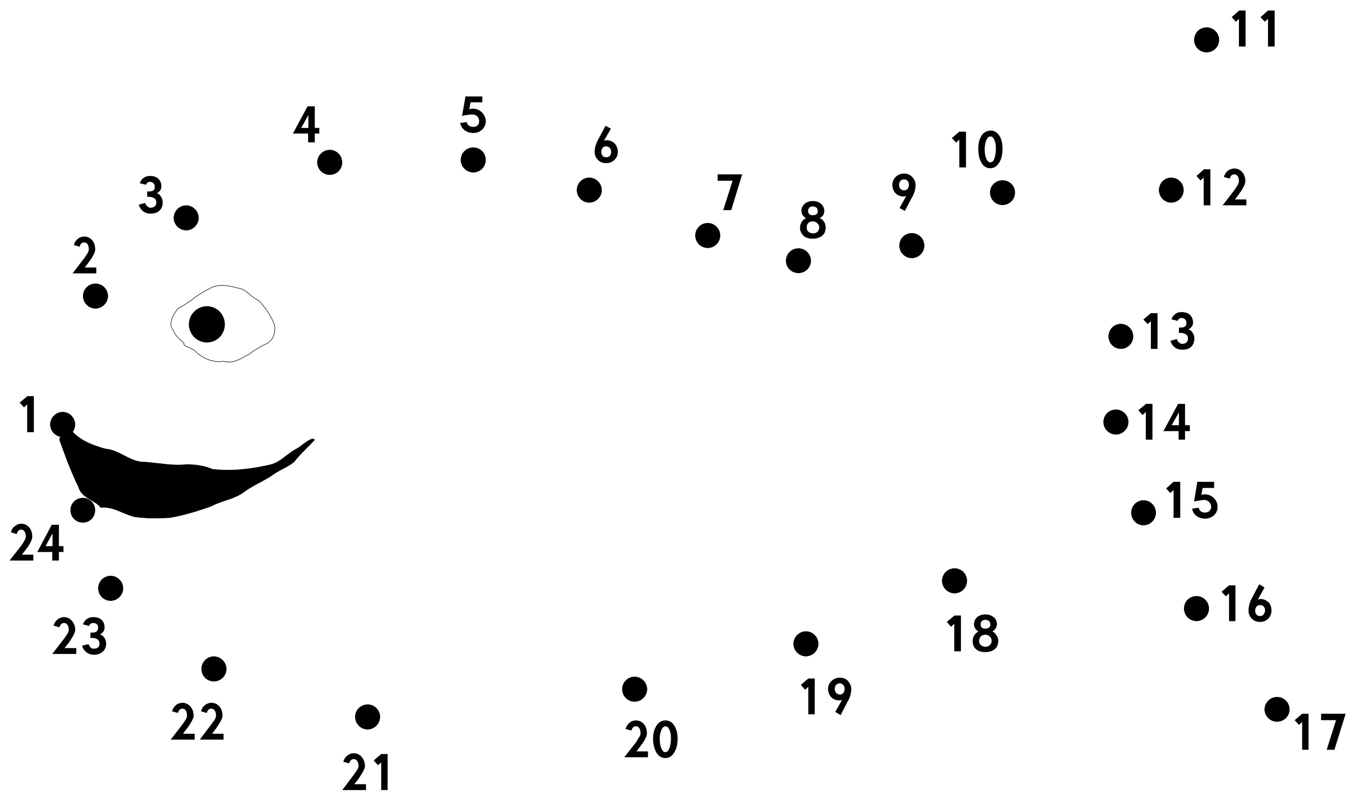
An Efficient Algorithm for Determining an Aesthetic Shape Connecting Unorganised 2D Points

Stefan Ohrhallinger[1,2] and Sudhir Mudur[2]

[1]Vienna University of Technology, [2]Concordia University, Montréal

Connect The Dots



Now Try Without The Numbers

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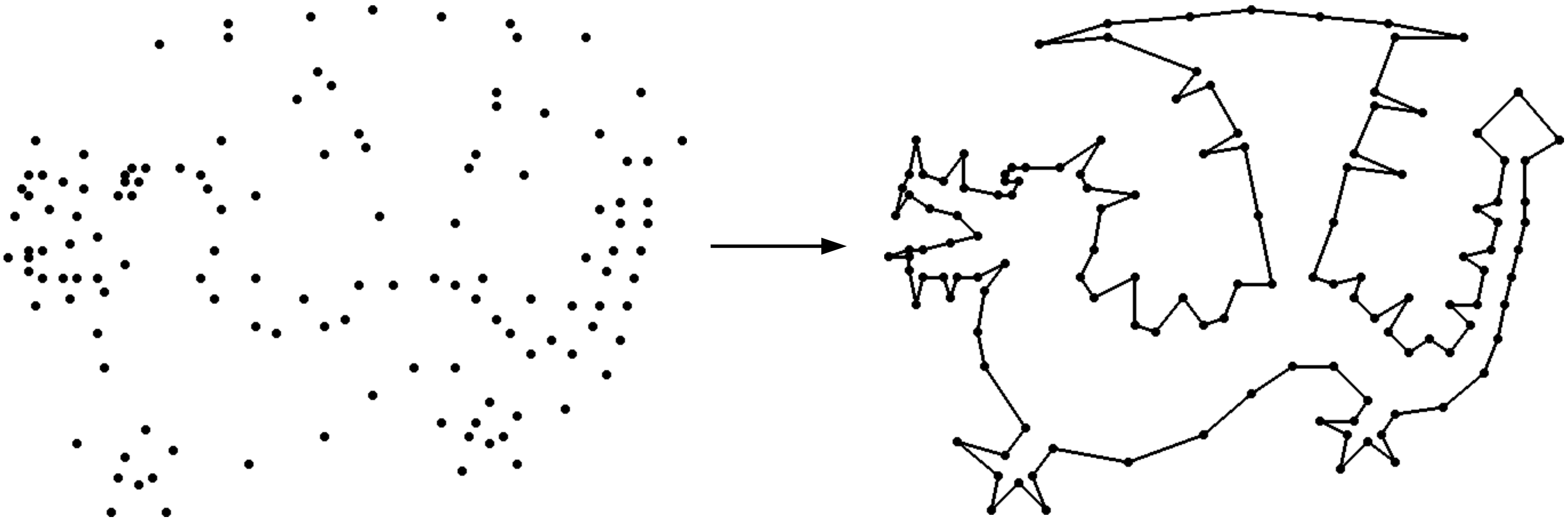
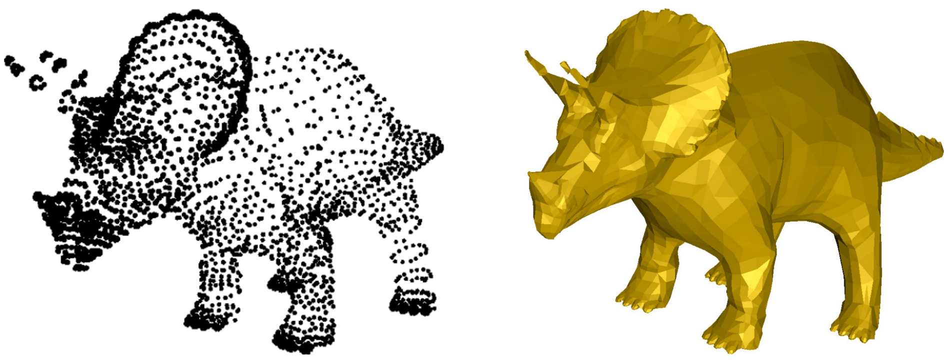
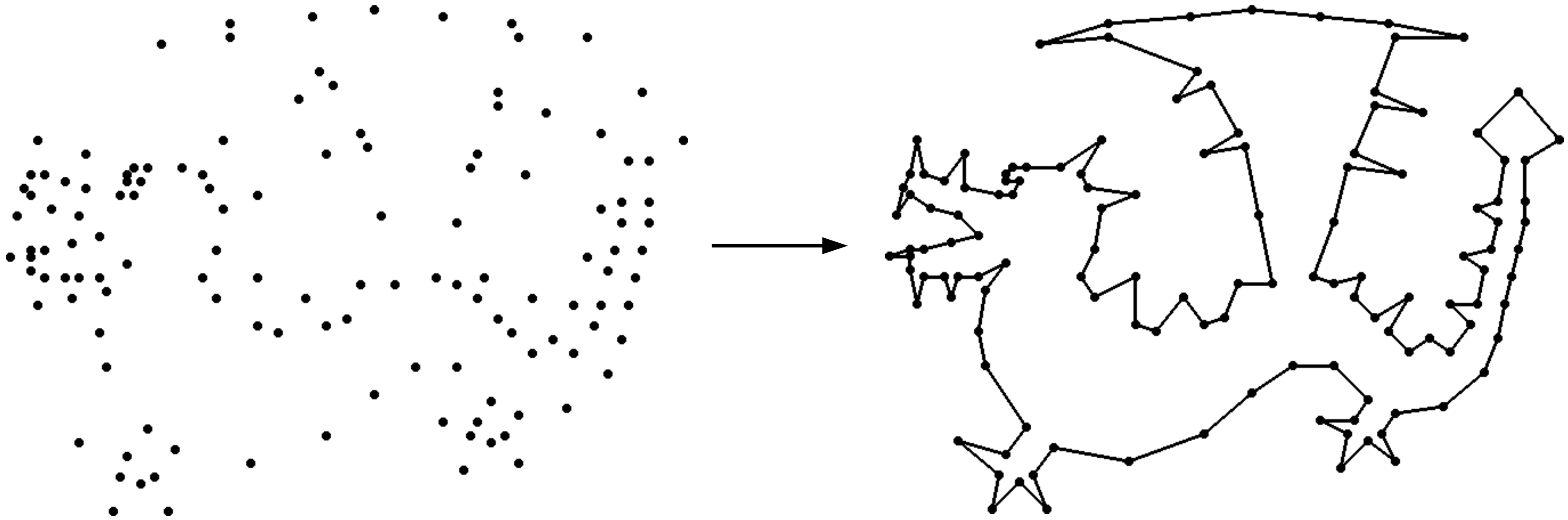


Boundary reconstruction = **recovering connectivity**

Now Try Without The Numbers



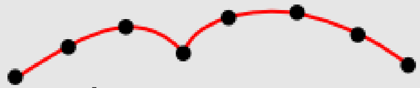


Boundary reconstruction = **recovering connectivity**

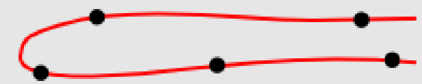
30 Years of Research Effort

α-shapes [EKS83], β-skeleton [KR85], γ-n'hood [Vel93], r-regular [Att97]

non-uniform



sharp corners



sparse sampling

α,β,γ,...
parameter-free

O(n log n)
fast

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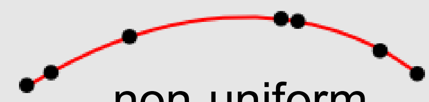

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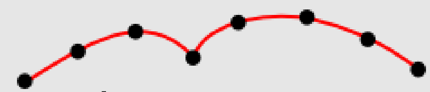

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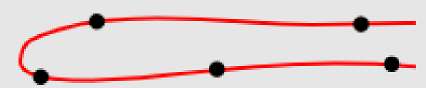

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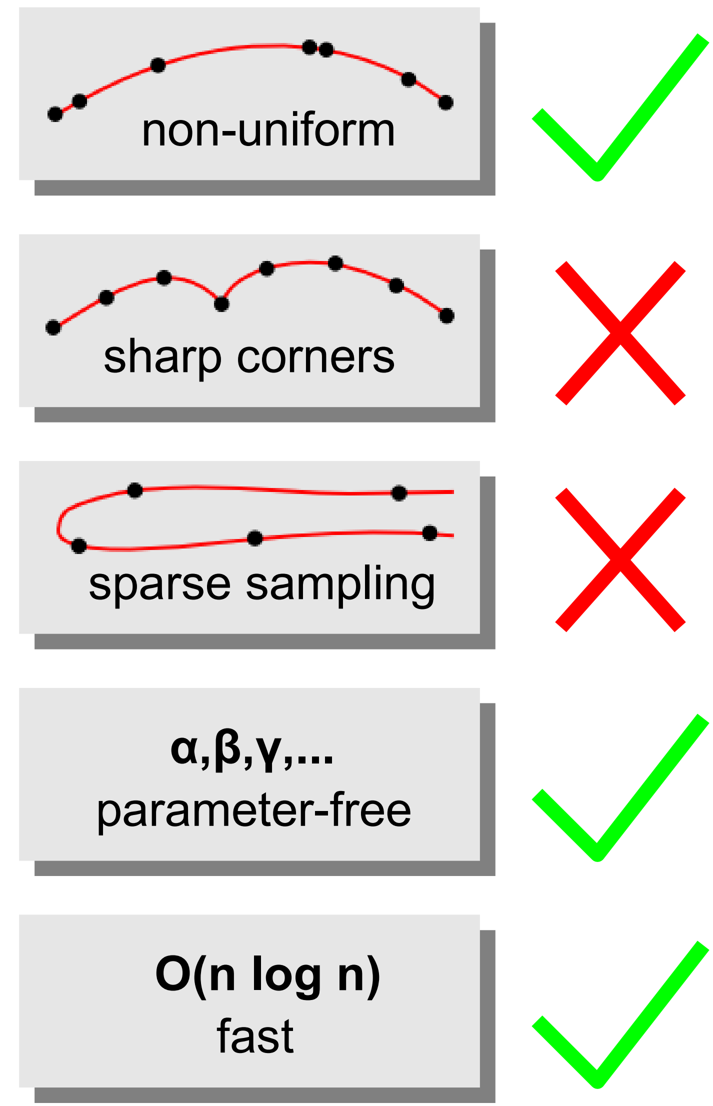

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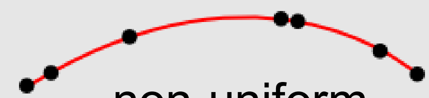
Gathan [DW01], GathanG [DW02]



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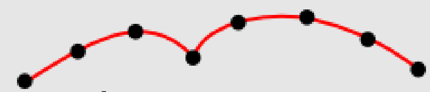

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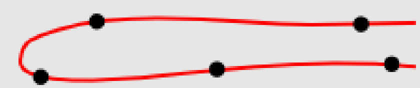

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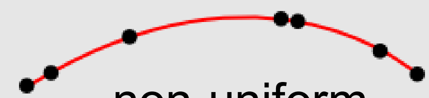
- DISCUR [ZNYL08], VICUR [NZ08]



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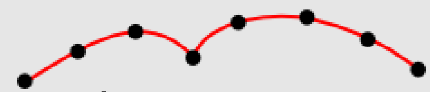

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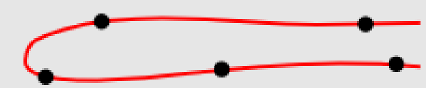

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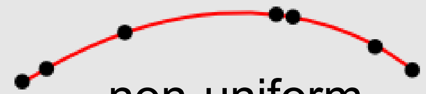
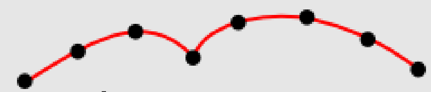
Global Approach for Closed Curves

Related: Travelling Salesman Problem

MST-based [OM11]


non-uniform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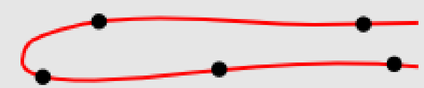
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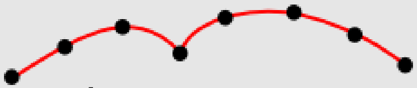

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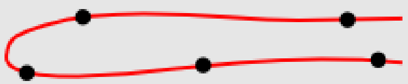

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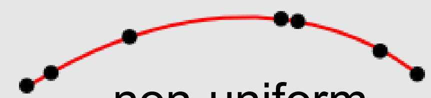
Exact solvers [ABCC11]



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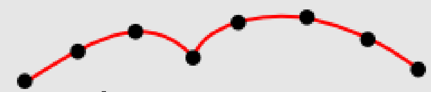

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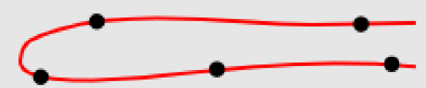

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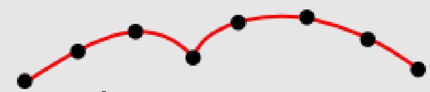

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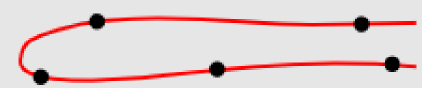

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Global Approach for Closed Curves

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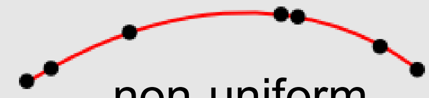
MST-based [OM11]

Our method [OM13]



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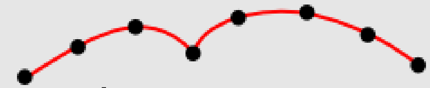

