

Computational Design for Consumer-Level Fabrication

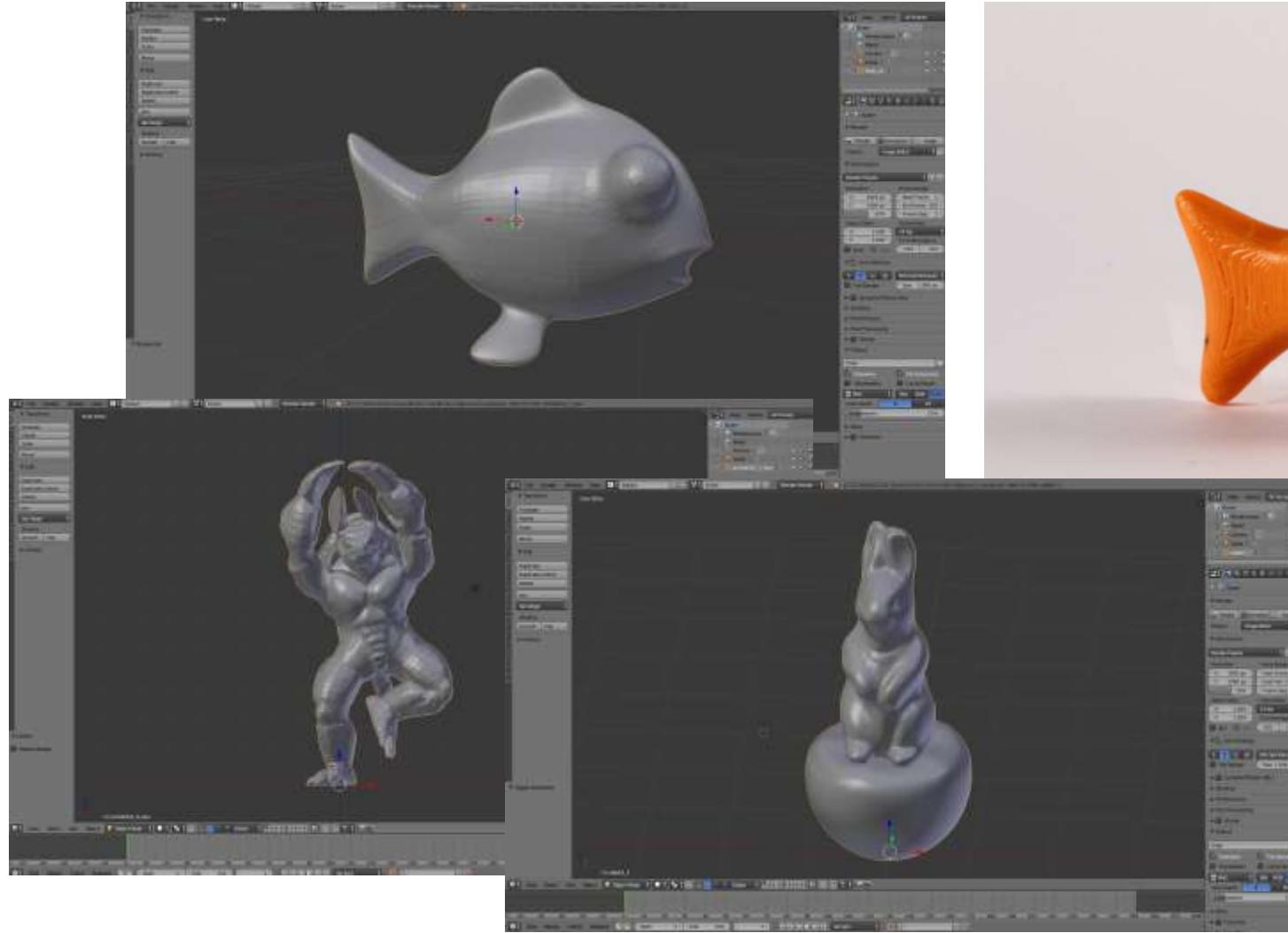
Przemyslaw Musalski

TU Wien

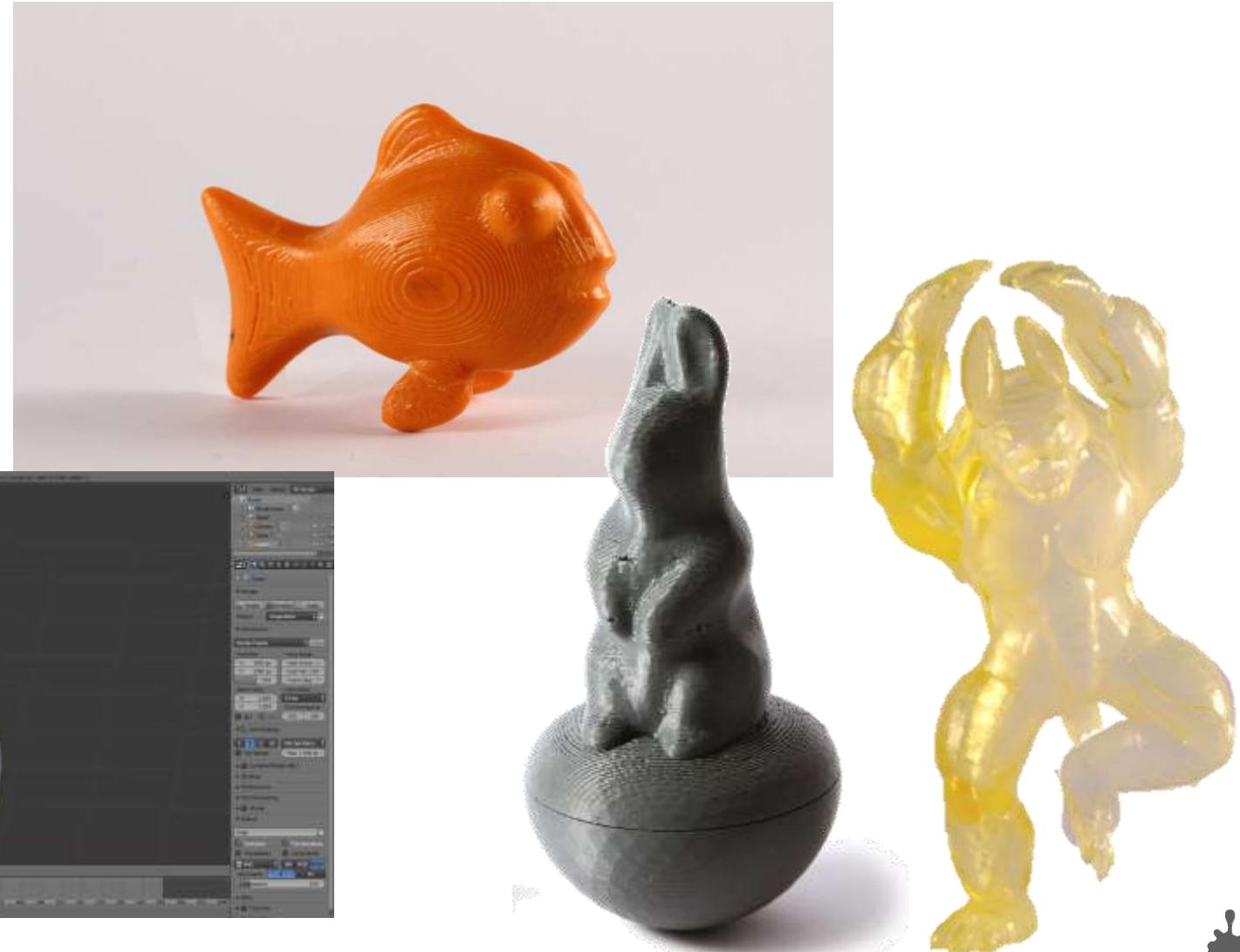


Motivation

3D Modeling...

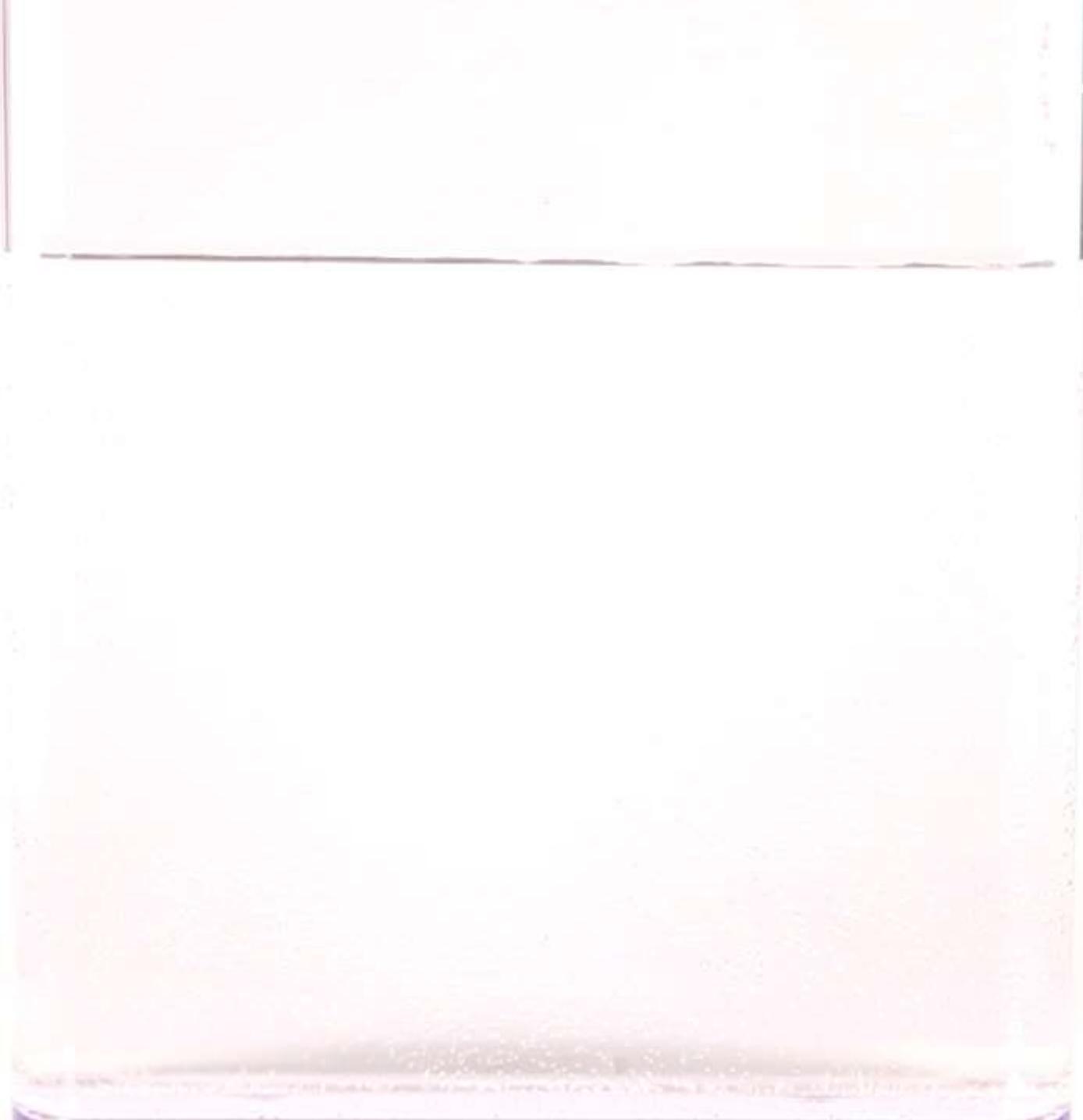


3D Printing...



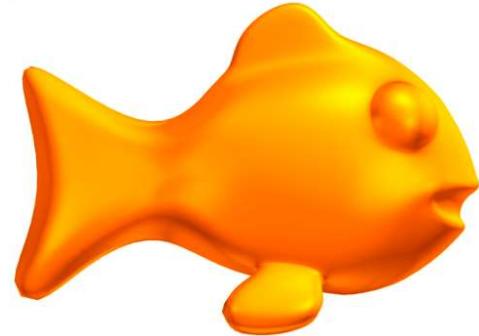
Motivation



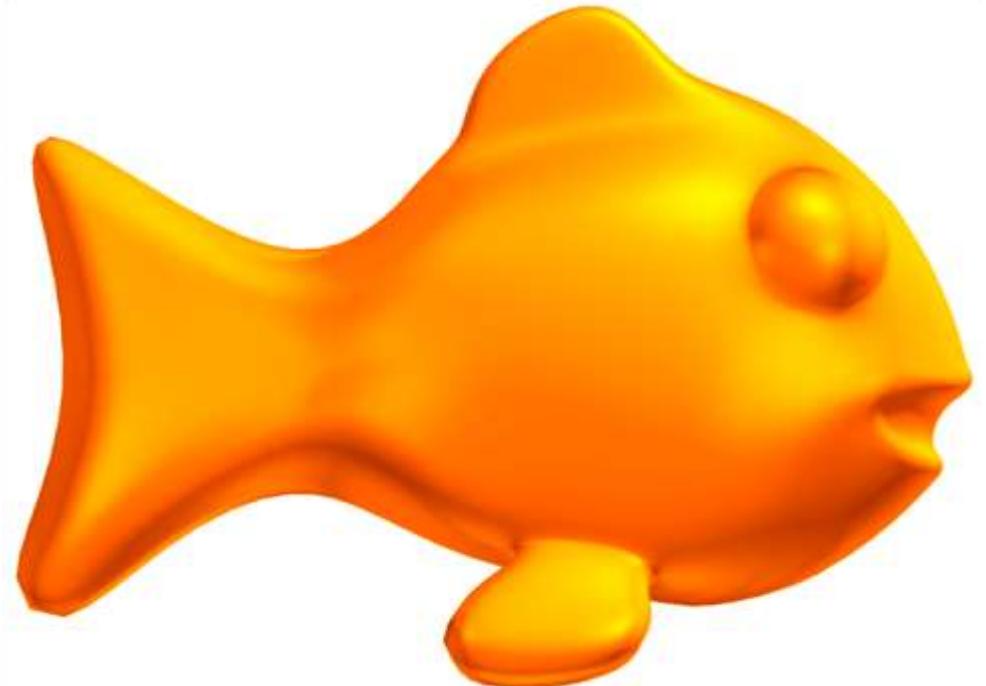


Goals

1. Optimize the shape to fulfill the desired goals



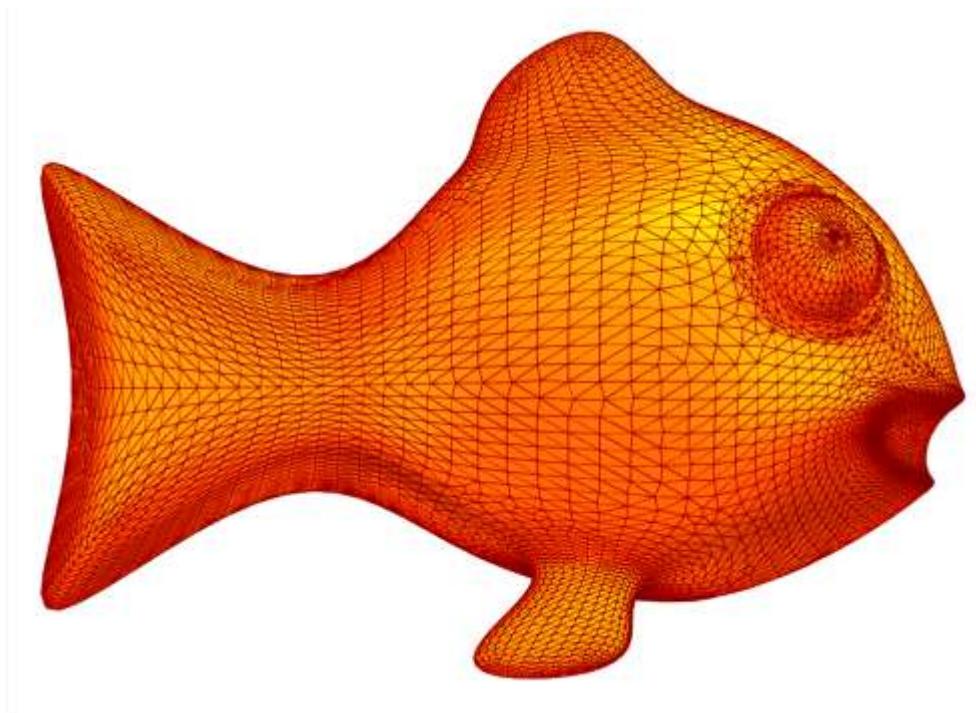
1. Optimize the shape to fulfill the desired goals
2. Keep the input shape deformation minimal



