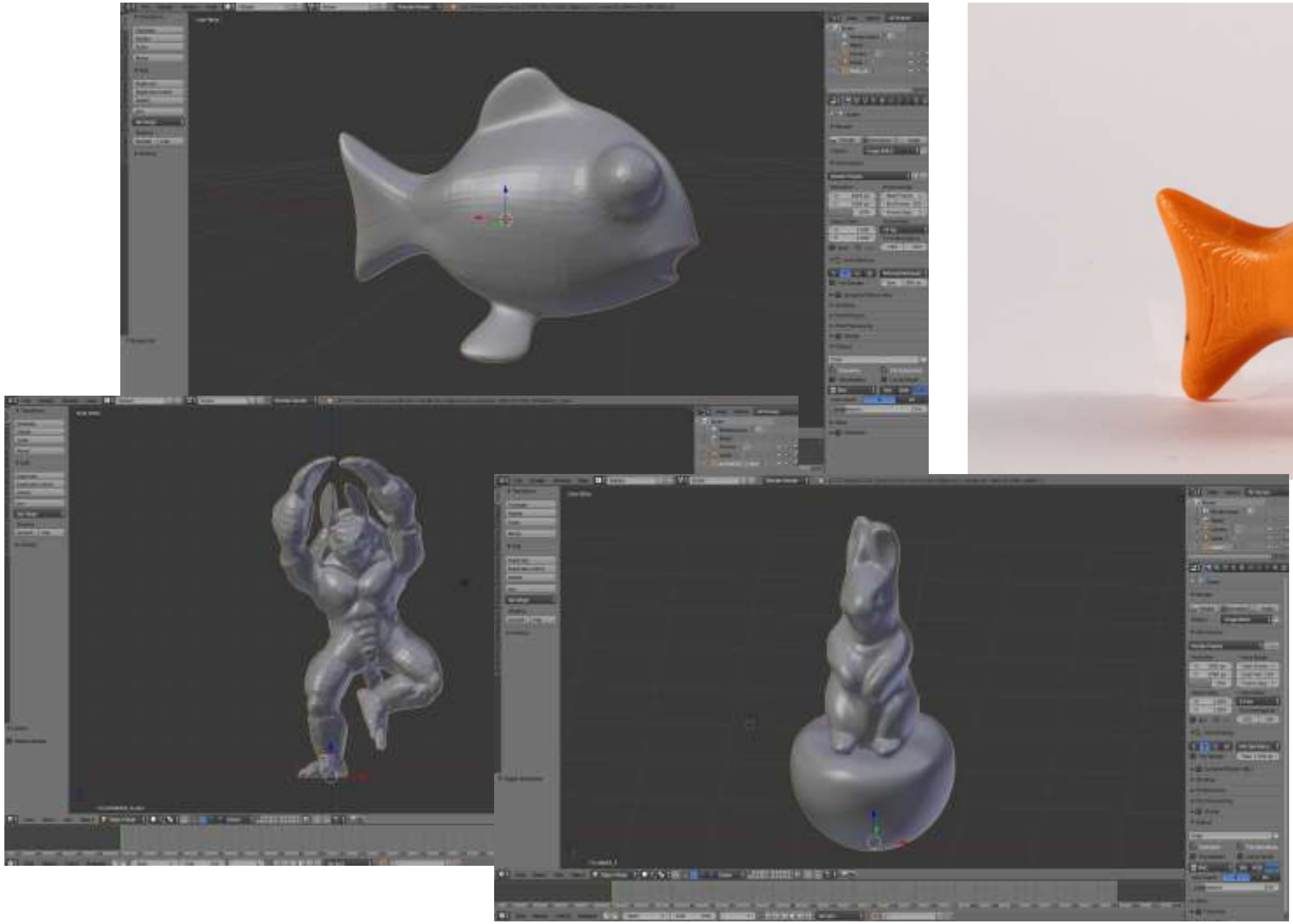
Shape Optimization for Consumer-Level 3D Printing

Przemyslaw Musialski

TU Wien

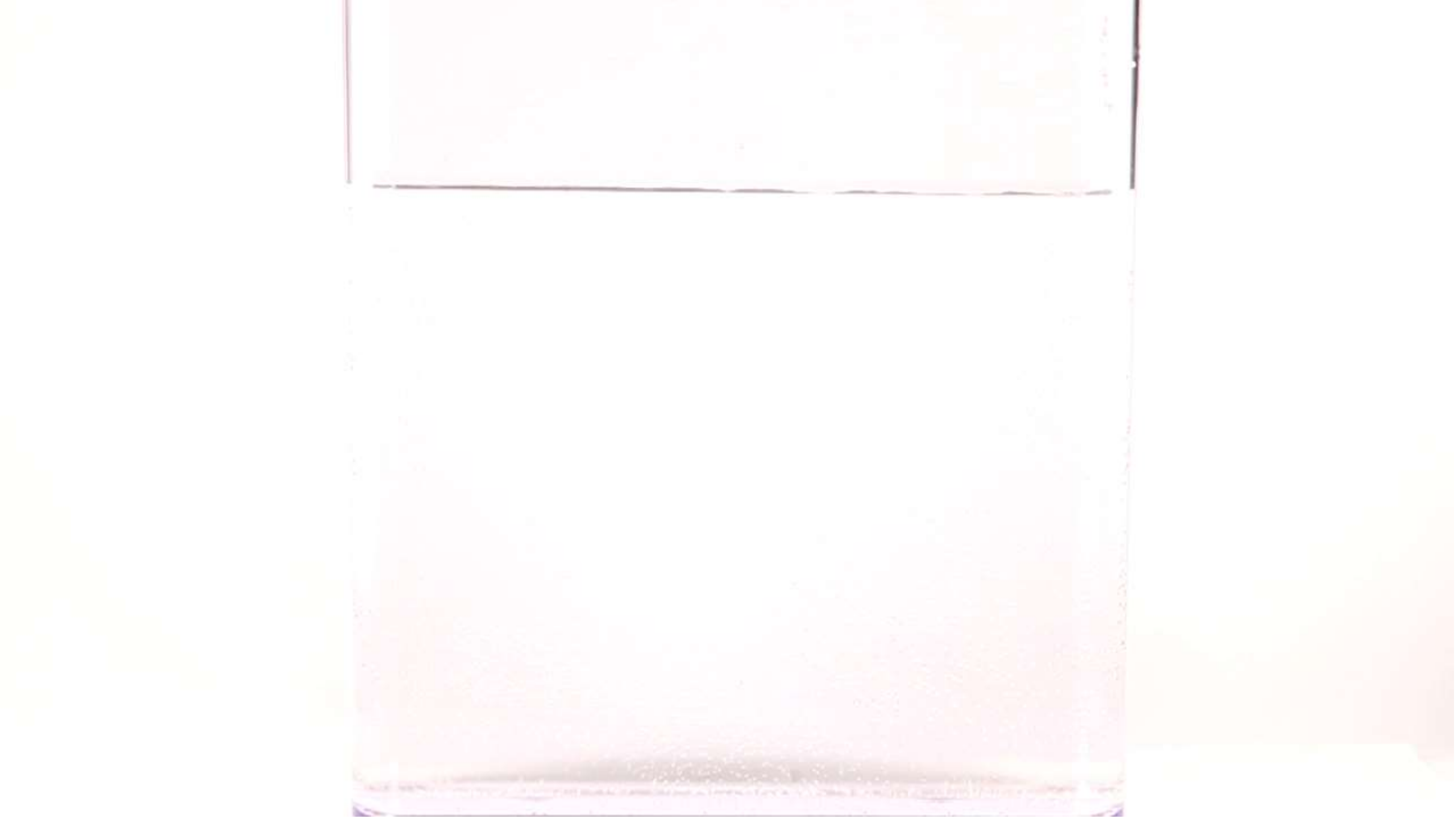
Motivation

3D Modeling…



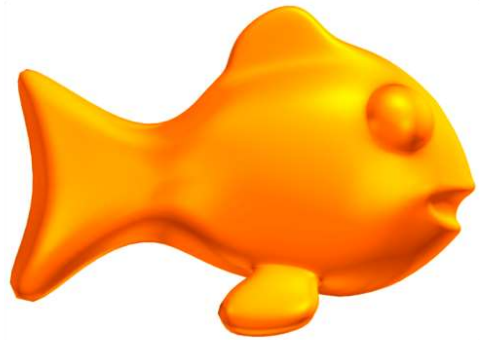
3D Printing…

Motivation



Goals

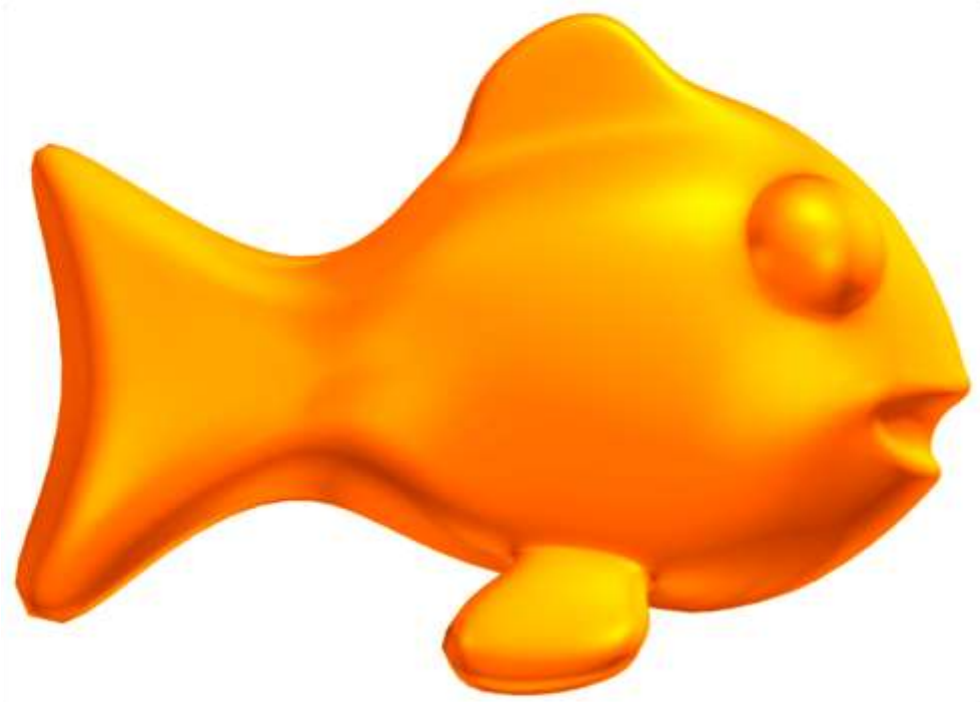
1. Optimize the shape to fulfill the desired goals



Goals

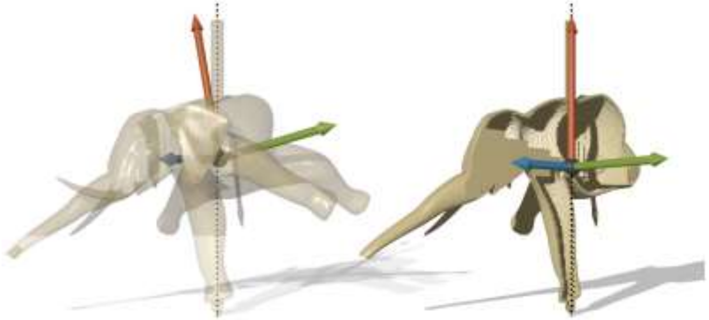
1. Optimize the shape to fulfill the desired goals

2. Keep the input shape deformation minimal

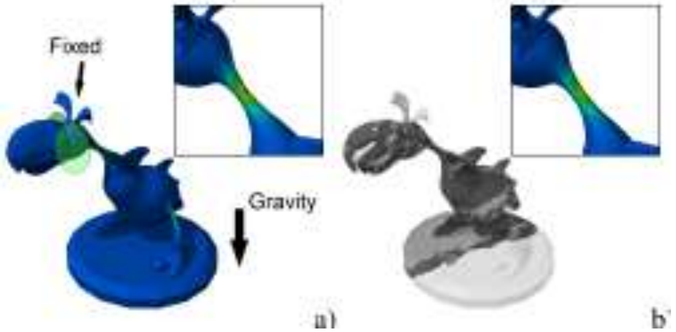


Related Work

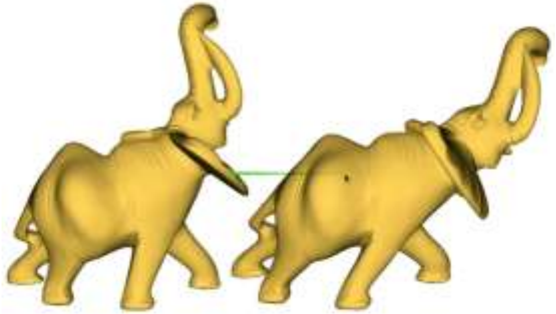
- Optimization of Mass Properties
  - [Prevost et al. 2013]
  - [Baecher et al. 2014]



- Structural Optimization
  - [Stava et al. 2012]
  - [Lu et al. 2014]

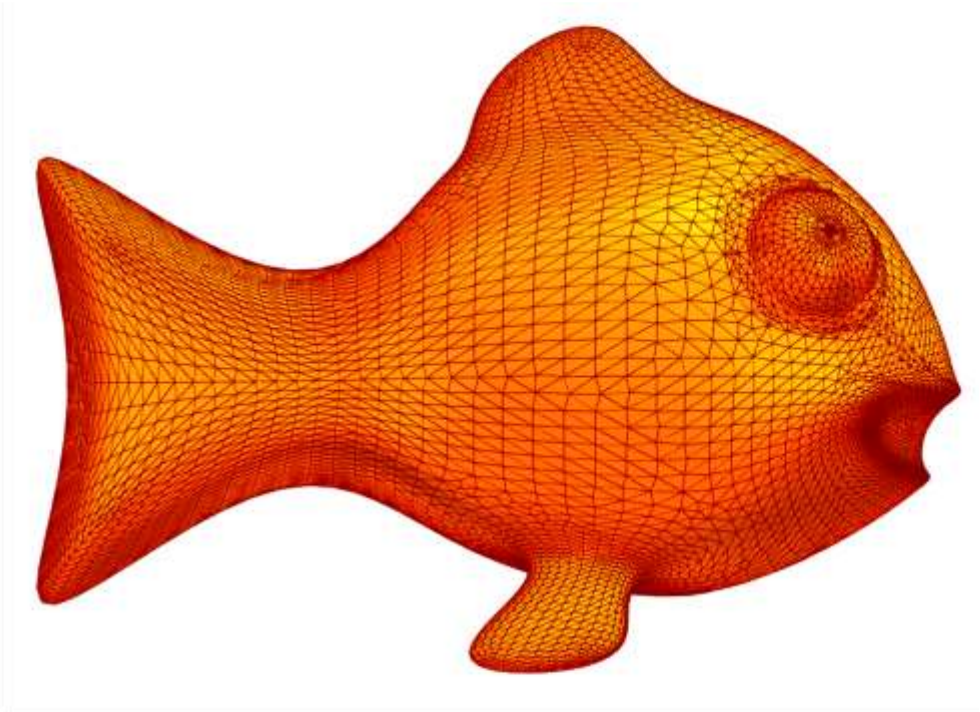


- Reduced Order Models
  - [Pentland and Williams 1989]
  - [von Tycowitch et al. 2013]



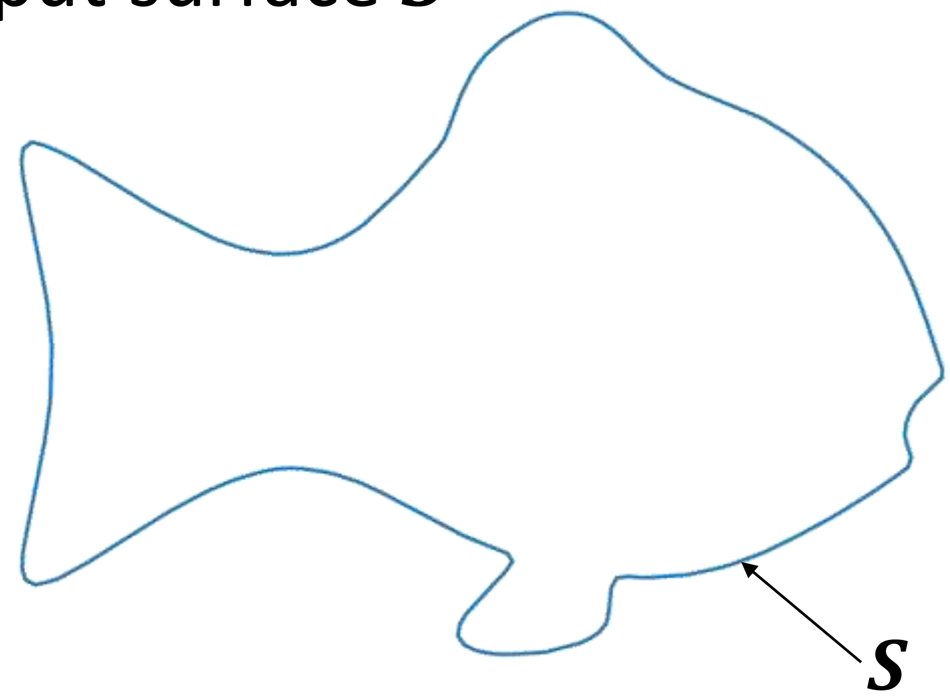
Shape Optimization

Input and Output



Input and Output

- Input surface $\boldsymbol{S}$



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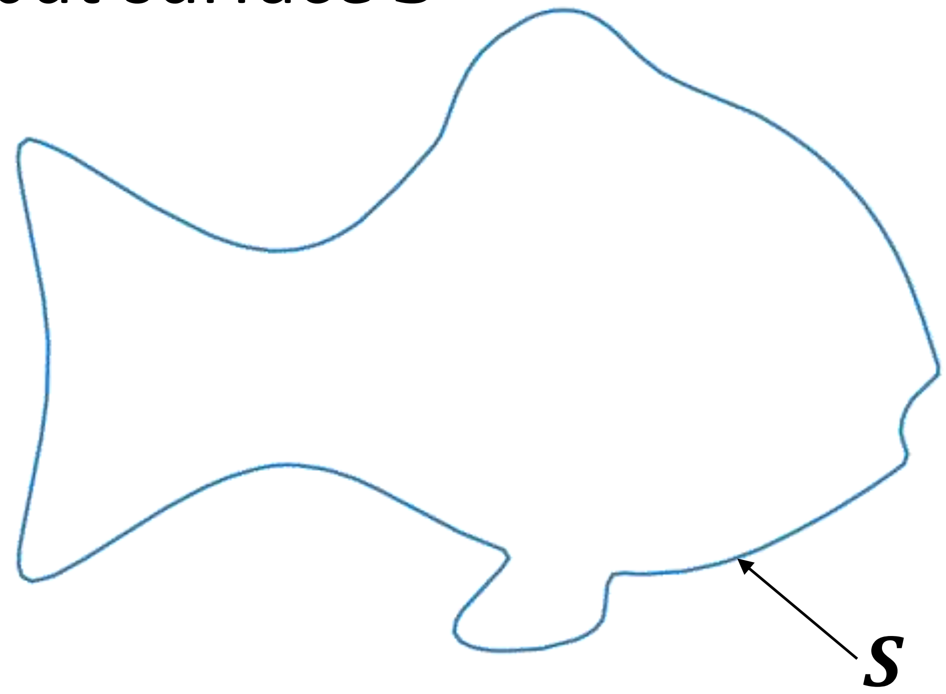
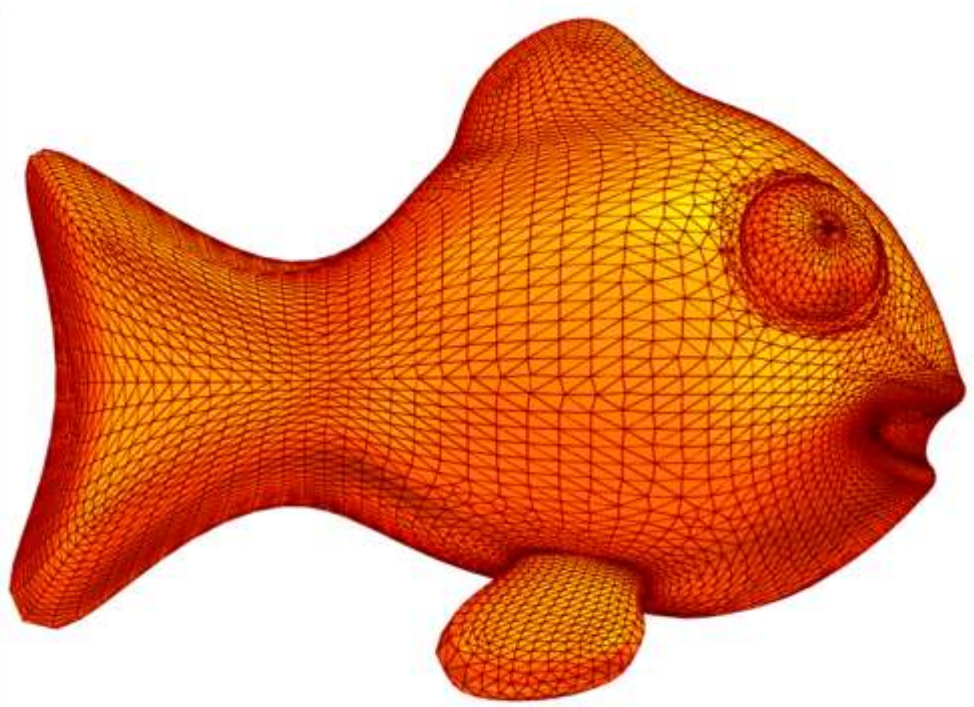
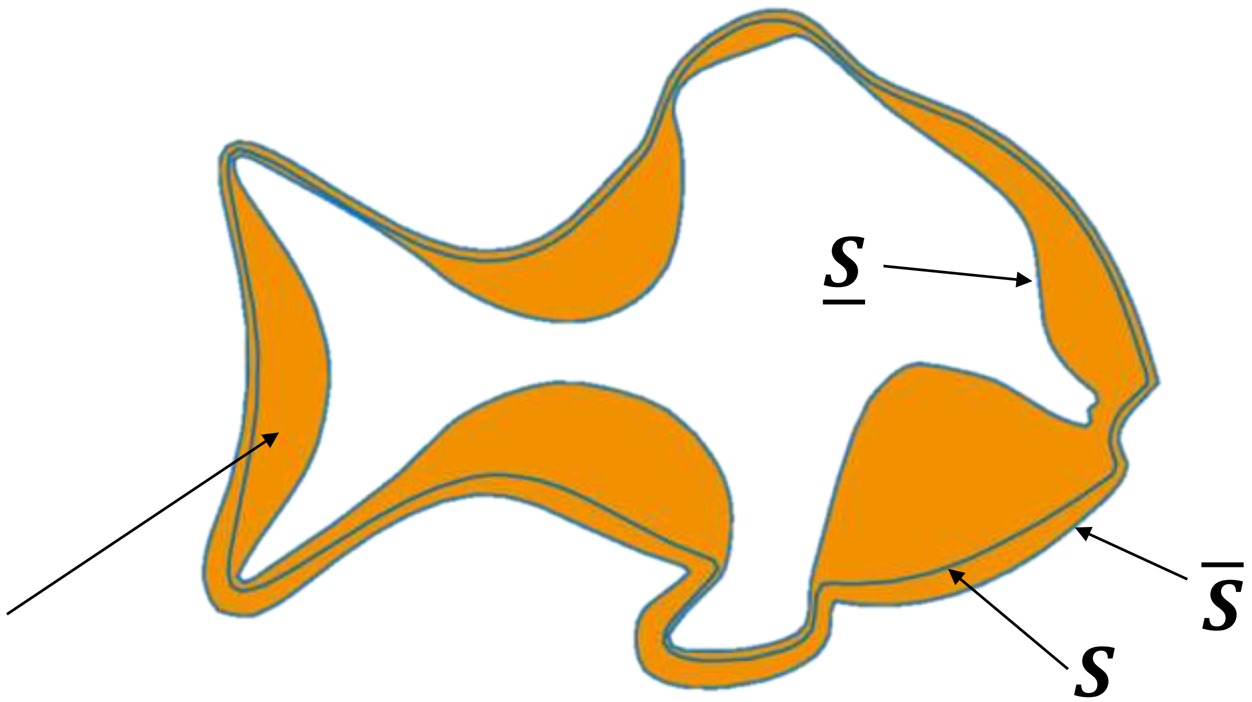