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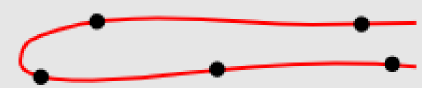

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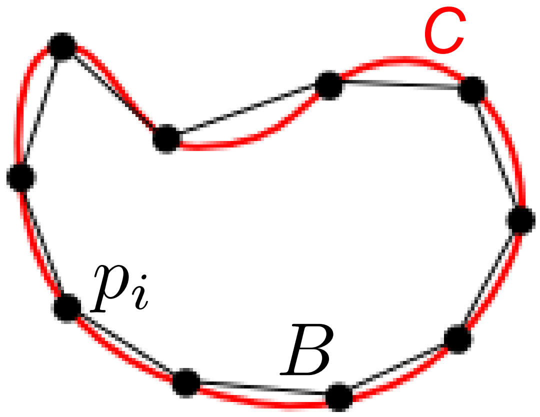
The Fundamental Problem



B reconstructs *C*

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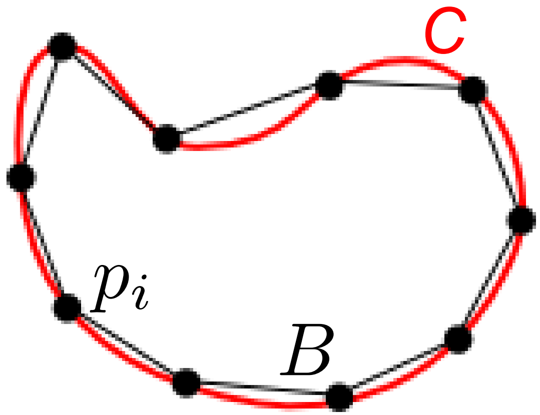

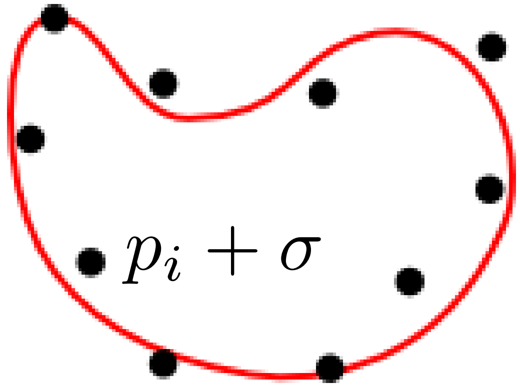

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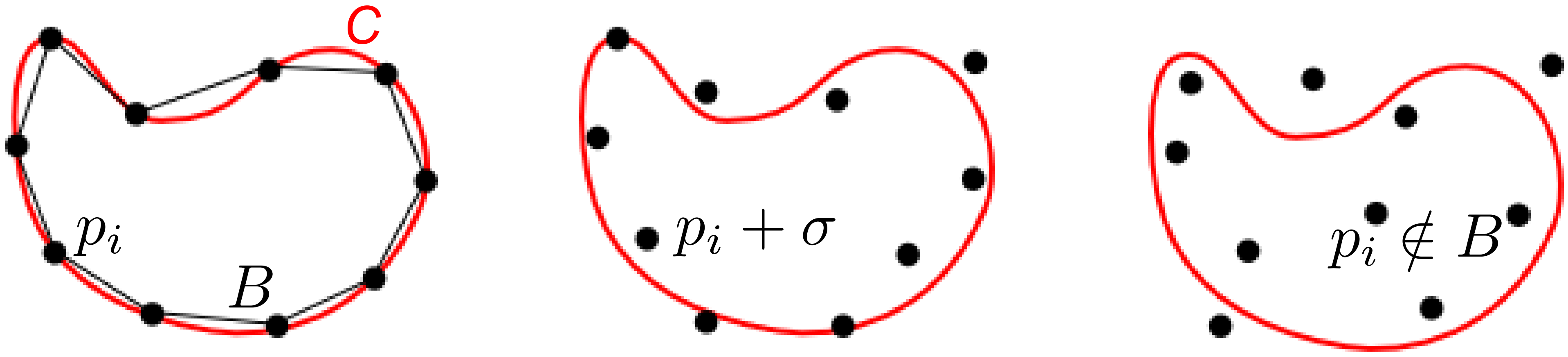

B reconstructs *C*   noisy samples   very noisy + outliers

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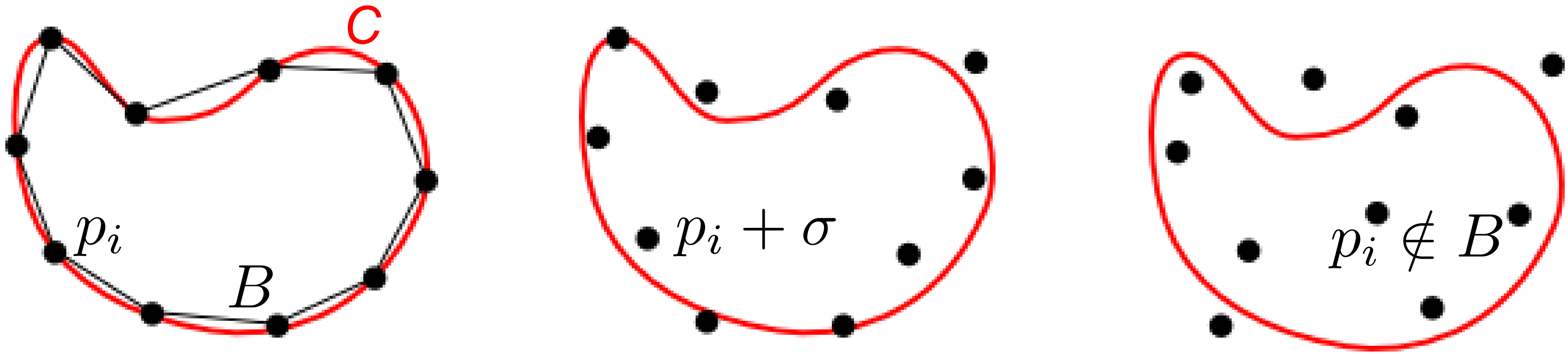

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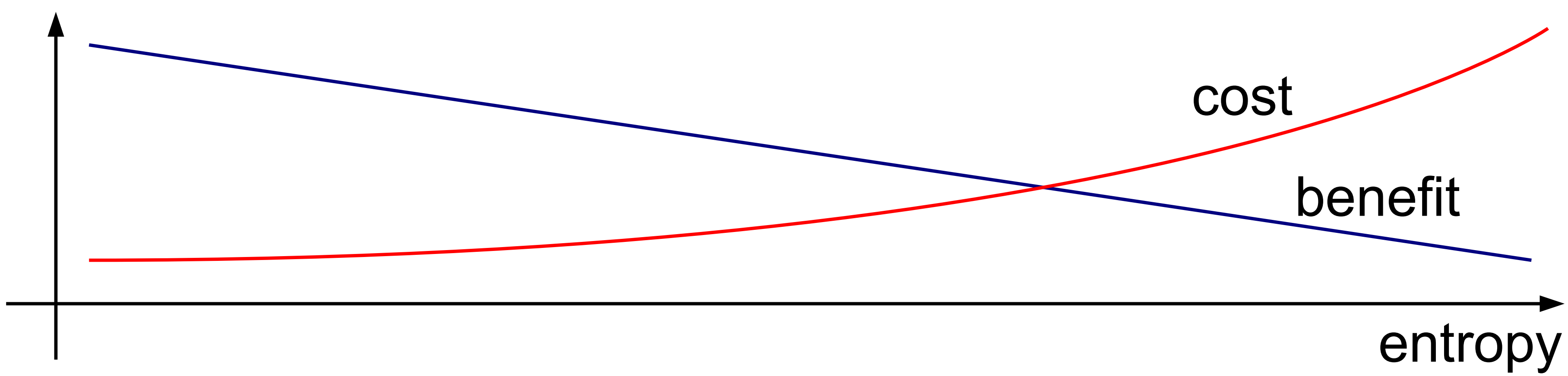

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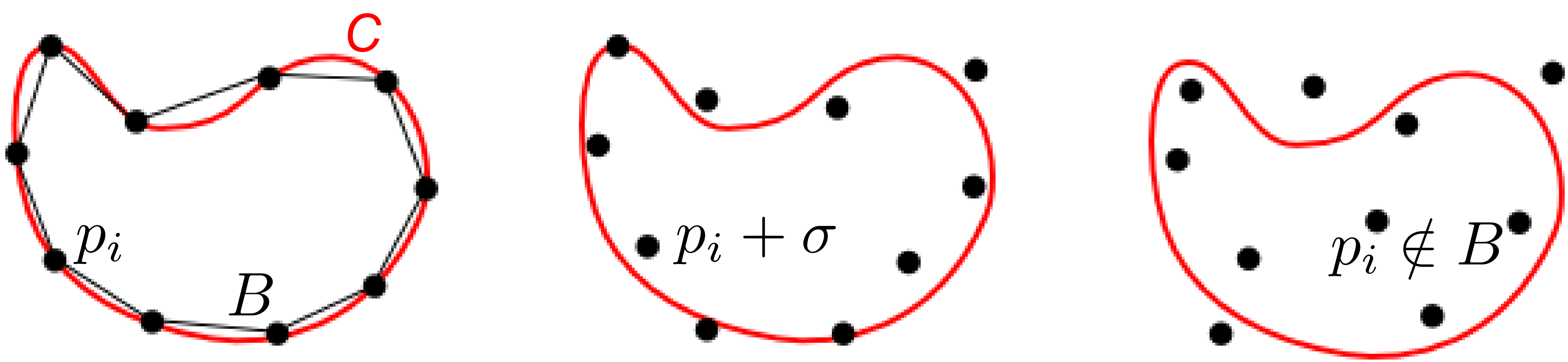
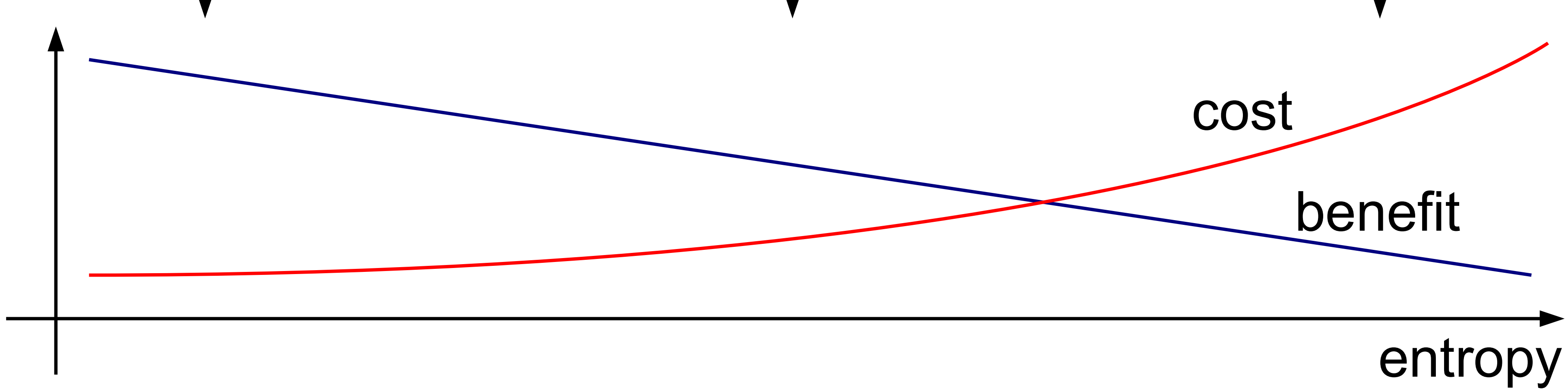


B reconstructs *C* — noisy samples — very noisy + outliers



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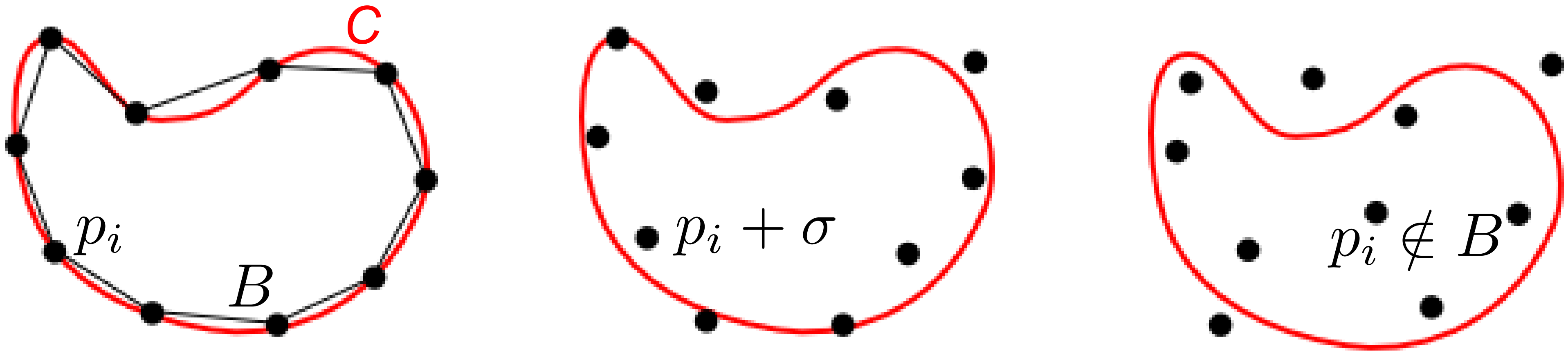

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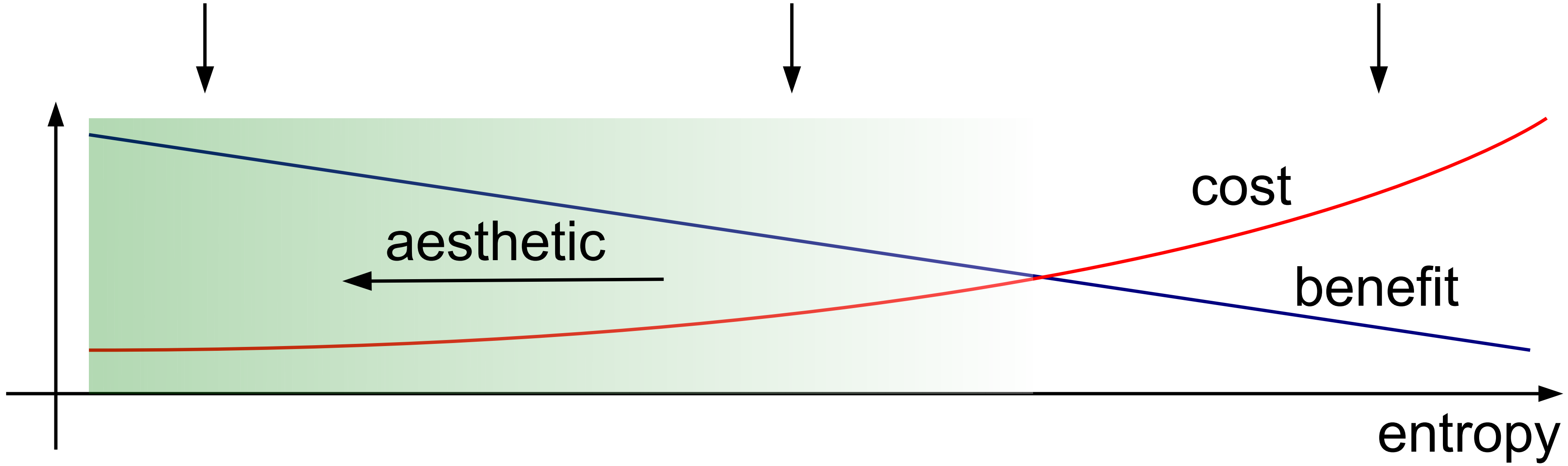

Our Approach

What do we know?	Sampling process ?	