Shape Optimization

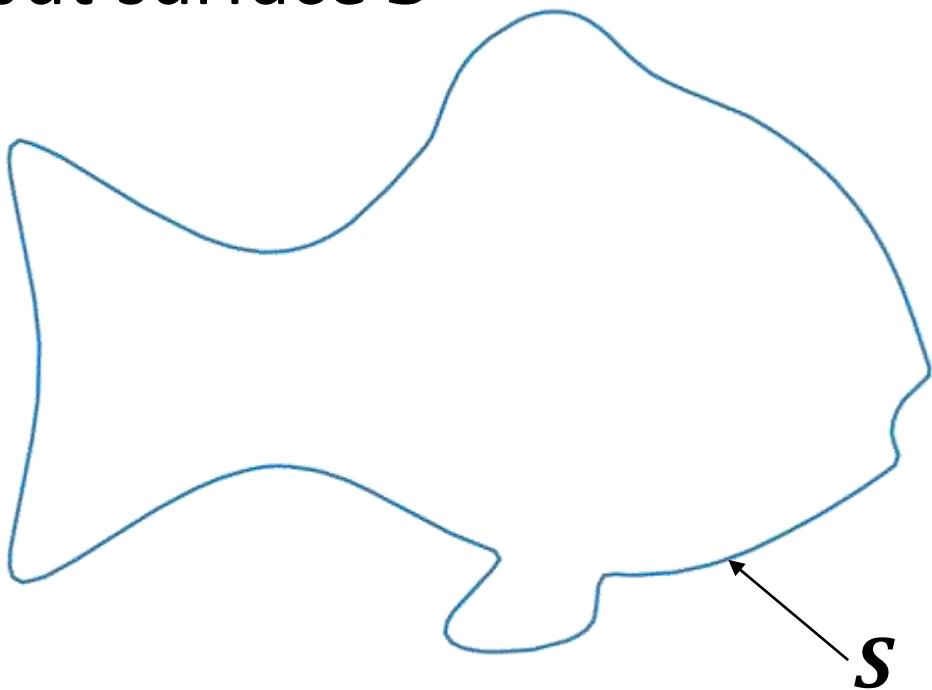


Input and Output



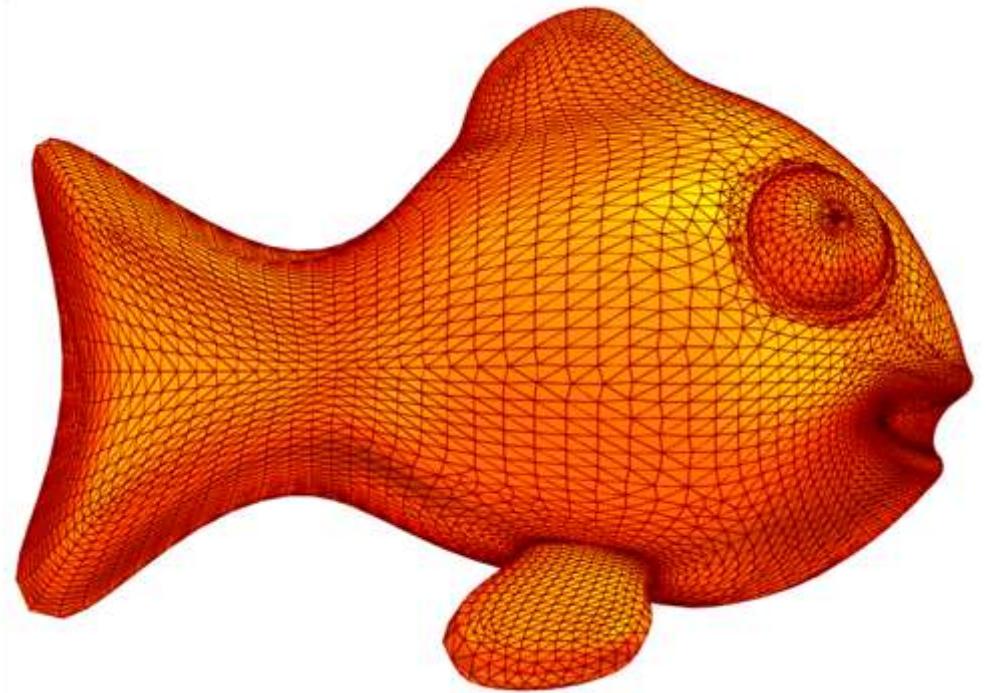
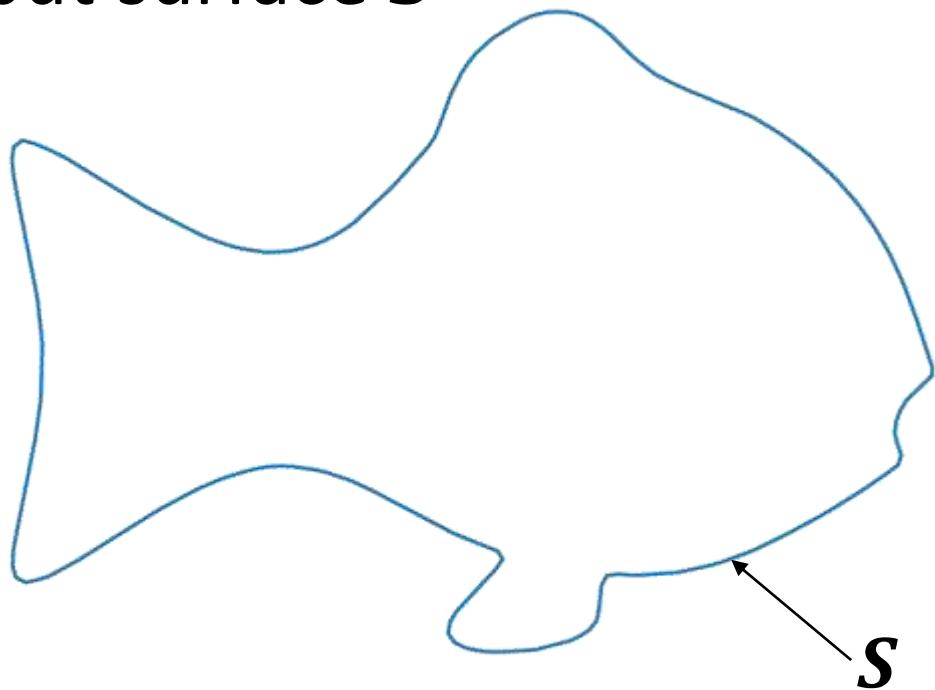
Input and Output

- Input surface S



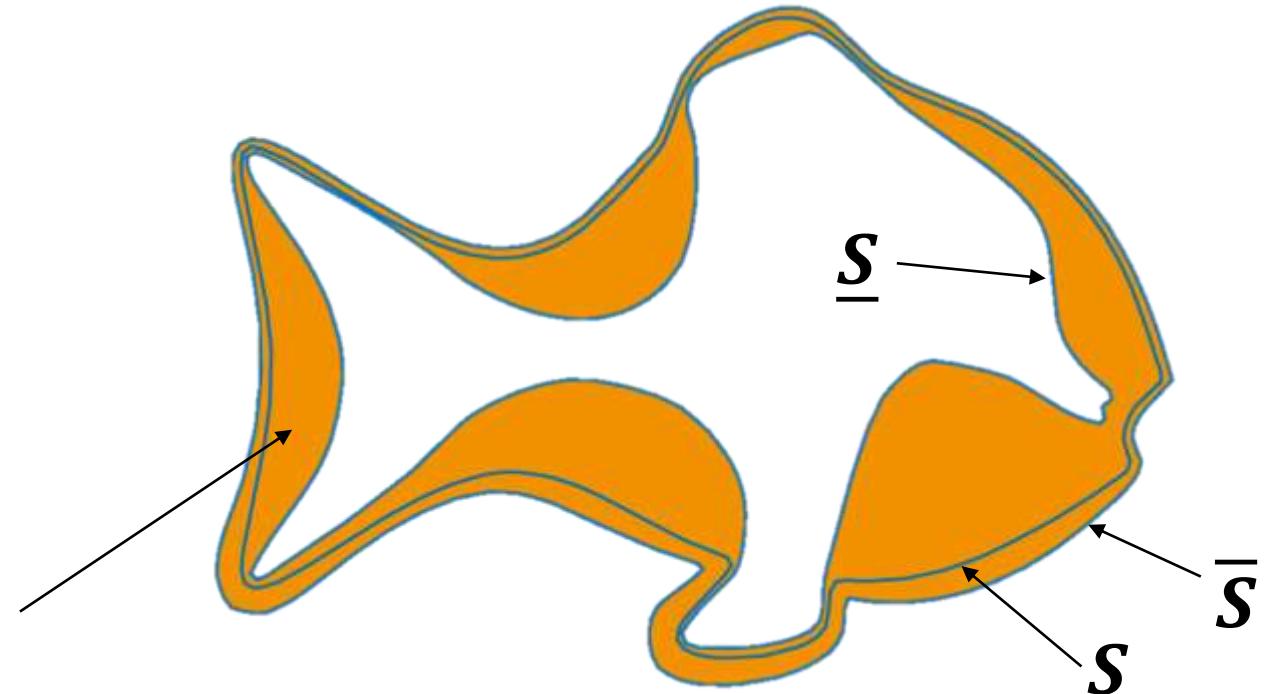
Input and Output

- input surface S



Input and Output

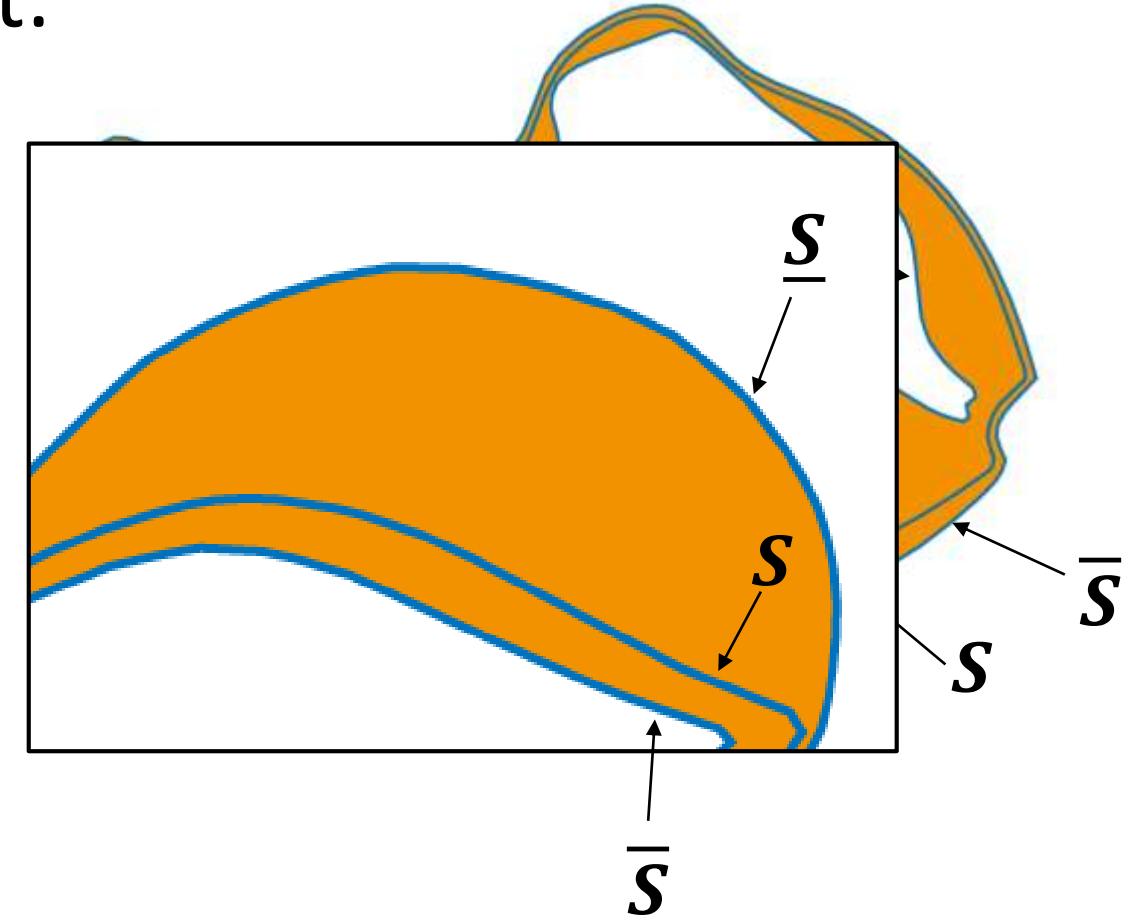
- input surface S
- output: two surfaces
 - outer offset surface \bar{S}
 - inner offset surface \underline{S}
- **solid body** between \bar{S} and \underline{S}



Offset Surfaces

- surface deformation by offset:

$$\underline{x}_i = x_i + \underline{\delta}_i \mathbf{v}_i$$

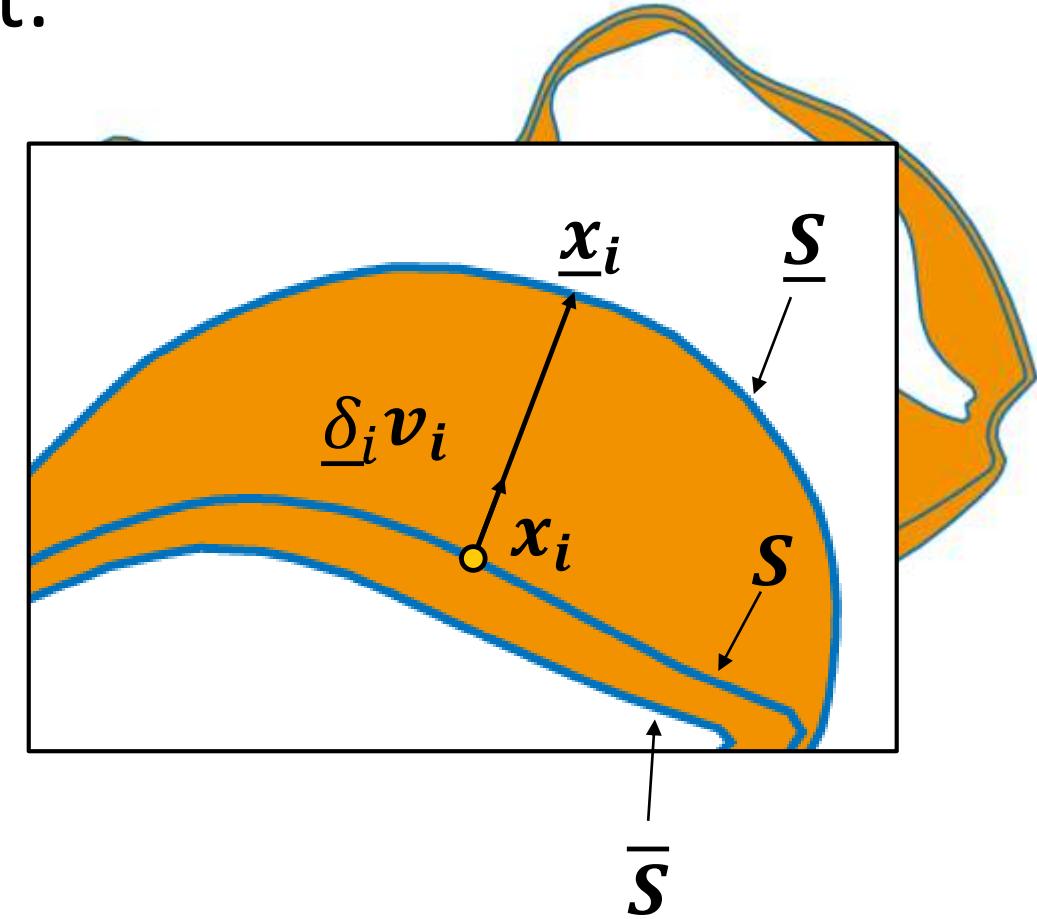


Offset Surfaces

- surface deformation by offset:

$$\underline{x}_i = \underline{x}_i + \underline{\delta}_i \boldsymbol{v}_i$$

- for each vertex \underline{x}_i
- along \boldsymbol{v}_i
- add an individual offset $\underline{\delta}_i$

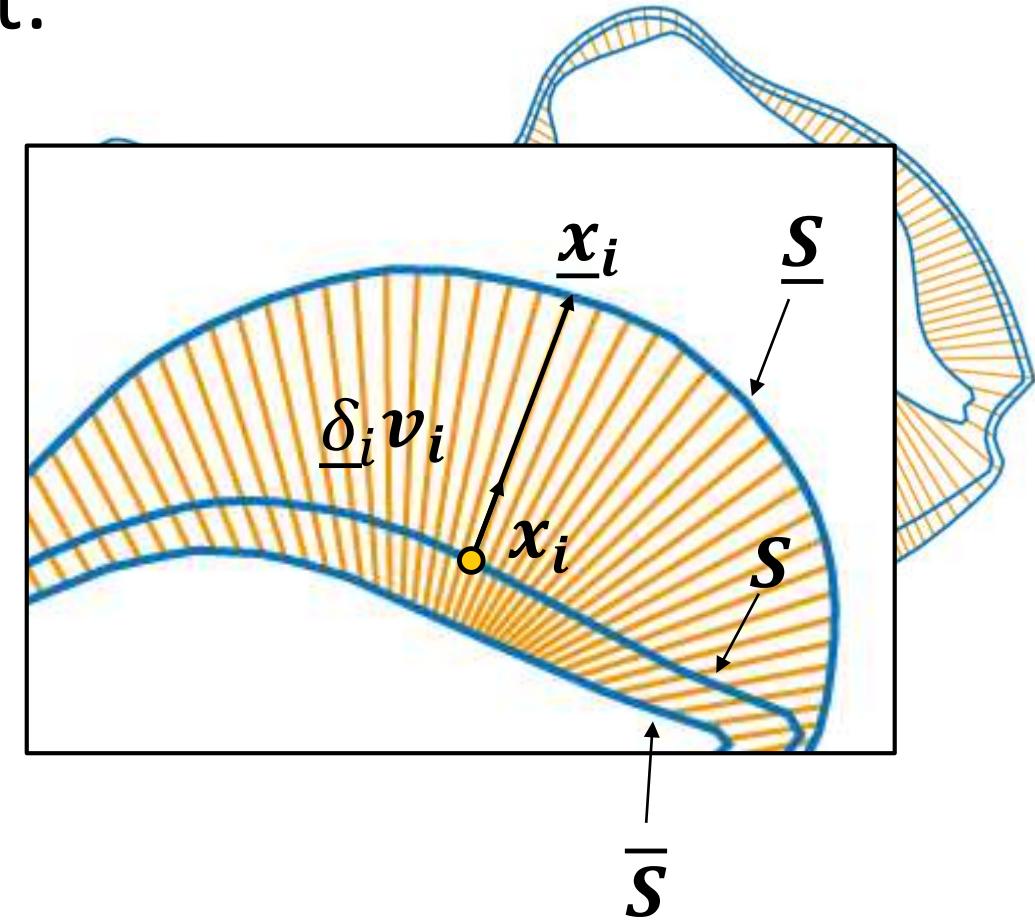


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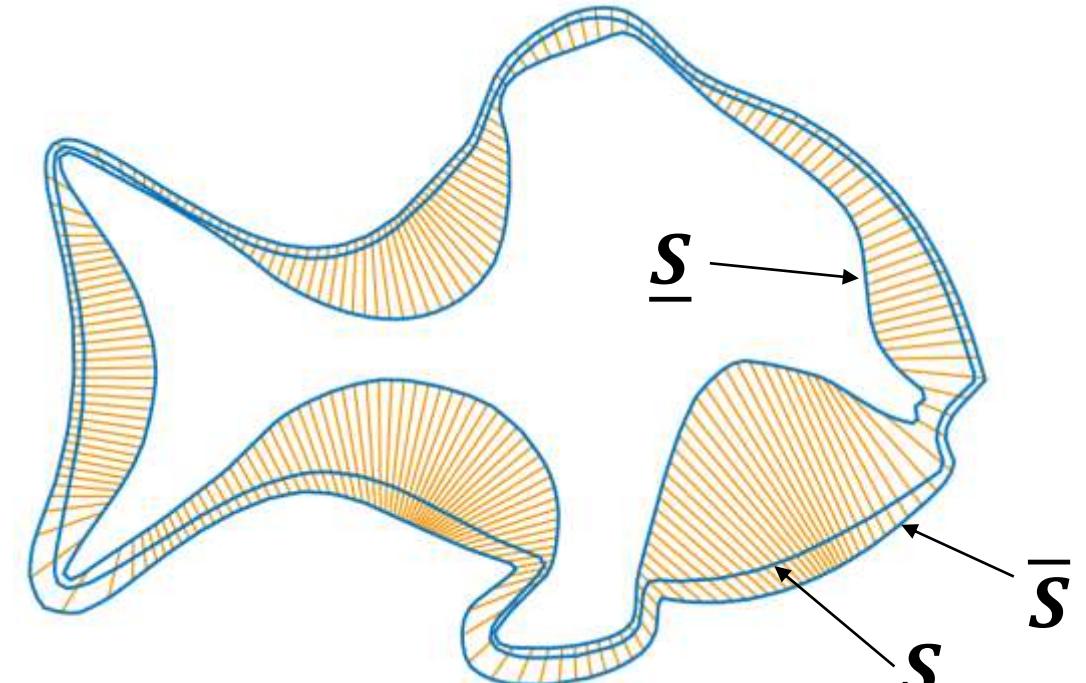


