SIGGRAPH ASIA 2017 BANGKOK
CONFERENCE 27 – 30 November 2017
EXHIBITION 28 – 30 November 2017
BITEC, Bangkok, Thailand
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Information Theory in Visualization

Mateu Sbert, Han-Wei Shen, Ivan Viola, Min Chen, Anton Bardera, and Miquel Feixas

AGENDA

1. **09:00-09:45 Mateu Sbert**
   Introduction to Information Theory and its Applications in Visual Computing
2. **09:45-10:30 Ivan Viola**
   Information Theory in Volume Visualization
3. **11:00-11:45 Soumya Dutta (Han-Wei Shen)**
   Information Theory in Flow Visualization
4. **11:45-12:30 Mateu Sbert (Min Chen)**
   Information Theory and Visualization

COURSE NOTES AND FURTHER MATERIAL

- USB ???
- Course URL: www.cg.tuwien.ac.at/research/publications/2017/sbert-2017-sa_course_0023/
- Textbook

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Tutorial on Information Theory in Visualization

presenters: Mateu Sbert, Han-Wei Shen, Ivan Viola
co-authors: Min Chen, Anton Bardera, Miquel Feixas

UdG UNIVERSITY OF OXFORD TU WIEN

Introduction to Information Theory

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OVERVIEW

- Introduction
- Information measures
  - entropy, conditional entropy
  - mutual information
- Information channel
- Relative entropy
- Mutual information decomposition
- Inequalities
- Information bottleneck method
- Entropy rate

INTRODUCTION (1)

- Claude Elwood Shannon, 1916-2001
- "A mathematical theory of communication", Bell System Technical Journal, July and October, 1948
- The significance of Shannon's work
- Transmission, storage and processing of information
- Applications: physics, computer science, mathematics, statistics, biology, linguistics, neurology, computer vision, etc.

INTRODUCTION (2)

- Certain quantities, like entropy and mutual information, arise as the answers to fundamental questions in communication theory
- Shannon entropy is the ultimate data compression or the expected length of an optimal code
- Mutual information is the communication rate in presence of noise

- Book: T.M. Cover and J.A. Thomas, Elements of Information Theory, Wiley, 1991, 2006

INTRODUCTION (3)

- Shannon introduced two fundamental concepts about "information" from the communication point of view
  - information is uncertainty
    - information source is modeled as a random variable or a random process
    - probability is employed to develop the information theory
  - information to be transmitted is digital
    - Shannon's work contains the first published use of "bit"
- Book: R.W. Yeung, Information Theory and Network, Springer, 2008

INFORMATION MEASURES (1)

- Random variable X taking values in an alphabet $\mathcal{X}$

$$\mathcal{X}:\{x_1,x_2,\ldots,x_n\},p(x)=\Pr\{X=x\},p(X)=\{p(x),x\in\mathcal{X}\}$$

- Shannon entropy $H(X)$, $H(p)$: uncertainty, information, homogeneity, uniformity

$$H(X)=-\sum_{x\in\mathcal{X}}p(x)\log p(x)\equiv-\sum_{i=1}^{n}p(x_i)\log p(x_i)$$

  - information associated with x: $-\log p(x)$; base of logarithm: 2; convention: $0 \log 0 = 0$; unit: bit: uncertainty of the toss of an ordinary coin

INFORMATION MEASURES (2)

- Examples:
  - Entropy of a fair coin toss:

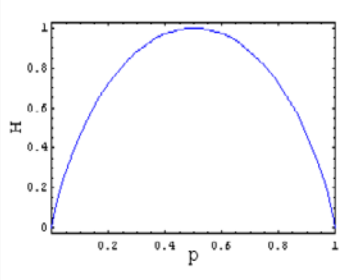
    $H(X) = -(1/2) \log (1/2) - (1/2) \log(1/2) = 1$ bit

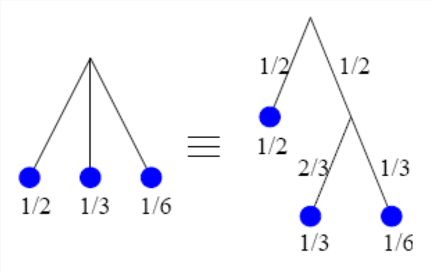
  - Entropy of a fair die toss:

    $H(X) = -(1/6) \log (1/6) - (1/6) \log(1/6) \ldots = 2.58$ bits

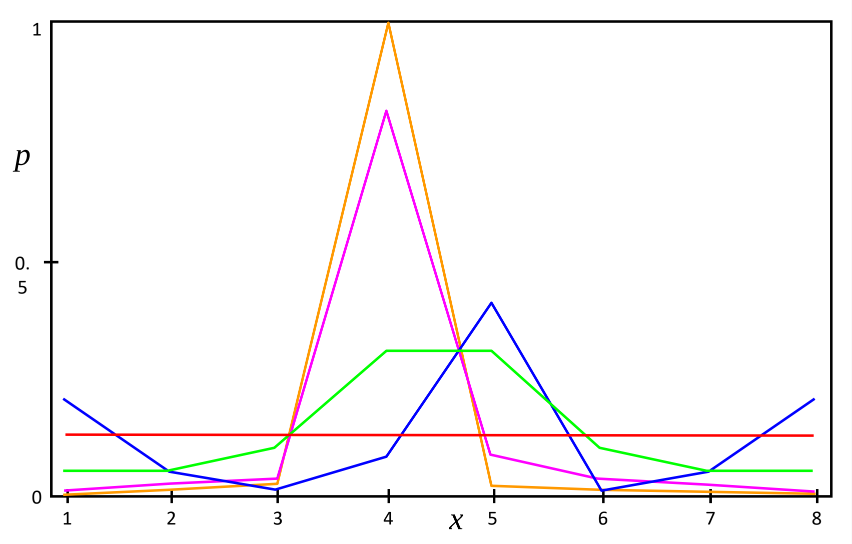
INFORMATION MEASURES (3)

- Properties of Shannon entropy
  - $0 \le H(X) \le \log|\mathcal{X}|$
  - binary entropy: $H(X) = -p\log p - (1-p)\log(1-p)$
  - $H(p_1, p_2, p_3) = H(p_1, p_2 + p_3) + (p_2 + p_3)H\left(\frac{p_2}{p_2 + p_3}, \frac{p_3}{p_2 + p_3}\right)$





INFORMATION MEASURES (4)



H(0.001, 0.002, 0.003, 0.980, 0.008, 0.003, 0.002, 0.001) = 0.190

H(0.010, 0.020, 0.030, 0.800, 0.080, 0.030, 0.020, 0.010) = 1.211

H(0.200, 0.050, 0.010, 0.080, 0.400, 0.010, 0.050, 0.200) = 2.314

H(0.125, 0.125, 0.125, 0.125, 0.125, 0.125, 0.125, 0.125) = 3.000

INFORMATION MEASURES (5)

- Discrete random variable Y in an alphabet Y

$$\mathrm{Y}:\{y_1,y_2,\ldots,y_n\},\ p(y)=\Pr\{Y=y\}$$

- Joint entropy

$$H(X,Y)=-\sum_{x\in \mathrm{X}}\sum_{y\in \mathrm{Y}} p(x,y)\log p(x,y)$$

- Conditional entropy

$$H(Y|X)=\sum_{x\in \mathrm{X}} p(x)H(Y|x)=-\sum_{x\in \mathrm{X}} p(x)\sum_{y\in \mathrm{Y}} p(y|x)\log p(y|x)$$

$$=-\sum_{x\in \mathrm{X}}\sum_{y\in \mathrm{Y}} p(x,y)\log p(y|x)$$

INFORMATION CHANNEL

- Communication or information channel $X \rightarrow Y$

$$X \xrightarrow{p(Y|X)} Y$$

$p(X)$ $p(Y)$

$p(X)$

$$\begin{pmatrix} p(x_1) \\ p(x_2) \\ \ldots \\ p(x_n) \end{pmatrix}$$

$p(Y|X)$

$$\begin{pmatrix} p(y_1|x_1) & p(y_2|x_1) & \ldots & p(y_m|x_1) \\ p(y_1|x_2) & p(y_2|x_2) & \ldots & p(y_m|x_2) \\ \ldots & \ldots & \ldots & \ldots \\ p(y_1|x_n) & p(y_2|x_n) & \ldots & p(y_m|x_n) \end{pmatrix}$$

$p(Y|x)$

$$\begin{pmatrix} p(y_1) & p(y_2) & \ldots & p(y_m) \end{pmatrix}$$

$p(Y)$

$$\sum_{y \in Y} p(y|x) = 1$$

$$p(y) = \sum_{x \in X} p(x)p(y|x)$$

Bayes' rule

$$p(x,y) = p(x)p(y|x) = p(y)p(x|y)$$

INFORMATION MEASURES (6)

- Mutual information $I(X;Y)$: shared information, correlation, dependence, information transfer

$$I(X;Y) = H(Y) - H(Y \mid X) = \sum_{x \in X} \sum_{y \in Y} p(x,y) \log \frac{p(x,y)}{p(x)p(y)}$$

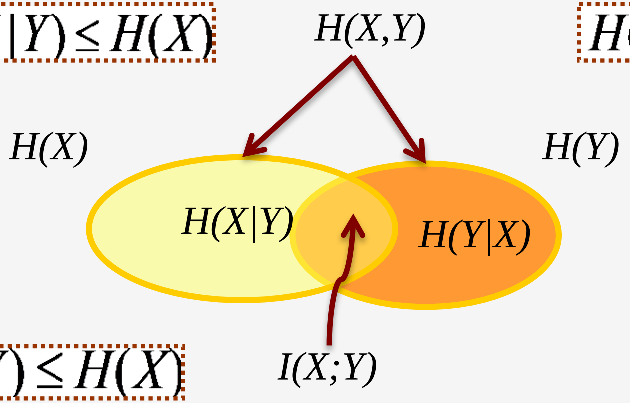
$$= \sum_{x \in X} p(x) \sum_{y \in Y} p(y \mid x) \log \frac{p(y \mid x)}{p(y)}$$

INFORMATION MEASURES (7)

- Relationship between information measures

$$0 \le H(X|Y) \le H(X)$$

$$H(X,Y) = H(X) + H(Y|X)$$



$$I(X;Y) \le H(X)$$

$$I(X;Y) = I(Y;X) \ge 0$$

$$H(X,Y) = H(X) + H(Y) - I(X;Y)$$

Yeung's book: Chapter 3 establishes a one-to-one correspondence between Shannon's information measures and set theory. A number of examples are given to show how the use of information diagrams can simplify the proofs of many results in information theory.

INFORMATION MEASURES EXAMPLE

As an example, we consider the joint distribution $p(X,Y)$ represented in Fig. 1.3.*left*. The marginal probability distributions of X and Y are given by $p(X)=\{0.25, 0.25, 0.5\}$ and $p(Y)=\{0.375, 0.625\}$, respectively. Thus, $H(X) = -0.25 \log 0.25 - 0.25 \log 0.25 - 0.5 \log 0.5 = 1.5$ bits, $H(Y) = -0.375 \log 0.375 - 0.625 \log 0.625 = 0.954$ bits, and $H(X,Y) = -0.125 \log 0.125 - 0.125 \log 0.125 - 0.25 \log 0.25 - 0 \log 0 - 0 \log 0 - 0.5 \log 0.5 = 1.75$ bits.

$p(X,Y)$		$\mathcal{Y}$ y_1	y_2	$p(X)$
$\mathcal{X}$	x_1	0.125	0.125	0.25
	x_2	0.25	0	0.25
	x_3	0	0.5	0.5
$p(Y)$		0.375	0.625	

$H(X,Y) = 1.75$

$p(Y\|X)$		$\mathcal{Y}$ y_1	y_2	$H(Y\|x \in \mathcal{X})$
$\mathcal{X}$	x_1	0.5	0.5	$H(Y\|x_1) = 1$
	x_2	1	0	$H(Y\|x_2) = 0$
	x_3	0	1	$H(Y\|x_3) = 0$

$H(Y|X) = 0.25$

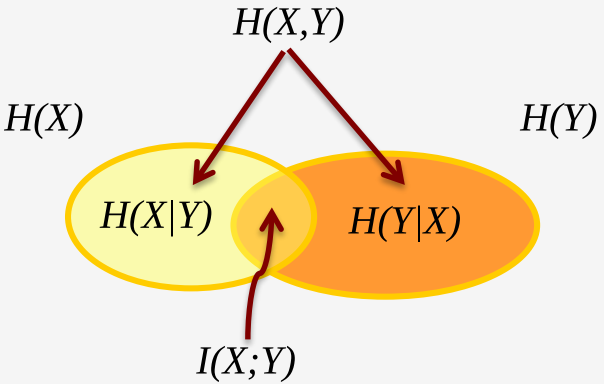
$$
\begin{aligned}
H(Y|X) &= \sum_{i=1}^{3} p(x_i) H(Y|X = x_i) \\
&= 0.25\ H(Y|X = x_1) + 0.25\ H(Y|X = x_2) + 0.5\ H(Y|X = x_3) \\
&= 0.25 \times 1 + 0.25 \times 0 + 0.5 \times 0 = 0.25 \text{ bits.}
\end{aligned}
$$

INFORMATION MEASURES (8)

- Normalized mutual information: different forms

$$\frac{I(X;Y)}{H(X,Y)} \qquad \frac{I(X;Y)}{H(X)+H(Y)}$$

$$\frac{I(X;Y)}{\min\{H(X),H(Y)\}} \qquad \frac{I(X;Y)}{\max\{H(X),H(Y)\}}$$



- Information distance

$$H(X|Y)+H(Y|X)$$

RELATIVE ENTROPY

- Relative entropy, informational divergence, Kullback-Leibler distance $D_{KL}(p,q)$: how much *p* is different from *q* (on a common alphabet X)

$$D_{KL}(p,q) = \sum_{x \in X} p(x) \log \frac{p(x)}{q(x)}$$

  - convention: $0 \log 0/q = 0$ and $p \log p/0 = \infty$
  - $D_{KL}(p,q) >= 0$
  - it is not a true metric or "distance" (non-symmetric, triangular inequality is not fulfilled)
  - $I(X;Y) = D_{KL}(p(X,Y), p(X)p(Y))$

MUTUAL INFORMATION

$$I(X;Y) = H(Y) - H(Y \mid X) = \sum_{x \in X} \sum_{y \in Y} p(x,y) \log \frac{p(x,y)}{p(x)p(y)}$$

$$= \sum_{x \in X} p(x) \sum_{y \in Y} p(y \mid x) \log \frac{p(y \mid x)}{p(y)}$$

$$D_{KL}(p,q) = \sum_{x \in X} p(x) \log \frac{p(x)}{q(x)}$$

$$I(X;Y) = D_{KL}(p(X,Y), p(X)p(Y))$$

MUTUAL INFORMATION DECOMPOSITION

- Information associated with x

$$I(X;Y)=\sum_{x\in X}p(x)\sum_{y\in Y}p(y|x)\log\frac{p(y|x)}{p(y)}=\sum_{x\in X}p(x)\left(H(Y)-H(Y|x)\right)$$

$$I_1(x;Y)=\sum_{y\in Y}p(y|x)\log\frac{p(y|x)}{p(y)}$$

$$I_2(x;Y)=H(Y)-H(Y|x)$$

[DeWeese]

$$I_3(x;Y)=\sum_{y\in Y}p(y|x)I_2(X;y)$$

[Butts]

$$I(X;Y)=\sum_{x\in X}p(x)I_k(x;Y)$$

$$k=1,2,3$$

INEQUALITIES

- Data processing inequality: if X -> Y -> Z is a Markov chain, then

$$I(X;Y) \geq I(X;Z)$$

No processing of Y can increase the information that Y contains about X, i.e., further processing of Y can only increase our uncertainty about X on average

- Jensen's inequality: a function $f(x)$ is said to be convex over an interval *(a,b)* if for every x_1, x_2 in *(a,b)* and $0<=\lambda<=1$

$$f(\lambda x_1 + (1-\lambda)x_2) \leq \lambda f(x_1) + (1-\lambda) f(x_2)$$

JENSEN-SHANNON DIVERGENCE

- From the concavity of entropy, Jensen-Shannon divergence

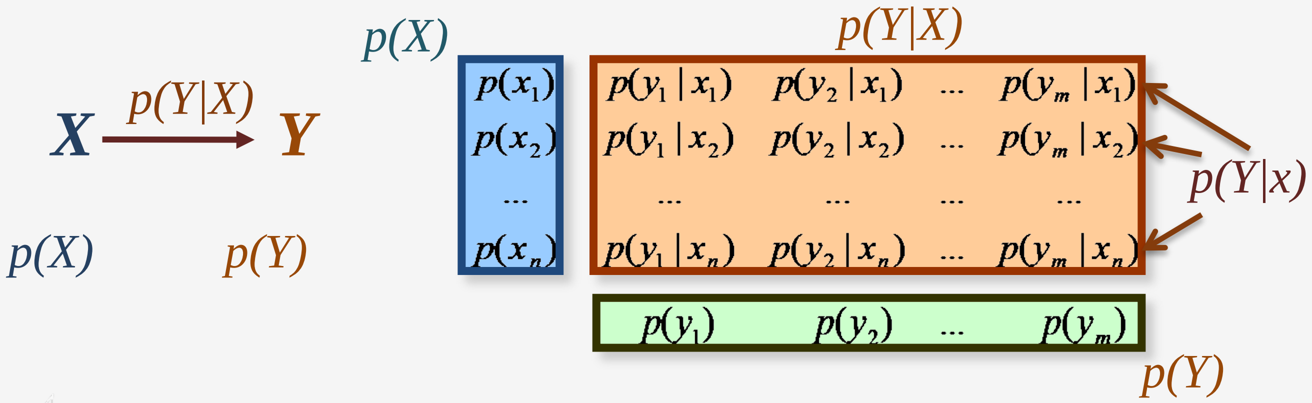
$$JS(\pi_1,\ldots,\pi_N;p_1,\ldots,p_N) = H\left(\sum_{i=1}^{N}\pi_i p_i\right) - \sum_{i=1}^{N}\pi_i H(p_i) \geq 0$$

[Burbea]

- $JS(\pi_1,\ldots,\pi_N;p_1,\ldots,p_N) = \sum_{i=1}^{N}\pi_i D_{KL}\left(p_i, \sum_{i=1}^{N}\pi_i p_i\right)$
- $JS(p(x_1),\ldots,p(x_n);p(Y|x_1),\ldots,p(Y|x_n)) = I(X;Y)$

INFORMATION CHANNEL, MI AND JS

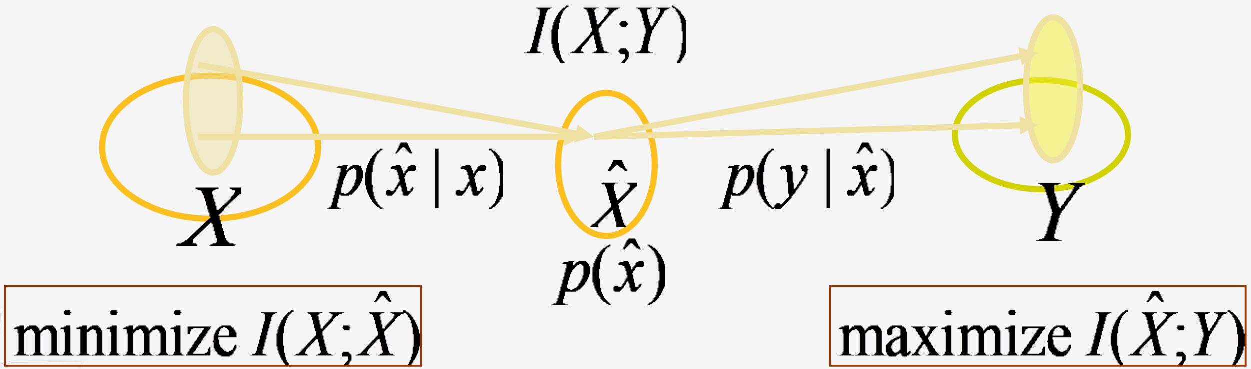
- Communication or information channel $X \to Y$



$$JS(p(x_1),\ldots,p(x_n);p(Y|x_1),\ldots,p(Y|x_n)) = I(X;Y)$$

INFORMATION BOTTLENECK METHOD (1)

- Tishby, Pereira and Bialek, 1999
- To look for a compressed representation of X which maintains the (mutual) information about the relevant variable Y as high as possible



INFORMATION BOTTLENECK METHOD (2)

- Agglomerative information bottleneck method: clustering/merging is guided by the minimization of the loss of mutual information

$$I(X;Y) \geq I(\hat{X};Y)$$

- Loss of mutual information

$$I(X;Y) - I(\hat{X};Y) =$$
$$p(\hat{x}) JS(p(x_1)/p(\hat{x}), \ldots, p(x_m)/p(\hat{x}); p(Y \mid x_1), \ldots, p(Y \mid x_m))$$

where $p(\hat{x}) = \sum_{k=1}^{m} p(x_k)$

[Slonim]

- The quality of each cluster is measured by the Jensen-Shannon divergence between the individual distributions in the cluster

INFORMATION CHANNEL AND IB

- Communication or information channel $X \longrightarrow Y$

$$X \xrightarrow{p(Y|X)} Y$$

$p(X)$ $p(Y)$

$p(X)$ $p(Y|X)$

$p(X)$	$p(Y\|X)$			
$p(x_1)$	$p(y_1 \| x_1)$	$p(y_2 \| x_1)$	...	$p(y_m \| x_1)$
$p(x_2)$	$p(y_1 \| x_2)$	$p(y_2 \| x_2)$	...	$p(y_m \| x_2)$
...	...	...	...	...
$p(x_n)$	$p(y_1 \| x_n)$	$p(y_2 \| x_n)$	...	$p(y_m \| x_n)$
	$p(y_1)$	$p(y_2)$	...	$p(y_m)$

$p(Y|x)$

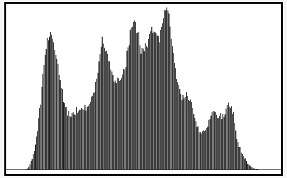
$p(Y)$

$$I(X;Y) - I(\hat{X};Y) =$$
$$p(\hat{x}) JS(p(x_1)/p(\hat{x}), p(x_2)/p(\hat{x}); p(Y|x_1), p(Y|x_2))$$

$$p(\hat{x}) = p(x_1) + p(x_2)$$

EXAMPLE: ENTROPY OF AN IMAGE

- The information content of an image is expressed by the Shannon entropy of the (normalized) intensity histogram



- The entropy disregards the spatial contribution of pixels

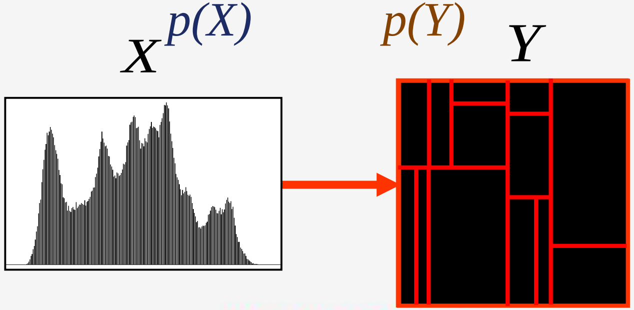
EXAMPLE: IMAGE PARTITIONING (1)

- Information channel $X \rightarrow Y$ defined between the intensity histogram and the image regions

$$X \xrightarrow{p(Y|X)} Y$$

b_i = number of pixels of bin i;
r_j = number of pixels of region j
N = total number of pixels

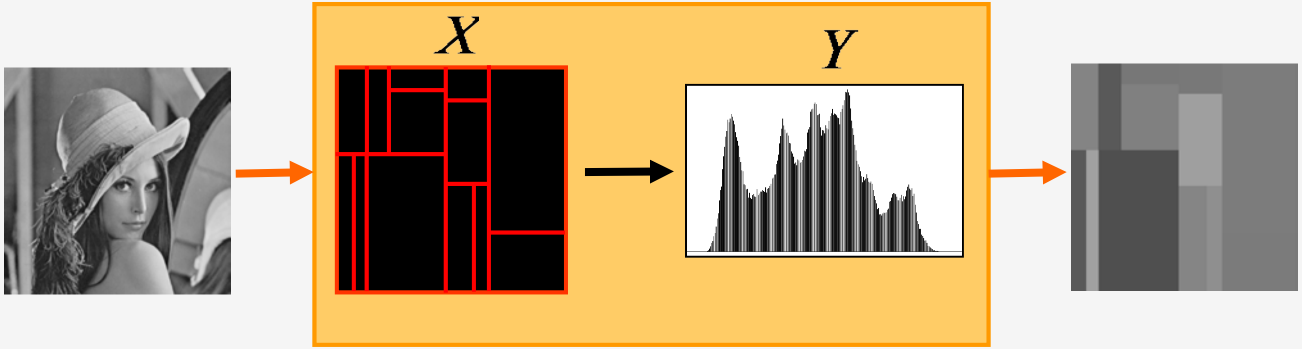
$$p(X) = \left\{ \frac{b_i}{N} \right\}$$



$$p(Y) = \left\{ \frac{r_j}{N} \right\}$$

EXAMPLE: IMAGE PARTITIONING (2)

information bottleneck method



information gain

$$I(X;Y) - I(\hat{X};Y) = p(\hat{x}) JS(p(x_1)/p(\hat{x}), p(x_2)/p(\hat{x}); p(Y|x_1), p(Y|x_2))$$

at each step, increase of $I(X;Y)$ = decrease of $H(X|Y)$

$$H(X) = I(X;Y) + H(X|Y)$$

EXAMPLE: IMAGE PARTITIONING (3)

$$MIR = \frac{I(\hat{X};Y)}{I(X;Y)} \text{ ; number of regions ; \% of regions}$$

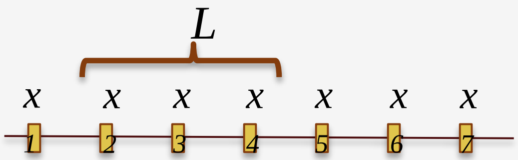
0.1; 13; 0.00 | 0.2; 64; 0.02 | 0.3; 330; 0.13 | 0.4; 1553; 0.59 | 0.0; 5597; 2.14

1; 234238; 89.35 | 0.9; 129136; 49.26 | 0.8; 67291; 25.67 | 0.7; 34011; 12.97 | 0.6; 15316; 5.84

ENTROPY RATE

- Shannon entropy

$$H(X) = -\sum_{x \in \mathcal{X}} p(x) \log p(x)$$



- Joint entropy

$$H(X^L) = -\sum_{x^L \in \mathcal{X}^L} p(x^L) \log p(x^L)$$

- Entropy rate or information density

$$h = \lim_{L \to \infty} \frac{H(X^L)}{L}$$

$$= \lim_{L \to \infty} (H(X^L) - H(X^{L-1}))$$

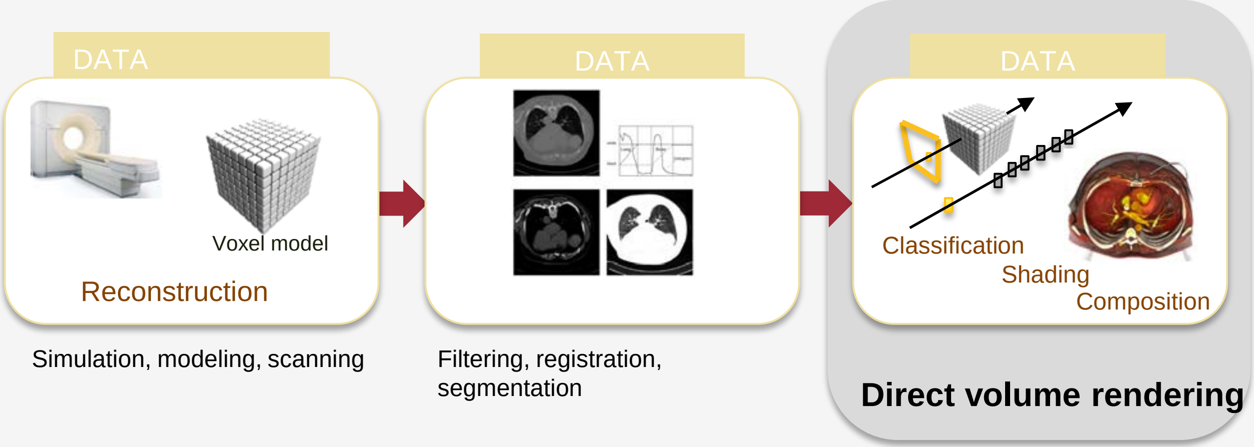
Viewpoint metrics and applications

Mateu Sbert
University of Girona, Tianjin University

VIEWPOINT SELECTION

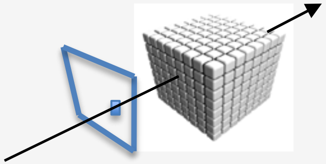
- Automatic selection of the most informative viewpoints is a very useful focusing mechanism in visualization
- It can guide the viewer to the most interesting information of the data set
- A selection of most informative viewpoints can be used for a virtual walkthrough or a compact representation of the information the data contains
- Best view selection algorithms have been applied to computer graphics domains, such as scene understanding and virtual exploration, N best views selection , image-based modeling and rendering, mesh simplication, molecular visualization, and camera placement
- Information theory measures have been used as viewpoint metrics since the work of Vazquez et al. [2001], see also [Sbert et al. 2009]

THE VISUALIZATION PIPELINE



DIRECT VOLUME RENDERING (DVR)

- Volume dataset is considered as a transparent gel with light travelling through it



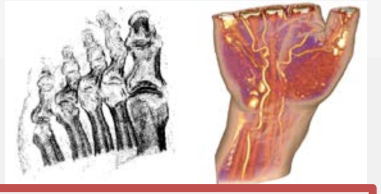
- classification maps primitives to graphical attributes

Transfer function definition

- shading (illumination) models shadows, light scattering, absorption…
  - usually absorption + emission optical model

Local or global illumination

- compositing integrates samples with optical properties along viewing rays



Both realistic and illustrative rendering

VIEWPOINT SELECTION

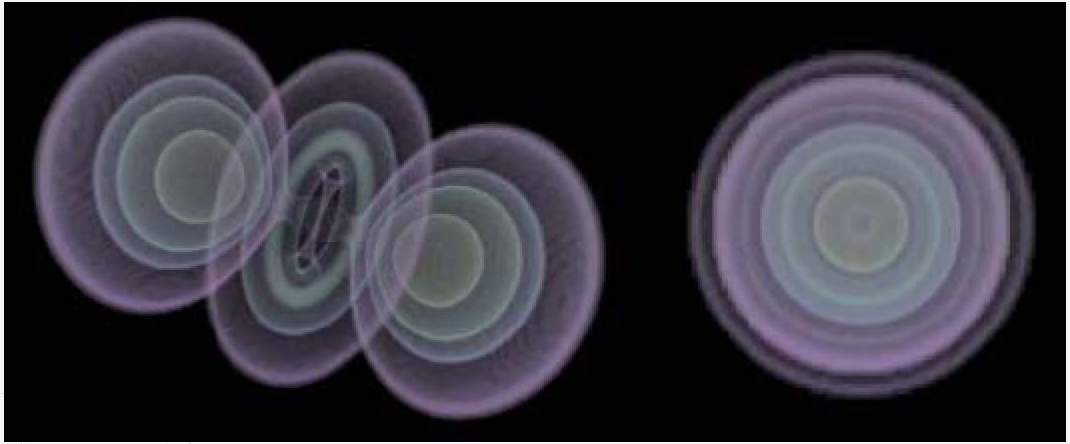
- Takahashi et al. 2005
  - Evaluation of viewpoint quality based on the visibility of extracted isosurfaces or interval volumes.
  - Use as viewpoint metrics the average of viewpoint entropies for the extracted isosurfaces.

$$E_i^{iso}(v) = \frac{-1}{\log(m_i + 1)} \sum_{j=0}^{m_i} \frac{A_{ij}}{S} \log \frac{A_{ij}}{S}$$

$$E^{iso}(v) = \sum_{i=1}^{n} \frac{E_i^{iso}(v)}{n}$$

VIEWPOINT SELECTION

- Takahashi et al. 2005



Best and worst views of interval volumes extracted from a data set containing simulated electron density distribution in a hydrogen atom

VIEWPOINT SELECTION

- Bordoloi and Shen 2005
  - Best view selection: use entropy of the projected visibilities distribution

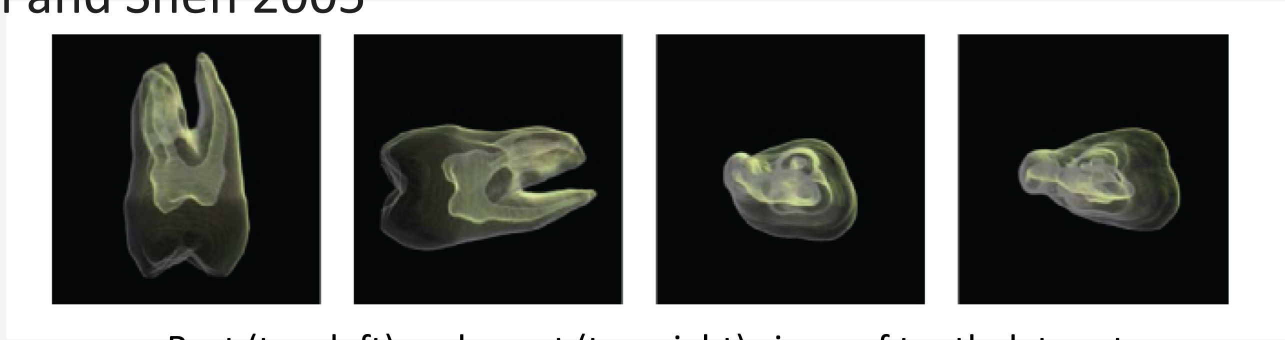
$$H(v) = -\sum_{i=1}^{n} q_i(v) \log q_i(v)$$

  - Representative views: cluster views according to Jensen-Shannon similarity measure

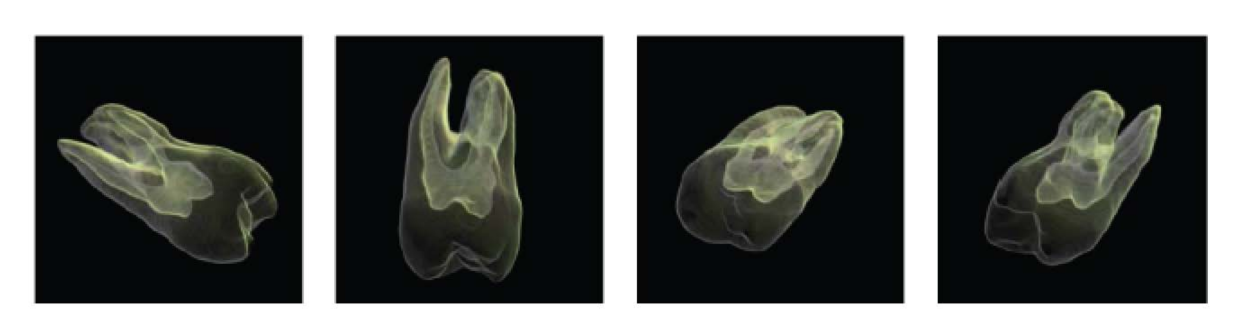
$$JS(\frac{1}{2}, \frac{1}{2}; q(v_1), q(v_2)) = H(\frac{1}{2}q(v_1) + \frac{1}{2}q(v_2)) - H(q(v_1)) - H(q(v_2))$$

VIEWPOINT SELECTION

- Bordoloi and Shen 2005



Best (two left) and worst (two right) views of tooth data set

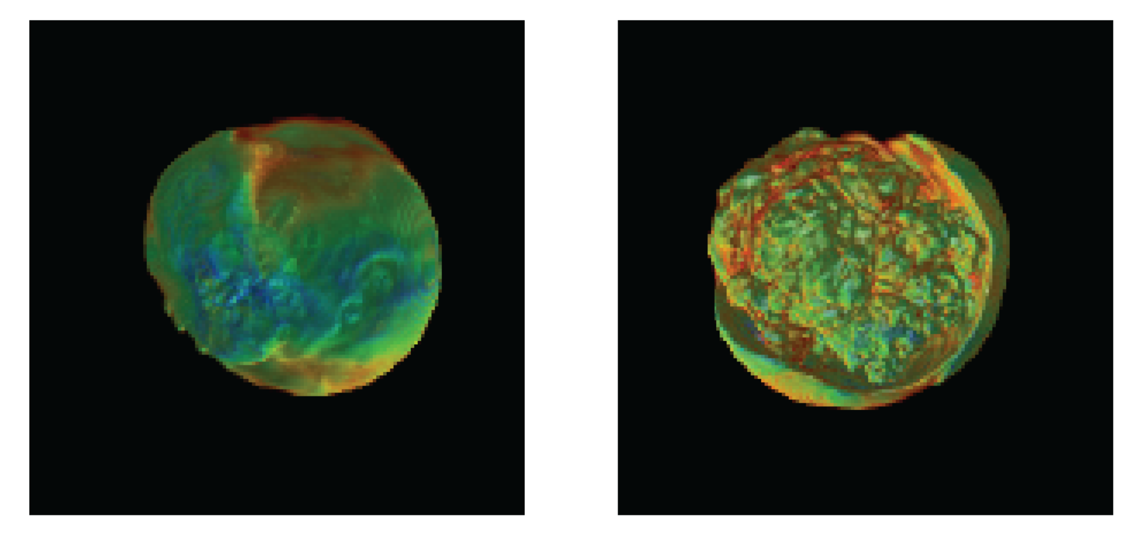


Four representative views

VIEWPOINT SELECTION

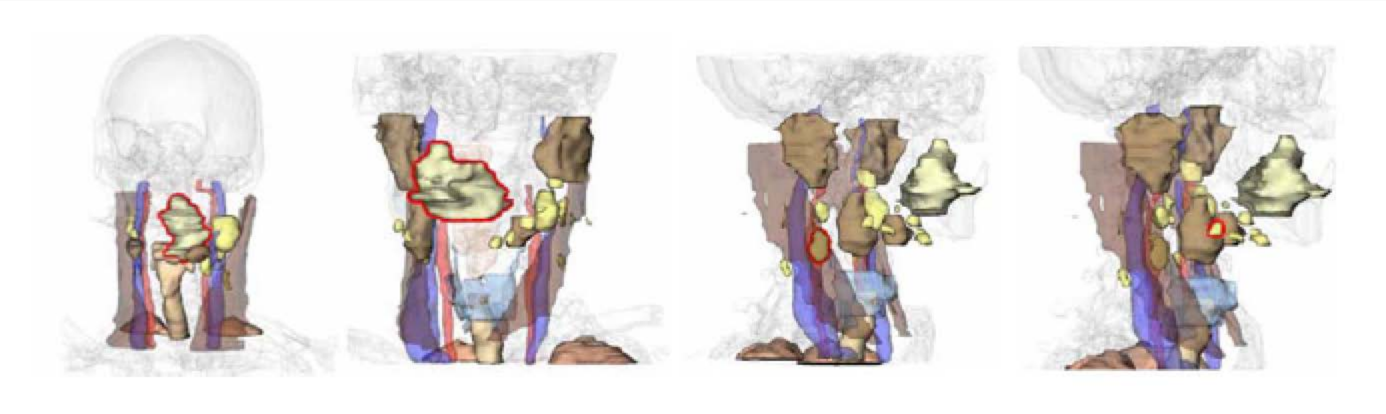
- Ji and Shen 2006
  - Quality of viewpoint *v*, *u(v)*, is a combination of three values

$$u(v) = \beta_1 opacity(v) + \beta_2 color(v) + \beta_3 curvature(v)$$



VIEWPOINT SELECTION

- Mühler et al. 2007
  - Semantics-driven view selection. Entropy, between other factors, used to select best views.
  - Guided navigation through features assists studying the correspondence between focus objects.



