direction indicated by $\overline{qr'}$ in the geographic layout as represented by the dotted line in Figure 2. This suggests that the geographic line segment $\overline{qr'}$ falls into Sector 2, which is one of the eight sectors illustrated in the figure, and is then fit to Sector 3 instead, through the schematization process. The difference between the original sector and the aligned sector in the octilinear layout is denoted by $\mathrm{rpos}\left(q,r\right)$, which is $\left|2-3\right| = 1$ in this case. The total cost of the relative positions is then given by:

$$c_{(P)} = \sum_{pq \in E} \mathrm{rpos}\left(p,q\right)$$

where E represents the set of edges in the railway network.

Figure 2. The sector index of each octilinear direction ranges from 0 to 7. In this illustration, the sector indices of $\mathrm{dir}\left(\mathrm{q},\mathrm{p}\right)$, $\mathrm{dir}\left(\mathrm{q},\mathrm{r}\right)$ and label (i.e., $\alpha\left(q,u\right)$) are 6, 3, and 1, respectively.



The last cost, which is referred to as *edge length*, is prepared for minimizing the total length of the railway network edges. In other words, the cost allows researchers to pursue the compact representation of the railway network by reducing the total length as much as possible. This is accomplished by minimizing the following cost:

$$c_{(L)} = \sum_{pq \in E} \lambda\left(p,q\right)$$

where $\lambda\left(p,q\right)$ represents the length of the edge $\overline{pq}$ and E is again the set of railway network edges on the map. Note that $\lambda\left(p,q\right)$ can be a real value because the coordinates of each station vertex are also real in this formulation. In this way, unnecessary extensions of the edges can be penalized in this formulation. Note that the constant value D is imposed as the lower bound of the edge length of $\overline{pq}$ (i.e., $\lambda\left(p,q\right)$), where D is set to be 1.0 by default.

In this formulation, the total cost is defined as a weighted sum of these three types of costs as:

$$c_{(\mathrm{total})} = w_{(\mathrm{B})}c_{(\mathrm{B})} + w_{(\mathrm{P})}c_{(\mathrm{P})} + w_{(\mathrm{L})}c_{(\mathrm{L})}$$

where $w_{(\mathrm{B})}$, $w_{(\mathrm{P})}$, and $w_{(\mathrm{L})}$ are the weights assigned to the three costs, respectively. In the implementation of this approach, $w_{(\mathrm{B})} : w_{(\mathrm{P})} : w_{(\mathrm{L})} = 1:1:100$ by default.

Refer to the literature (Nöllenburg & Wolff, 2001) for more details about the constraints for aligning railway network edges to the octilinear directions while preserving the relative geographic positions and avoiding unwanted conflicts among railway lines. The formulation for this constrained optimization provides the initial schematic layouts of railway networks as shown in Figures 5(a), 6(a), 7(a), 8(a), 9(a), and 10(a) as well as on the top-left of Figure 1.

Progressive Annotation of Railway Stations

Once the railway network has successfully been schematized, the next step is to place a station name label in the neighborhood of each station vertex. As described earlier, the central downtown area of a railway network is inherently crowded with multiple railway lines and interchange stations, and thus it is often hard to find enough labeling space around the stations in that area without changing the network layout. The problem becomes even more difficult if the station name labels are forced into alignment along the underlying octilinear directions. The cost of the associated computation becomes considerably high due to an exhaustive search for all possible combinations of label alignment patterns over the entire railway map under a complicated set of constraints.

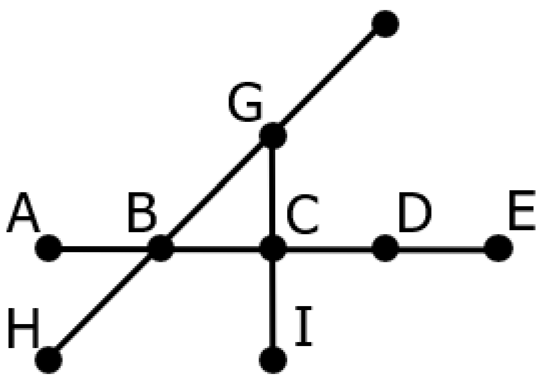
As a solution to this problem, a novel algorithm is introduced to annotate station vertices progressively from the map center to its borders. Here, the input of the algorithm is assumed to be an octilinear layout of the railway network, which is obtained using the aforementioned MIP-based formulation (Nöllenburg & Wolff, 2001).

Sorting Station Vertices for Progressive Annotation

The first solution that comes to mind is to simultaneously explore the best placements of station name labels over the entire railway network. However, as mentioned earlier, this inevitably leads to a large solution space to search for the optimal placements and usually results in the high computational complexity. This suggests the need to reduce this search space and make the annotation problem practical from a computational point of view. The basic idea is to progressively place annotation labels one by one from the crowded downtown area at the center of the map to sparse rural areas around the boundary of the map domain. In this approach, this idea is implemented by sorting the station vertices according to the sum of geodesic (i.e., shortest topological) distances within the railway network.