Offset Surfaces

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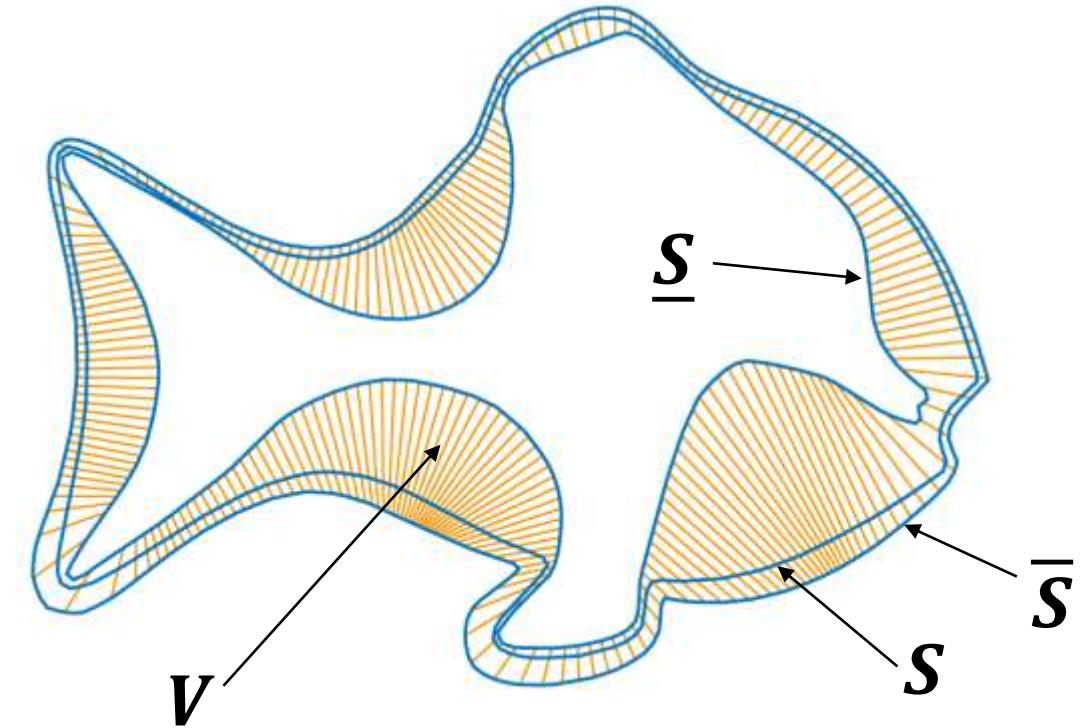
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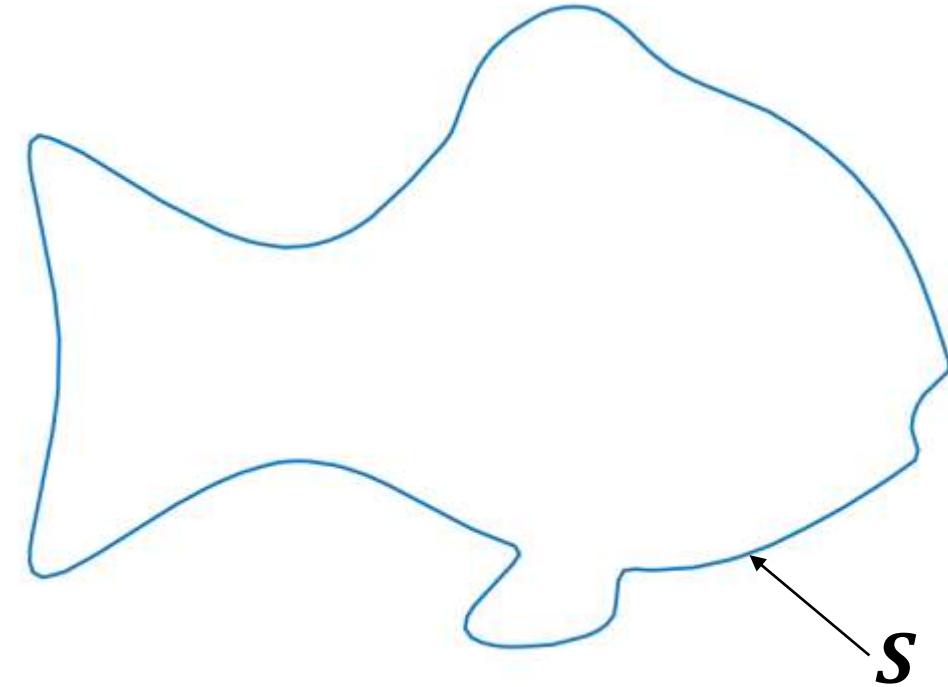


Offset Surfaces

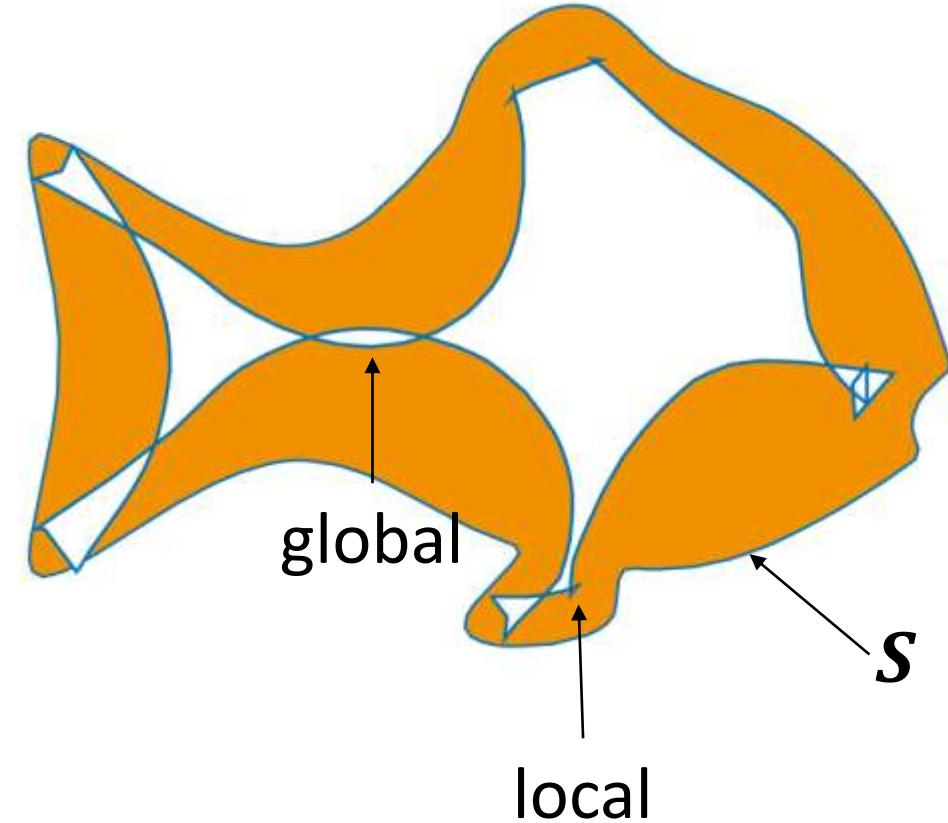
- How far can we offset?
- Along which directions V ?



Offset Bounds

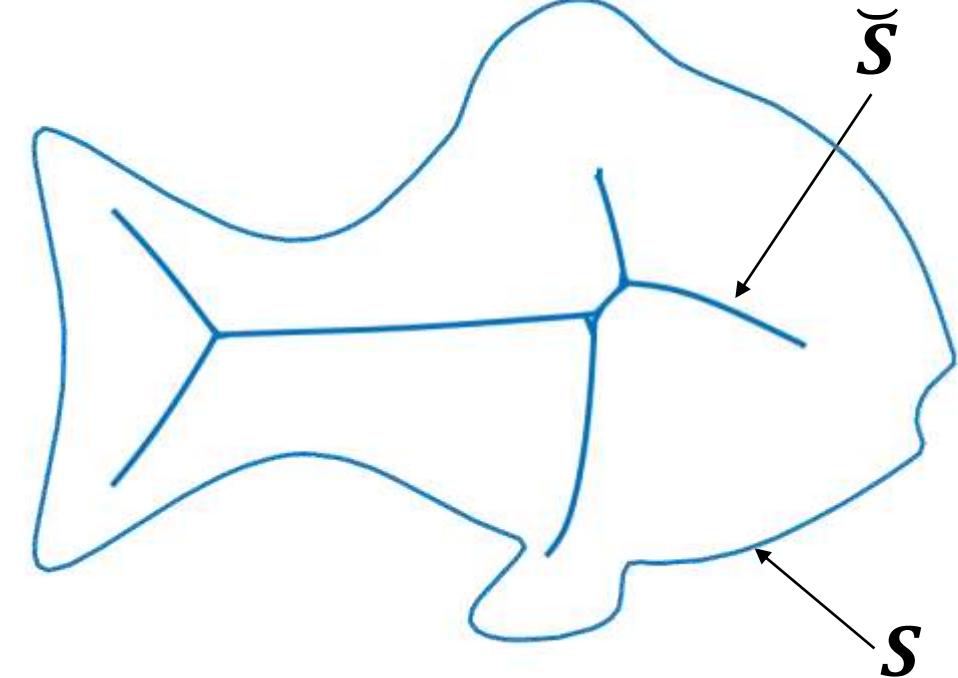


Offset Bounds



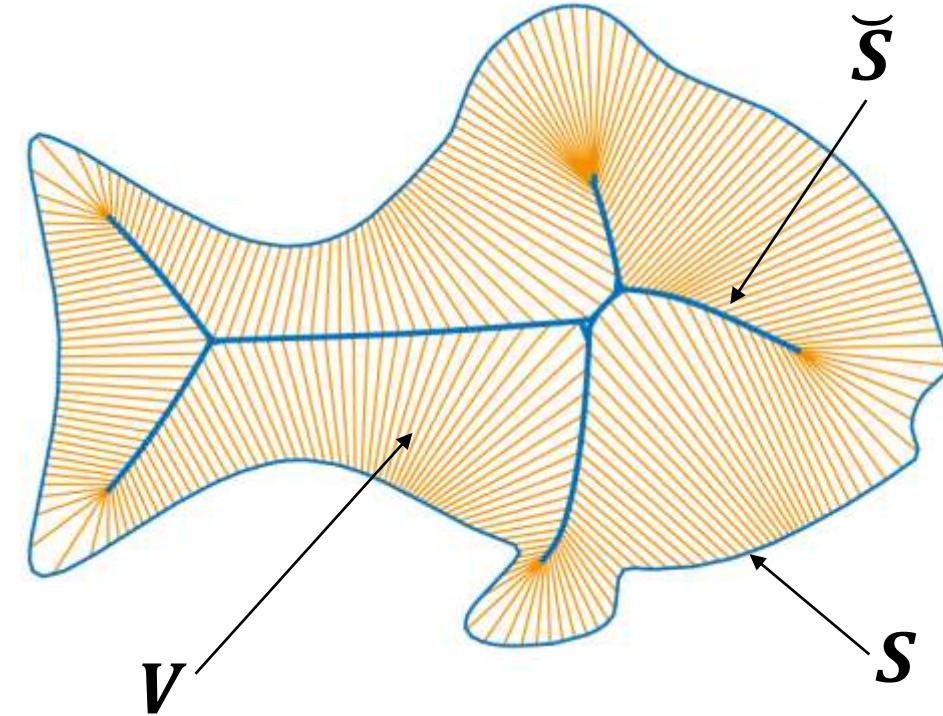
Offset Bounds

- inside: skeleton \check{S}
 - Mean Curvature Flow
[Tagliasacchi et al. 2012]



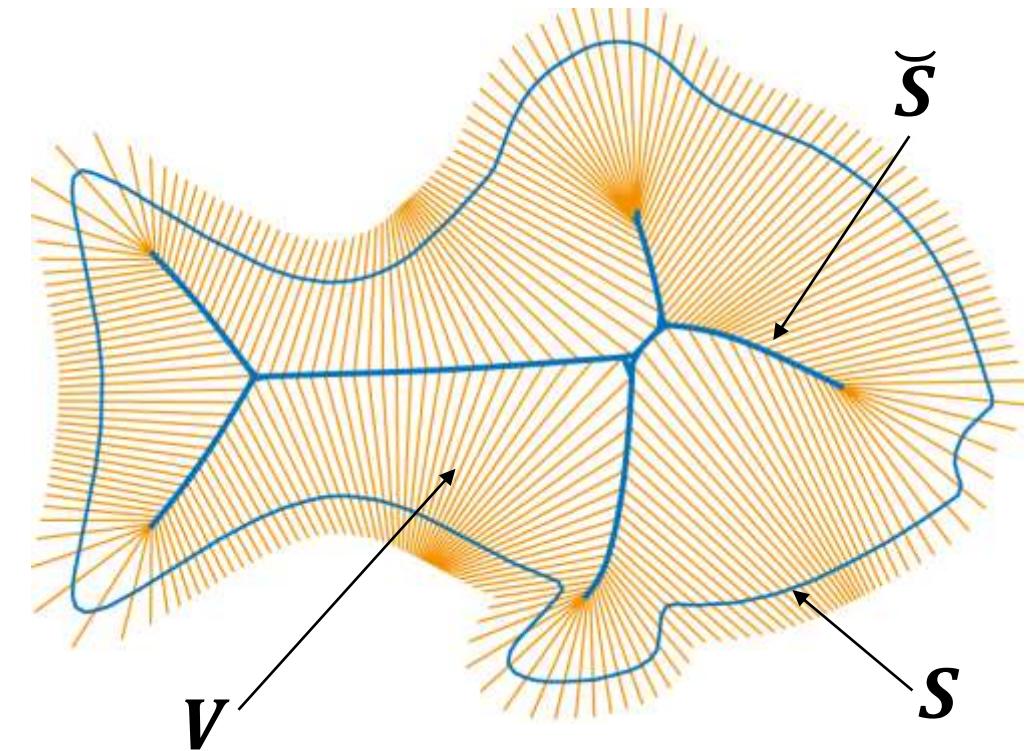
Offset Vectors

- inside: skeleton \tilde{S}
 - Mean Curvature Flow
[Tagliasacchi et al. 2012]
- offset along vectors $v_i \in V$



Offset Vectors and Bounds

- inside: skeleton \tilde{S}
 - Mean Curvature Flow
[Tagliasacchi et al. 2012]
- offset along vectors $v_i \in V$
- outside a constant max. value

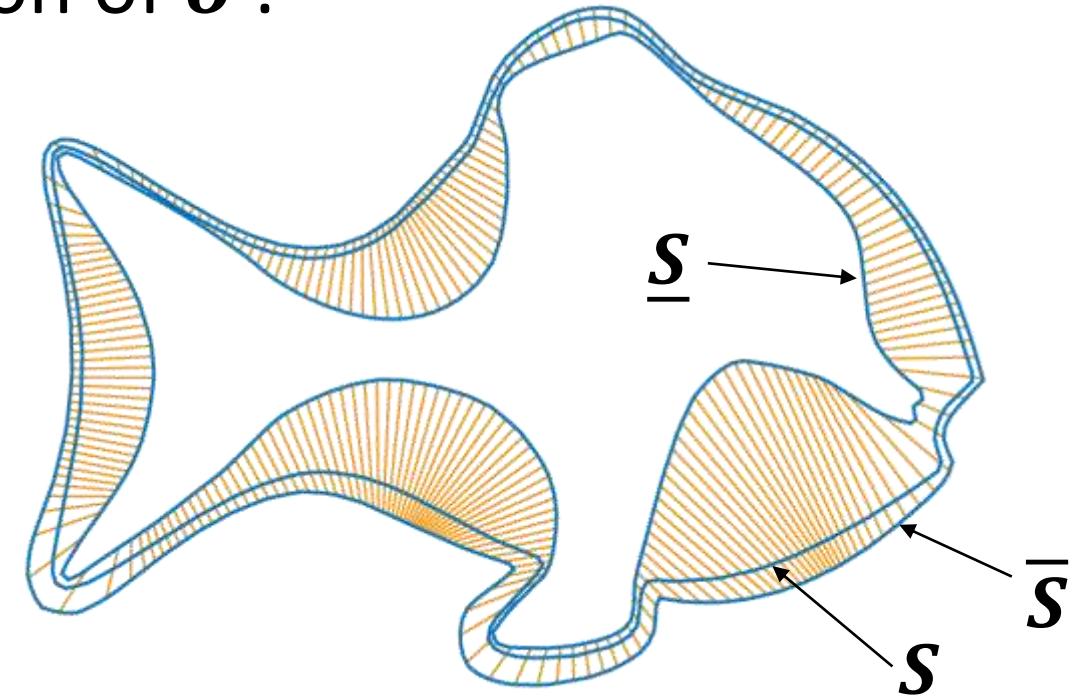


Shape Optimization Problem

- minimize objective $f(\delta)$ as a function of δ :

$$\min_{\delta} f(\delta) \text{ s.t. } g(\delta)$$

- subject to constraints $g(\delta)$
- for example:
 - $f :=$ make shape float subject to
 - $g :=$ keep upright orientation