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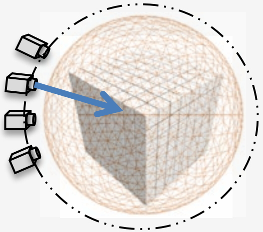
VISIBILITY CHANNEL

- Viola et al. 2006, Ruiz et al. 2010

$p(Z|V)$

V → **Z**

$p(V)$ viewpoints $\quad$ $p(Z)$ voxels



$$p(v) = \frac{vis(v)}{\sum_{i \in \mathcal{V}} vis(i)} \qquad p(z|v) = \frac{vis(z|v)}{vis(v)}$$

$p(V)$	$p(Z\|V)$			
$p(v_1)$	$p(z_1\|v_1)$	$p(z_2\|v_1)$	$\cdots$	$p(z_m\|v_1)$
$p(v_2)$	$p(z_1\|v_2)$	$p(z_2\|v_2)$	$\cdots$	$p(z_m\|v_2)$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
$p(v_n)$	$p(z_1\|v_n)$	$p(z_2\|v_n)$	$\cdots$	$p(z_m\|v_n)$
$p(Z)$	$p(z_1)$	$p(z_2)$	$\cdots$	$p(z_m)$

$$p(z) = \sum_{v \in \mathcal{V}} p(v) p(z|v)$$

- How a viewpoint sees the voxels
- Mutual information

$$I(V;Z) = \sum_{v \in \mathcal{V}} p(v) \sum_{z \in \mathcal{Z}} p(z|v) \log \frac{p(z|v)}{p(z)} = \sum_{v \in \mathcal{V}} p(v) I(v;Z)$$

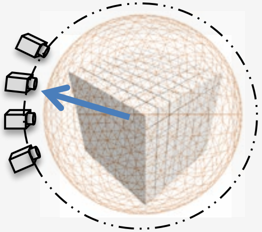
- Viewpoint mutual information (VMI): $I(v;Z) = \sum_{z \in \mathcal{Z}} p(z|v) \log \frac{p(Z|V)}{p(z)}$

REVERSED VISIBILITY CHANNEL

- Ruiz et al. 2010

Z → **V** ($p(V|Z)$)

$p(Z)$ voxels — $p(V)$ viewpoints



$$p(v|z) = \frac{p(v)p(z|v)}{p(z)}$$

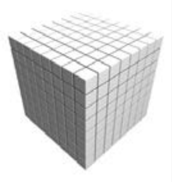
$p(Z)$	$p(V\|Z)$			
$p(z_1)$	$p(v_1\|z_1)$	$p(v_2\|z_1)$	$\cdots$	$p(v_m\|z_1)$
$p(z_2)$	$p(v_1\|z_2)$	$p(v_2\|z_2)$	$\cdots$	$p(v_m\|z_2)$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
$p(z_n)$	$p(v_1\|z_n)$	$p(v_2\|z_n)$	$\cdots$	$p(v_m\|z_n)$
$p(V)$	$p(v_1)$	$p(v_2)$	$\cdots$	$p(v_m)$

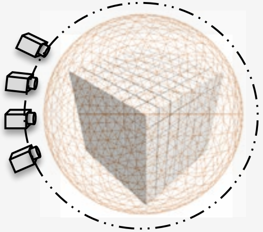
- How a voxel "sees" the viewpoints
- Mutual information

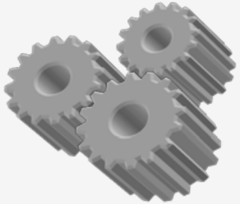
$$I(Z;V) = \sum_{z\in\mathcal{Z}} p(z) \sum_{v\in\mathcal{V}} p(v|z) \log \frac{p(v|z)}{p(v)} = \sum_{z\in\mathcal{Z}} p(z) I(z;V)$$

- Voxel mutual information (VOMI): $I(z;V) = \sum_{v\in\mathcal{V}} p(v|z) \log \frac{p(v|Z)}{p(v)}$

VOMI MAP COMPUTATION







Transfer function

Volume dataset → Classified data → Ray casting → Visibility histogram for each viewpoint → Probabilities computation → **VOMI map**

VISIBILITY CHANNEL

- Viola et al. 2006
- Adding importance to VMI for viewpoint navigation with focus of interest. Objects instead of voxels

$$I(v;O) = D_{KL}(p(O|v)||p(O))$$

$$I'(v;O) = D_{KL}(p(O|v)||p'(O)) = \sum_{o \in \mathbb{O}} p(o|v) \log \frac{p(o|v)}{p'(o)}$$

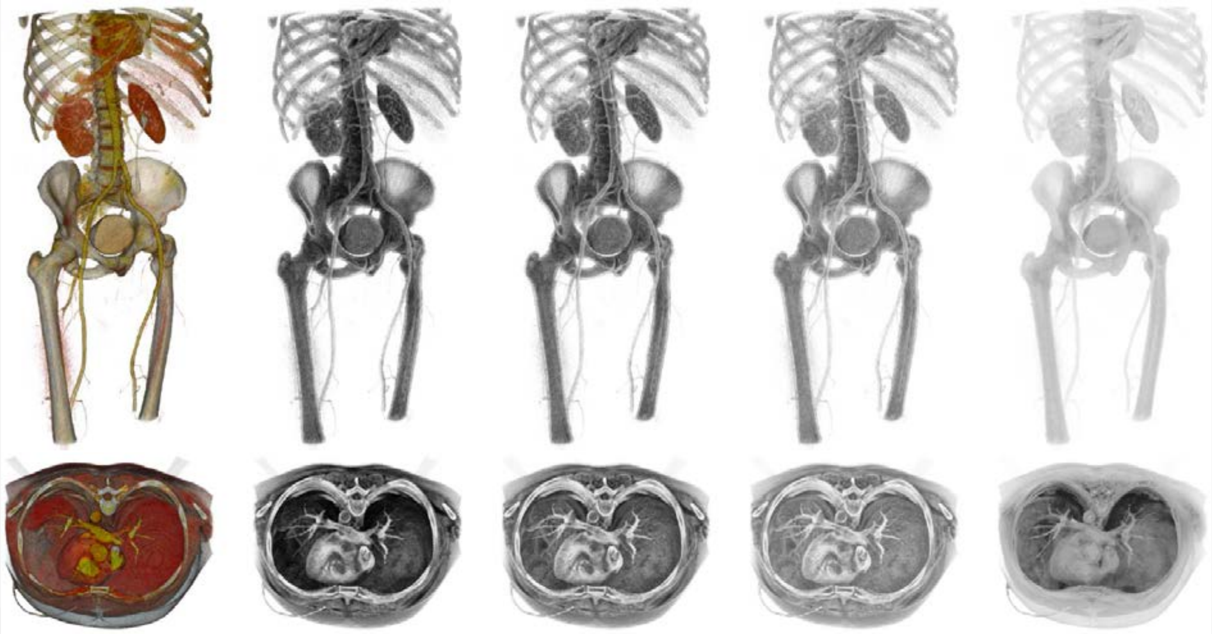
VOMI APPLICATIONS

- Interpret VOMI as ambient occlusion
  - $AO(z) = 1 - \overline{I(z;V)}$
  - Simulate global illumination
  - Realistic and illustrative rendering
  - Color ambient occlusion
  - $CAO_\alpha(z;V) = \sum_{v \in \mathcal{V}} \left(p(v|z) \log \frac{p(v|z)}{p(v)} \right) \left(1 - C_\alpha(v)\right)$

VOMI APPLICATIONS

- Interpret VOMI as importance
  - Modulate opacity to obtain focus+context effects emphasizing important parts
- "Project" VOMI to viewpoints to obtain informativeness of each viewpoint
  - $INF(v) = \sum_{z \in Z} p(v|z) I(z; V)$
  - Viewpoint selection

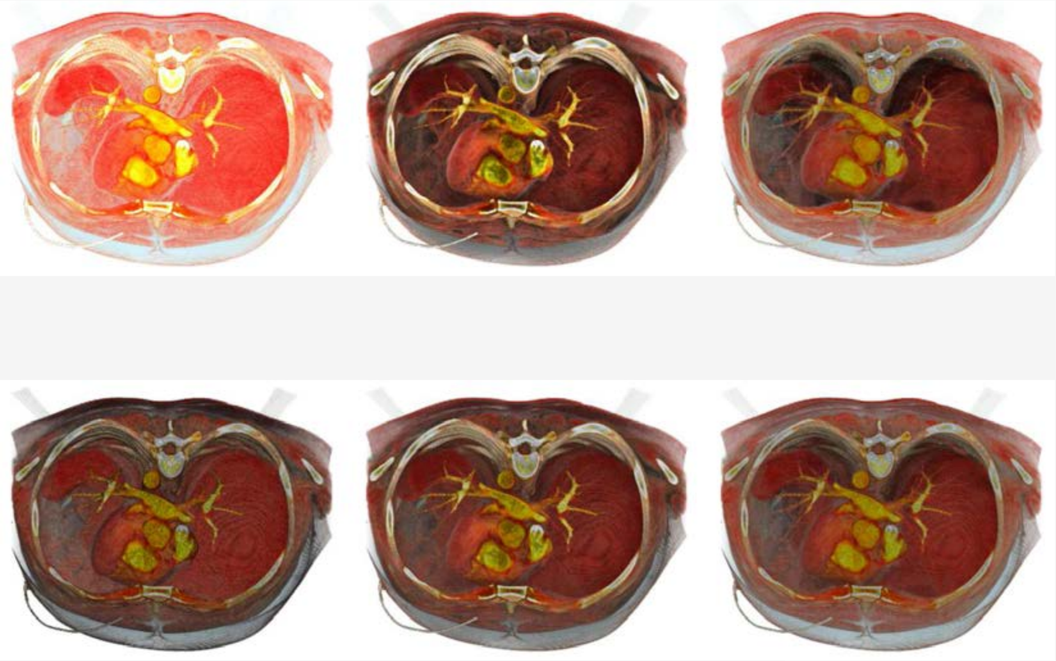
VOMI AS AMBIENT OCCLUSION MAP



Original | Ambient Occlusion, Landis 2002 | Vicinity shading, Stewart 2003 | Obscurances, Iones et al. 98 | **VOMI**

VOMI APPLIED AS AMBIENT OCCLUSION

- Ambient lighting term
- Additive term to local lighting

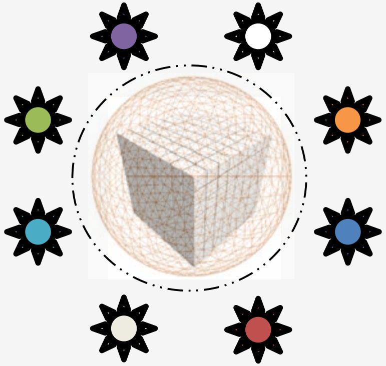


Original

Vicinity shading, Stewart 2003

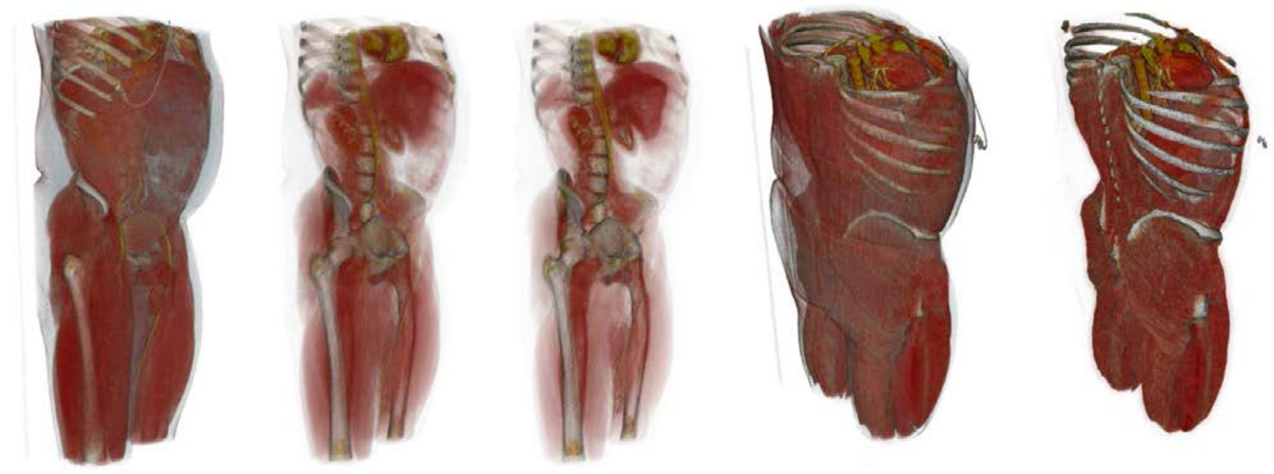
VOMI

COLOR AMBIENT OCCLUSION



CAO map | CAO map with contours | CAO maps with contours and color quantization

OPACITY MODULATION



Original | Modulated to emphasize skeleton | Original | Modulated to emphasize ribs

VIEWPOINT SELECTION

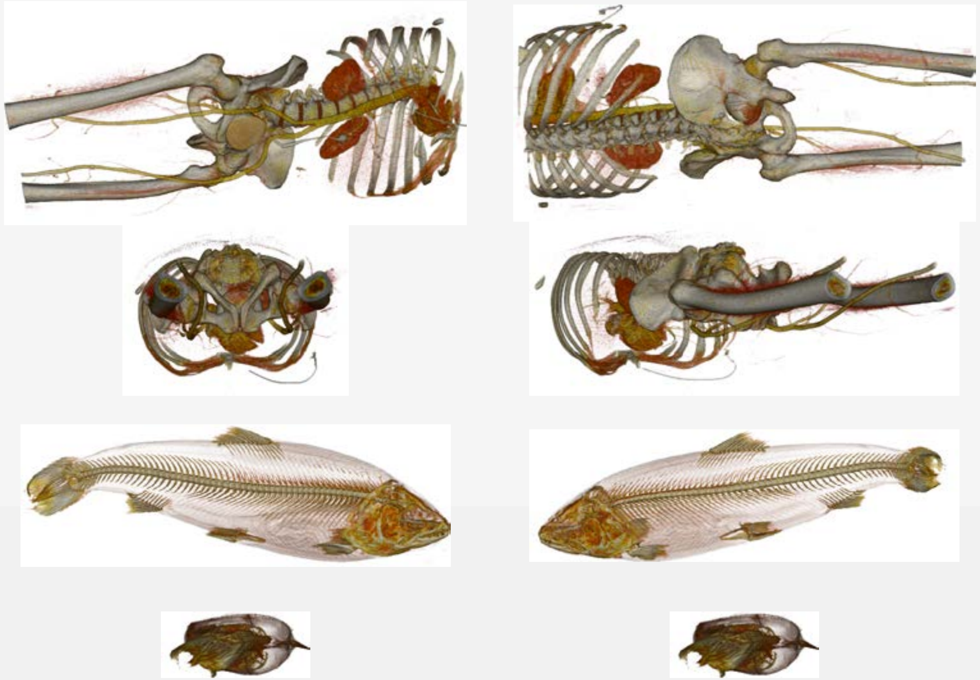
- VMI versus Informativeness

Min VMI — Max INF

Max VMI — Min INF

Min VMI — Max INF

Max VMI — Min INF



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THANKS FOR YOUR ATTENTION!

Mateu Sbert

University of Girona, Tianjin University

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Volume Visualization

Information Theory in Visualization

Ivan Viola

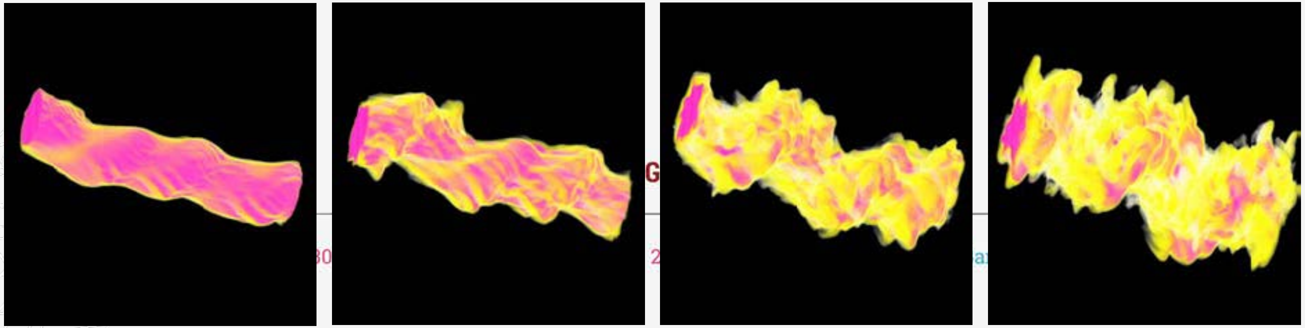
AGENDA

- Time-varying data
- Level-of-detail selection
- Isosurface extraction
- Volume splitting
- Transfer function specification
- Multimodal volume visualization

VIEW SELECTION FOR VOLUME DATA

- 3D scalar fields over time
- Viewpoint quality: visibility of voxels
- Measure: Joint viewpoint entropy

$$
\begin{aligned}
H(X) &= H(X_{t_1}, X_{t_2}, \ldots, X_{t_n}) \\
&= H(X_{t_1}) + H(X_{t_2}|X_{t_1}) + \ldots + H(X_{t_n}|X_{t_n-1})
\end{aligned}
$$



[Bordoloi and Shen 2005]

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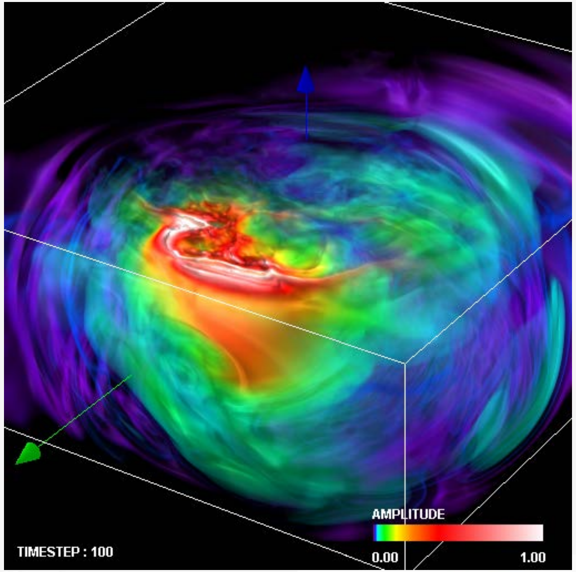
IMPORTANCE-DRIVEN VISUALIZATION

- Quantify data-block importance using entropy within a block and mutual information wrt. neighboring blocks

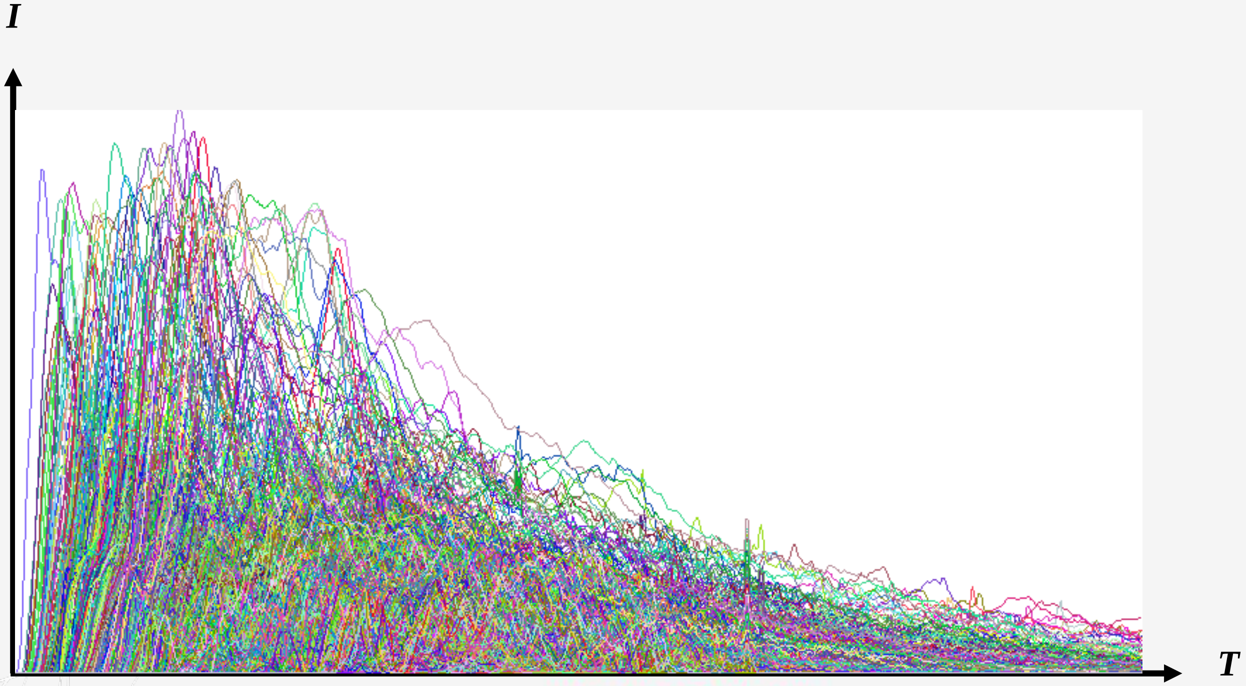
$$H(X) - I(X,Y) = H(X|Y)$$

- Concatenating data-block importance
- over time
- Clustering time-importance curves
- Selecting spatial region corresponding to the curve set



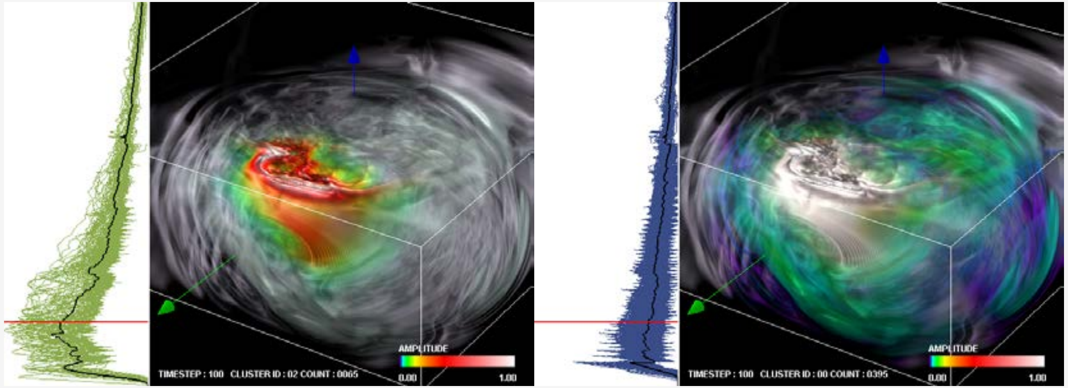
[Wang et al. 2008]

IMPORTANCE-DRIVEN VISUALIZATION

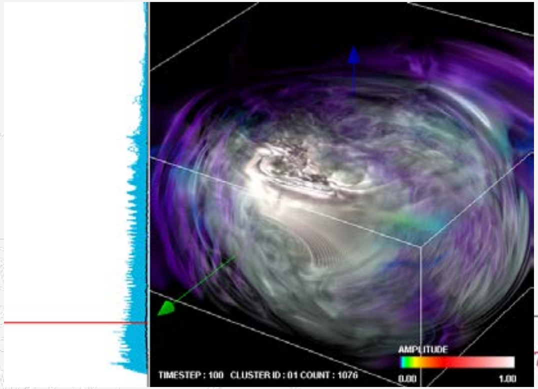


[Wang et al. 2008]

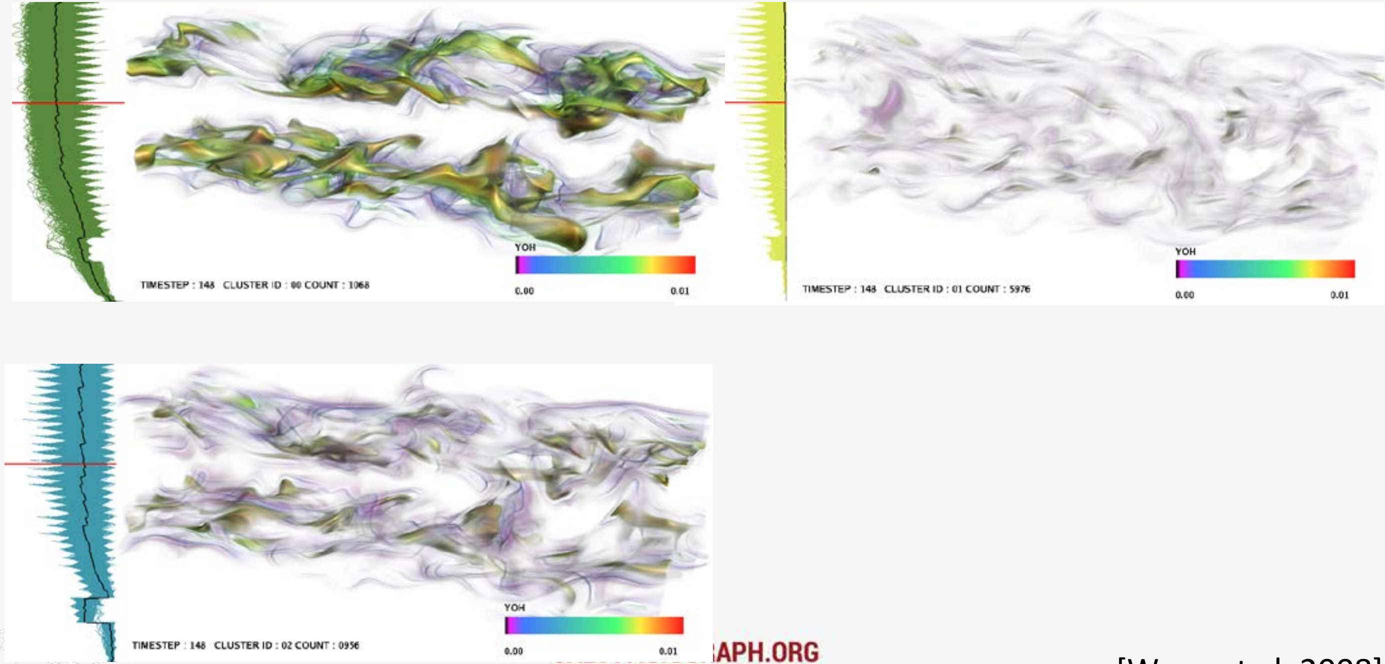
IMPORTANCE-DRIVEN VISUALIZATION



[Wang et al. 2008]



IMPORTANCE-DRIVEN VISUALIZATION



[Wang et al. 2008]

MULTIRESOLUTION VOLUMES

- Distortion (D) and Contribution (C) characteristics of a multiresolution block
- Level-of-Detail quality evaluated via Entropy measure

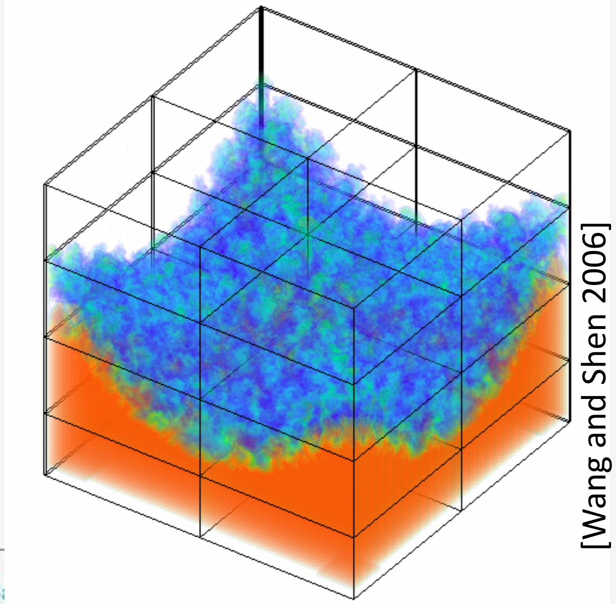
$$d_{ij} = \sigma_{ij} \frac{\mu_i^2 + \mu_j^2 + c_1}{2\mu_i\mu_j + c_1} \frac{\sigma_i^2 + \sigma_j^2 + c_2}{2\sigma_i\sigma_j + c_2}$$

$$p_i = \frac{C_i \cdot D_i}{\sum_{i=1}^{M} C_i \cdot D_i}$$

$$D_i = \sum_{j=0}^{7} d_{ij} + max(D_j|_{j=0}^{7})$$

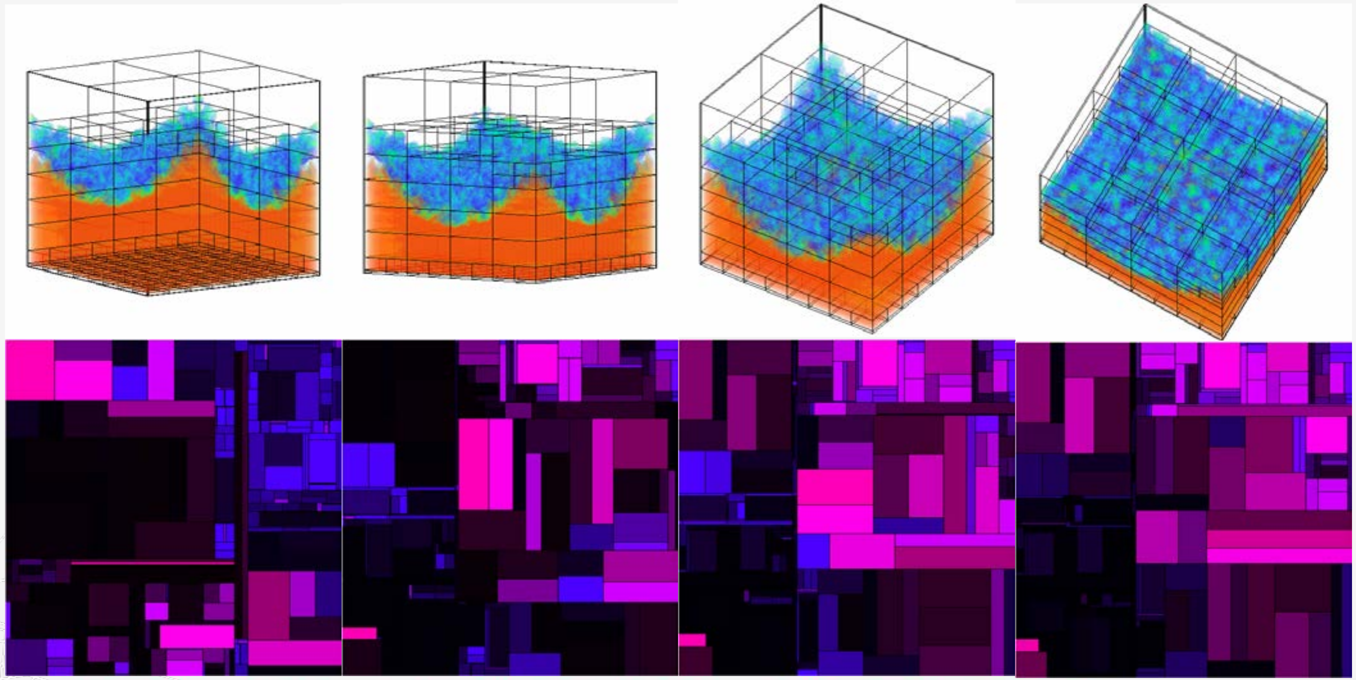
$$C_i = \mu \cdot l \cdot a \cdot v$$

- Join and split optimizations driven by entropy changes



[Wang and Shen 2006]

