"Can we reduce [...]? Yes we can!"

"I enjoyed reading this mathematically very sound paper."

"... an advance to an important problem often encountered ..."

Curve Reconstruction with Many Fewer Samples

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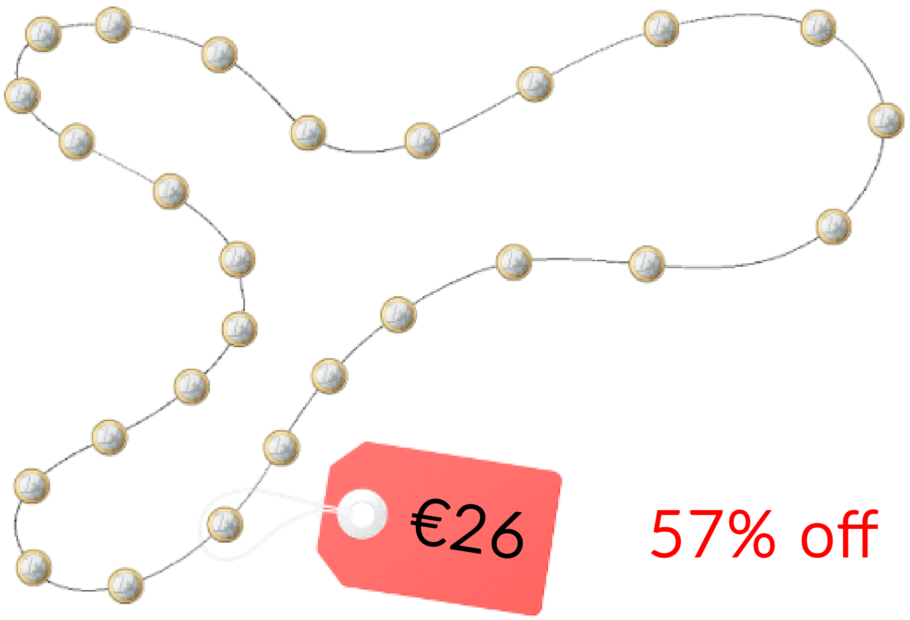
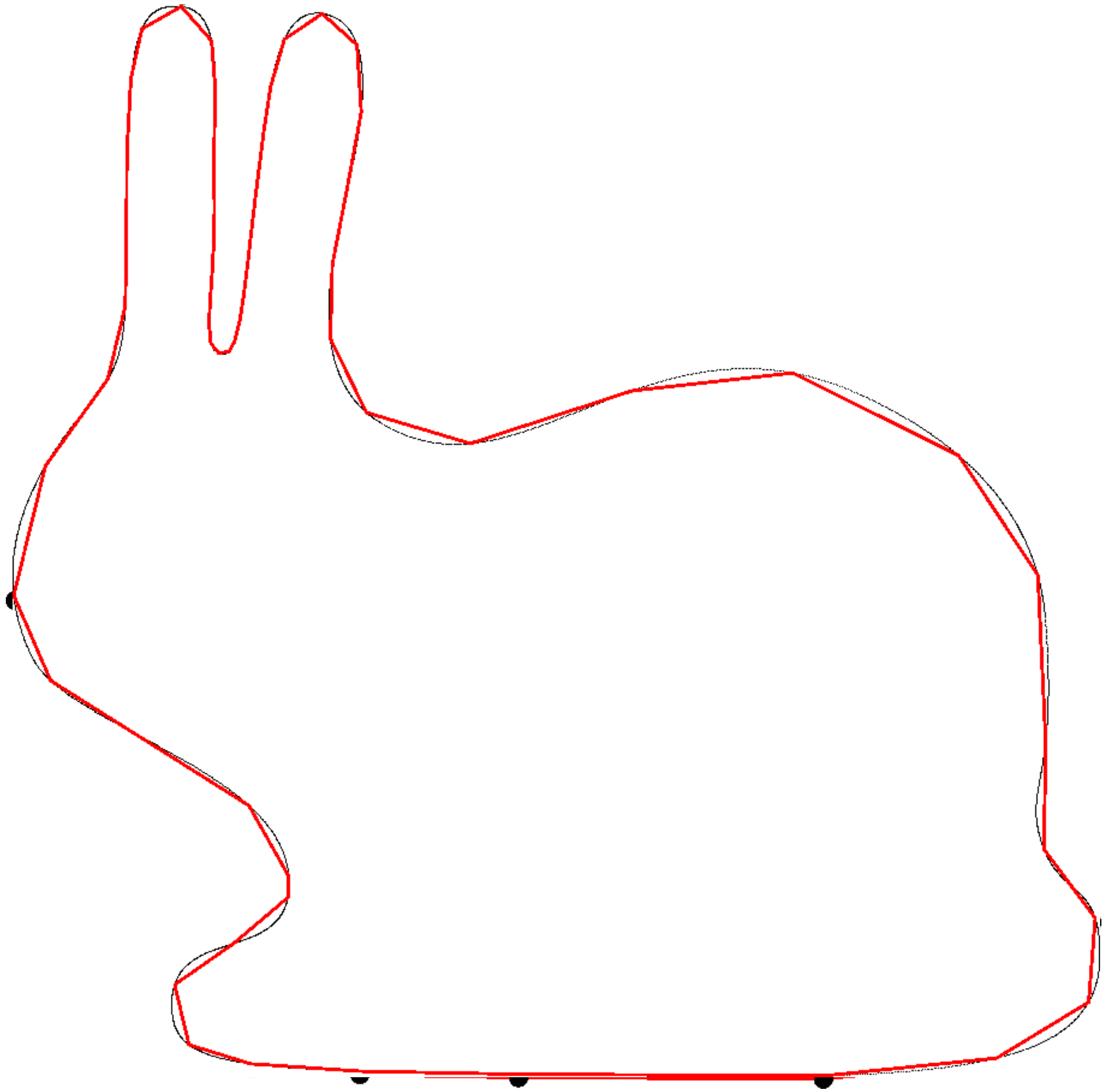
Why Sample Curves with Fewer Points?

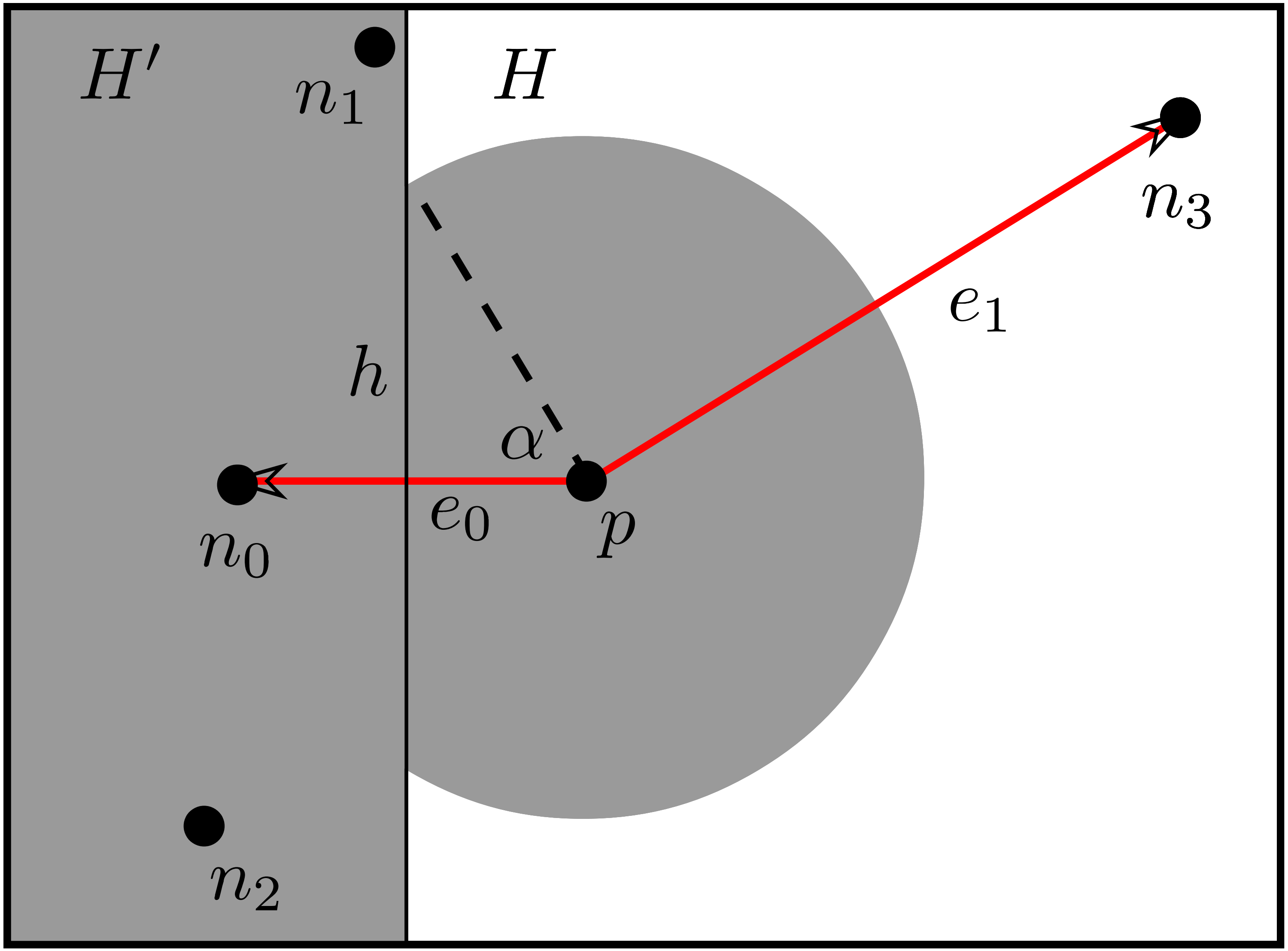
Each sample costs: 