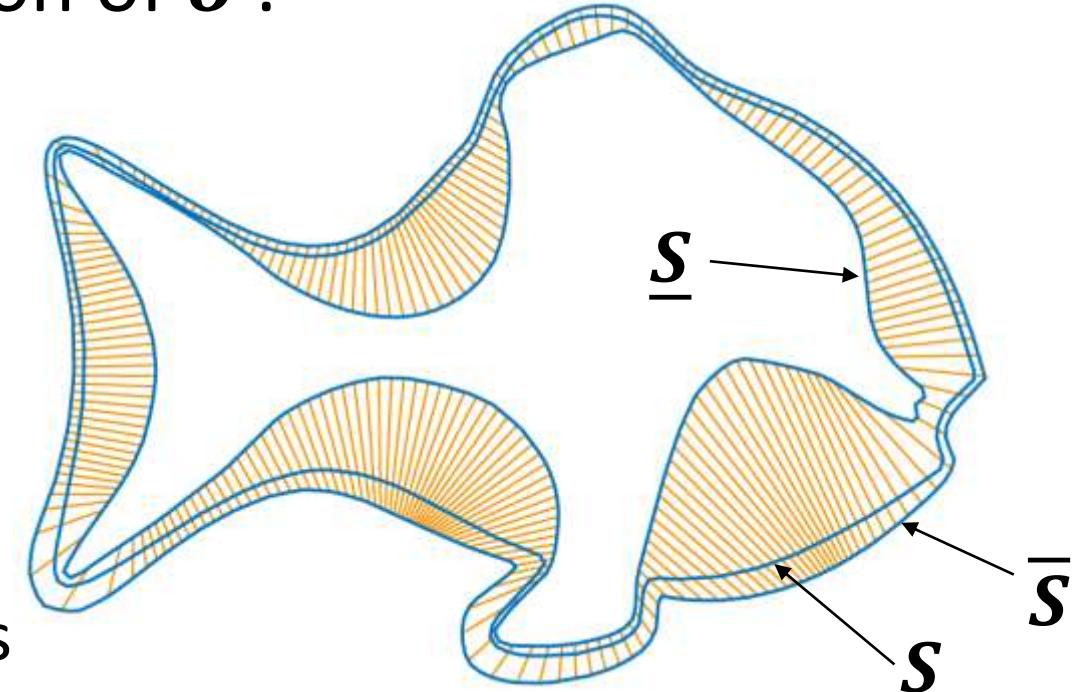


Shape Optimization Problem

- minimize objective $f(\delta)$ as a function of δ :

$$\min_{\delta} f(\delta) \rightarrow n \text{ unknowns}$$

- issues:
 - problem is huge for large meshes
→ scales very badly
 - problem is underdetermined
→ there exist many solutions (regularization needed)

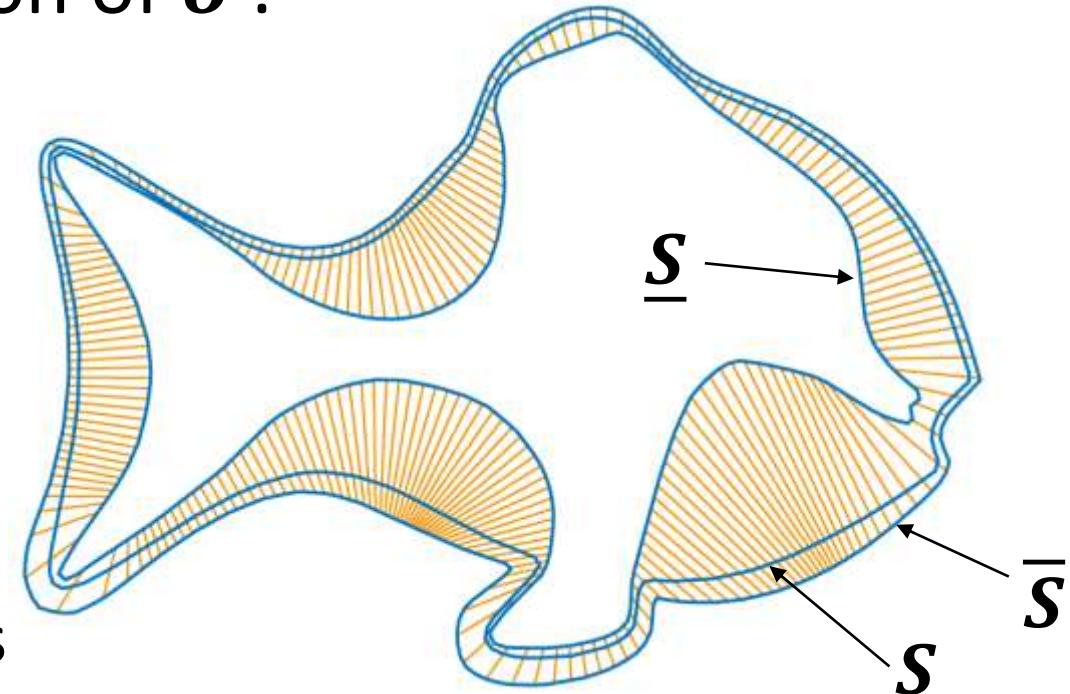


Shape Optimization Problem

- minimize objective f as a function of δ :

ergo: formulation is not suitable for practice

- issues:
 - problem is huge for large meshes
→ scales very badly
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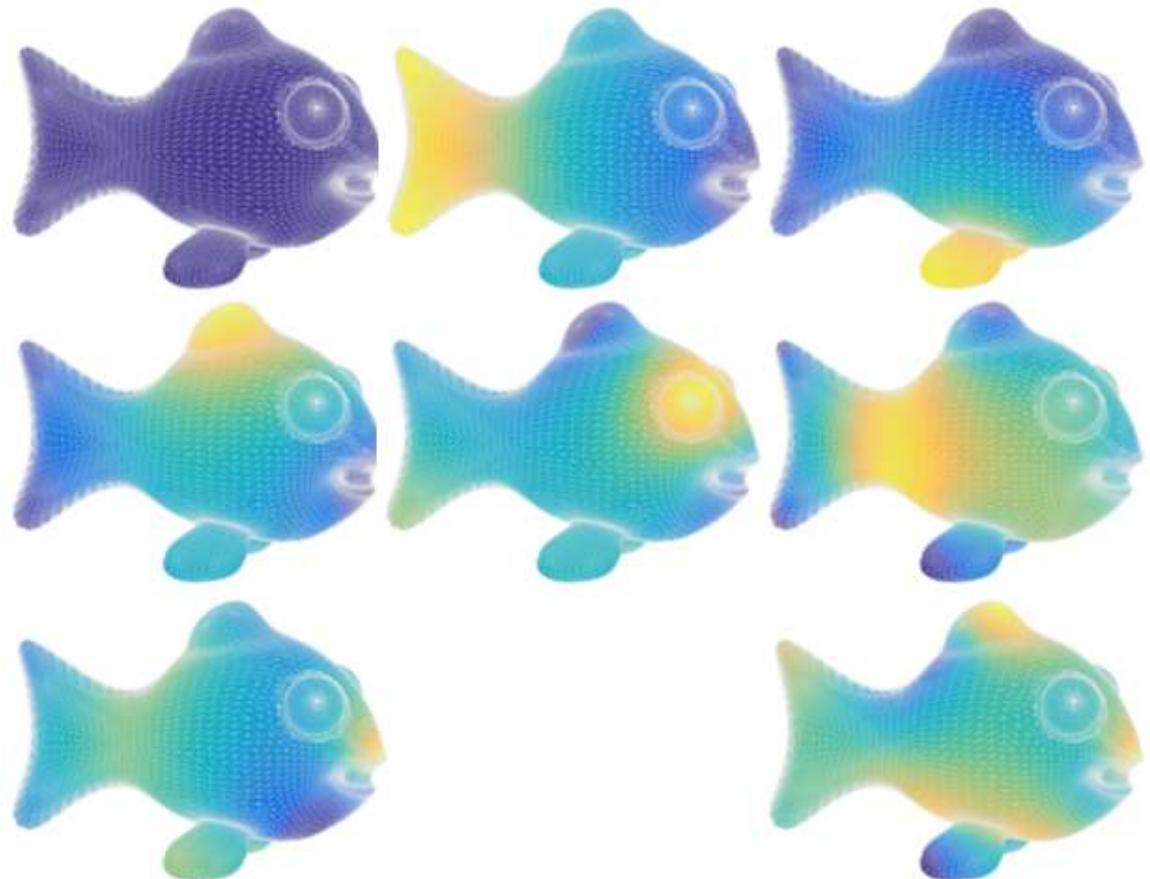


Order Reduction



Order Reduction

- order reduction:
 - lower the dimensionality while preserving input-output behavior
- idea:
 - project problem onto a lower dimensional space
- → Manifold Harmonics

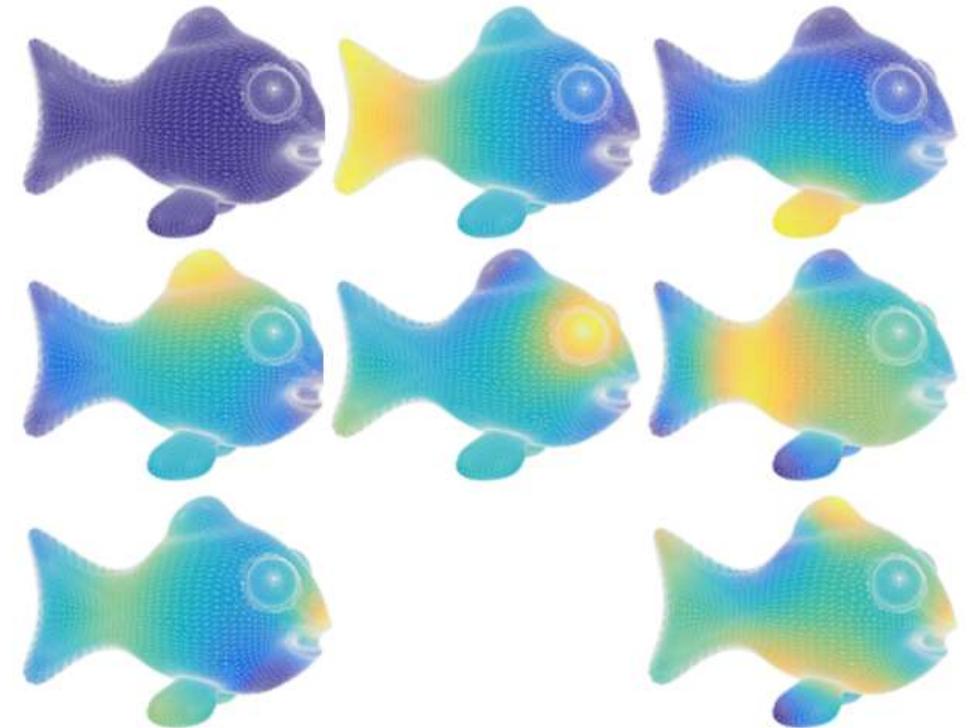


Manifold Harmonics

- diagonalization of the Laplacian matrix \mathbf{L}
→ Spectral Theorem:

$$\mathbf{L} = \boldsymbol{\Gamma} \boldsymbol{\Lambda} \boldsymbol{\Gamma}^T$$

- generalization of the **Fourier Transform** for scalar functions on surfaces



[VALLET, B. AND LÉVY, B. 2008. Spectral Geometry Processing with Manifold Harmonics. *Computer Graphics Forum* 27, 2, 251–260.]



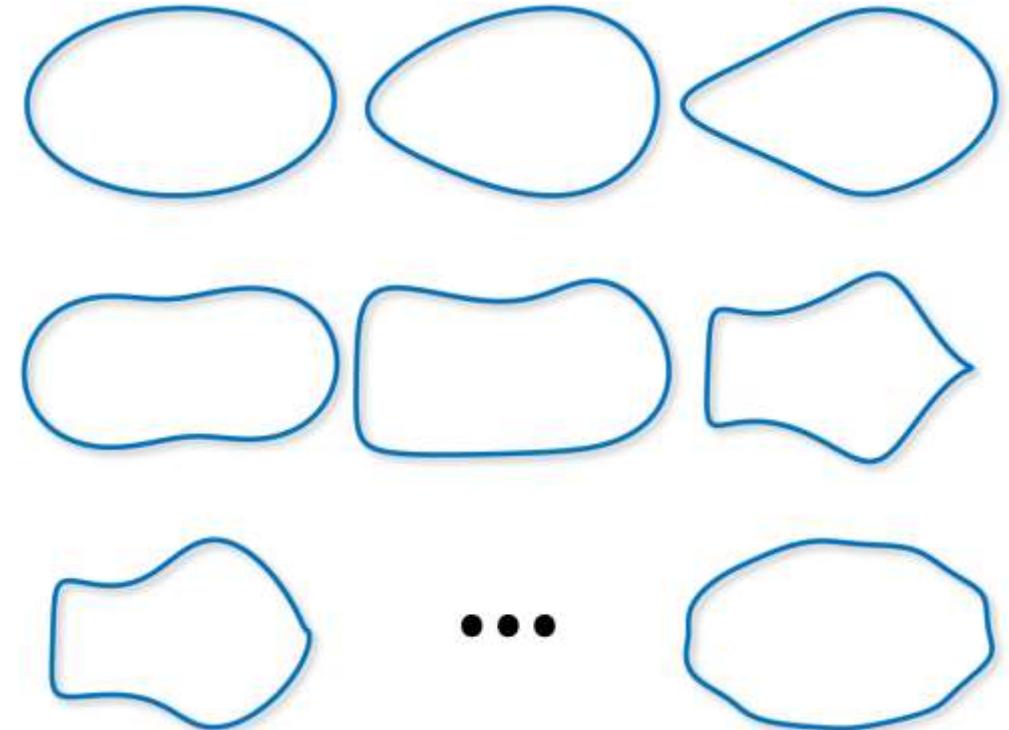
Manifold Harmonics

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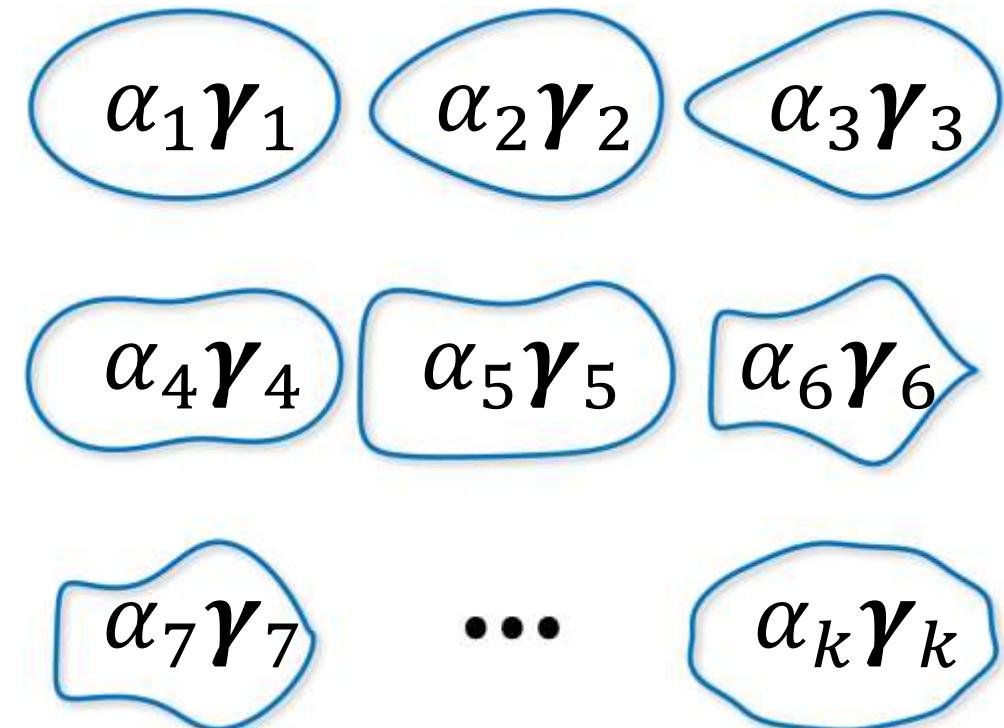
Order Reduction

- project unknown offsets

$\delta = [\delta_1, \delta_2, \dots, \delta_n]^T$ onto Γ_k :

$$\delta = \Gamma_k \alpha = \sum$$

- vector $\alpha = [\alpha_1, \alpha_2, \dots, \alpha_k]^T$
now contains the unknowns!