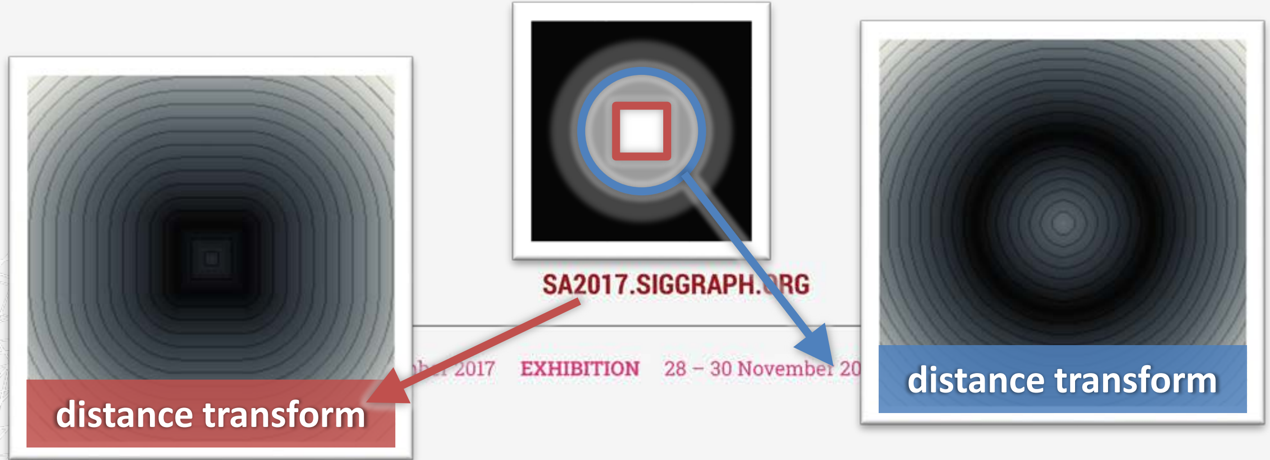
MULTIRESOLUTION VOLUMES



[Wang and Shen 2006]

ISOSURFACE SIMILARITY MAPS

- Quantifying the most significant iso-surfaces in volume
- Compare iso-surfaces through evaluating mutual information of their distance volume
  - *X* and *Y* are independent: $I(X,Y) = 0$
  - *X* and *Y* are identical: $I(X,Y) = H(X) = H(Y)$

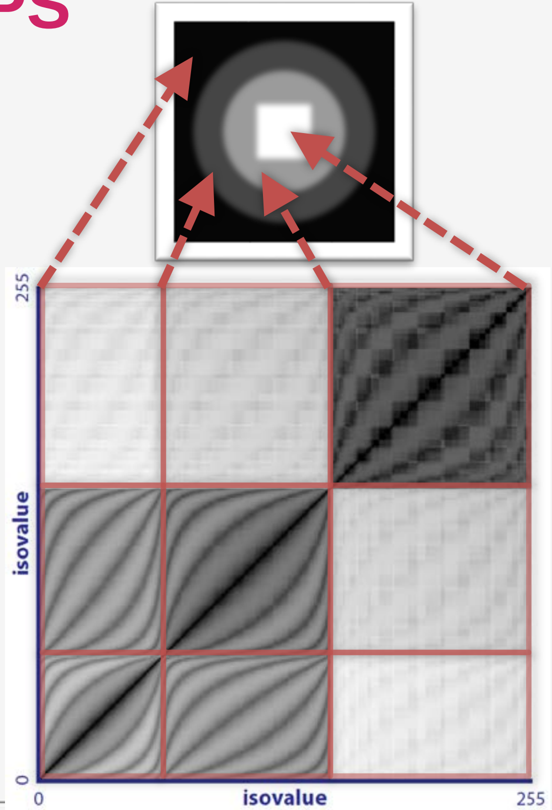


[Bruckner and Möller 2010]

ISOSURFACE SIMILARITY MAPS

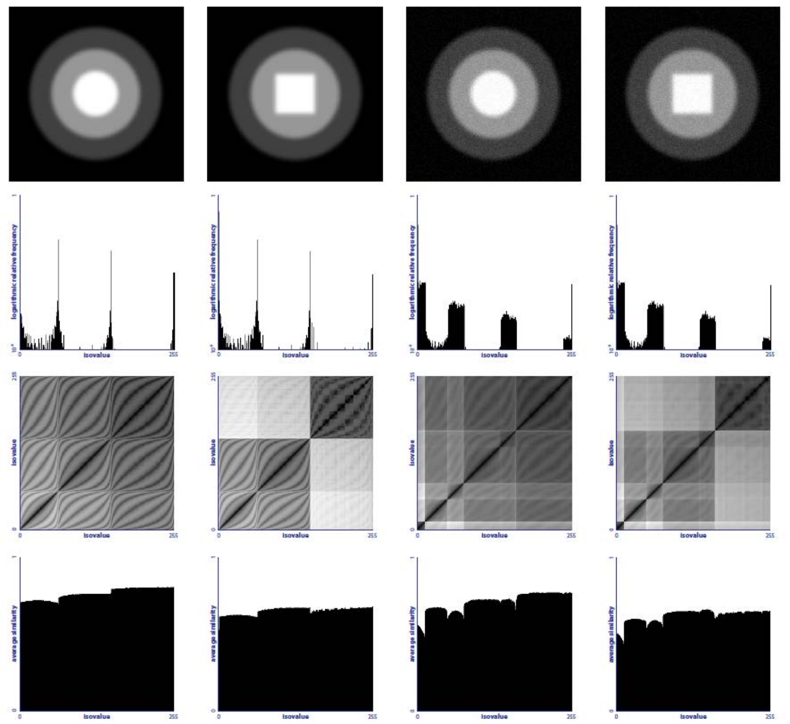
- **Normalized measure**

$$\hat{I}(X,Y) = \frac{2I(X,Y)}{H(X)+H(Y)}$$



[Bruckner and Möller 2010]

ISOSURFACE SIMILARITY MAPS

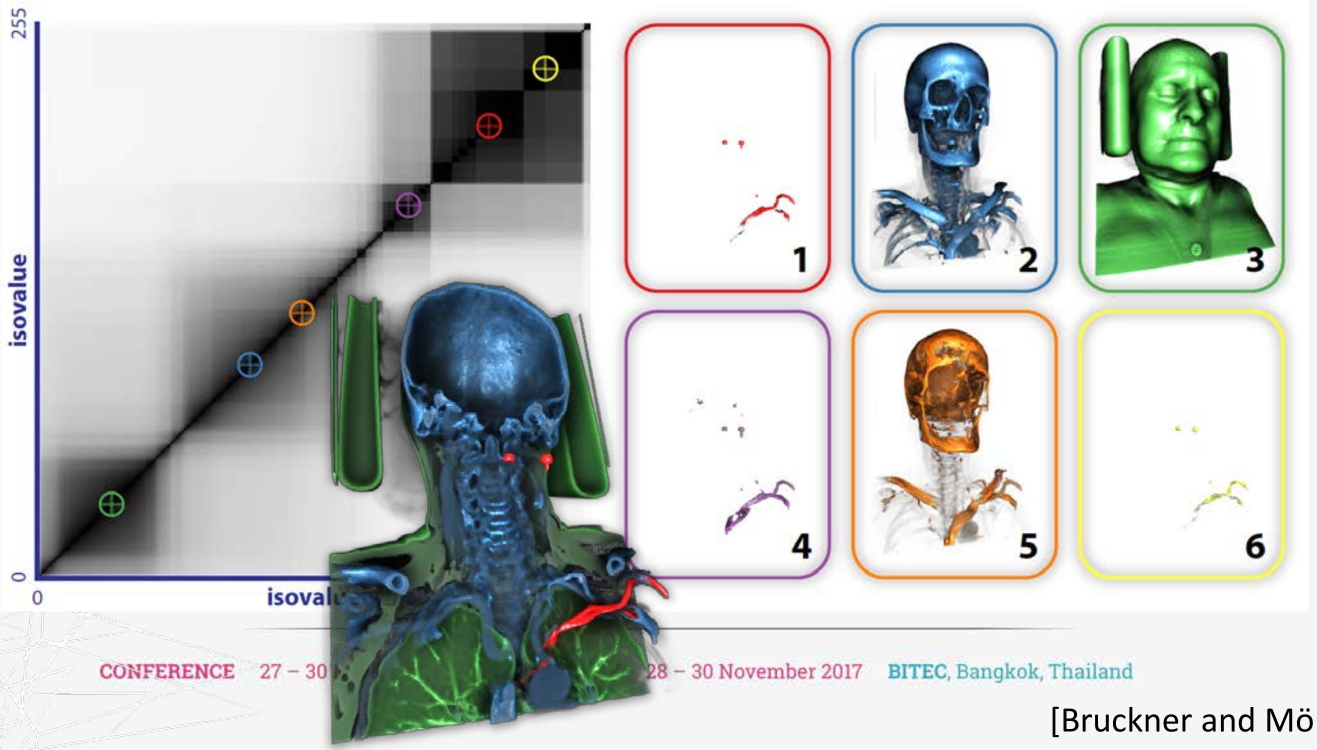


$$SD(i) = \frac{1}{|V|}\sum_{j=1}^{|V|} SM(i,j)$$

[Bruckner and Möller 2010]

ISOSURFACE SIMILARITY MAPS

- Selection of characteristic iso-surfaces



[Bruckner and Möller 2010]

SIMILARITY-BASED EXPLODED VIEWS

- A two step process is proposed to automatically obtain the partitioning planes:
  - Explosion axis: selection of the most structured view
  - Partitioning of the data: slices are grouped according to the maximization of a similarity criterion

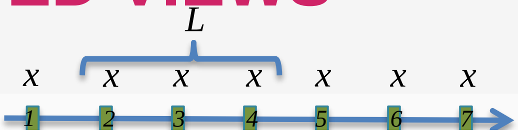
[Ruiz et al. 2008]

SIMILARITY-BASED EXPLODED VIEWS

- **Structured View measured through Entropy Rate**
  measure of the randomness or unpredictability of a system

[Ruiz et al. 2008]

SIMILARITY-BASED EXPLODED VIEWS



- **Explosion Axis:**
  Joint entropy of L vector

$$H(X^L) = -\sum_{x^L \in \mathrm{X}^L} p(x^L) \log p(x^L)$$

- **Entropy rate represents the average information content per symbol in a stochastic process**

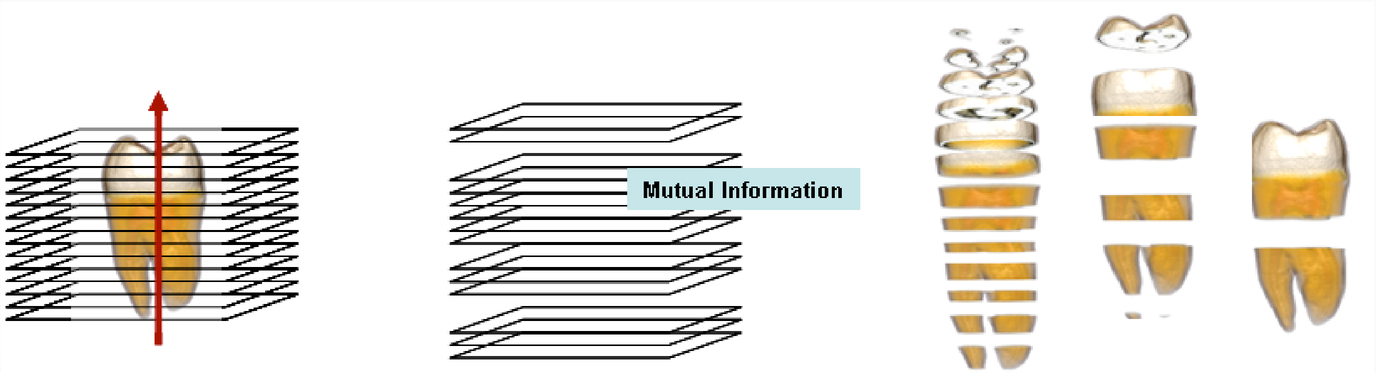
$$h = \lim_{L \to \infty} \frac{H(X^L)}{L} = \lim_{L \to \infty} (H(X^L) - H(X^{L-1}))$$

- **For a set of viewpoints we perform ray casting to compute entropy rate for each ray**
- **Summing up entropy rates per direction quantifies how much structural change particular direction exhibit**

[Ruiz et al. 2008]

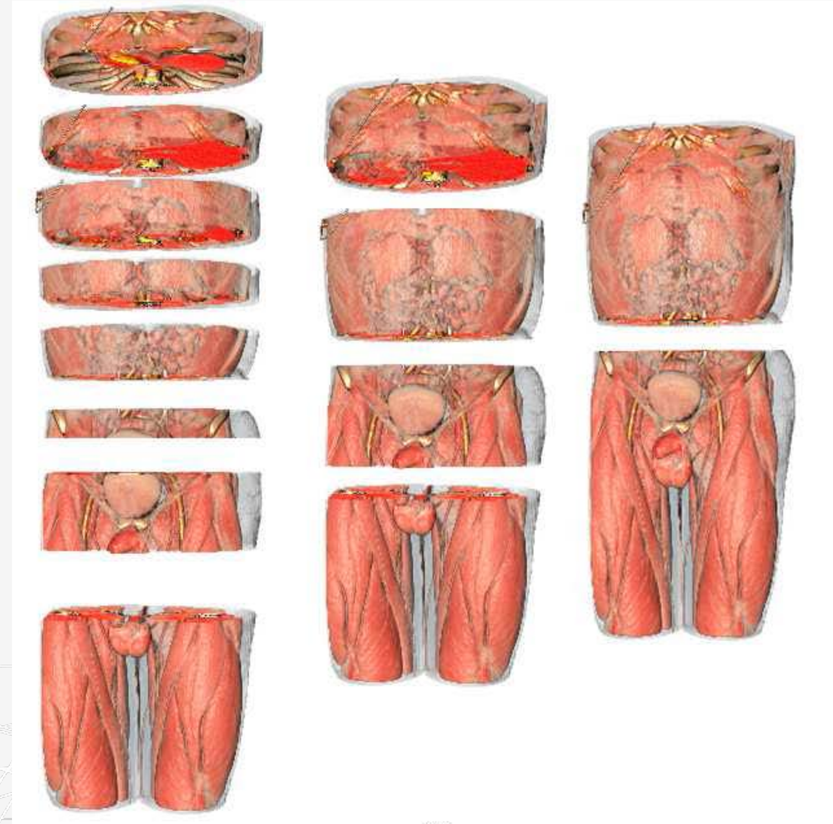
SIMILARITY-BASED EXPLODED VIEWS

- **Bottom-up grouping:** group the most similar slices or slabs through normalized mutual information
  degree of similarity or shared information between two slices or slabs



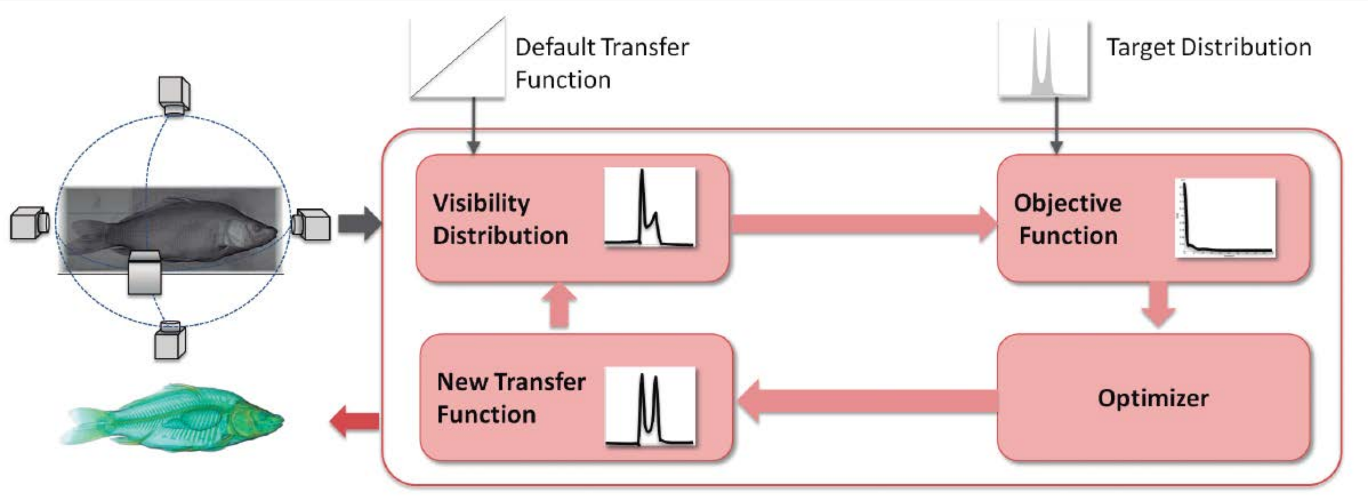
[Ruiz et al. 2008]

SIMILARITY-BASED EXPLODED VIEWS



[Ruiz et al. 2008]

TRANSFER FUNCTIONS FOR SCALAR FIELDS



[Ruiz et al. 2011]

TRANSFER FUNCTIONS FOR SCALAR FIELDS

- **Measuring visibility** $vis(b|v)$ **binned data-values *b* from viewpoint *v***
- **Constructing a communication channel between viewpoints and bins with transition probability matrix and marginal probabilities**

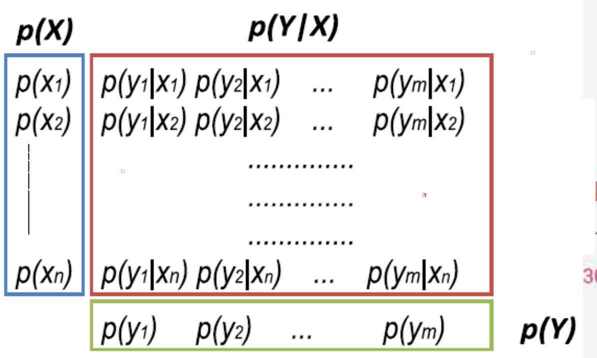
$$p(b|v) = vis(b|v)/vis(v)$$

$$vis(v) = \sum_{b \in \mathcal{B}} vis(b|v)$$

$$p(v) = vis(v)/\sum_{i \in \mathcal{V}} vis(i)$$

$$p(b) = \sum_{v \in \mathcal{V}} p(v)p(b|v)$$

- **Average projected**



[Ruiz et al. 2011]

TRANSFER FUNCTIONS FOR SCALAR FIELDS

- Target distribution *q(B)*:
  - Uniform for all data values
  - Proportional to bin size
  - Proportional to value
  - Proportional to gradient magnitude
  - Proportional to importance
- Informational divergence (KL-distance): quantifies the distance between true *p(B)* and target *q(B)* distributions

$$D_{KL}(p(B),q(B)) = \sum_{b \in \mathscr{B}} p(b) \log \frac{p(b)}{q(b)}$$

[Ruiz et al. 2011]

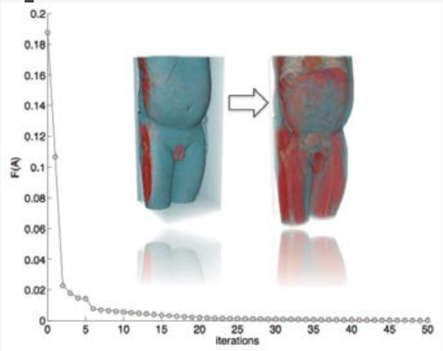
TRANSFER FUNCTIONS FOR SCALAR FIELDS

- **Iterative Transfer Function Design**

$$A = \{\alpha_1, \alpha_2, \ldots \alpha_n\}$$

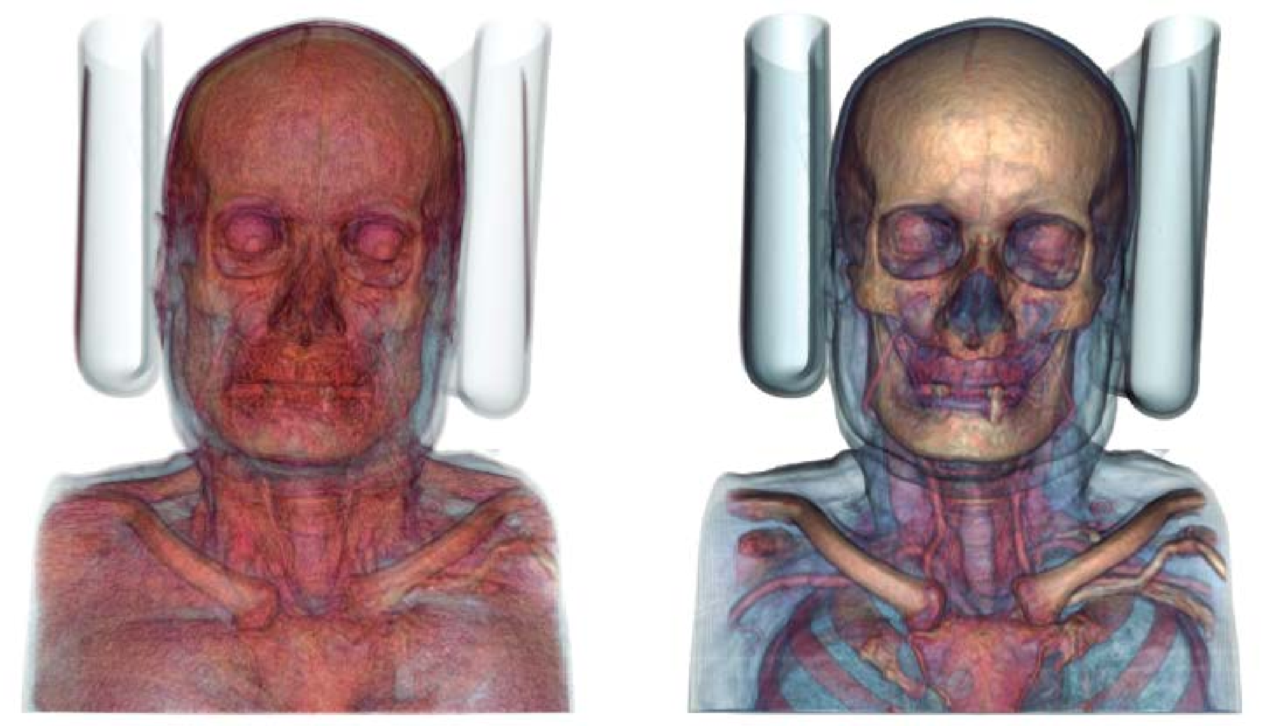
- **Minimize informational divergence between the average projected visibility distribution from all viewpoints and a target distribution**
- **Transforming the opacity vector**
- **Optimizer: Steepest Gradient Descent**

$$A_{t+1} = A_t - \nabla D_{KL}(p||q)$$



[Ruiz et al. 2011]

TRANSFER FUNCTIONS FOR SCALAR FIELDS



[Ruiz et al. 2011]

TRANSFER FUNCTIONS FOR SCALAR FIELDS

[Ruiz et al. 2011]

MULTIMODAL DATA FUSION

- **Fuse two modalities based on how much information a data bin is associated with**

$$I(f) = -log_2(P(f))$$

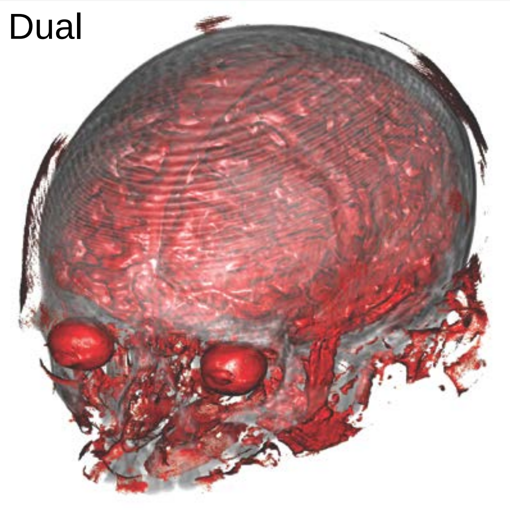
$$\gamma(f_1, f_2) = \frac{I(f_2)}{I(f_1) + I(f_2)}$$

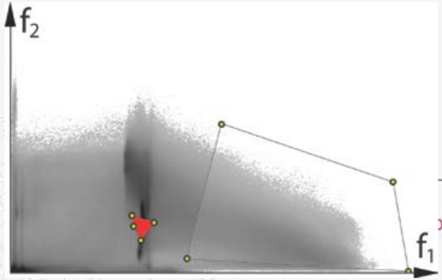
$$f_{fused} = (1-\gamma) * f_1 + \gamma * f_2$$

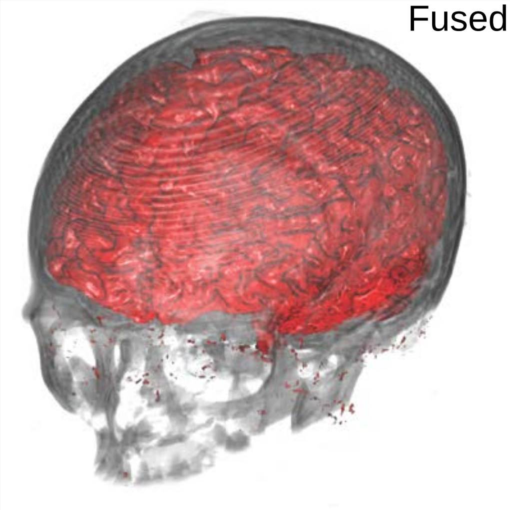
$$\nabla f_{fused} = (1-\gamma) * \nabla f_1 + \gamma * \nabla f_2$$

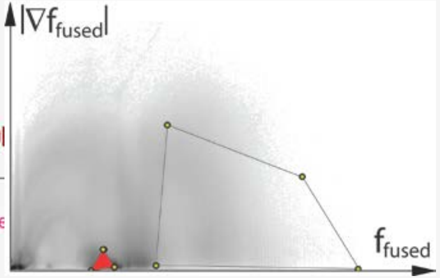
[Haidacher et al. 2008]

MULTIMODAL DATA FUSION





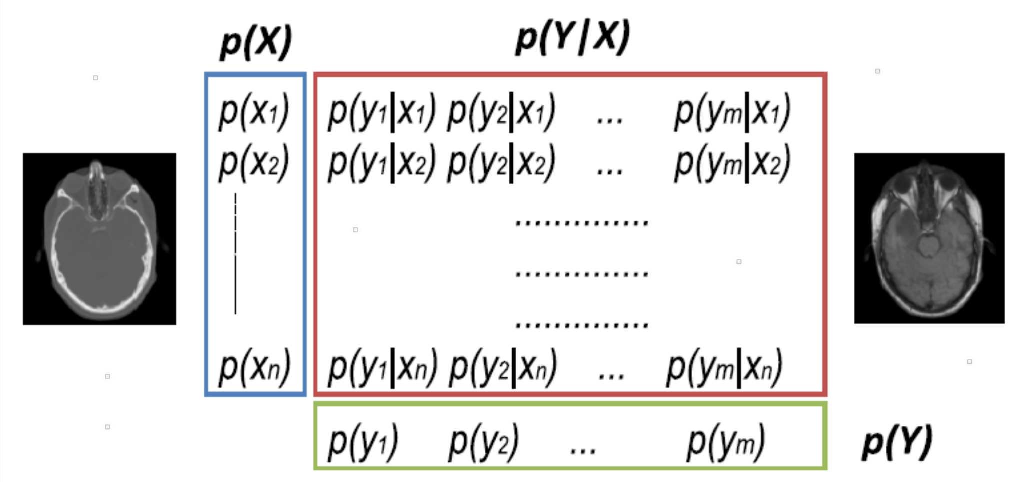




[Haidacher et al. 2008]

MULTIMODAL DATA FUSION

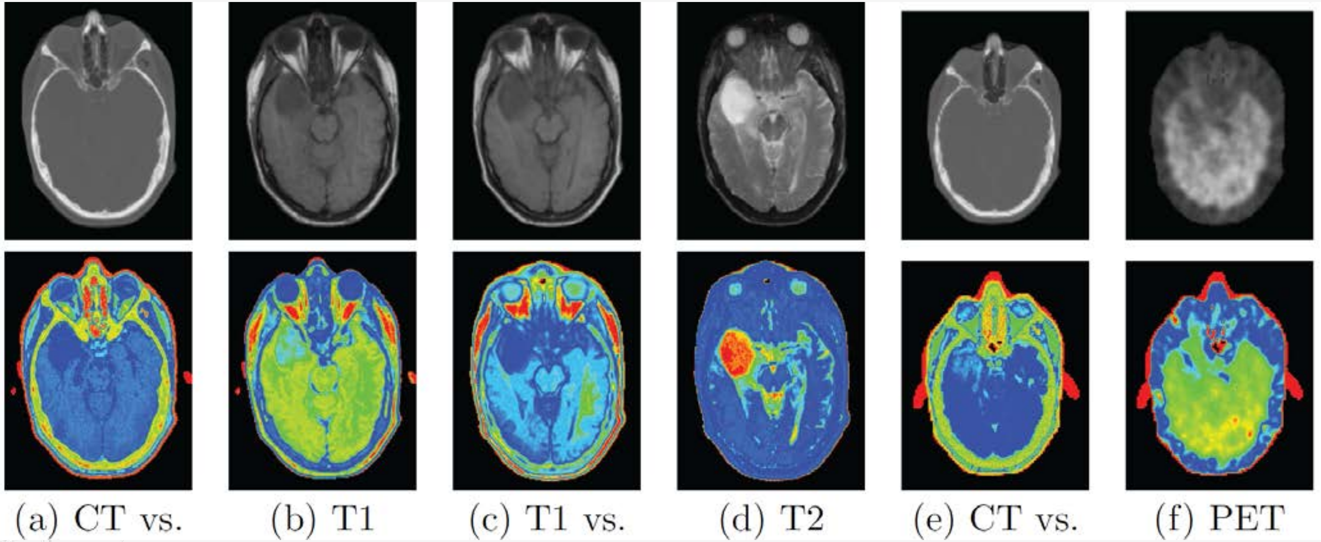
- Fuse two modalities by a winner



[Bramon et al. 2012]

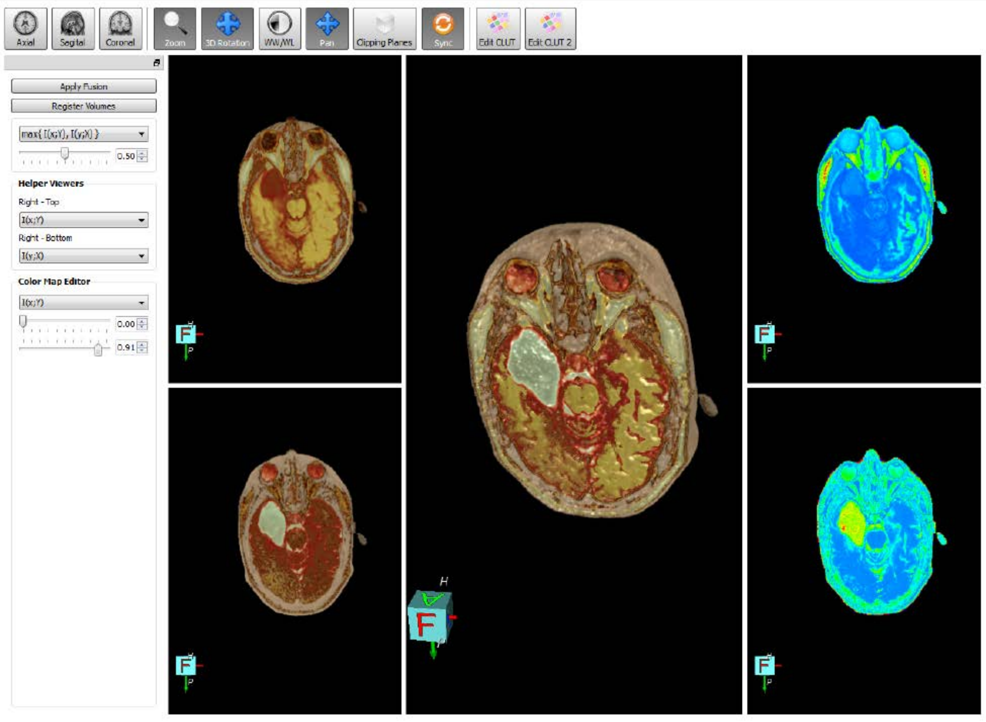
MUTUAL INFORMATION DECOMPOSITIONS

- **Predictability, surprise, entanglement**



[Bramon et al. 2012]

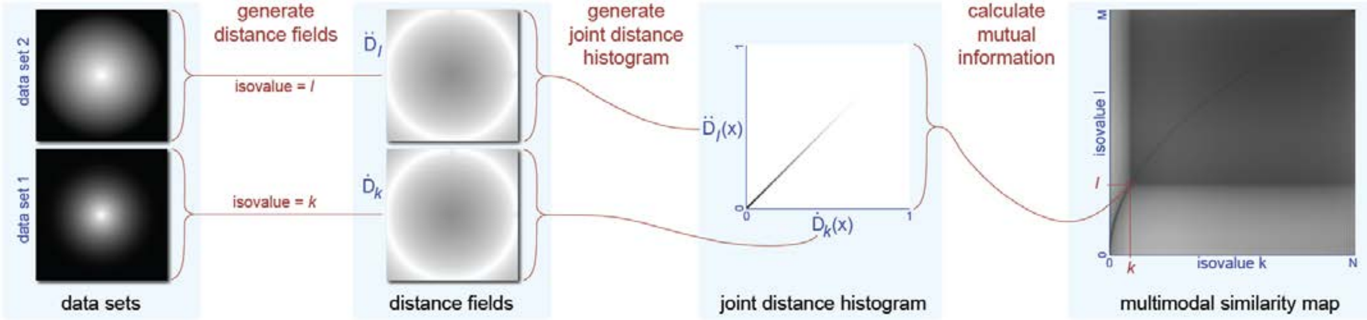
MULTIMODAL VISUAL FUSION



[Bramon et al. 2012]

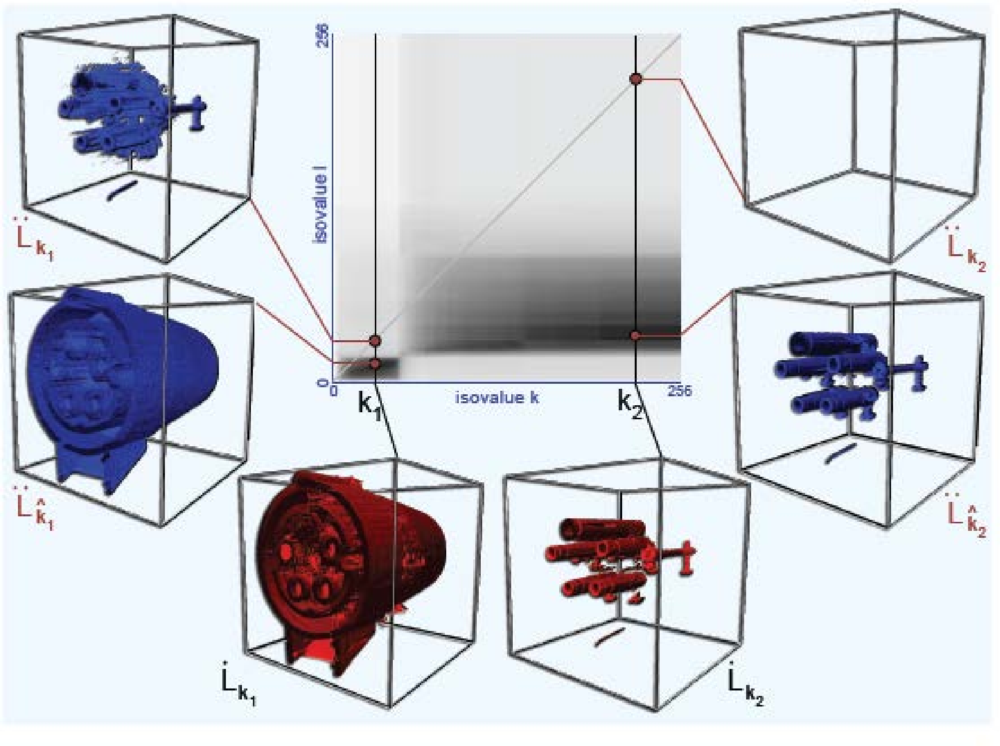
MULTIMODAL SIMILARITY MATRIX

- **Isosurface Similarity Maps extended to support Multi-Modal Data**



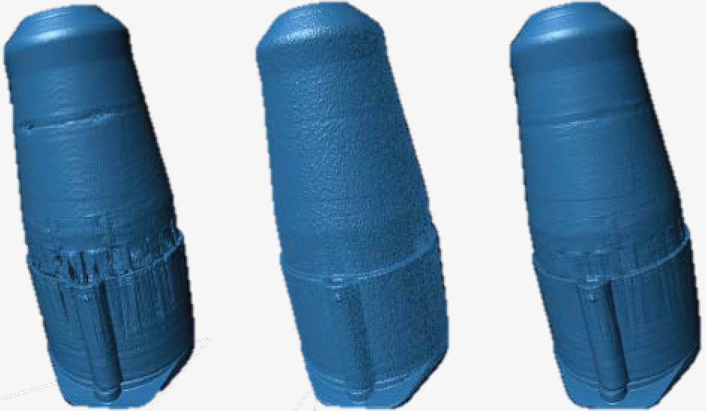
[Haidacher et al. 2011]