To find the proper order of station vertices to be annotated, the proposed approach computes for each vertex the sum of its geodesic distances to other vertices over the railway network. In general, the sum becomes smaller as the vertex is closer to the center of the network. This measure has been used to retrieve 3D shapes from the shape database (Osada, Funkhouser, Chazelle, & Dobkin, 2001), in which the distribution of the geodesic distance sums over the 3D shape is employed as the shape descriptor. Figure 3 shows an example. Here, each element of the matrix on the right represents the shortest topological distance between the pair of vertices at the corresponding row and column, which can be obtained by applying the Dijkstra algorithm (Cormen, Leiserson, Rivest, & Stein, 2009) to the network on the left. Note that the weight of all the edges in the network is assumed to be 1. The bottom row of the matrix lists the sum of such geodesic distances for the respective vertices. Actually, the vertices are sorted in an ascending order in terms of this distance sum, which means that the order of vertices is $\left\{ \mathrm{C},\ \mathrm{B},\ \mathrm{G},\ \mathrm{D},\ldots \right\}$ in this example. In general, as shown in this figure, the researchers' vertex ordering process successfully locates interchange stations in the central downtown area in such a way that they appear earlier in the list and terminal stations in the rural areas around the end of the list. Following this order allows for the effective placement of annotation labels in the dense downtown

Figure 3. Computation of the sum of pairwise geodesic distances over the railway network. The element of the matrix on the right shows the respective pairwise shortest distance between the corresponding end vertices over the railway network on the left.



	A	B	C	D	E	F	G	H	I
A	0	1	2	3	4	3	2	2	3
B	1	0	1	2	3	2	1	1	2
C	2	1	0	1	2	2	1	1	1
D	3	2	1	0	1	3	2	3	2
E	4	3	2	1	0	4	3	4	3
F	3	2	2	3	4	0	1	3	3
G	2	1	1	2	3	1	0	2	2
H	2	1	2	3	4	3	2	0	3
I	3	2	1	2	3	3	2	3	0
	20	13	12	17	24	21	14	20	19

area, first by sparing additional labeling space around the corresponding stations, and then handling those in sparse rural areas.

Octilinear Alignment of Annotation Labels

In this approach, two design rules are employed to newly formulate an algorithm for progressively annotating stations of the octilinear railway network. The first rule is to spare additional labeling space when necessary by stretching the edges of the railway network while retaining their original octilinear directions. The second rule is to align each label along one of the eight octilinear directions in such a way that it emanates from the corresponding station vertex. To optimize the label placement, as shown in Figure 2, researchers find its best direction that spans the largest angle at the corresponding station (e.g., q) with existing railway lines (e.g., $\overline{pqr}$), while the segment $\overline{qu}$ represents the name label placement. Suppose that one denotes the sector indices of the edge $\overline{qp}$ and the label by $\mathrm{dir}\left(q,p\right)$ and $\alpha\left(q,u\right)$, respectively. According to the work by Wu et al. (2013), the label placement can be formulated by minimizing $\beta\left(q,u\right)$ that satisfies the following equation:

$$\min\left\{\left|\mathrm{dir}\left(q,p\right)-\alpha\left(q,u\right)\right|,8-\left|\mathrm{dir}\left(q,p\right)-\alpha\left(q,u\right)\right|\right\}=4-\beta\left(q,u\right)$$

Since the section index $\mathrm{dir}\left(q,\cdot\right)$ goes from 0 to 7, and then begins with 0 again when the edge circulates around the vertex q, either of the two terms $\left|\mathrm{dir}\left(q,p\right)-\alpha\left(q,u\right)\right|$ and $8-\left|\mathrm{dir}\left(q,p\right)-\alpha\left(q,u\right)\right|$ corresponds to the smaller angle spanned by $\overline{qp}$ and $\overline{qu}$ and, thus, the upper limit of the left side of the above equation is 4. The same consideration can be applied to the network edge $\overline{qr}$ and label edge $\overline{qu}$. This implies the need to introduce the following new cost term for progressive annotation of station vertices:

$$c_{(D)}=\sum_{q\in V,\overline{qp},\overline{qr}\in E_q,\overline{qu}\in L_q}\beta\left(q,u\right)$$

where V, E_q, and L_q represent the sets of the station vertices, network edges incident to the station vertex q, and label edges emanating from q. Note that $\overline{qp}$ and $\overline{qr}$ are assumed to be adjacent to each other around the vertex q in this case. For implementing the proposed progressive annotation, the aforementioned total cost is replaced with:

$$c_{(\mathrm{total})}=w_{(\mathrm{B})}c_{(\mathrm{B})}+w_{(\mathrm{P})}c_{(\mathrm{P})}+w_{(\mathrm{L})}c_{(\mathrm{L})}+w_{(\mathrm{D})}c_{(\mathrm{D})}$$

where $w_{(\mathrm{B})}:w_{(\mathrm{P})}:w_{(\mathrm{L})}:w_{(\mathrm{D})}=1:1:100:10$ by default.