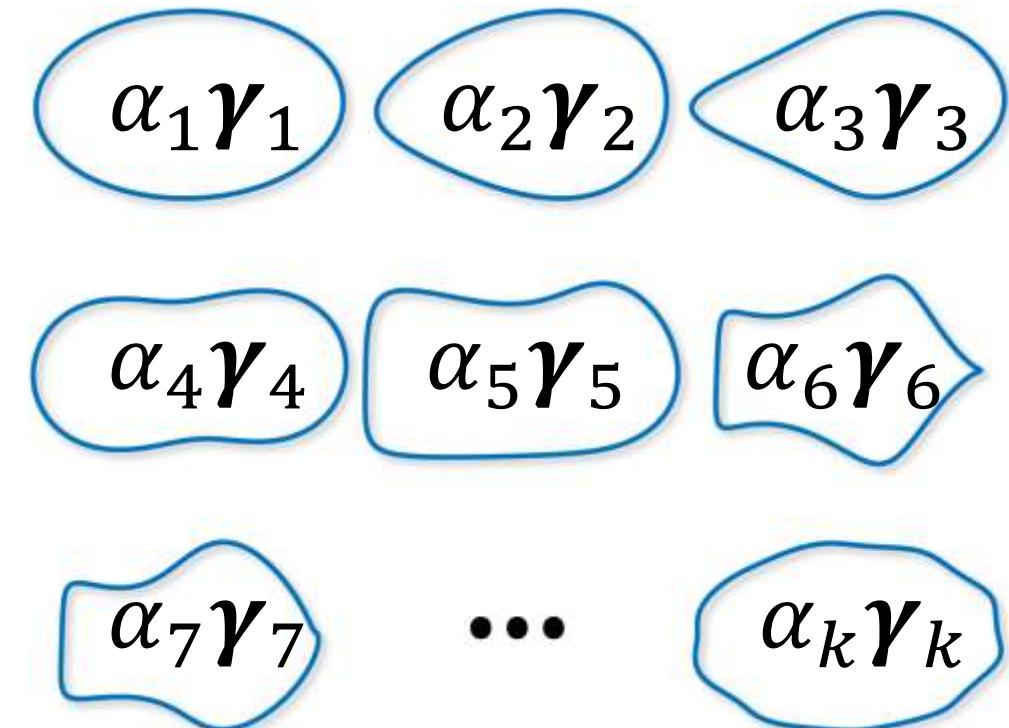
Order Reduction

- project unknown offsets

$\delta = [\delta_1, \delta_2, \dots, \delta_n]^T$ onto Γ_k :

$$\bar{x}_i = x_i + \delta_i v_i$$

$$\bar{x}_i = x_i + \sum_{j=1}^k \alpha_j \gamma_{ij} v_i$$

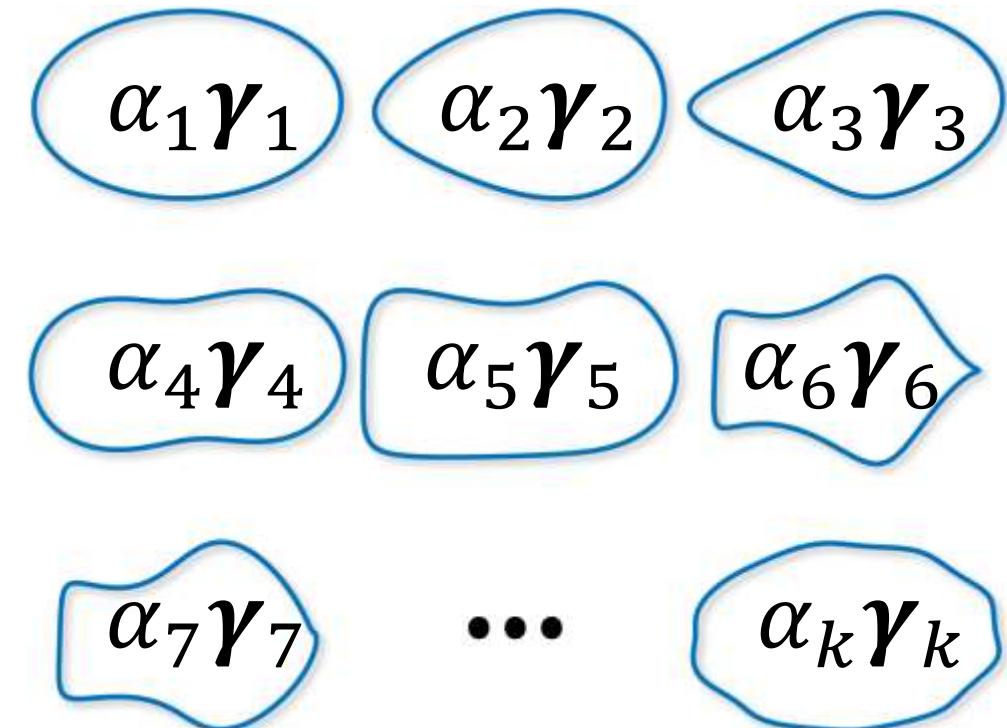


Reduced Shape Optimization Problem

- minimize objective f as a function of α :

$$\min_{\alpha} f(\alpha)$$

- (subject to constraints)

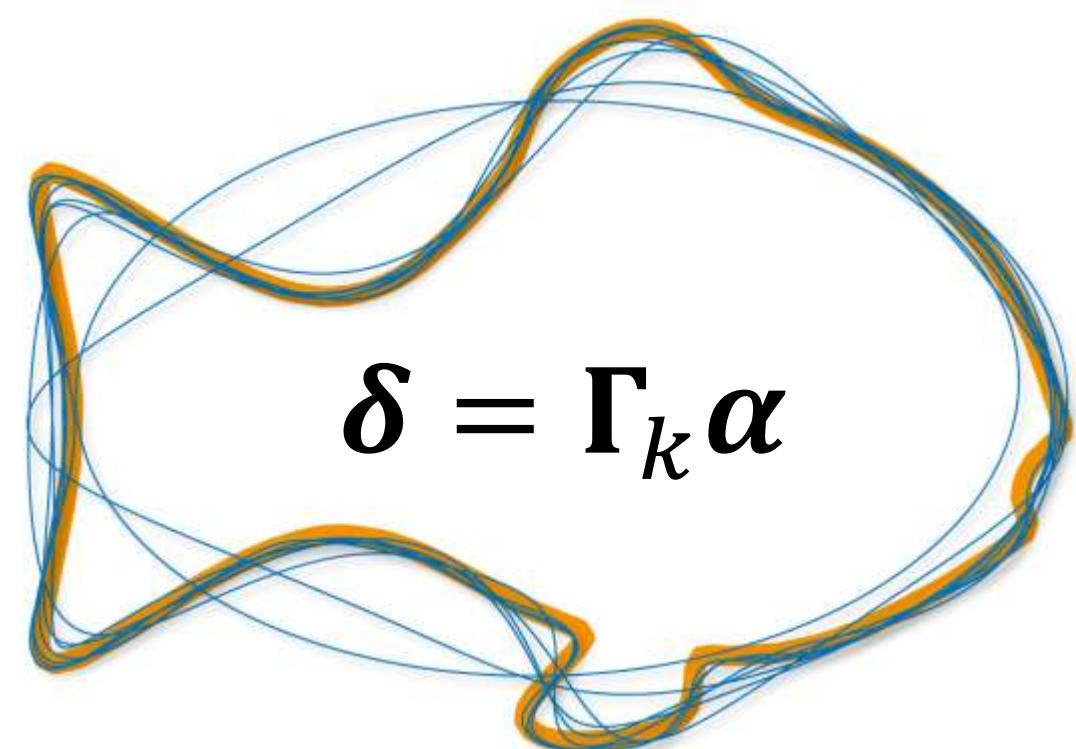


Reduced Shape Optimization Problem

- minimize objective f as a function of α :

$$\min_{\alpha} f(\alpha) \rightarrow k \text{ unknowns, } k \ll n$$

We **deform only**
the **low-frequencies** and
leave high-frequency details
untouched!

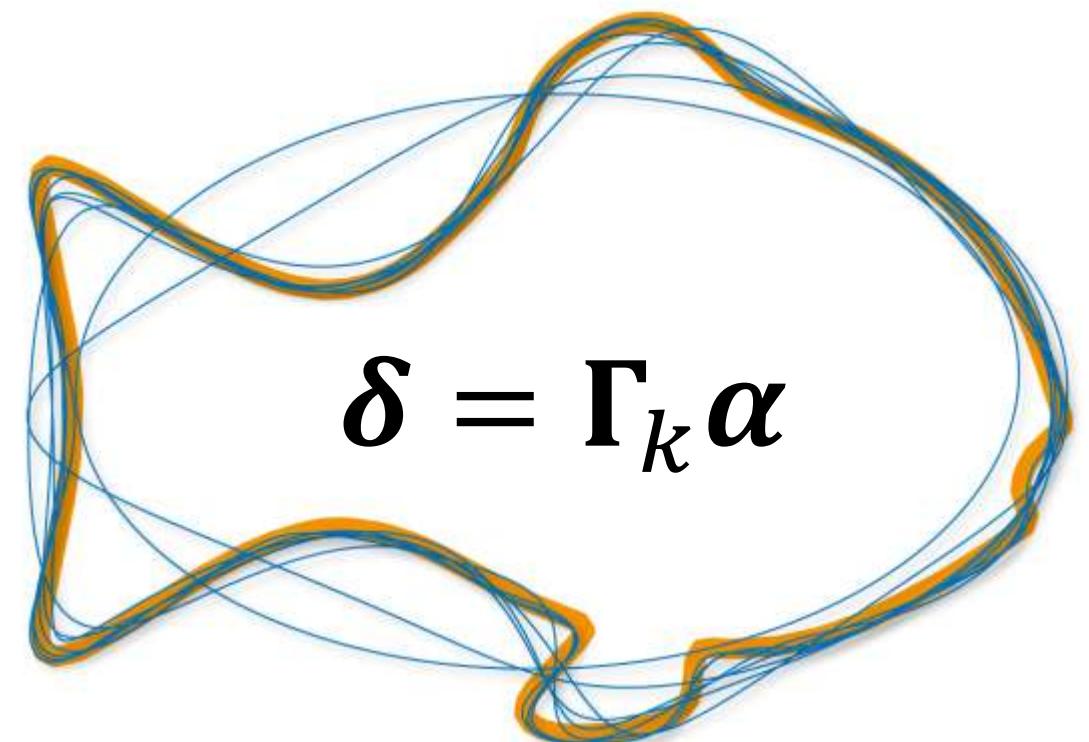


Reduced Shape Optimization Problem

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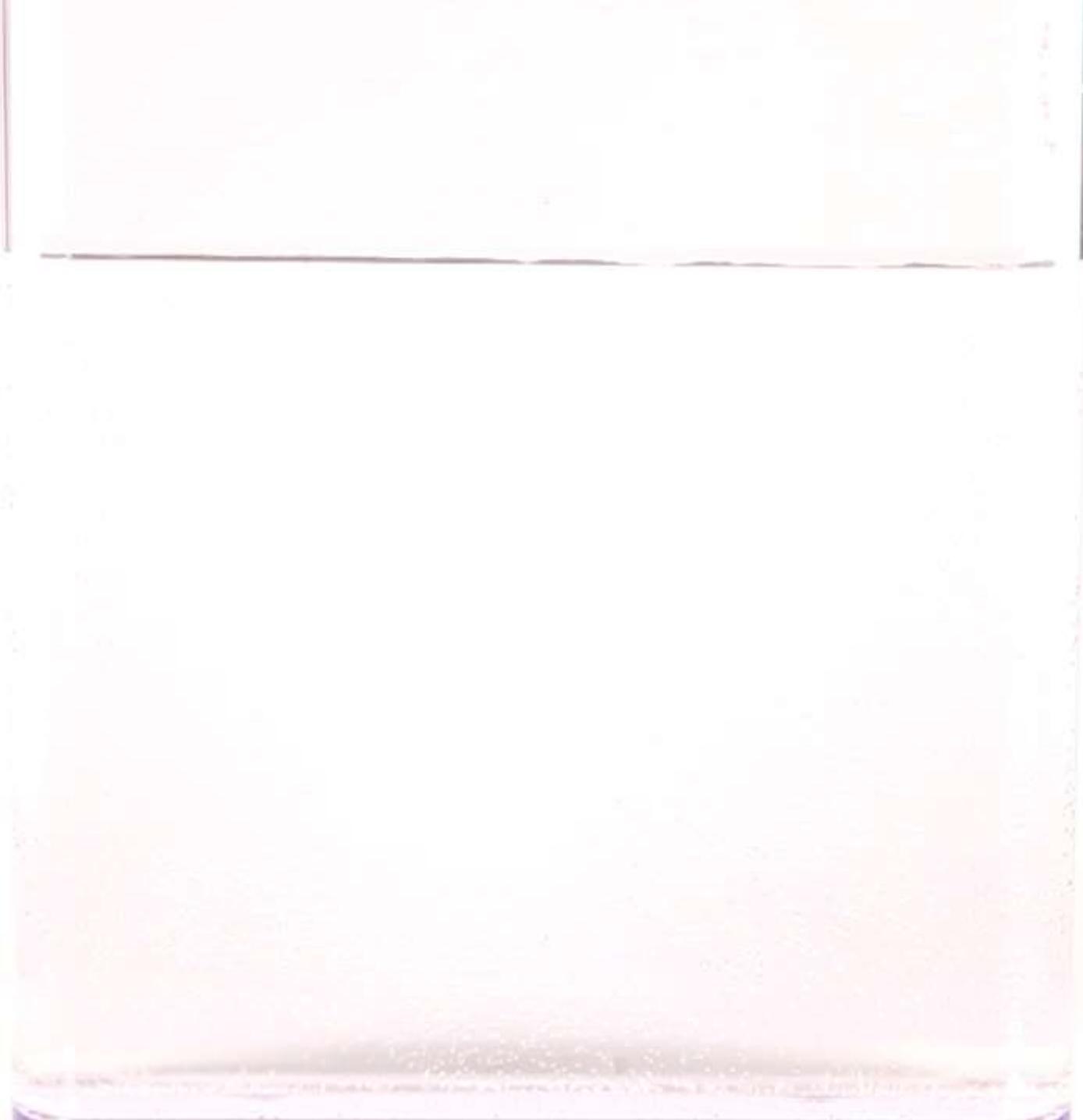
- independent of mesh resolution
- implicit regularization
- numerically stable
- easy to implement



Applications I:

Mass Properties





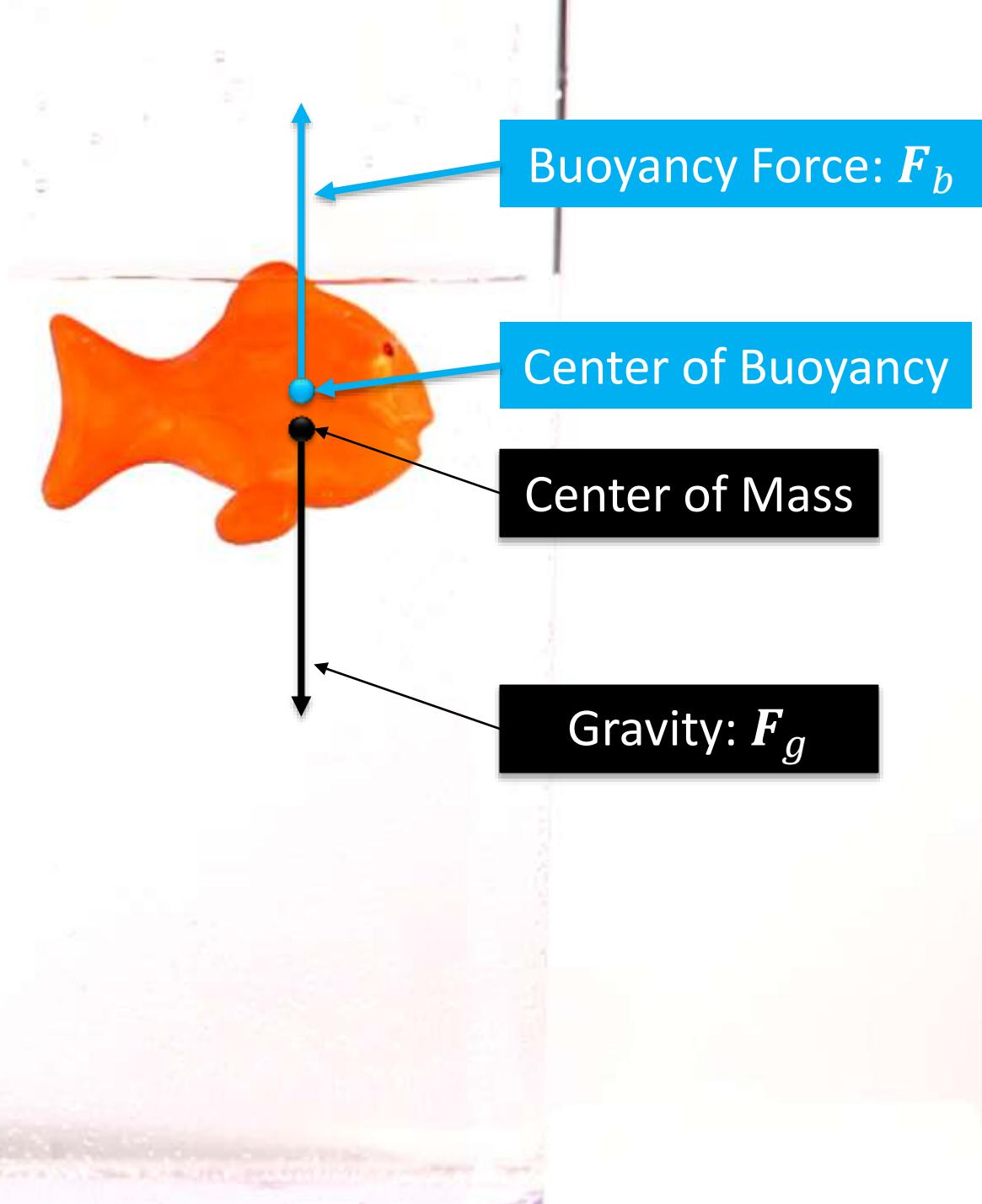
Equilibrium

$$F_g = F_b$$



Mass Properties

$$P(S)$$



Applications

- Gauss' Divergence Theorem

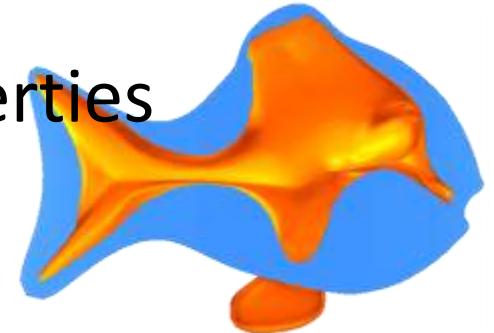
- allows us to compute mass properties as a function of the surface

$$\mathbf{P}_m(S) = M$$

$$\mathbf{P}_{x,y,z}(S) = \mathbf{CoM} = [c_x \quad c_y \quad c_z]^T$$

$$\mathbf{P}_{x^2,xy,\dots,z^2}(S) = \mathbf{I} = \begin{bmatrix} I_{x^2} & I_{xy} & I_{xz} \\ I_{xy} & I_{y^2} & I_{yz} \\ I_{xz} & I_{yz} & I_{z^2} \end{bmatrix}$$