Our Approach

	Sampling process	Sampled object
What do we know?	?	is a solid

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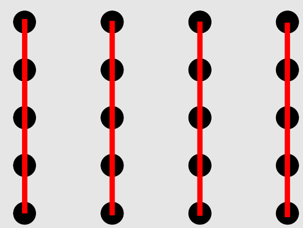
What do we know?	?	is a solid
Correlates to Gestalt principles	 Proximity	

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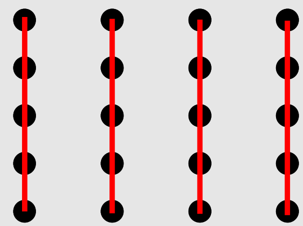
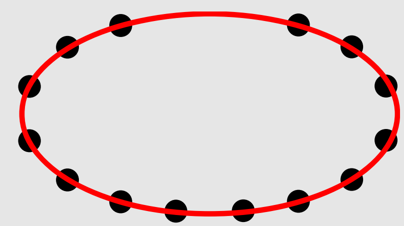
Correlates to Gestalt principles	 Proximity	 Closure

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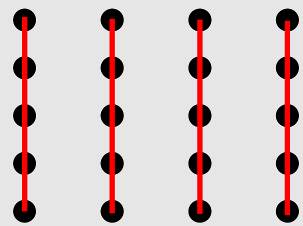
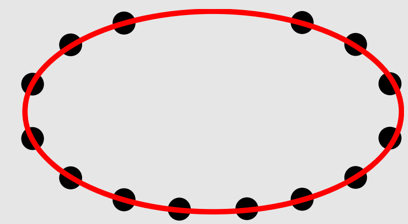
Correlates to Gestalt principles	 Proximity	 Closure
Formalized as B_{min}	$\underset{B_i \in DT}{\operatorname{argmin}} \lvert\lvert B_i \rvert\rvert$	

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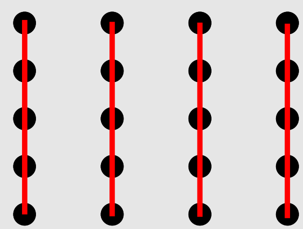
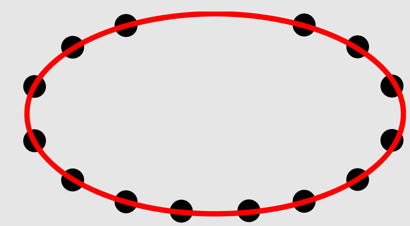
Correlates to Gestalt principles	 Proximity	 Closure
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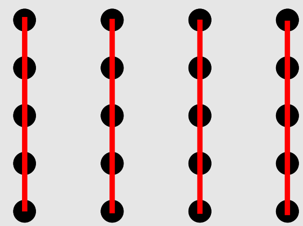
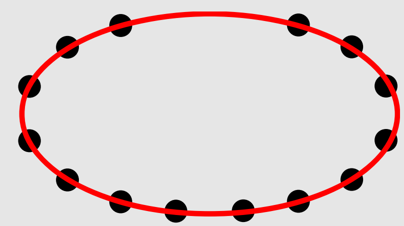
Correlates to Gestalt principles	 Proximity	 Closure
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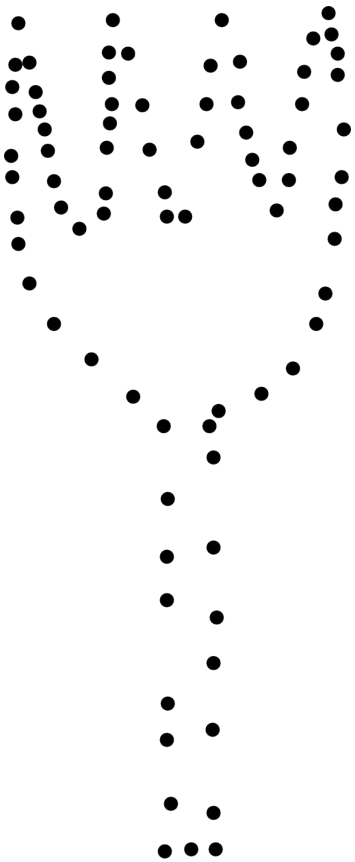
Goal: **O(n log n)** for aesthetic point sets, heuristic for remaining class

A Naïve Attempt



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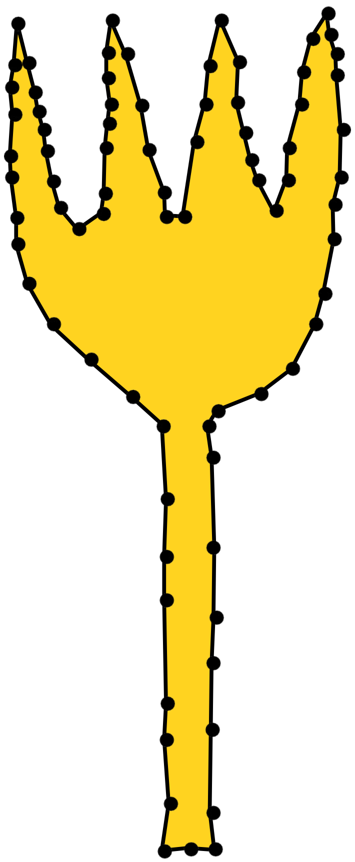

B_{min}

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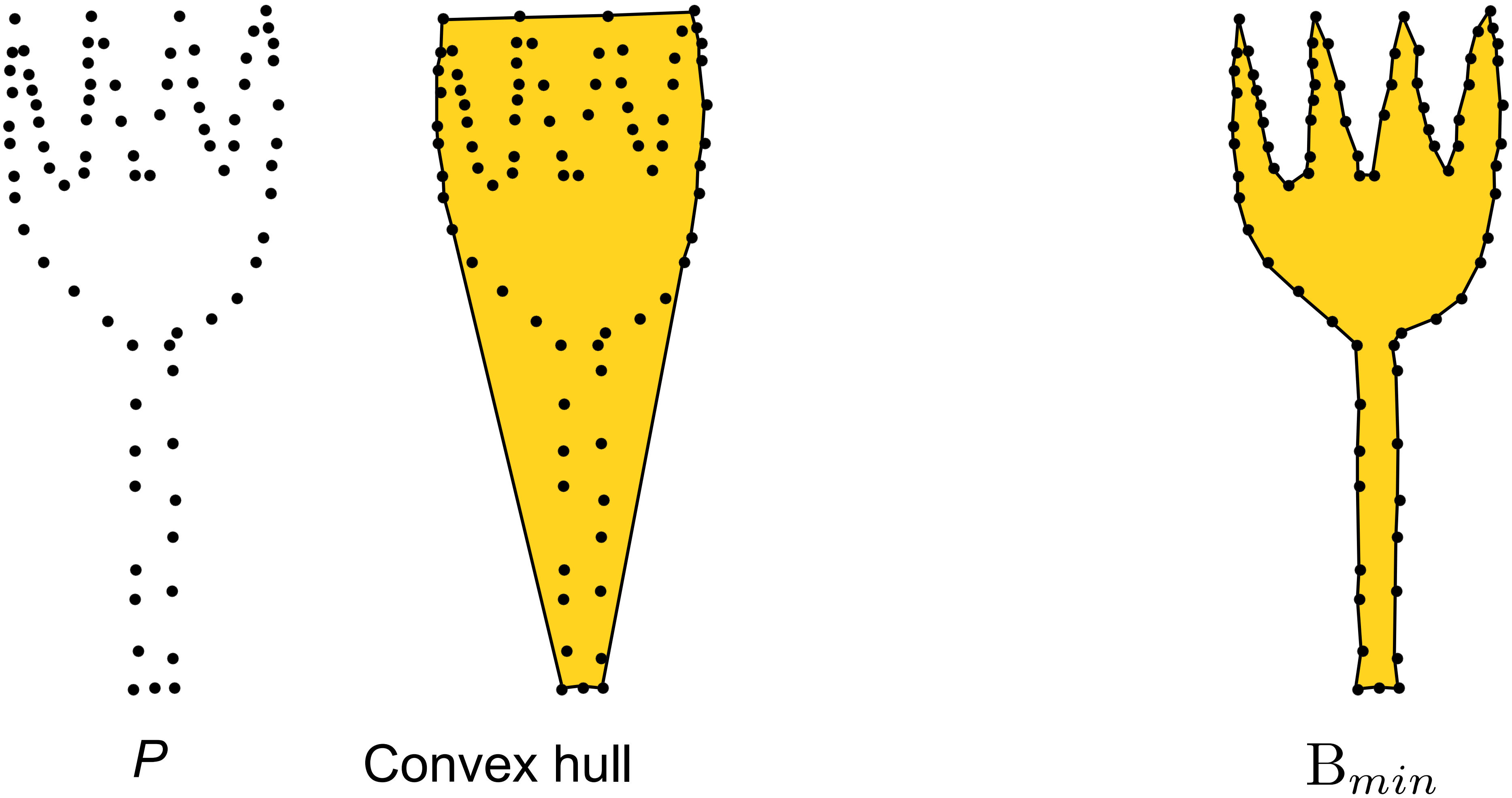

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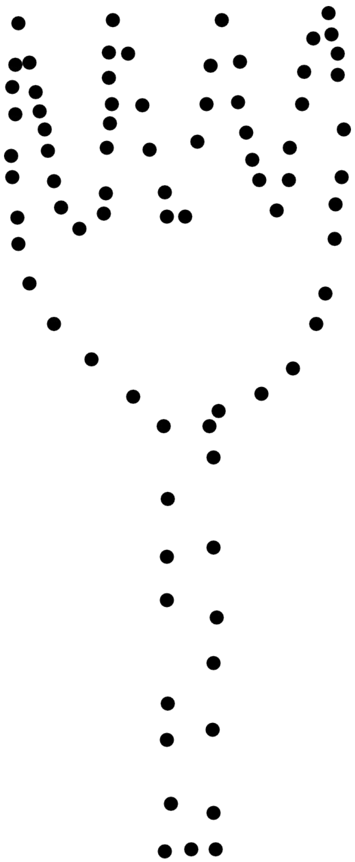

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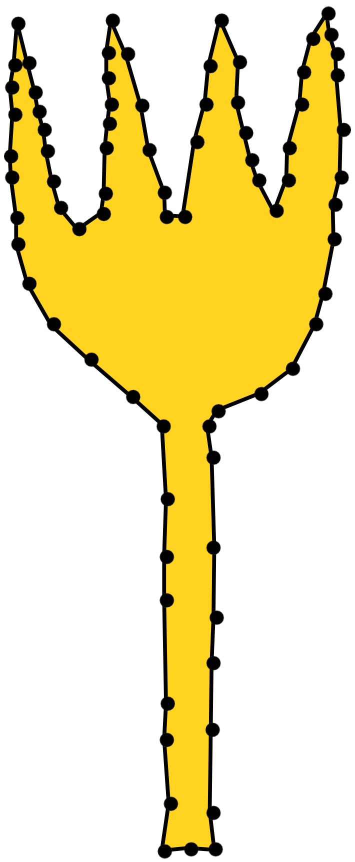
Convex hull: Sculpturing [Boi84]



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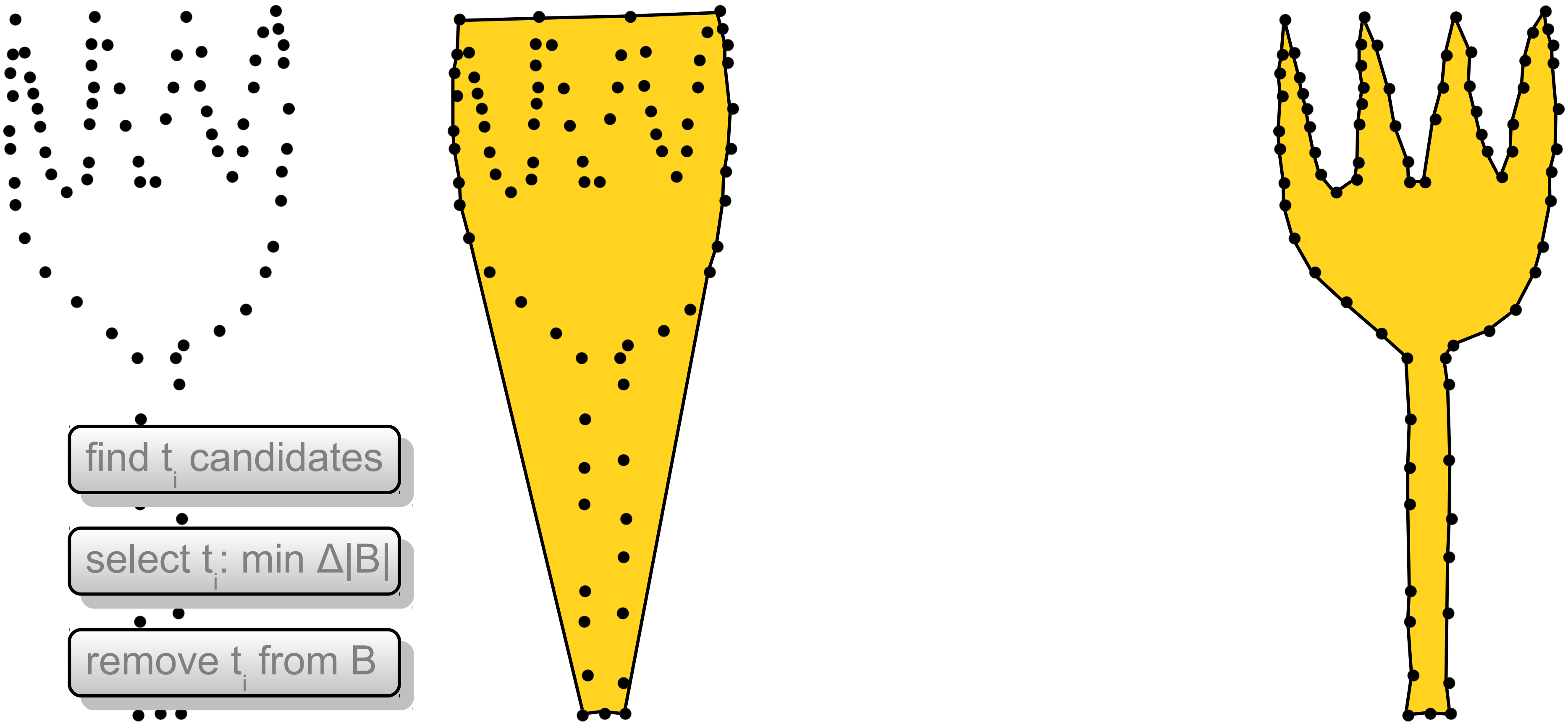

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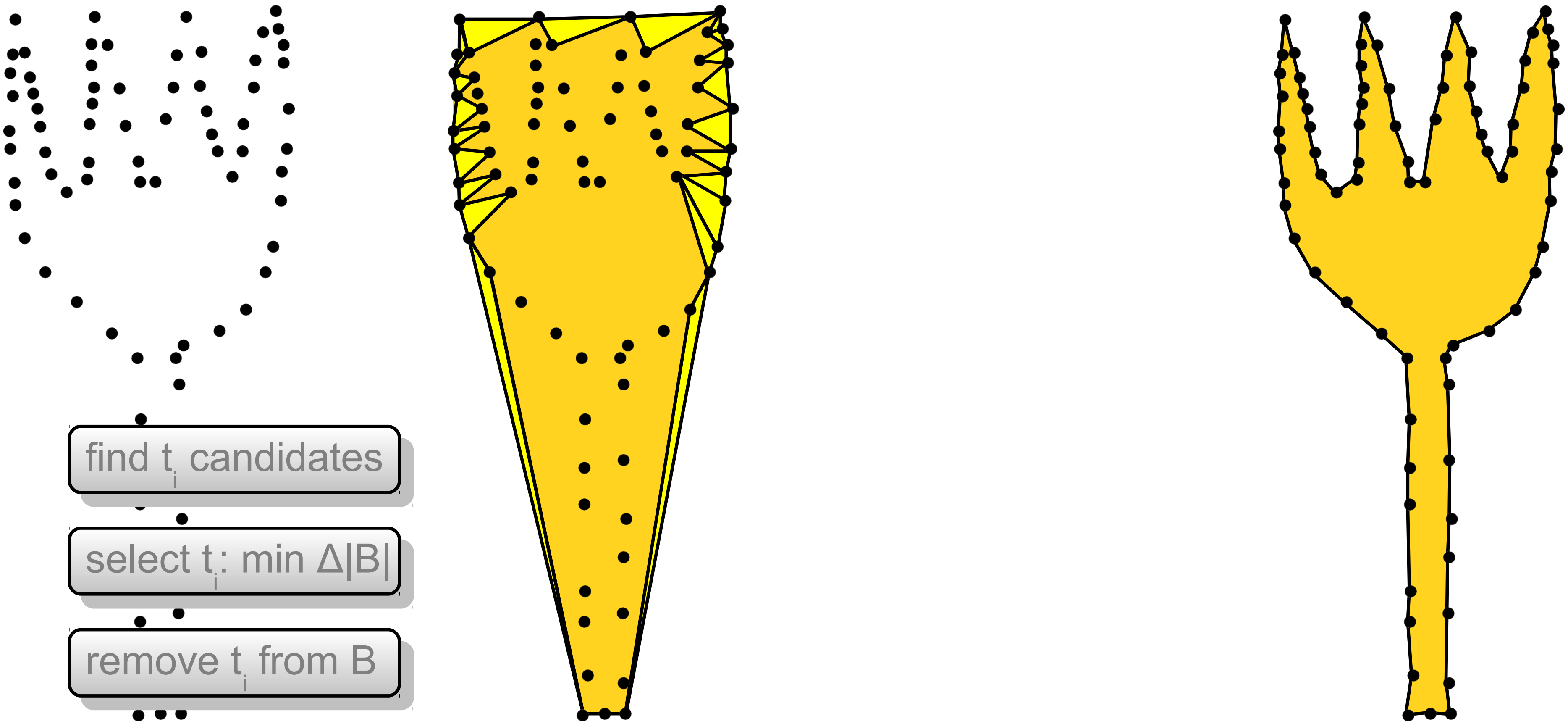