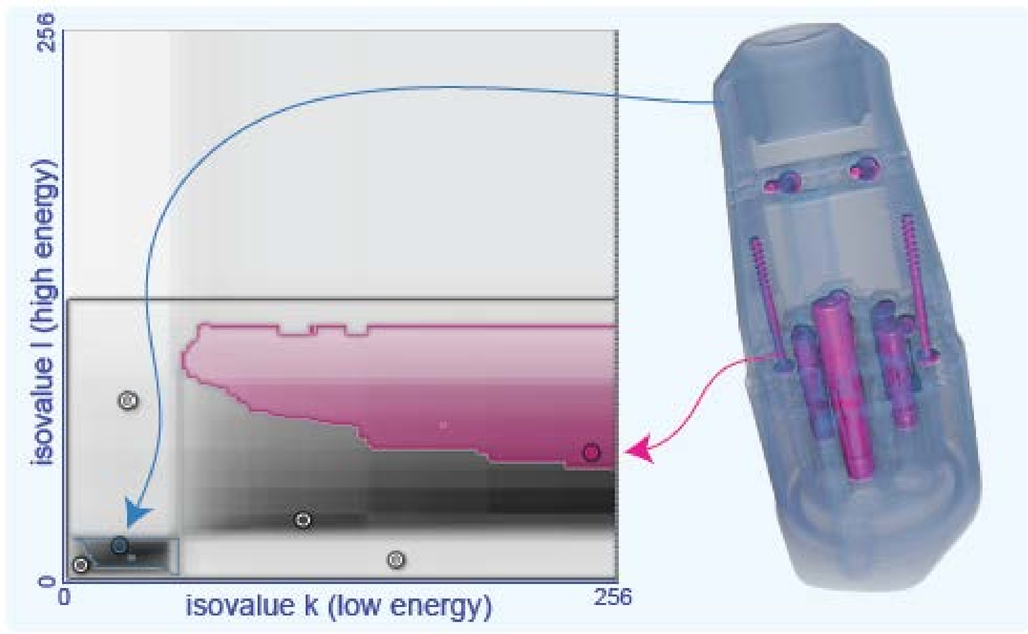
MULTIMODAL SIMILARITY MATRIX



[Haidacher et al. 2011]

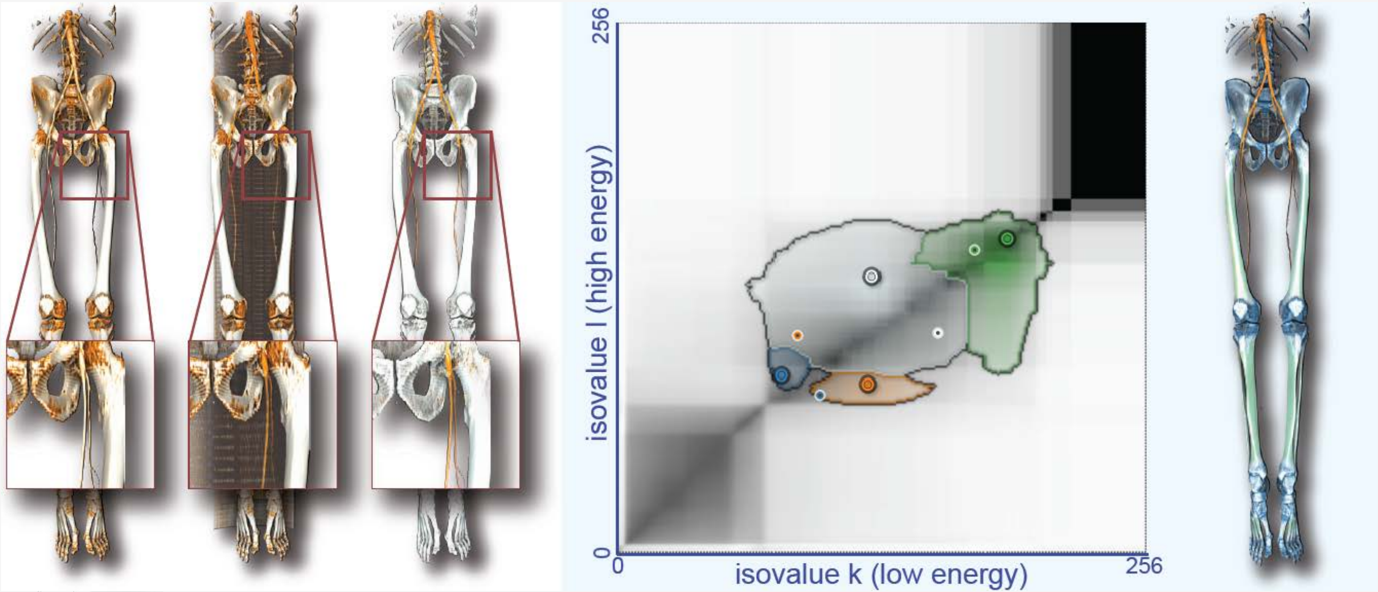
MULTIMODAL SIMILARITY MATRIX





[Haidacher et al. 2011]

MULTIMODAL SIMILARITY MATRIX



[Haidacher et al. 2011]

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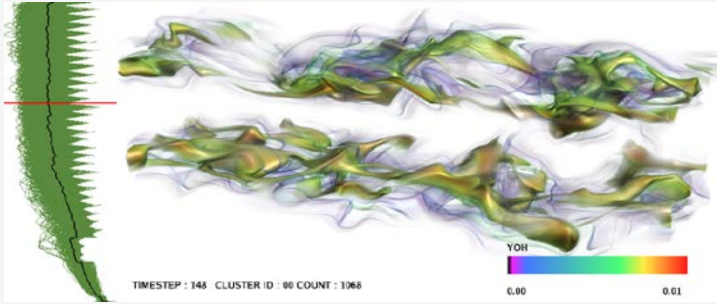
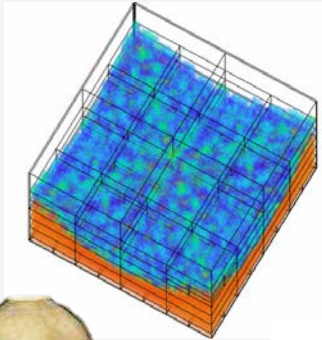
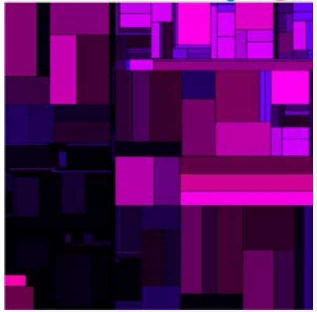
- [Haidacher et al. 2011] M. Haidacher, S. Bruckner, M. E. Gröller: **Volume Analysis Using Multimodal Surface Similarity**, *IEEE TVCG (Proceedings of IEEE Visualization)*, 2011

THANK YOU







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Flow Visualization

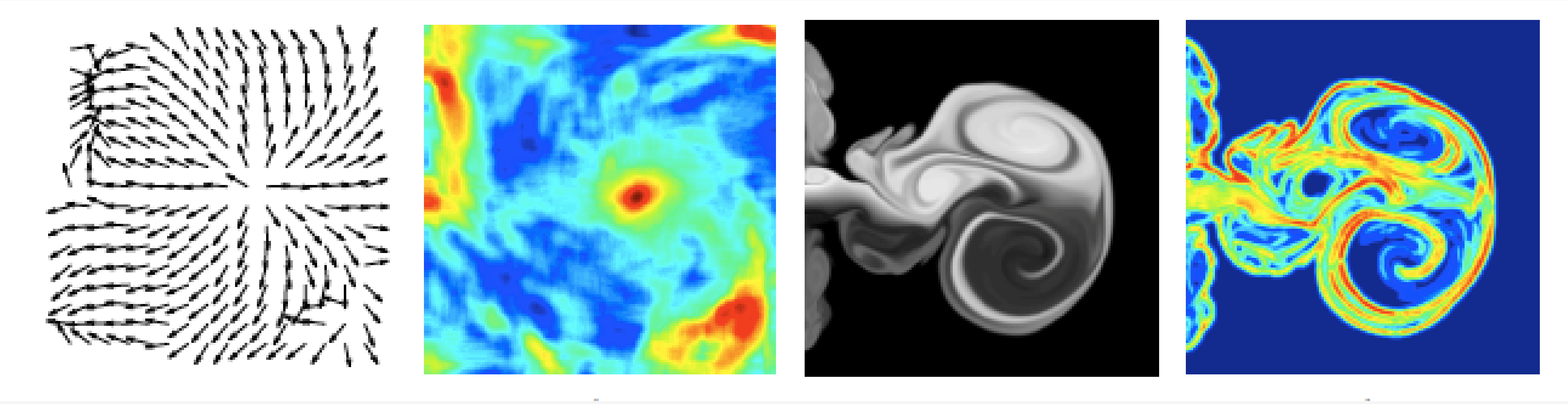
Tutorial on Information Theory in Visualization

Han-Wei Shen

The Ohio State University

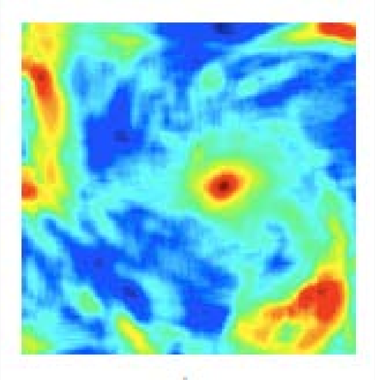
Background

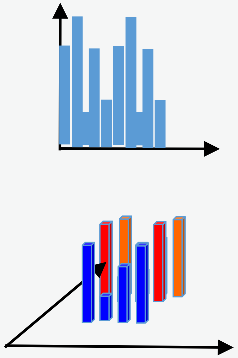
- A data set can be considered as a random variable
- Each data point can be considered as an outcome of the random variable
- We can estimate the information content of the whole data set or of local regions



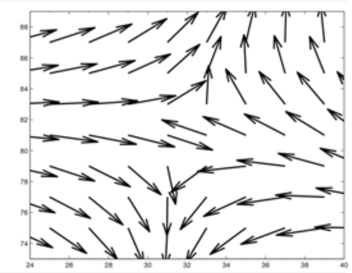
Distributions from Scientific Data

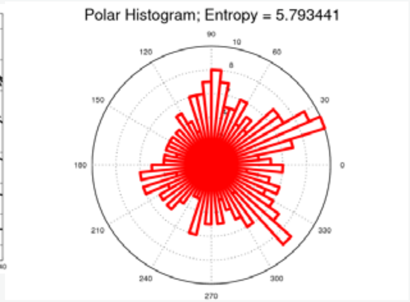
Scalar Distributions



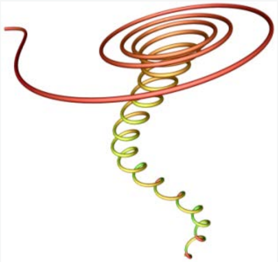


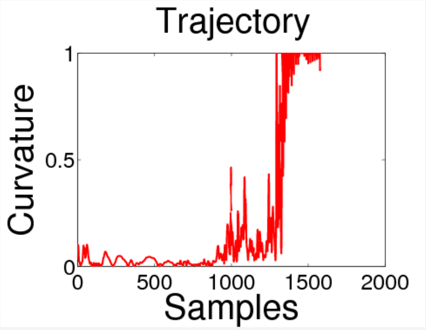
Vector Distributions



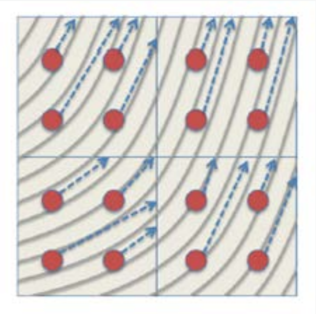


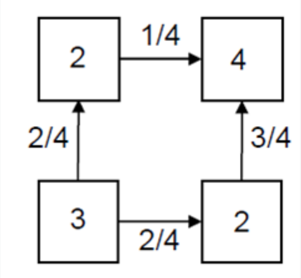
Feature Distributions





State Transitions





Data Sets with Multiple Variables

- Assuming your data set contains two variables X and Y
- You want to know the relationship between X and Y
- You can calculate the conditional entropy, mutual information, etc between these two variables
- Some of the metrics can be used as the ‘information distance’ between two variables

Entropy for Multiple Variables

- Joint Entropy

$$H(X,Y) = -\sum_{x\in\mathcal{X}}\sum_{y\in\mathcal{Y}} p(x,y)\log p(x,y)$$

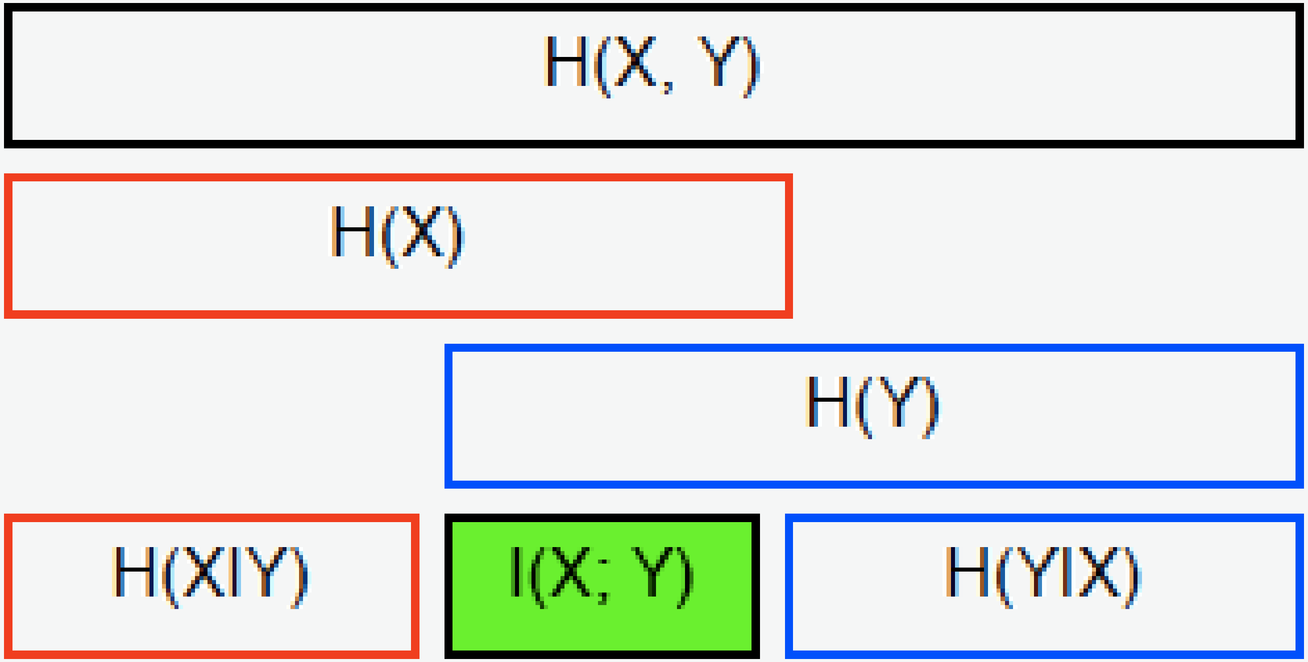
- Conditional Entropy

$$H(X|Y) = \sum_{y\in\mathcal{Y}} p(y)H(X|Y=y) = -\sum_{y\in\mathcal{Y}}\sum_{x\in\mathcal{X}} p(x,y)\log p(x|y)$$

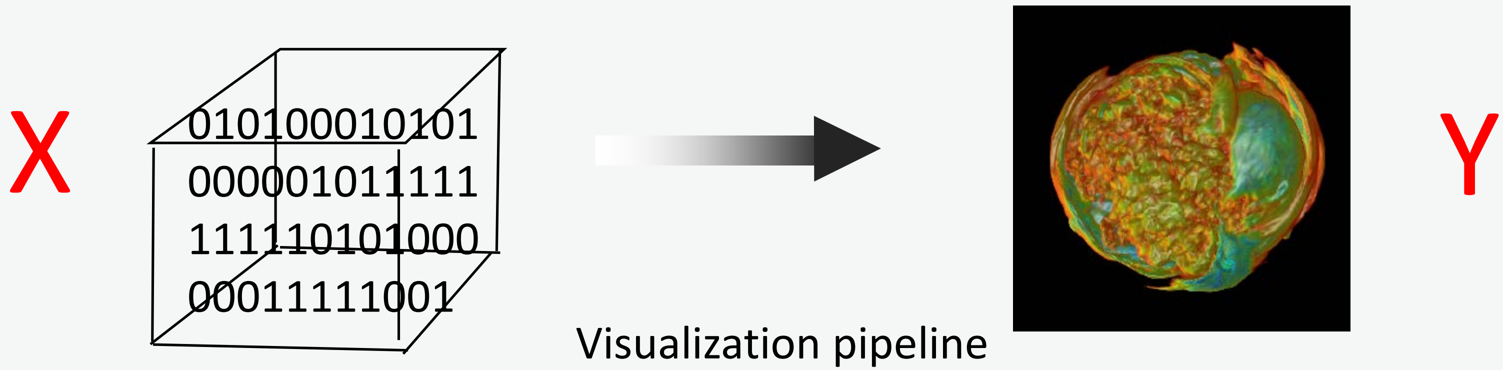
- Mutual Information

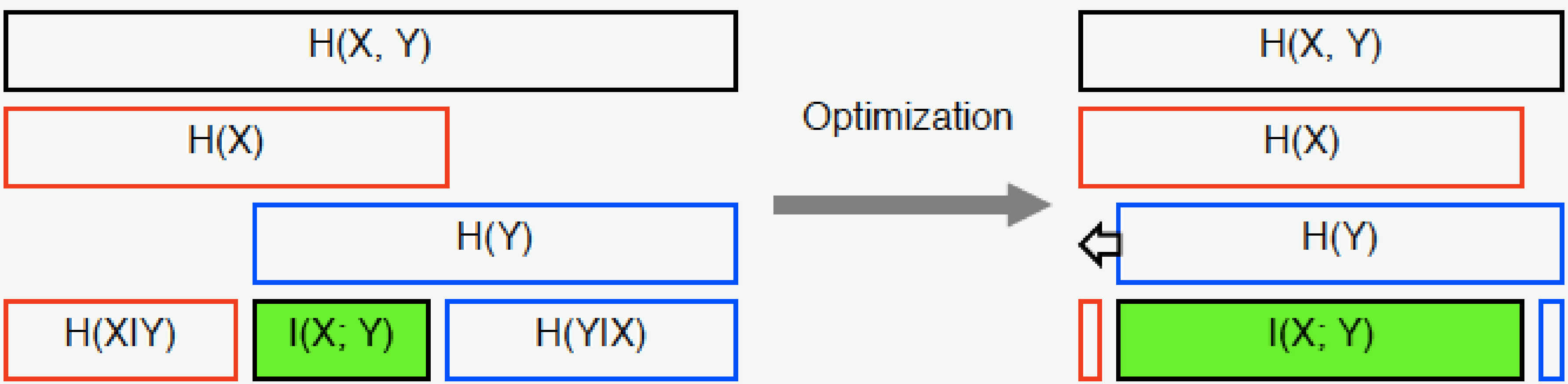
$$I(X;Y) = H(X) + H(Y) - H(X,Y) = H(X) - H(X|Y) = H(Y) - H(Y|X)$$

Relations of Entropy Measures



Evaluating Visualization



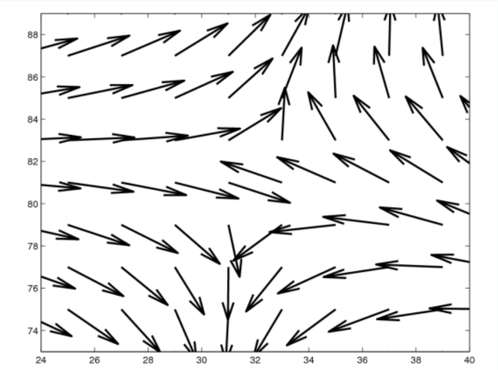


$$H(x) = -\sum_{i=1}^{n} p_i \log p_i$$

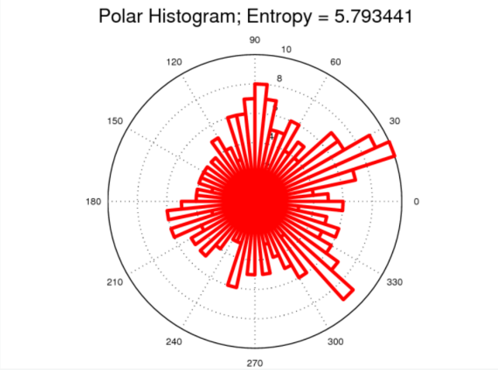
$$I(X;Y) = H(X) + H(Y) - H(X,Y) = H(X) - H(X|Y) = H(Y) - H(Y|X)$$

Vector Field Analysis

- Concept
  - Treat the vector field as a data source that generates vector orientation as outcome
  - The more diversified are the vector orientations, the more information is contained in the vector field
- Measurement
  - Estimate the distribution of the vector orientation
  - Compute the entropy of this distribution as the measurement

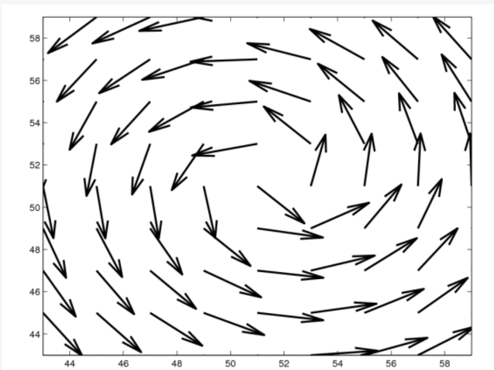


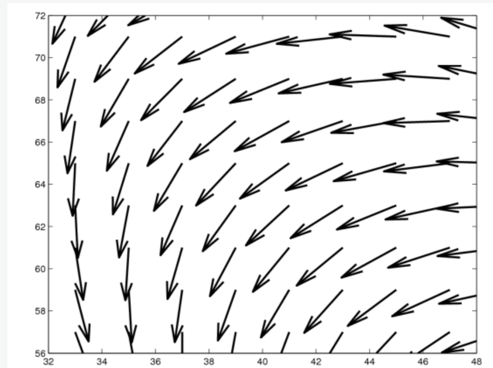
Vector field

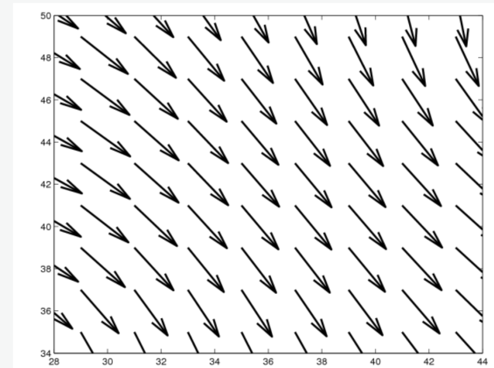


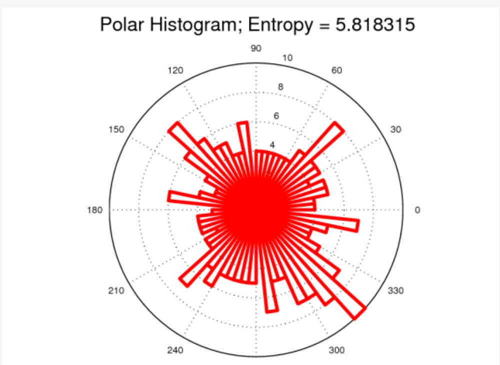
Polar Histogram

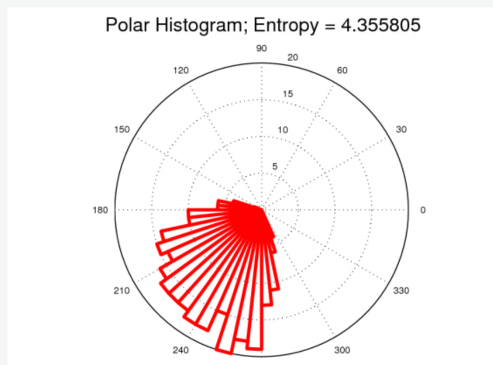
Information in Vector Fields

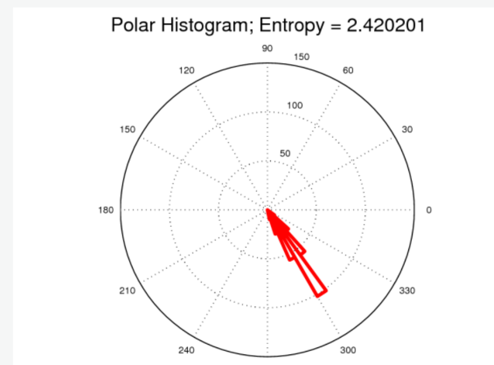






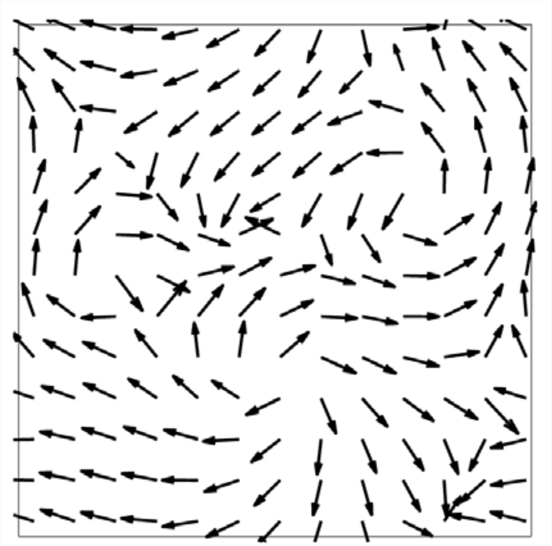




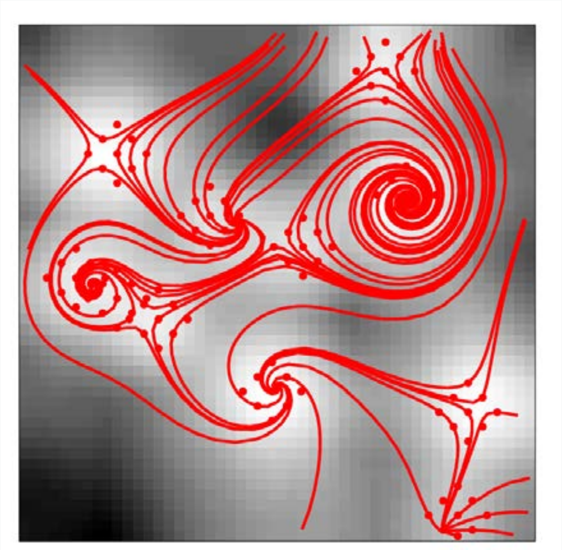


Entropy Field and Seeding

Measure the entropy around each point's neighborhood



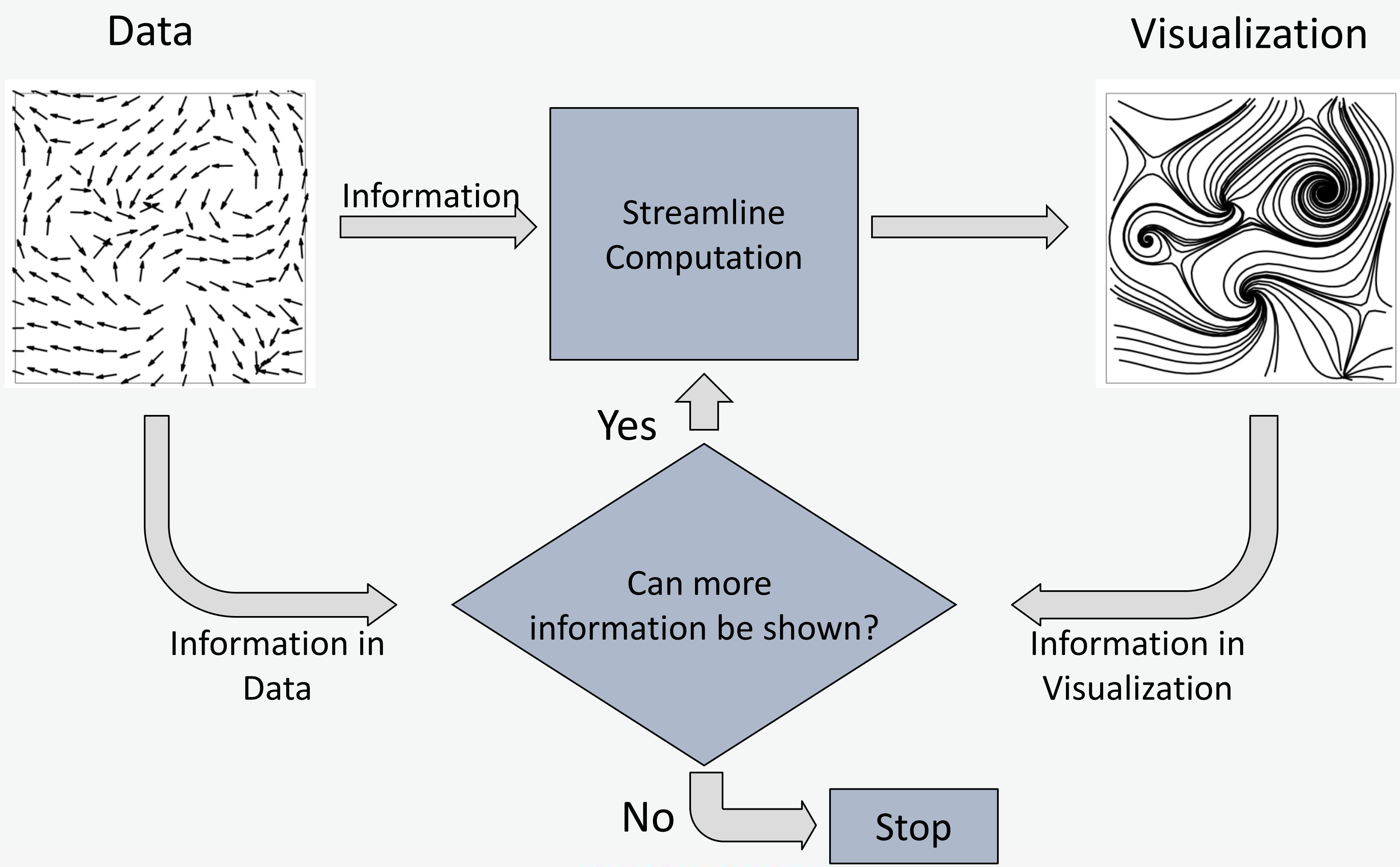
Vector Field



Entropy field: higher value means more information in the corresponding region

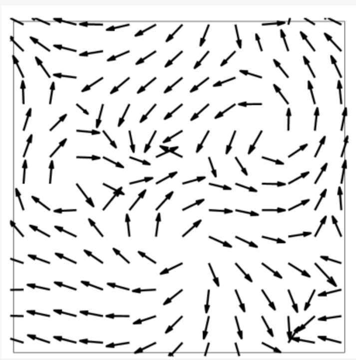
Entropy-based seeding: Places streamlines on the region with high entropy

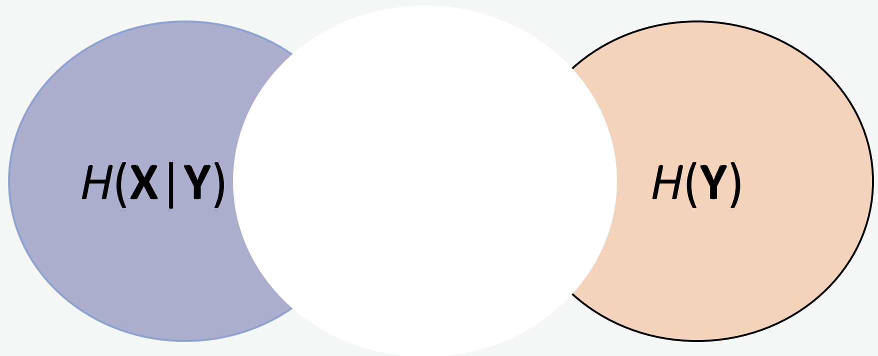
Evaluation of Visualization



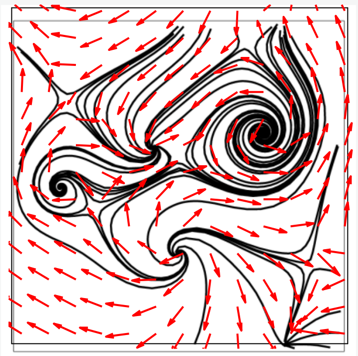
Information Comparison Between Data/Visualization

Vector Field **X**





Streamlines **Y**

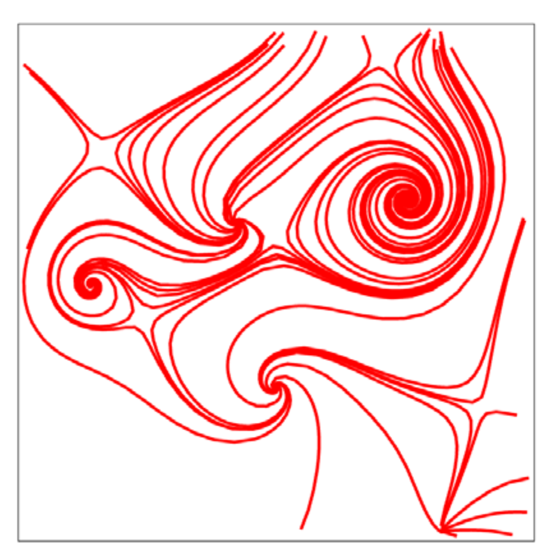


Conditional entropy $H(\mathbf{X}|\mathbf{Y})$:
The information in **X** not represented by **Y**

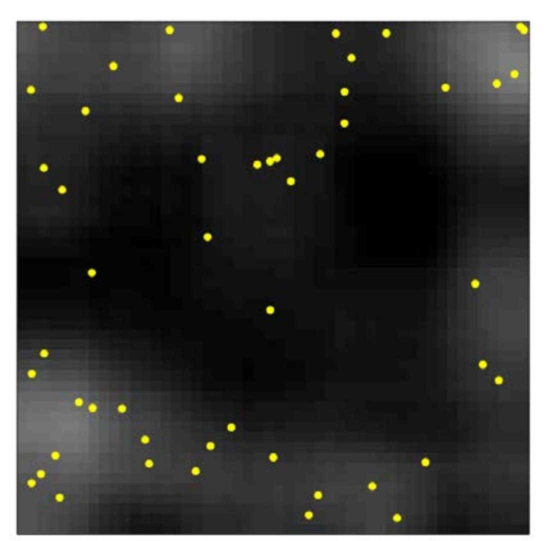
An effective visualization should represent most information in the data, i.e. $H(\mathbf{X}|\mathbf{Y})$ should be small

Conditional Entropy Field and Seeding

Measure the under-represented information in local regions



Streamlines

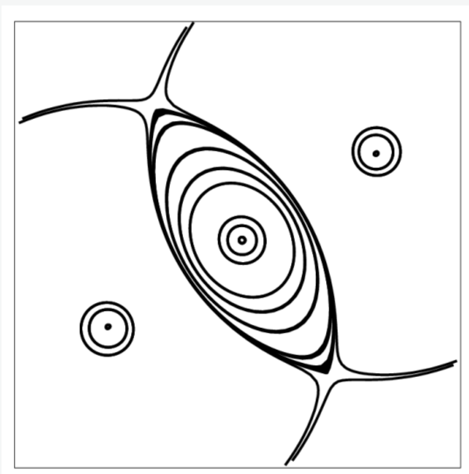


Conditional entropy field

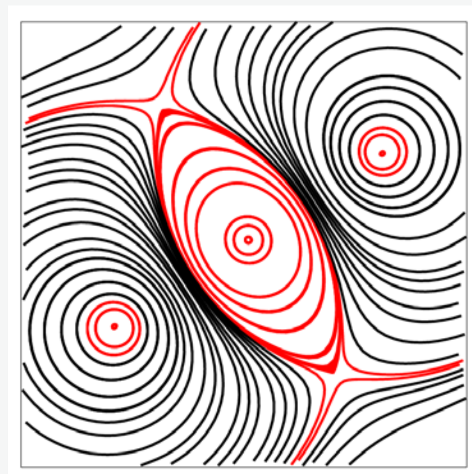
Conditional-entropy-based seeding: Place more seeds on regions with higher under-represented information