In the proposed approach, the optimal alignments of station name labels are computed one-by-one according to the aforementioned order of vertices based on the sum of the pairwise geodesic distances. The size of each label is adjusted by counting the number of characters in the corresponding station name. If the approach cannot avoid unwanted overlaps among railway network edges and labels after having solved the constrained optimization problem, it additionally incorporates constraints that prohibit such conflicts and continue to resolve the optimization problem until the resultant annotated network becomes free of conflicts. This scheme effectively spares minimum necessary space for the label placement by adaptively elongating the railway network edges. The actual computation has been implemented by replacing the lower bound of the length for each edge with the new length, whenever the edge is stretched for sparing the additional labeling space in the progressive annotation process. This process has been carried out individually for each station vertex until all the name labels are appropriately placed. Furthermore, the overall computation is relatively fast when compared with the conventional simultaneous annotation process, which is often realized to be unfeasible. Figure 1 shows this step-by-step process for progressively annotating stations over a schematic railway map.

Formulating Constraints for Preserving Mental Maps

In most cases, the aforementioned approach can successfully place station names as annotation labels while maintaining the mental map of the original schematic layout. Nonetheless, the approach still fails to sufficiently retain the mental map through the progressive annotation because no constraints are explicitly imposed to control the overall design of the railway map. Such failure cases can be found in Figures 7(b), 8(b), 9(b), and 10(b), where the shapes of the corresponding original layouts are undesirably distorted in the respective cases. In particular, the congested downtown areas around the center of the map are excessively stretched horizontally in the Vienna metro map (Figure 9(b)) and vertically in the Taipei metro map (Figure 10(b)), respectively, in which the underlying mental maps are not fully respected. This chapter describes how this type of unwanted distortion can be avoided by introducing a new set of constraints.

Extracting Closed Regions From the Railway Network

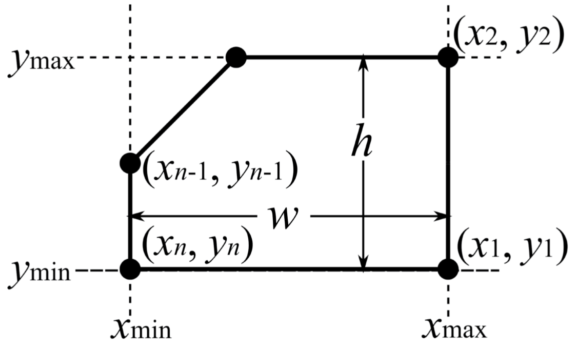
The authors consider that this distortion problem arises from that fact that the progressive annotation technique cannot adequately retain the original aspect ratio of the specific features in the schematic railway map. More specifically, the unwanted distortion in the central downtown area definitely disturbs our mental map of the railway network and keeps us from making consistent matching between the original schematic map and its annotated version. This inspires us to focus on the regions surrounded by the railway network in the downtown area, because the downtown area is usually crowded with railway lines and interchange stations and thus is more likely to contain feature regions confined by the railway lines. If the proposed approach can limit the change in the aspect ratios of such feature regions, it can successfully mitigate the distortion of the underlying mental map of the railway map. This actually matches with the aesthetic criteria for preserving the mental maps proposed in the pioneering work on mental map preservation (Eades et al., 1991; Misue et al., 1995). In this approach, this is accomplished by extracting the cycles by traversing the railway network as a planar graph and then imposing a new set of constraints that restrict the variation in the aspect ratios of regions bounded by the extracted cycles.

The extraction of these regions is achieved by faithfully reproducing the spatial embedding of the railway network on the plane and retrieving faces confined by the network. Although the faces can be categorized into open and closed, our approach just collects closed faces and enumerate the vertices on the boundary of each face because open faces are usually associated with boundary rural areas in the map and thus have less influence on the mental map of the railway map. This idea inspires us to restrict the degree of deformation of each closed region and thus retain the mental map of the original layout of the railway map as a result.

Preserving the Aspect Ratio of Each Closed Region

We are now ready to understand the formulation of the new constraints introduced to preserve the aspect ratios of the closed regions. Suppose that a closed region is confined by a cycle as shown in Figure 4, where the cycle contains n vertices each of which has $\left(x_i, y_i\right)$ coordinates as its position $\left(i=1,\ldots,n\right)$. To evaluate the aspect ratio of this closed region, let us denote the upper and lower limits for the x-coordinates of vertices by the variables $x_{\max}$ and $x_{\min}$, and those for the y-coordinates