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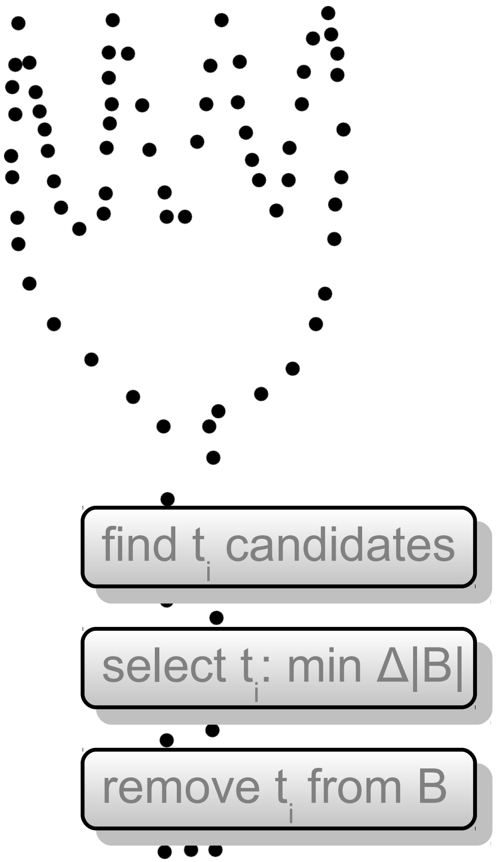

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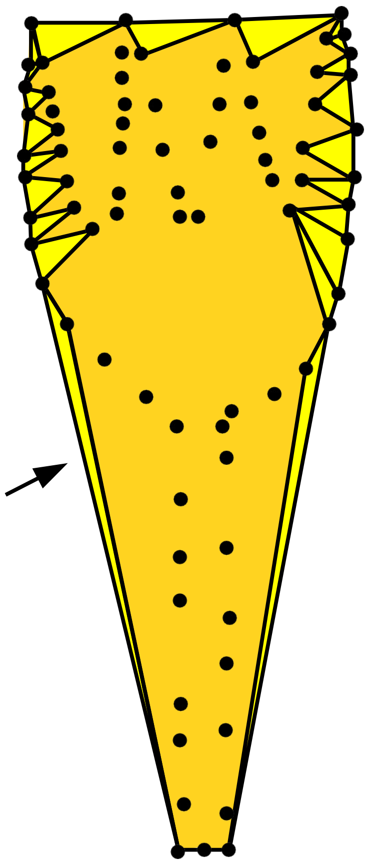

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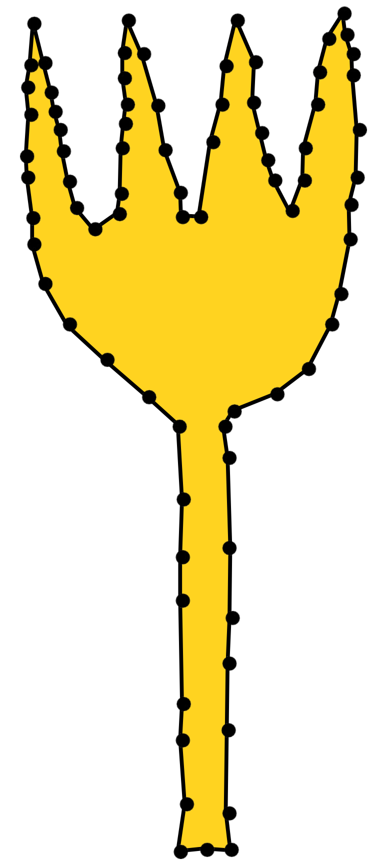

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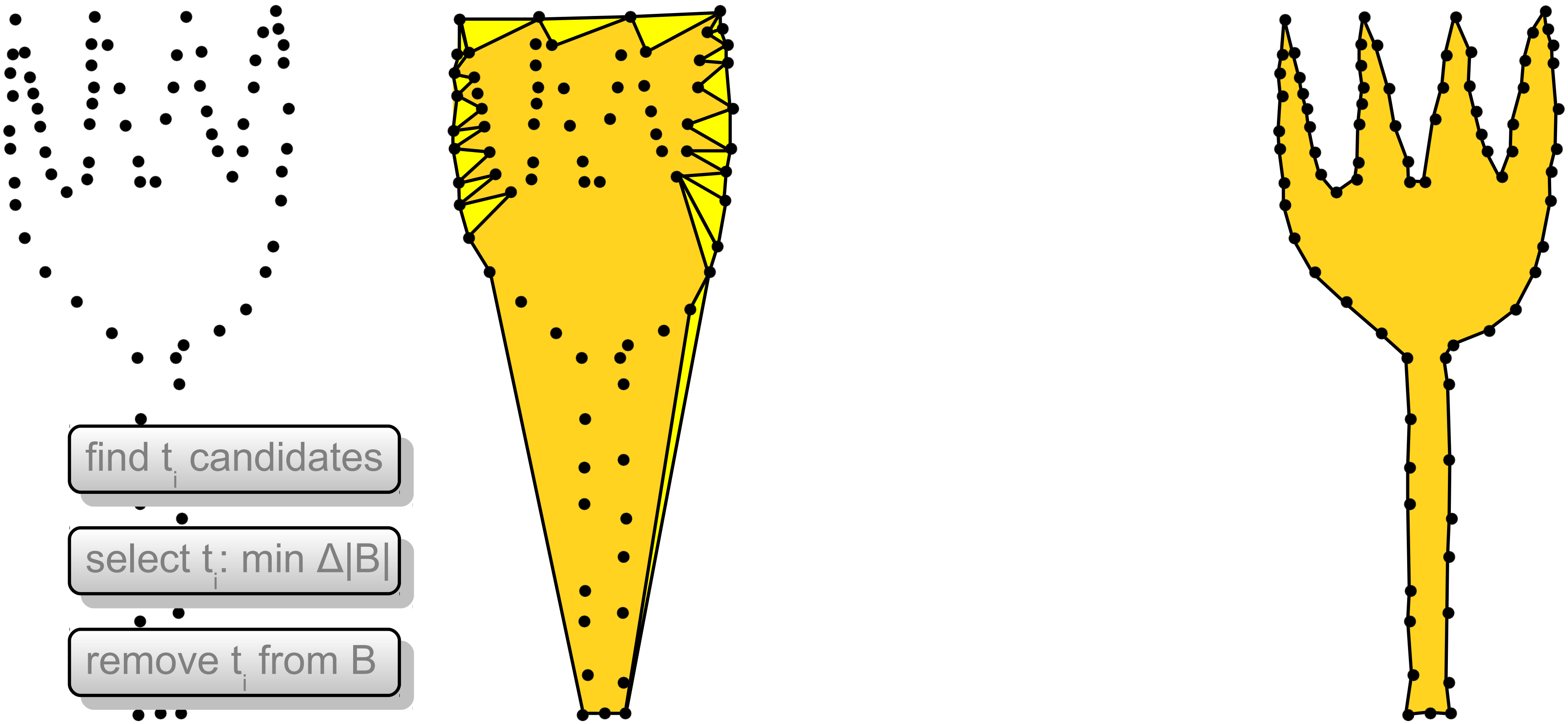

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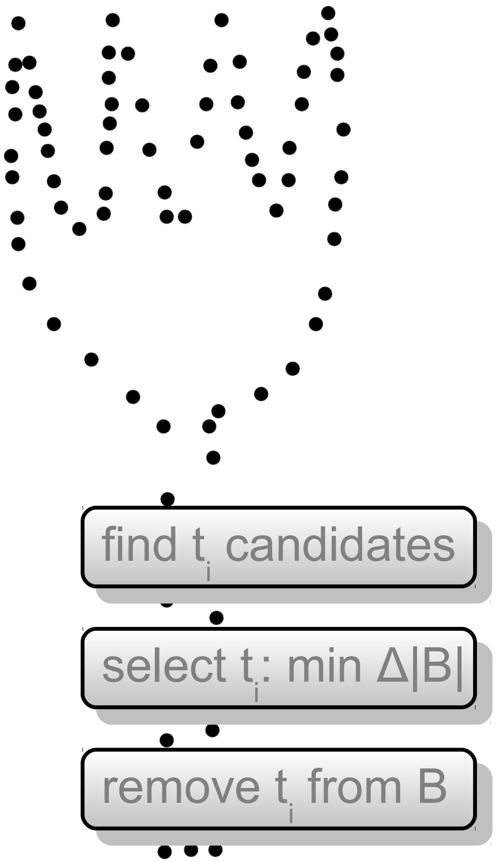

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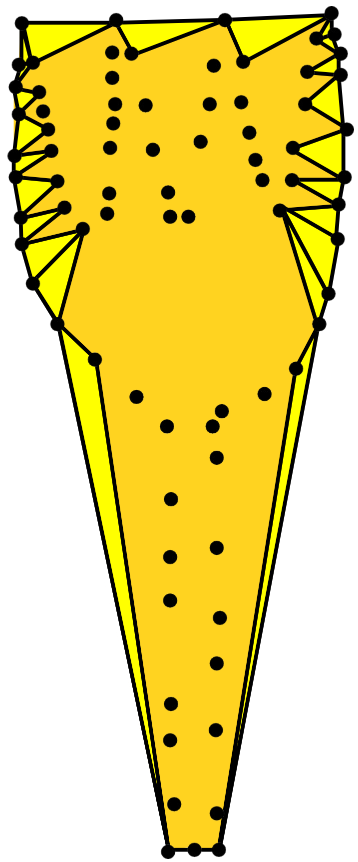

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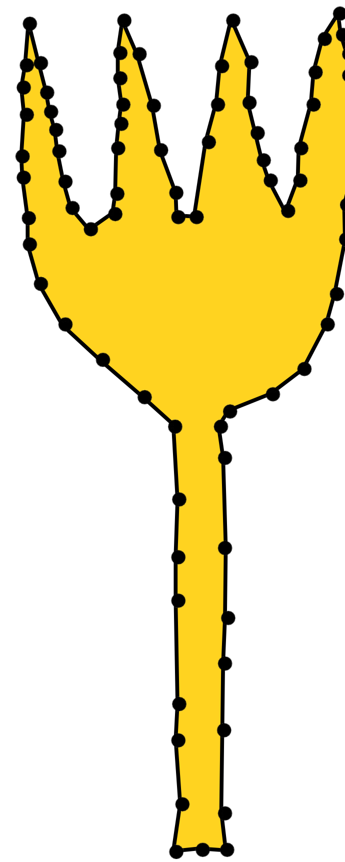
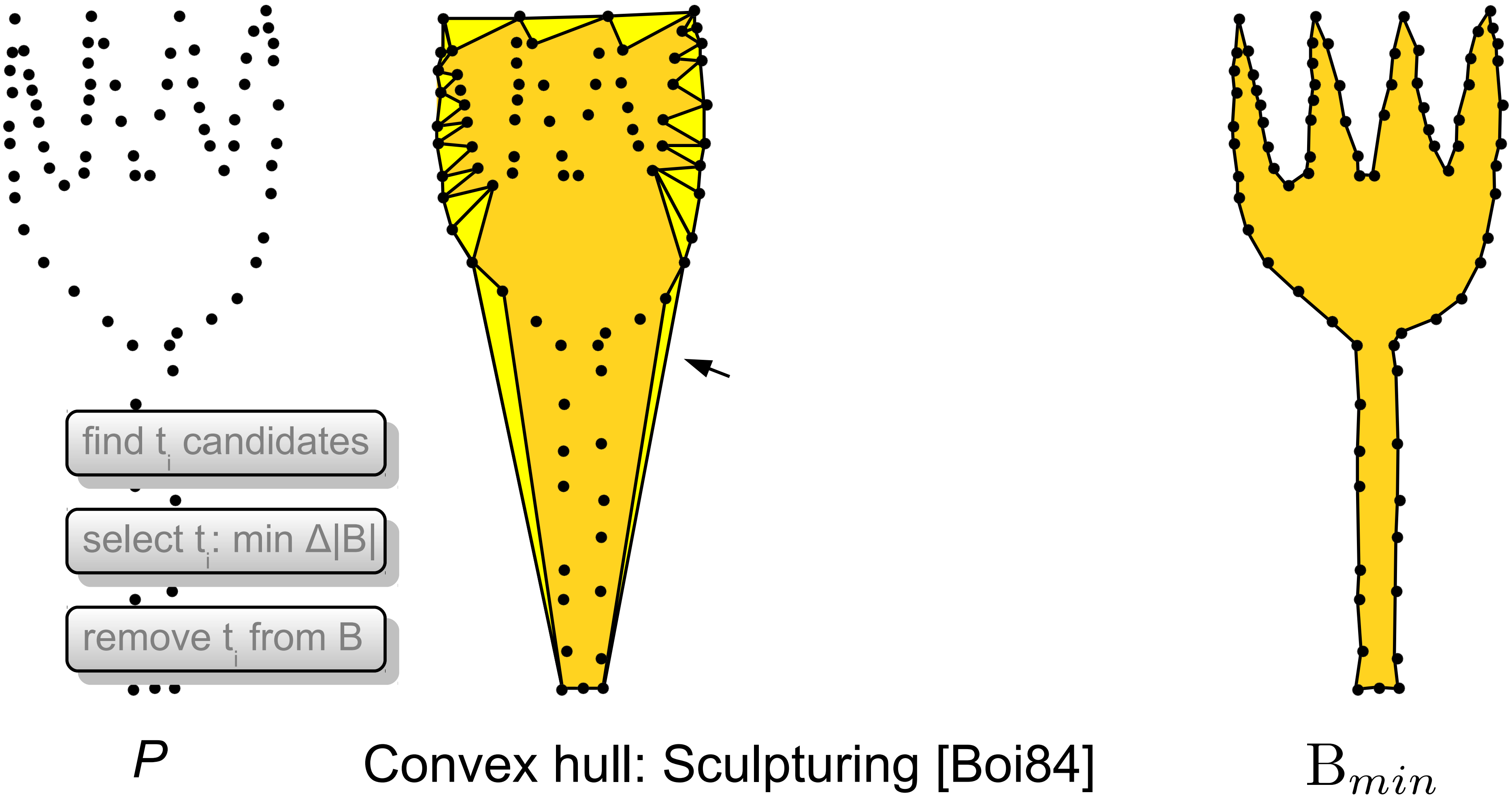
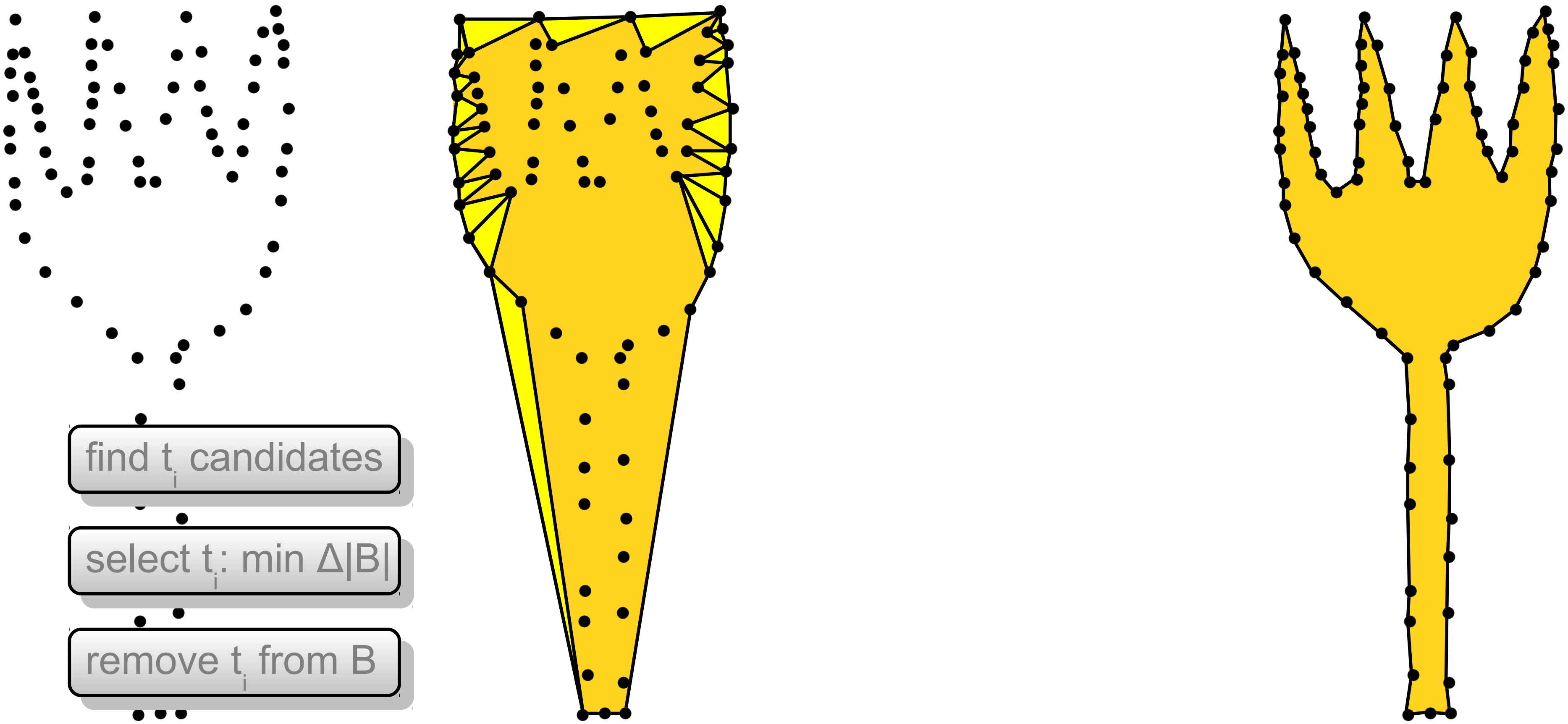