Input and Output

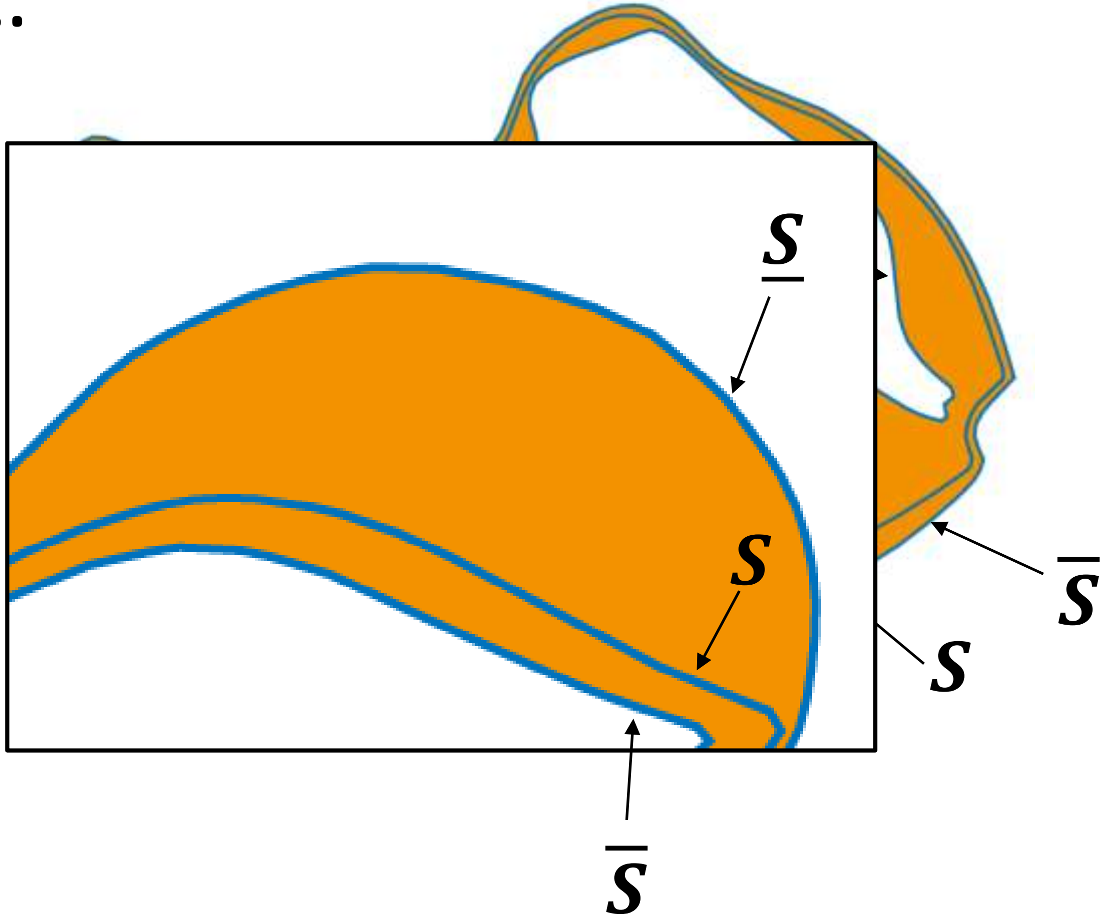
- input surface $\boldsymbol{S}$
- output: two surfaces
  - outer offset surface $\overline{\boldsymbol{S}}$
  - inner offset surface $\underline{\boldsymbol{S}}$
- **solid body** between $\overline{\boldsymbol{S}}$ and $\underline{\boldsymbol{S}}$



Offset Surfaces

- surface deformation by offset:

$$\underline{\boldsymbol{x}}_{\boldsymbol{i}} = \boldsymbol{x}_{\boldsymbol{i}} + \underline{\delta}_i \boldsymbol{v}_{\boldsymbol{i}}$$

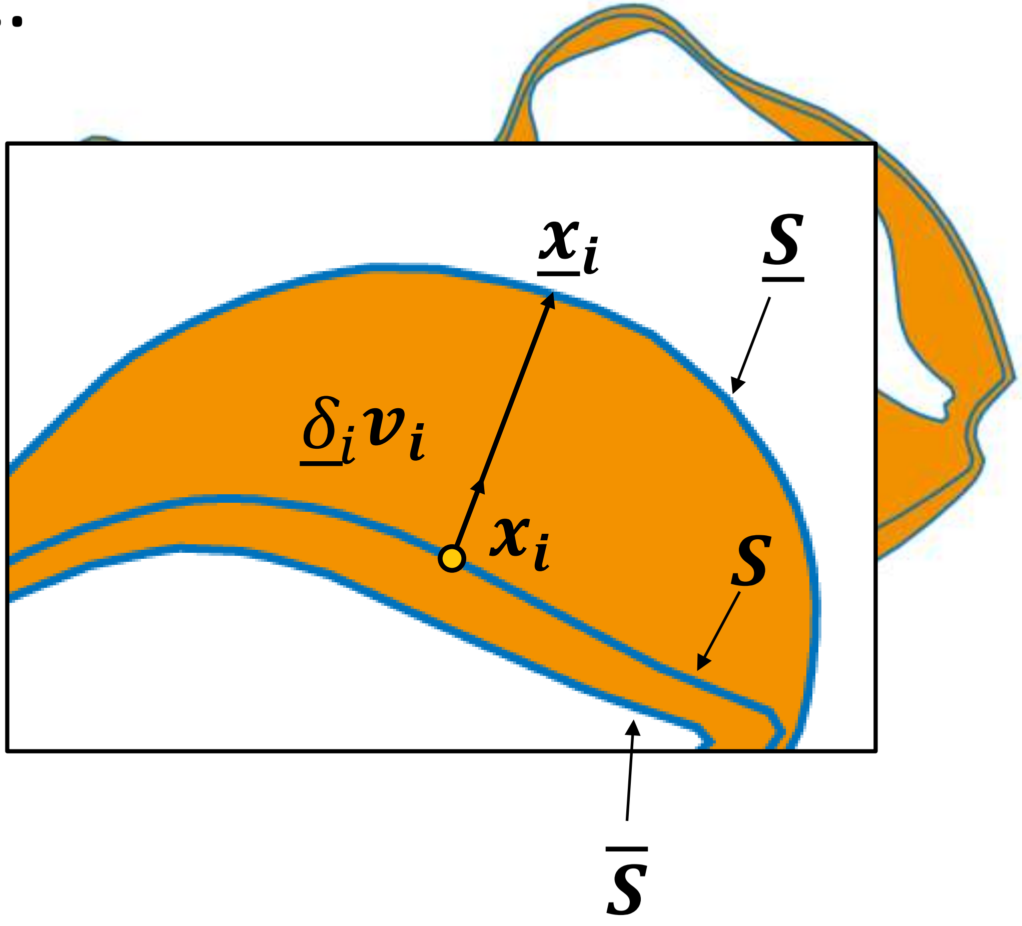


Offset Surfaces

- surface deformation by offset:

$$\underline{\boldsymbol{x}}_i = \boldsymbol{x}_i + \underline{\delta}_i \boldsymbol{v}_i$$

  - for each vertex $\boldsymbol{x}_i$
  - along $\boldsymbol{v}_i$
  - add an individual offset $\underline{\delta}_i$



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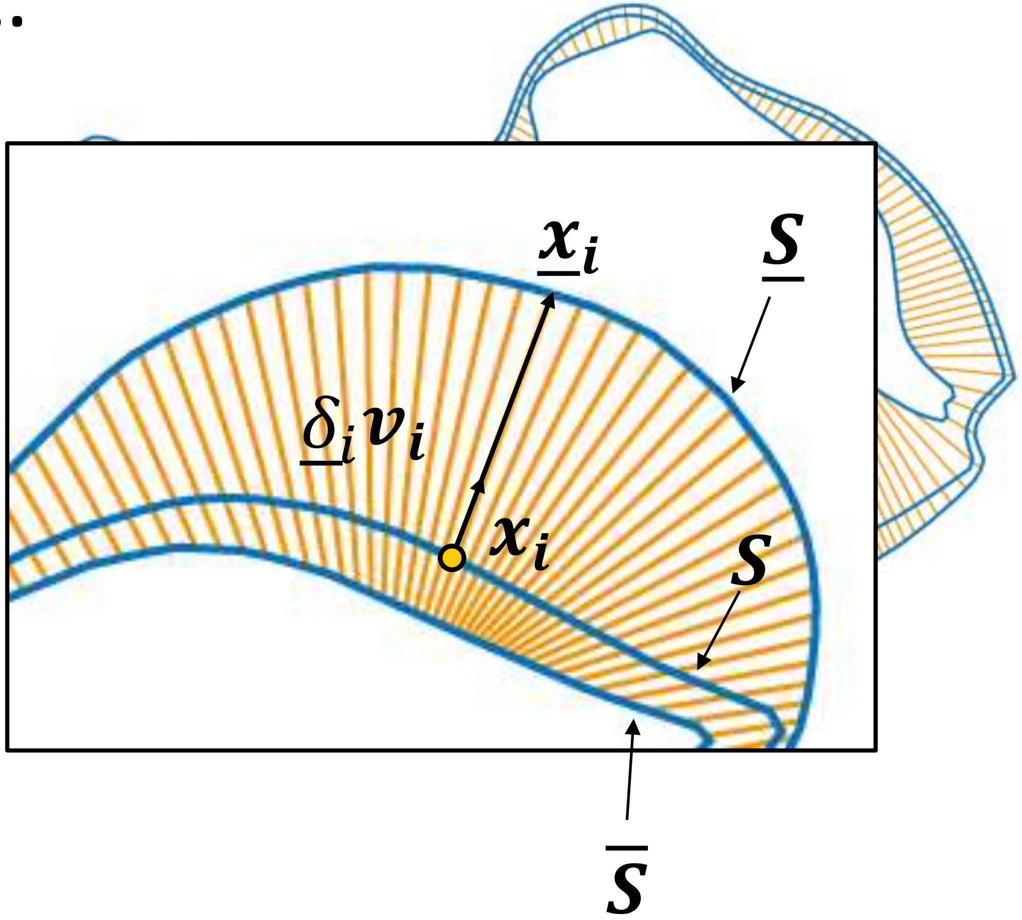

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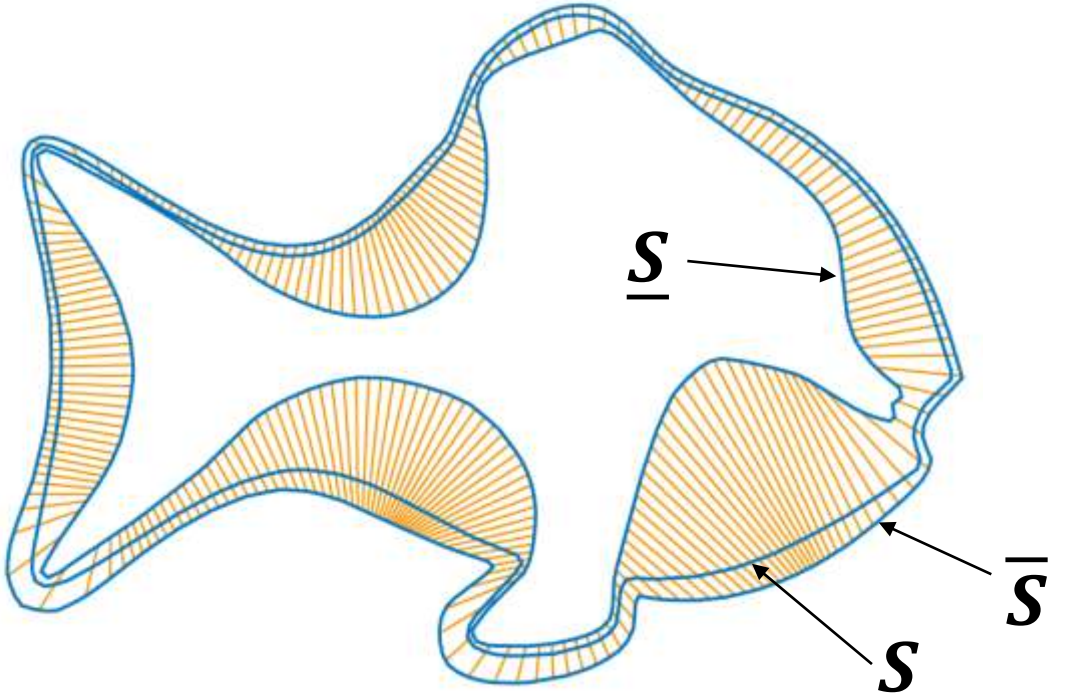
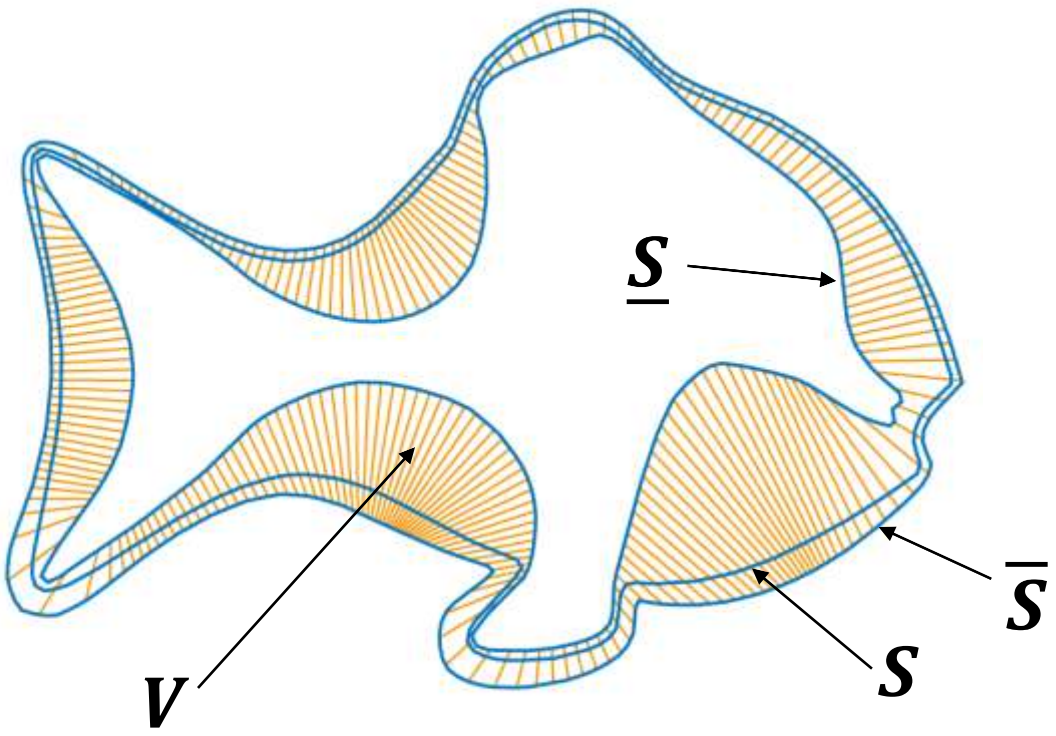


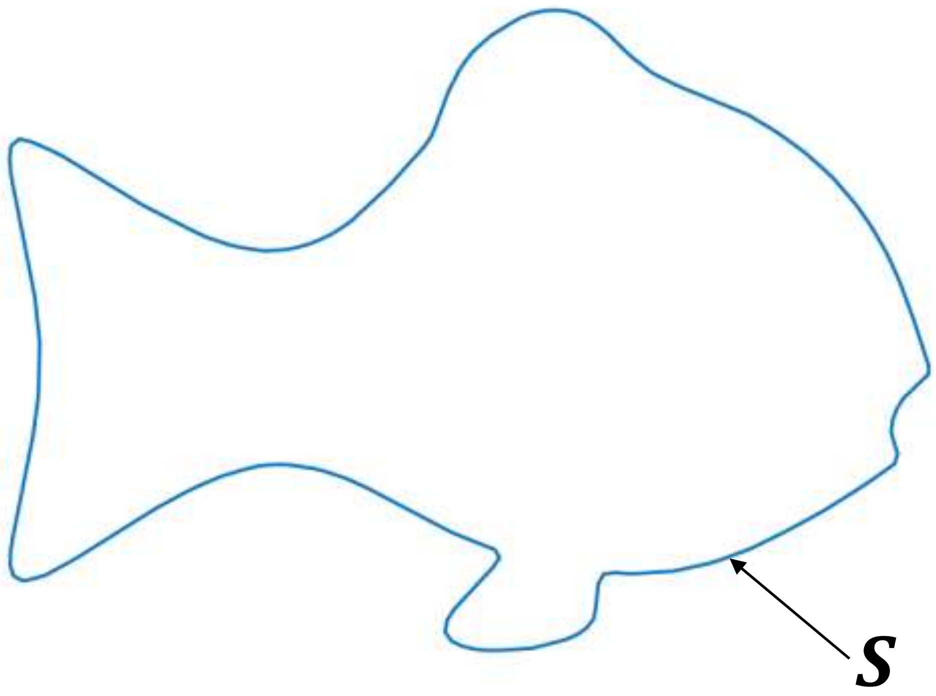
Offset Surfaces

- How far can we offset?

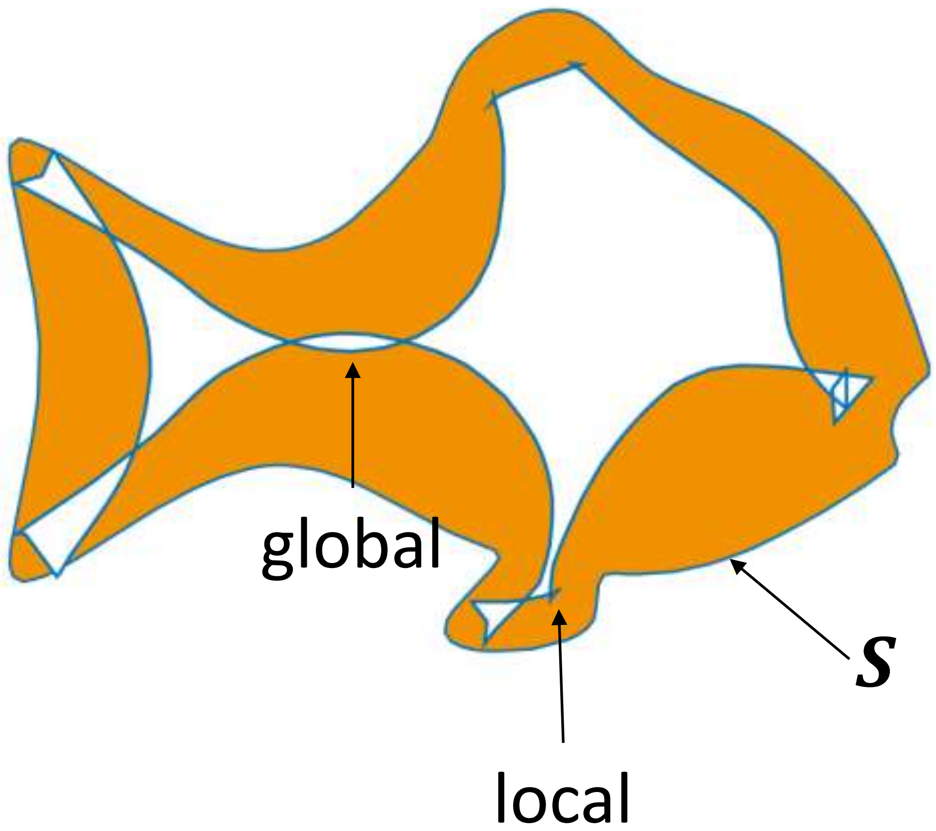
- Along which directions $\boldsymbol{V}$?