Figure 4. Setup for constraining the aspect ratio of the region



by the variables $y_{\max}$ and $y_{\min}$. Since the x - and y . -coordinates of all the vertices stay within in the intervals $\{x_{\min}, x_{\max}\}$ and $\{y_{\min}, y_{\max}\}$, respectively, the following inequalities hold:

$$x_{\min} \le x_i \le x_{\max} \qquad y_{\min} \le y_i \le y_{\max} \qquad (i = 1,\ldots,n)$$

Although these are necessary conditions, they are not sufficient conditions to guarantee the existence of the limits $x_{\min}$, $x_{\max}$, $y_{\min}$, and $y_{\max}$. To make sure that these variables exactly limit the range of vertex coordinates $\{(x_i, y_i)\}(i = 1,\ldots,n)$, a binary variable $\theta_{\text{xmin}}(i)$, $\theta_{\text{xmax}}(i)$, $\theta_{\text{ymin}}(i)$, and $\theta_{\text{ymax}}(i)(\in\{0,1\})$ are additionally introduced for the i -th vertex. For example, since $x_{\min}$ must exactly coincide with at least one of x_1, x_2, $\ldots$, x_{n-1}, and x_n, it satisfies:

$$x_i - x_{\min} \le M\left(1-\theta_{x\min}(i)\right) \text{ and } x_{\min} - x_i \le M\left(1-\theta_{x\min}(i)\right) \text{ for } i = 1,\ldots,n$$
$$\text{while } \theta_{x\min}(1) + \theta_{x\min}(2) + \cdots + \theta_{x\min}(n) \ge 1$$

where M is a sufficiently large constant value as suggested in (Nöllenburg & Wolff, 2001). The same can be applied to the variables $x_{\max}$, $y_{\min}$, and $y_{\max}$, and thus the following conditions are also satisfied:

$$x_i - x_{\max} \le M\left(1-\theta_{x\max}(i)\right) \text{ and } x_{\max} - x_i \le M\left(1-\theta_{x\max}(i)\right) \text{ for } i = 1,\ldots,n$$
$$\text{while } \theta_{x\max}(1) + \theta_{x\max}(2) + \cdots + \theta_{x\max}(n) \ge 1$$

$$y_i - y_{\min} \le M\left(1-\theta_{y\min}(i)\right) \text{ and } \mathrm{y}_{\min} - y_i \le M\left(1-\theta_{y\min}(i)\right) \text{ for } i = 1,\ldots,n$$
$$\text{while } \theta_{y\min}(1) + \theta_{y\min}(2) + \cdots + \theta_{y\min}(n) \ge 1$$

$$y_i - y_{\max} \le M\left(1-\theta_{y\max}(i)\right) \text{ and } y_{\max} - y_i \le M\left(1-\theta_{y\max}(i)\right) \text{ for } i = 1,\ldots,n$$
$$\text{while } \theta_{y\max}(1) + \theta_{y\max}(2) + \cdots + \theta_{y\max}(n) \ge 1$$

Let us denote the width and height of a rectangle that tightly encloses the closed region by w and h, respectively, as shown in Figure 4. These variables can naturally be derived from the upper and lower limits of the closed region along the x - and y -axes, as $w = x_{\max} - x_{\min}$ and $h = y_{\max} - y_{\min}$.

Assume that the constant values W and H represent the width and height of a rectangle defining the original closed region just after the network schematization, respectively. Since the aspect ratio of the closed region $\frac{w}{h}$ should be kept close to original aspect $\frac{W}{H}$, the following inequality conditions are introduced as the new constraints:

$$\frac{W}{\left(1+E\right)H} \le \frac{w}{h} \le \frac{\left(1+E\right)W}{H}$$

where E is the constant value that represents the error tolerance ratio. These conditions can be rewritten as $Wh \le \left(1+E\right)Hw$ and $Hw \le \left(1+E\right)Wh$.

It is clear that the aspect ratio of the closed region $\frac{w}{h}$ changes more significantly if the corresponding error tolerance ratio E increases. Thus, our new formulation allows to control the degree of distortion in the progressive annotation process by adjusting the error tolerance ratio E. In what follows, E is set to be 0.5 and 0.1. The effects of this new set of constraints can be found in Figures 7(c), (d), 8(c), (d), 9(c), (d), 10(c) and (d).

RESULTS

The authors' prototype system has been implemented on a laptop PC with 3.1 GHz 6-Core Intel Core i5 CPU and 16GB RAM. The source code was written in C++ using OpenGL for drawing maps, Boost Graph Library for constructing the graph data structure, CPLEX for the MIP-based optimization, and GLUI for the user interface. Table 1 shows computation times necessary for progressively labeling schematic railway maps, which was obtained by averaging three trials.

In the system implementation, all the results were generated as raster images of the resolution 1280×1024 primarily due to the limitation of the screen space, and the entire railway network was automatically fit to the available screen size. Thus, the fonts for the station names were also adjusted according to the domain size of the railway network after the annotation process. Nonetheless, it is technically possible that the results can be produced in a vector format by taking advantage of accessible software libraries since the authors' employed outline fonts in their system implementation.