B_{min}

A Naïve Attempt

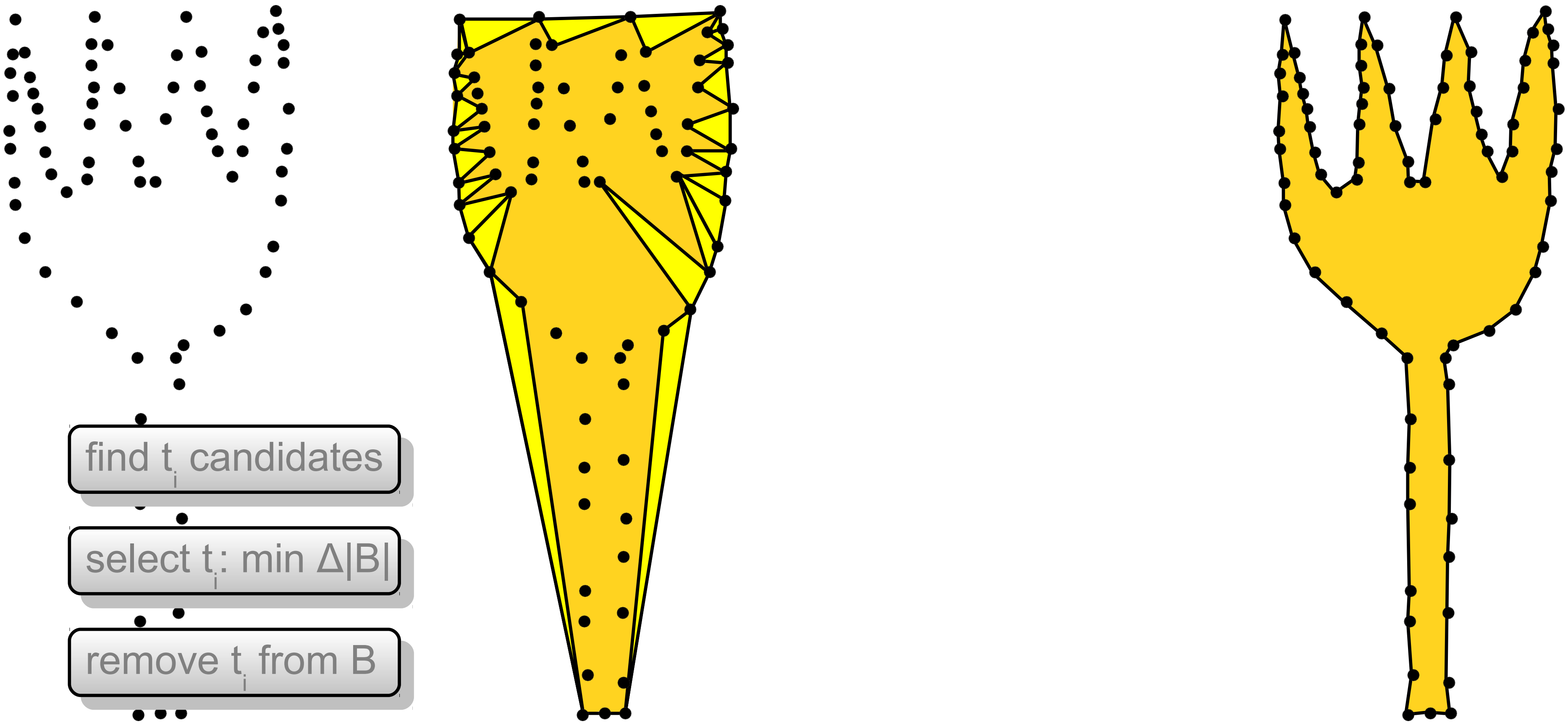


A Naïve Attempt



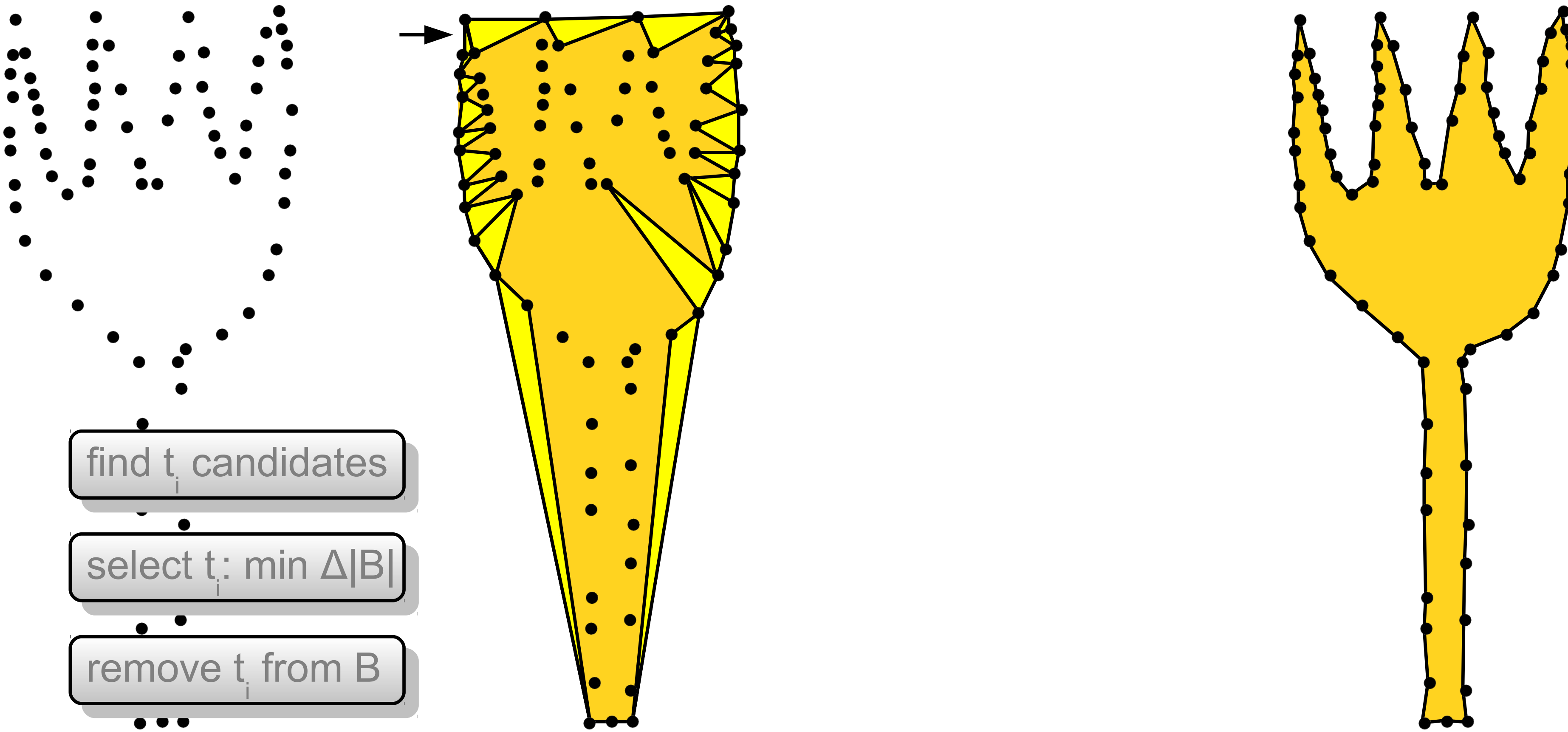
P   Convex hull: Sculpturing [Boi84] B_{min}

A Naïve Attempt



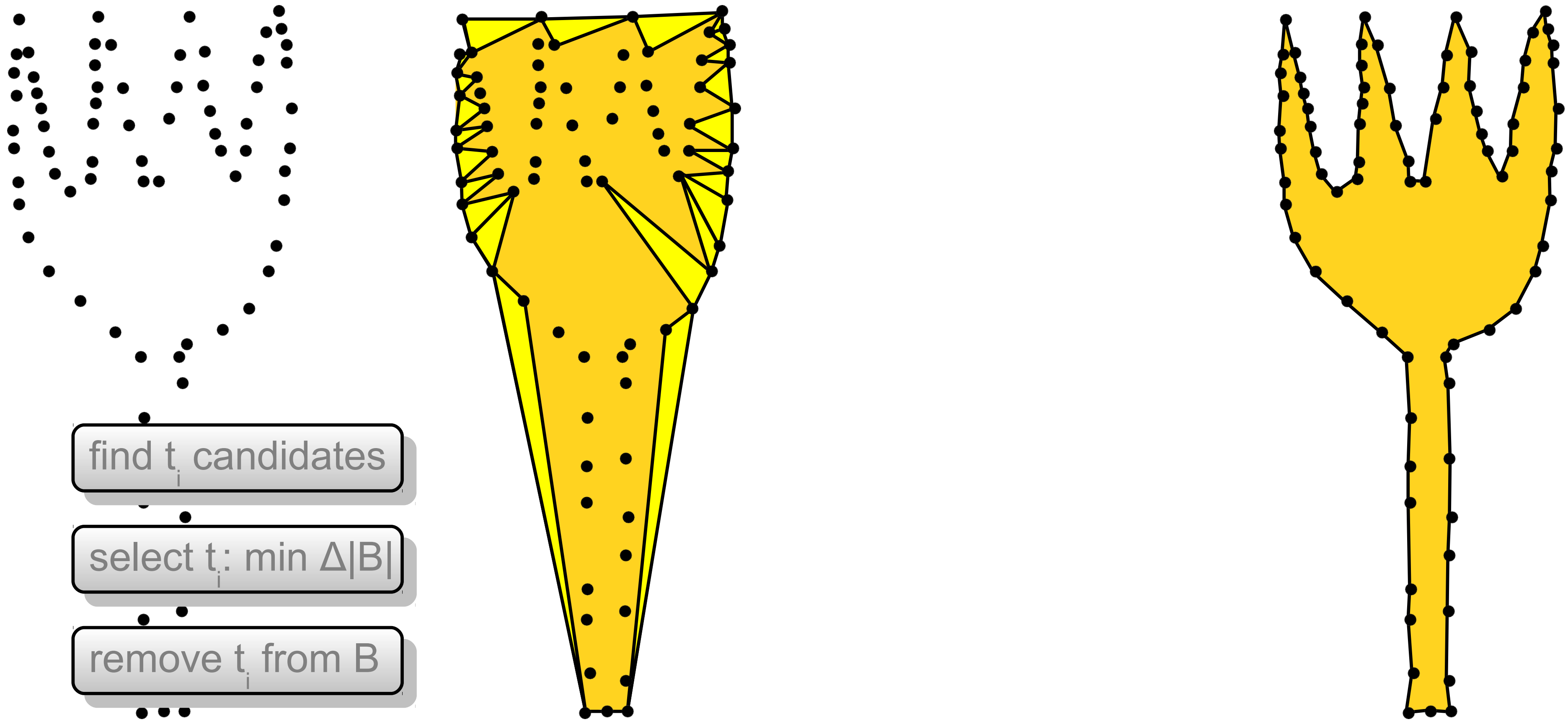
P Convex hull: Sculpturing [Boi84] B_{min}

A Naïve Attempt



P   Convex hull: Sculpturing [Boi84] B_{min}

A Naïve Attempt

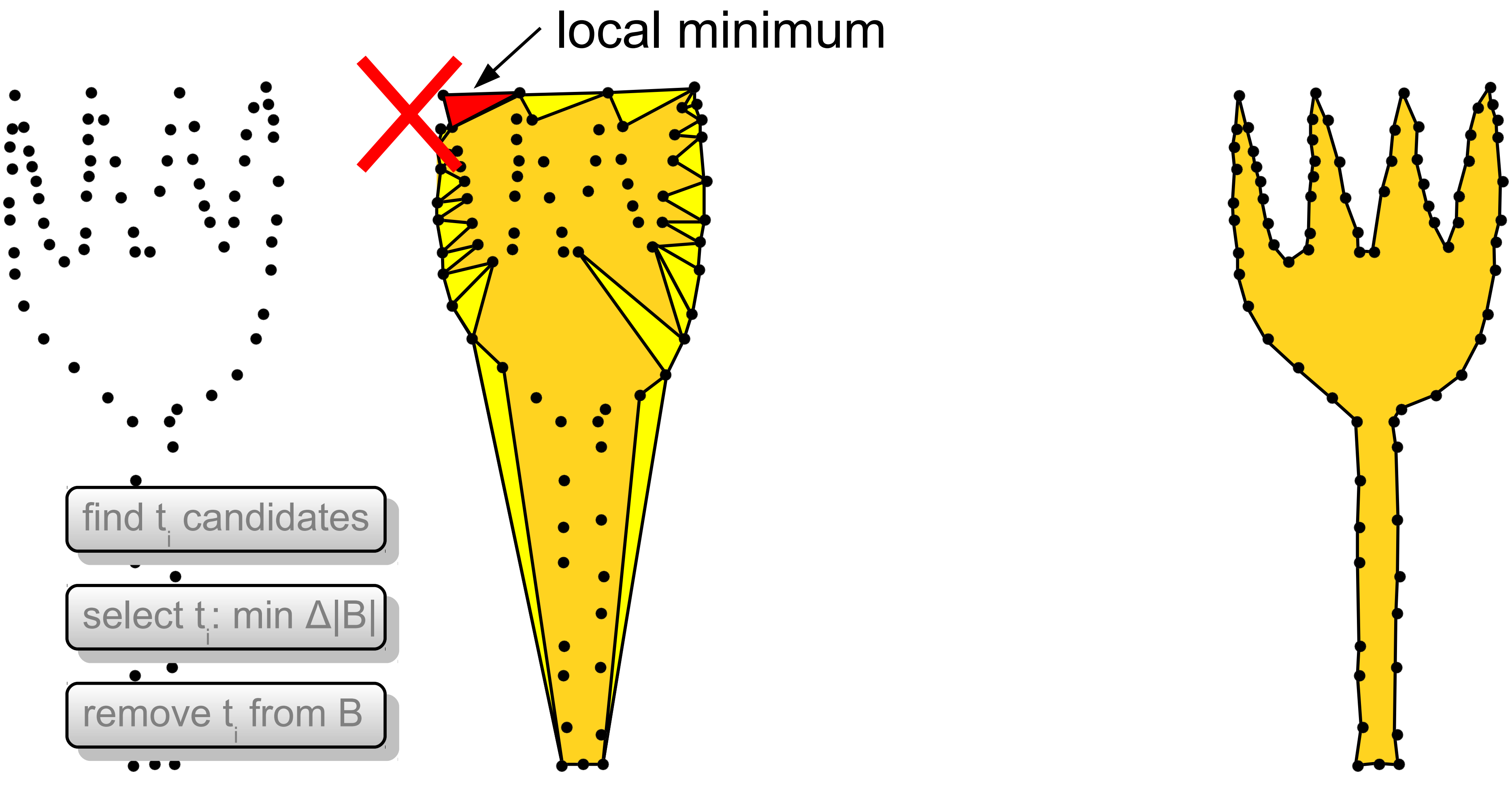


P

Convex hull: Sculpturing [Boi84]