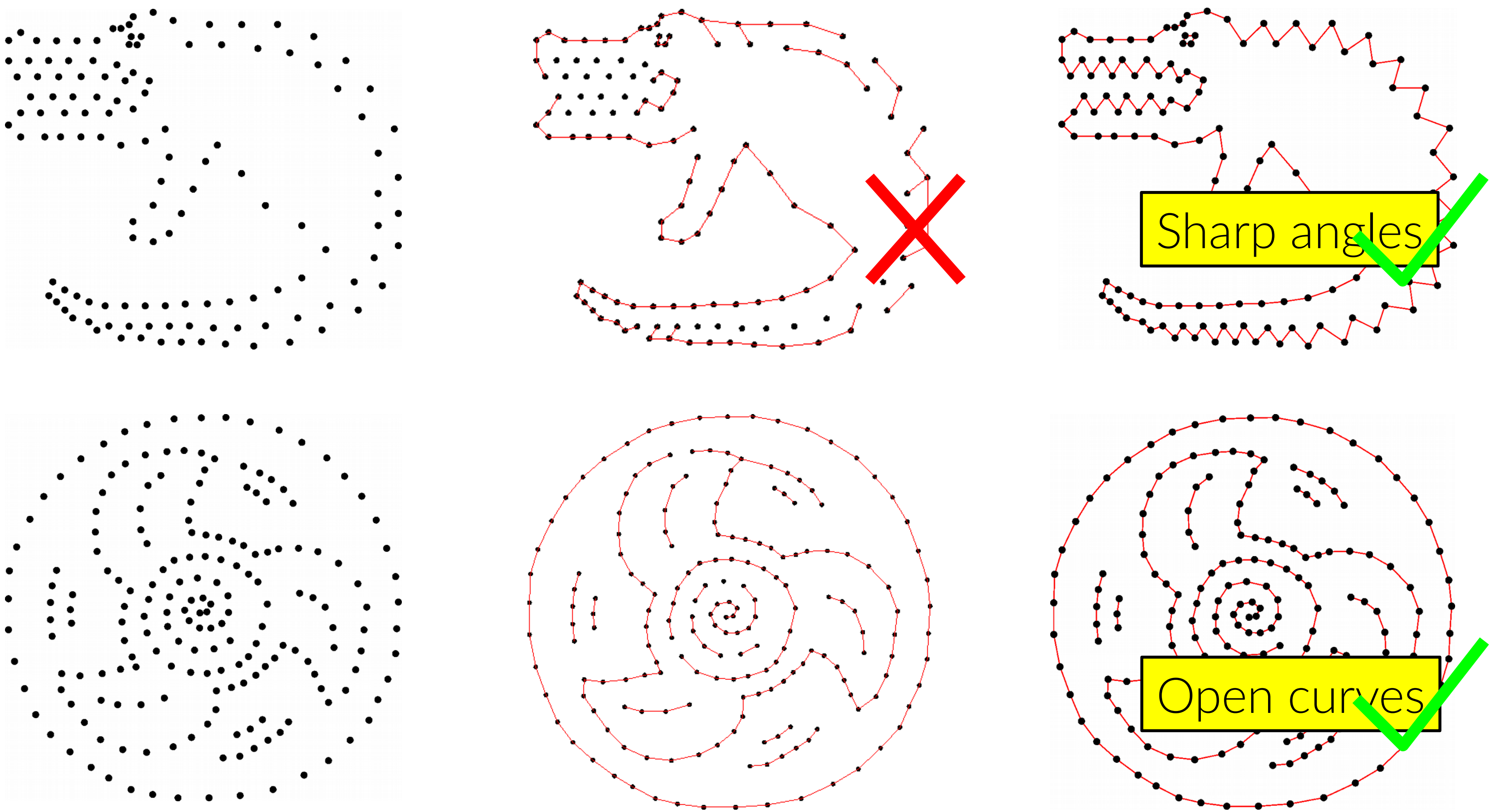
57% off



Algorithm HNN-CRUST

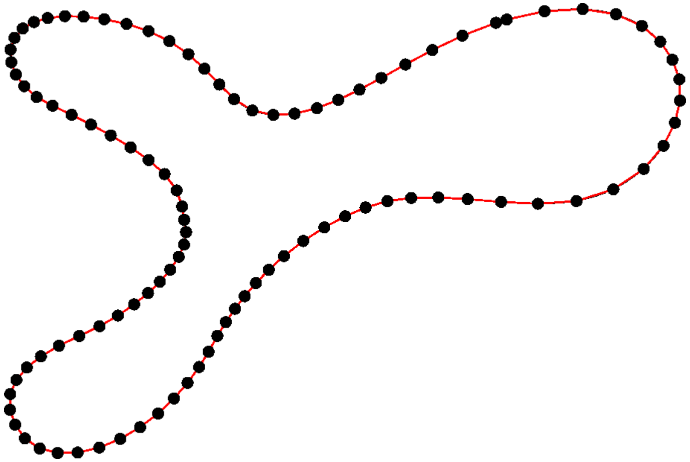


HNN-CRUST Reconstruction Results

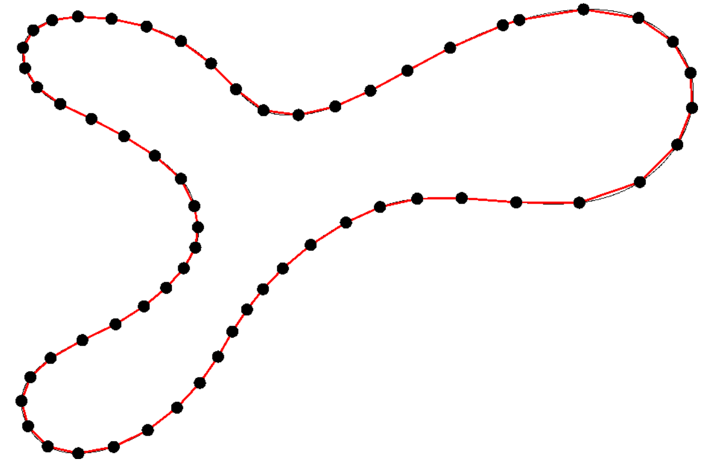


Samples | CRUST [Amenta et al. '98] | HNN-CRUST

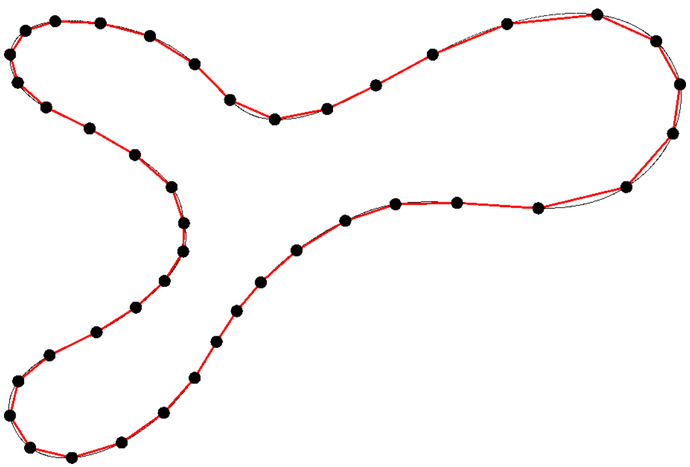
Earlier Sampling Conditions



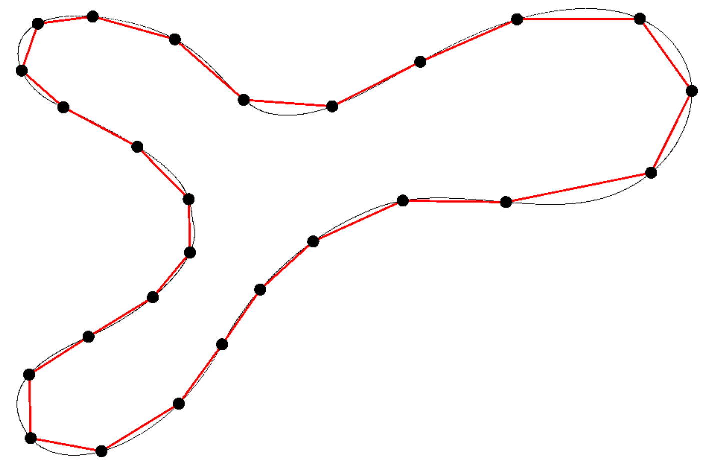
$\varepsilon<0.2$: CRUST [Amenta et al. ‘98]



