“Can we reduce [...]? Yes we can!”

“I enjoyed reading this mathematically very sound paper.”

“... an advance to an important problem often encountered ...”

Curve Reconstruction with Many Fewer Samples

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Why Sample Curves with Fewer Points?

Each sample costs:

- €61
- €26

57% off
Sampling Condition ↔ Reconstruction
Algorithm HNN-CRUST
HNN-CRUST Reconstruction Results

Samples | CRUST [Amenta et al. ‘98] | HNN-CRUST

Sharp angles

Open curves
Earlier Sampling Conditions

- $\epsilon < 0.2$: CRUST [Amenta et al. '98]
- $\epsilon < 0.3$: NN-CRUST [Dey, Kumar '99]
- $\epsilon < 0.47$: Our HNN-CRUST
- $\rho < 0.9$: Our HNN-CRUST
What is $\varepsilon$-Sampling?

- $M = \text{medial axis}$ [Blum ’67]
- $lfs = \text{local feature size}$ [Ruppert ‘93]
- $D = \text{disk empty of } C$

$$||s,p|| < \varepsilon \cdot lfs(p)$$
The Problem of Large $\varepsilon$

Required $\text{Lfs}$ vanishes at samples $\rightarrow s_1$ connects wrongly to $s_i$
So We Designed $\rho$-Sampling

Interval $I(p_0, p_1)$:
$\text{reach}(I) = \min lfs(I)$
[Federer '59]

$||s, p|| < \rho \ast \text{reach}(I)$

reach does not vanish at samples!
Works for Large $\rho$

\[ \rho \approx 1 \]

\[ s_0 \quad s_1 \quad s_2 \]

$C \quad M$
Results for ρ<0.9 Sampling

ε<0.3

Samples: 61

ρ<0.9

Samples: 26
Bounding Reconstruction Distance

ε < 0.3: 
131 samples

ρ < 0.9: 
58 samples

ρ < 0.9, d = 1%: 
60 samples (+2)

d = bounded Hausdorff distance (in % of larger axis extent)
Reconstruction Distances Compared

$\varepsilon < 0.3$

$\rho < 0.9$

$d$  

$\infty$  

1%  

0.3%  

0.1%  

0.03%
Improved Bound for $\varepsilon$-Sampling

$\varepsilon < 0.3$, 131 samples

$\varepsilon < 0.47$, 94 samples

$\rho < 0.9$, 58 samples

$\varepsilon < r$-sampling $\rightarrow$ $\rho < r/(1 - r)$-sampling

Proof: $\text{reach}(l) \geq (1-r)\text{lfs}(p)$

$\rho < 0.9 \rightarrow \varepsilon < 0.47$ (or $\varepsilon < 0.9$ at constant curvature)
Limits of HNN-CRUST

Samples | GathanG [Dey, Wenger ‘02] | HNN-CRUST

Sharp angles

Very sharp angles
Conclusion and Outlook

1) Simple variant HNN-CRUST
2) Sampling cond. ≡ reconstruction
3) $\rho < 0.9$ close to tight bound
4) Corollary: $\varepsilon < 0.\overline{3} \rightarrow \varepsilon < 0.47$

All figures/tables reproducible from open source (link in paper)

Now extending it to:

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noisy samples
3D
Computer-Assisted Proof of $\rho<0.9$

Blue disks = exclusion zone of $C$, must contain point $z$ (=farthest connected to $s_1$ instead of $s_2$ by HNN-CRUST)

$C$ is defined by points $x$, $s_1$, $y$, $s_2$

$C$ is bounded by parameters: $r=|s_0s_1|/|s_1s_2|$, in $[0..1]$ $\alpha$, $\beta$ with $s_1$-tangent, $[0^\circ..27^\circ]$

Sample parameter space in tiny steps, worst case combinations

Case $r=1$, $\alpha=\beta=27^\circ$
Computer-Assisted Proof – More Cases

$r=1, \alpha=\beta=27^\circ$

$r=\frac{1}{3}, \alpha=\beta=27^\circ$

$r=\frac{1}{\sqrt{2}}, \alpha=\beta=27^\circ$

$r=\frac{1}{\sqrt{2}}, \alpha=27^\circ, \beta=0^\circ$

$r=\frac{1}{\sqrt{2}}, \alpha=13^\circ, \beta=27^\circ$

$r=\frac{1}{\sqrt{2}}, \alpha=\beta=0^\circ$

$r=0, \alpha=\beta=0^\circ$

$r=1, \alpha=0^\circ, \beta=27^\circ$