Table 1. Computation times (in seconds)

City	Figure	Time	Figure	Time	Figure	Time
Lyon	Figure 1	16.2	N/A		N/A	
Prague	Figure 5(b)	101.1	N/A		N/A	
Montreal	Figure 6(b)	29.2	Figure 6(c)	27.7	Figure 6(d)	57.6
Lisbon	Figure 7(b)	17.2	Figure 7(c)	35.6	Figure 7(d)	36.0
Milan	Figure 8(b)	144.1	Figure 8(c)	185.9	Figure 8(d)	278.0
Vienna	Figure 9(b)	140.8	Figure 9(c)	164.3	Figure 9(d)	235.5
Taipei	Figure 10(b)	101.1	Figure 10(c)	180.6	Figure 10(d)	409.7

The prototype system was also equipped with an interface for adjusting the error tolerance ratio E to control how tightly the original mental map is preserved in the annotated railway maps, together with the weights assigned to the four cost terms described earlier.

Design Examples

Figure 1 presents a sequence of the metro maps of Lyon, in which the proposed approach progressively introduces station name labels one-by-one, from the central downtown area to the exterior rural areas. The results show that the approach can spare additional labeling space for around the next station to be annotated by elongating the railway network edges while retaining their original directions.

Figure 5(a) and (b) exhibit the original schematic layout of the Prague metro and its annotated version, respectively. The results reveal that the overall shape of the metro map is relatively preserved using the proposed progressive annotation only. Figure 6 exposes the influence of the change in the weight assignment on the resultant layout of the annotated railway map. Here, $w_{(B)} : w_{(P)} : w_{(L)} : w_{(D)} = 1:1:100:1$ in Figure 6(b), $w_{(B)} : w_{(P)} : w_{(L)} : w_{(D)} = 1:1:100:10$ in Figure 6(c), and $w_{(B)} : w_{(P)} : w_{(L)} : w_{(D)} = 1:1:100:100$ in Figure 6(d). In Figure 6(b), the weight for the label assignment is the smallest, and thus the alignments of the station names are not fully perpendicular to the metro line segments. Compared to this example, the alignments of several labels become orthogonal to the metro line segments in Figure 6(c) since the weight for the label assignment increases. On the other hand, the larger weight for the label assignment forces more labels to be perpendicular to the metro lines and finally results in the distortion of the network layout, as shown in Figure 6(d). The last case takes more computation time since it requires more significant deformation of the original schematic layout of the metro network.

Figures 7 through 10 demonstrate how the proposed set of constraints preserves the underlying mental map of the respective metro maps in major cities. In the case of the Lisbon metro map, the simple progressive annotation scheme enlarges the annotated metro network excessively in the horizontal direction (Figure 7(b)), and thus cannot adequately maintain the overall shape of the original schematic layout (Figure 7(a)). Newly introduced constraints for retaining the aspect ratios of the closed regions successfully alleviate this unwanted distortion according to the imposed error tolerance ratios (Figures 7(c) and (d)). As for the Milan metro map in Figure 8, the progressive annotation again stretched the metro map along the horizontal direction while constraints for preserving the mental map reproduce the aspect ratios of the closed regions in the original schematic layout. Figure 9 presents the Vienna metro map, in which the network layout in the central downtown area is strongly distorted.

This undesirable deformation is again minimized thanks to the constraints that preserve the aspect ratios of the closed regions confined by the network. Another drastic change in the layout can

Figure 5. Annotating the Prague metro network: (a) Original schematic map; (b) A progressively annotated map without any constraints on the entire shape