Result

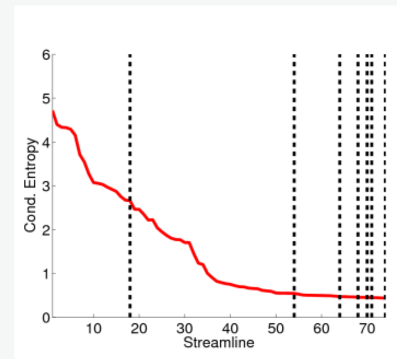
1st iteration: Entropy-based seeding



2nd iteration: Cond.-entropy-based seeding



Conditional entropy



When conditional entropy converges

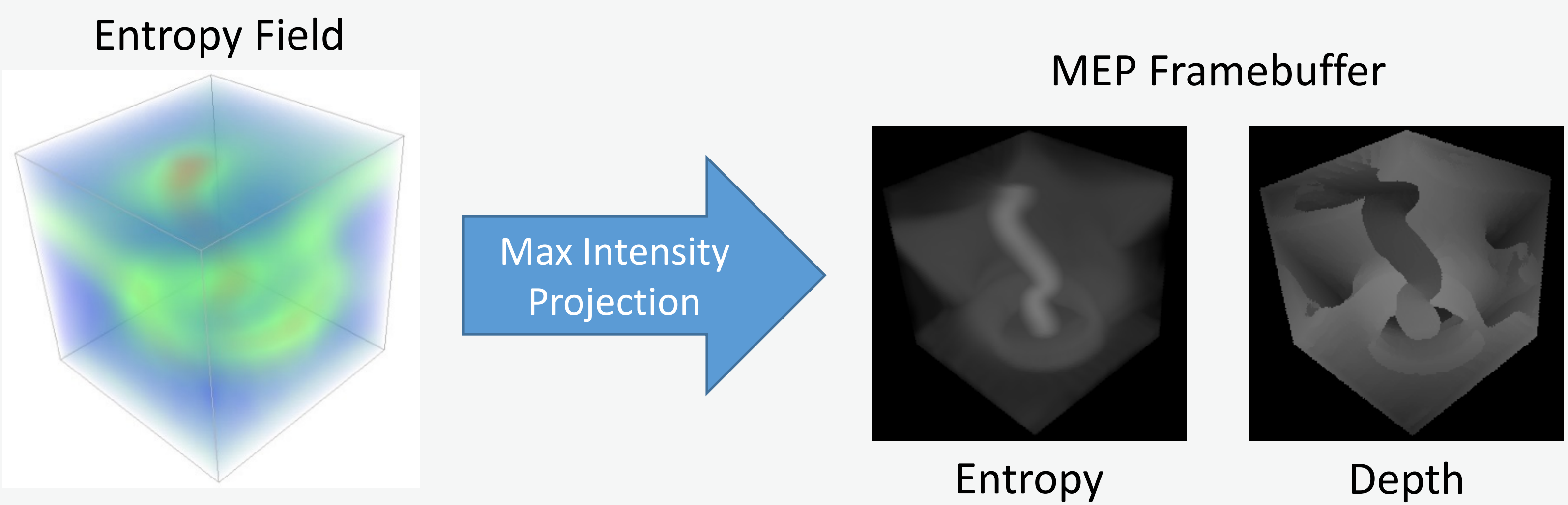
View-dependent Flow Visualization

- Goal: create a clear view of important features in 3D flow fields
- Issue: occlusion of the flow features
- Approaches
  - Evaluate flow field in screen space with information theory
  - Place streamline to highlight salient flow features with reduced occlusion

Maximal Entropy Projection (MEP)

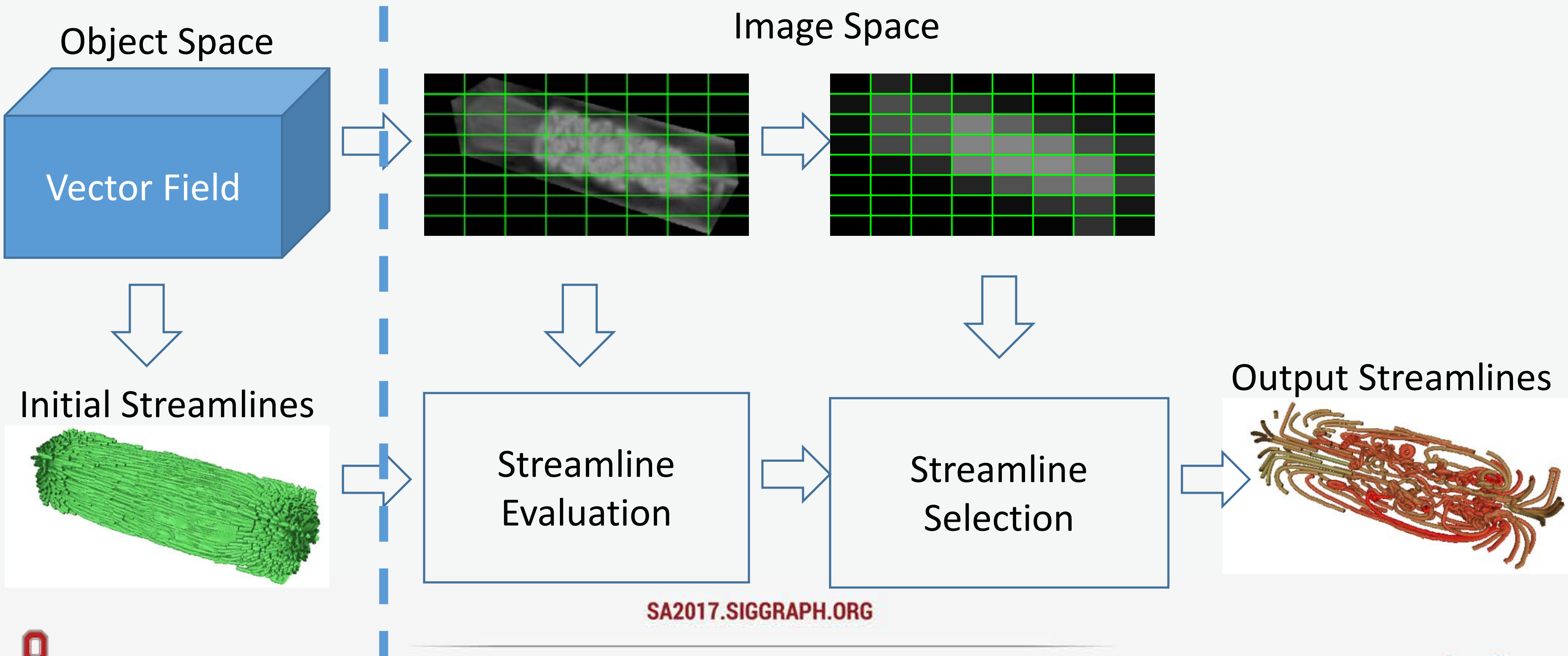
MEP: Project the entropy field to the screen via Maximal Intensity Projection (MIP)

- Measure the maximal entropy visible to each pixel
- Store the sampled entropy and depth in the MEP Framebuffer



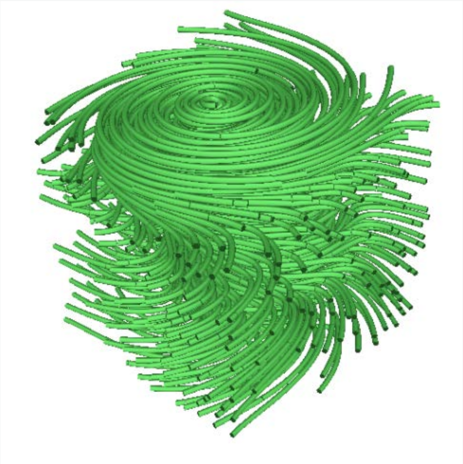
MEP-based Streamline Placement

- Highlight salient flow features
- Reduce occlusion to these features

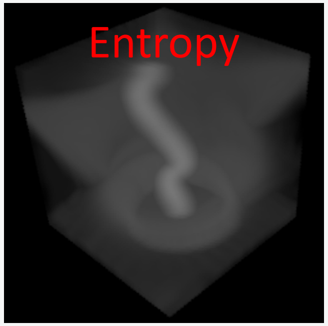


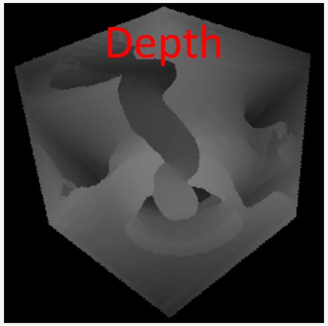
Streamline Evaluation

Input Streamlines



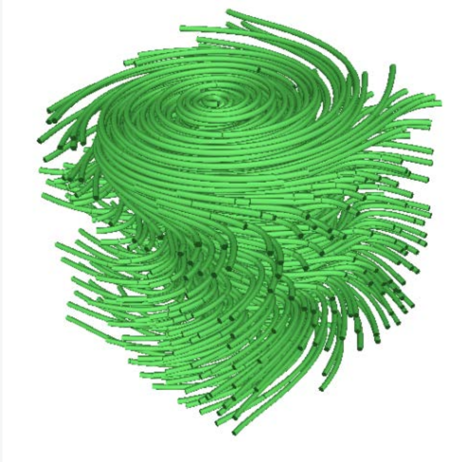
MEP Framebuffer

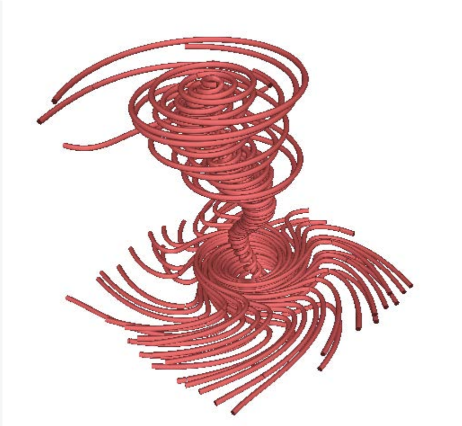




Streamline Evaluation

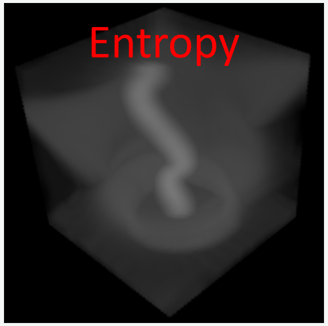
Input Streamlines

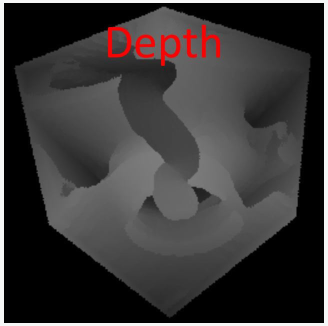




Streamlines w/ less occlusion to the MEP Framebuffer

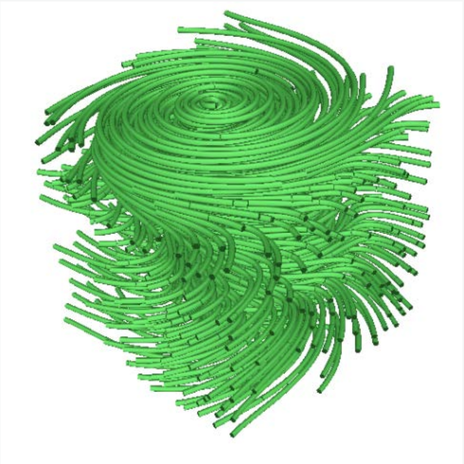
MEP Framebuffer



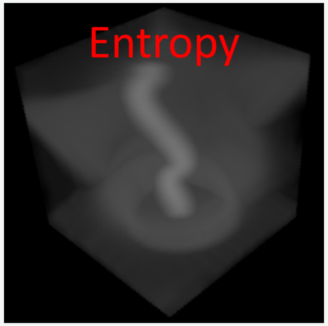


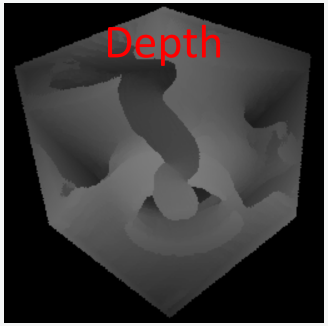
Streamline Evaluation

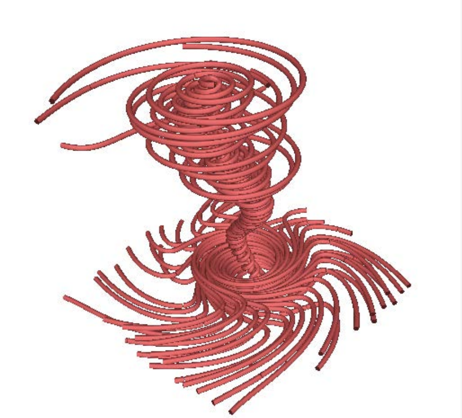
Input Streamlines



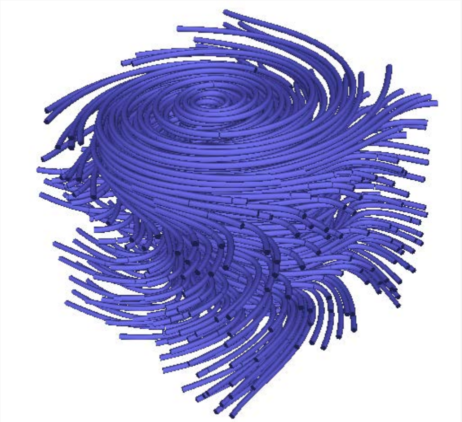
MEP Framebuffer





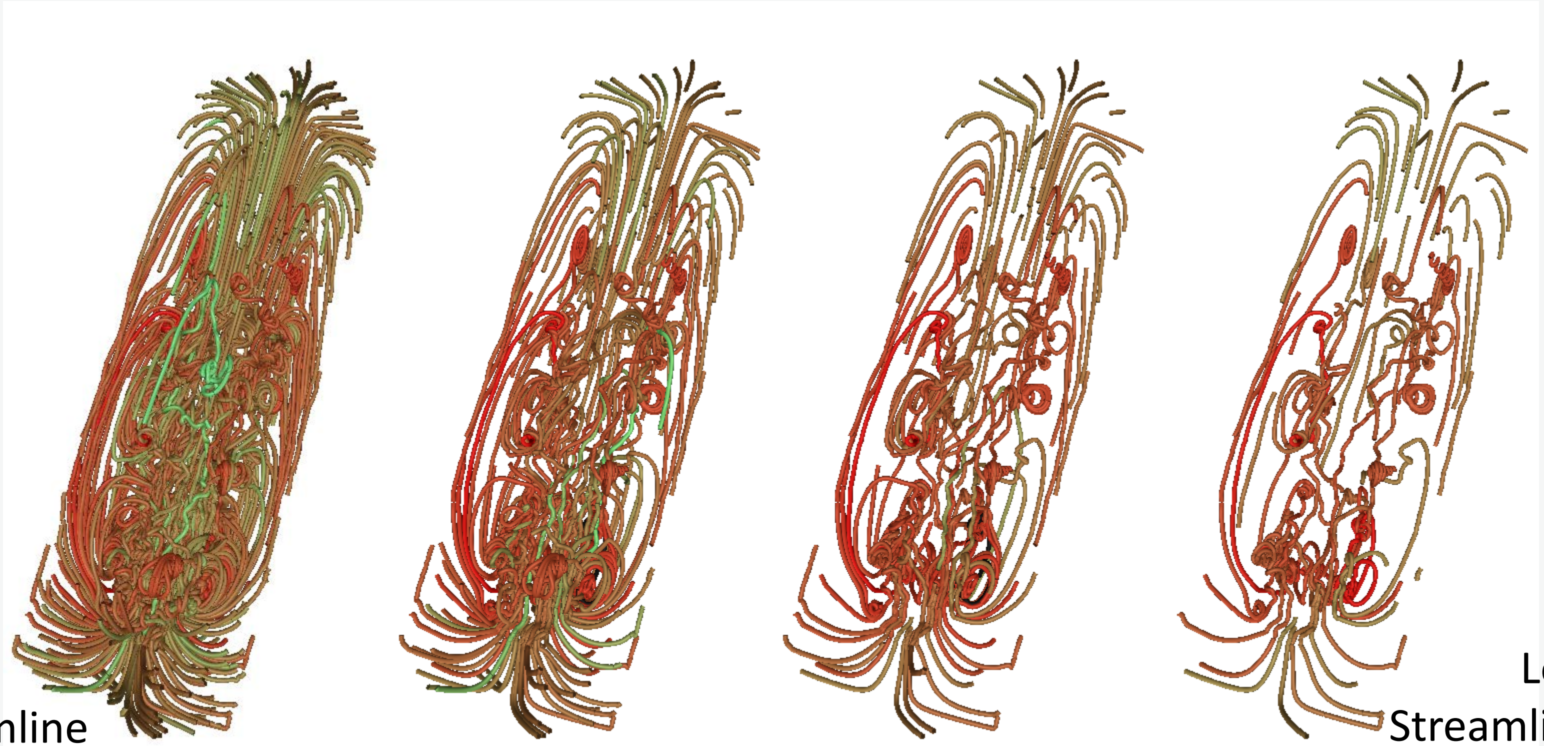


Streamlines w/ less occlusion to the MEP Framebuffer



Streamlines that occluded the MEP Framebuffer

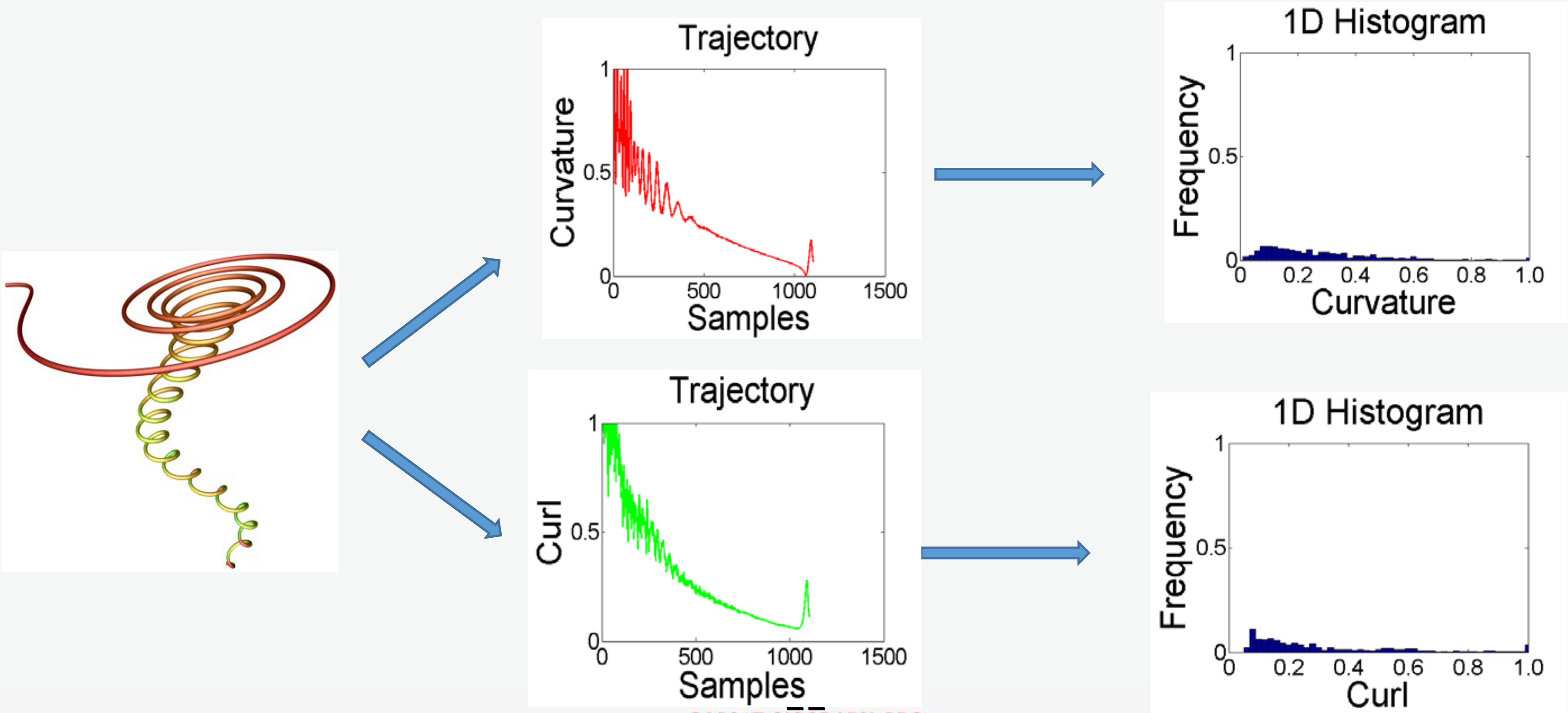
MEP-based Streamline Placement



High Streamline Density → Low Streamline Density

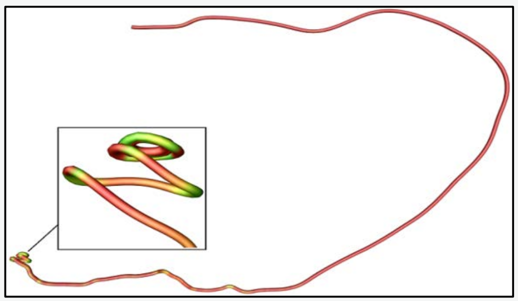
Streamline Statistical Feature Descriptors

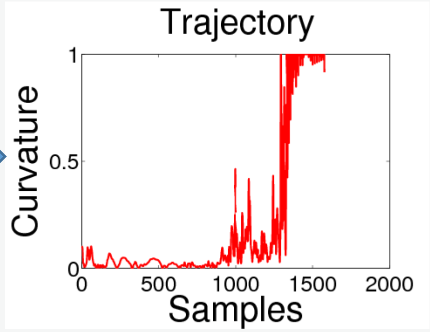
- Each streamline is represented as one or more distributions of feature measures such as curvature, curl and torsion

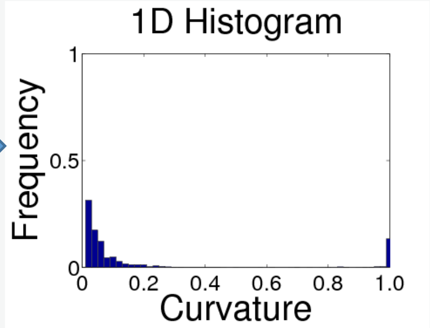


Streamline Statistical Feature Descriptors

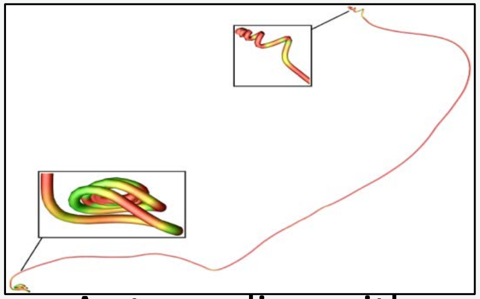
- Problem of 1D histograms
  - The order of features is not preserved in the final histogram

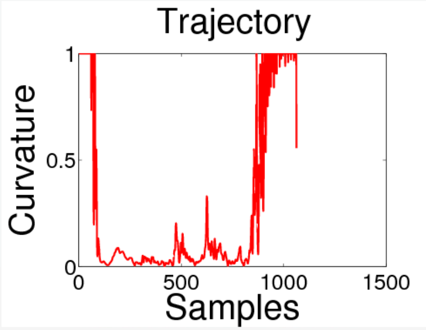


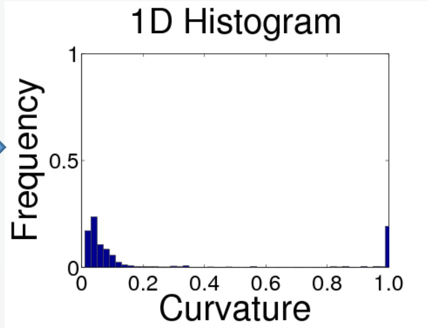




A streamline with
only one high curvature zone



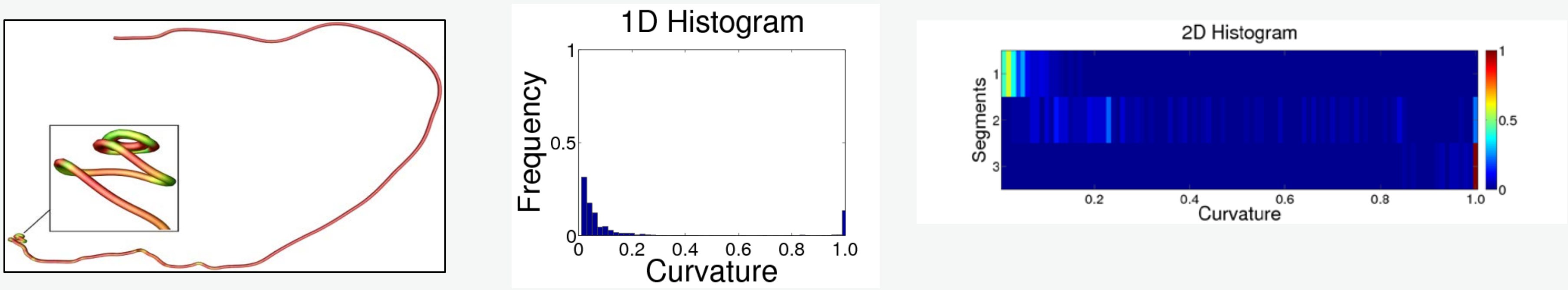


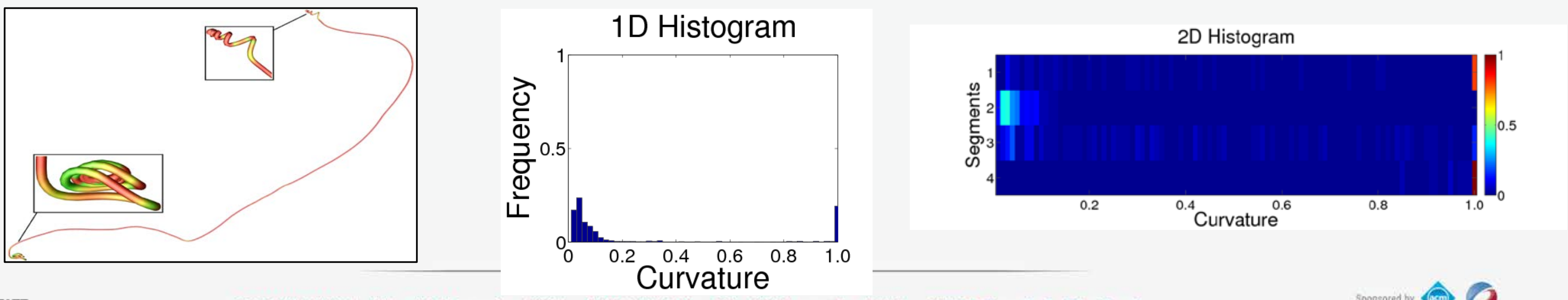


A streamline with
two high curvature zone

Streamline Statistical Feature Descriptors

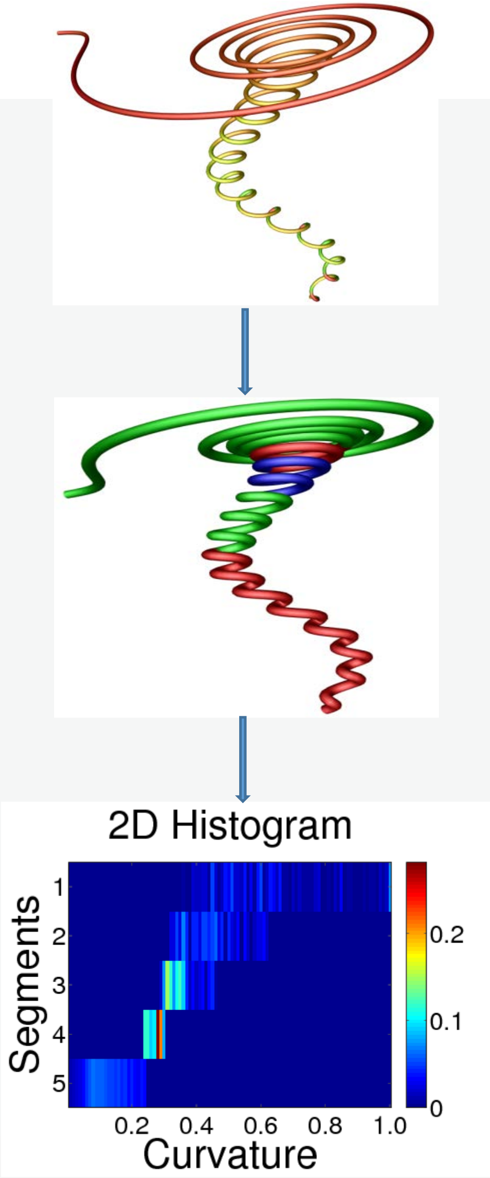
- Solution: 2D Histograms
  - Decompose the streamline into a fixed number of segments
  - Create 1D histogram of appropriate quantity for each segment
  - Stack the 1D histograms to form a 2D histogram which preserve the order between segments





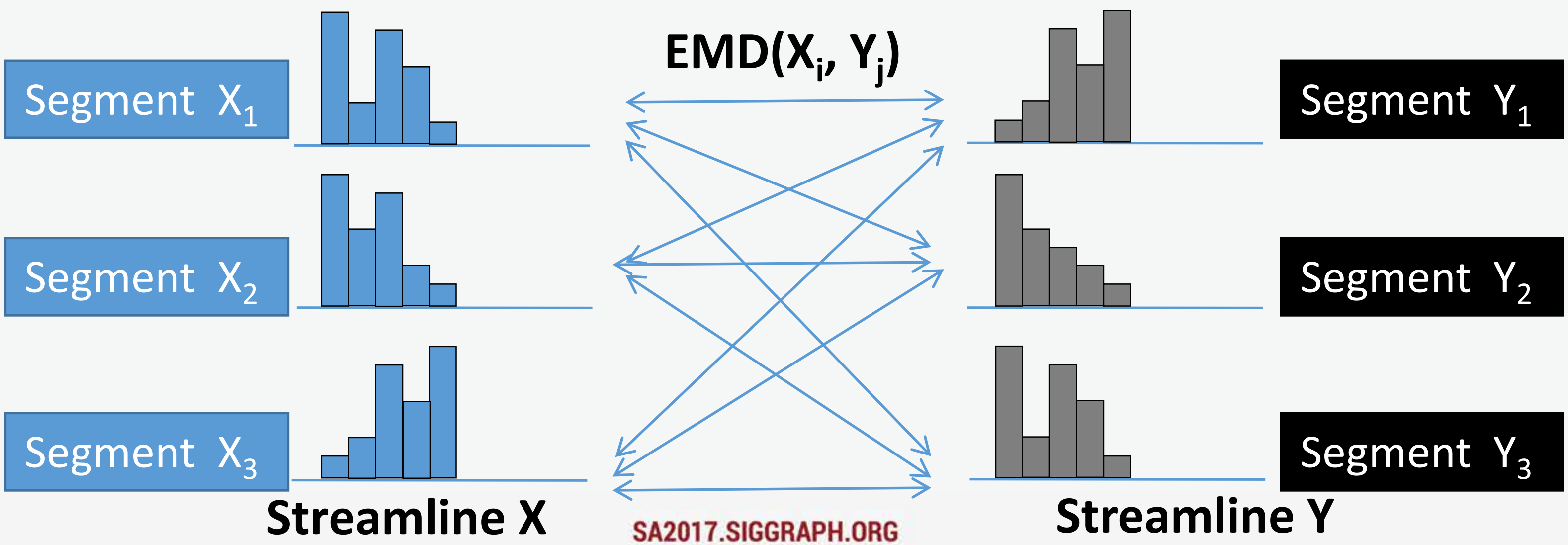
Streamline Decomposition

- An iterative segmentation algorithm
- Recursively divide into segments until:
  - ➢The difference in the 1D histograms between two halves is smaller than a threshold
  - ➢Streamline segment is too short to be further segmented



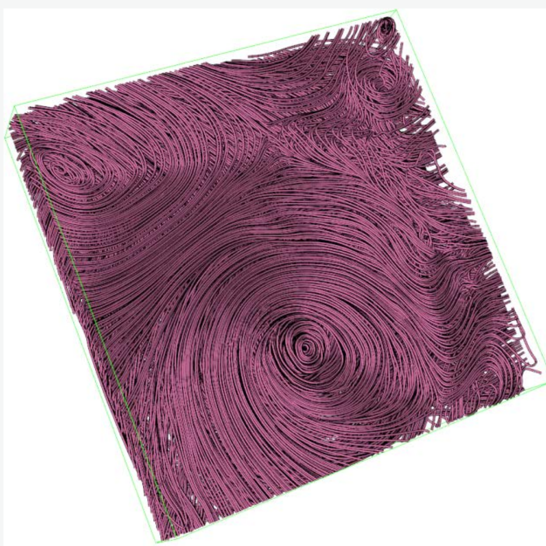
Measure Similarity Between Two Streamlines

- Compute similarity between the 2D histograms of two streamlines
  - As two streamline have different number of segments,
    - Apply **Dynamic Time Warping (DTW)** to find an optimal mapping between segments
  - For each pair of segments,
    - Use **Earth Mover's Distance** to measure the distance of their 1D histograms

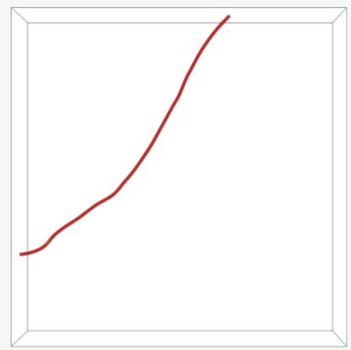


Similarity-based Streamline Query
(Hurricane Isabel Data Set)

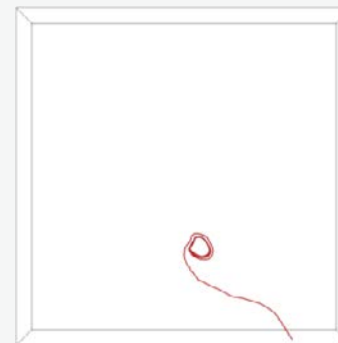
- Streamlines having similar features as the one selected by the user are displayed to highlight features in the data
- Histograms based on Curvature and Torsion are used to answer query in this particular case



Hurricane Isabel

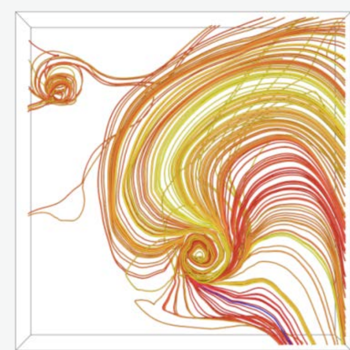


User selected target



User selected target

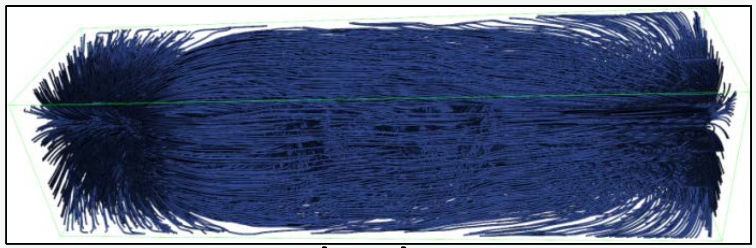
Top 400 matches



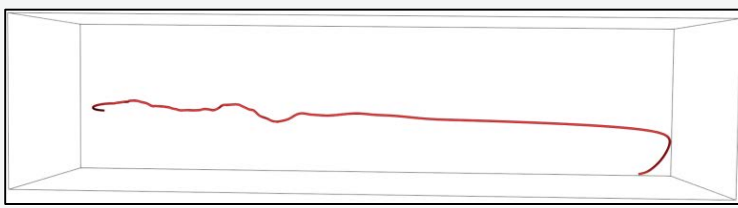
Top 200 matches

Similarity-based Streamline Query
(Solar Plume Data Set)