B_{min}

A Naïve Attempt

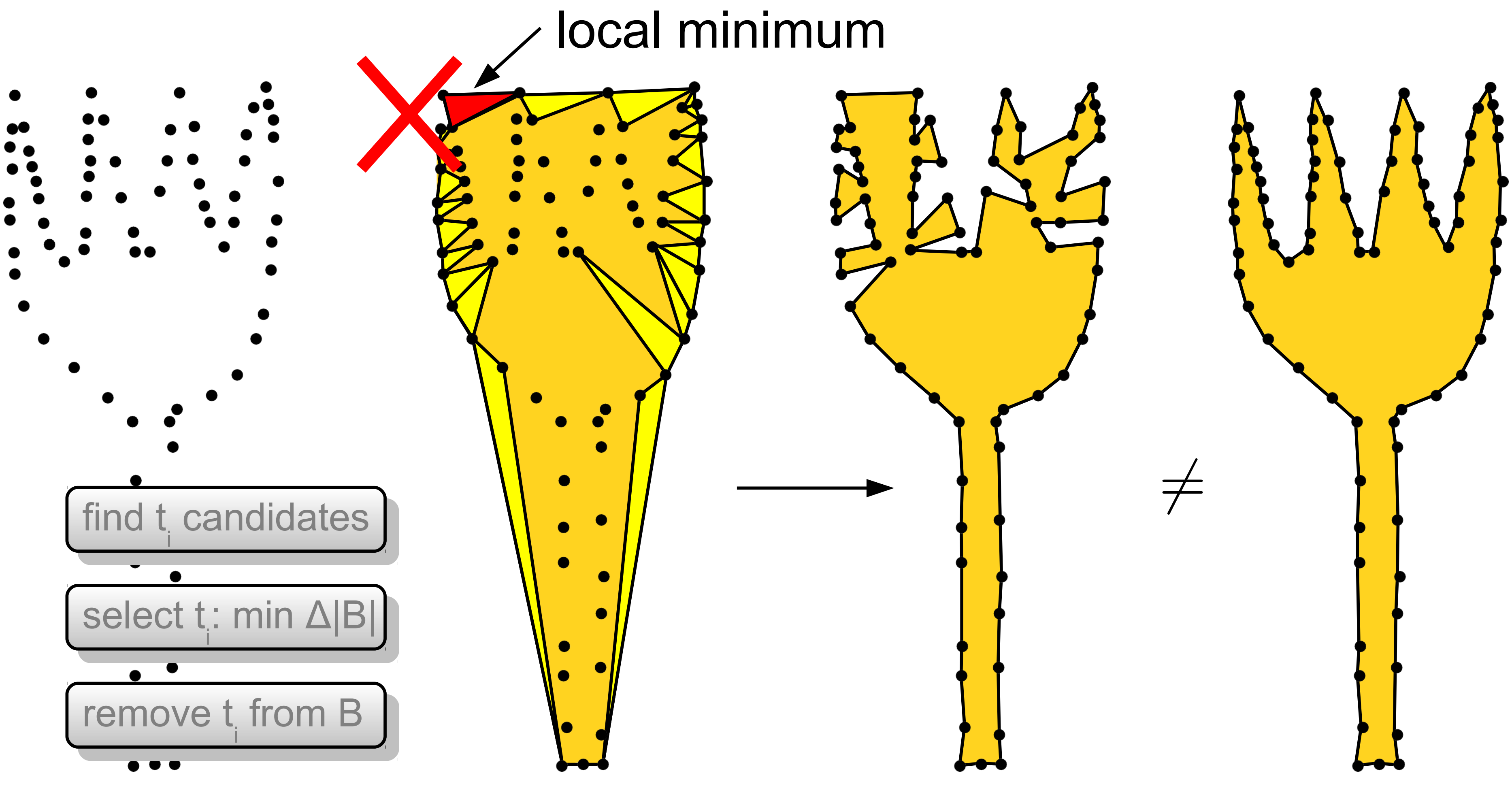


P

Convex hull: Sculpturing [Boi84]

B_{min}

A Naïve Attempt

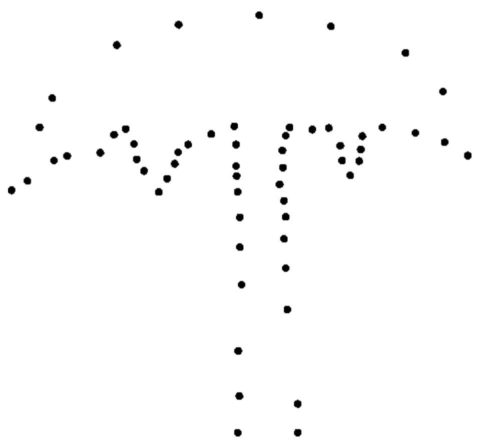


P

Convex hull: Sculpturing [Boi84]

B_{min}

Start With A Minimum Set



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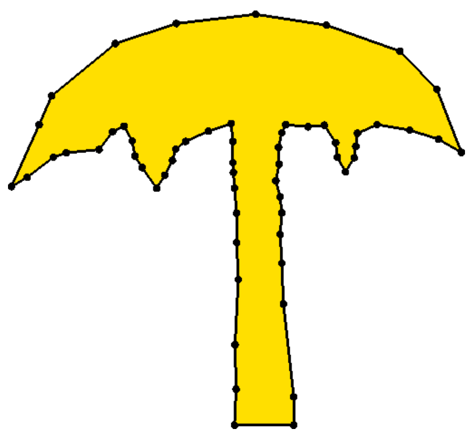
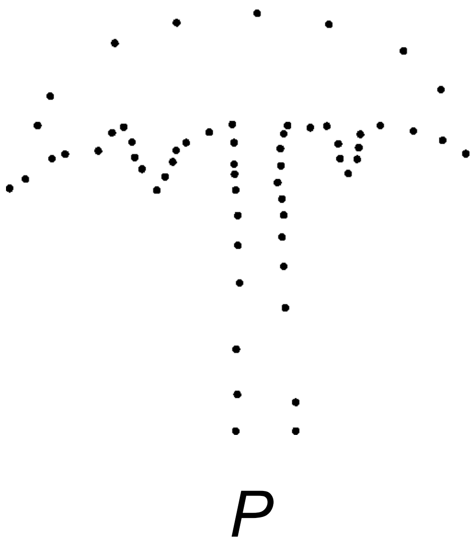
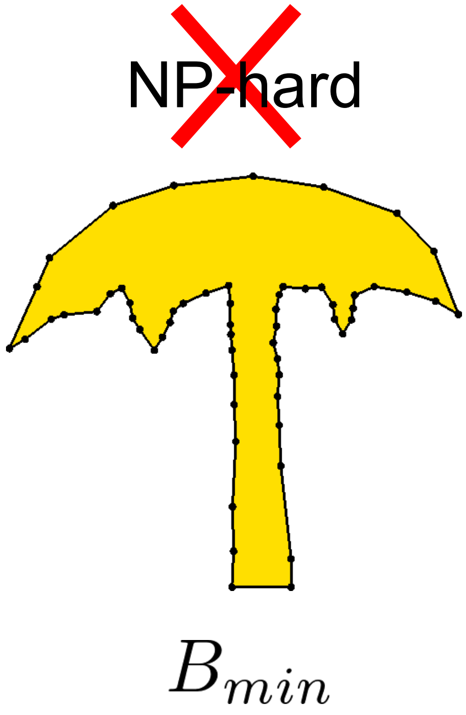


B_{min}

Start With A Minimum Set

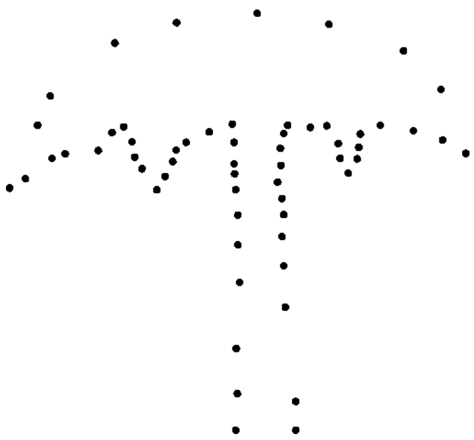




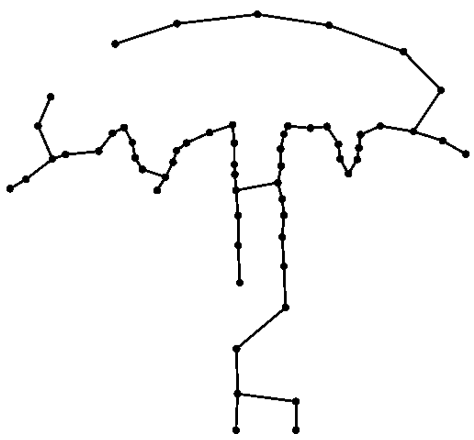
Start With A Minimum Set

$O(n \log n)$

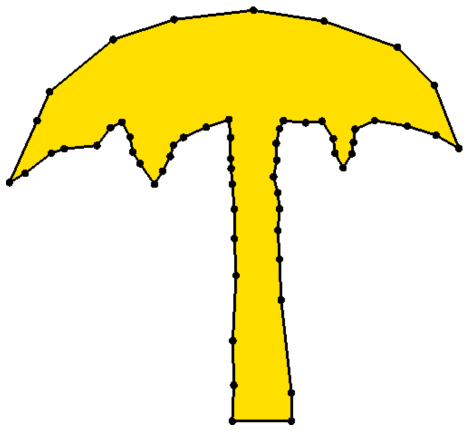
~~NP-hard~~



P



MST

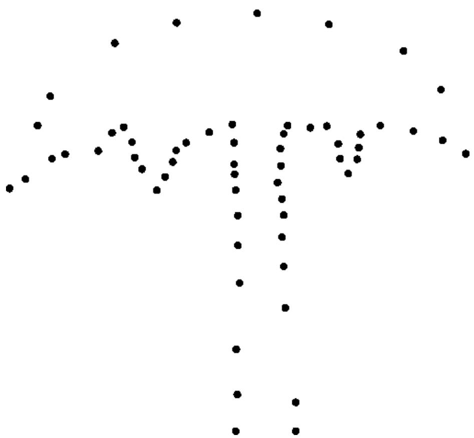


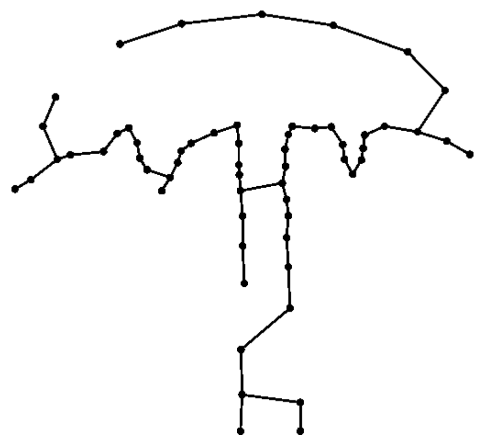
B_{min}

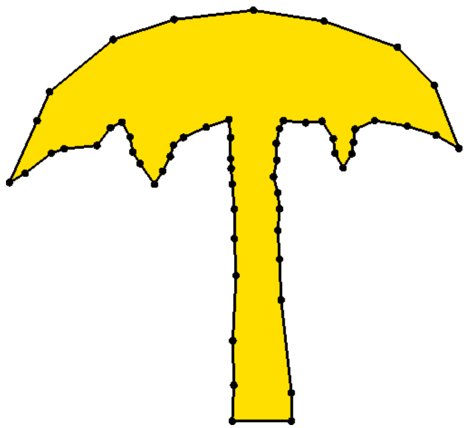
Start With A Minimum Set

$O(n \log n)$

~~NP-hard~~

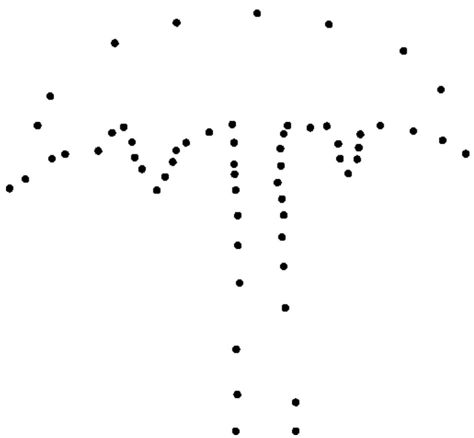






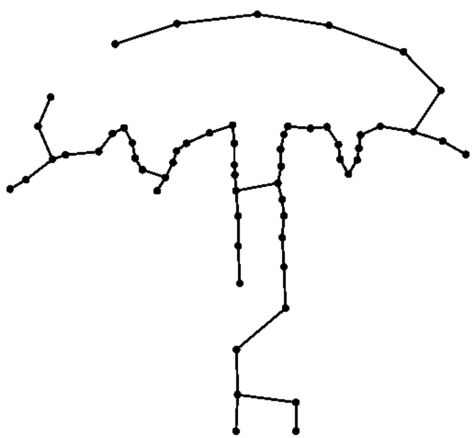
P	MST	B_{min}
Vertex degree c	$c \geq 1$	c=2