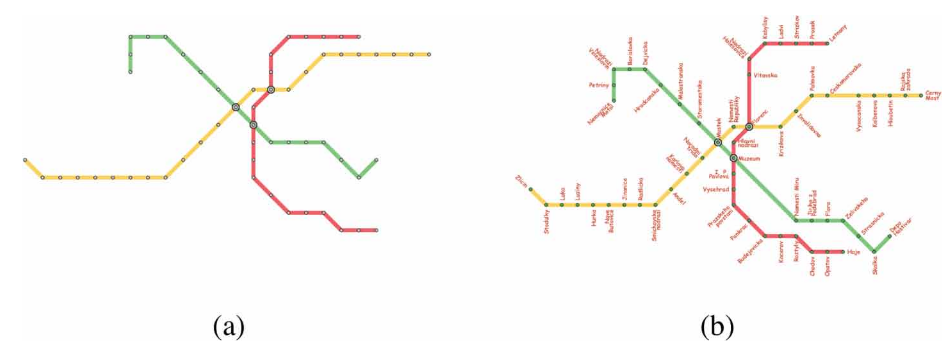
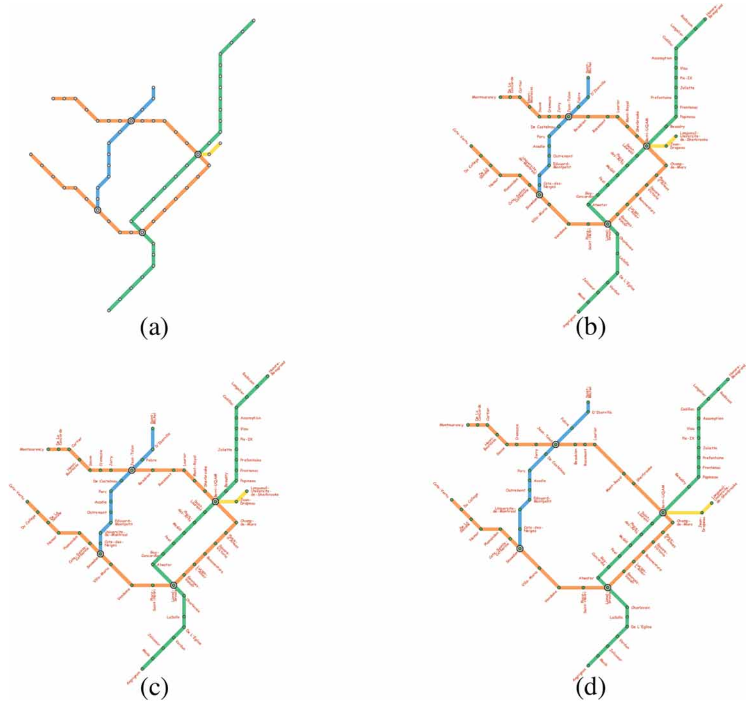


Figure 6. Montreal metro map: (a) Schematic metro map; (b) Annotated metro map with a small weight for the label alignment $\left(w\left(B\right):w\left(P\right):w\left(L\right):w\left(D\right)=1:1:100:1\right)$; (c) Annotated metro map with a medium weight for the label alignment $\left(w\left(B\right):w\left(P\right):w\left(L\right):w\left(D\right)=1:1:100:10\right)$; (d) Annotated metro map with a large weight for label alignment $\left(w\left(B\right):w\left(P\right):w\left(L\right):w\left(D\right)=1:1:100:100\right)$



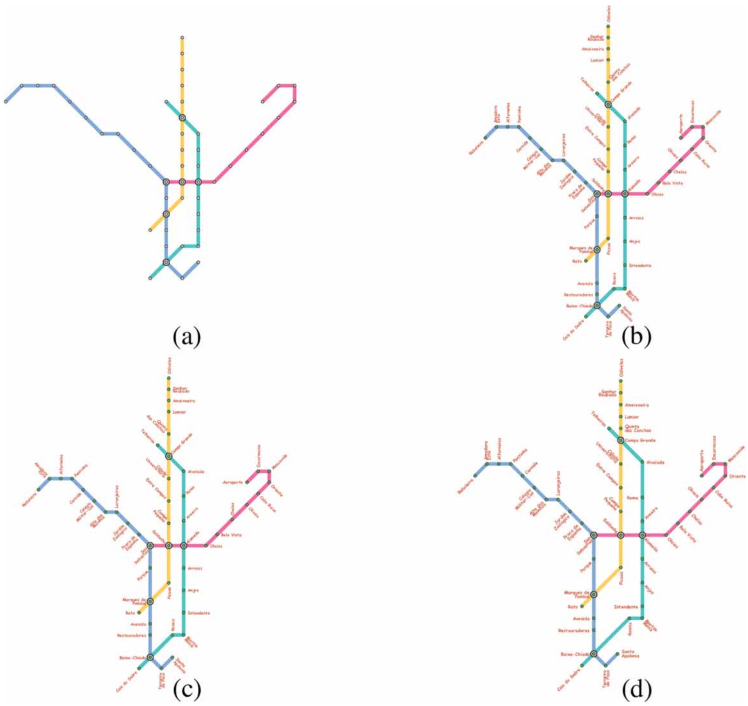
be found in the annotated version of the Taipei metro map shown in Figure 10, in which the spaces between the metro lines in parallel cannot be preserved. Also, in this case, such unacceptable changes in the layout can be avoided using the proposed formulation for keeping the underlying mental map. Note that the computation complexity usually becomes higher if more severe constraints are imposed for preserving the mental map. However, such severe constraints sometimes effectively spare the necessary labeling space in an early stage of the progressive annotation process, and the associated computation time becomes short in that case.

These results demonstrate that the proposed approach can successfully spare enough labeling space in the crowded downtown area by elongating the railway network edges with the underlying mental map fully preserved. See the demonstration video by visiting https://youtu.be/Eoqwj2XQiBg.

