# Optimization of Natural Frequencies for Fabrication-Aware Shape Modeling

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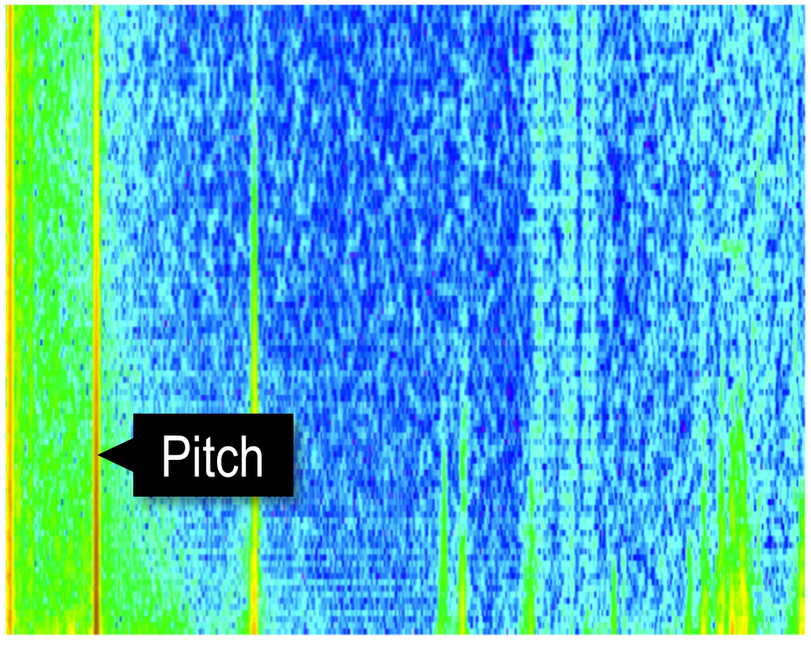
## Introduction

If a bell-like solid is struck with a hammer, it emits a **sound at a characteristic pitch**, i.e., its fundamental mode.

We can analyze the pitch in Hz using a microphone, or **predict it computationally**.

The computation requires a digital representation of the solid (e.g. a mesh) and a **finite element solver** [2].




## Problem Statement

We tackle the inverse problem–**shape synthesis**.

Given a **target pitch**, a **target shape**, and a **target material**, our algorithm computes a solid that matches both the pitch and the shape.

Challenges include choosing a **shape design space**, a **finite element model**, and a **fabrication method**.

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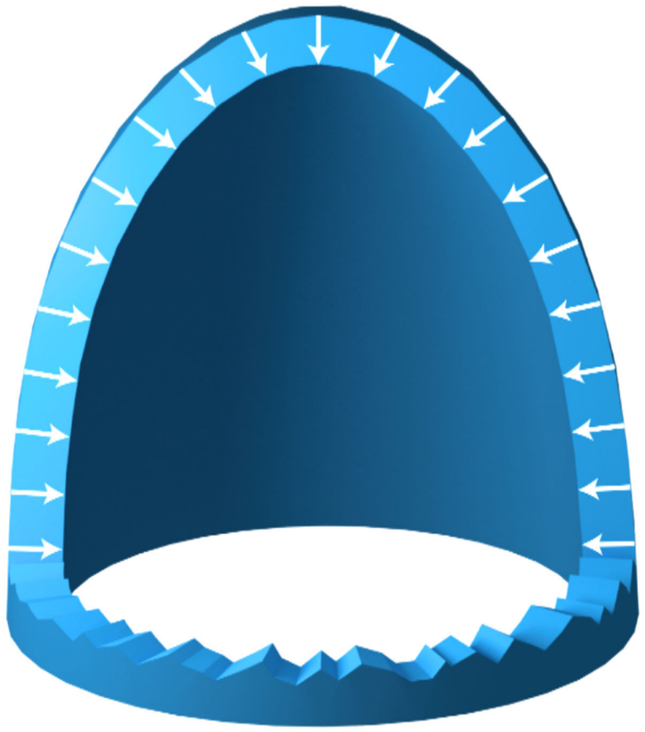
## Shape Design Space

Our target shape is a manifold with boundary.