$\varepsilon<0.\overline{3}$: NN-CRUST [Dey, Kumar ‘99]

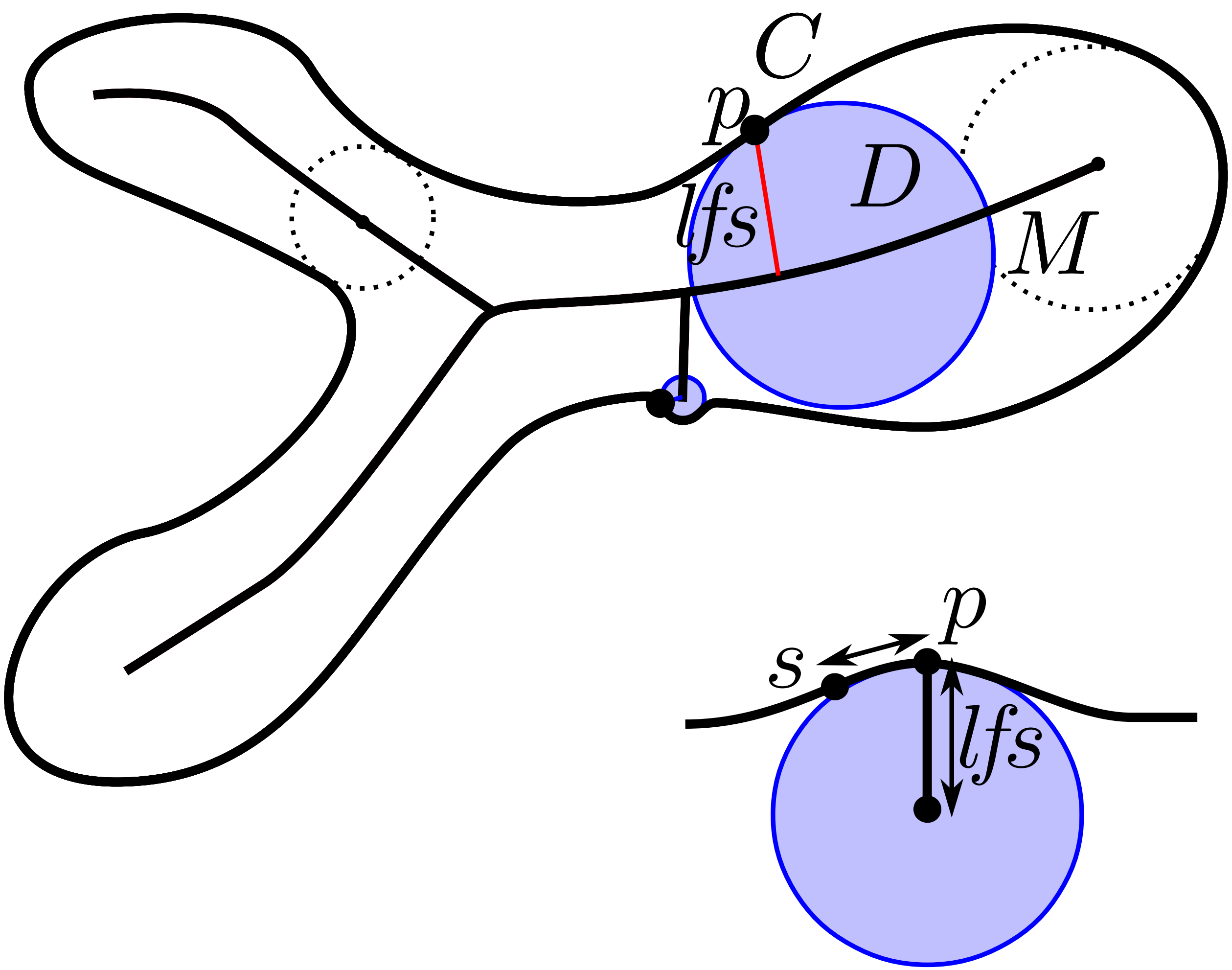


$\varepsilon<0.47$: Our HNN-CRUST



$\rho<0.9$: Our HNN-CRUST

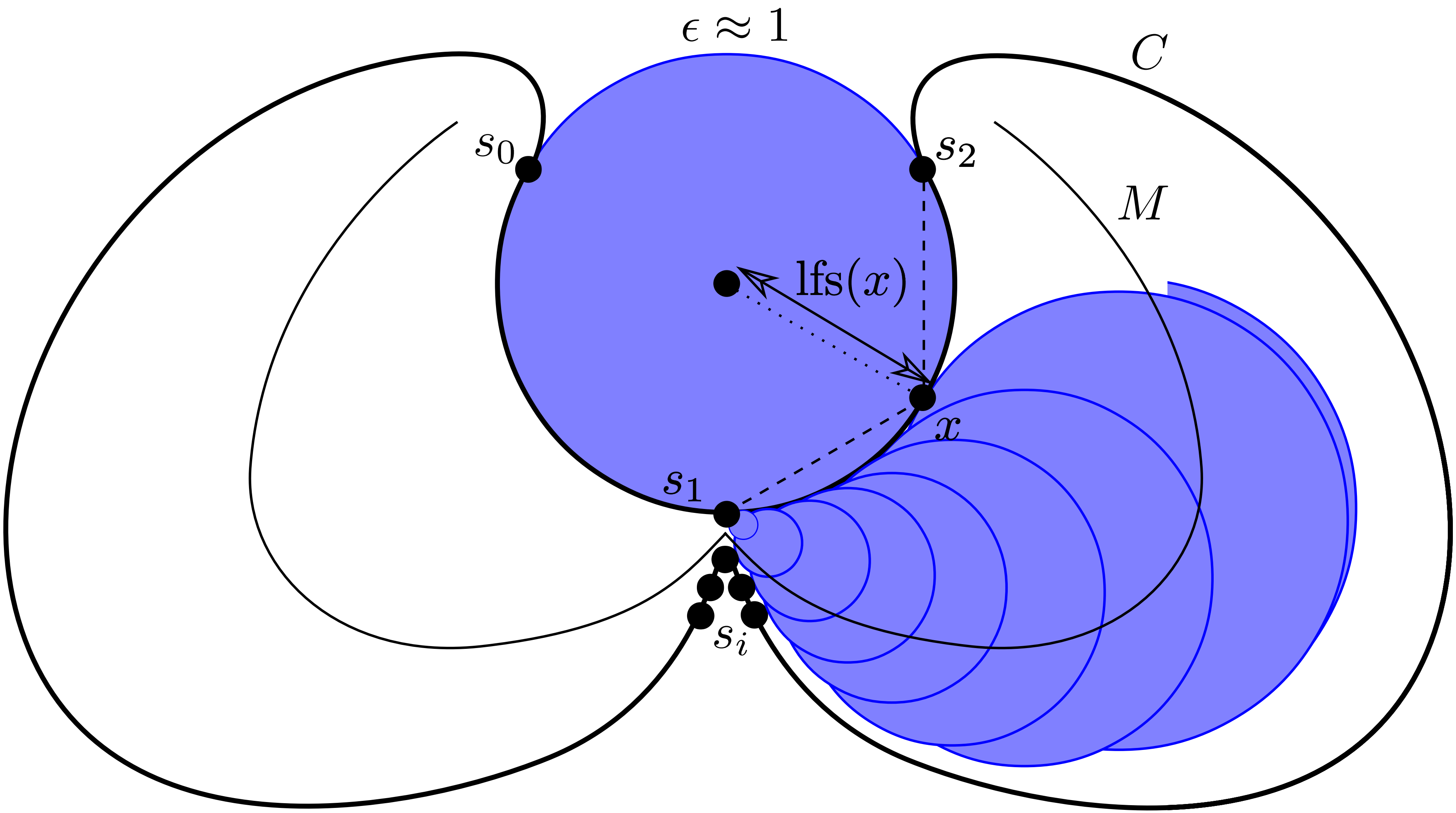
What is ε-Sampling?