Offset Bounds

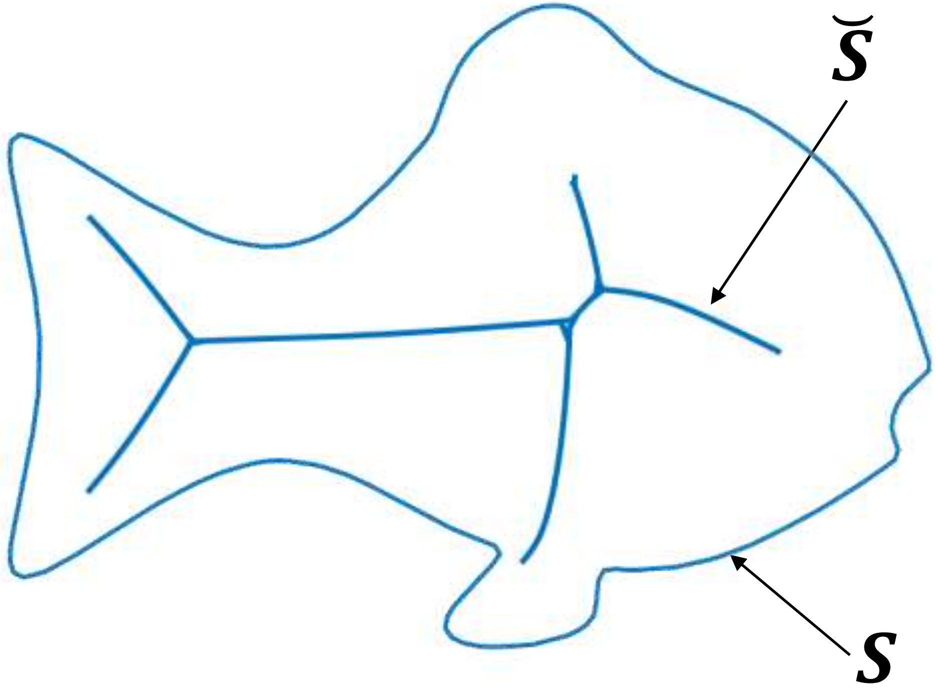


Offset Bounds



Offset Bounds

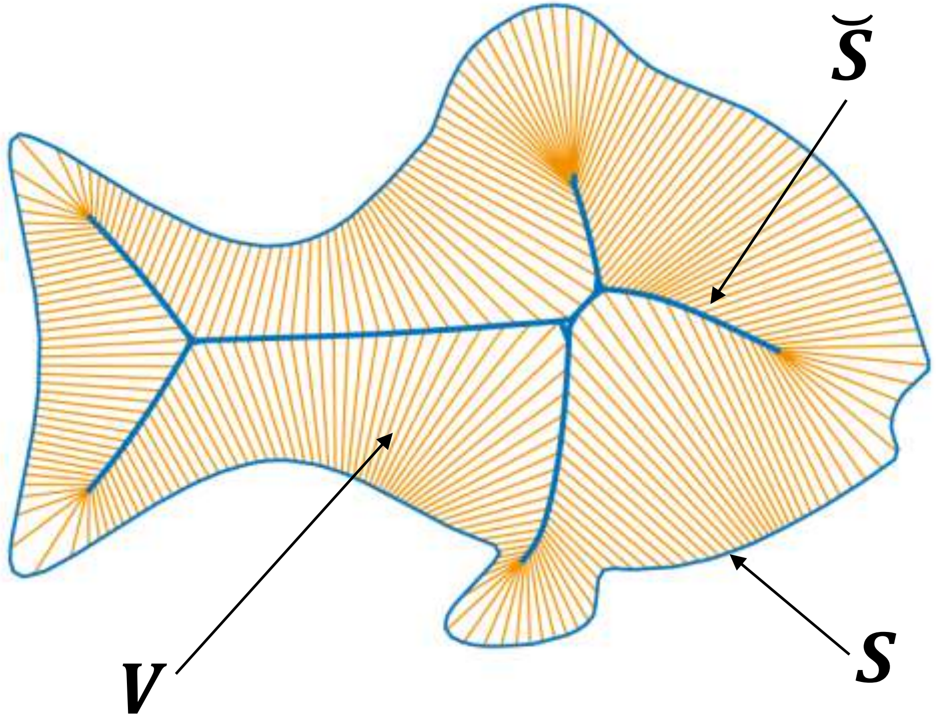
- inside: skeleton $\breve{\boldsymbol{S}}$
  - Mean Curvature Flow [Tagliasacchi et al. 2012]



Offset Vectors

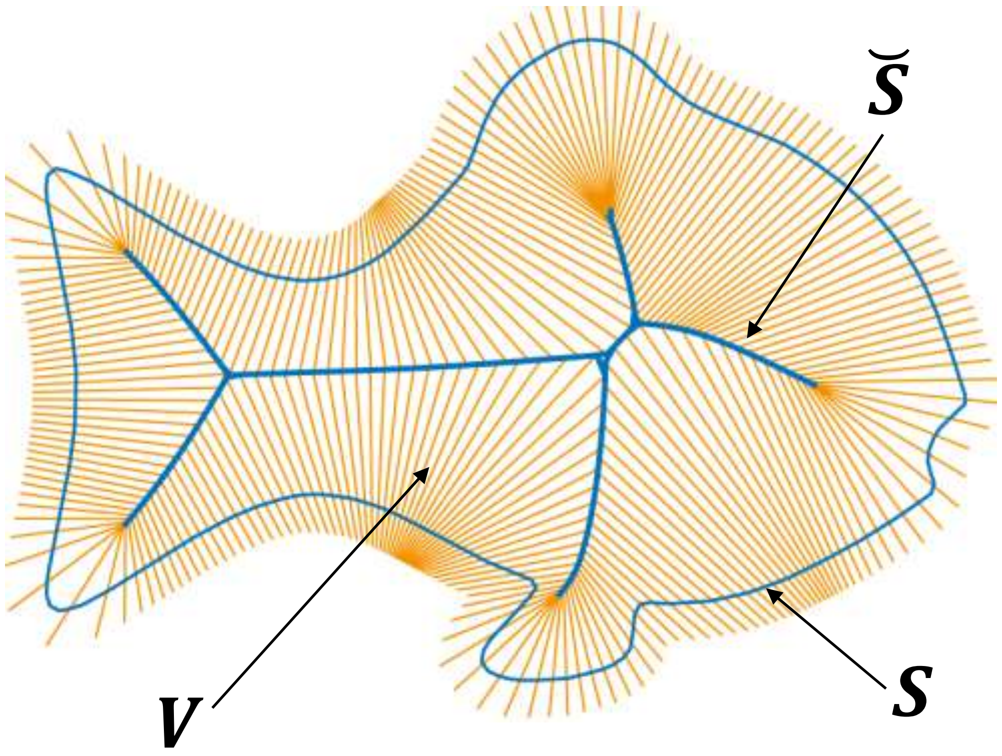
- inside: skeleton $\breve{\boldsymbol{S}}$
  - Mean Curvature Flow [Tagliasacchi et al. 2012]

- offset along vectors $\boldsymbol{v_i} \in \boldsymbol{V}$



Offset Vectors and Bounds

- inside: skeleton $\breve{\boldsymbol{S}}$
  - Mean Curvature Flow [Tagliasacchi et al. 2012]
- offset along vectors $\boldsymbol{v_i} \in \boldsymbol{V}$
- outside a constant max. value



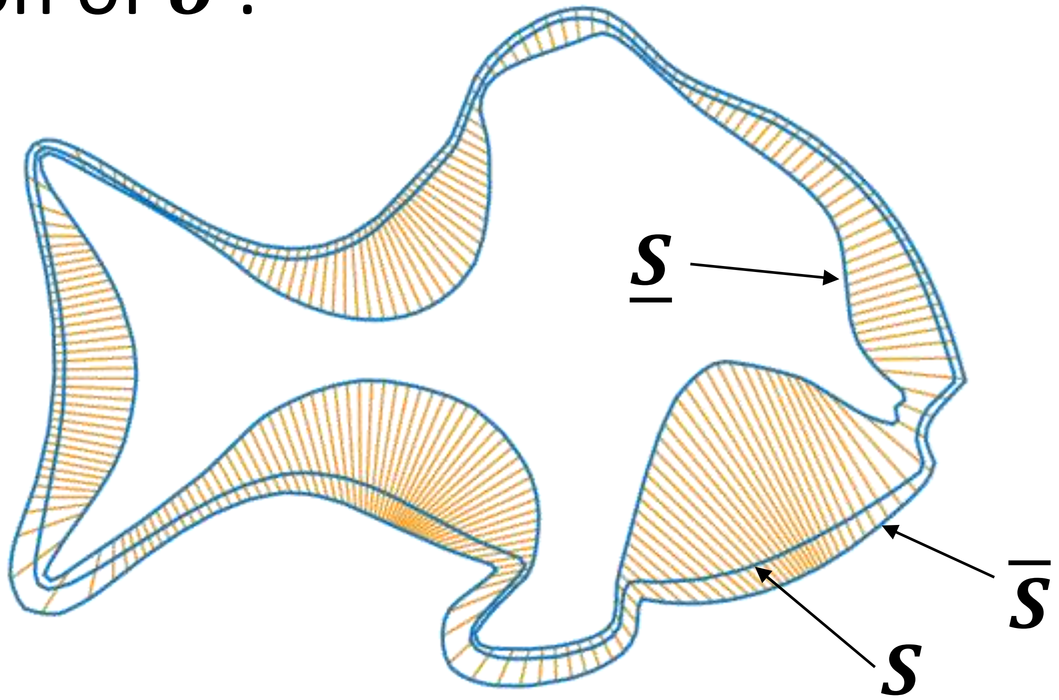
Shape Optimization Problem

- minimize objective f as a function of $\boldsymbol{\delta}$:

$$\min_{\boldsymbol{\delta}} f\,(\boldsymbol{\delta}) \;\; \text{s.t.} \;\; g(\boldsymbol{\delta})$$

- subject to constraints $g(\boldsymbol{\delta})$

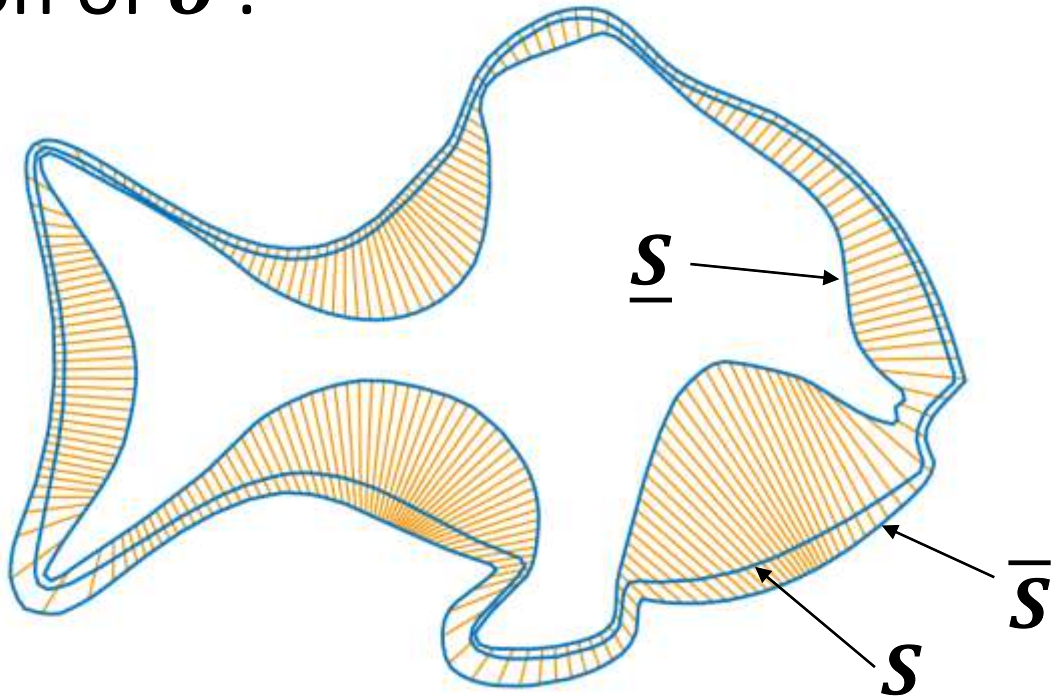
- for example:
  - f := make shape float
    subject to
  - g := keep upright orientation



Shape Optimization Problem

- minimize objective f as a function of $\boldsymbol{\delta}$:

$$\min_{\delta} f(\boldsymbol{\delta}) \quad \rightarrow n \text{ unknowns}$$



- issues:
  - problem is huge for large meshes

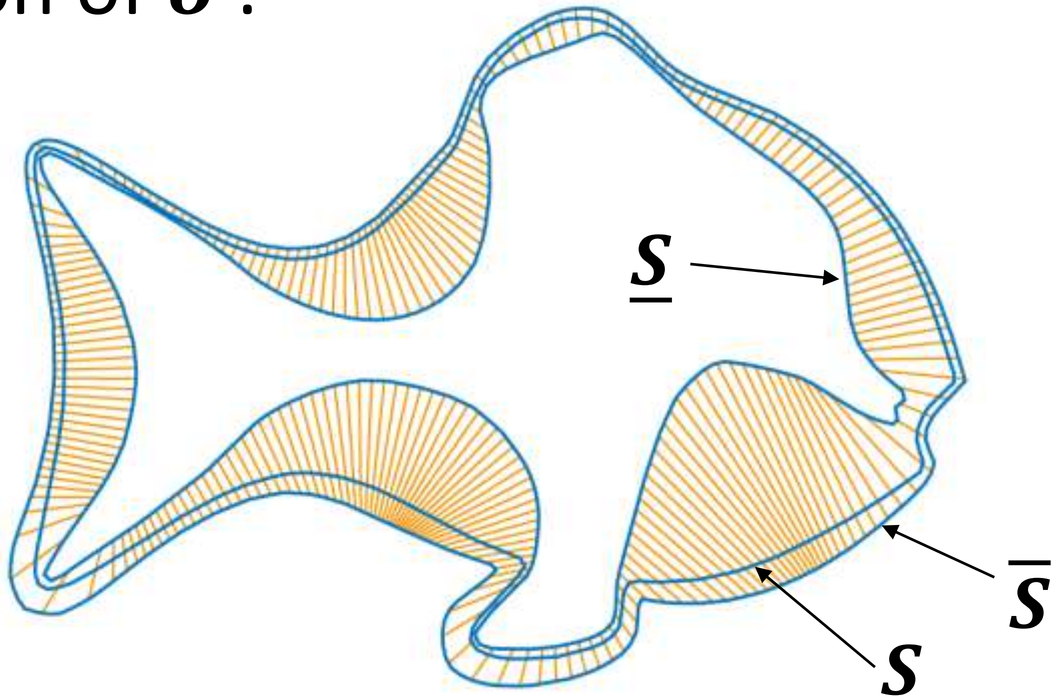
  → **scales very badly**
  - problem is underdetermined

  → **there exist many solutions** (regularization needed)

Shape Optimization Problem

- minimize objective f as a function of $\boldsymbol{\delta}$:

ergo: formulation is not suitable for practice



- issues:
  - problem is huge for large meshes

  → **scales very badly**
  - problem is underdetermined

  → **there exist many solutions** (regularization needed)

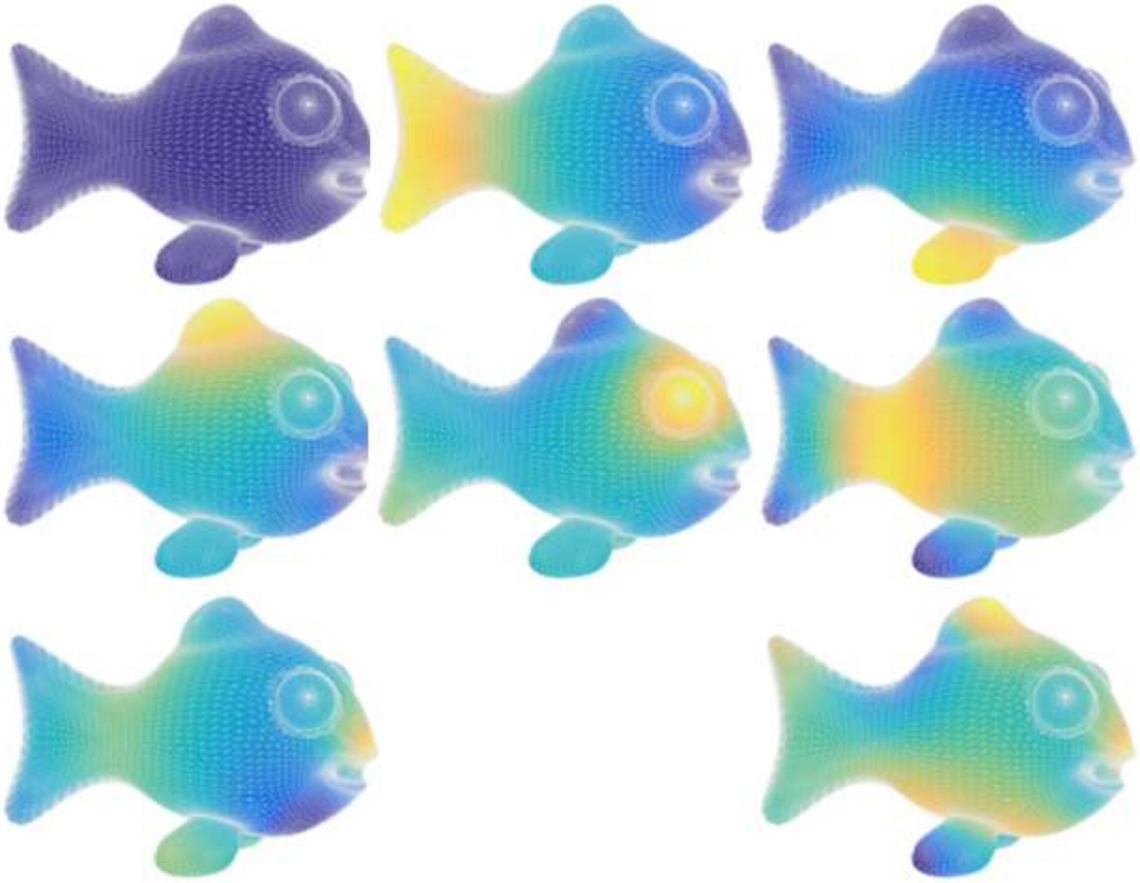
Order Reduction

Order Reduction

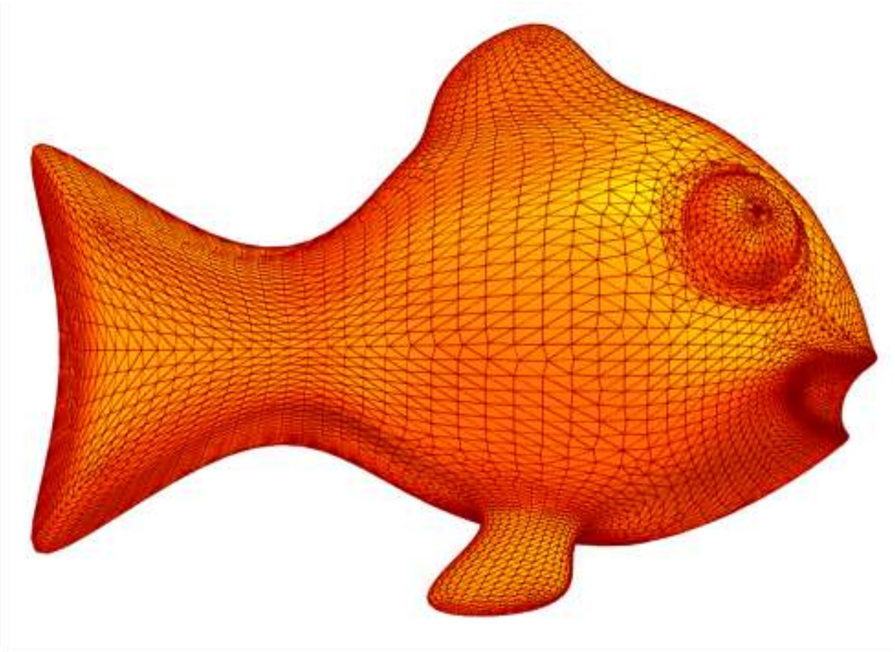
- order reduction:
  - lower the dimensionality while preserving input-output behavior

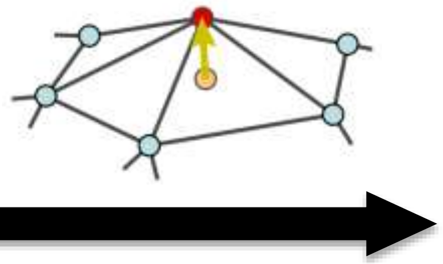
- idea:
  - project problem onto a lower dimensional space

- → Manifold Harmonics



Mesh Laplacian





$$\mathbf{L}_M = \begin{bmatrix} 4 & -1 & -1 & & -1 & -1 & & & & \\ -1 & 3 & -1 & -1 & & & & & & \\ -1 & -1 & 5 & -1 & & -1 & -1 & & & \\ & -1 & -1 & 4 & & & -1 & & & -1 \\ -1 & & & & 3 & -1 & & -1 & & \\ -1 & & -1 & & & 4 & -1 & -1 & & \\ & & -1 & -1 & & -1 & 6 & -1 & -1 & -1 \\ & & & & -1 & -1 & -1 & 6 & -1 & -1 \\ & & & & & & -1 & -1 & 3 & -1 \\ & & & -1 & & & -1 & -1 & -1 & 4 \end{bmatrix}$$