Start With A Minimum Set



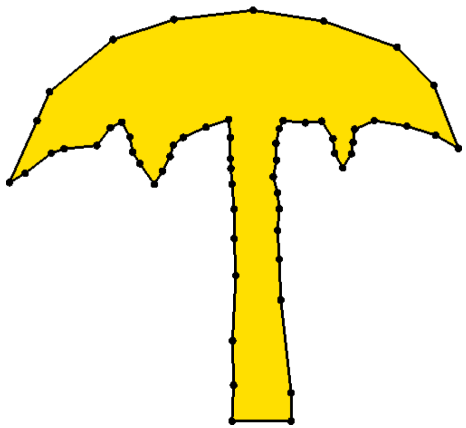
P

$O(n \log n)$



MST

~~NP-hard~~

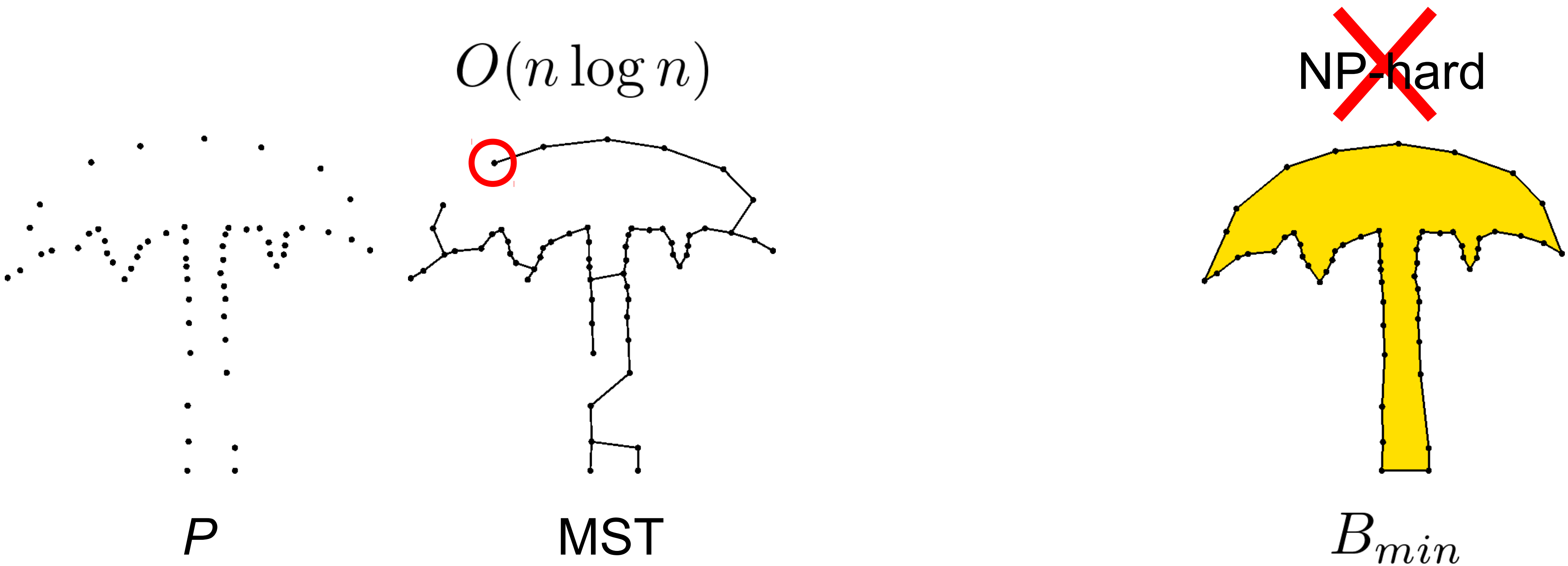


B_{min}

Vertex degree c | $c \geq 1$ | c=2

$$\underset{G \in DT}{\operatorname{argmin}} \sum_{e_i \in G} ||e_i||$$

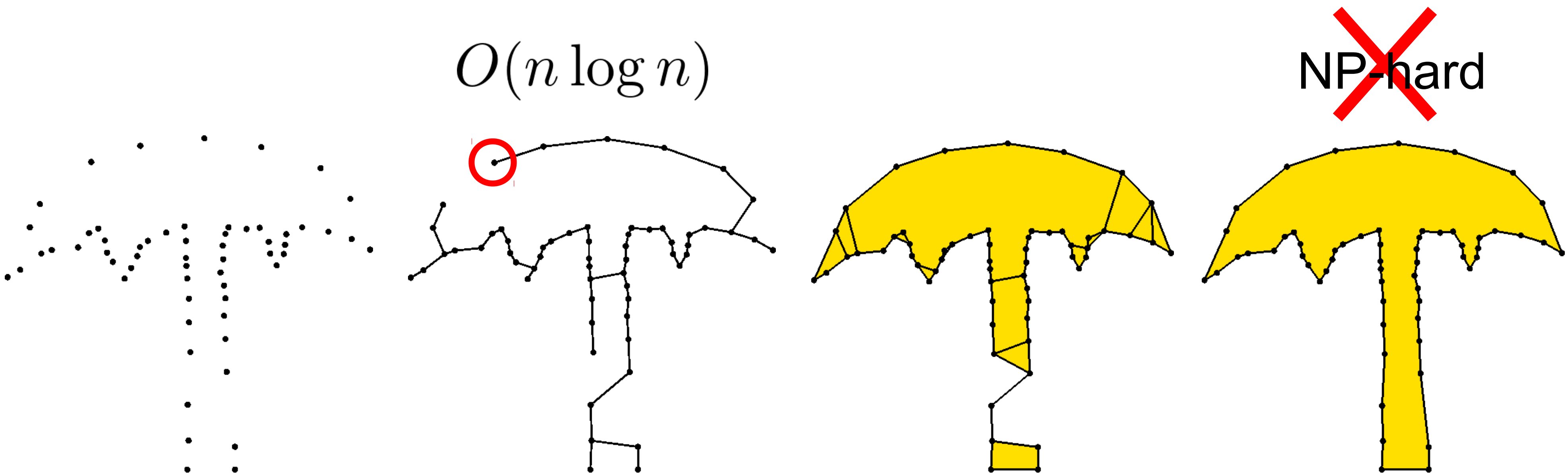
Start With A Minimum Set



Vertex degree c $c \geq 1$ c=2

$$\underset{G \in DT}{\operatorname{argmin}} \sum_{e_i \in G} ||e_i||$$

Start With A Minimum Set

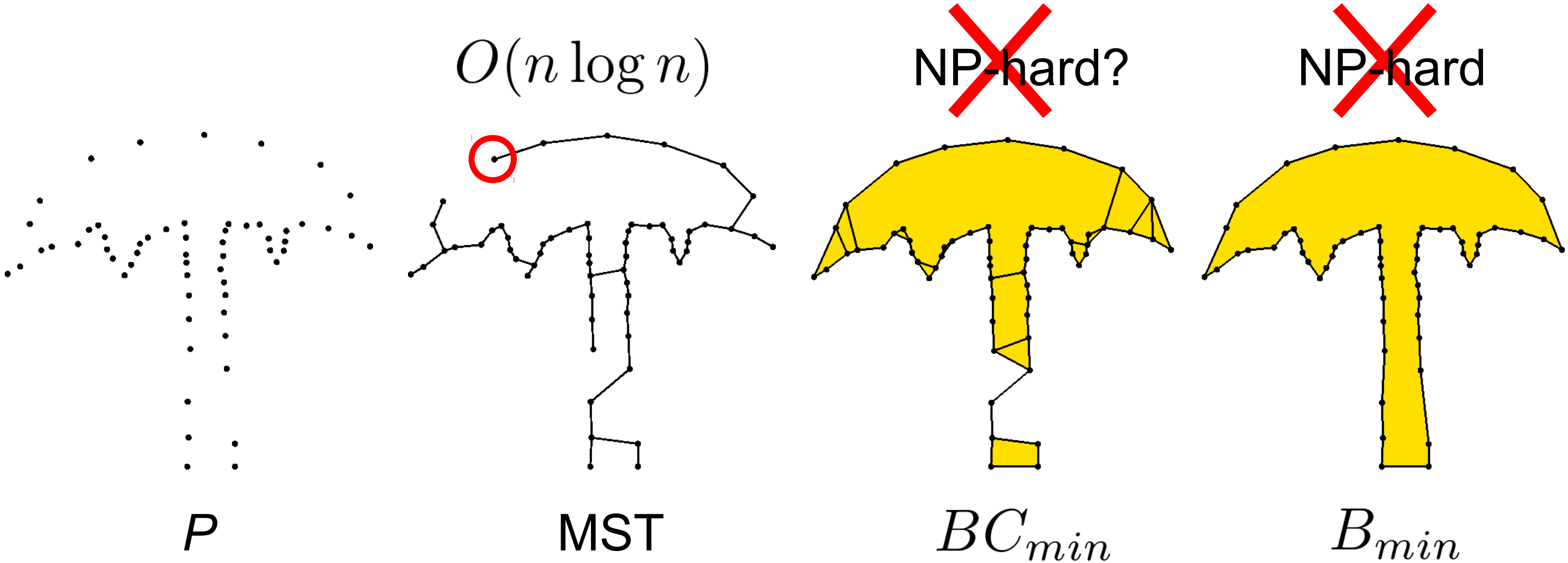


P | MST | BC_{min} | B_{min}

Vertex degree c | c $\geq$ 1 | c $\geq$ 2 | c=2

$$\operatorname*{argmin}_{G \in DT} \sum_{e_i \in G} ||e_i||$$

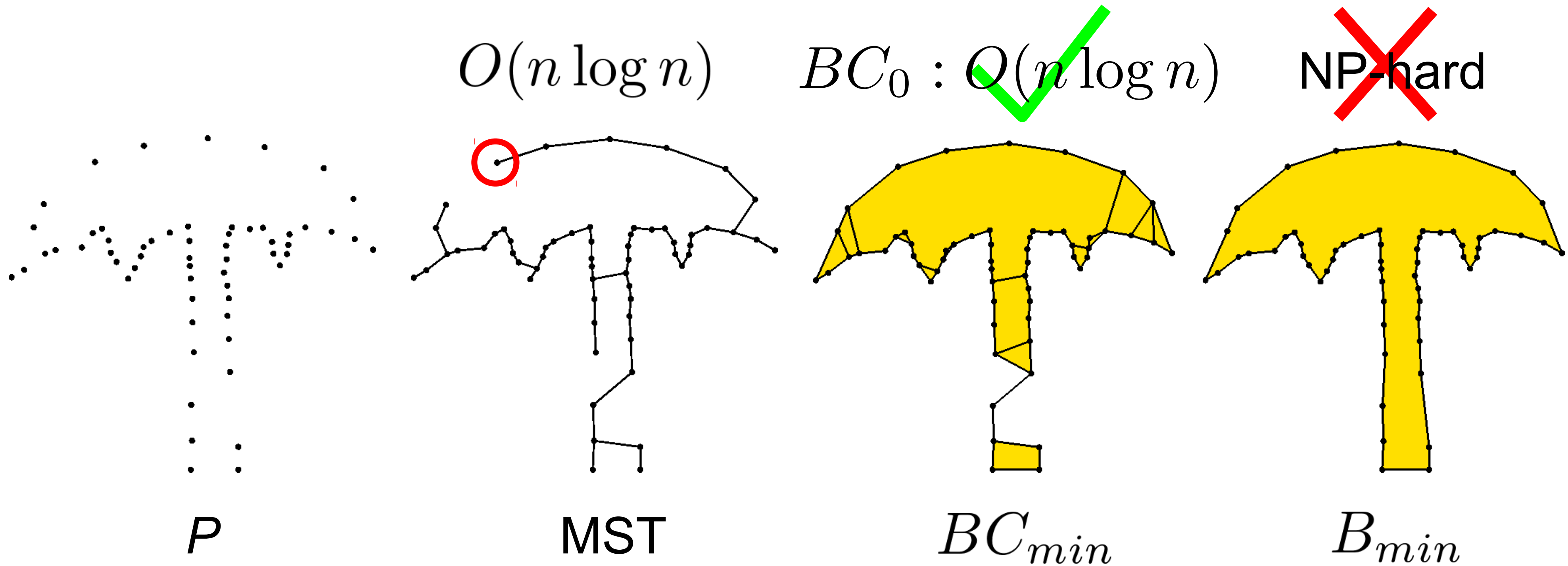
Start With A Minimum Set



Vertex degree c | c $\geq$ 1 | c $\geq$ 2 | c=2

$$\underset{G \in DT}{\operatorname{argmin}} \sum_{e_i \in G} ||e_i||$$

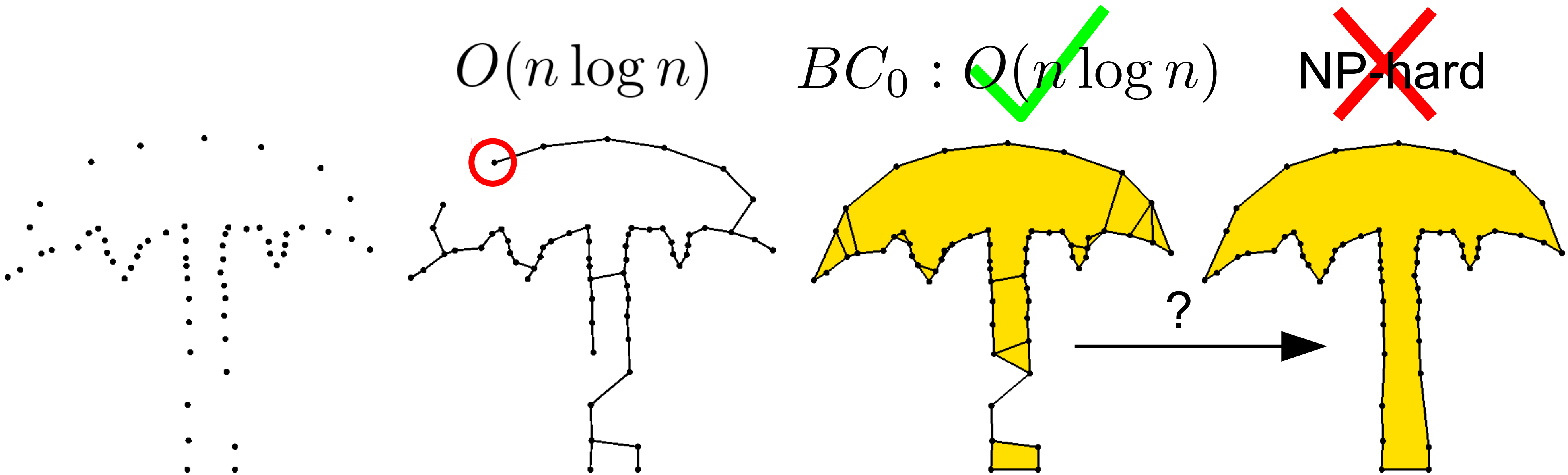
Start With A Minimum Set



Vertex degree c   c $\geq$ 1   c $\geq$ 2   c=2

$$\underset{G \in DT}{\operatorname{argmin}} \sum_{e_i \in G} ||e_i||$$

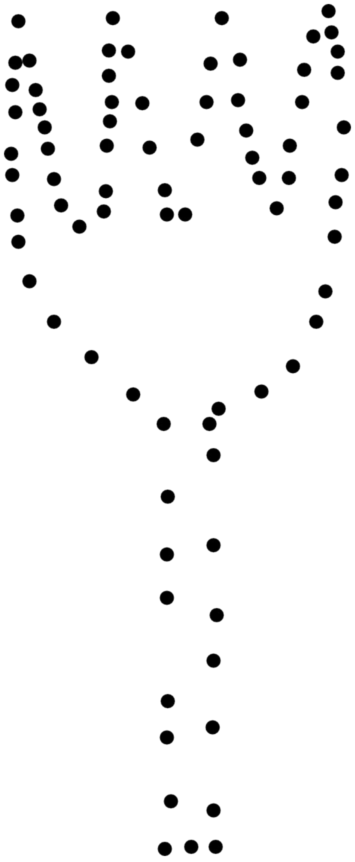
Start With A Minimum Set



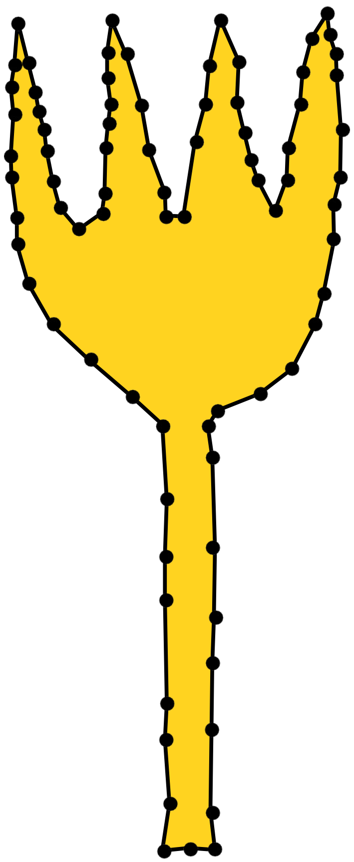
P	MST	BC_{min}	B_{min}
Vertex degree c	c ≥ 1	c ≥ 2	c=2

$$\underset{G \in DT}{\operatorname{argmin}} \sum_{e_i \in G} ||e_i||$$

Overview Of Our **'Connect2D'** Algorithm

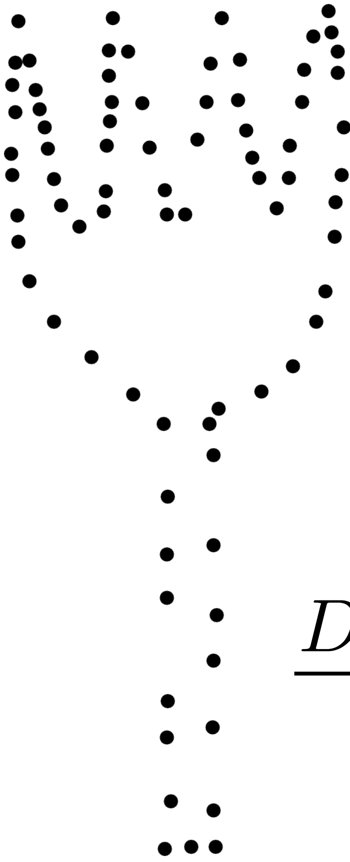


Input

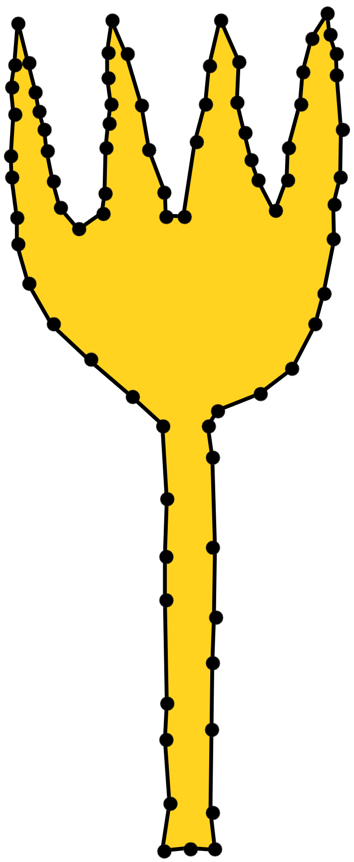


B_{out}

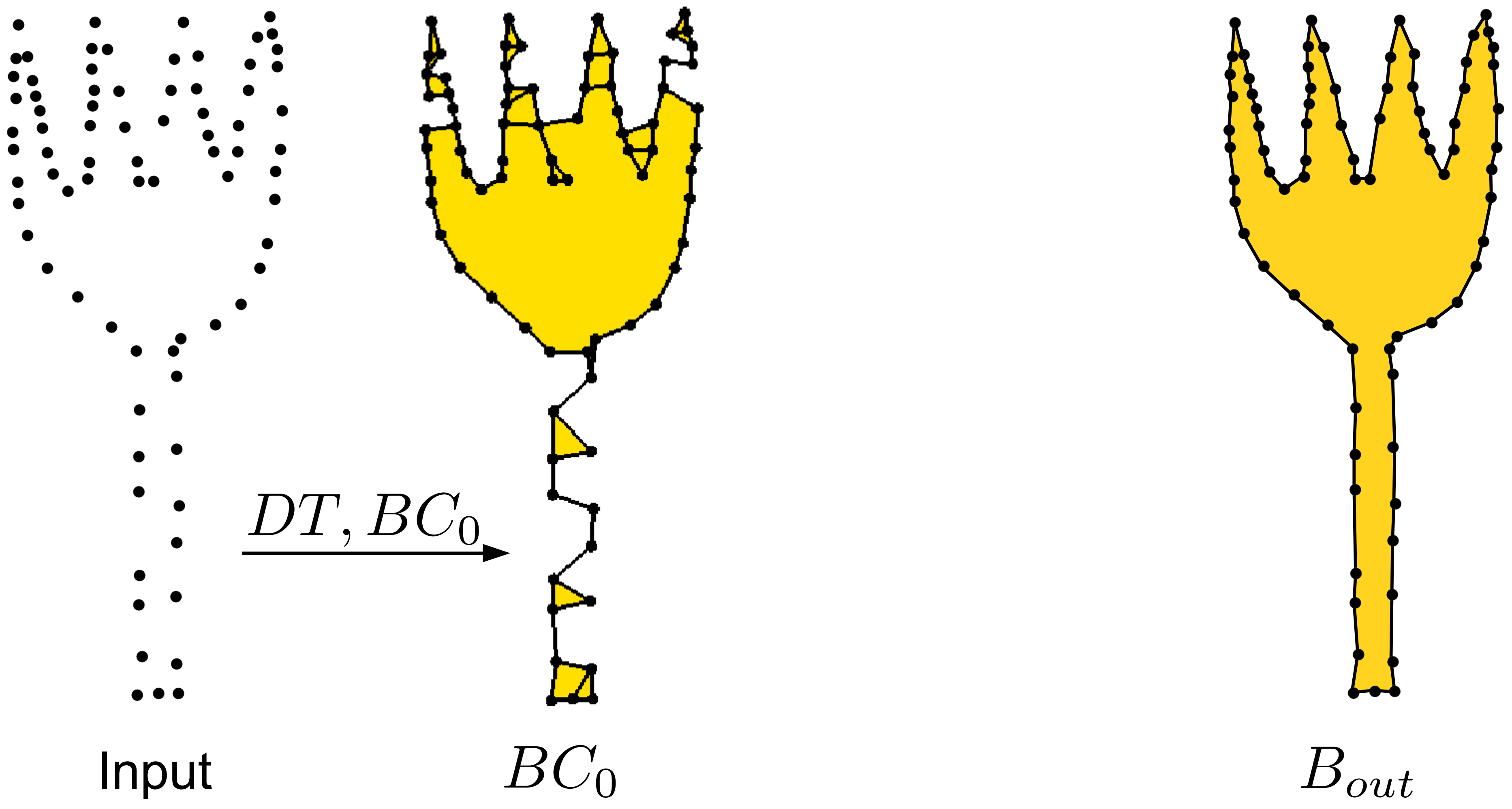
Overview Of Our **'Connect2D'** Algorithm