M = medial axis
[Blum '67]
lfs = local feature size
[Ruppert '93]
D = disk empty of C

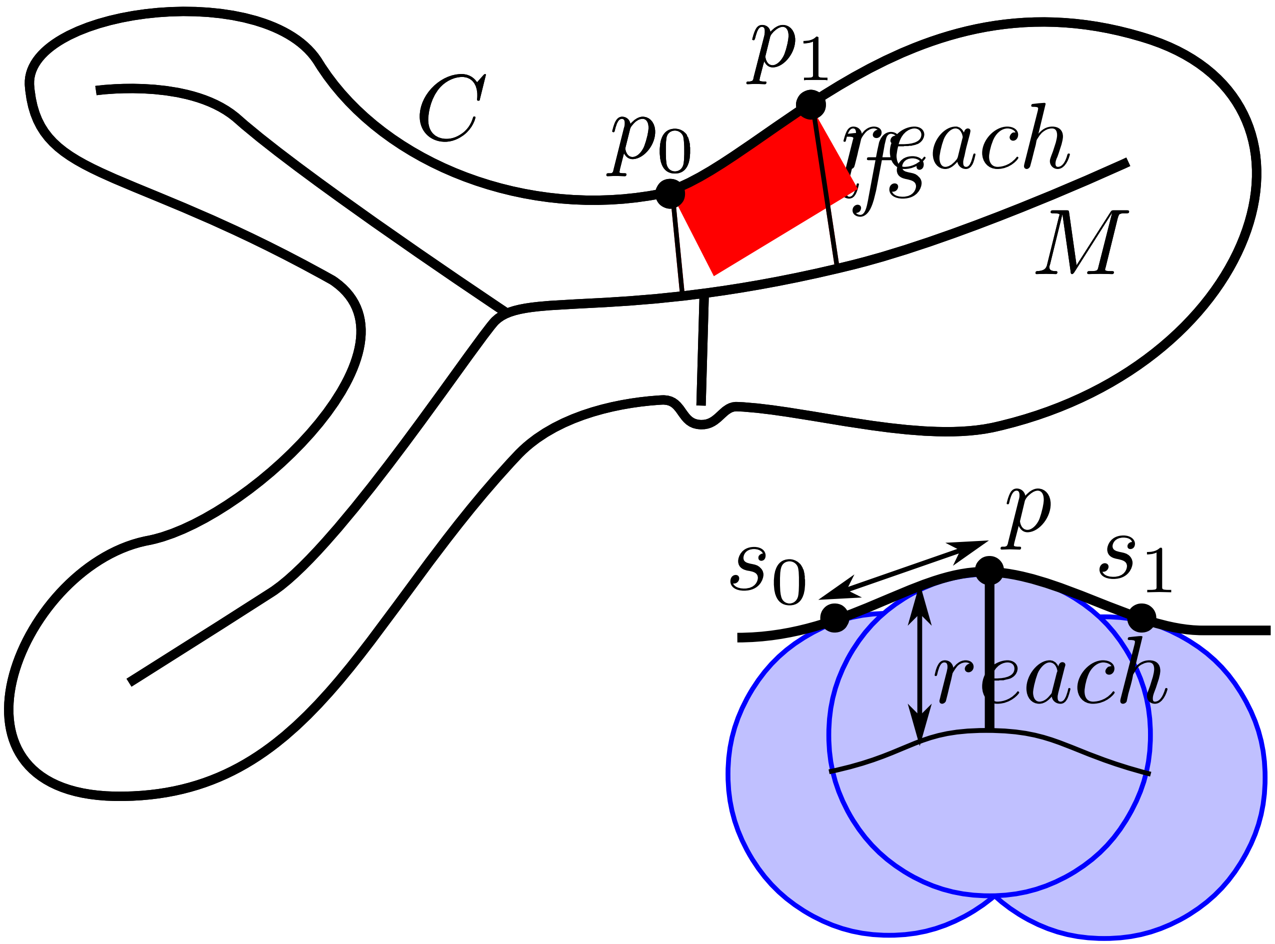
$||s,p|| < \varepsilon * lfs(p)$

The Problem of Large ε



Required lfs vanishes at samples → s_1 connects wrongly to s_i

So We Designed ρ-Sampling



Interval $I(p_0,p_1)$:

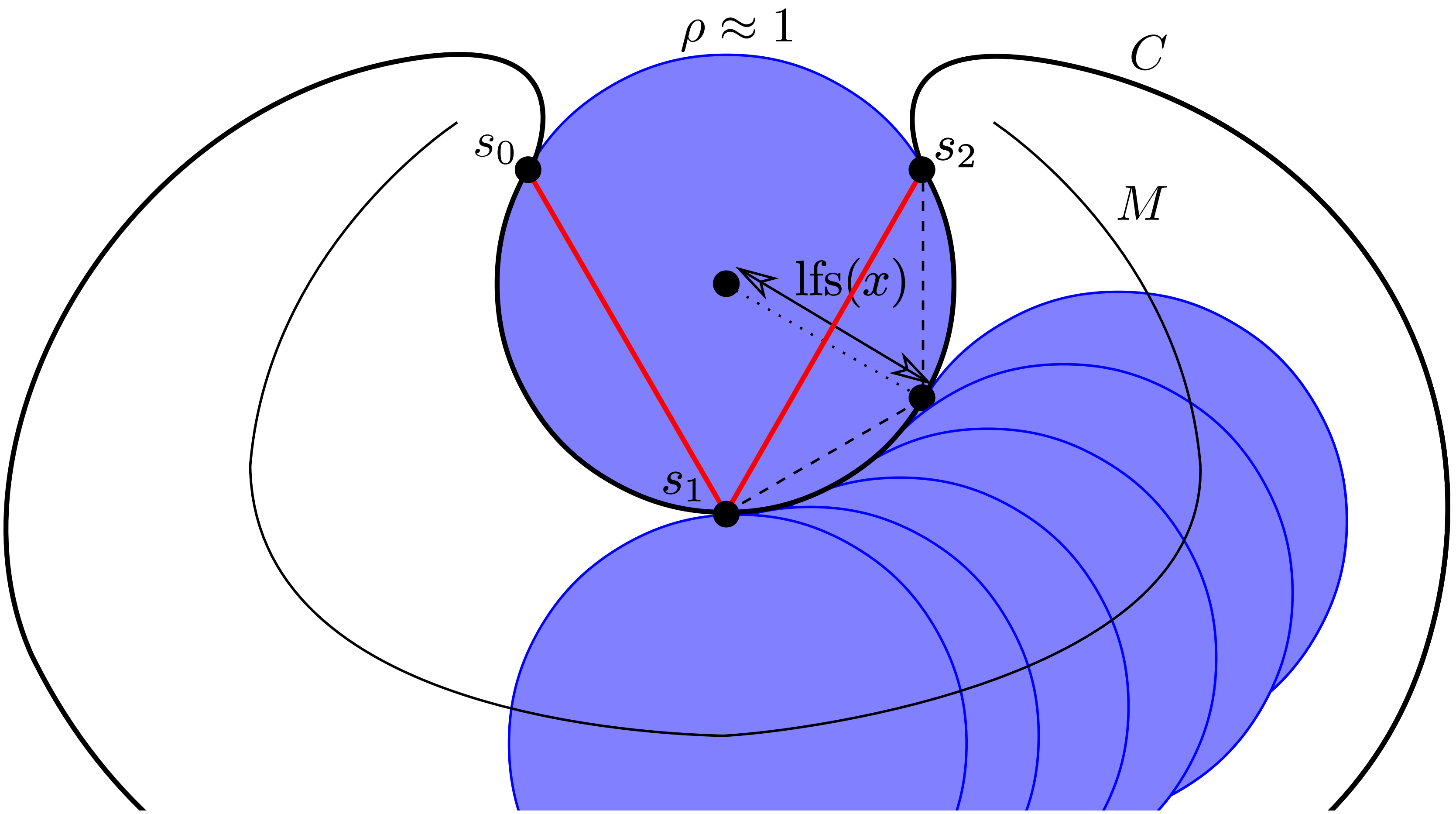
reach(I)=min *lfs*(I)