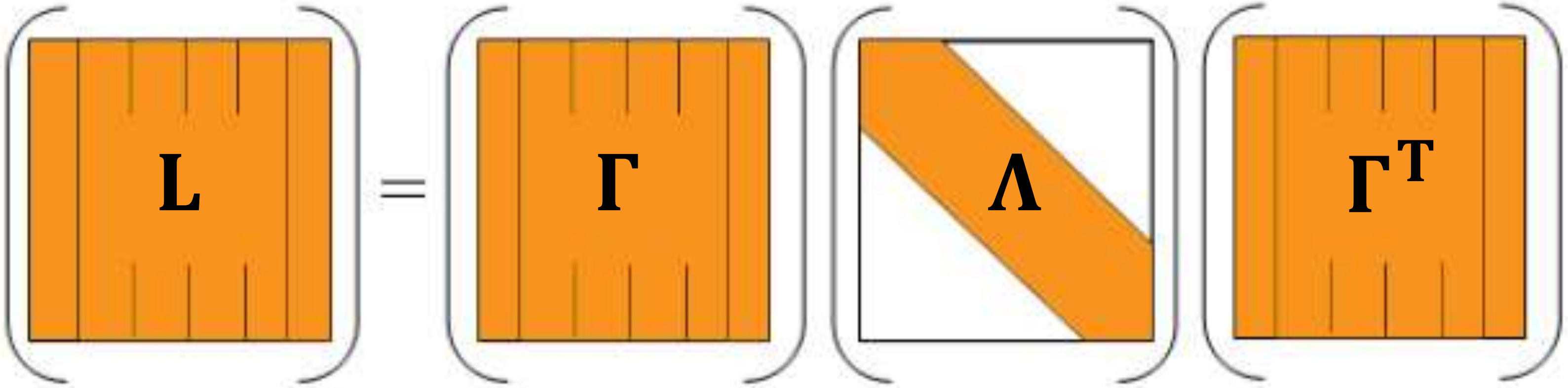
Input Mesh M Differential Operator $\boldsymbol{\Delta}_M$ Mesh Laplacian $\mathbf{L}_M$

Manifold Harmonics

- diagonalization of the Laplacian matrix **L**

  → Spectral Theorem:

$$\mathbf{L} = \mathbf{\Gamma}\mathbf{\Lambda}\mathbf{\Gamma}^{\mathbf{T}}$$



Manifold Harmonics

- diagonalization of the Laplacian matrix **L**

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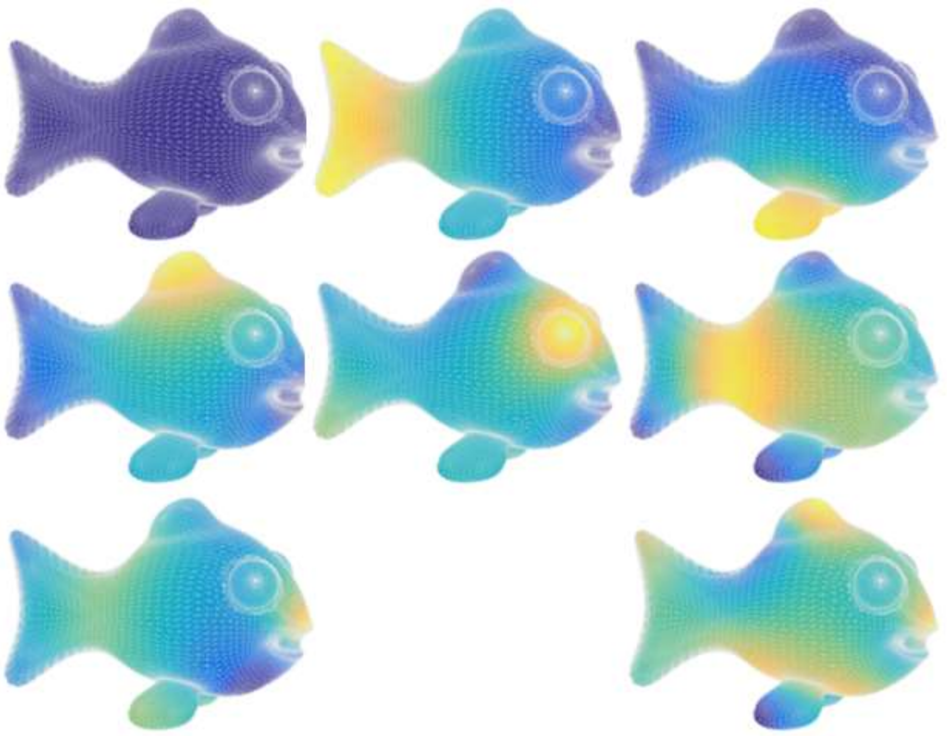
- generalization of the **Fourier Transform** for scalar functions on surfaces



[VALLET, B. AND LÉVY, B. 2008. Spectral Geometry Processing with Manifold Harmonics. *Computer Graphics Forum 27*, 2, 251–260.]

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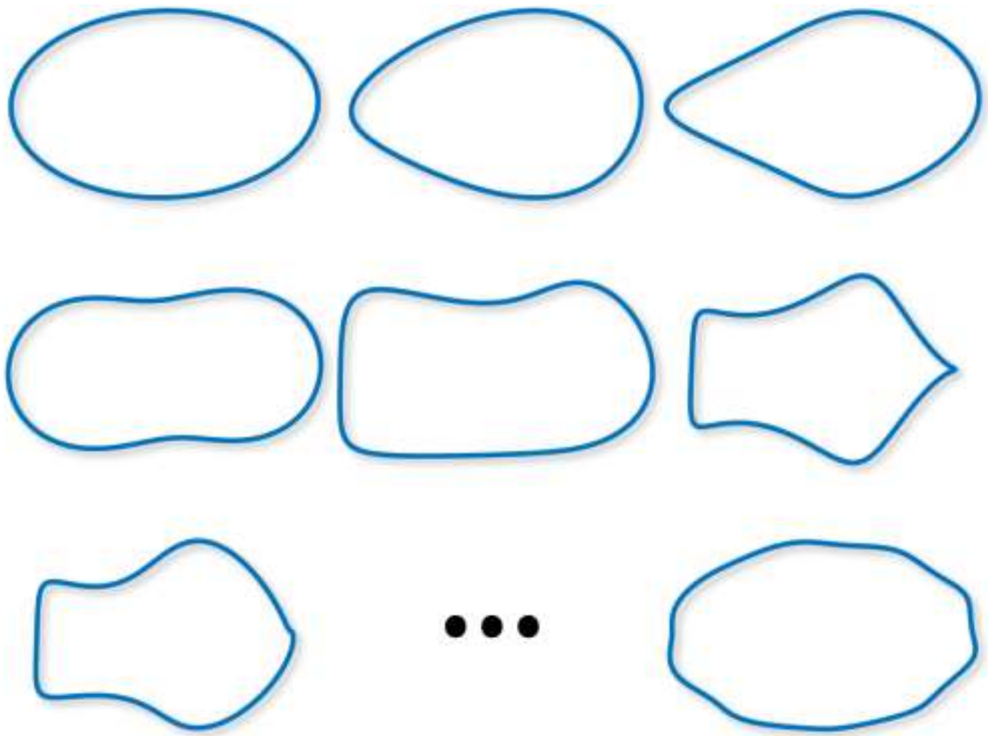

- generalization of the **Fourier Transform** for scalar functions on surfaces

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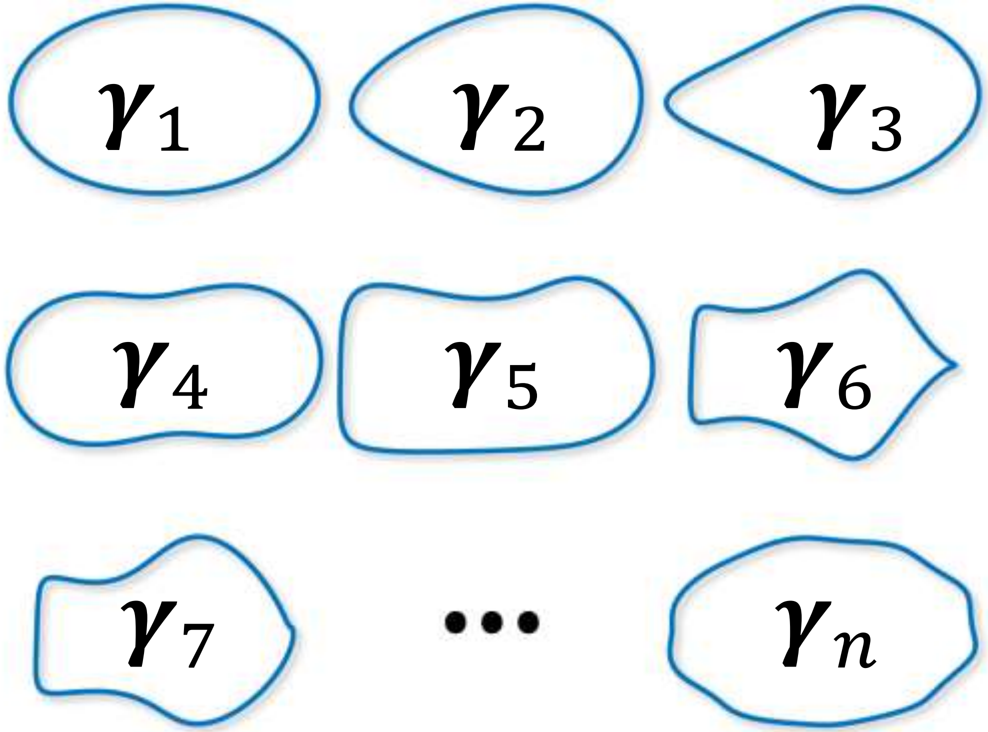
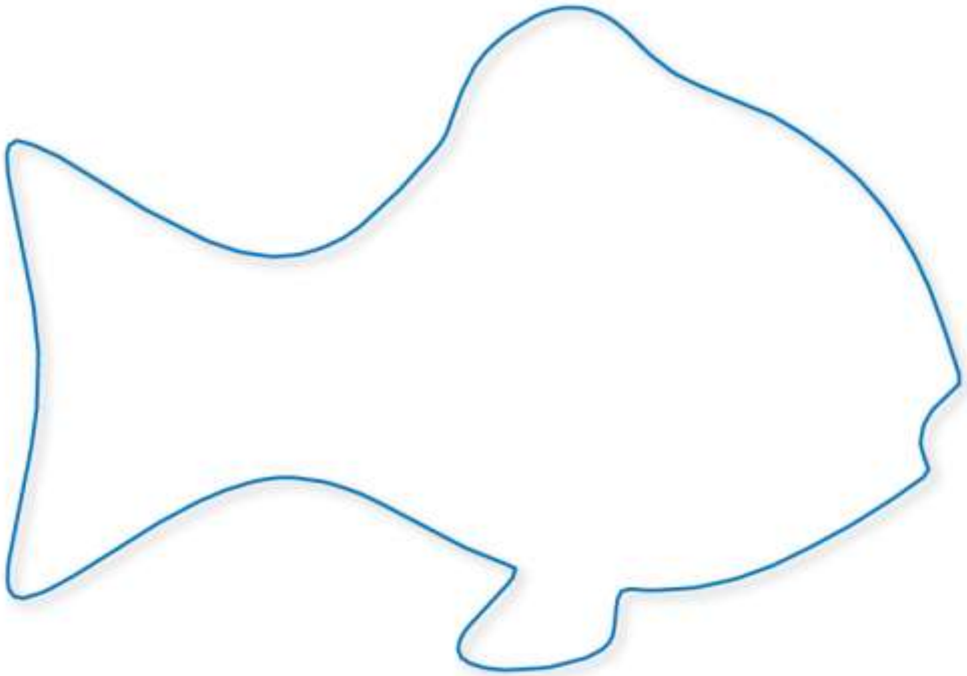
Manifold Harmonics

- eigenfunctions
  - $\mathbf{\Gamma} = [\,\boldsymbol{\gamma}_1\,\boldsymbol{\gamma}_2\ \dots\ \boldsymbol{\gamma}_n]$
- shape can be transformed to
  - $\widetilde{\boldsymbol{X}} = \mathbf{\Gamma}^{\mathrm{T}}\boldsymbol{X}$



Manifold Harmonics

- eigenfunctions
  - $\mathbf{\Gamma} = [\,\boldsymbol{\gamma}_1\,\boldsymbol{\gamma}_2\ \dots\ \boldsymbol{\gamma}_n]$
- shape can be transformed to
  - $\widetilde{\boldsymbol{X}} = \mathbf{\Gamma}^{\mathrm{T}}\boldsymbol{X}$
- reconstruction
  - $\boldsymbol{X}_k = \mathbf{\Gamma}_k\ \widetilde{\boldsymbol{X}}_k$
  - with $\mathbf{\Gamma}_k = [\,\boldsymbol{\gamma}_1\,\boldsymbol{\gamma}_2\ \dots\ \boldsymbol{\gamma}_k]$



Manifold Harmonics

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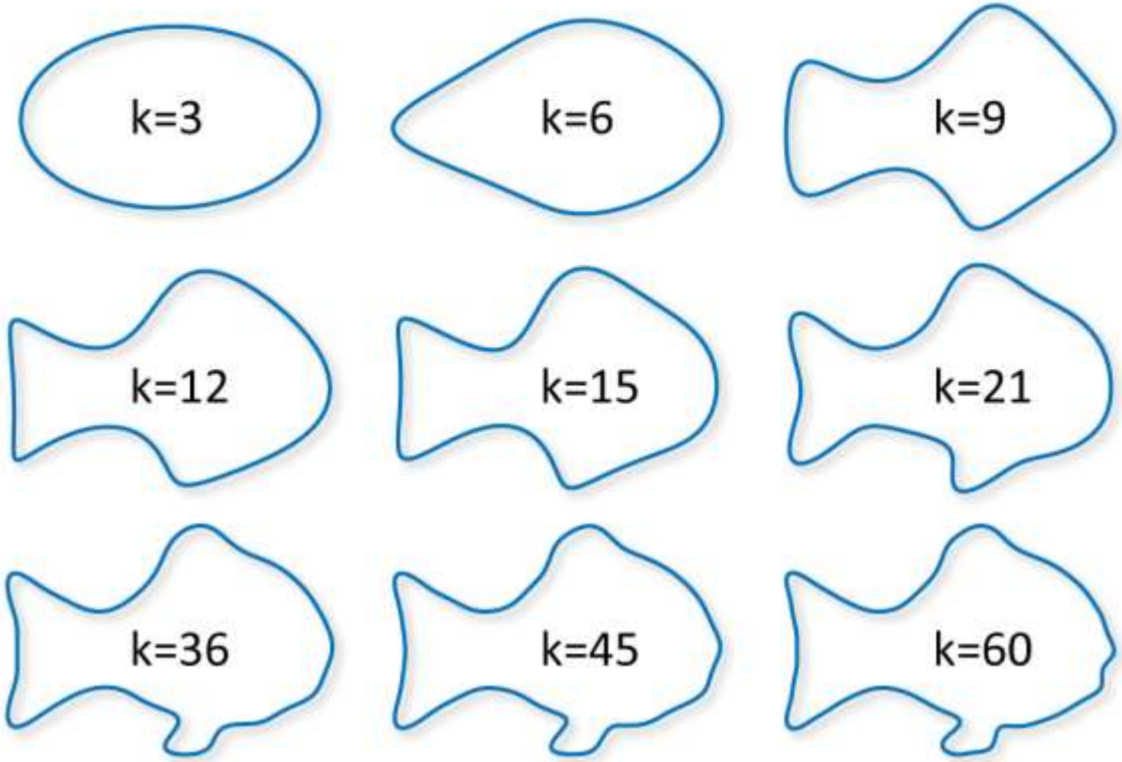
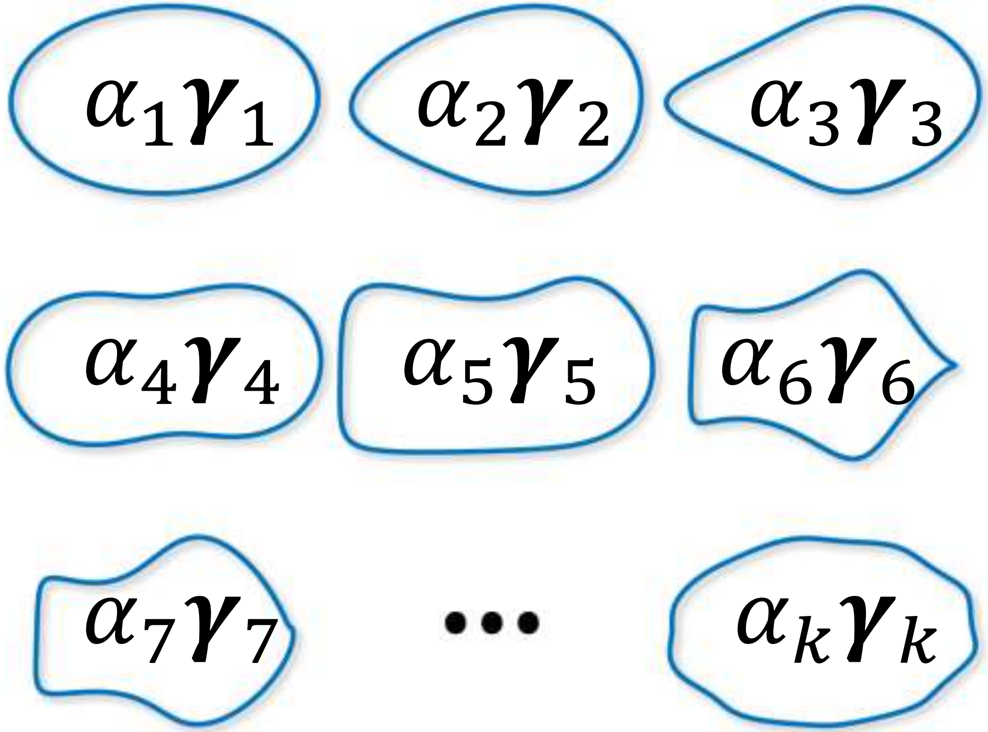


Order Reduction

- **project unknown offsets** $\boldsymbol{\delta} = [\delta_1, \delta_2, \dots, \delta_n]^T$ onto $\boldsymbol{\Gamma}_k$:

$$\boldsymbol{\delta} = \boldsymbol{\Gamma}_k \boldsymbol{\alpha} = \sum$$



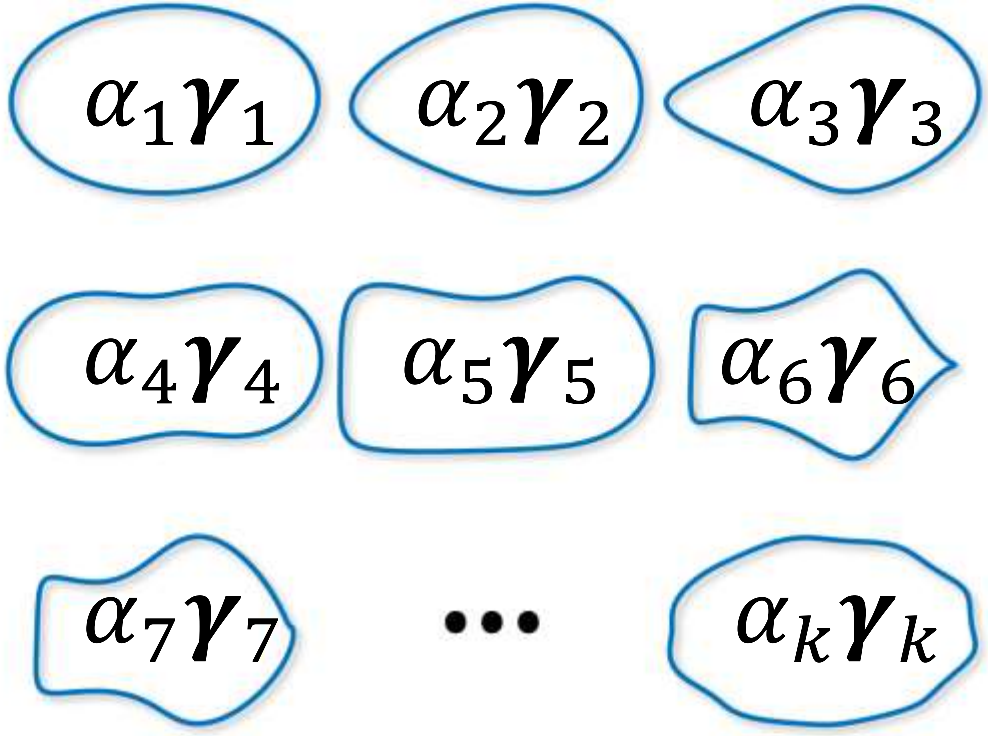
- vector $\boldsymbol{\alpha} = [\alpha_1, \alpha_2, \dots, \alpha_k]^T$ now contains the unknowns!