Evaluation Through a User Study

To evaluate the feasibility of the proposed formulation for preserving the mental map, 54 participants ages 20 to 65 (9 females and 45 males, including 41 university students) were recruited to conduct an informal user study. The participants of the user study were assumed to have the standard or corrected visual acuity while not specifically familiar with the fundamentals of visualization research. They were then requested to compare the original schematic layout of the synthesized metro map with the three annotated versions, and select from the three the best match for the underlying mental map of the original schematic layout. Here, the three annotated layouts are those obtained with (A)

Figure 7. Metro network in Lisbon. Station names are placed from the map center while adaptively extending railway line segments for sparing enough labeling space while annotating the Lisbon metro network. (a) Original schematic map, (b) A progressively annotated map without any constraints on the entire shape. Progressively annotated maps while the aspect ratios of closed regions are constrained by the new constraints, where the tolerance error ratio E is set to be (c) $\mathrm{E} = 0.5$ and (d) $\mathrm{E} = 0.1$.



no constraints for preserving the aspect ratios of closed regions, (B) constraints with the tolerance error ratio $E = 0.5$, and (C) constraints with the tolerance error ratio $E = 0.1$.

Table 2 presents the results of the informal user study. As for the cases in which the simple progressive annotation can successfully annotation stations of the schematic map without excessively distorting the original layout, the participants are likely to choose the answer randomly, and the votes are uniformly distributed for the three layouts. The metro maps of Lyon, Prague, and Montreal are such examples. To the contrary, the participants liked the annotated layouts optimized with newly formulated constraints in the other four cases, including the metro maps in Lisbon, Milan, Vienna, and Taipei. In particular, for the metro maps in Milan and Taipei, the participants prefer the small error tolerance ratio that strictly preserves the aspect ratios of closed regions in the central downtown area. However, they did not discriminate the constrained version with different error tolerance ratios for the metro maps in Lisbon and Vienna. This is probably because the difference between the two versions is small and the severe constraints can sometimes, a little excessively, enlarge the closed regions to suffice the given conditions. Understanding the perceived degree of mental map preservation in terms of the error tolerance ratio is left as future work.

CONCLUSION

This article presented an approach to progressively annotating stations in schematic railway maps. The approach can automatically place name labels in the neighborhoods of stations one-by-one from the crowded downtown area to the external rural areas. This is accomplished by elongating the railway network edges to spare the necessary labeling space while retaining their original directions in the

Figure 8. Annotating the Milan metro network. (a) Original schematic map, (b) A progressively annotated map without any constraints on the entire shape. Progressively annotated maps while the aspect ratios of closed regions are constrained by the new constraints, where the tolerance error ratio E is set to be (c) $E = 0.5$ and (d) $E = 0.1$.

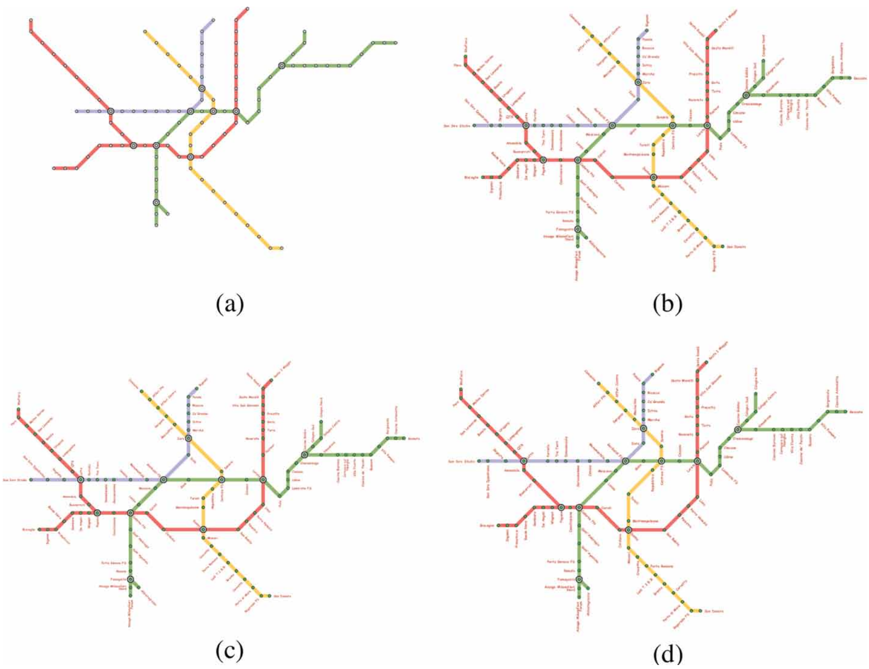


Figure 9. Annotating the Vienna metro network. (a) Original schematic map, (b) A progressively annotated map without any constraints on the entire shape. Progressively annotated maps while the aspect ratios of closed regions are constrained by the new constraints, where the tolerance error ratio E is set to be (c) $E = 0.5$ and (d) $E = 0.1$.