Center of Buoyancy



Center of Mass



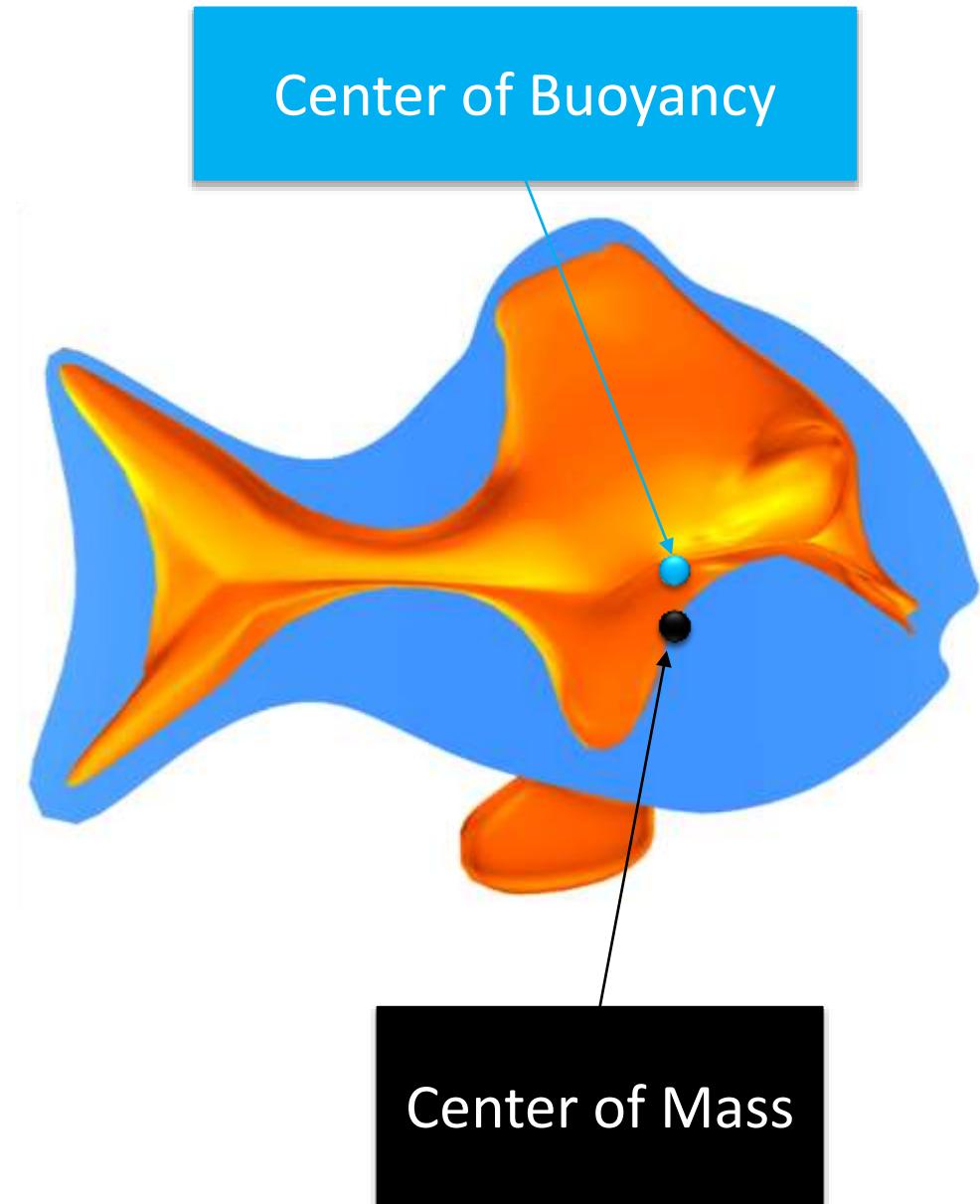
Applications

- optimization problem

$$\min_{\alpha} f \left(P(s(\delta(\alpha))) \right)$$

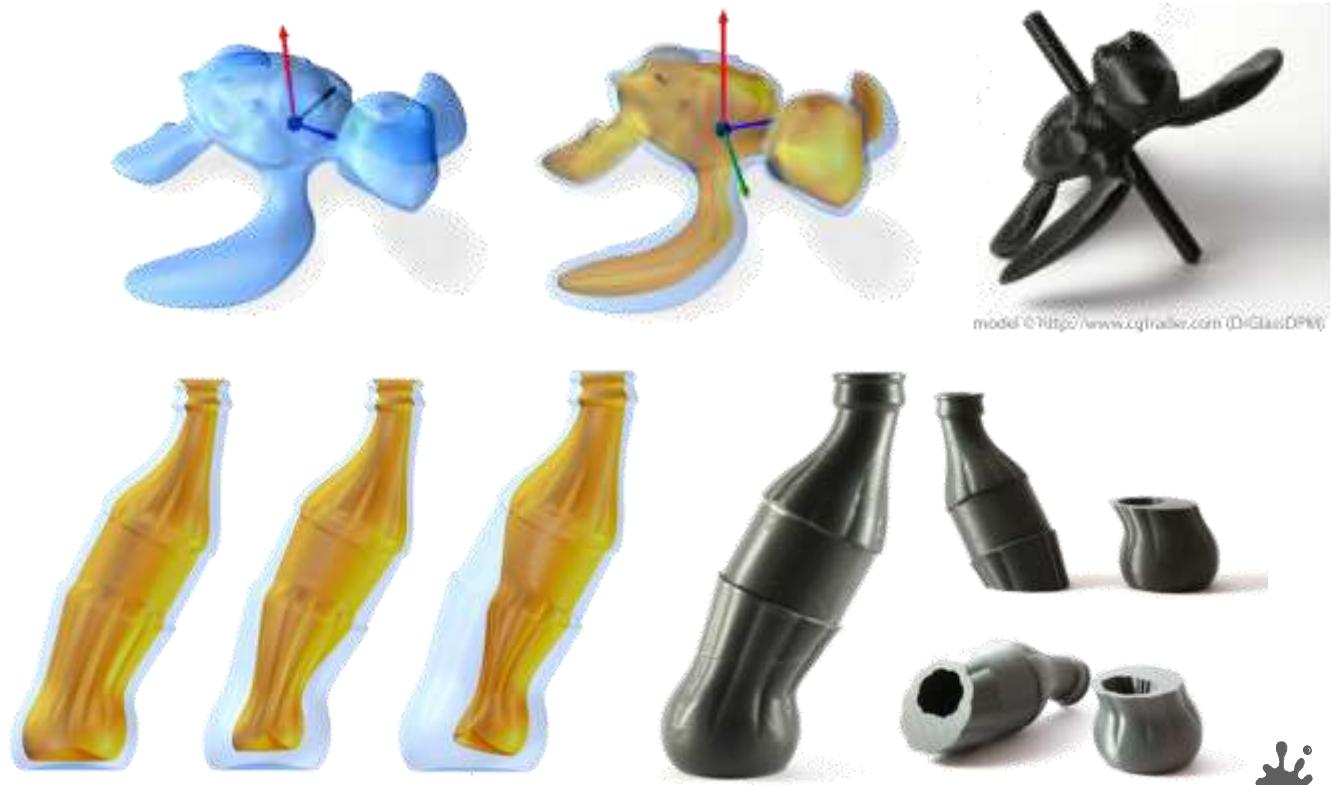
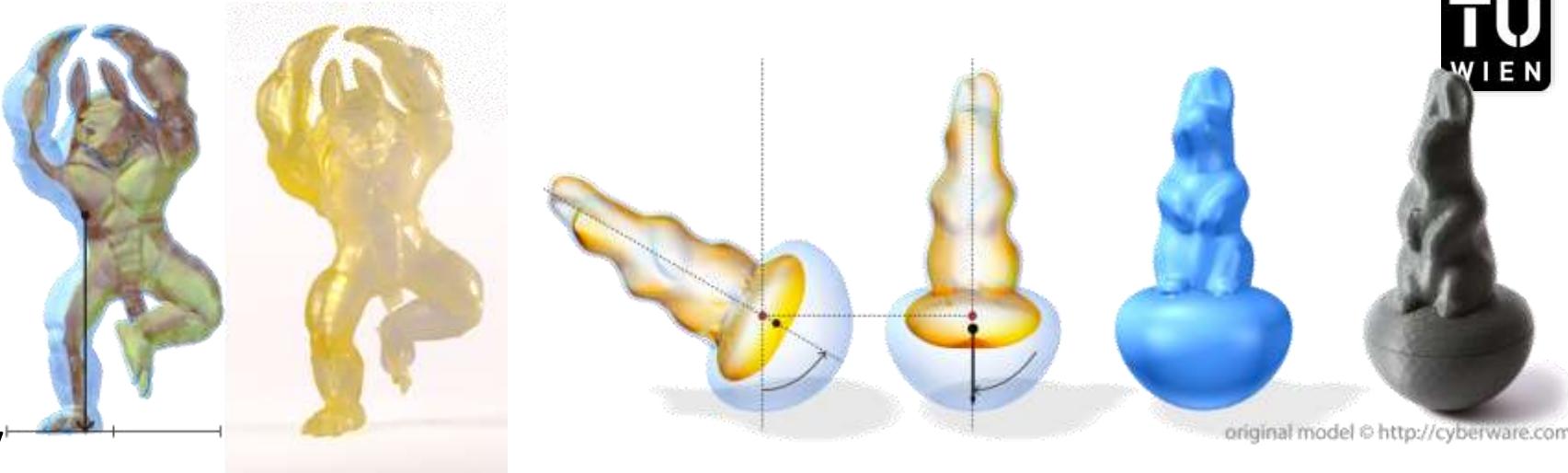
- an analytical gradient

$$\nabla f = \frac{\partial f}{\partial P} \frac{\partial P}{\partial S} \frac{\partial S}{\partial \delta} \frac{\partial \delta}{\partial \alpha}$$

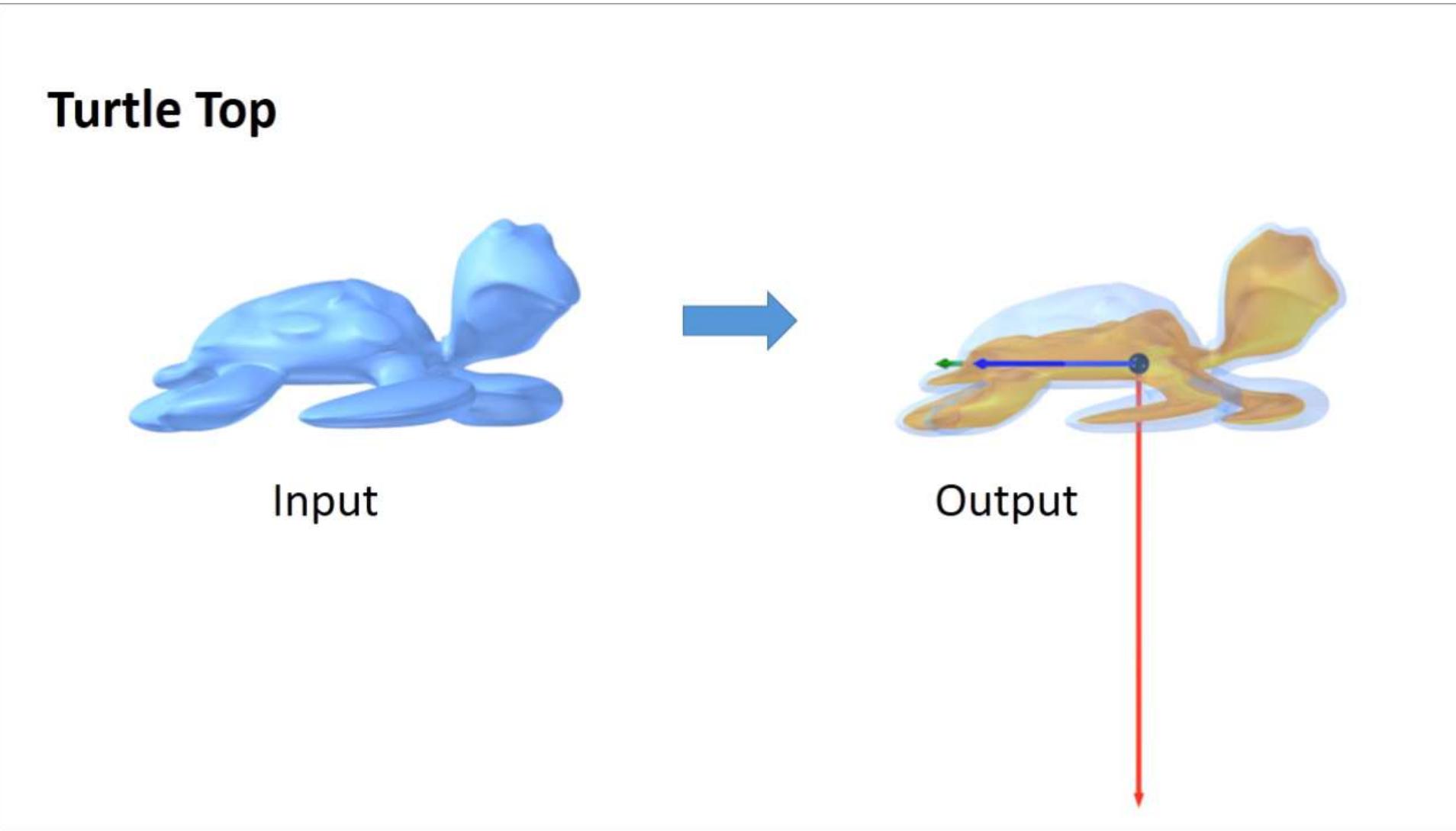


Applications

- static stability
- monostatic stability
- rotational stability
- static stability under storage
- volume and buoyancy