[Federer '59]

$||s,p|| < \rho^* reach(I)$

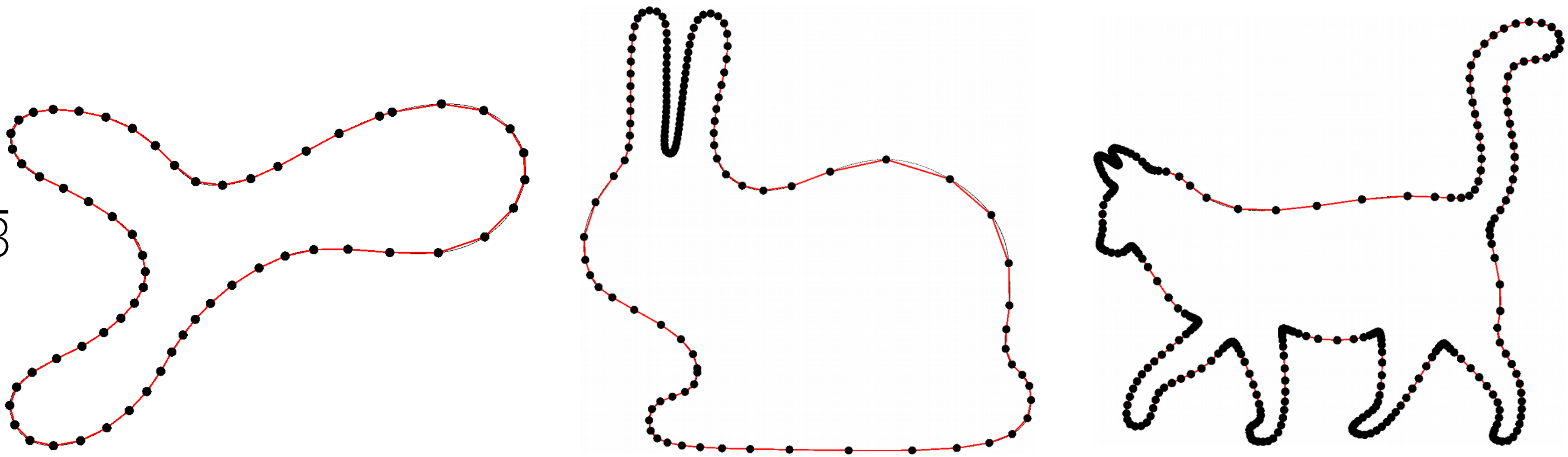
reach does not vanish at samples!