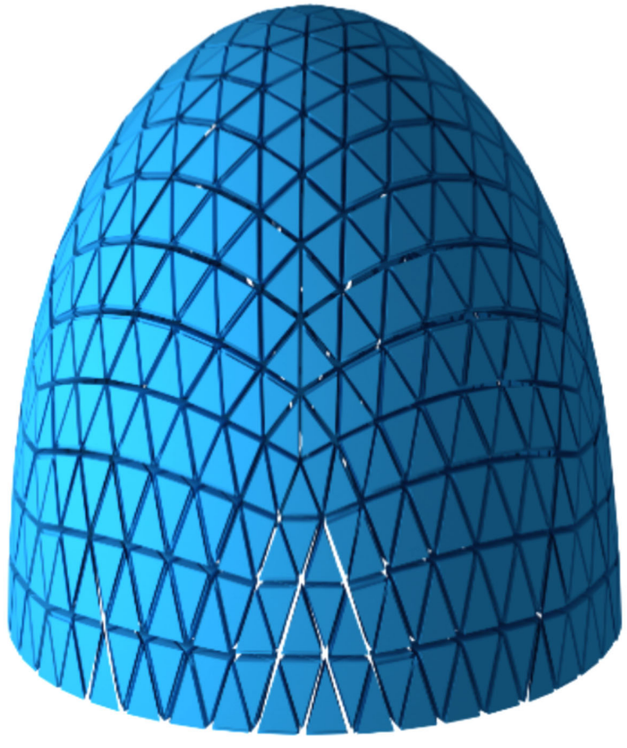
We construct an **inner offset surface**, using the **wall thickness** as a parameter that spans the design space.

To **prevent self-intersections**, the offset vectors target a **mean-curvature skeleton**.



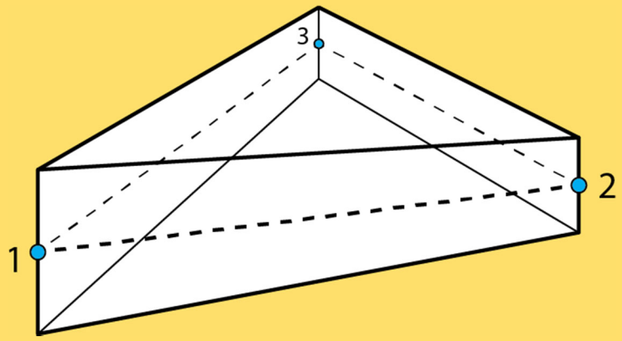
## Optimization

Given a target pitch, we find the **optimal wall thickness** using an off-the-shelf **non-linear optimization routine**.



## Finite Element Method

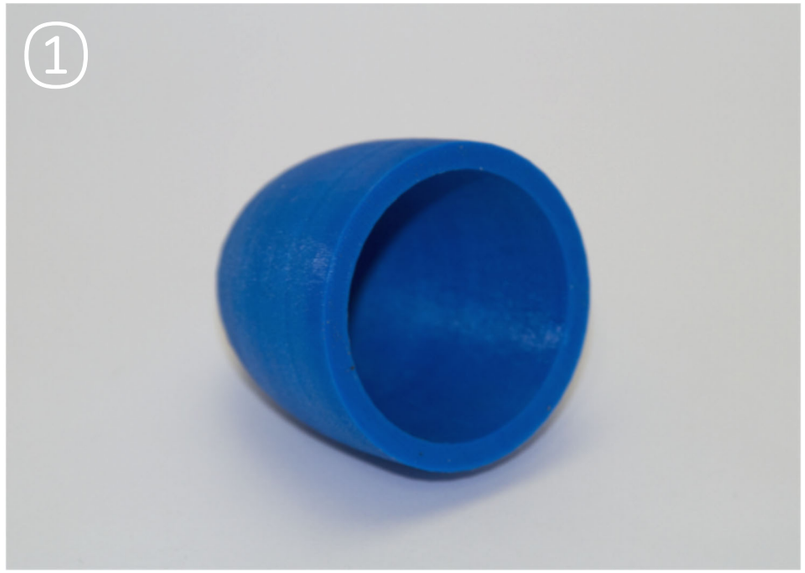
To predict the pitch of the solid, we construct a finite element mesh with **quadratic thin-shell elements** to capture bending.