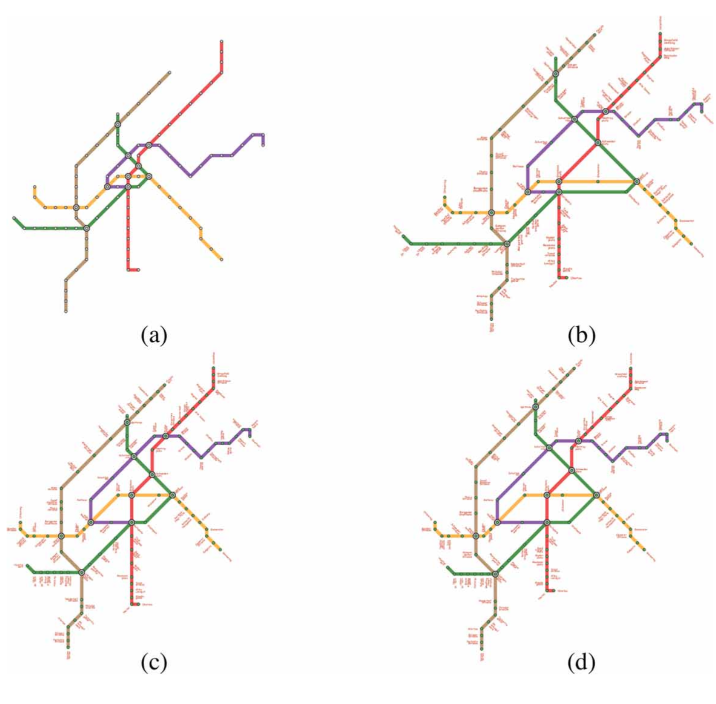


Figure 10. Annotating the Taipei metro network. (a) Original Taipei map, (b) A progressively annotated map without any constraints on the entire shape. Progressively annotated maps while the aspect ratios of closed regions are constrained by the new constraints, where the tolerance error ratio E is set to be (c) $E = 0.5$ and (d) $E = 0.1$.

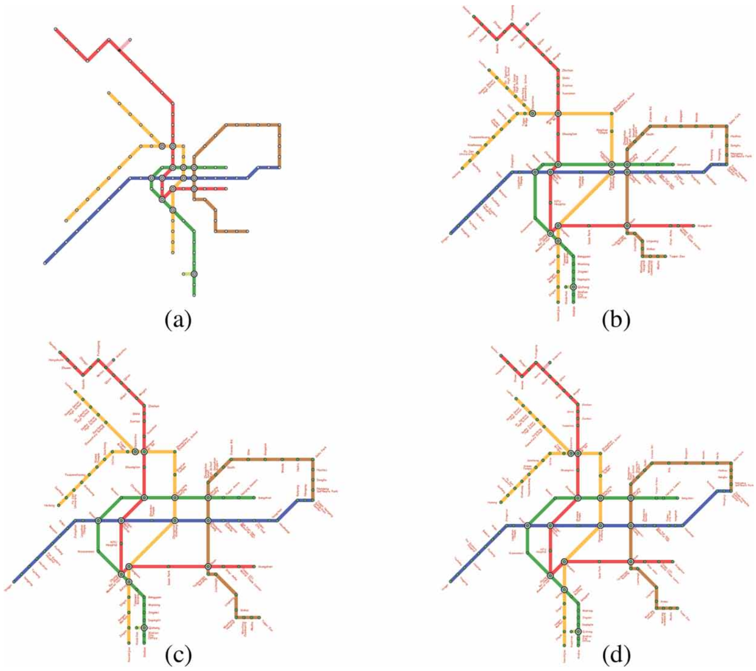


Table 2. Choices of annotated network layout that best preserves the mental map

City	(A) No Constraints	(B) $E \leq 0.5$	(C) $E \leq 0.1$
Lyon	27	22	5
Prague	16	25	13
Montreal	21	19	14
Milan	9	10	35
Lisbon	9	26	19
Vienna	1	46	7
Taipei	9	13	32

schematic railway map. Furthermore, a set of constraints is newly formulated to keep the aspect ratios of the featured closed regions, which allows researchers to maximally preserve the underlying mental map of the schematic railway network. Experimental results are presented to demonstrate the feasibility of the proposed approach, which is followed by the evaluation of the proposed mental map preservation through an informal user study.

Possible future extension includes placing station name labels consistently along each railway line in order to explore more aesthetic and compact map layouts. For this purpose, clustering stations by inferring the underlying map context will be important. In this work, the proposed approach extracts all closed faces confined by the railway network as the feature regions and imposes the proposed constraints on them to limit the variation in their aspect ratio. More careful selection of such feature regions, including open faces, may further improve the capability to maintain the underlying mental map embedded in the original schematic layout. It is also an interesting idea to employ the proposed constraints to keep shapes and layouts inherent in the design when applying deformation techniques in the problems of information visualization. This will be useful especially in other types of maps, such as road and residential maps, in which researchers have to rearrange the geometric positions of geographic components according to the change in the map scale (Maruyama, Takahashi, Wu, Misue, & Arikawa, 2019).

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