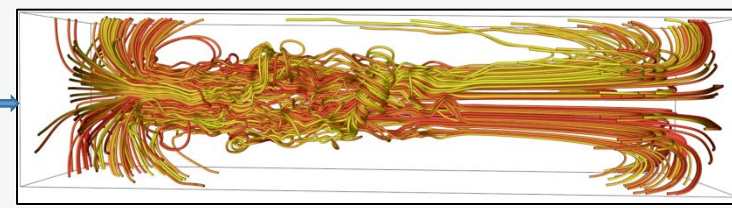
- Query response using curvature and torsion based histograms



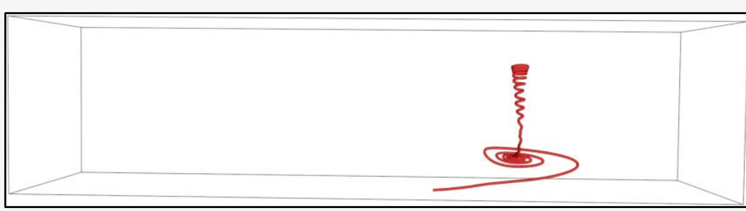
Solar Plume



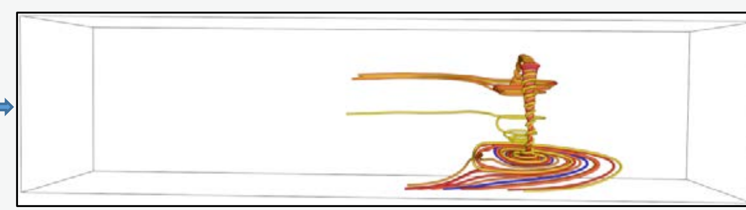
User selected streamline



Top 200 matches



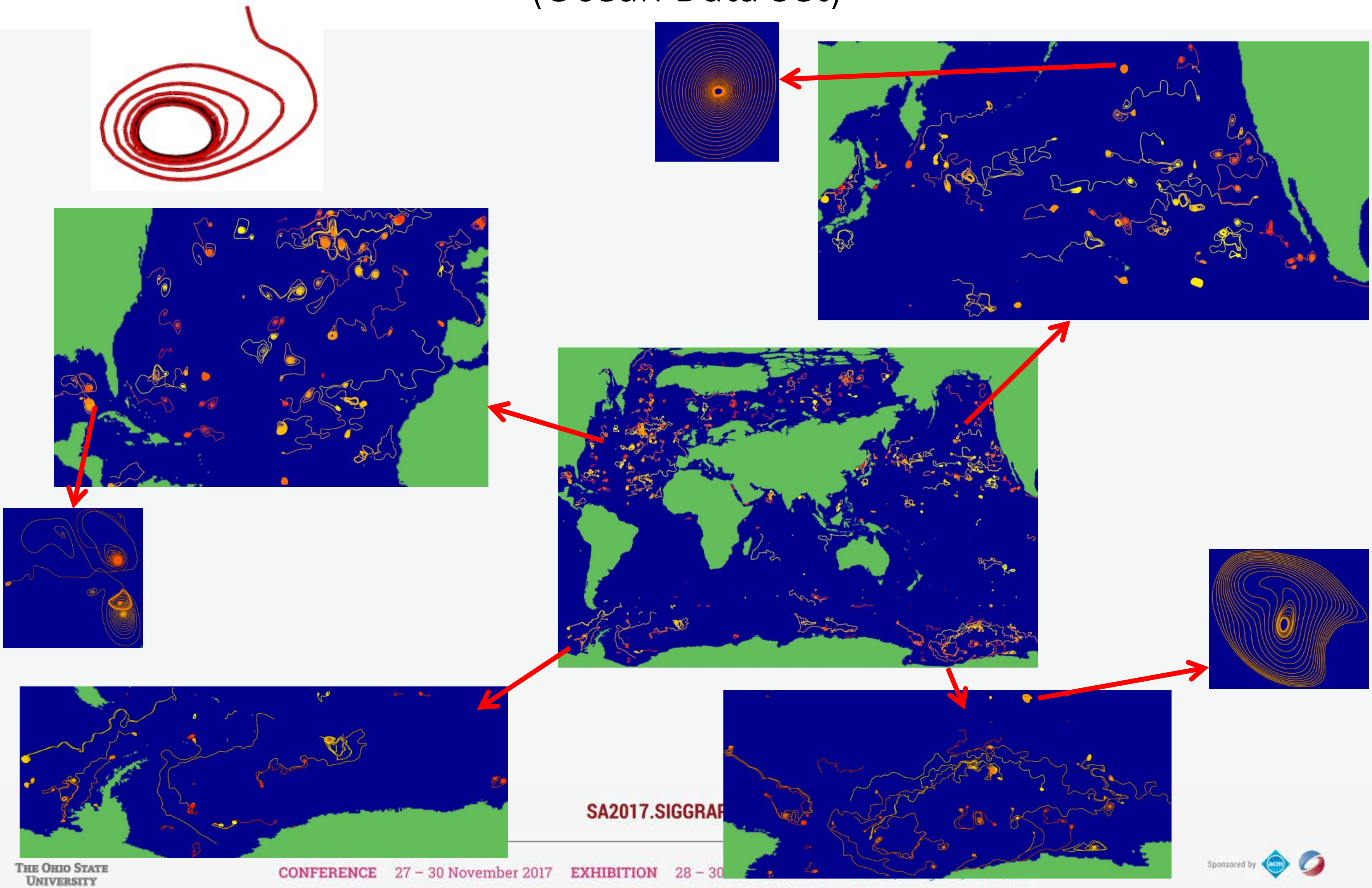
User selected streamline



Top 20 matches

Similarity-based Streamline Query

(Ocean Data Set)

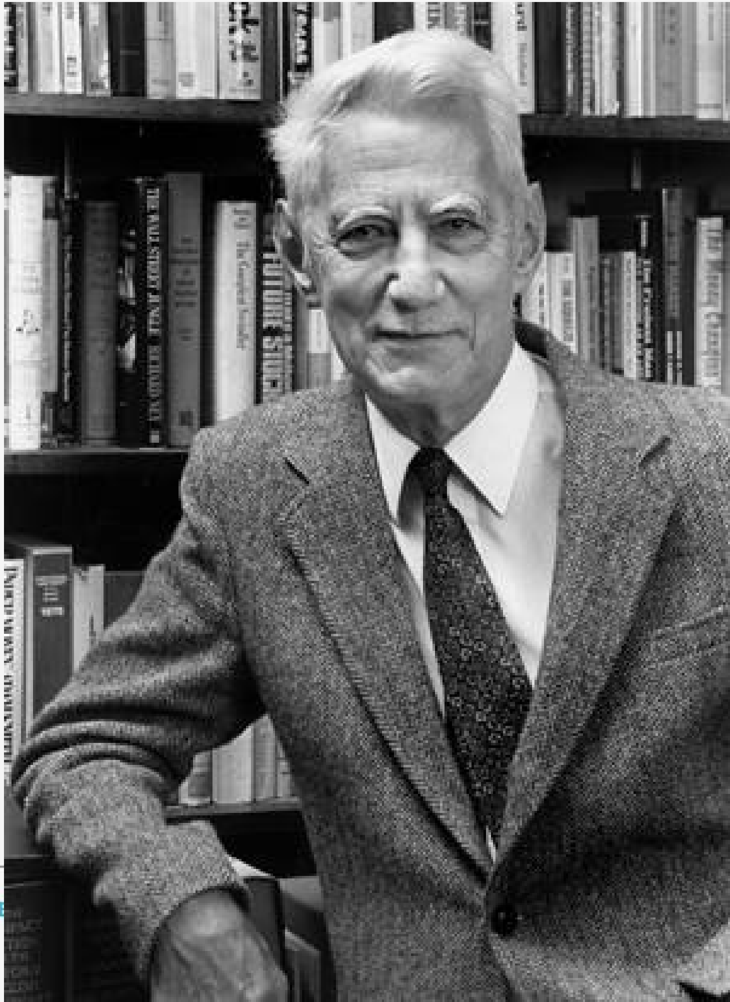


Visualization & Information Theory

author: Min Chen, University of Oxford
presenter: Mateu Sbert, University of Girona, Tianjin University

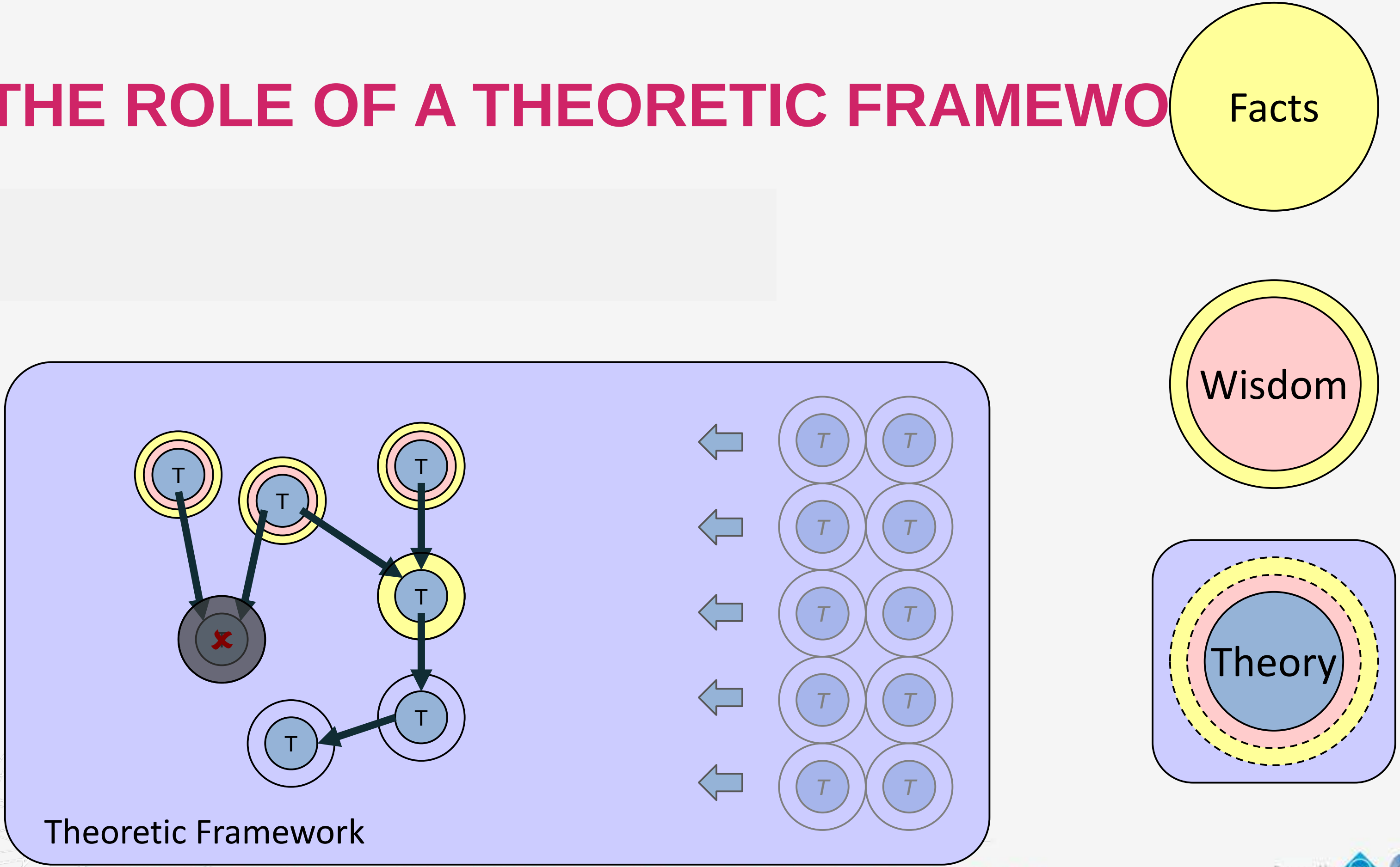
ENTROPY

Claude E. Shannon (1916-2001)



- Random variable (**alphabet**)
  - X
- It takes values (**letters**)
  - $x_1, x_2, \ldots, x_m$
- Probability mass function
  - $p(x_i)$
- Entropy (**uncertainty**)
  - $$\mathbf{H}(X) = -\sum_{i}^{m} p(x_i) \log_2 p(x_i)$$

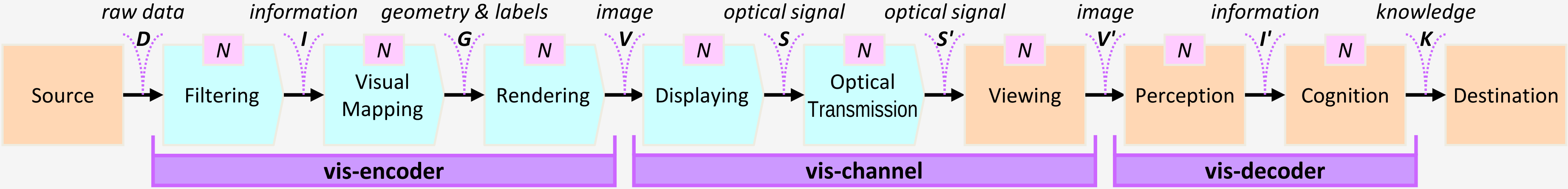
THE ROLE OF A THEORETIC FRAMEWO

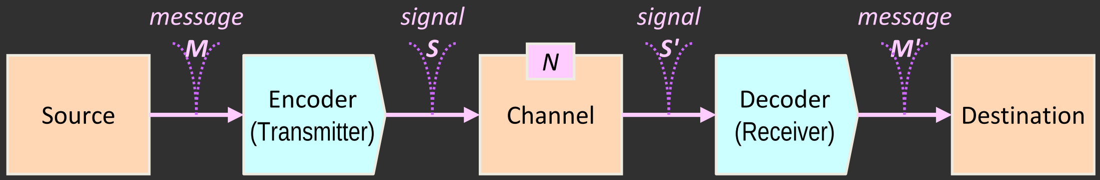


AN INFORMATION-THEORETIC VIEW OF VISUALIZATION

M. Chen and H. Jänicke, **An information-theoretic framework for visualization**, *IEEE Transactions on Visualisation and Computer Graphics*, 2010

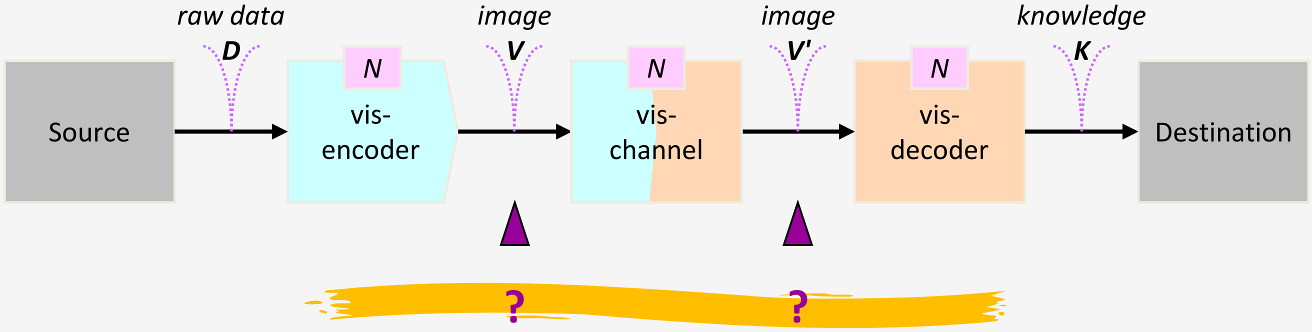
A General Visualization System

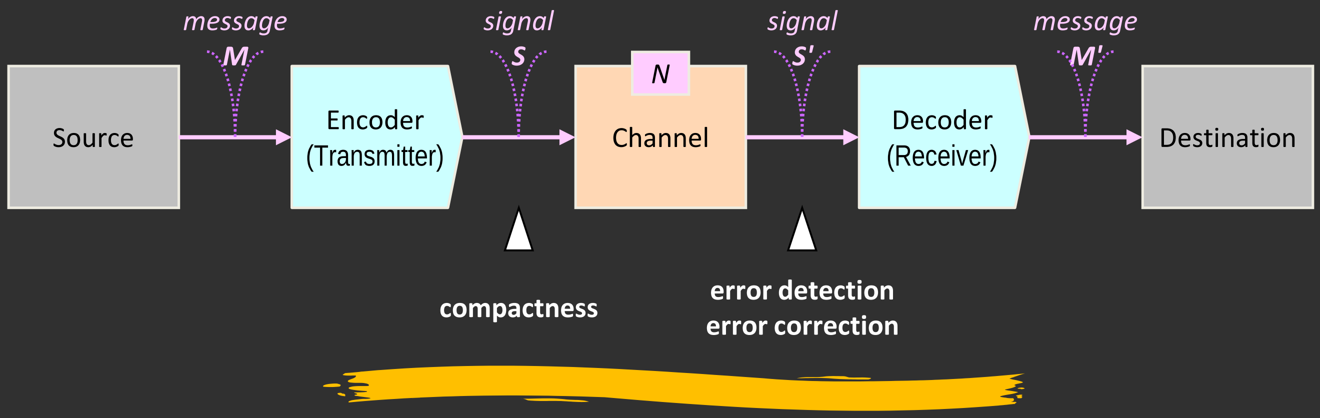




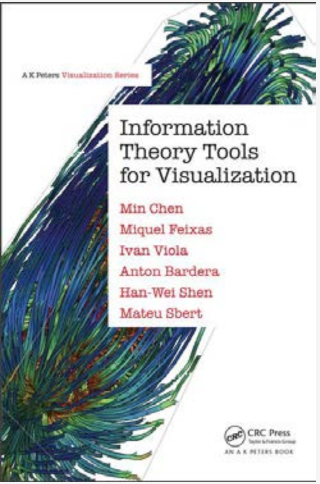
A General Communication System

SOURCE ENCODING AND CHANNEL ENCODING





Three Communication Subsystems


A K Peters Visualization Series
Information Theory Tools for Visualization
Min Chen
Miquel Feixas
Ivan Viola
Anton Bardera
Han-Wei Shen
Mateu Sbert
CRC Press

OUTLINE

1. Data Intelligence —— a big picture
2. Visualization —— a small picture
3. Measurement, Explanation, and Prediction
4. Example: Visual Multiplexing
5. Example: Error Detection and Correction
6. Example: Process Optimization
7. Summary

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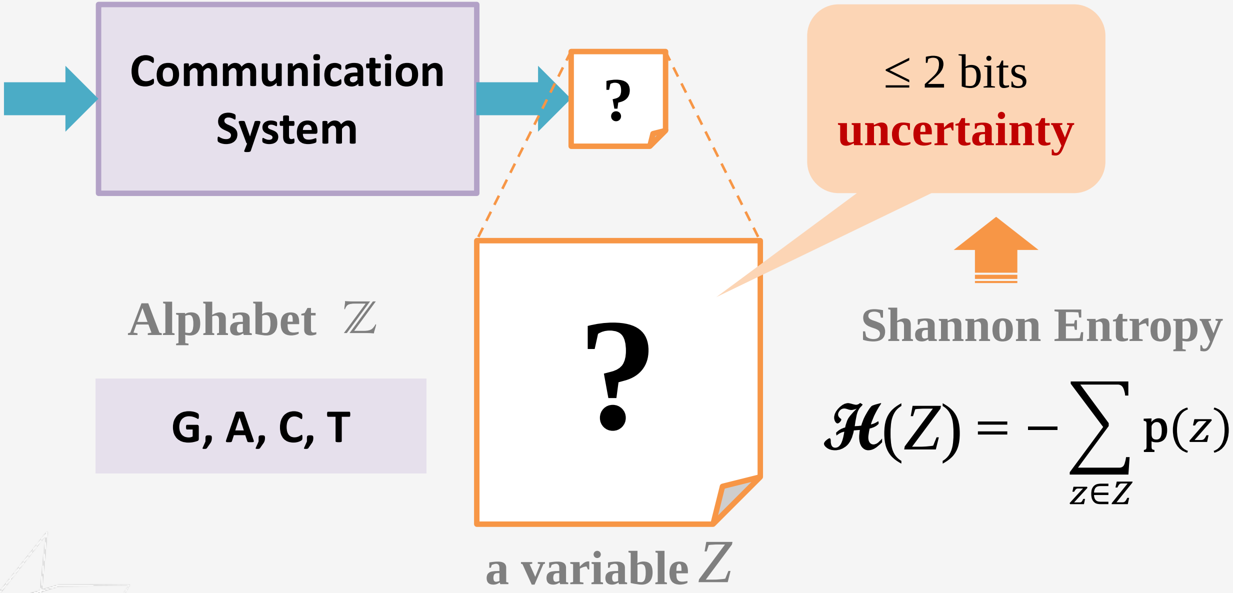
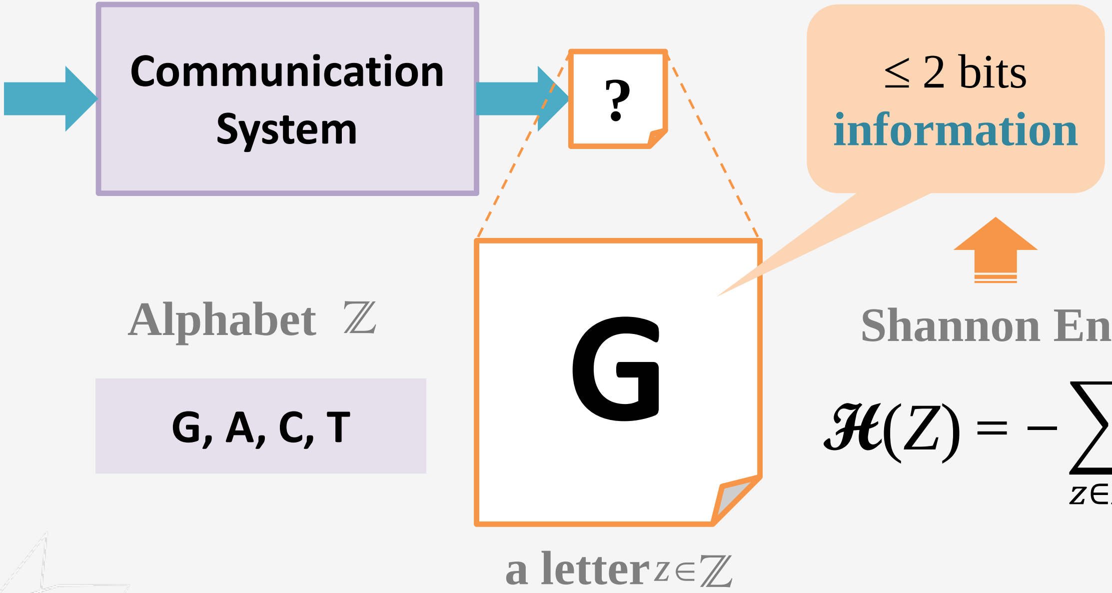
SHANNON'S DEFINITION OF INFORMATION



$$\mathcal{H}(Z) = -\sum_{z \in Z} \mathrm{p}(z) \log_2 \mathrm{p}(z)$$

SHANNON'S DEFINITION OF INFORMATION



Shannon Entropy

$$\mathcal{H}(Z) = -\sum_{z \in Z} \mathrm{p}(z) \log_2 \mathrm{p}(z)$$

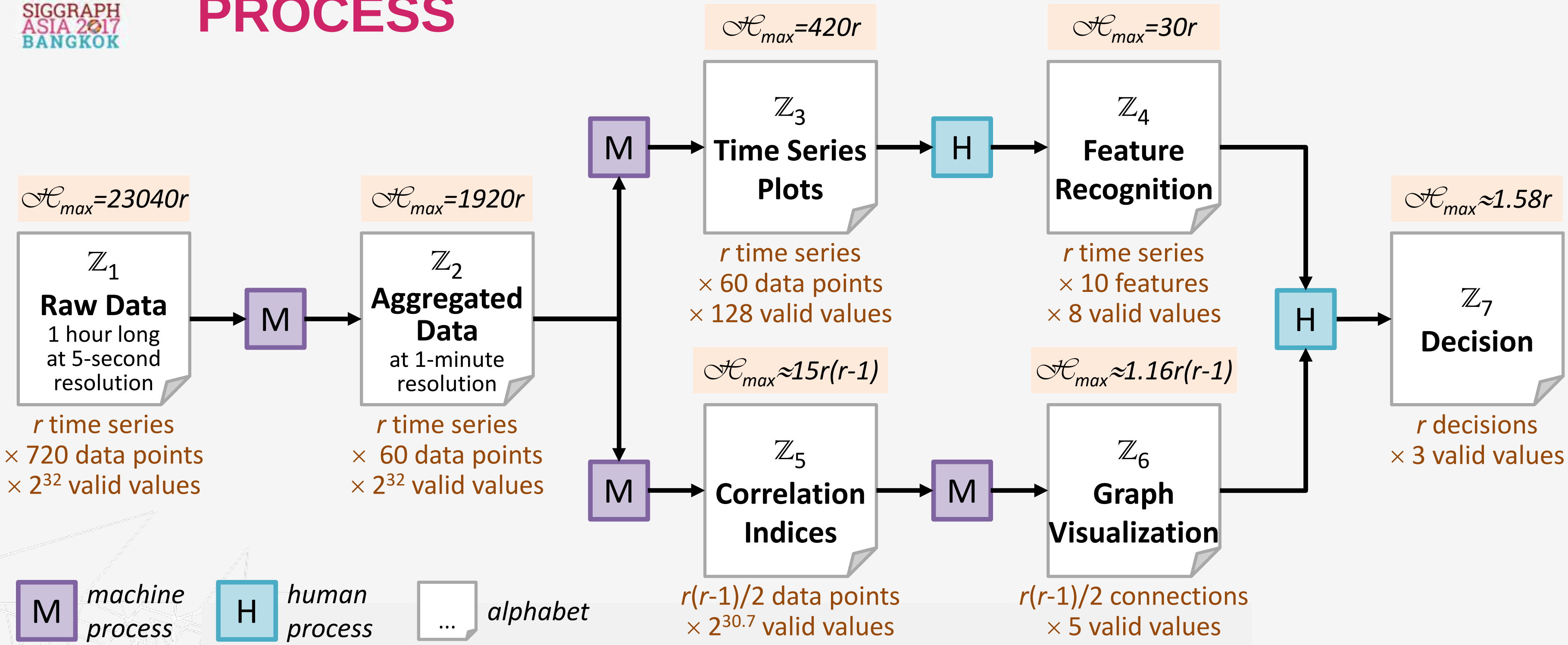
AN ALPHABET AND ITS LETTERS

- English alphabet — A, B, C, ..., X, Y, Z
- All English prefixes — bio, geo, pre, pro, ..., un
- All English words — a, ..., silicosis, ..., titin, ...
- All sentences in a text corpus — ...
- ...
- All published Siggraph papers — ...
- ...

45 letters
pneumonoultramicroscopicsilicovolcanoconiosis

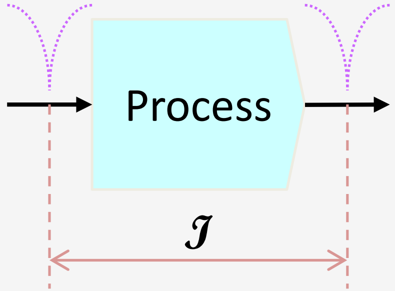
189,819 letters
a word or a formula?

AN EXAMPLE OF VISUALIZATION (OR VA) PROCESS

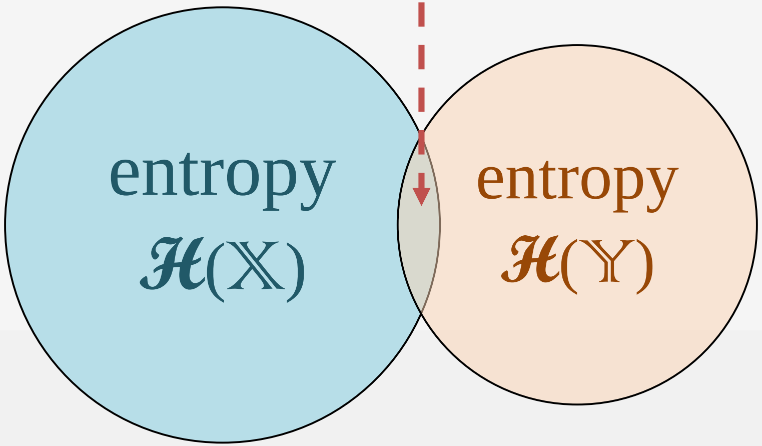


M. Chen and A. Golan, **What May Visualization Processes Optimize?** accepted by *IEEE Transactions on Visualization and Computer Graphics*, 2015.

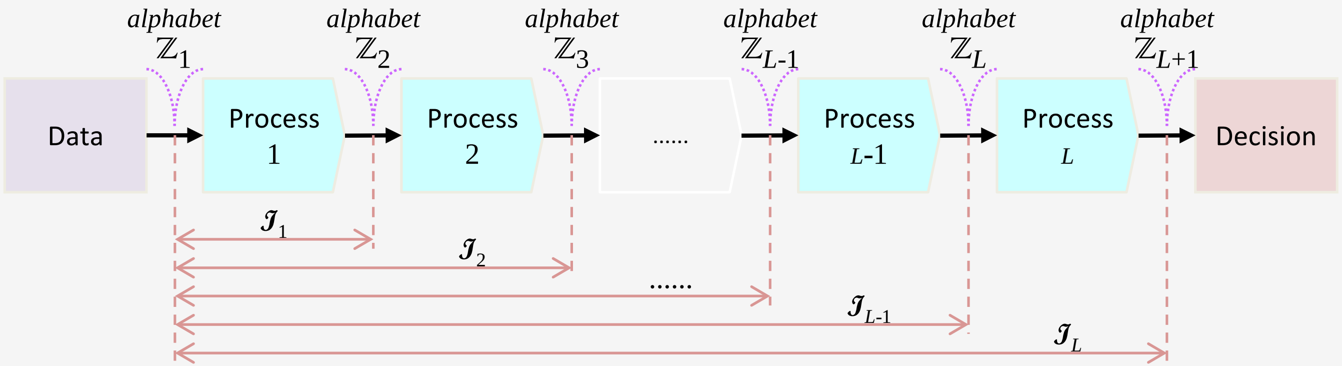
MUTUAL INFORMATION (SHARED UNCERTAINTY)



$$\mathcal{I}(X;Y) = \sum_{x \in X} \sum_{y \in Y} \mathbf{p}(x,y) \log_2 \frac{\mathbf{p}(x,y)}{\mathbf{p_X}(x)\mathbf{p_Y}(y)}$$



DATA PROCESSING INEQUALITY

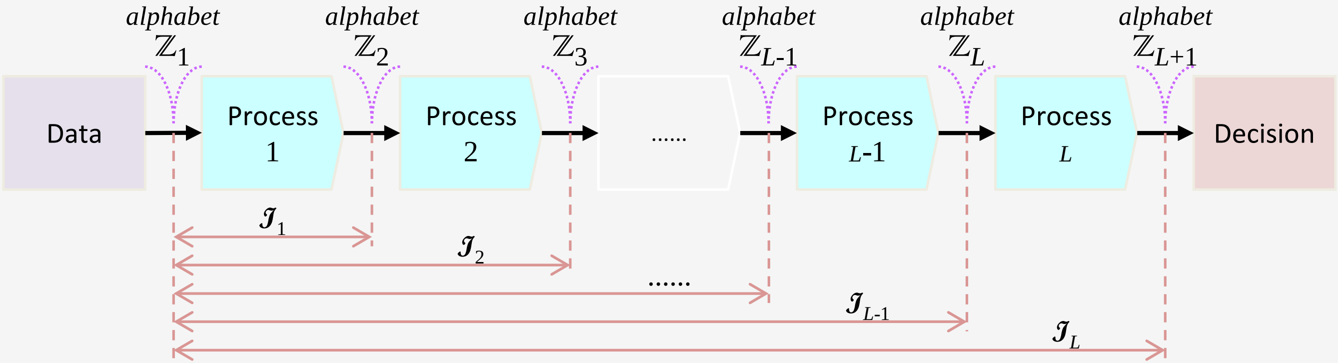


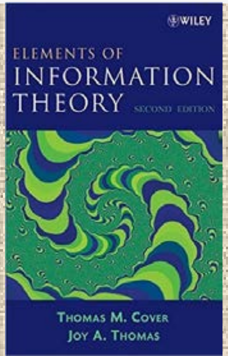
Decreasing of Mutual Information

$$\mathcal{I}_1 \geq \mathcal{I}_2 \geq \ldots \geq \mathcal{I}_{L\text{-}1} \geq \mathcal{I}_L$$

M. Chen and H. Jänicke, **An information-theoretic framework for visualization**,
IEEE Transactions on Visualisation and Computer Graphics, 2010

DATA PROCESSING INEQUALITY

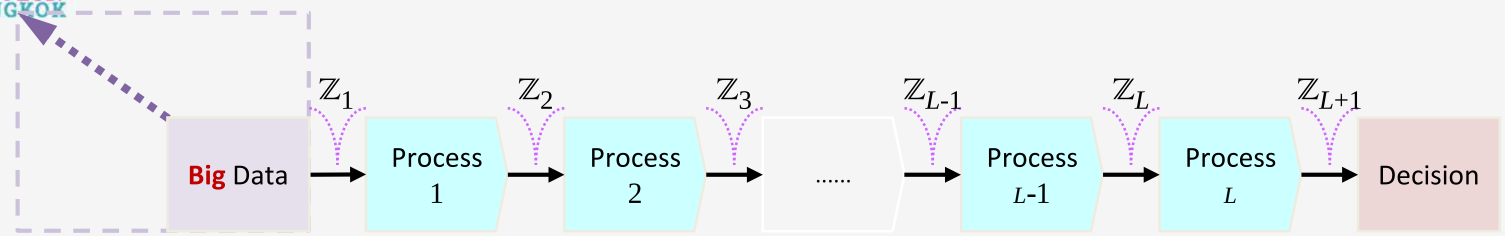


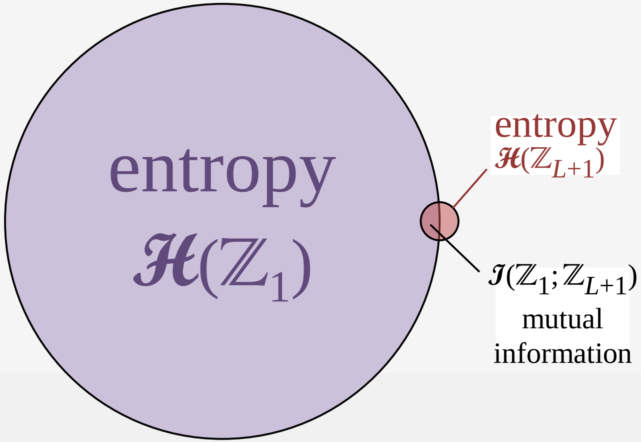


"No clever manipulation of data can improve the inferences that can be made from the data"

[Cover and Thomas, 2006]

DATA PROCESSING INEQUALITY WITH "BIG DATA"

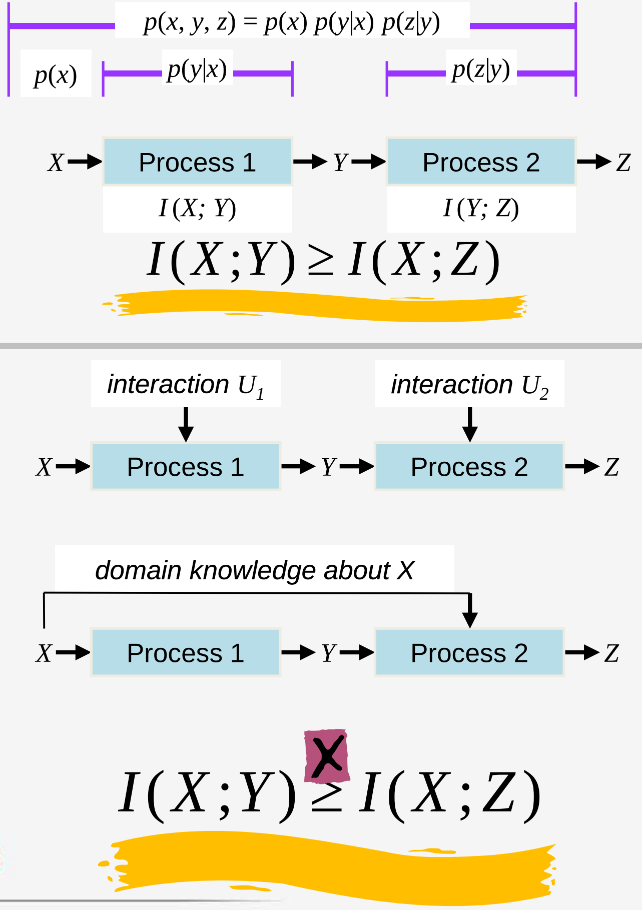




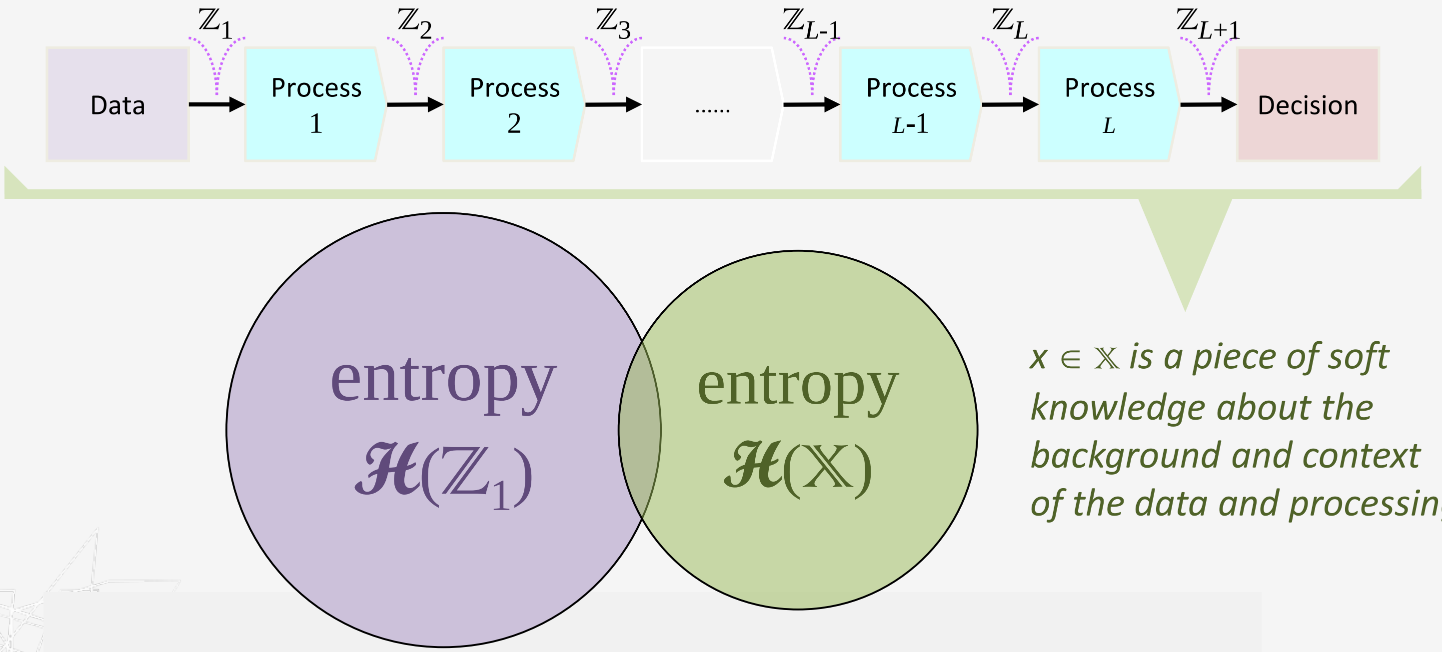
Can one trust a decision with a very small amount of mutual information with the data?