Order Reduction

- **project unknown offsets** $\boldsymbol{\delta} = [\delta_1, \delta_2, \ldots, \delta_n]^T$ onto $\boldsymbol{\Gamma}_k$:

$$\bar{\boldsymbol{x}}_i = \boldsymbol{x}_i + \delta_i \boldsymbol{v}_i$$

$$\bar{\boldsymbol{x}}_i = \boldsymbol{x}_i + \sum_{j=1}^{k} \alpha_j \gamma_{ij} \boldsymbol{v}_i$$

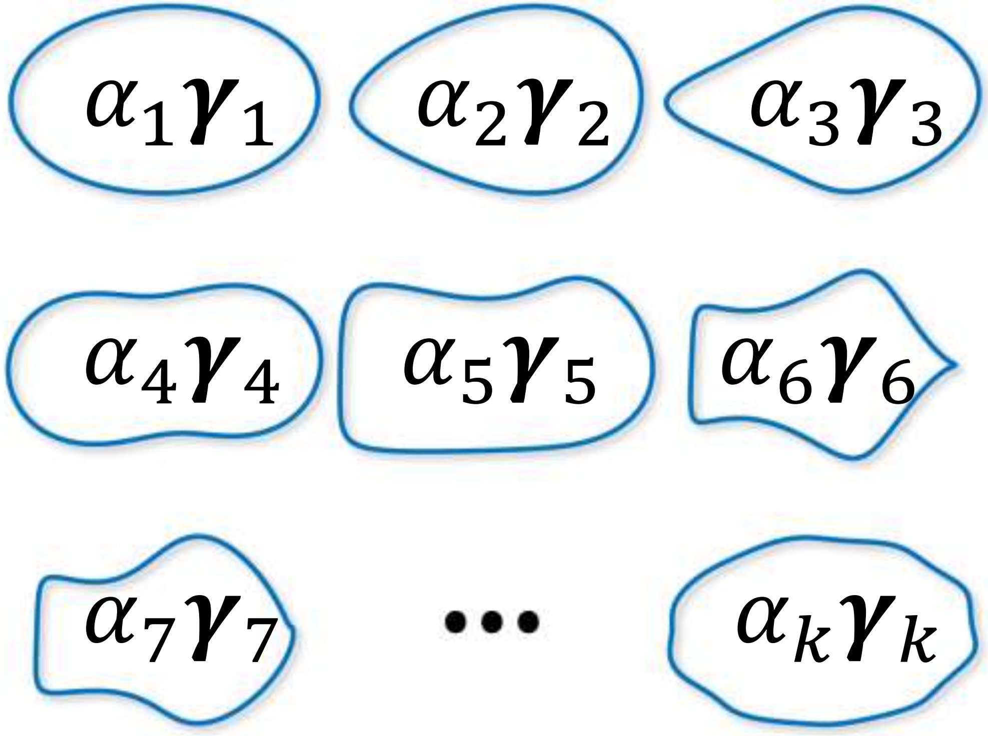


Reduced Shape Optimization Problem

- minimize objective f as a function of $\boldsymbol{\alpha}$:

$$\min_{\boldsymbol{\alpha}} f\,(\boldsymbol{\alpha})$$

- (subject to constraints)

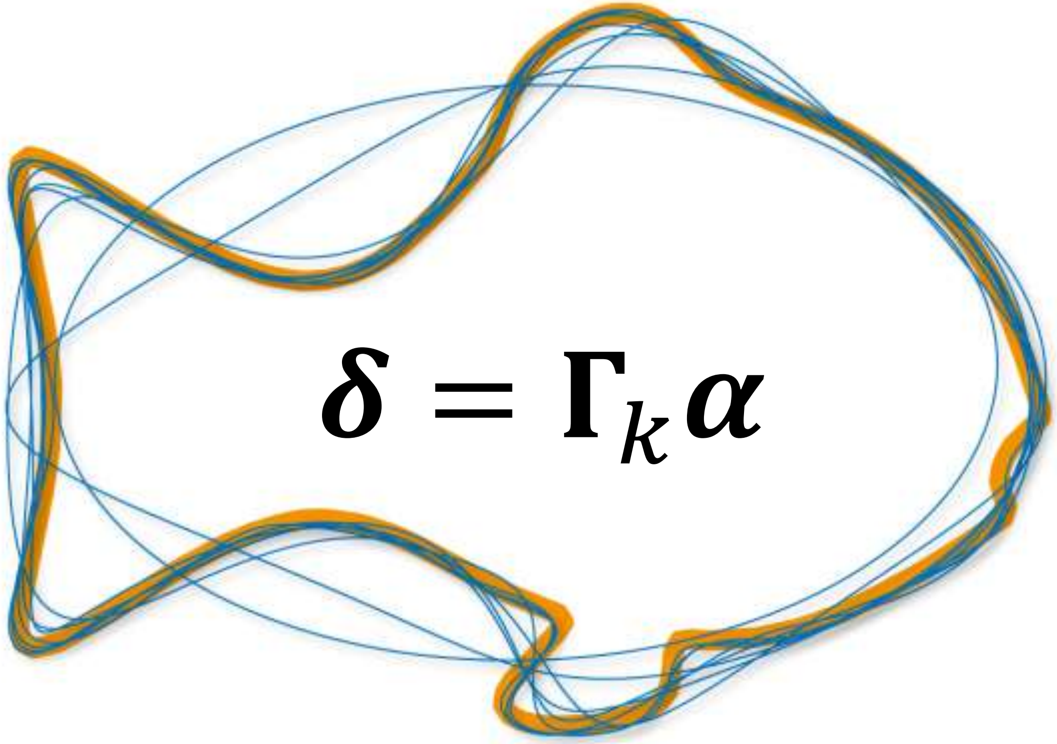


Reduced Shape Optimization Problem

- minimize objective f as a function of $\boldsymbol{\alpha}$:

$$\min_{\boldsymbol{\alpha}} f\,(\boldsymbol{\alpha}) \;\rightarrow k \text{ unknowns, } k \ll n$$

We **deform only** the **low-frequencies** and leave high-frequency details untouched!



$$\boldsymbol{\delta} = \boldsymbol{\Gamma}_k \boldsymbol{\alpha}$$

Reduced Shape Optimization Problem

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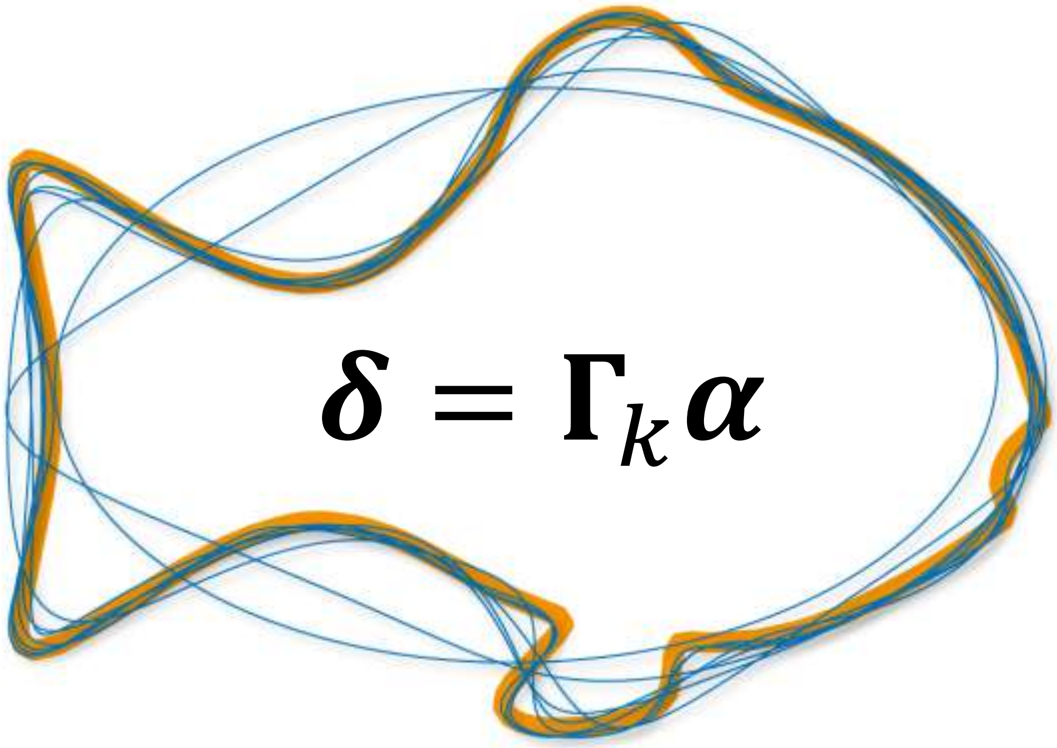


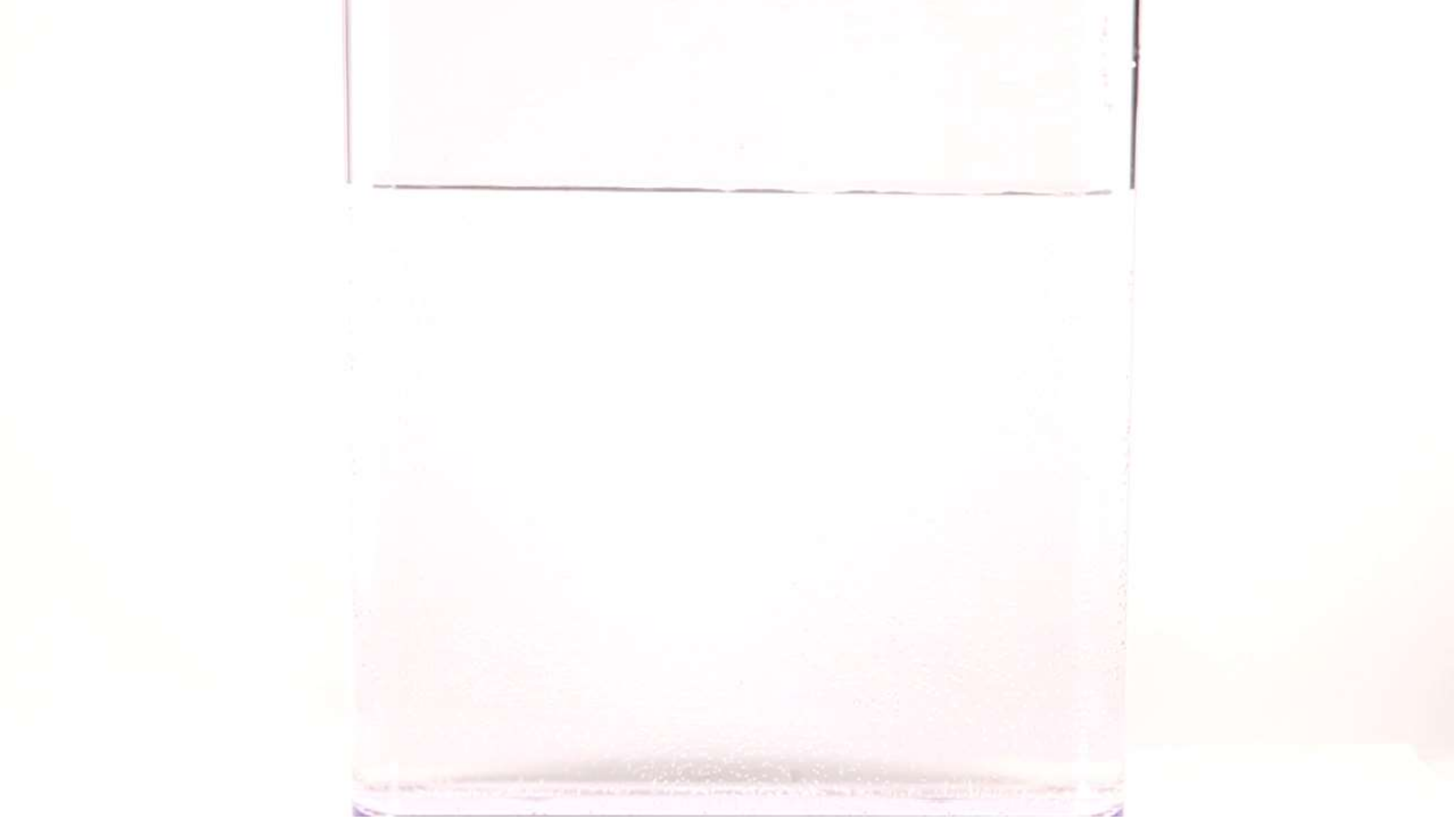
→ independent of mesh resolution

→ implicit regularization

→ numerically stable

→ easy to implement

Applications I: Mass Properties



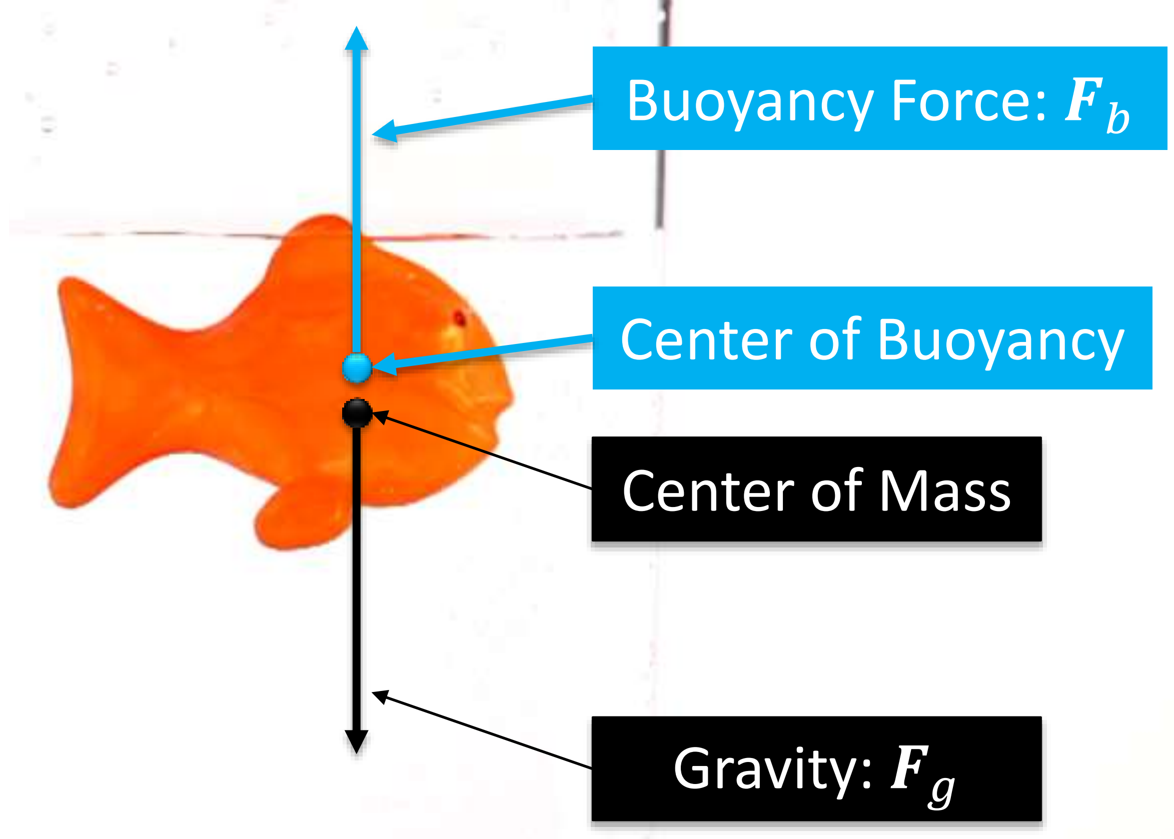
Equilibrium

$$\boldsymbol{F}_g = \boldsymbol{F}_b$$

↓

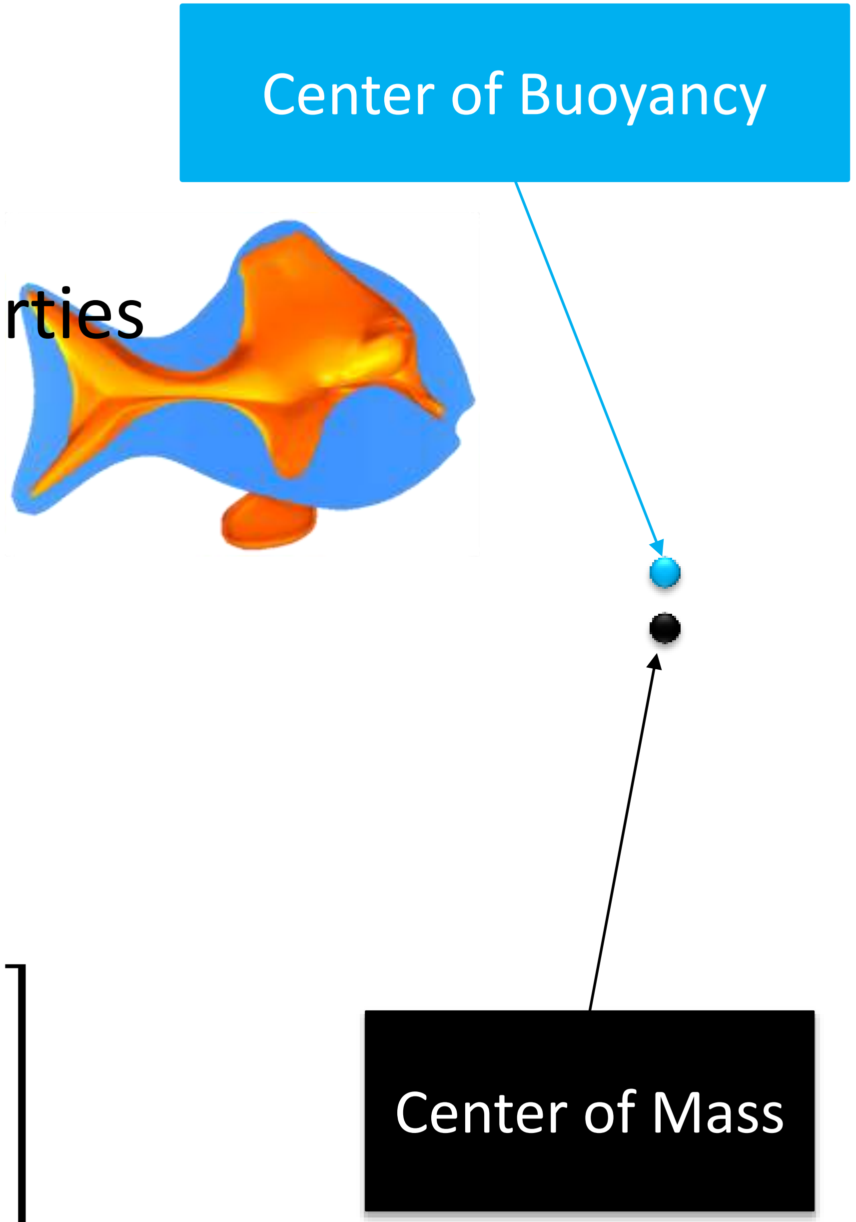
Mass Properties

$$\boldsymbol{P}(\boldsymbol{S})$$



Applications

- Gauss' Divergence Theorem
  - allows us to compute mass properties as a function of the surface



$$\boldsymbol{P}_{\boldsymbol{m}}(\boldsymbol{S}) = M$$

$$\boldsymbol{P}_{\boldsymbol{x,y,z}}(\boldsymbol{S}) = \boldsymbol{CoM} = [c_x \quad c_y \quad c_z]^{\boldsymbol{T}}$$

$$\boldsymbol{P}_{\boldsymbol{x^2,xy,\dots,z^2}}(\boldsymbol{S}) = \boldsymbol{I} = \begin{bmatrix} I_{x^2} & I_{xy} & I_{xz} \\ I_{xy} & I_{y^2} & I_{yz} \\ I_{xz} & I_{yz} & I_{z^2} \end{bmatrix}$$

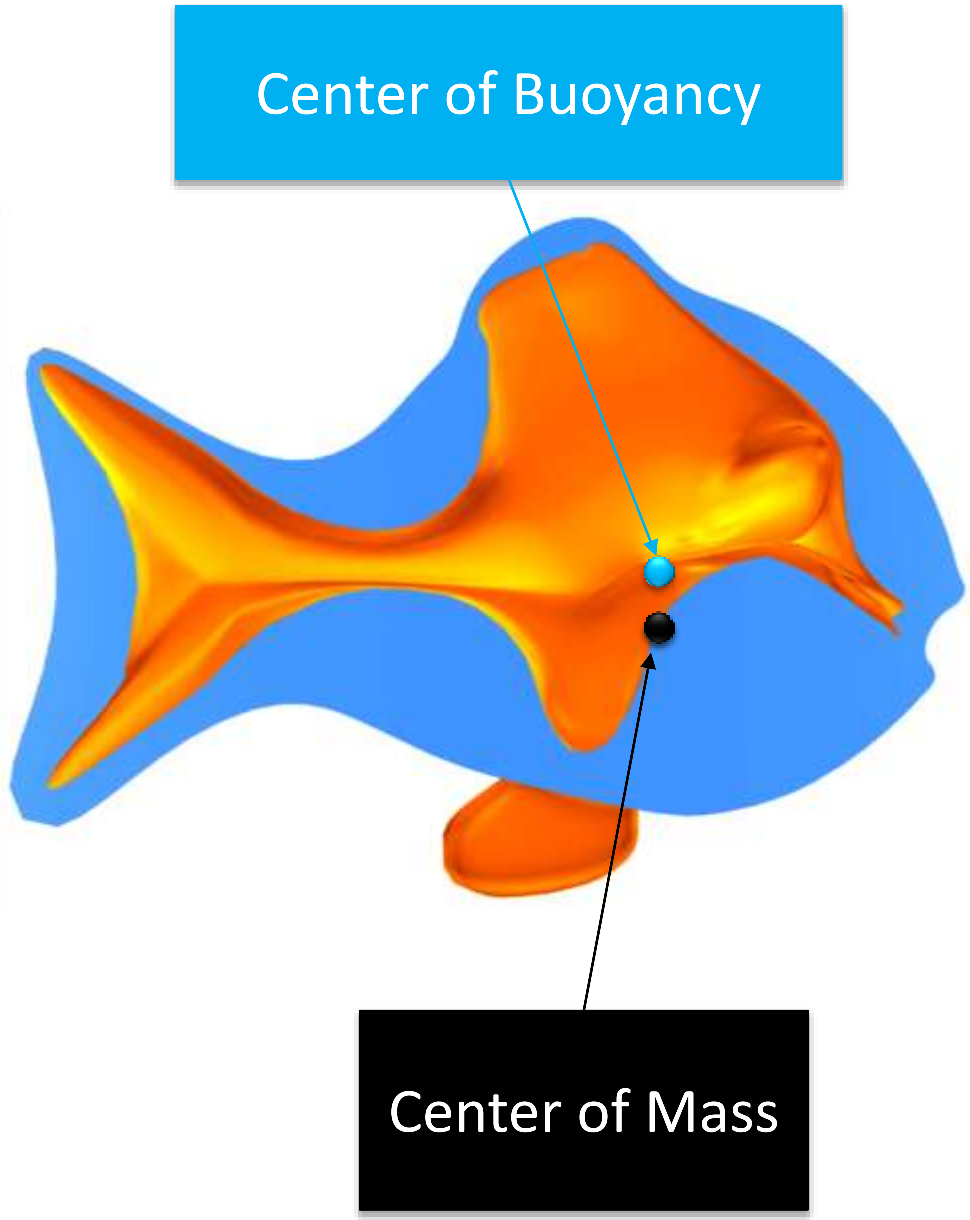
Applications

- optimization problem

$$\min_{\boldsymbol{\alpha}} f\left(\boldsymbol{P}(\boldsymbol{S}(\boldsymbol{\delta}(\boldsymbol{\alpha})))\right)$$