Works for Large ρ



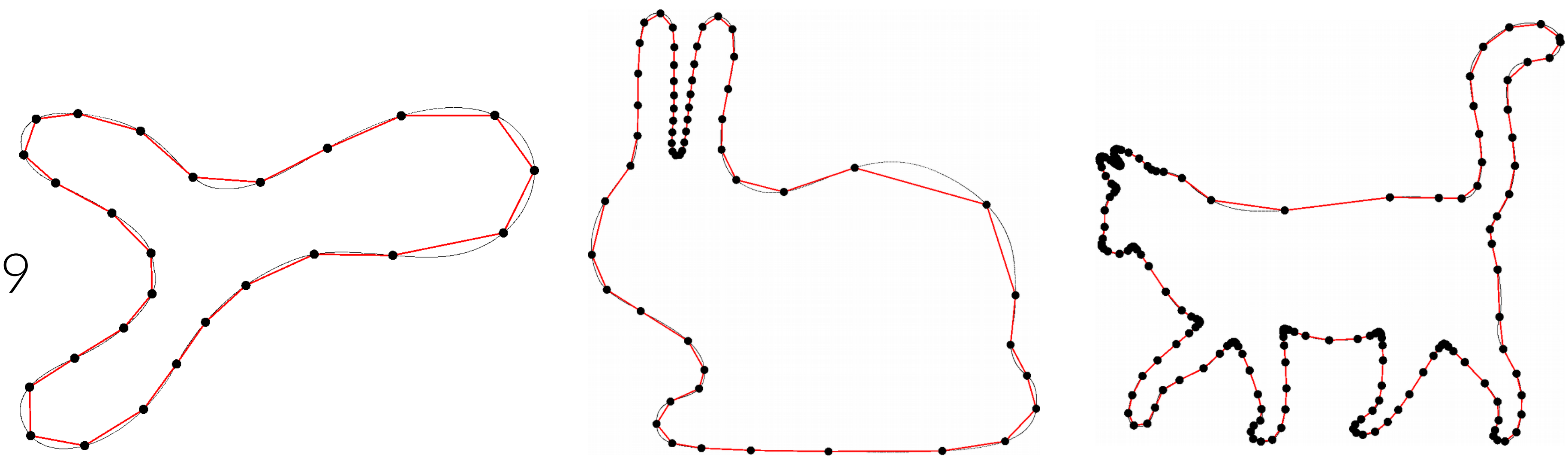
Results for ρ<0.9 Sampling

$\varepsilon<0.\overline{3}$



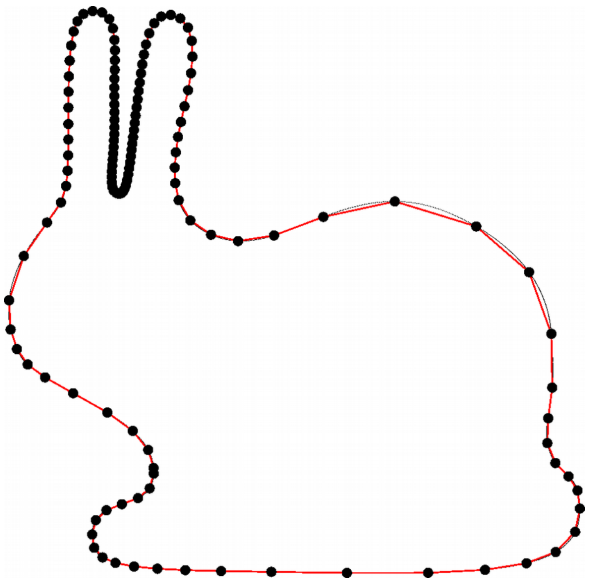
Samples: 61 131 356

$\rho<0.9$

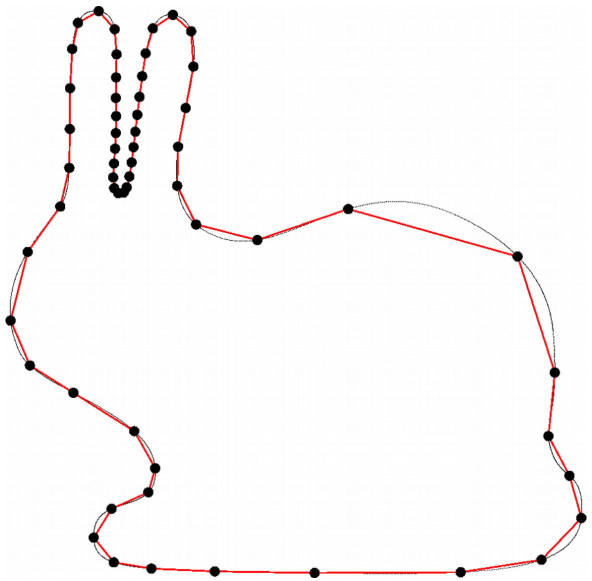


Samples: 26 58 180

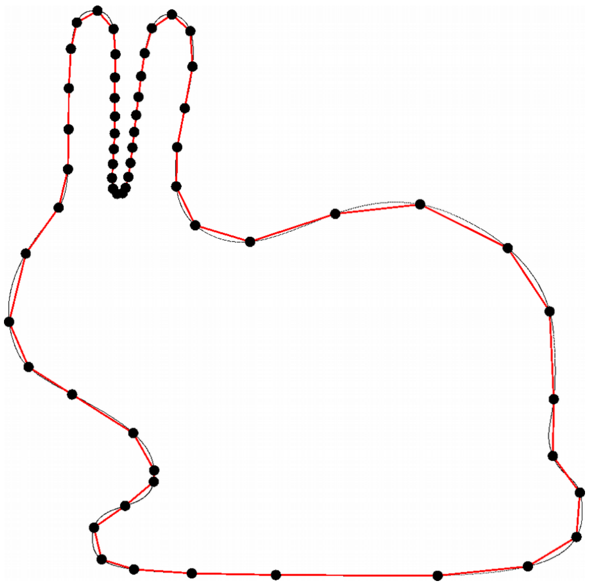
Bounding Reconstruction Distance



$\varepsilon<0.\overline{3}$:
131 samples



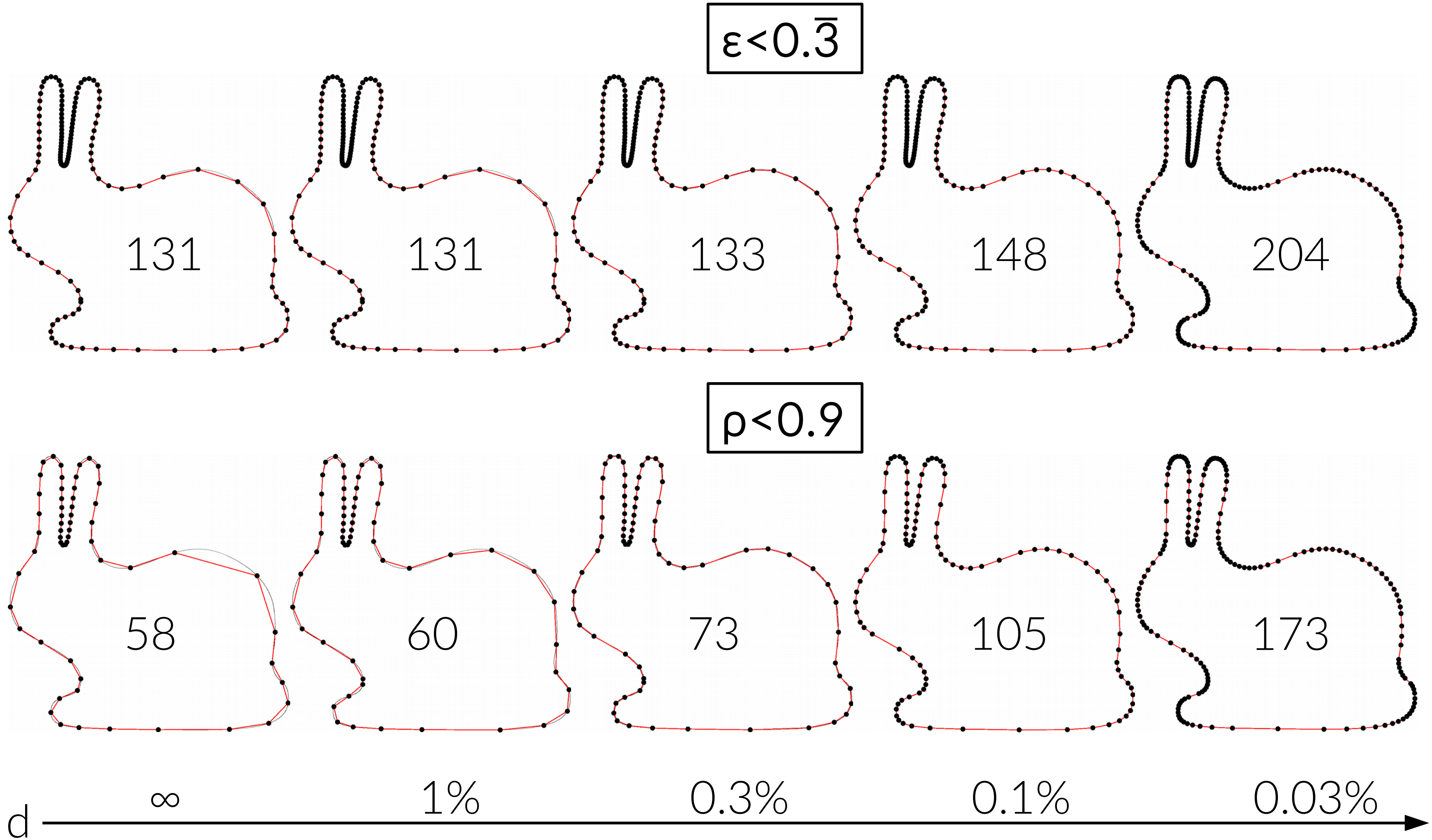
$\rho<0.9$:
58 samples



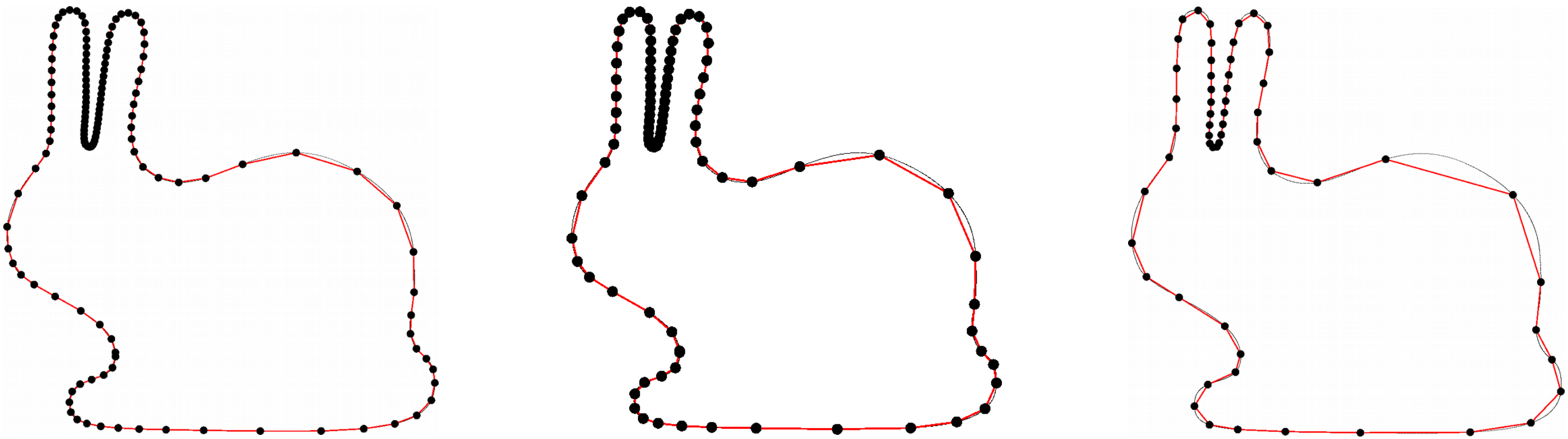
$\rho<0.9$, d=1%:
60 samples (+2)

d=bounded Hausdorff distance (in % of larger axis extent)