DPI IS NOT UBIQUITOUS

- **Markov chain conditions**
  - **Closed coupling: (X, Y), (Y,Z)**
  - **X and Z are conditionally independent**
- **What if one of the conditions is broken?**
- **In visual analytics, both conditions are usually broken.**

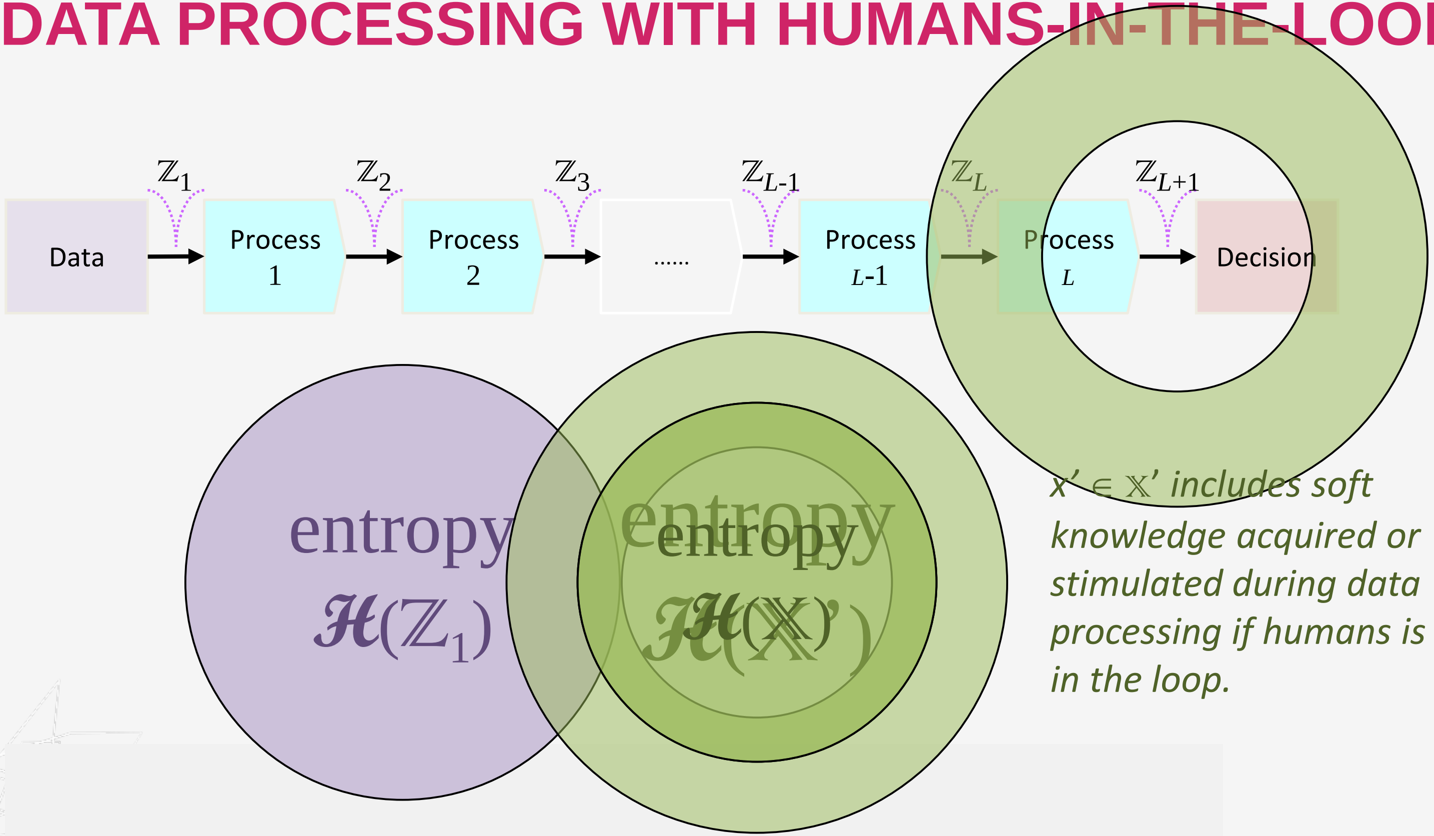


DATA PROCESSING WITH SOFT KNOWLEDGE



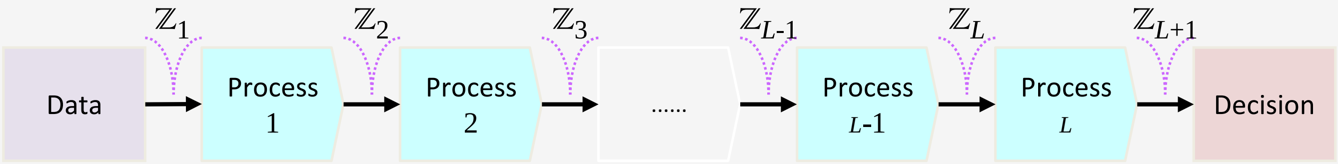
$x \in \mathbb{X}$ is a piece of soft knowledge about the background and context of the data and processing.

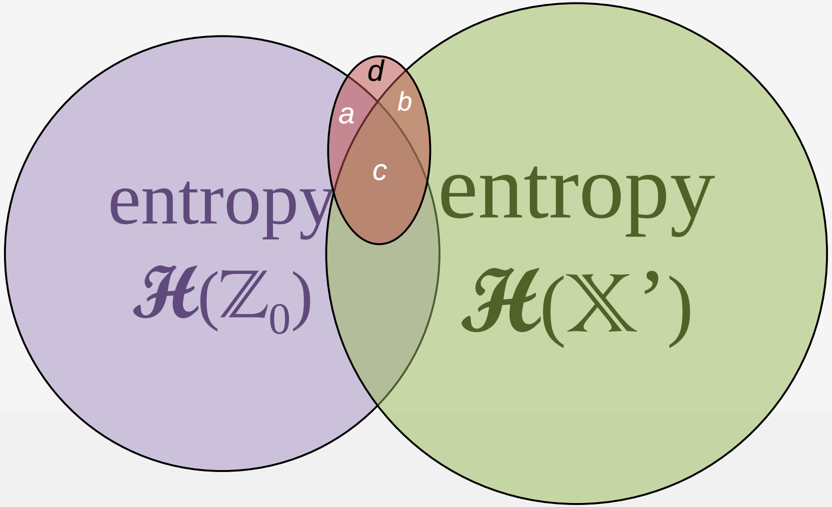
DATA PROCESSING WITH HUMANS-IN-THE-LOOP



x' ∈ $\mathbb{X}$' includes soft knowledge acquired or stimulated during data processing if humans is in the loop.

DATA PROCESSING WITH HUMANS-IN-THE-LOOP





All possible decisions under different conditions

a) totally data-driven

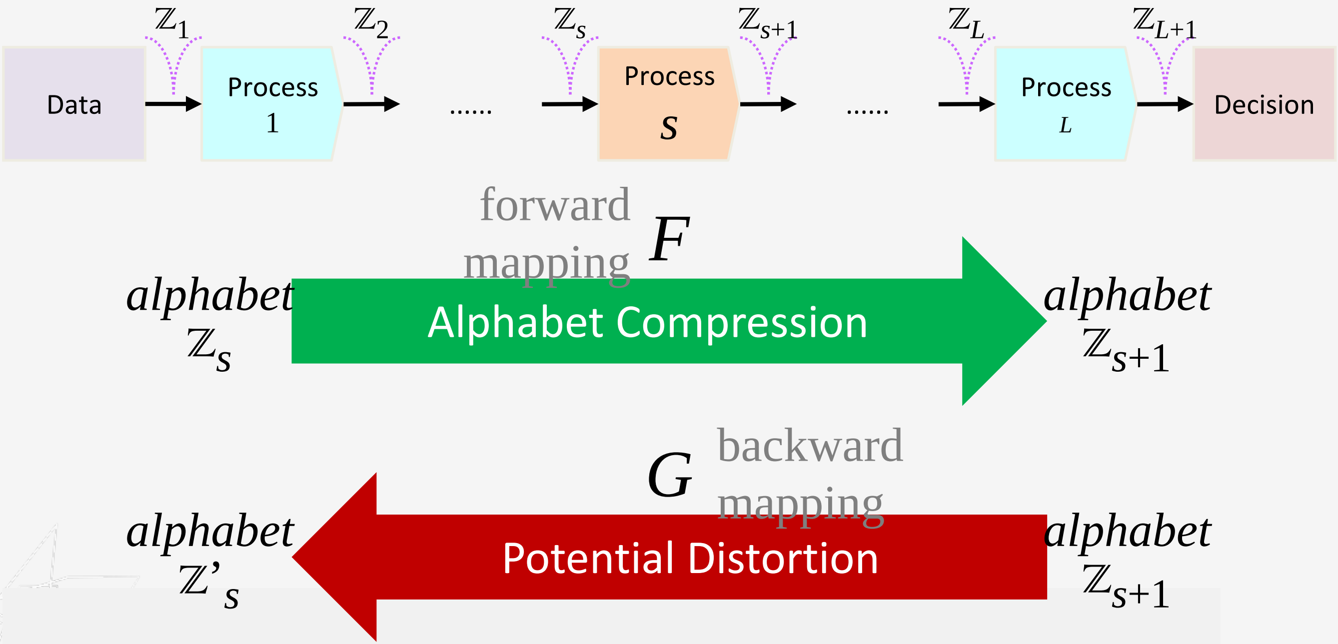
b) totally instinct-driven

c) data-informed

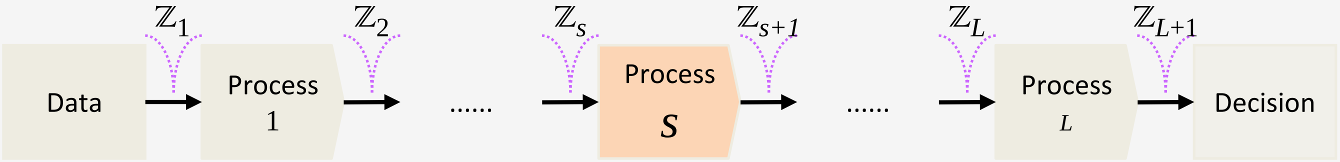
d) due to unknown or uncontrollable factors

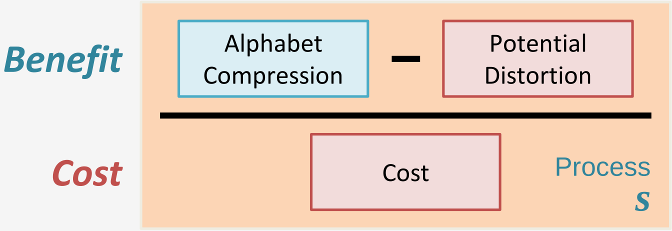
TRANSFORMATION

M. Chen and A. Golan, **What May Visualization Processes Optimize?**
IEEE Transactions on Visualization and Computer Graphics, 2016.



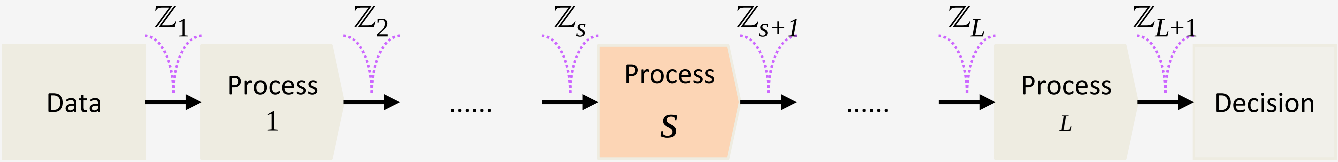
COST-BENEFIT RATIO





M. Chen and A. Golan, **What May Visualization Processes Optimize?** *IEEE Transactions on Visualization and Computer Graphics*, 2016.

COST-BENEFIT RATIO



Benefit

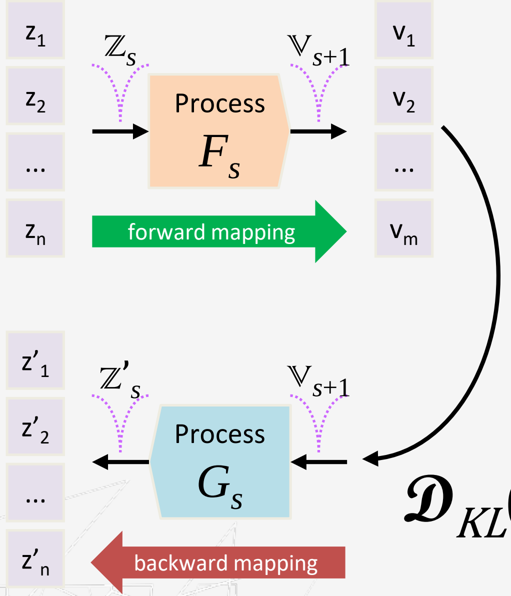
Alphabet Compression *Potential Distortion*

$$\frac{\mathscr{B}(F_s)}{\mathscr{C}(F_s)} = \frac{\mathscr{H}(Z_s) - \mathscr{H}(Z_{s+1}) - \mathscr{D}_{KL}(Z'_s || Z_s)}{\mathscr{C}(F_s)}$$

Cost

Cost

KULLBACK–LEIBLER DIVERGENCE

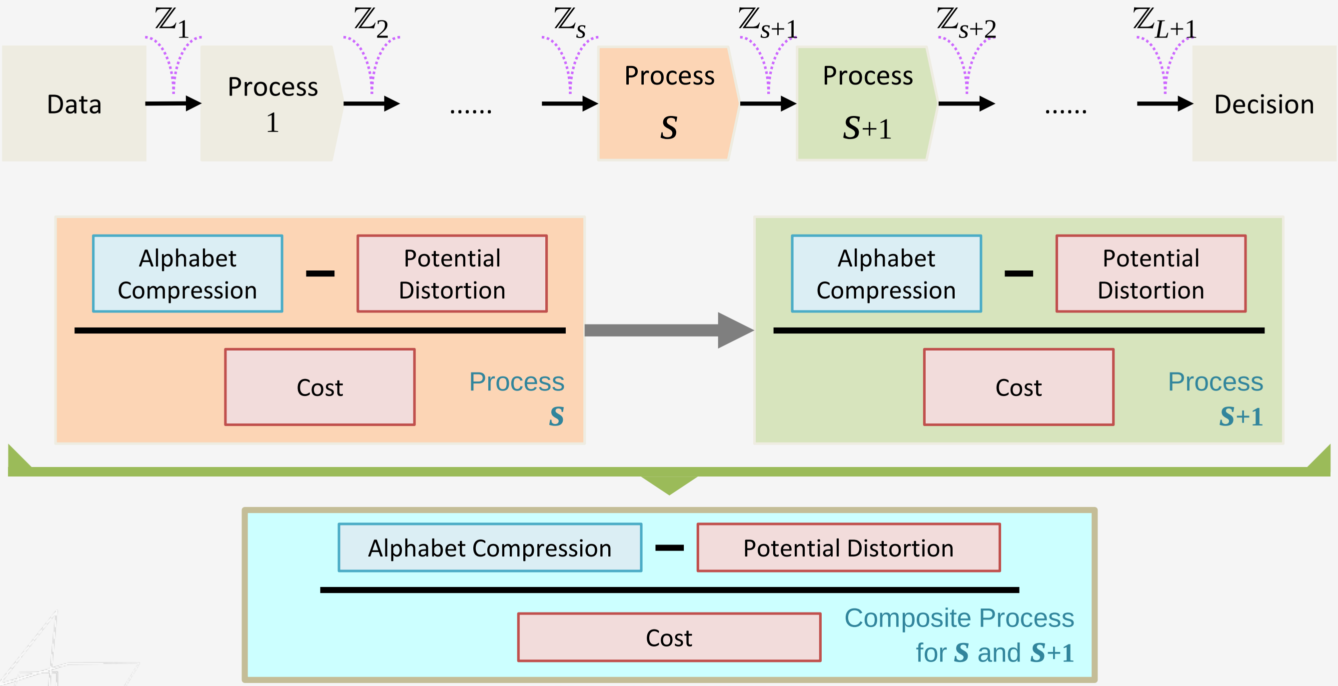


$$\mathcal{D}_{KL}(Z' \| Z) = \sum_{(z=z')\in Z} \mathrm{p}(z') \log_2 \frac{\mathbf{p}(z')}{\mathbf{q}(z)}$$

Solomon Kullback
1907-1994

Richard Leibler
1914-2003

COMPOSITION



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THREE SPACES AND THREE MEASURES

M. Chen and H. Jänicke, **An information-theoretic framework for visualization**, *IEEE Transactions on Visualisation and Computer Graphics*, 2010

- **Entropy of Input Data Space:** $H(X)$
- **Visualization Capacity:** $V(G)$
- **Display Capacity:** D

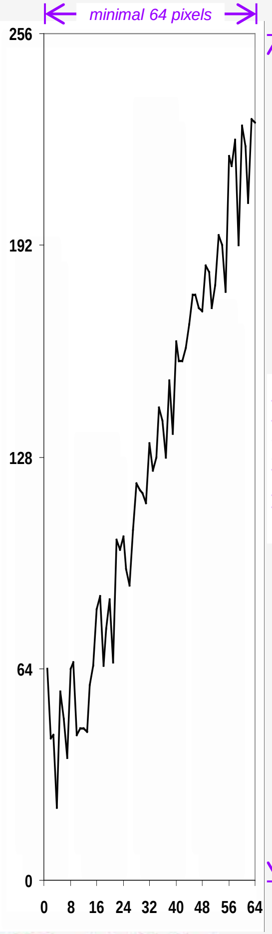
$$\text{Visual Mapping Ratio (VMR)} = \frac{\boldsymbol{V}(G)}{\boldsymbol{H}(X)}$$

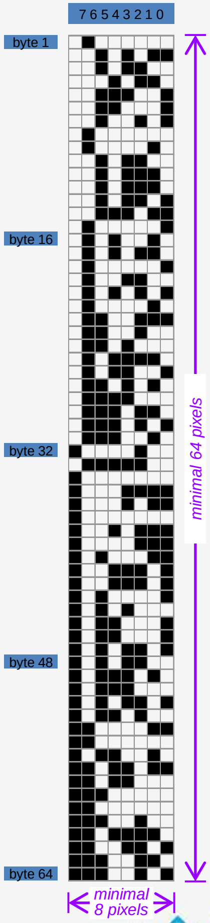
$$\text{Information Loss Ratio (ILR)} = \frac{\max(\boldsymbol{H}(X) - \boldsymbol{V}(G), 0)}{\boldsymbol{H}(X)}$$

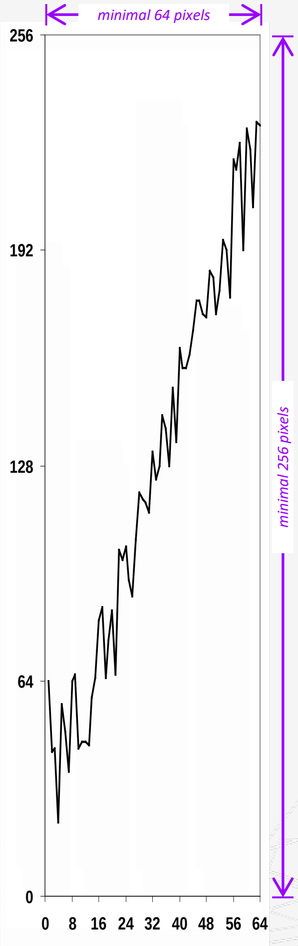
$$\text{Display Space Utilization (DSU)} = \frac{\boldsymbol{V}(G)}{\boldsymbol{D}}$$

EXAMPLE OF $V(G)$

- Entropy of the Data Alphabet: $$\mathbf{H}(Z) = -\sum_{t=0}^{64}\sum_{i=0}^{255}\frac{1}{256}\log_2\frac{1}{256} = 512$$
- $V(G) = H(Z)$
- Binary Pixel Plot: D
  - 4x4 pixels per bit $\rightarrow$ D: $2^9 \times 2^4 = 2^{13}$ bits
- Time Series Plot: D
  - Minimal 256x64 pixels $\rightarrow$ D: $2^8 \times 2^6 = 2^{14}$ bits
- The more compact, the better?
- Cost? Reconstructability?



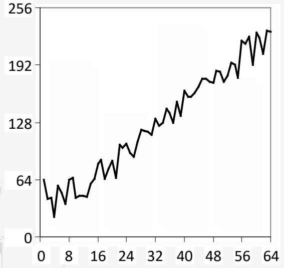




H(X) = 512 bits
V(G) = 512 bits
D = 16382 bits
VMR = 1
ILR = 0
DSU = 0.03

INFORMATION LOSS RATIO (ILR)

H(X) = 512 bits
V(G) = 384 bits
D = 4096 bits
VMR = 0.75
ILR = 0.25
DSU = 0.09



(a) evenly distributed *p*

- **Display Space Restriction**
  - **64x64 pixels**
- **Evenly distributed probability mass function**
- **Linear visual mapping**
- **ILR is a probabilistic measure about**
  - **a data space X, not an instance x_i**

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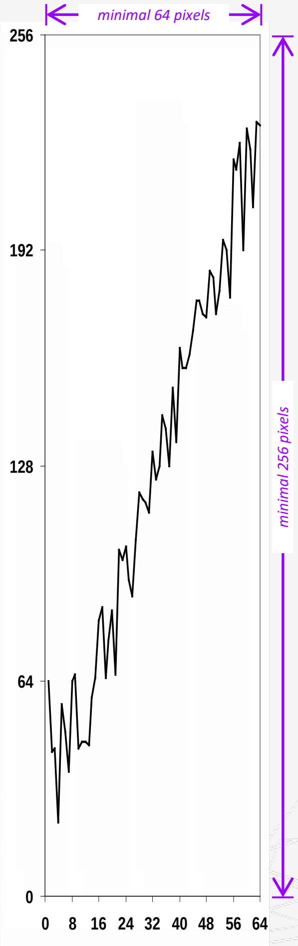
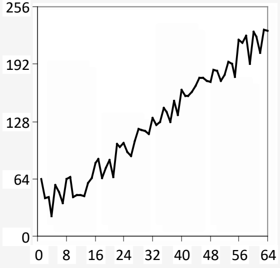


INFORMATION LOSS RATIO (ILR)

information loss:
25%



(a) evenly distributed *p*

M. Chen and H. Jänicke, **An information-theoretic framework for visualization**, *IEEE Transactions on Visualisation and Computer Graphics*, 2010

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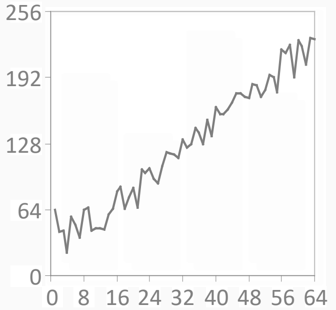
- **Evenly distributed probability mass function**
- **Linear visual mapping**
- **ILR is a probabilistic measure about**
  - **a data space X, not an instance x_i**

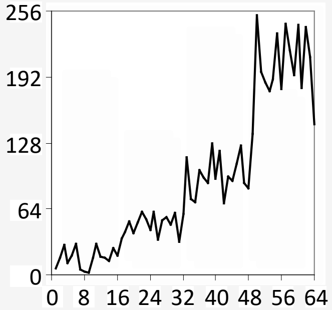
NON-UNIFORM DISTRIBUTION

information loss: 25%

information loss: 25.8%

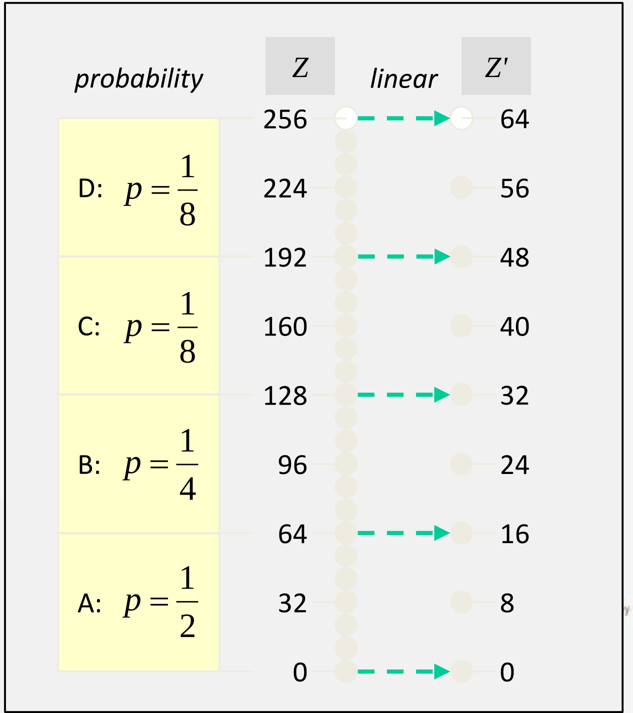


(a) evenly distributed *p*



(b) unevenly distributed *p*

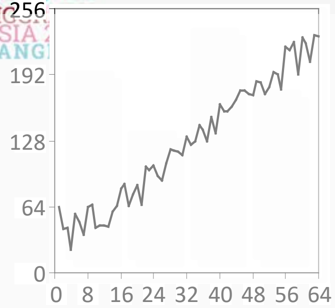
- **Linear visual mapping**



NON-UNIFORM DISTRIBUTION

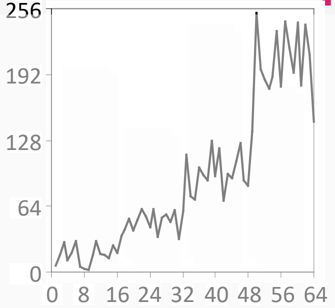
- **Non-linear visual mapping**

information loss:
25%



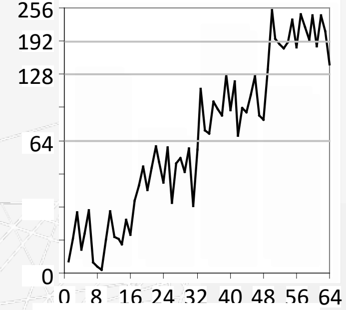
(a) evenly distributed *p*

information loss:
25.8%

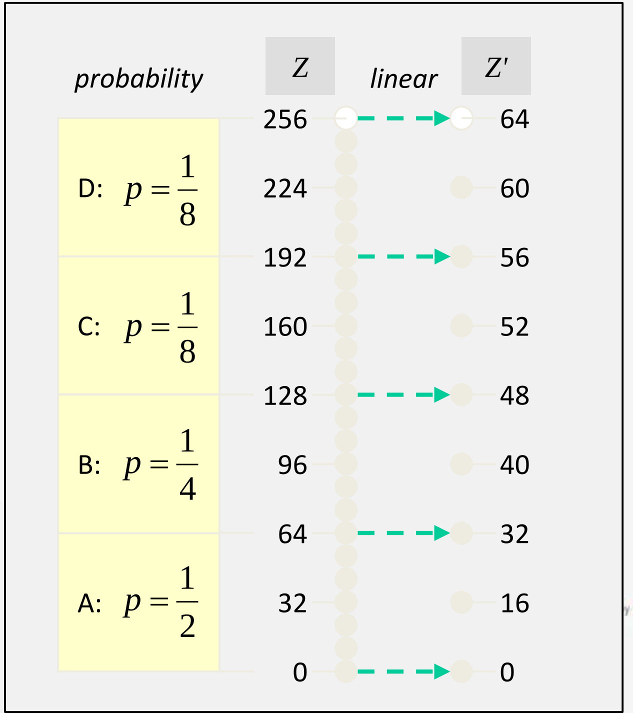


(b) unevenly distributed *p*

information loss:
22.6%

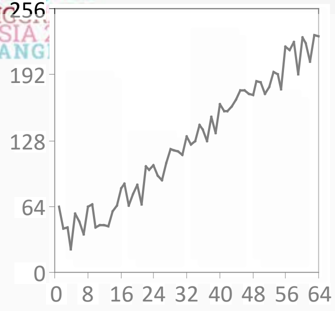


(c) 4 regional mappings



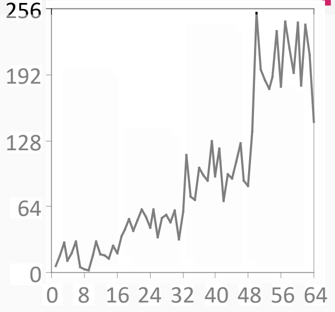
NON-UNIFORM DISTRIBUTION

information loss: 25%



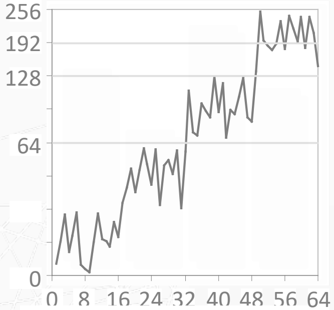
(a) evenly distributed *p*

information loss: 25.8%



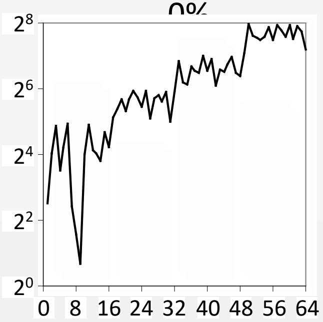
(b) unevenly distributed *p*

information loss: 22.6%



(c) 4 regional mappings

information loss: 0%



(d) logarithmic plot

- **Logarithmic visual mapping**

D: $p = \frac{1}{8}$

C: $p = \frac{1}{8}$

B: $p = \frac{1}{4}$

A: $p = \frac{1}{2}$

$k \to \infty$

$p = 1/2^k$

$p = 1/2^k$

$p = 1/2^{k-1}$

......

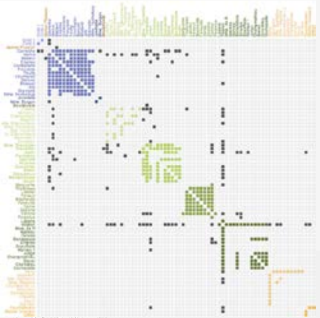
$p = 1/2^3$

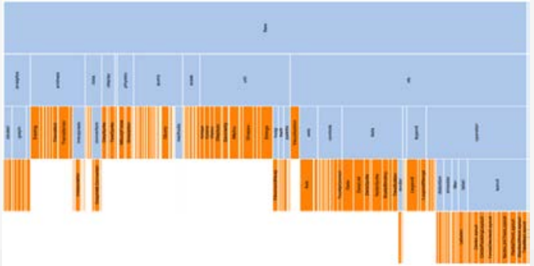
$p = 1/2^2$

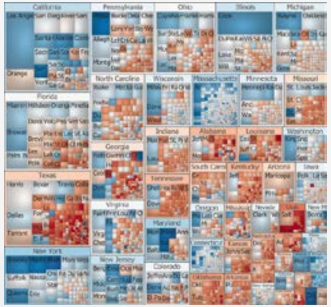
$p = 1/2$

A PREDICTION?