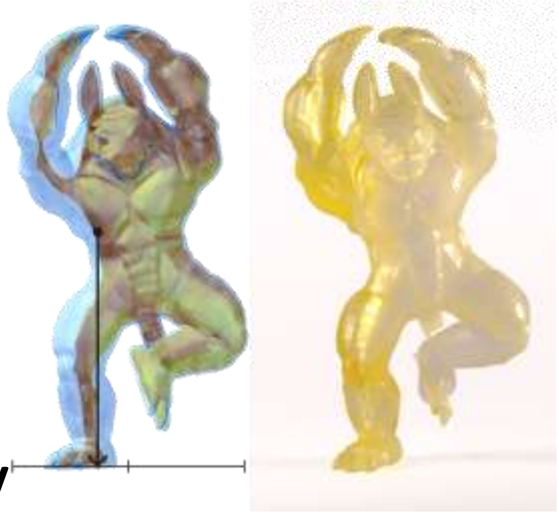
- an analytical gradient

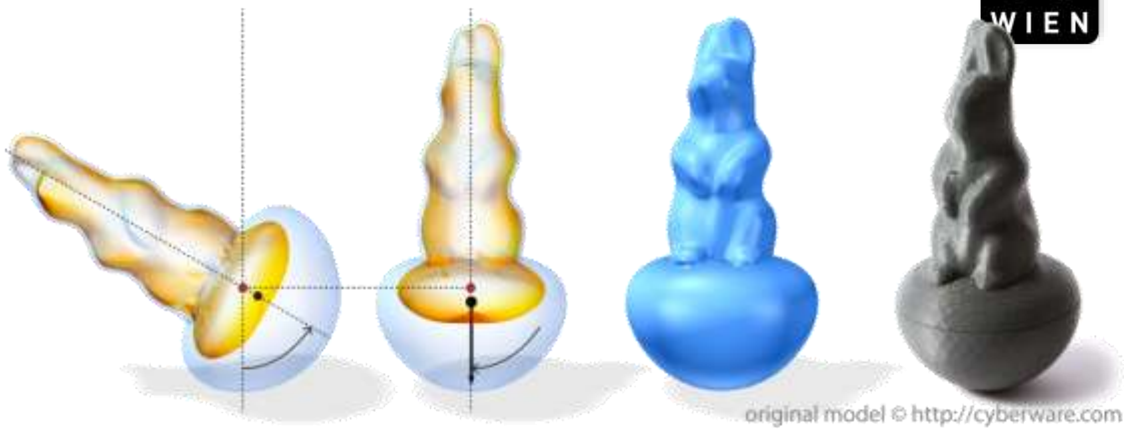
$$\nabla f = \frac{\partial f}{\partial \boldsymbol{P}} \frac{\partial \boldsymbol{P}}{\partial \boldsymbol{S}} \frac{\partial \boldsymbol{S}}{\partial \boldsymbol{\delta}} \frac{\partial \boldsymbol{\delta}}{\partial \boldsymbol{\alpha}}$$

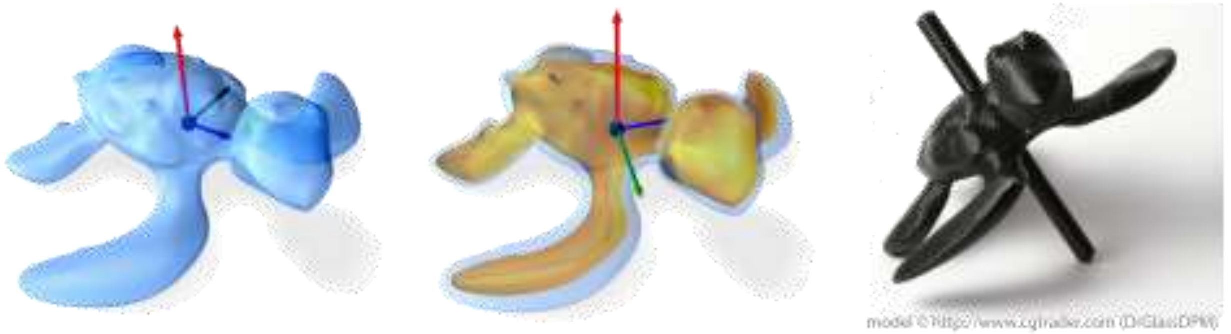


Applications

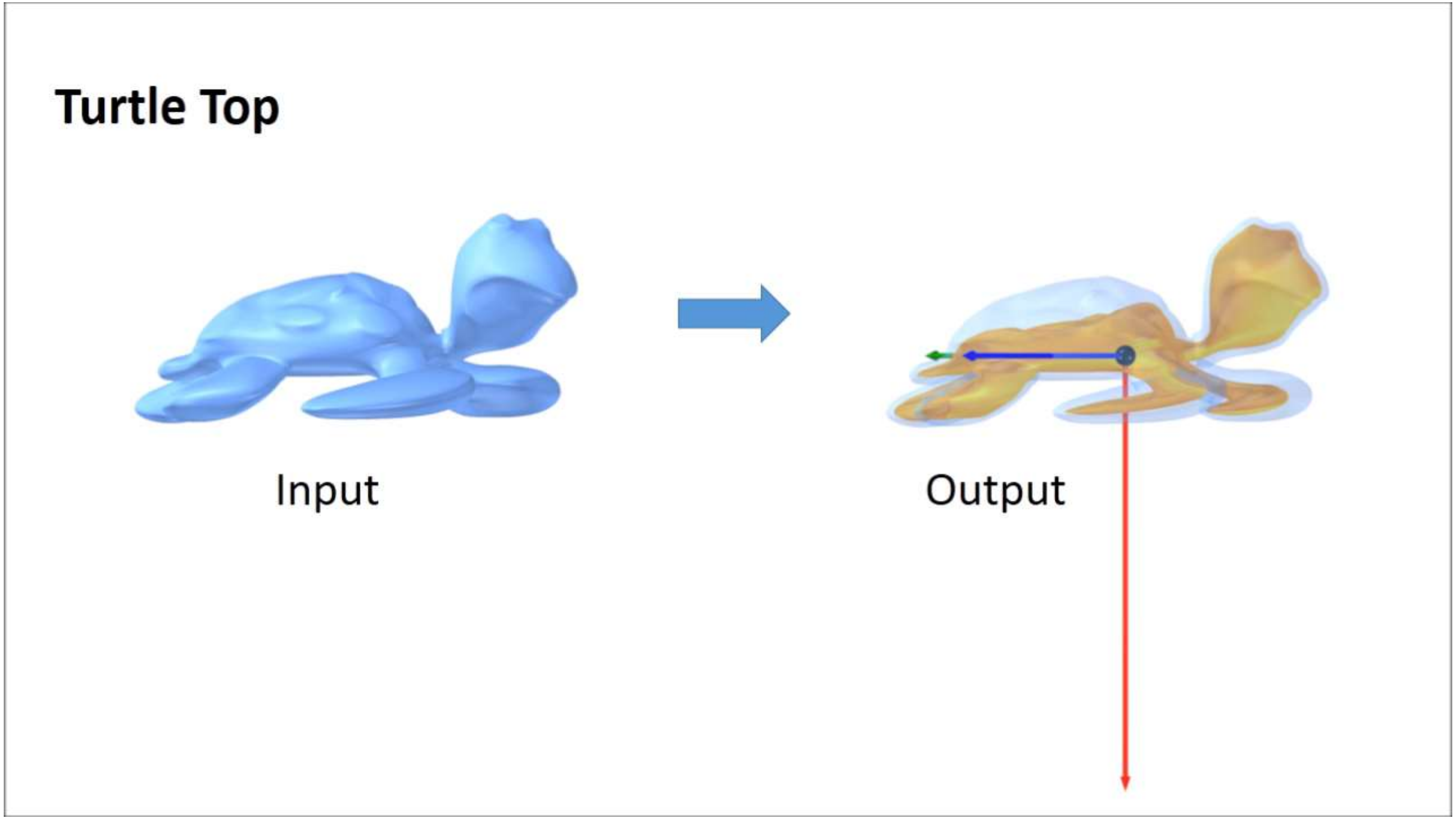
- static stability
- monostatic stability
- rotational stability
- static stability under storage
- volume and buoyancy