Reconstruction Distances Compared



Improved Bound for ε-Sampling

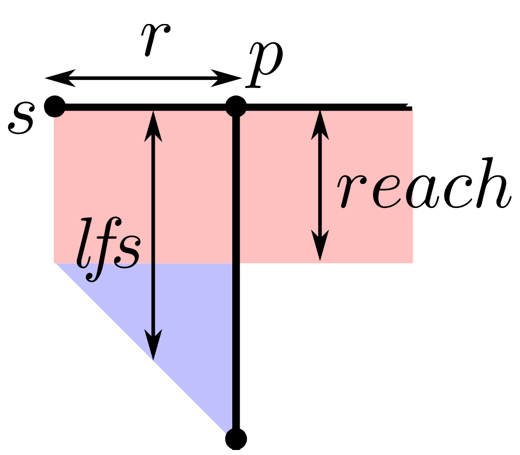


ε<0.$\overline{3}$, 131 samples ε<0.47, 94 samples ρ<0.9, 58 samples

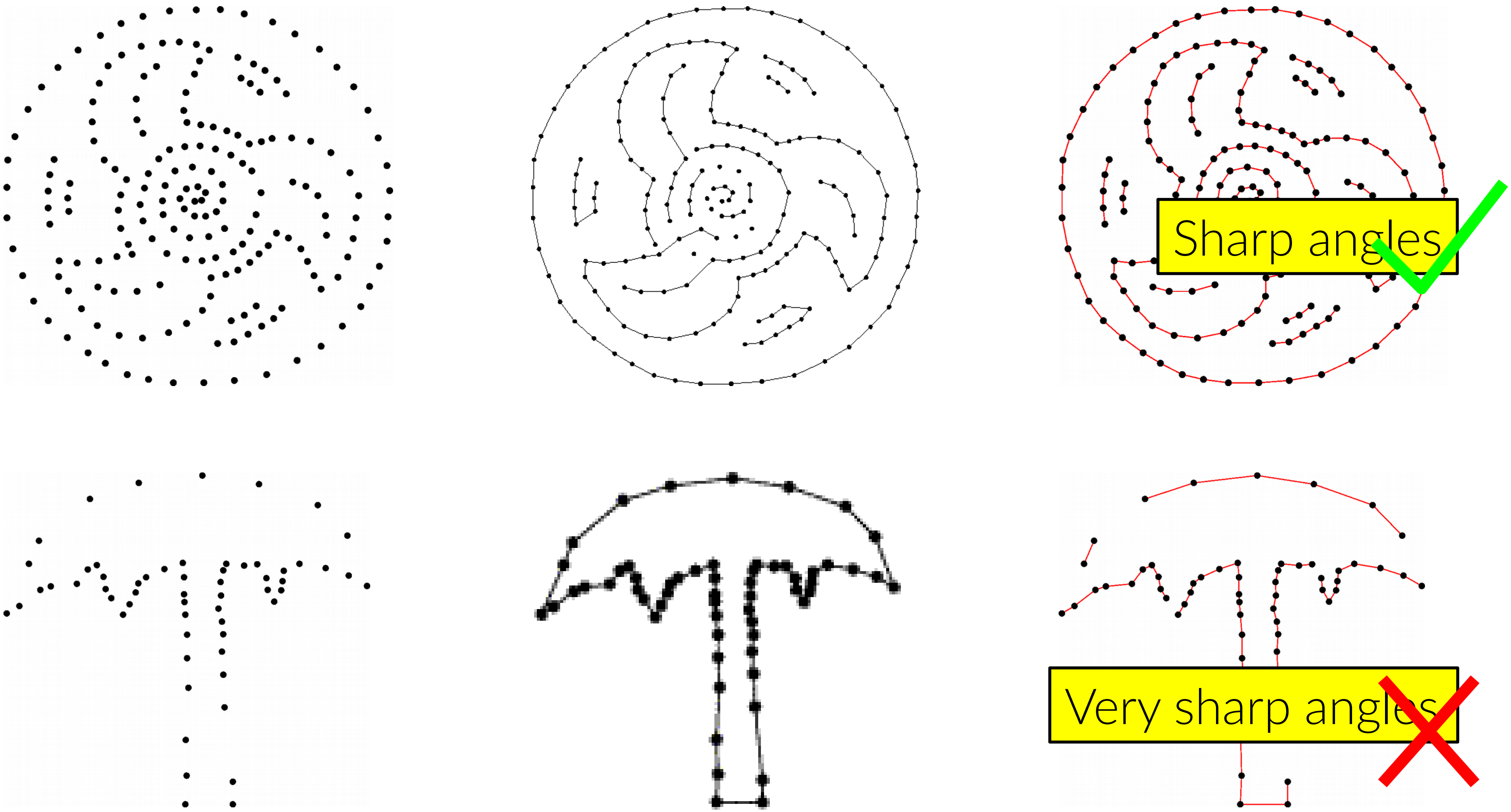
ε < r-sampling → ρ < r/(1 − r)-sampling

Proof: reach(I) ≥ (1-r)lfs(p)

ρ<0.9 → ε<0.47 (or ε<0.9 at constant curvature)

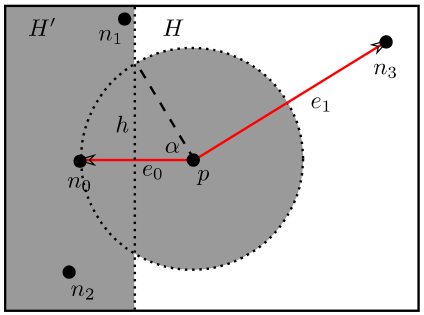


Limits of HNN-CRUST

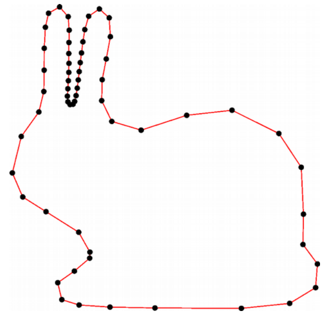


Samples | GathanG [Dey, Wenger ‘02] | HNN-CRUST

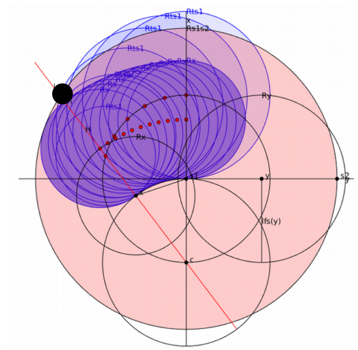
Conclusion and Outlook



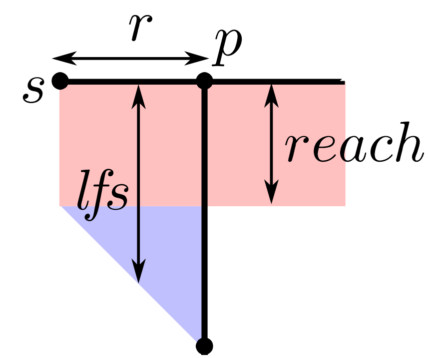
1) Simple variant HNN-CRUST



2) Sampling cond. ≡ reconstruction



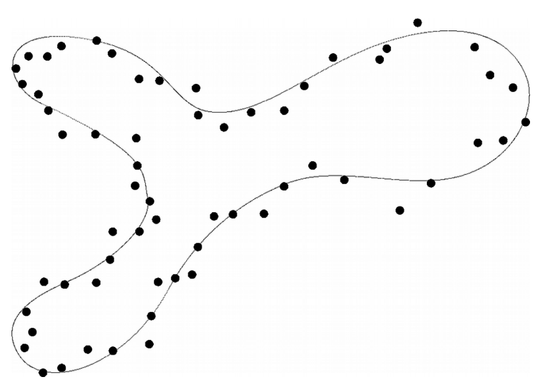
3) ρ<0.9 close to tight bound



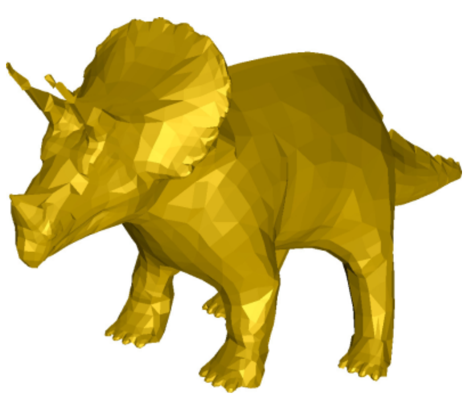
4) Corollary: $\varepsilon<0.\overline{3} \rightarrow \varepsilon<0.47$

All figures/tables reproducible from open source (link in paper)

Now extending it to:



noisy samples



3D

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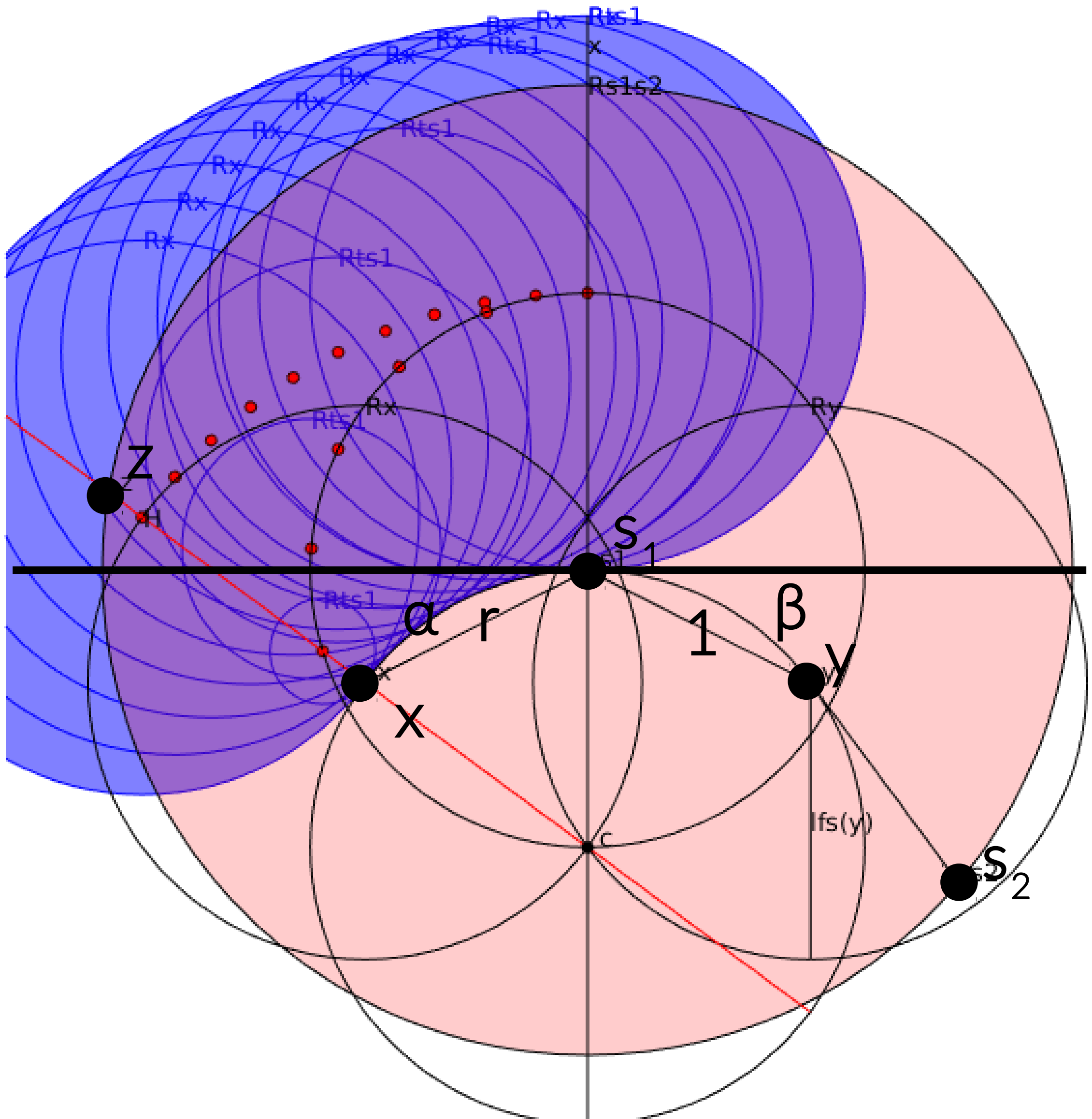
Computer-Assisted Proof of ρ<0.9