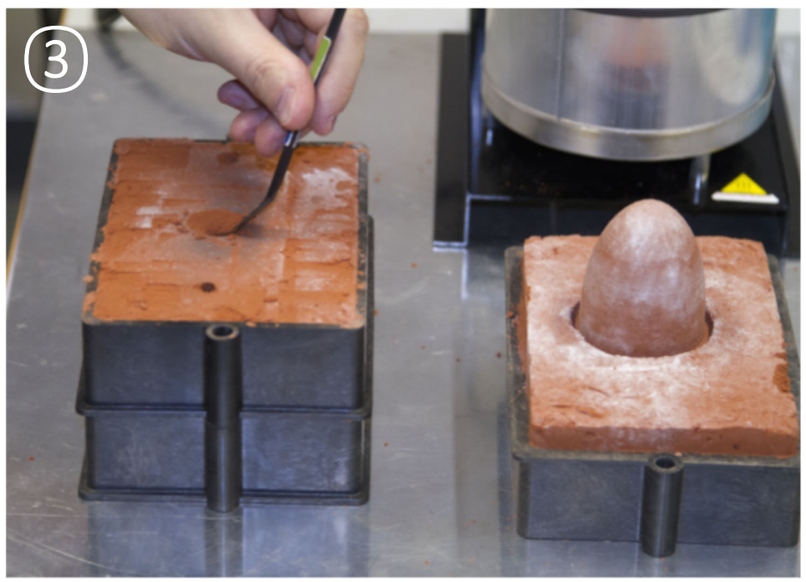

The pitch corresponds to the **smallest non-zero eigenvalue** $\lambda$ of the undamped vibration system [1]:

$$M\ddot{U} + KU = 0$$

$$\Rightarrow Kv = -\lambda^2 Mv$$








## Fabrication

① We print a **positive** of our solid with an **FDM 3D printer**.

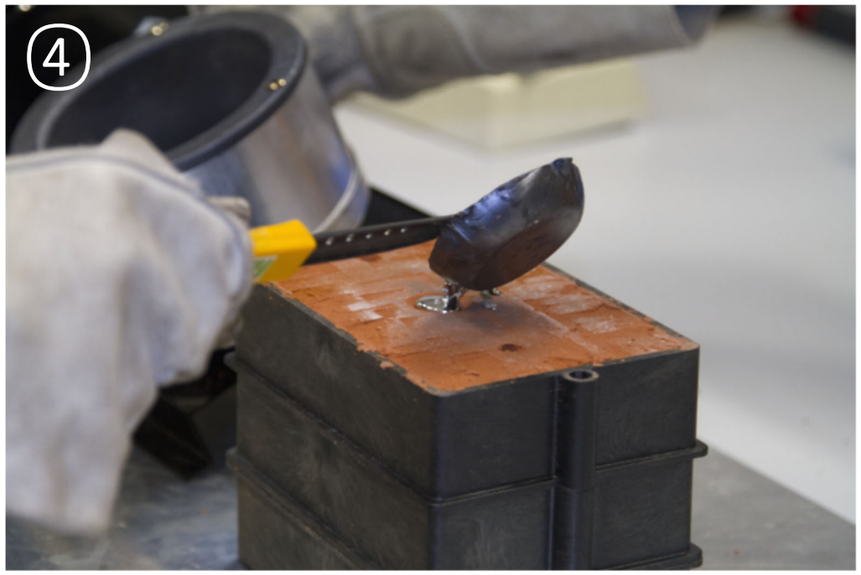
② Using **molding sand**, we create two halves of a **negative**.

③ The upper half is equipped with a **feed opening** and two air holes.

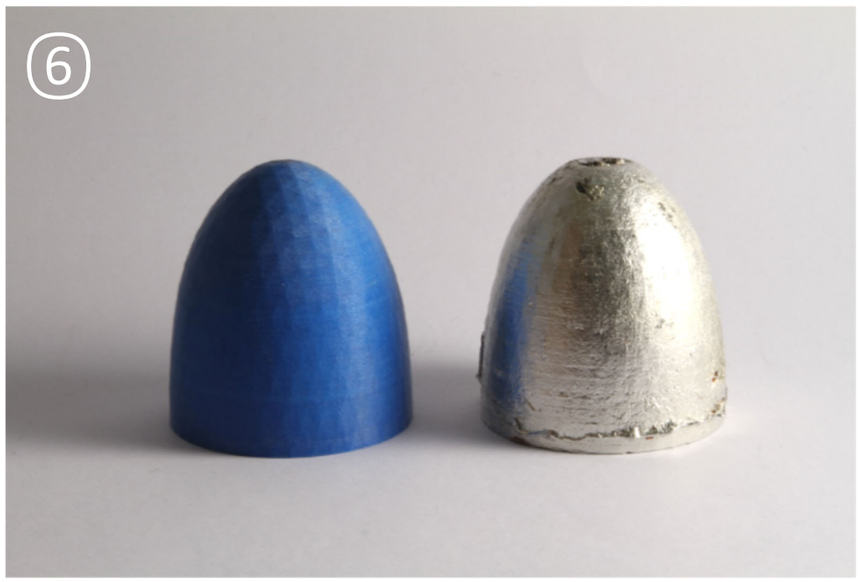
④ We pour **melted tin** into the feed opening until it fills up the three openings.

⑤ After a cooling period, the **sand is removed**.

⑥ Once we have **sawed off the appendages**, we obtain the final result.