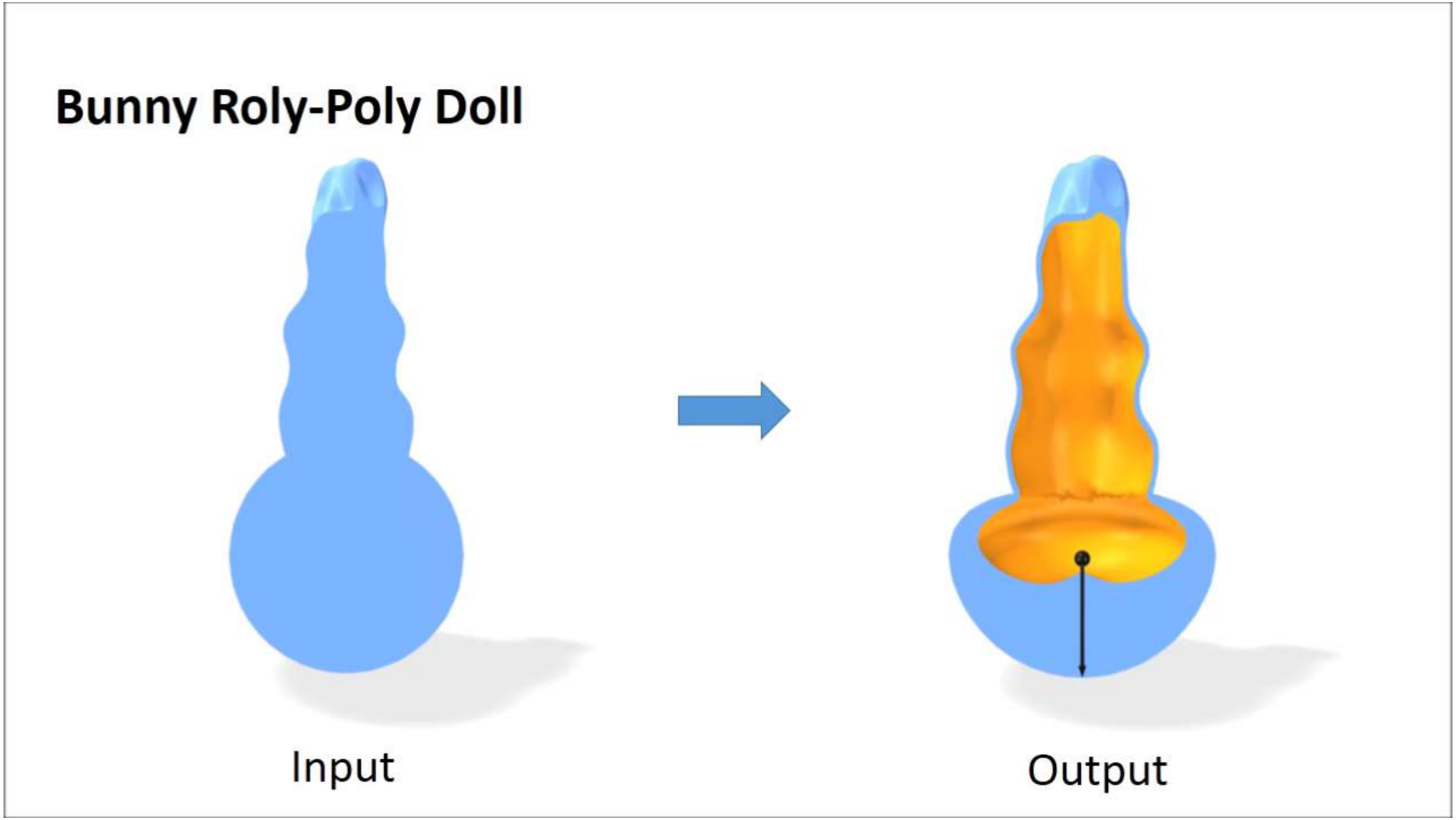




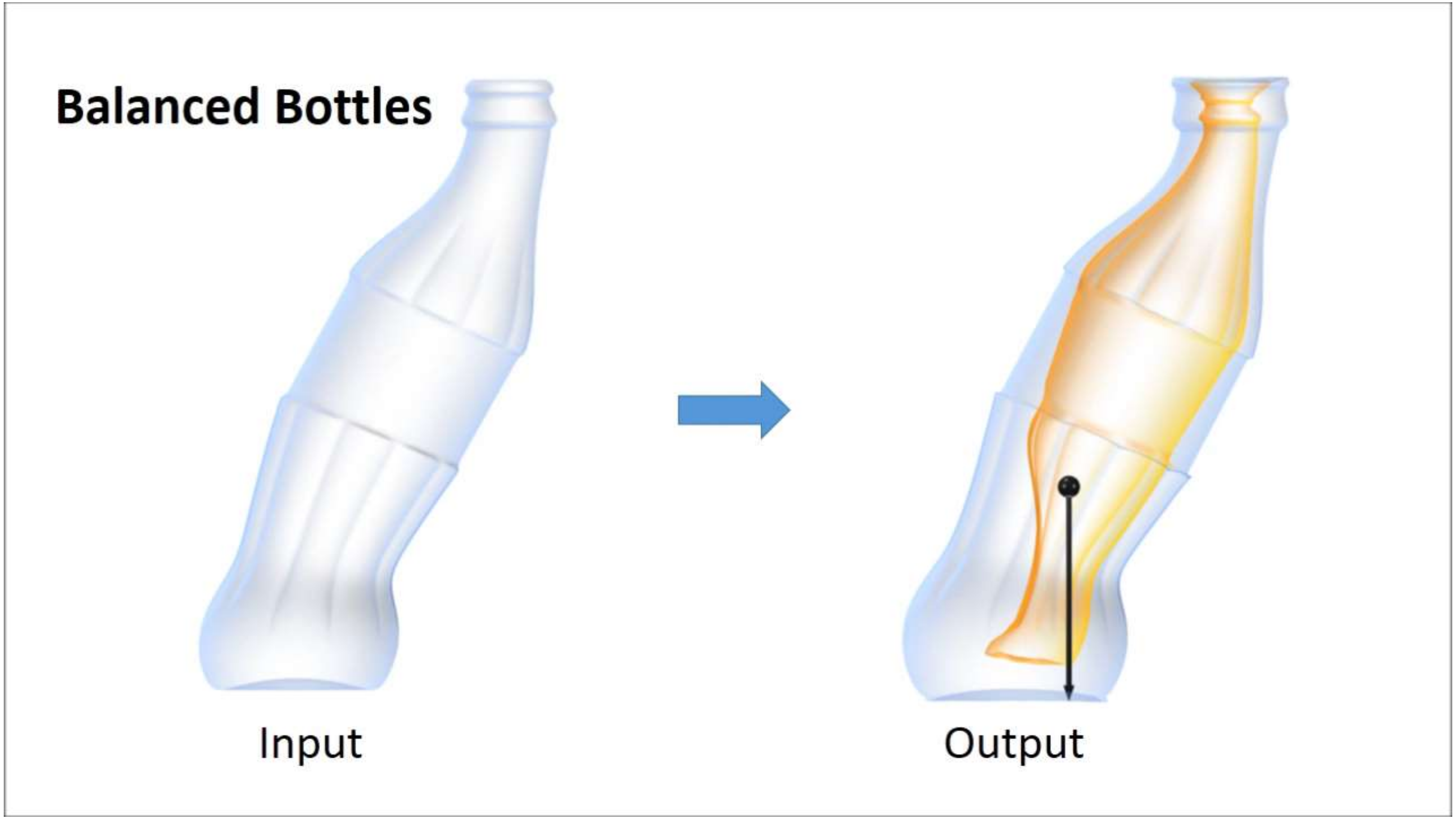
Spinning Turtle



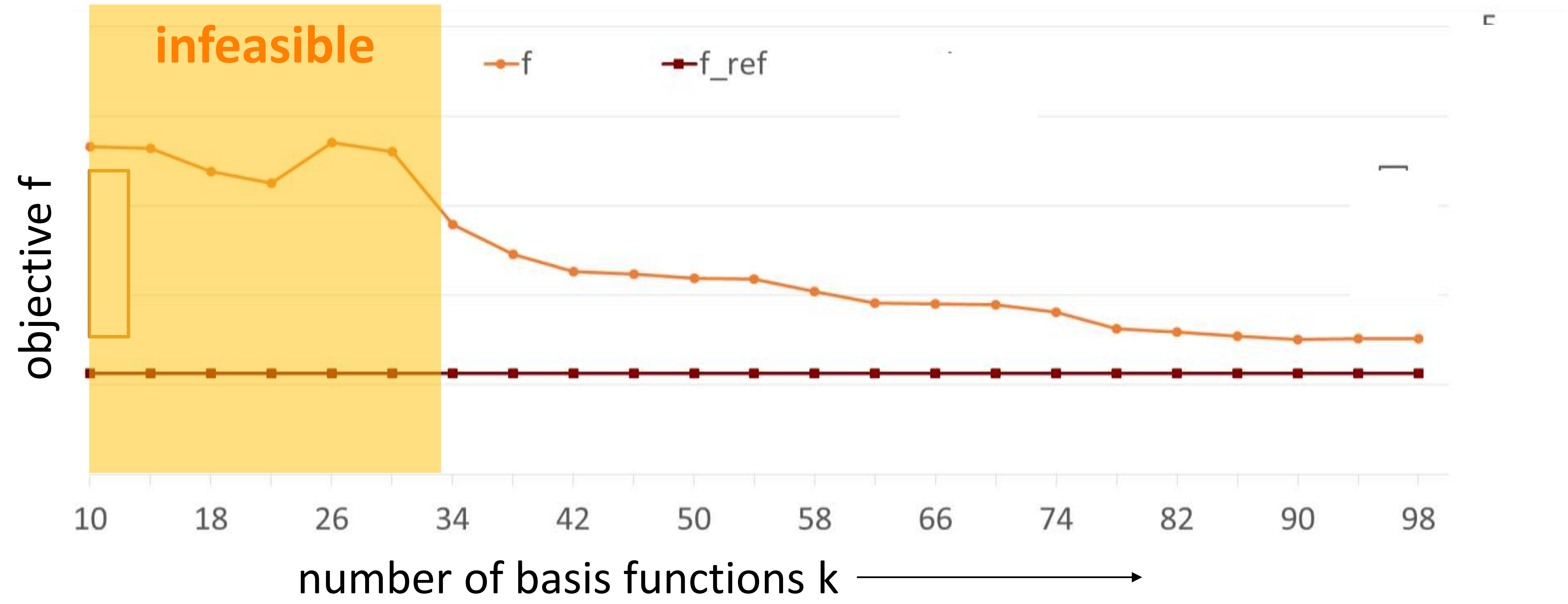
Rabbit Rolly-Polly



Balanced Bottles

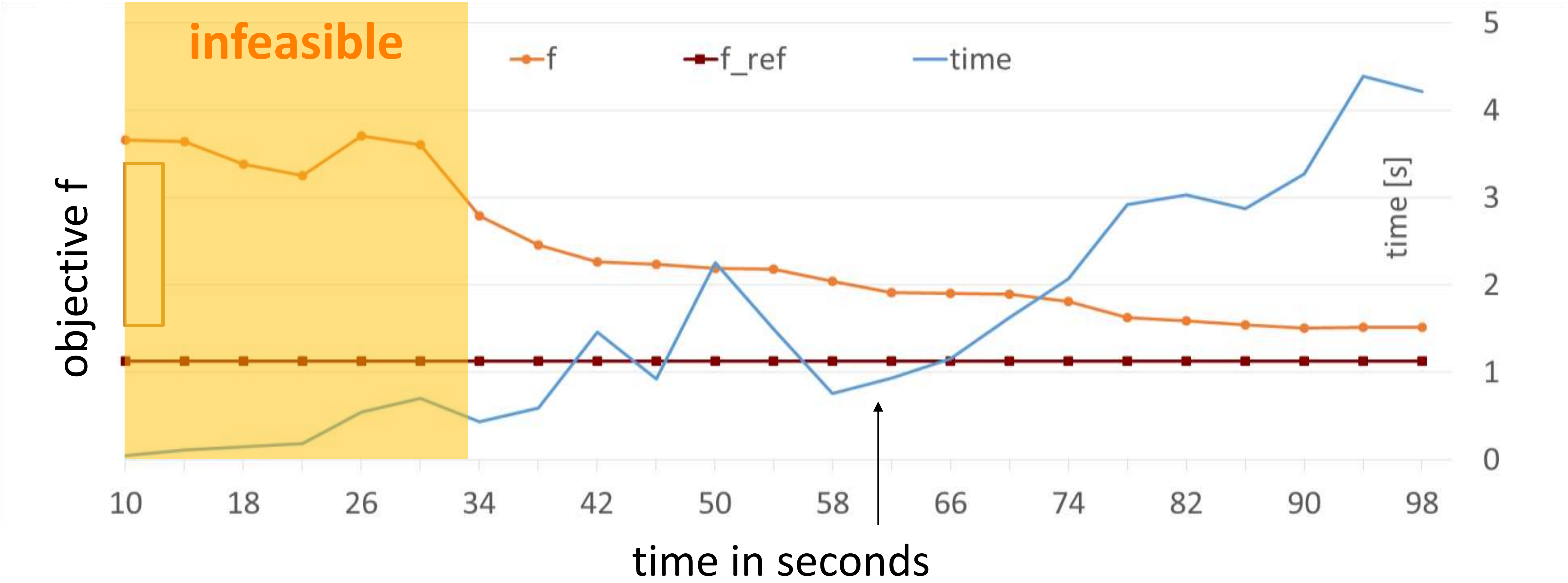


Evaluation

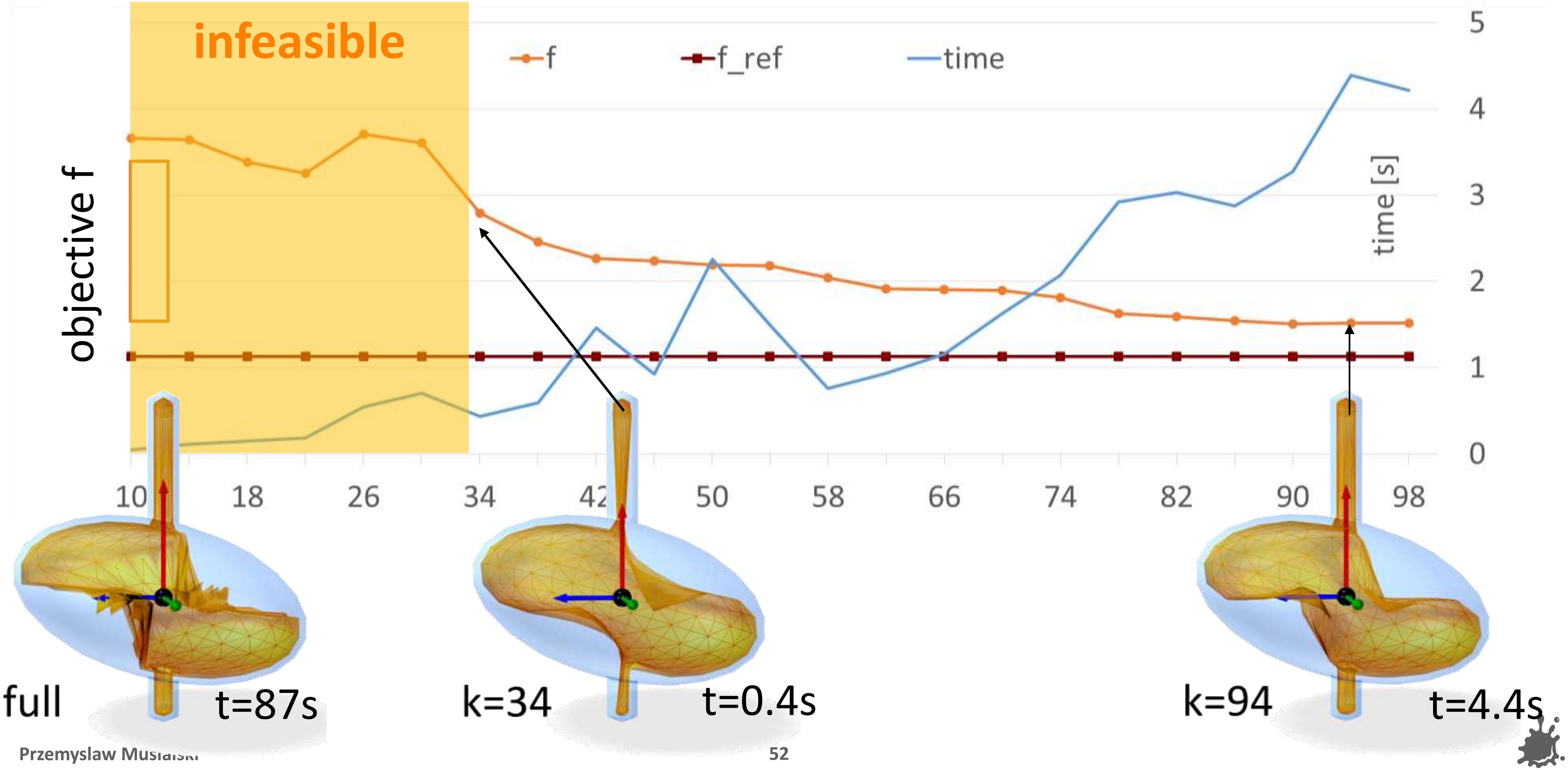


infeasible
f
f_ref
objective f
10
18
26
34
42
50
58
66
74
82
90
98
number of basis functions k

Evaluation and Performance

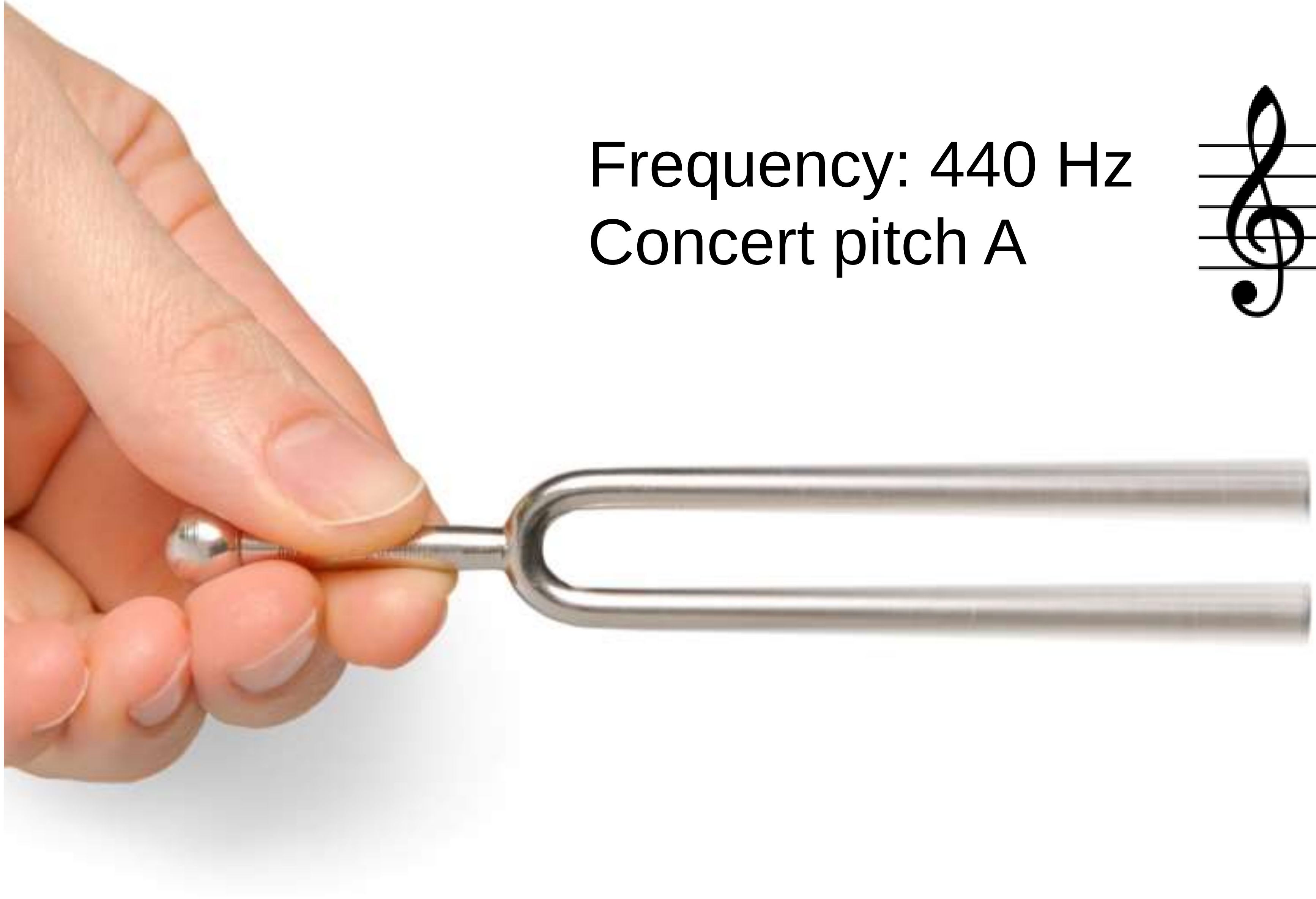


Evaluation and Performance

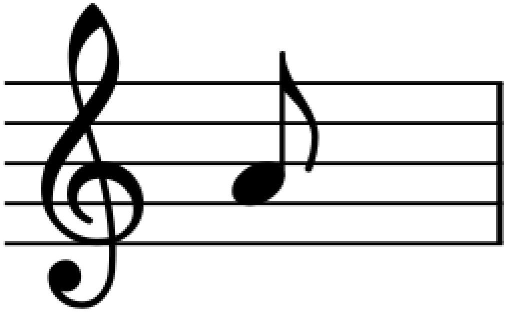


Applications II:
Modal Synthesis

Natural Frequencies I



Frequency: 440 Hz
Concert pitch A

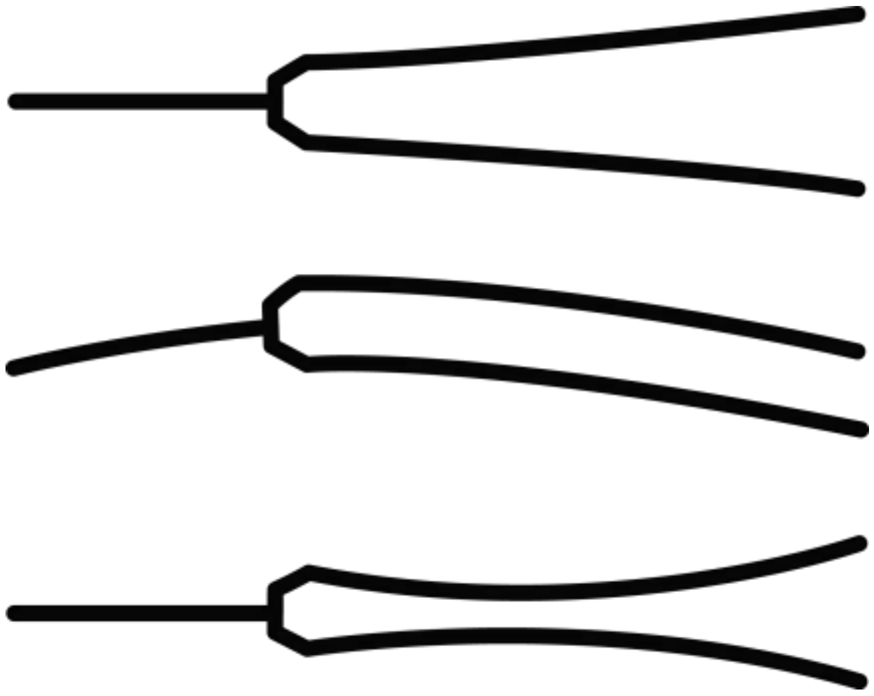


Natural Frequencies II

1st mode (pitch) — 440 Hz

2nd mode (1st overtone) — 1060 Hz

3rd mode (2nd overtone) — 2790 Hz



⋮

Overtone spectrum ⇔ characteristic sound of object

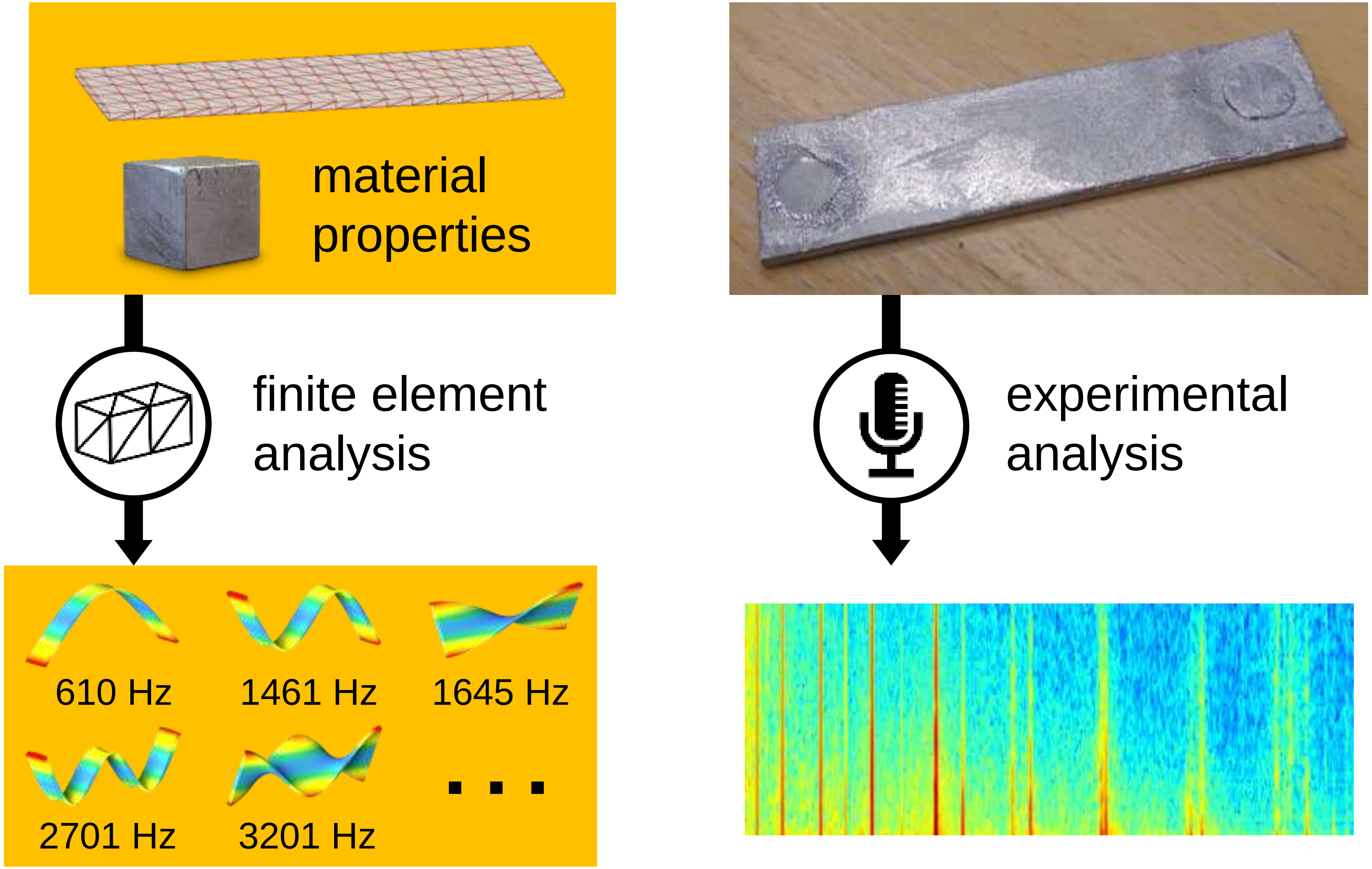
Natural Frequencies III

Natural modes depend on

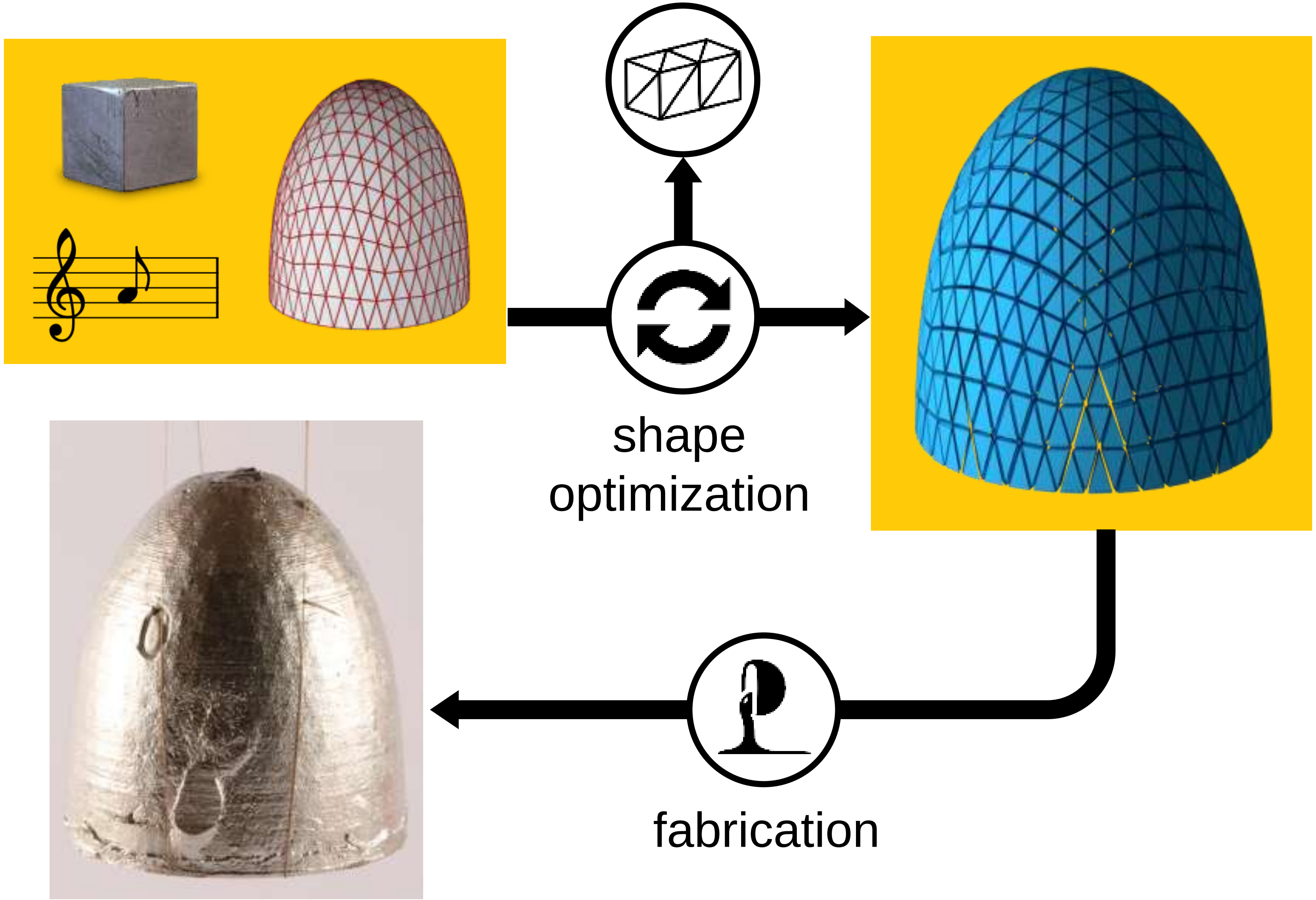
- material

- shape

Modal Analysis

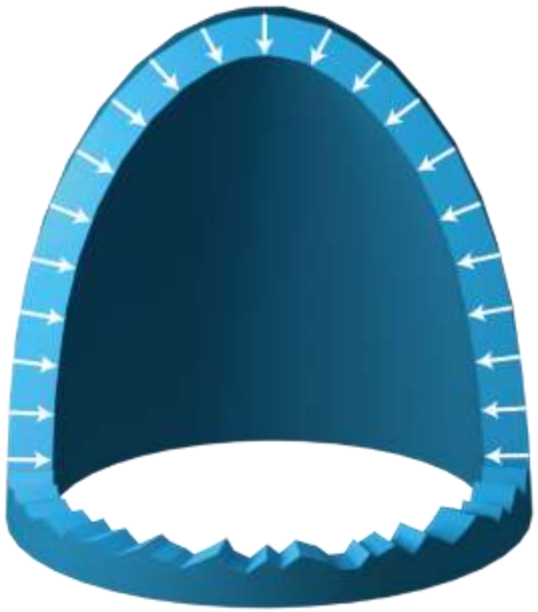


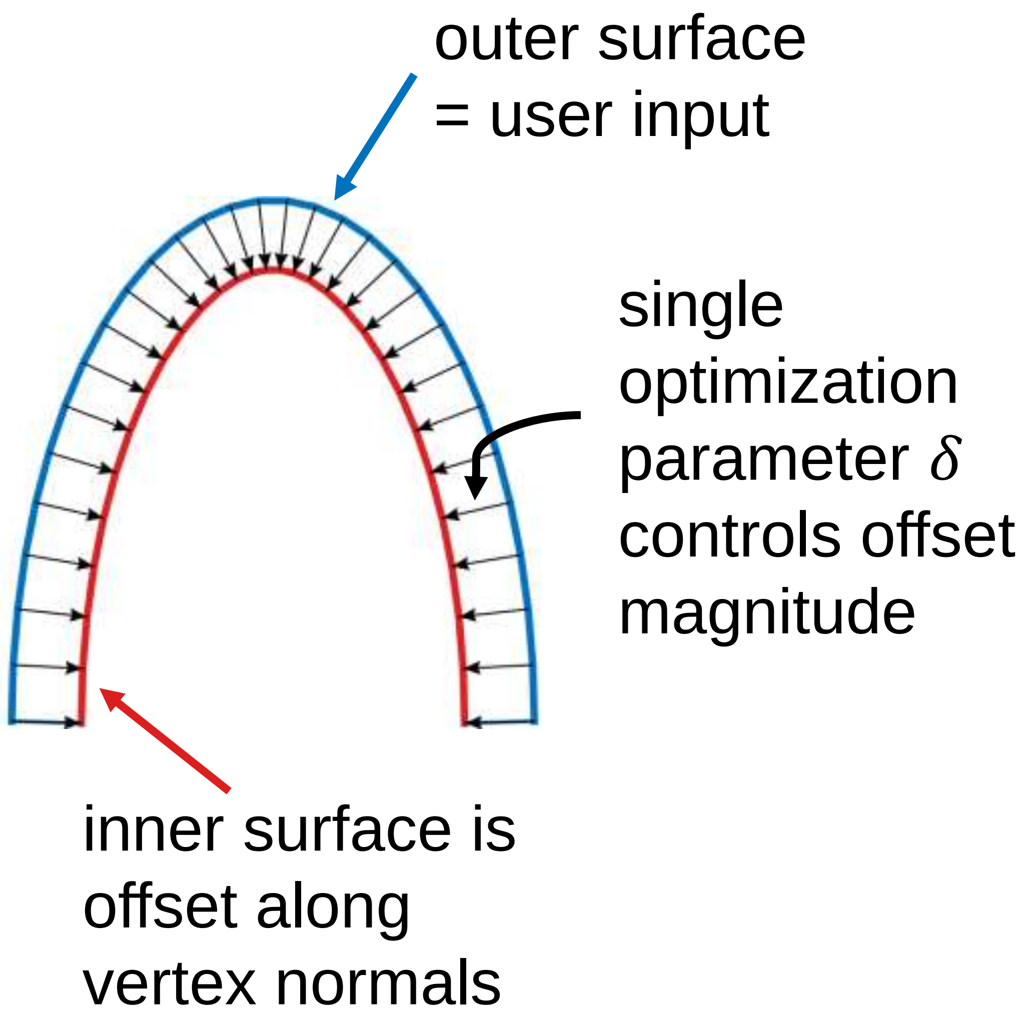
Goal: “Modal Synthesis”