Case r=1, α=β=27°

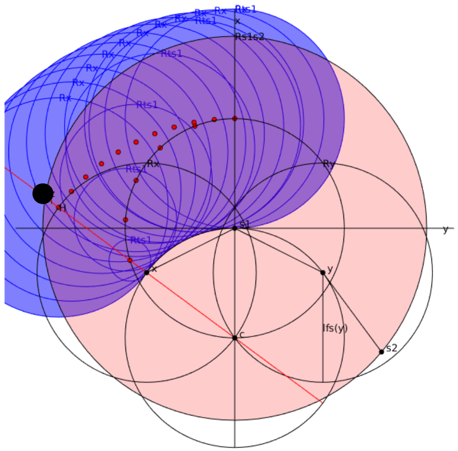
Blue disks = exclusion zone of C, must contain point z (=farthest connected to s_1 instead of s_2 by HNN-CRUST)

C is defined by points x, s_1, y, s_2

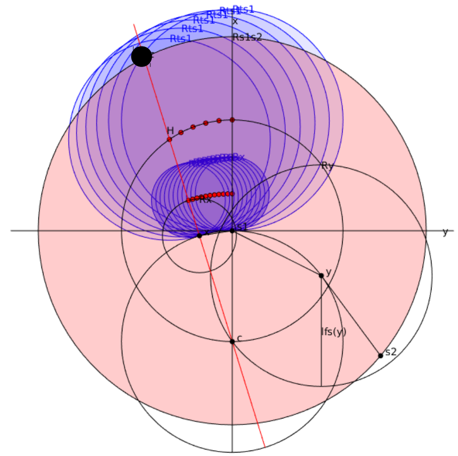
C is bounded by parameters:
$r=|s_0 s_1|/|s_1,s_2|$, in]0..1]
α, β with s_1-tangent, [0°..27°]

Sample parameter space in tiny steps, worst case combinations

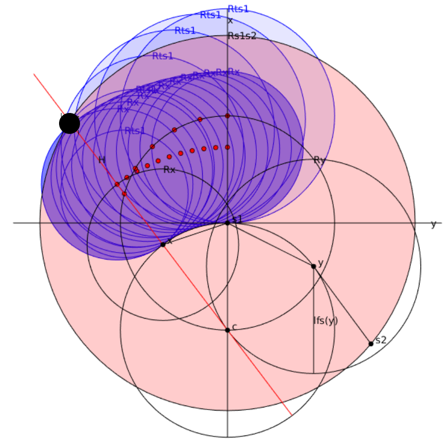
Computer-Assisted Proof – More Cases



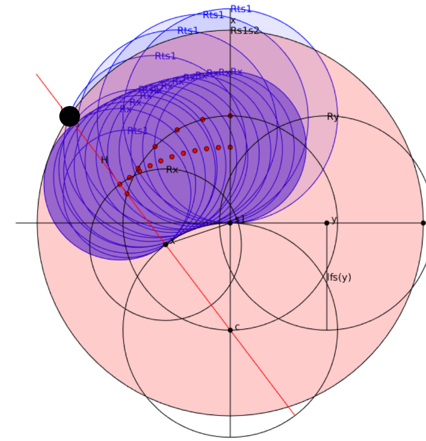
$r=1$, $\alpha=\beta=27°$



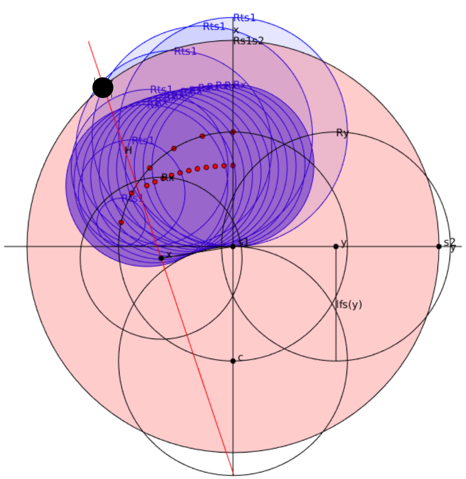
$r=\frac{1}{3}$, $\alpha=\beta=27°$



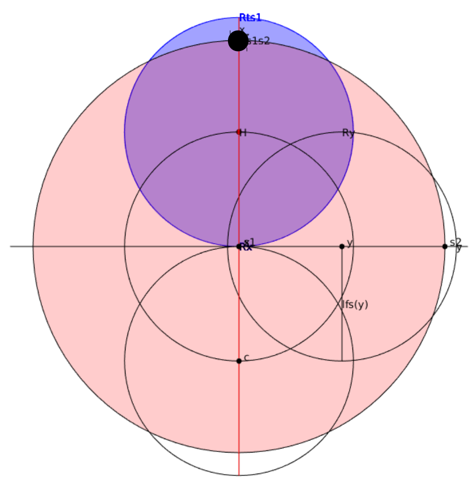
$r=\frac{1}{\sqrt{2}}$, $\alpha=\beta=27°$



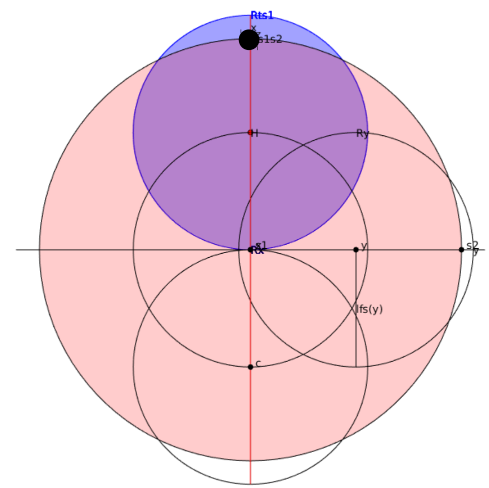
$r=\frac{1}{\sqrt{2}}$, $\alpha=27°$, $\beta=0°$



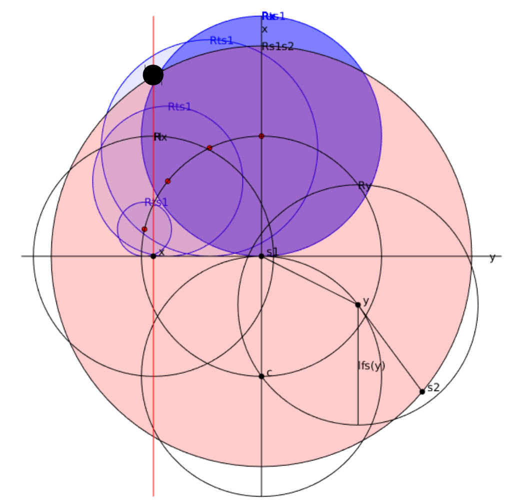
$r=\frac{1}{\sqrt{2}}$, $\alpha=13°$, $\beta=27°$



$r=\frac{1}{\sqrt{2}}$, $\alpha=\beta=0°$



$r=0$, $\alpha=\beta=0°$



$r=1$, $\alpha=0°$, $\beta=27°$