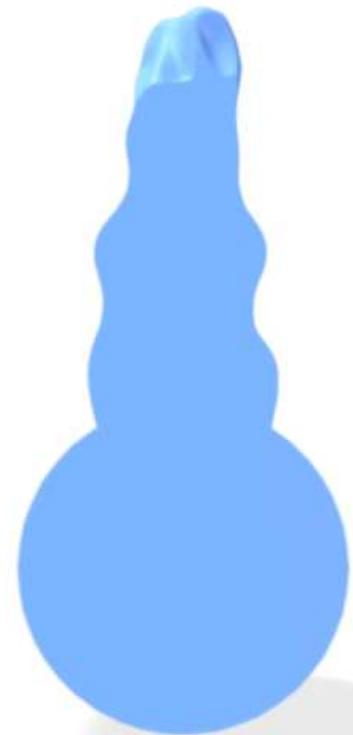


Spinning Turtle



Rabbit Rolly-Polly

Bunny Roly-Poly Doll



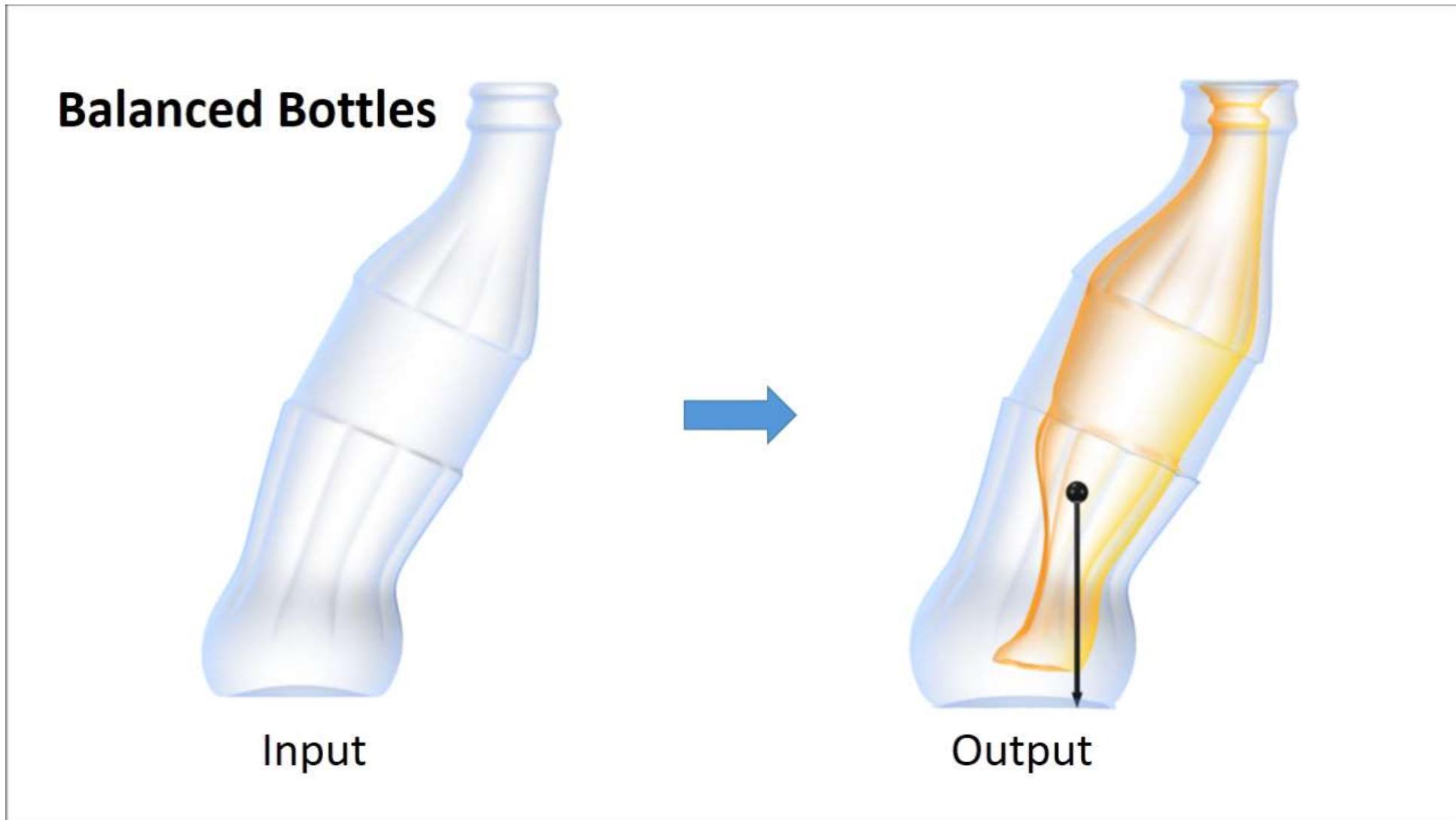
Input



Output



Balanced Bottles

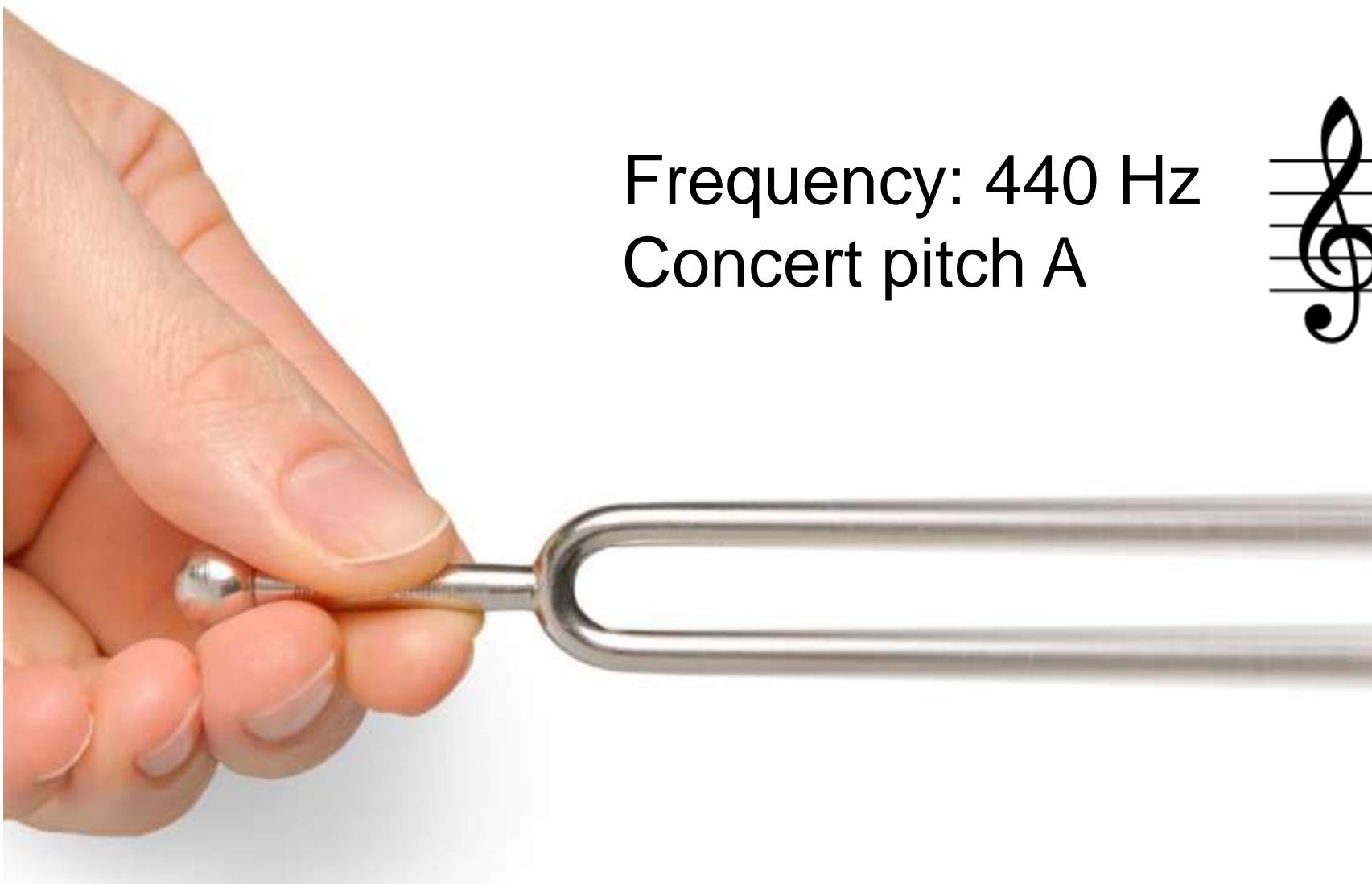


Applications II:

Modal Synthesis



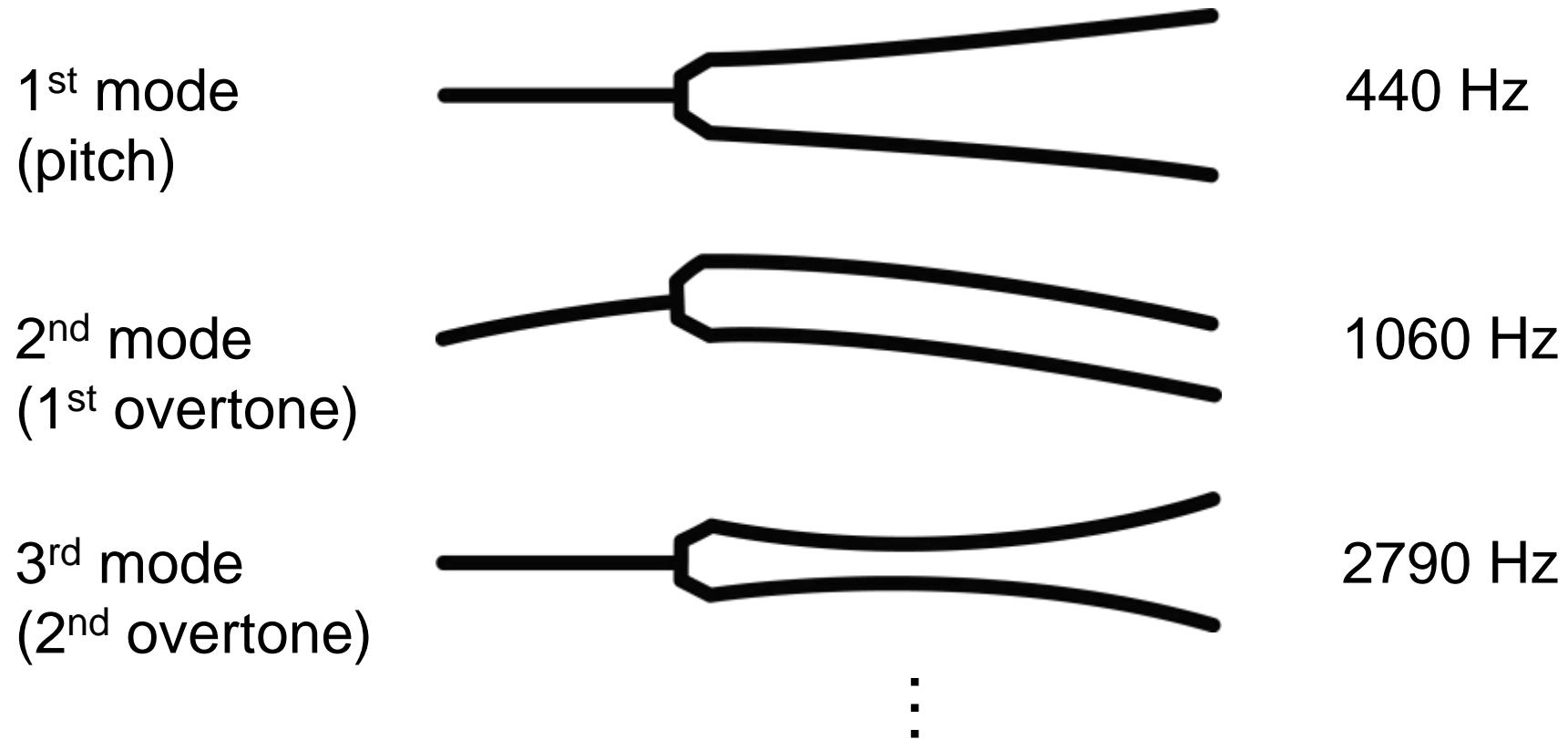
Natural Frequencies



Frequency: 440 Hz
Concert pitch A



Natural Frequencies



Overtone spectrum \Leftrightarrow characteristic sound of object



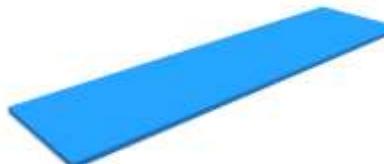
Natural Frequencies

Natural modes depend on

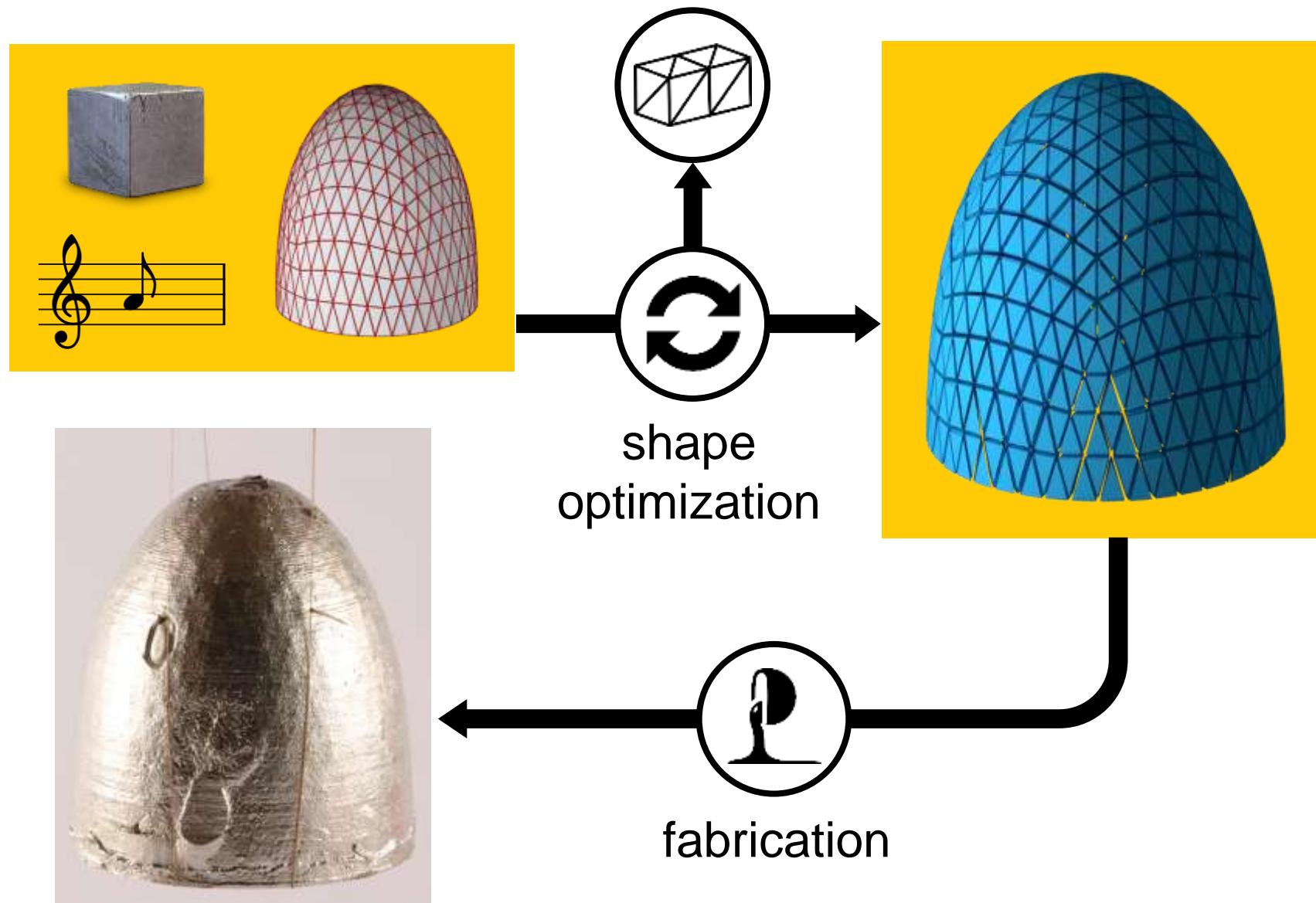
- material



- shape

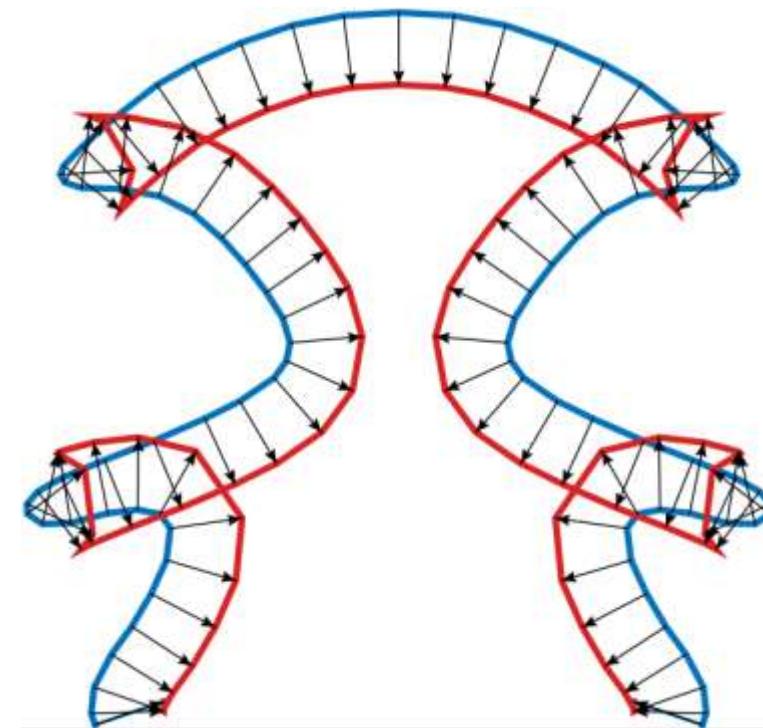


Goal: “Modal Synthesis”



Vertex-Normal Parametrization

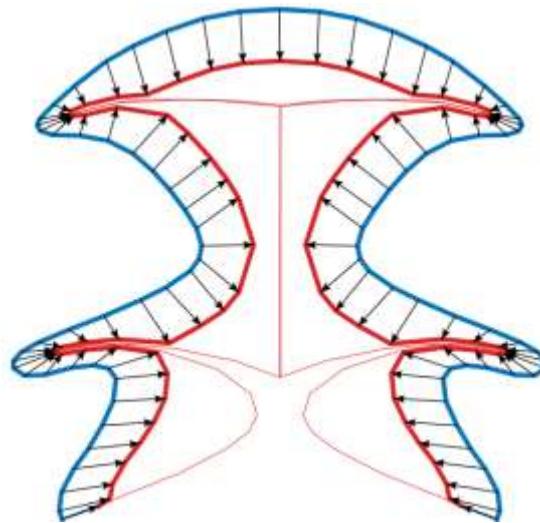
- Large offsets and high curvatures
 \Rightarrow self-intersections



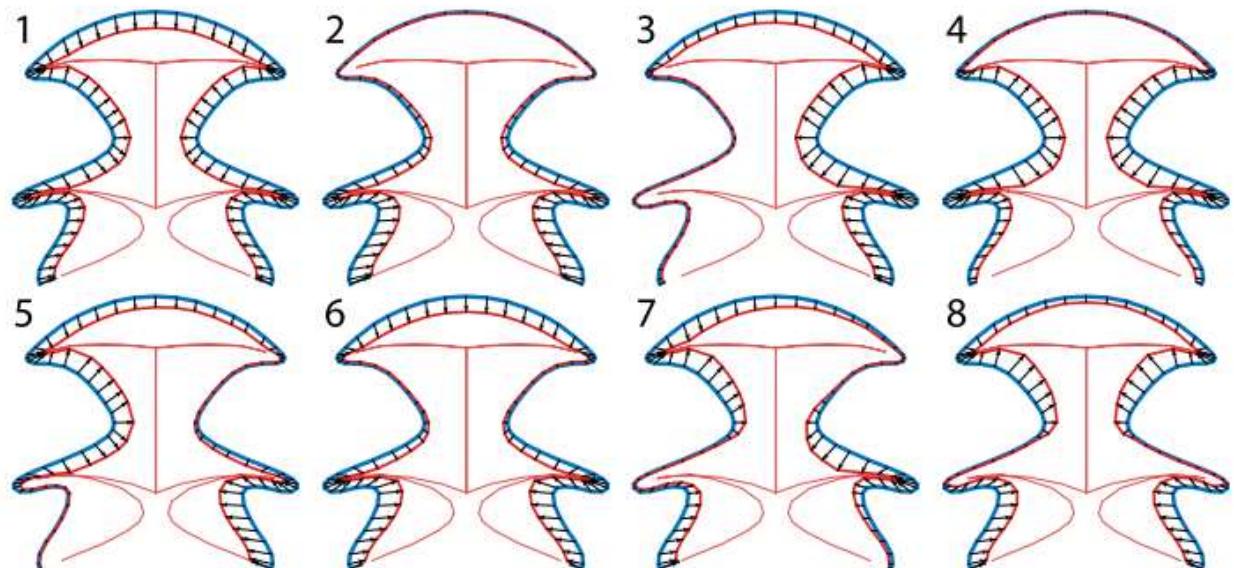
Shape Parametrization

- Use Reduced Basis with Manifold Harmonics
- Define offsets δ as linear combination of basis functions Γ_k

$$\delta = \Gamma_k \alpha$$



$$= X + \sum_{i=1}^k$$



Shape Optimization

- Use non-linear optimization routine (Matlab)

$$\min_{\alpha} f(\alpha) = (p - p_0)^2$$

- p_0 ... target pitch
- p ... pitch of incument solution
- α ... coefficient vector