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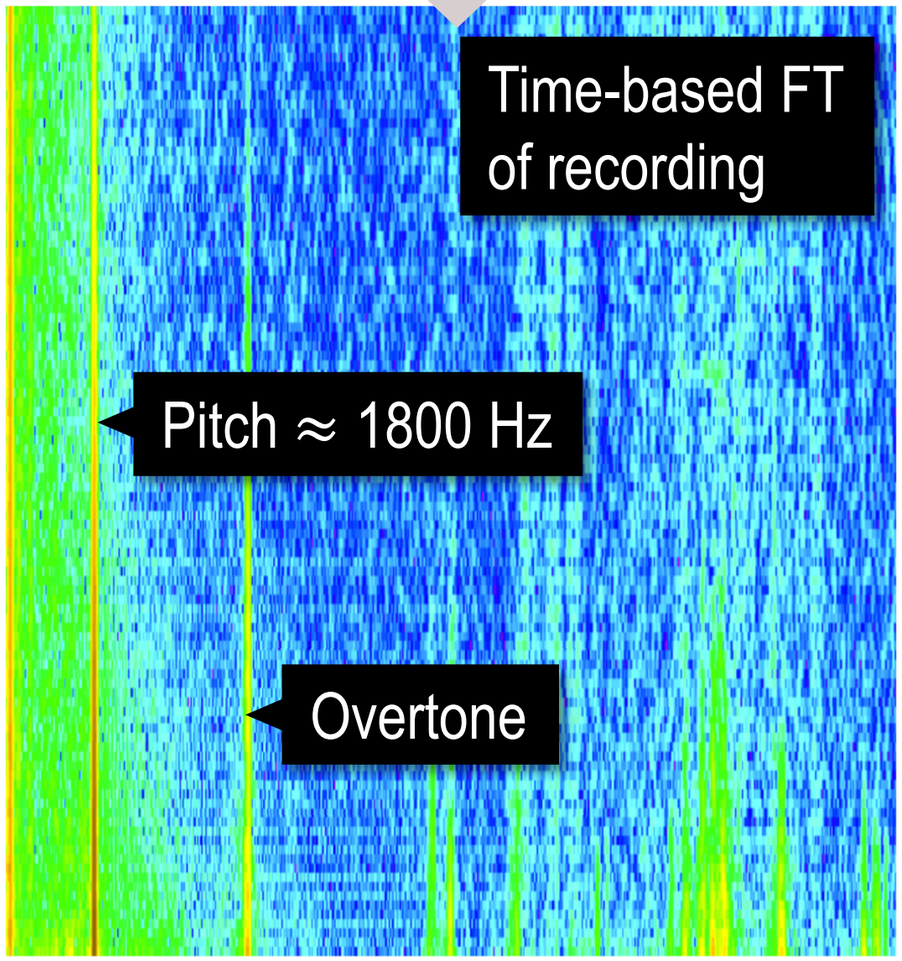


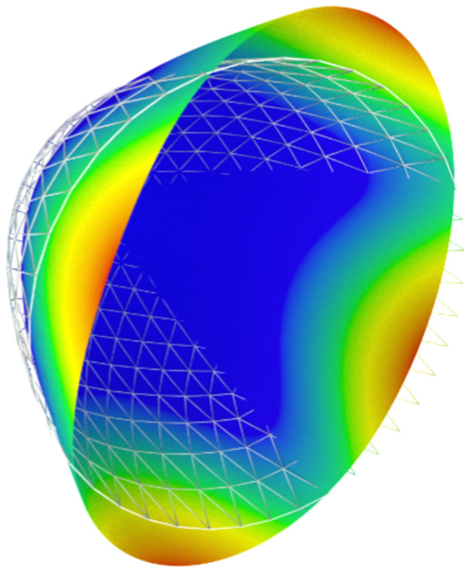

## Results

We mount the bell on two loops of yarn to allow it to **vibrate freely**. We strike it with a hammer and **record the sound**.

The **time-based Fourier Transform** of the signal reveals a **pitch of 1800 Hz**.

This measurement is in good accordance with the **1760 Hz predicted by our finite element model** (error < 2.5%).

Note that we are using **reference material parameters** for tin with 99.9% purity, **without any further tweaking**.



Mode shape found by FEM
Predicted pitch = 1760 Hz

[1] O'Brien, Book, Essl. 2001. Synthesizing sounds from physically based motion. In *Proceedings of ACM SIGGRAPH 2001*, ACM Press, 529-536.

[2] Umetani, Mitani, Igarashi. 2010. Desgining custom-made metallophones with concurrent eigenanalysis. In *Proceedings of the Conference on New Interfaces for Musical Expression (NIME)*.

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