- **Display capacity D is limited.**
- **Data space: noticeable non-uniform distribution: $H(X) << H_{max}(X)$**
- **Visualization capacity $V(G)$: a visual representation exhibiting a similar non-uniform distribution of space requirement can reduce information loss.**
- **Can adjacency matrices be improved for trees?**







http://hci.stanford.edu/jheer/files/zoo/

http://en.wikipedia.org/wiki/Treemapping

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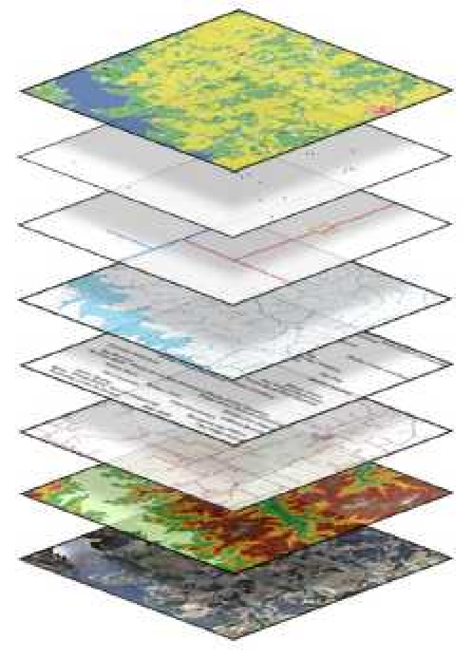
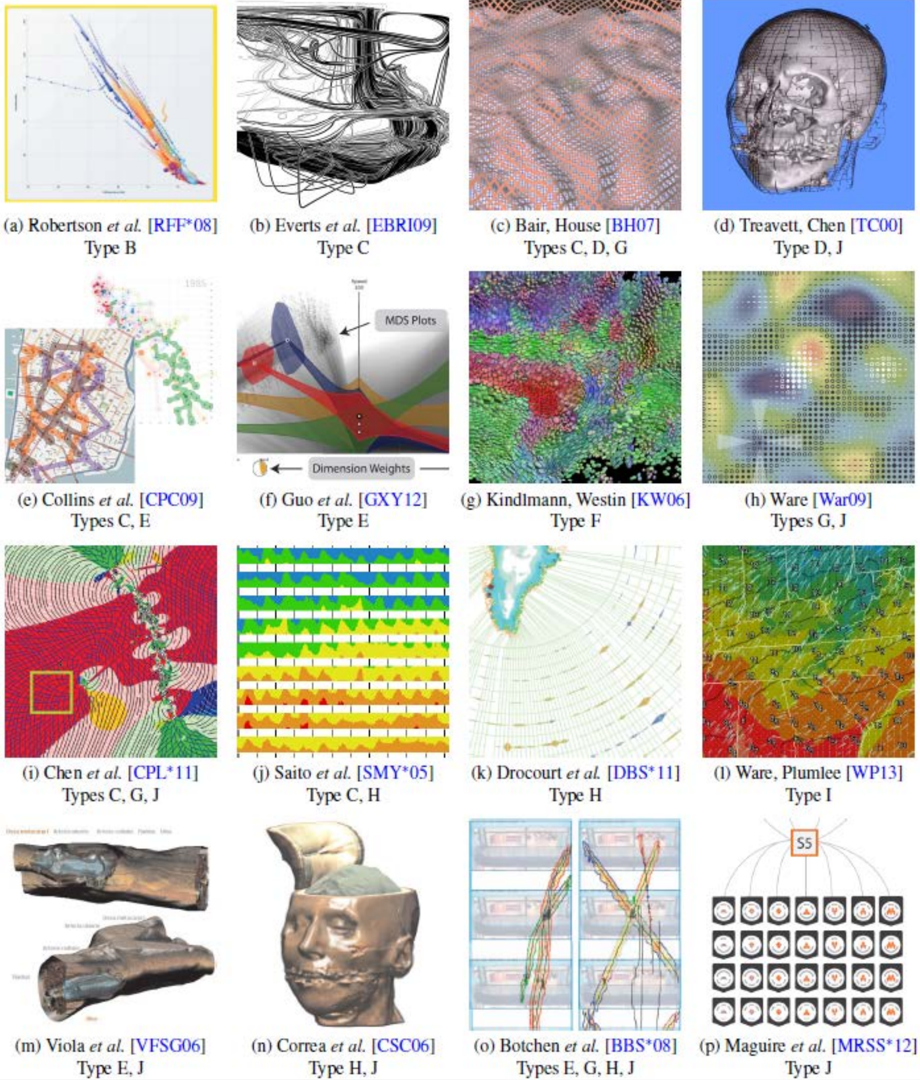
MAP OVERLAY AND ITS GENERALIZATION IN VISUALIZATION



http://learnpracticalgis.com/how-to-overlay-maps/

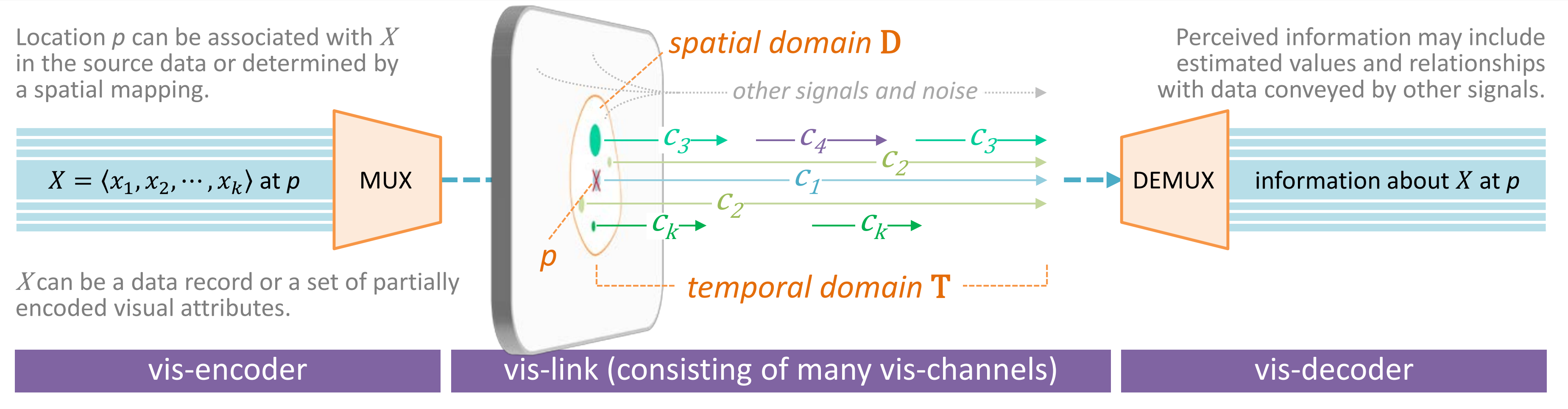


(a) Robertson *et al.* [RFF*08] Type B
(b) Everts *et al.* [EBRI09] Type C
(c) Bair, House [BH07] Types C, D, G
(d) Treavett, Chen [TC00] Type D, J
(e) Collins *et al.* [CPC09] Types C, E
(f) Guo *et al.* [GXY12] Type E
(g) Kindlmann, Westin [KW06] Type F
(h) Ware [War09] Types G, J
(i) Chen *et al.* [CPL*11] Types C, G, J
(j) Saito *et al.* [SMY*05] Type C, H
(k) Drocourt *et al.* [DBS*11] Type H
(l) Ware, Plumlee [WP13] Type I
(m) Viola *et al.* [VFSG06] Type E, J
(n) Correa *et al.* [CSC06] Type H, J
(o) Botchen *et al.* [BBS*08] Types E, G, H, J
(p) Maguire *et al.* [MRSS*12] Type J

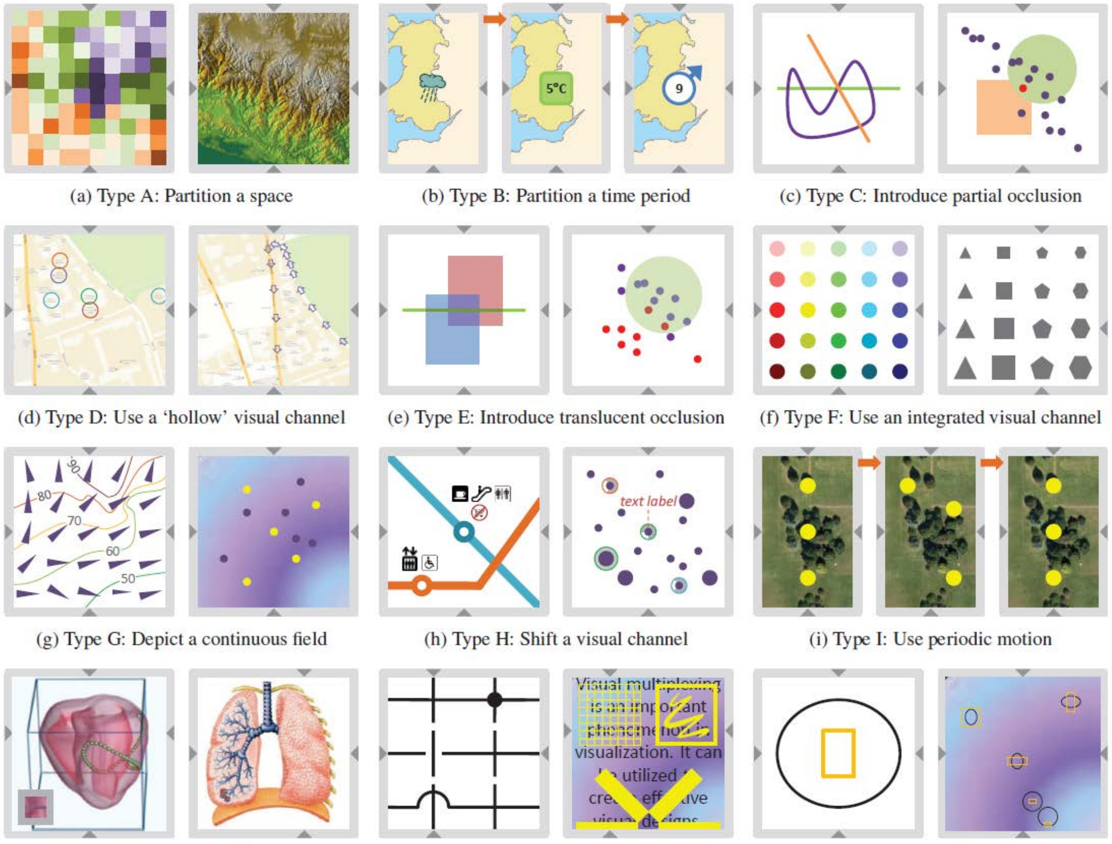
VISUAL MULTIPLEXING

M. Chen, S. Walton, K. Berger, J. Thiyagalingam, B. Duffy, H. Fang, C. Holloway, and A. E. Trefethen, **Visual multiplexing**, *Computer Graphics Forum*, 2014.

- Frequency-division multiplexing (FDM)?
- Time-division multiplexing (TDM)?
- Space-division multiplexing (SDM)?
- Code-division multiplexing (CDM)?

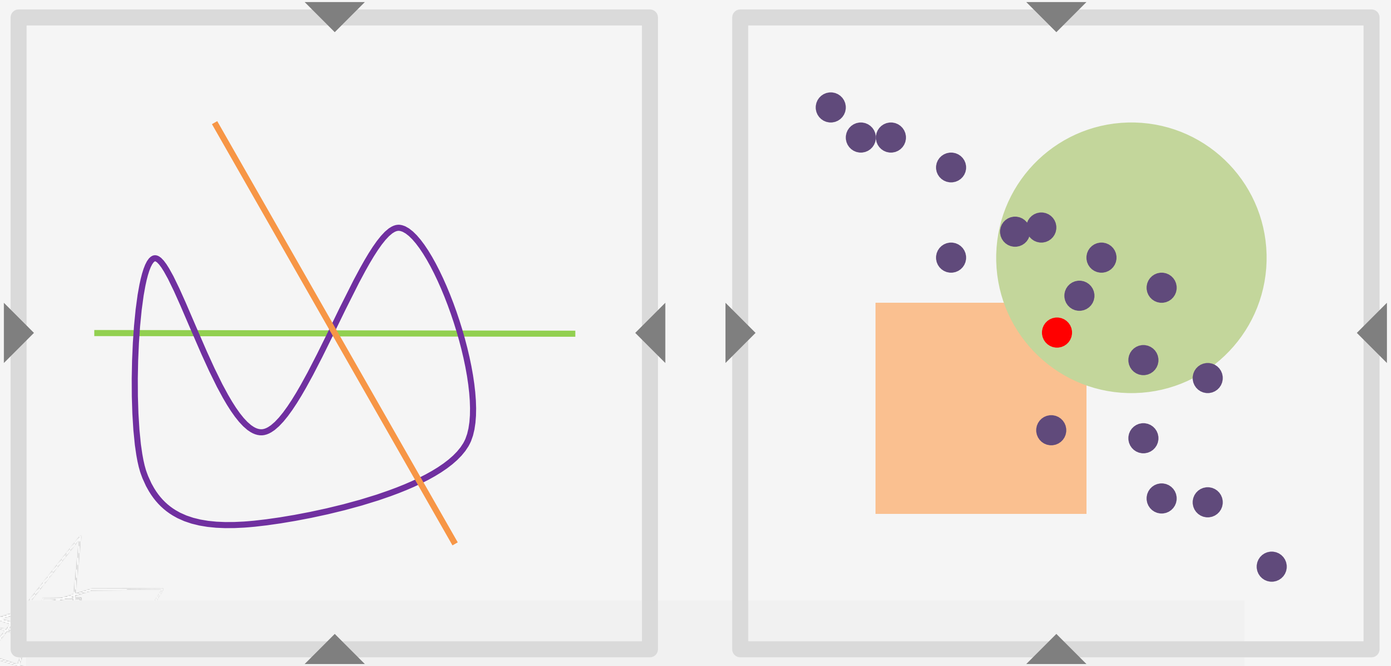


10 TYPES OF VISUAL MULTIPLEXING

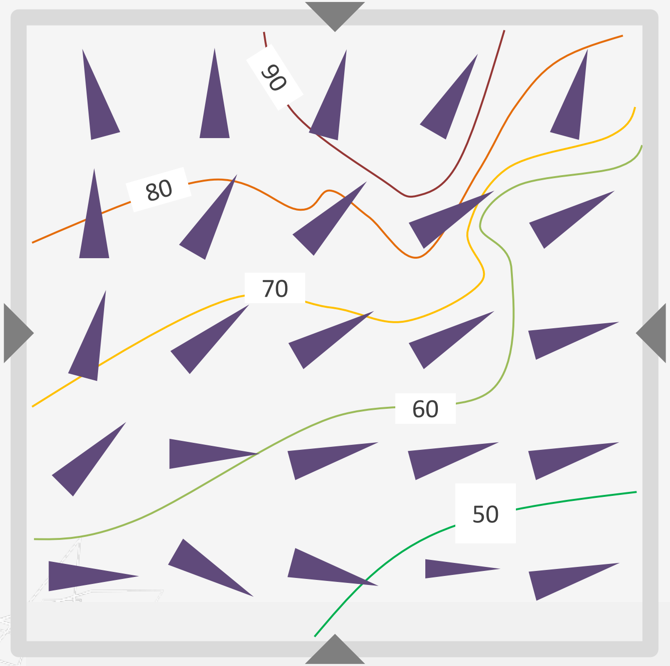


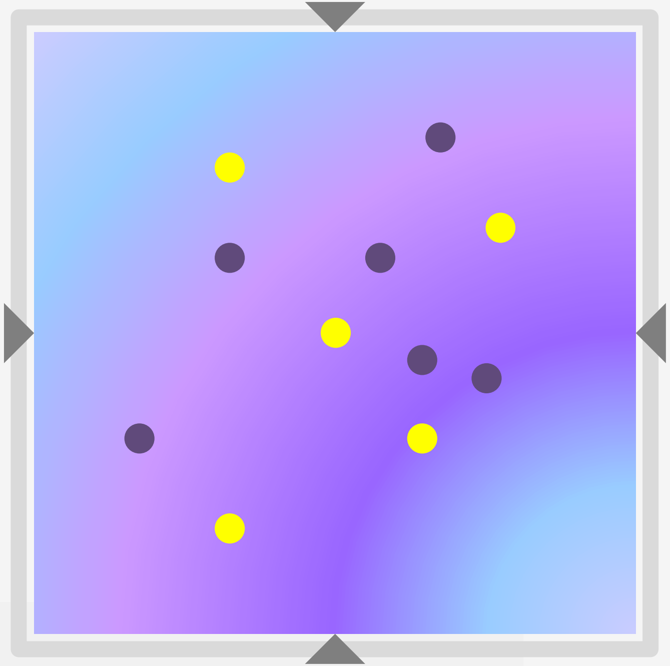
(a) Type A: Partition a space
(b) Type B: Partition a time period
(c) Type C: Introduce partial occlusion
(d) Type D: Use a 'hollow' visual channel
(e) Type E: Introduce translucent occlusion
(f) Type F: Use an integrated visual channel
(g) Type G: Depict a continuous field
(h) Type H: Shift a visual channel
(i) Type I: Use periodic motion
(j) Type J: Assume a priori knowledge
(k) Type J (continued): Acquired knowledge
(l) Type J (continued): Visual language

TYPE C: INTRODUCE PARTIAL OCCLUSION



TYPE G: DEPICT A CONTINUOUS FIELD





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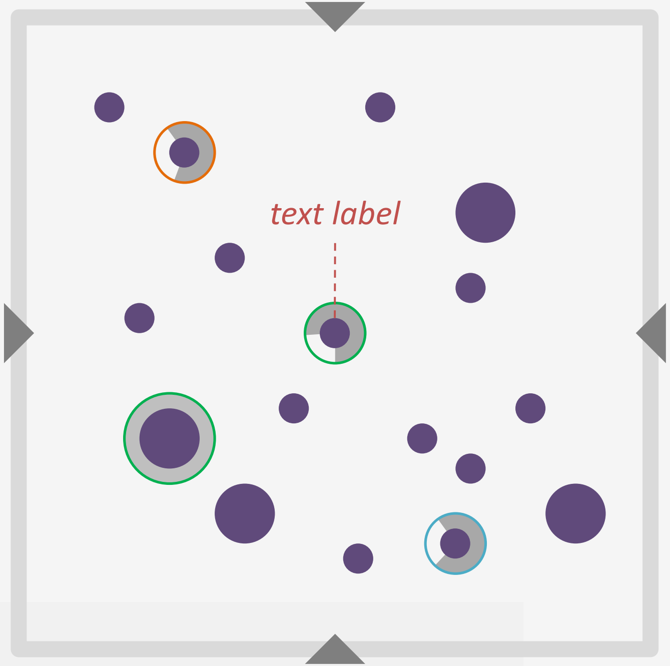
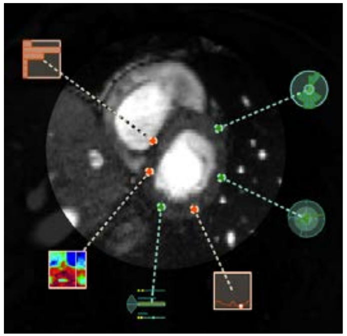
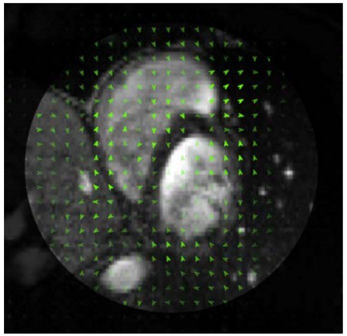
TYPE H: SHIFT A VISUAL CHANNEL



HOW ABOUT WHEN EVERY PIXEL IS USED?



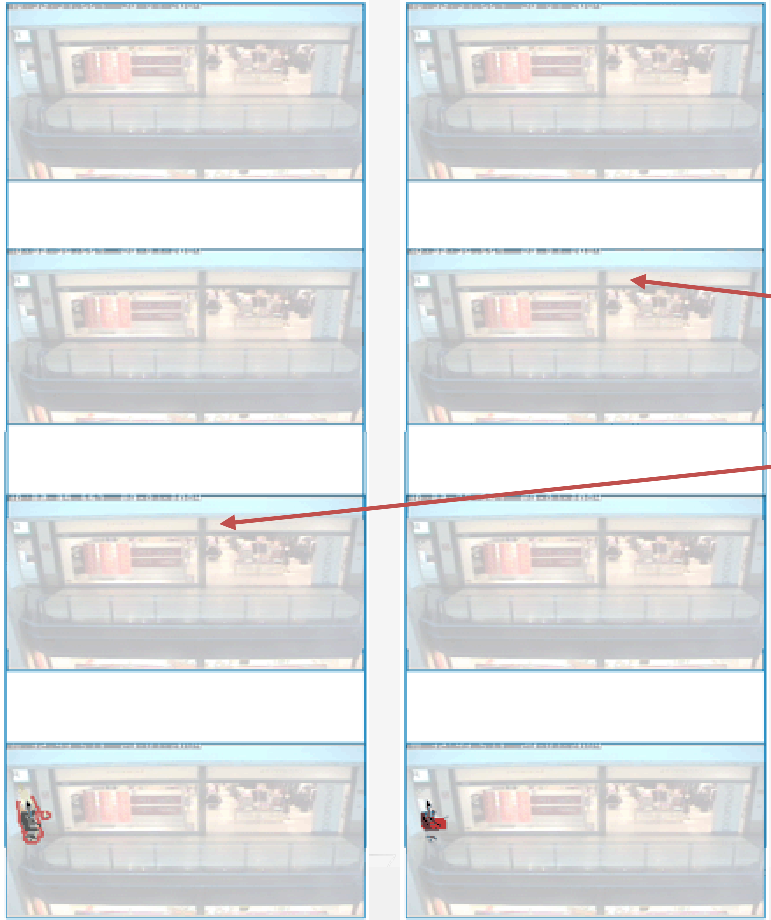


Data Space → Visualization Space → Display Space

Data Space	Visualization Space	Display Space
H	V(G)	D
Data Space Entropy	*Visualization Capacity (Visualization Space Entropy)*	*Display Space Capacity*

$$\frac{V(G)}{D} \ll 1$$

SYSTEMATIC APPLICATION



- C: Introduce Partial Occlusion
- E: Introduce Translucent Occlusion
- G: Depict a Continuous Field
- J: Assume A Priori Knowledge

R. P. Botchen, S. Bachthaler, F. Schick, M. Chen, G. Mori, D. Weiskopf and T. Ertl, **Action-based multi-field video visualization**, *IEEE Transactions on Visualization and Computer Graphics*, 2008.

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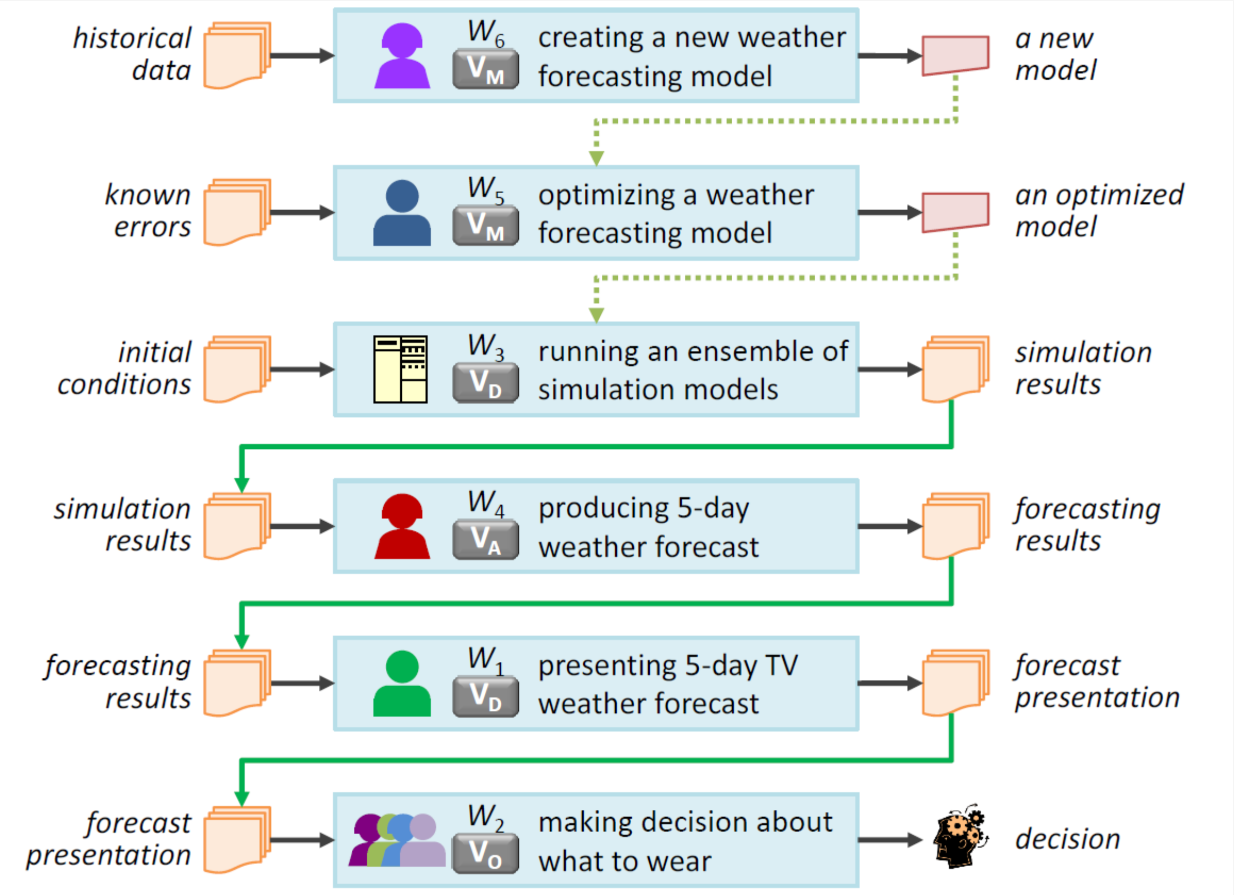
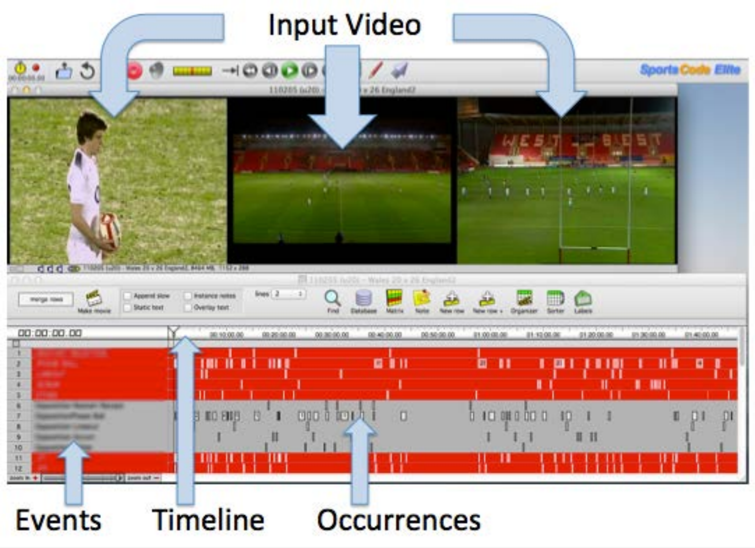
FOUR LEVELS OF VISUALIZATION

1. Disseminative Level — $O(1)$
   This is “a”!
2. Observational Level — $O(n)$
   “a”, “b”, “c”, … what, when, where?
3. Analytical Level — $O(n^k)$
   Are “a”, “b”, “c” related? Why?
4. Model-developmental Level — $O(k^n)$, $O(n!)$
   How does “a” lead to “b”?

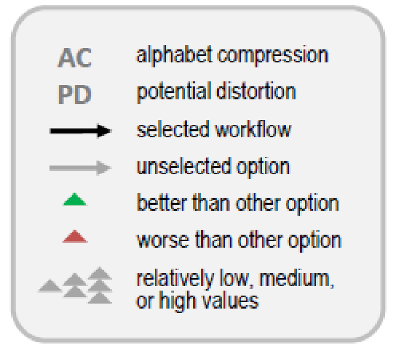
A COMPOSITE WORKFLOW

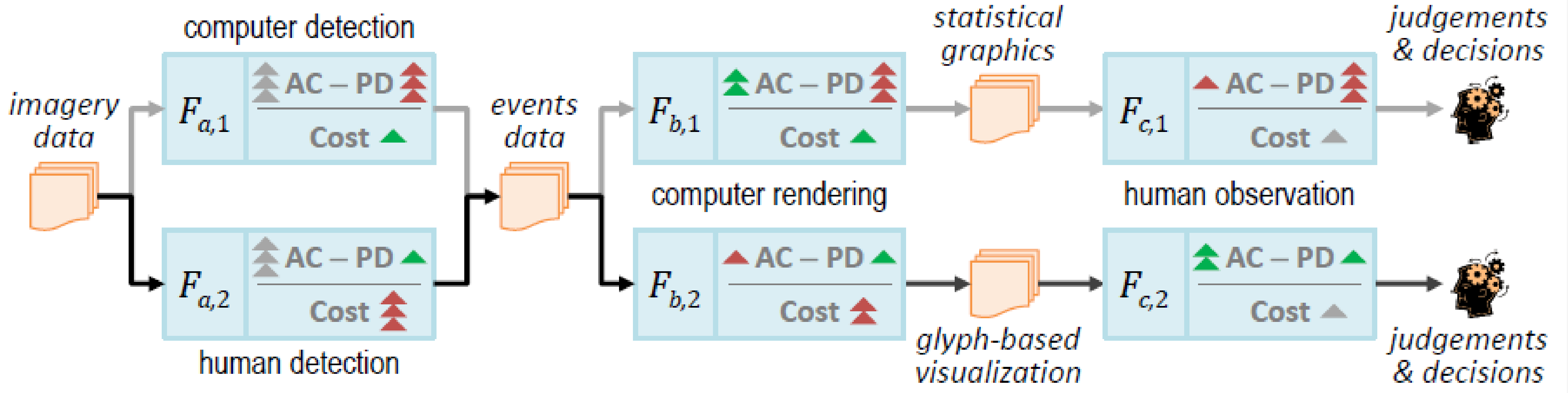




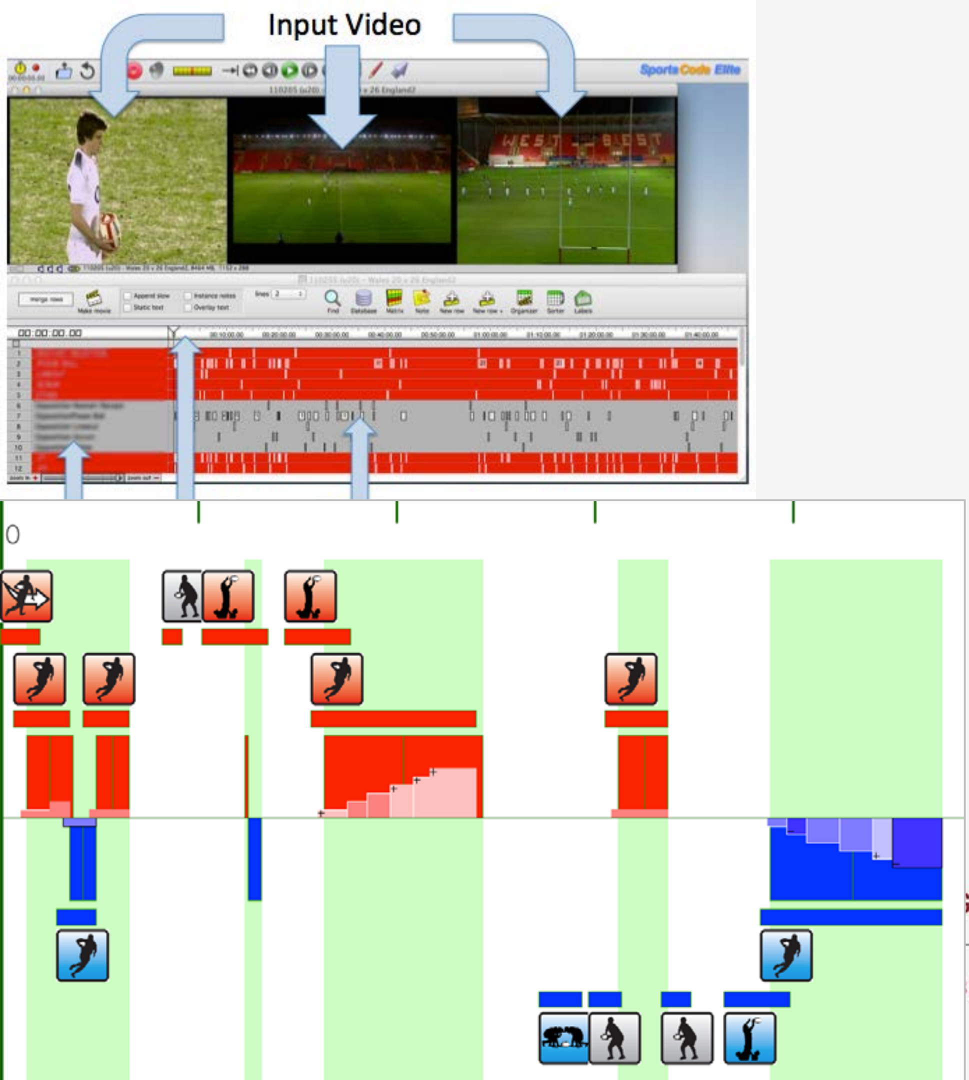
OBSERVATIONAL VISUALIZATION

P. A. Legg, D. H. S. Chung, M. L. Parry, M. W. Jones, R. Long, I. W. Griffiths and M. Chen, **MatchPad: Interactive Glyph-Based Visualization for Real-Time Sports Performance Analysis,** *Computer Graphics Forum*, 2012





OBSERVATIONAL



BBC News - Wales' Rugby World Cup team using Swansea Uni app - Mozilla Firefox

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29 September 2011 Last updated at 13:28

Wales' Rugby World Cup team using Swansea Uni app

Welsh coaches are using technology developed for them by experts at Swansea University to improve their match analysis at the Rugby World Cup.

The MatchPad was tested by the Welsh team at Cardiff's Millennium Stadium

Coach Warren Gatland and his backroom team are bombarded with statistics about scrums, line-outs and tackles during each game.

Now they are using an 'app' to simplify the information they need and understand the team's performance.

It also allows the coaches to review video of key moments during play.

The Welsh team employs three analysts whose job it is to collate data about all aspects of each game from the set pieces and restarts to tackles made or missed.

Dr Philip Legg of Swansea University's college of engineering and department of computer science said the problem was coaches were suffering from an "information overload."

The university was approached by the Welsh Rugby Union (WRU) and has developed an app it calls the MatchPad which runs on an Apple iPad.

It produces a visual timeline during the game so analysts and coaches can review video and additional detail on the events they are most interested in simply by pressing an icon.

The portability of the device means they can access the information in the analysis box, in the changing rooms, or even at pitch-side.

Dr Legg said: "During each game the team analysts are busy recording each of the events that happen.

"They look at each scrum, line out, restart, possession won and lost and tackles.

"They collect so much information - that's the basic problem and the app just tries to simplify

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"They look at each scrum, line out, restart, possession won and lost and tackles"

Dr Philip Legg
Swansea University

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Information Theory Taxonomy	Relevance in Visualization
Fundamental Concepts	**A possible mathematical framework that underpins the subject of visualization.**
Major Quantities and Properties	Quantitative measurements about the data and visualization space, and the relationship between input and output of a process or subsystem at different stages of a visualization pipeline.
Entropy	Measuring information content (see Section 5.1); salience in visualization.
Mutual Information	Uncertainty reduction in visualization (see Section 5.3); information-assisted visualization.
Major Theorems	Many theorems can be used to explain visualization phenomena and events.
Information balance (conservation law)	Given two visualizations, A and B, the amount of information about A contained in B is the same as that about B in A; overview + detail visualization; multi-view visualization.
Data processing inequality	After visual mapping, the visualization cannot contain more information than the original data (see Section 6.2); Information cannot be recovered after being degraded by some processes or subsystems in a visualization pipeline.
Channel Types	Providing a theoretical basis for classifying visualization subsystems (see Section 6).
Noiseless channel	Not common in practical visualization pipelines (see Section 3).
Noisy channel	Most visualization processes and subsystems can be affected by noise (see Section 3).
Channel Capacity	It can be adapted to define the maximum amount of information that can be visualized or dis-. played (see Sections 5.1 and 6)
Redundancy	Efficiency of a visual mapping; Error detection and correction (see Section 8).
Source Coding (for Noiseless Channels)	**Inspiration for developing new data abstraction and visual encoding techniques.**
Coding Schemes	Applicable, for example, to the following visualization algorithms:
Entropy coding (e.g., Huffman, arithmetic coding)	Logarithmic plots (see Section 7.1); importance-based visualization; information-assisted visualization; magic lens; illustrative deformation.