Vertex-Normal Parametrization

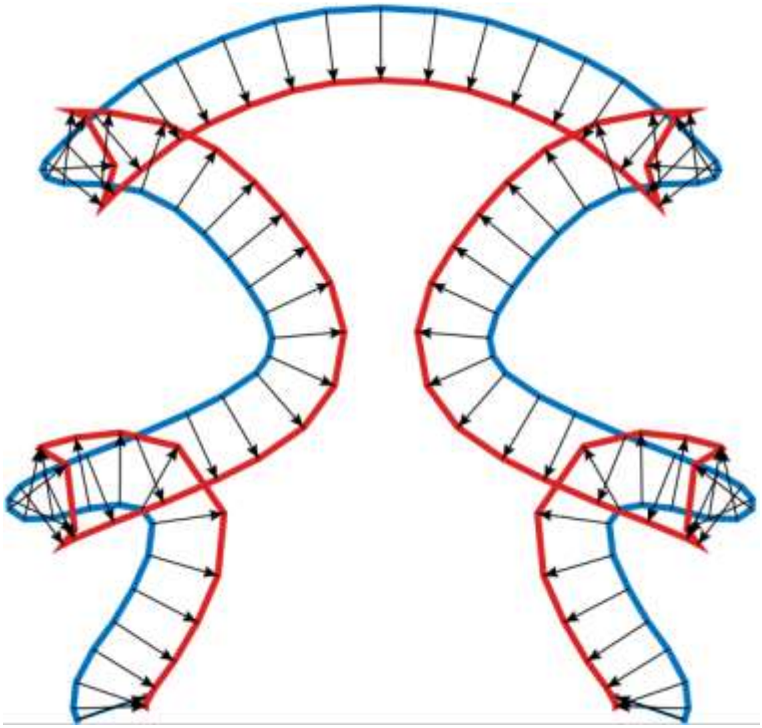
- Use offset surfaces
- Constant wall thickness





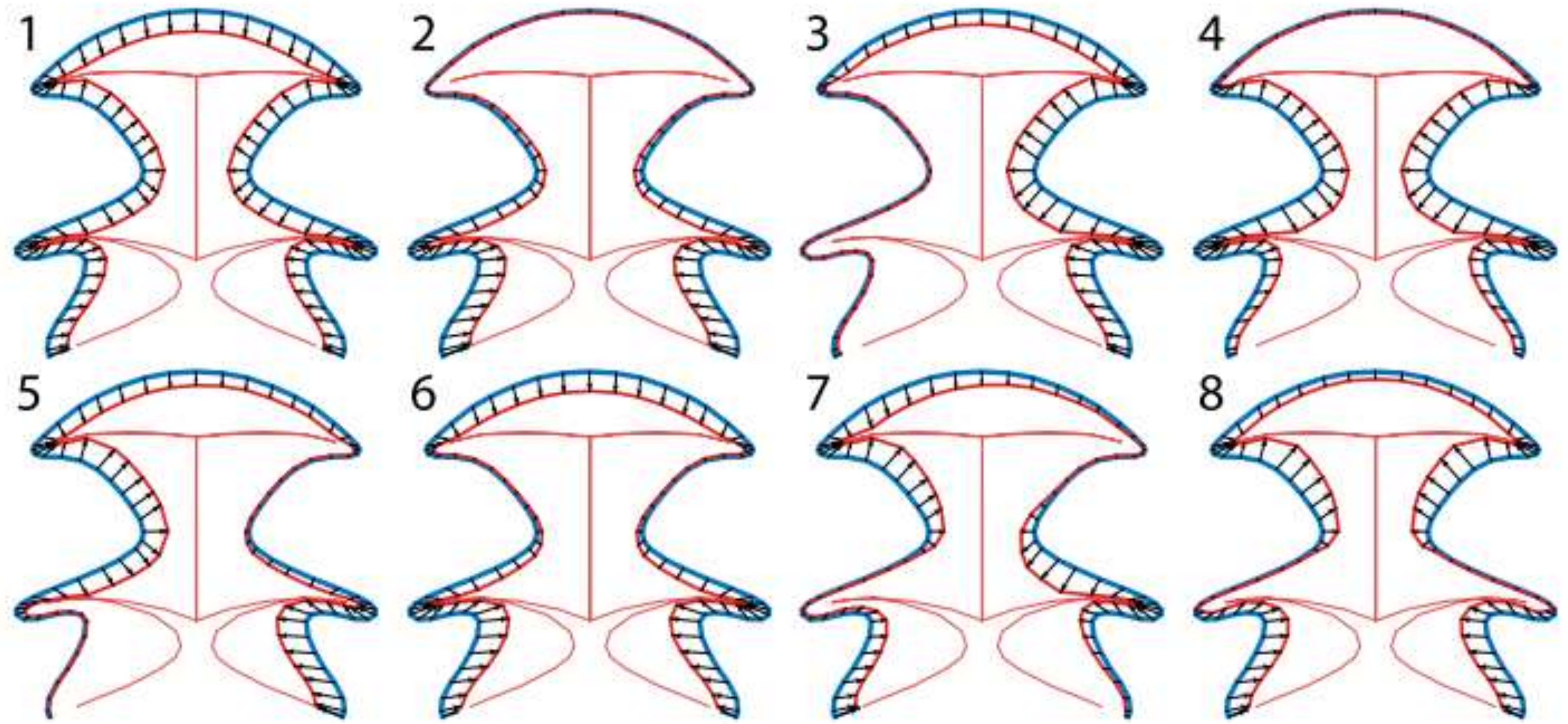
Vertex-Normal Parametrization

- Large offsets and high curvatures
  $\Longrightarrow$ self-intersections



Shape Parametrization

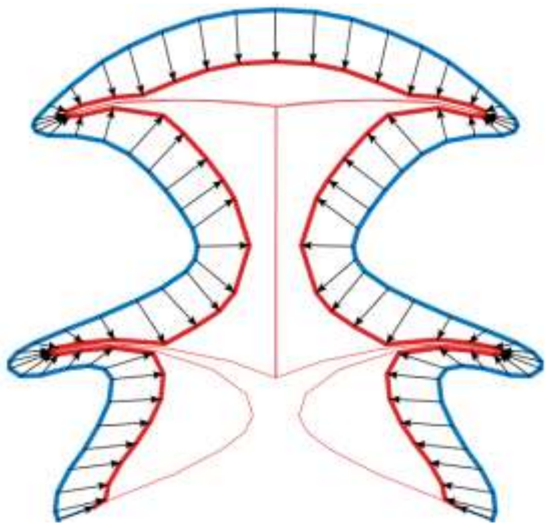
- Use Reduced Basis with Manifold Harmonics



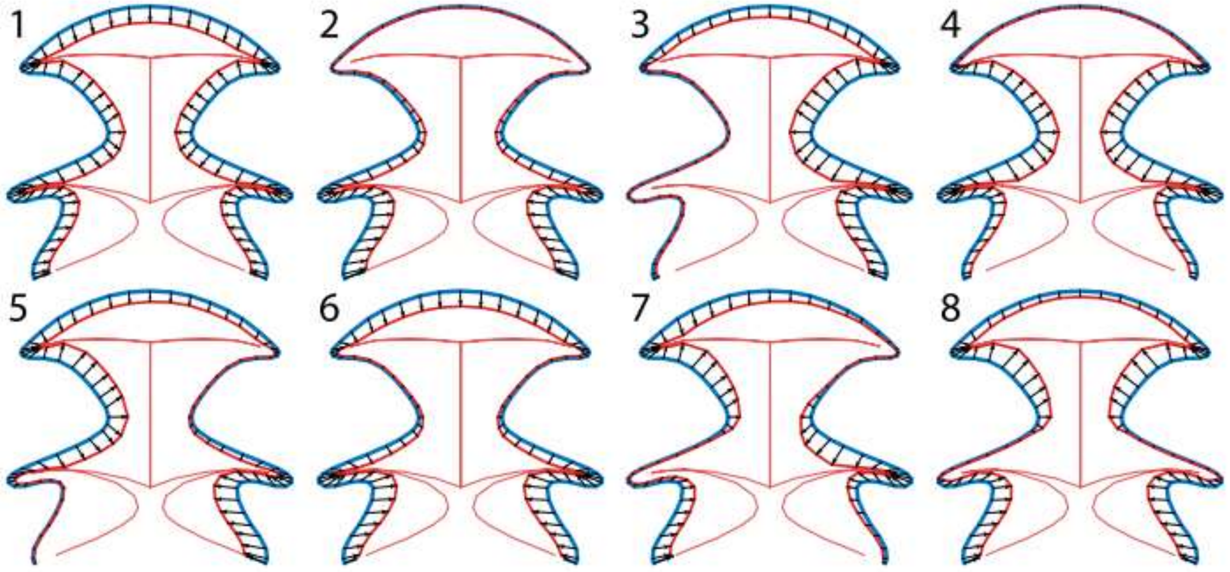
Shape Parametrization

- Use Reduced Basis with Manifold Harmonics
- Define offsets $\boldsymbol{\delta}$ as linear combination of basis functions $\boldsymbol{\Gamma}_{\boldsymbol{k}}$

$$\boldsymbol{\delta} = \boldsymbol{\Gamma}_k \boldsymbol{\alpha}$$



$$= \boldsymbol{X} + \sum_{i=1}^{k}$$



Shape Optimization

- Use non-linear optimization routine (Matlab)

$$\min_{\boldsymbol{\alpha}} f(\boldsymbol{\alpha}) = (p - p_0)^2$$

- p_0 … target pitch
- p … pitch of incument solution
- $\boldsymbol{\alpha}$ … coefficient vector

Fabrication

- Material
  - good acoustic properties
  - cast into complex shape

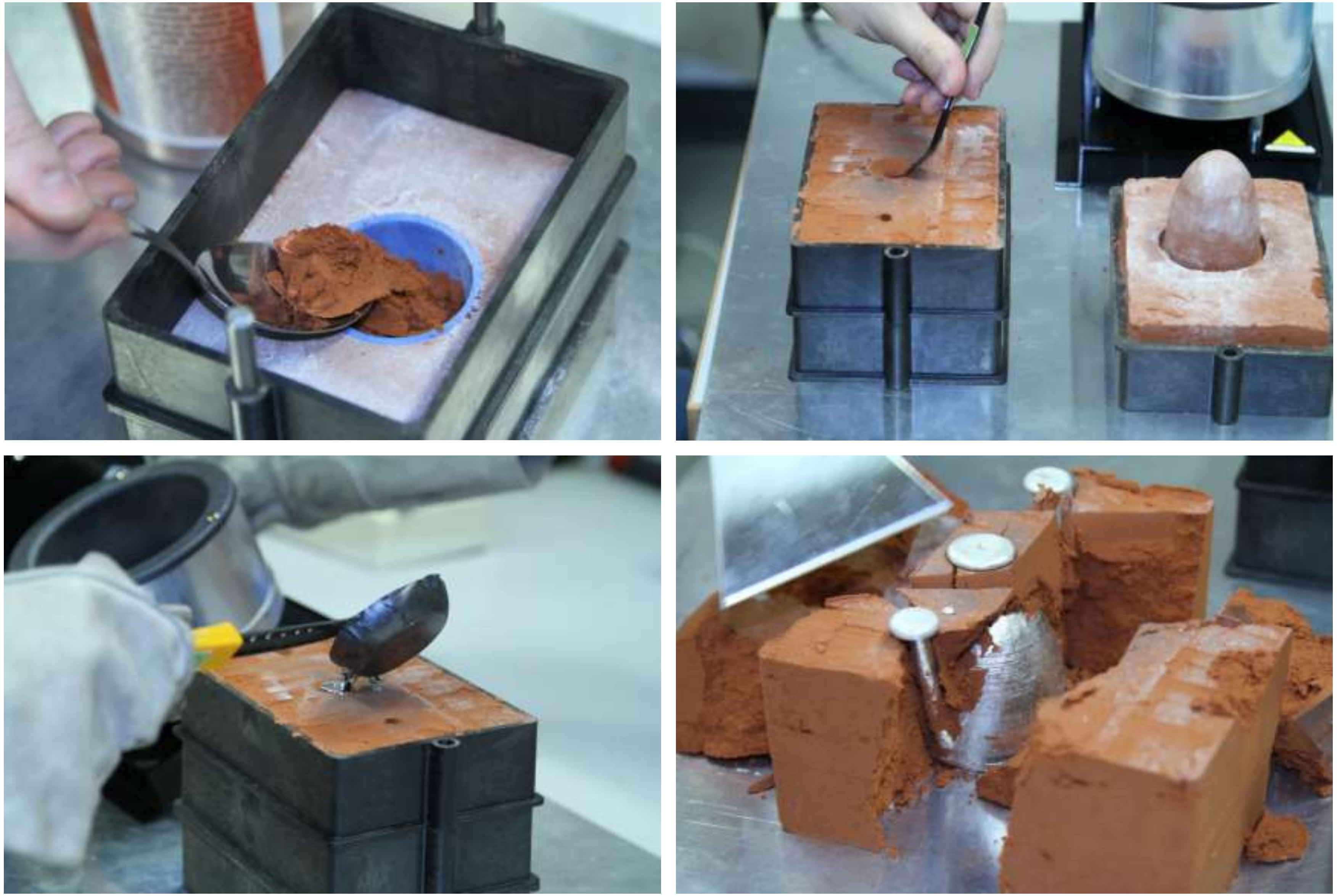
- Tin
  - melting point of 230°C
  - Young’s modulus of 50 GPa

Fabrication

- Oval bell
  - molds from sand

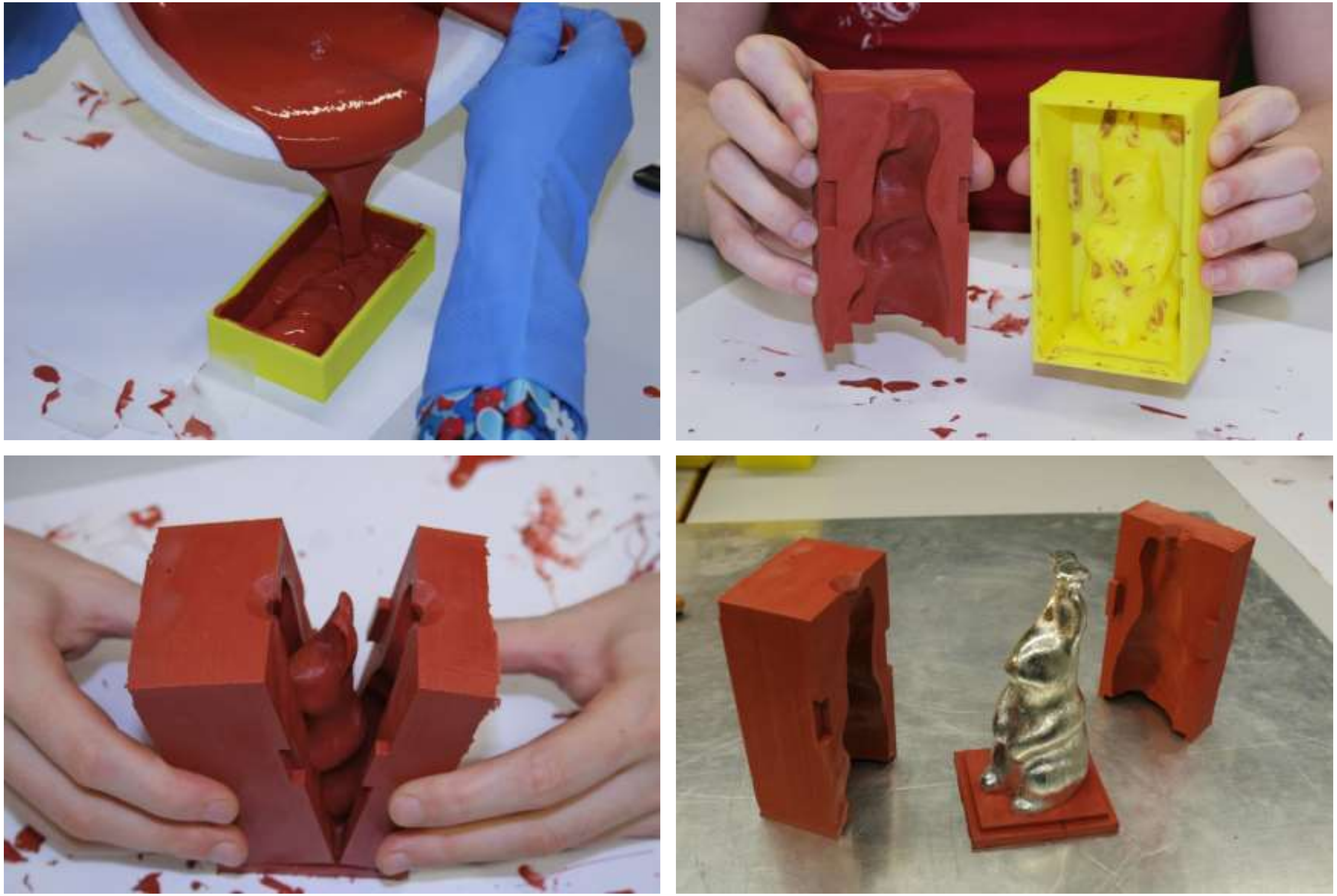
- Rabbit bell
  - molds from caoutchouc

Bell



Bell

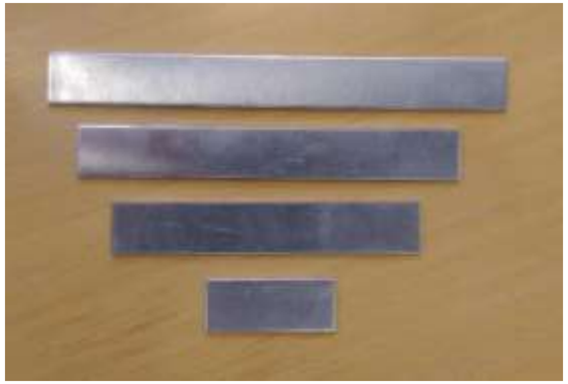
Rabbit



Rabbit

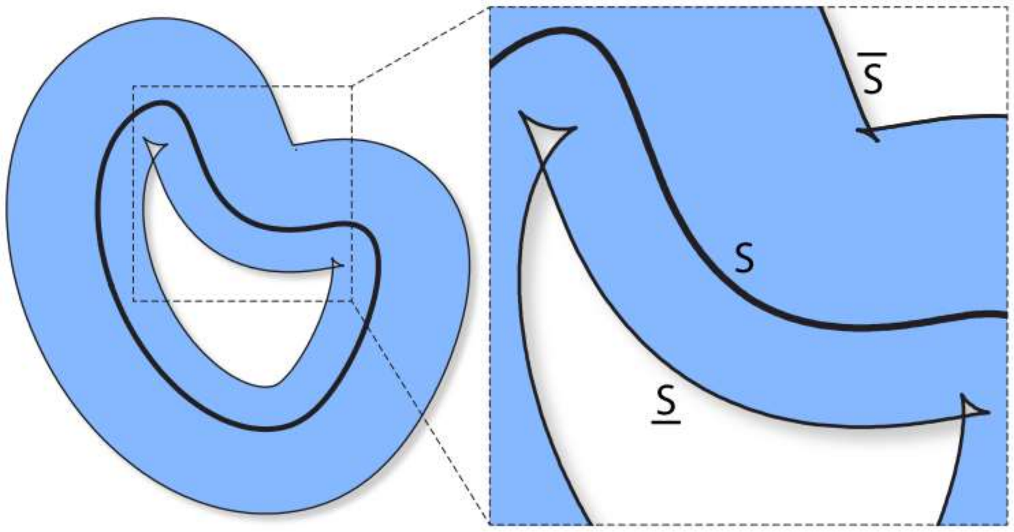
Results

- Aluminium plates
  - Median error of 1.7%
  - 0.7% with parameter estimation
- Bell
  - Error of 2.8%
- Rabbit Bell
  - Error of 11%
  - 6% with parameter estimation



Discussion & Limitations

- skeleton dependence
  - our method relies on the skeleton
  - we use iterative mesh contraction (Mean Curvature Flow)



- design space limitation
  - we can only offset a surface up to the skeleton

Conclusions

- we proposed a novel framework for shape optimization
- we provide an elegant and efficient basis-reduction
- we demonstrate the method by optimizing
  - mass properties
  - natural frequencies

Publications

- Musialski, P., Auzinger, T., Birsak, M., Wimmer, M. & Kobbelt, L. Reduced-Order Shape Optimization Using Offset Surfaces. *ACM Trans. Graph. (Proc. ACM SIGGRAPH 2015)* **34,** 102:1–102:9 (2015).
- Hafner, C., Musialski, P., Auzinger, T., Wimmer, M. & Kobbelt, L. Optimization of natural frequencies for fabrication-aware shape modeling. in *ACM SIGGRAPH 2015 Posters - SIGGRAPH '15* 1–1 (ACM Press, 2015).
- Hafner, C. Optimization of Natural Frequencies for Fabrication-Aware Shape Modeling, Master Thesis, TU-Wien (2015)

Thank you!