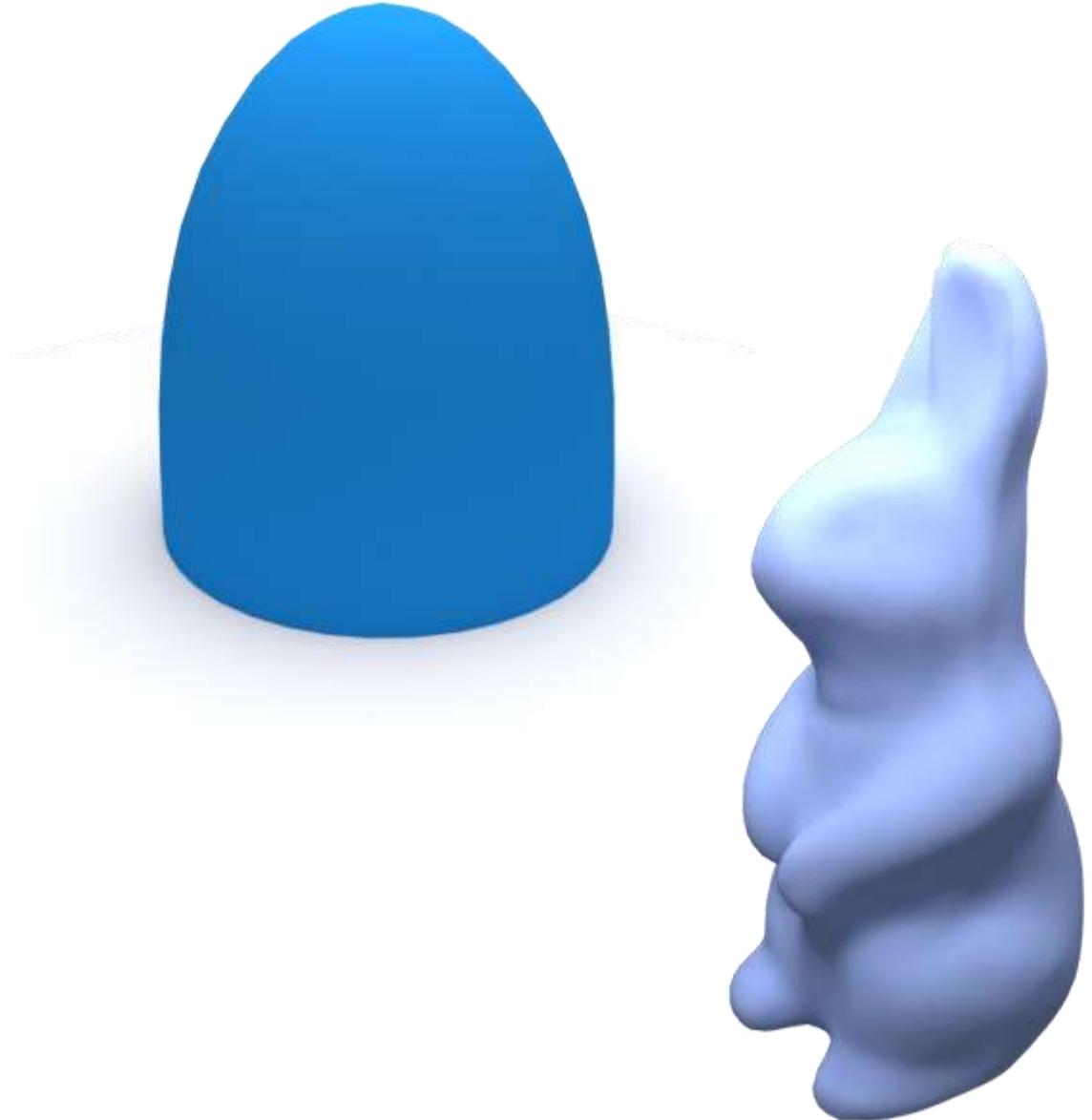
Fabrication

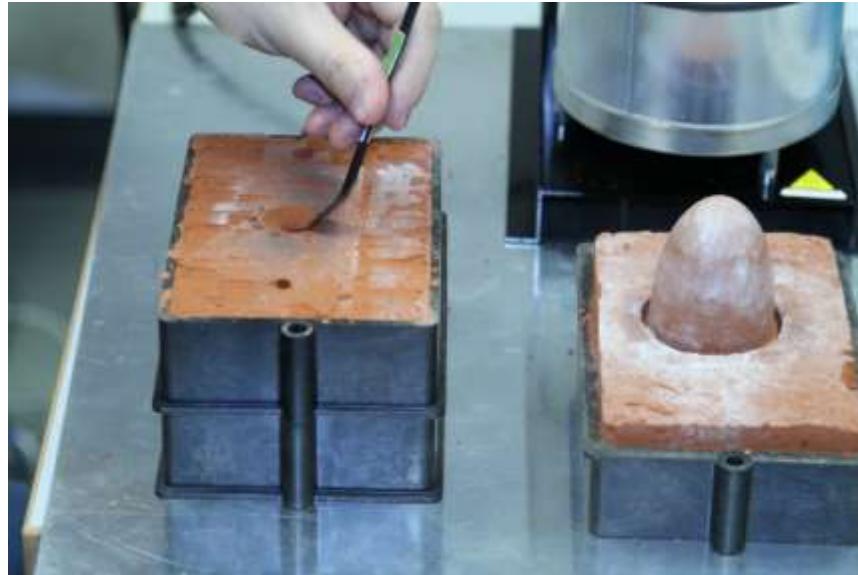
- Material
 - good acoustic properties
 - cast into complex shape
- Tin
 - melting point of 230°C
 - Young's modulus of 50 GPa



Fabrication

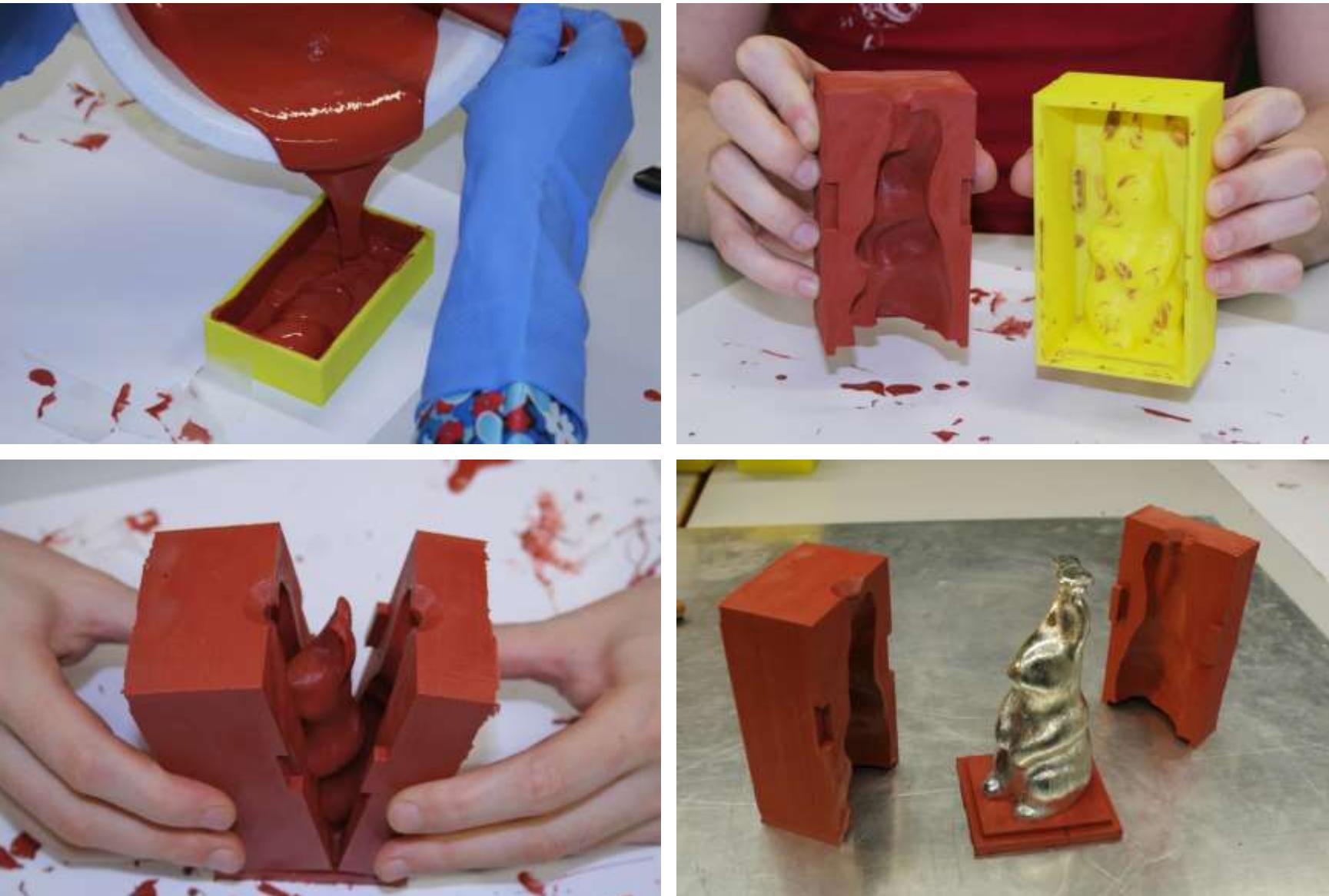
- Oval bell
 - molds from sand
- Rabbit bell
 - molds from caoutchouc







Rabbit

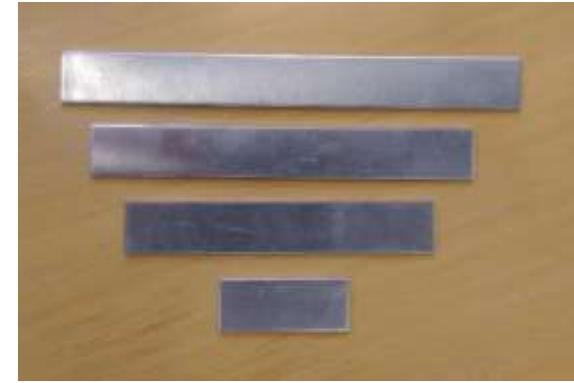


Rabbit



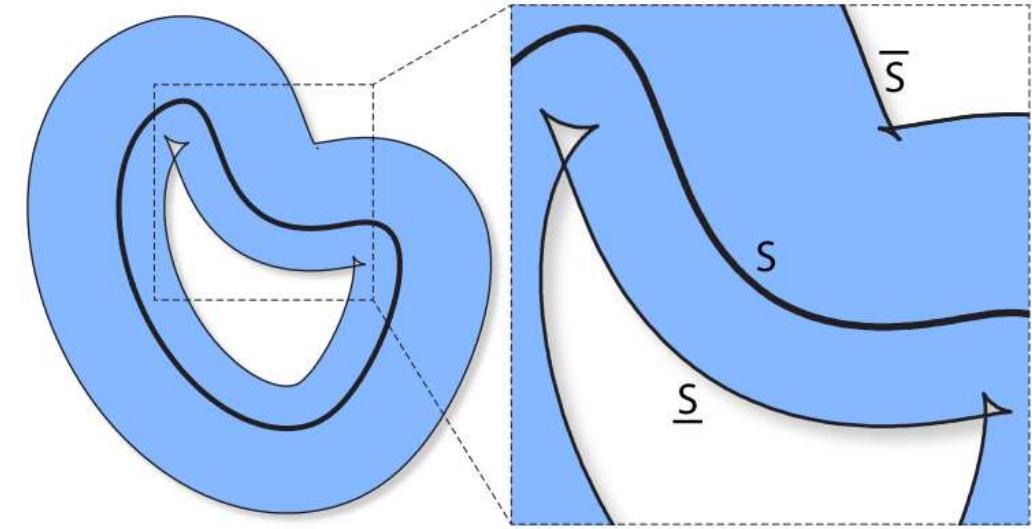
Results

- Aluminium plates
 - Median error of 1.7%
 - 0.7% with parameter estimation
- Bell
 - Error of 2.8%
- Rabbit Bell
 - Error of 11%
 - 6% with parameter estimation



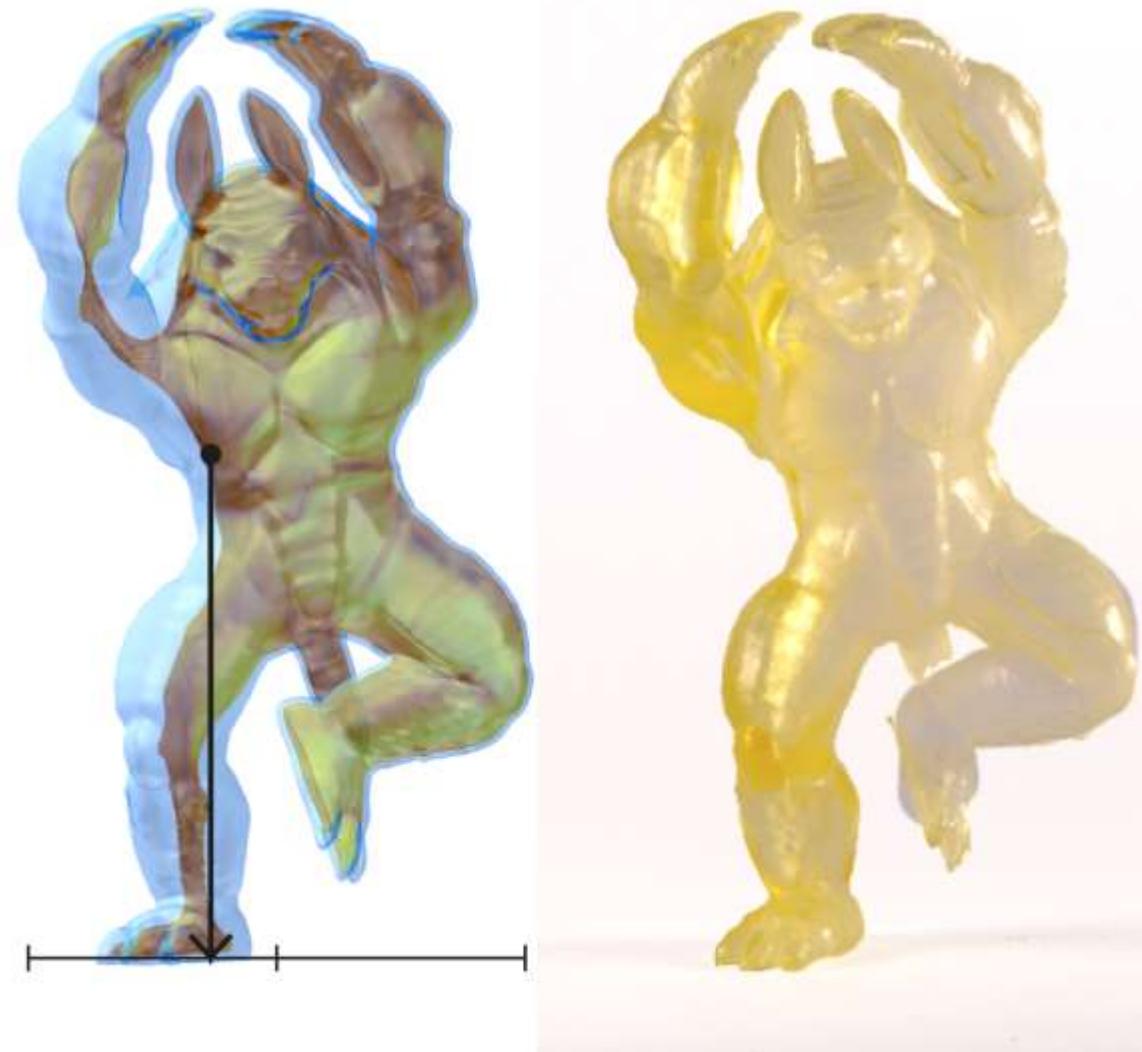
Discussion & Limitations

- skeleton dependence
 - our method relies on the skeleton
 - we use iterative mesh contraction (Mean Curvature Flow)
- design space limitation
 - we can only offset a surface up to the skeleton



Conclusions

- we proposed a novel framework for shape optimization
- we provide an elegant and efficient basis-reduction
- we demonstrate the method by optimizing
 - mass properties
 - natural frequencies



Publications

- Musalski, P., Auzinger, T., Birsak, M., Wimmer, M. & Kobbelt, L. Reduced-Order Shape Optimization Using Offset Surfaces. *ACM Trans. Graph. (Proc. ACM SIGGRAPH 2015)* **34**, 102:1–102:9 (2015).
- Hafner, C., Musalski, P., Auzinger, T., Wimmer, M. & Kobbelt, L. Optimization of natural frequencies for fabrication-aware shape modeling. in *ACM SIGGRAPH 2015 Posters - SIGGRAPH '15* 1–1 (ACM Press, 2015).
- Hafner, C. Optimization of Natural Frequencies for Fabrication-Aware Shape Modeling, Master Thesis, TU-Wien (2015)



Thank you!