Input

$\xrightarrow{DT, BC_0}$


B_{out}

Overview Of Our **'Connect2D'** Algorithm



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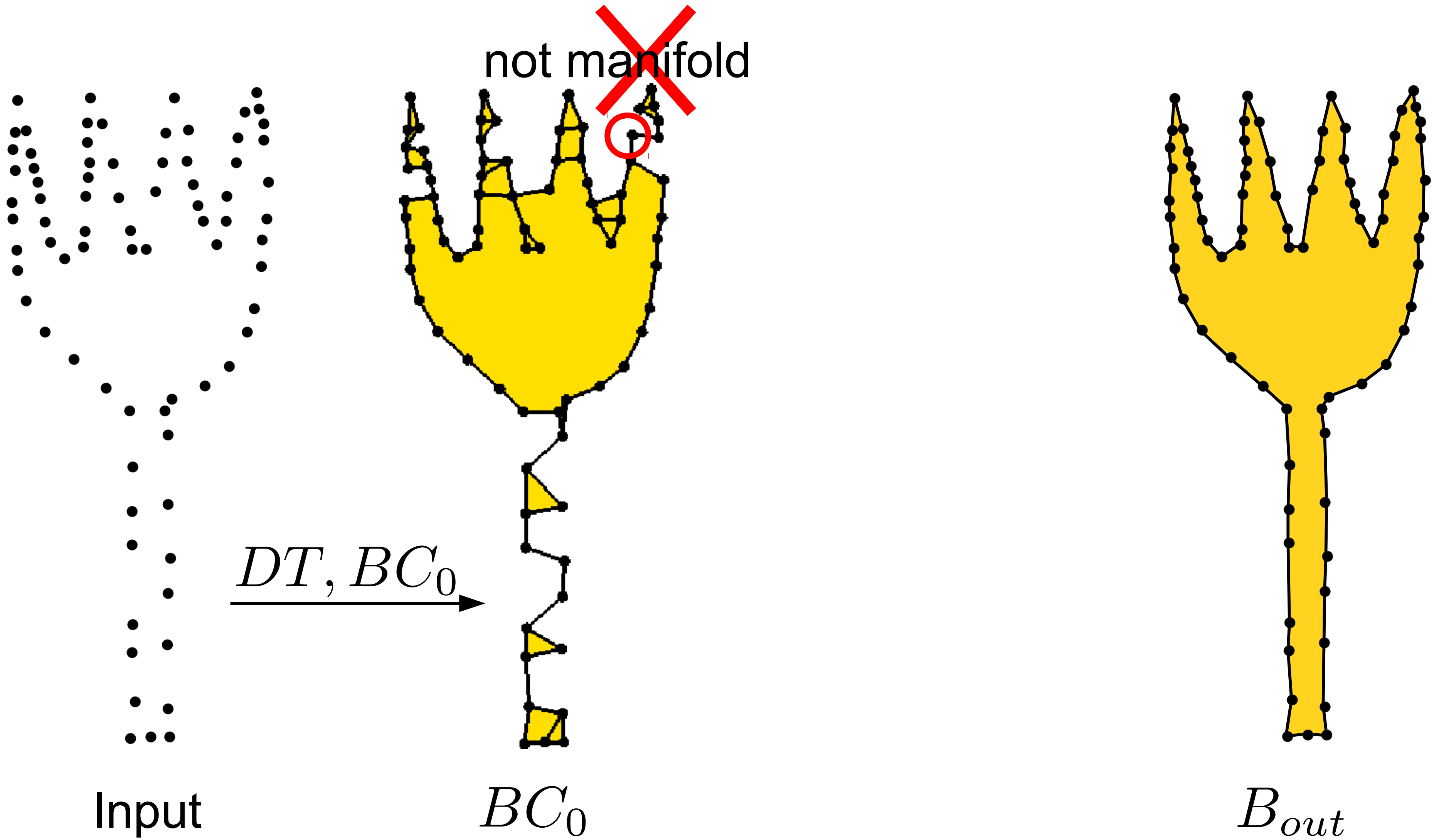

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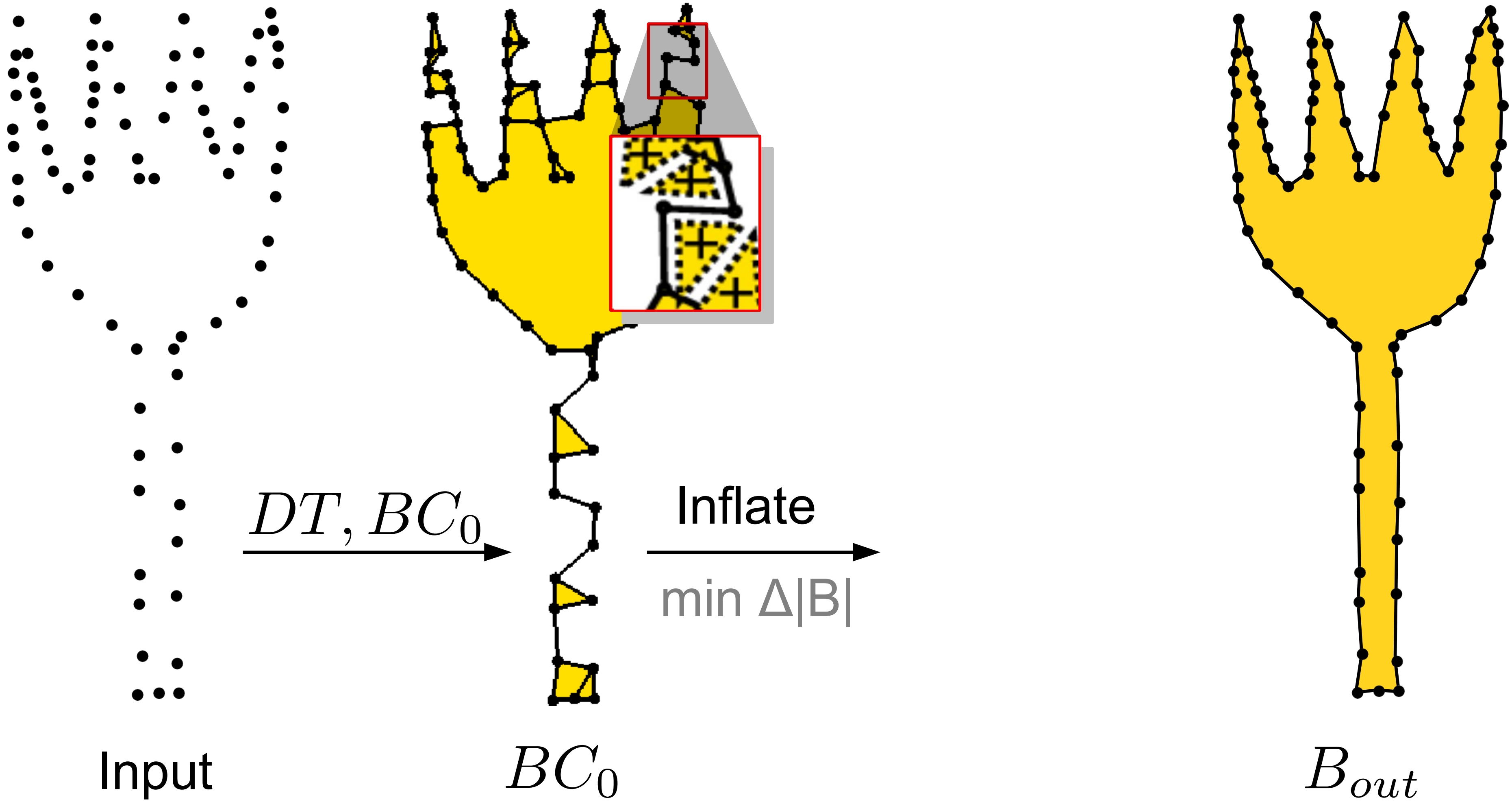
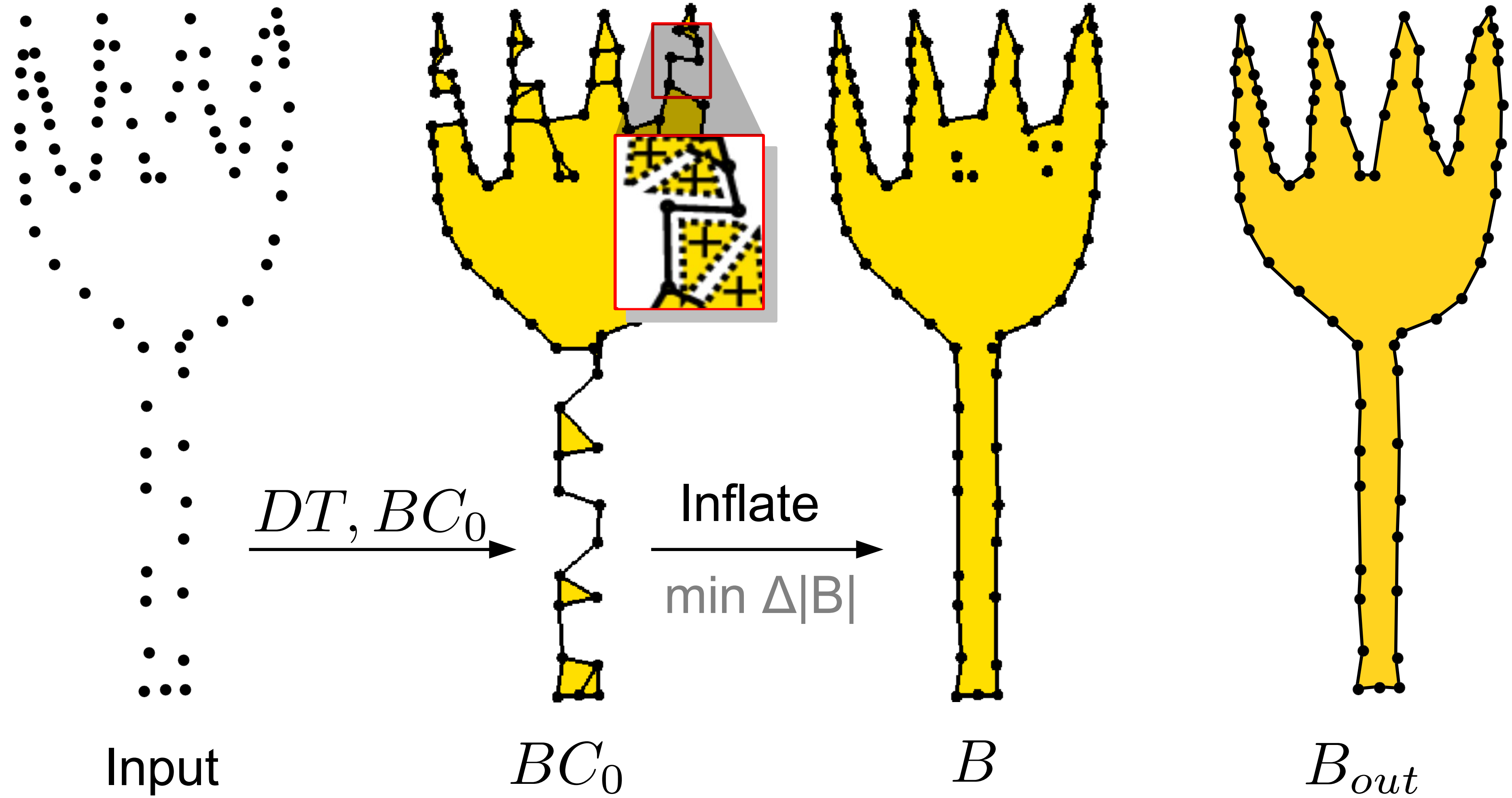
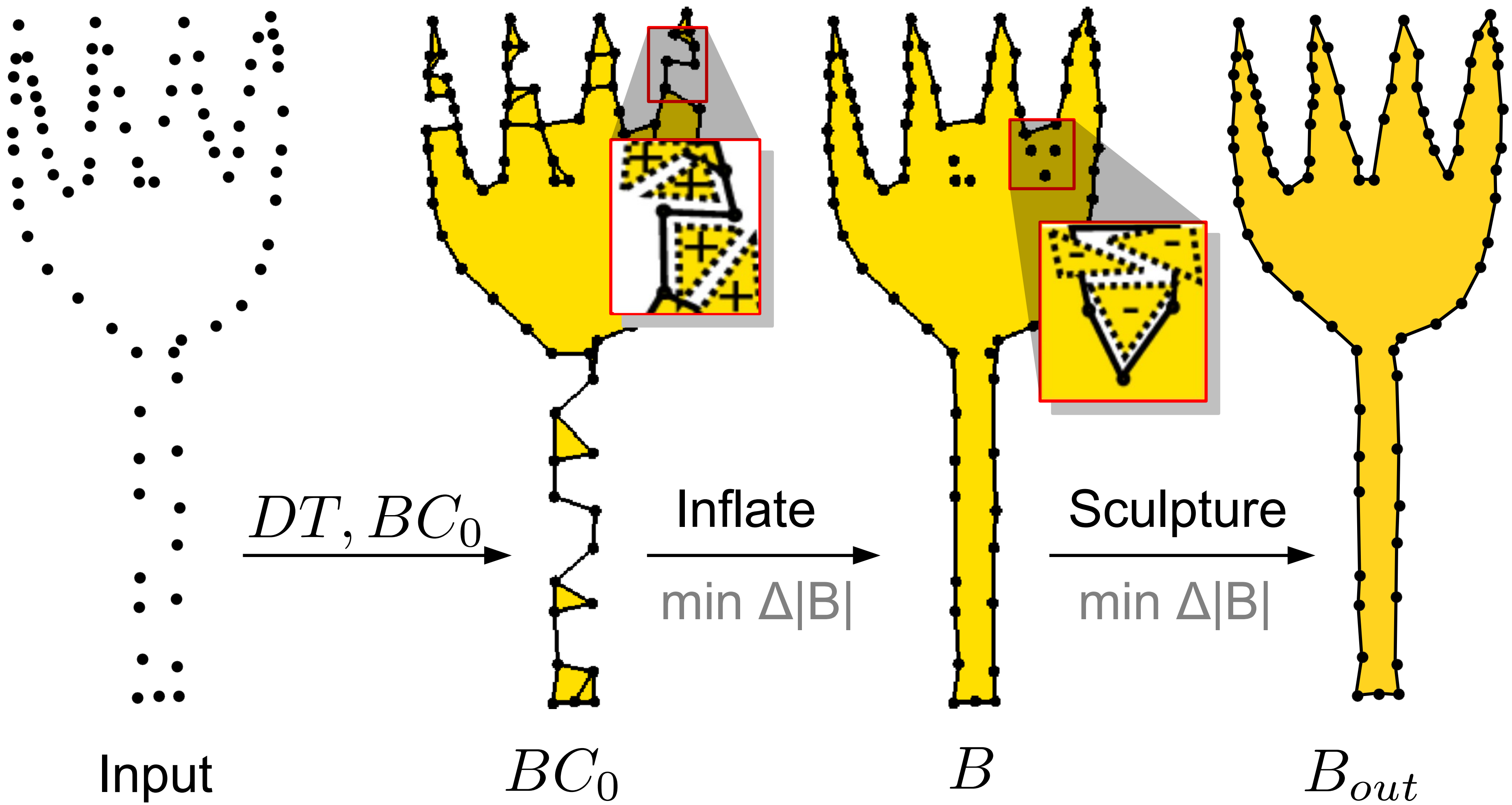


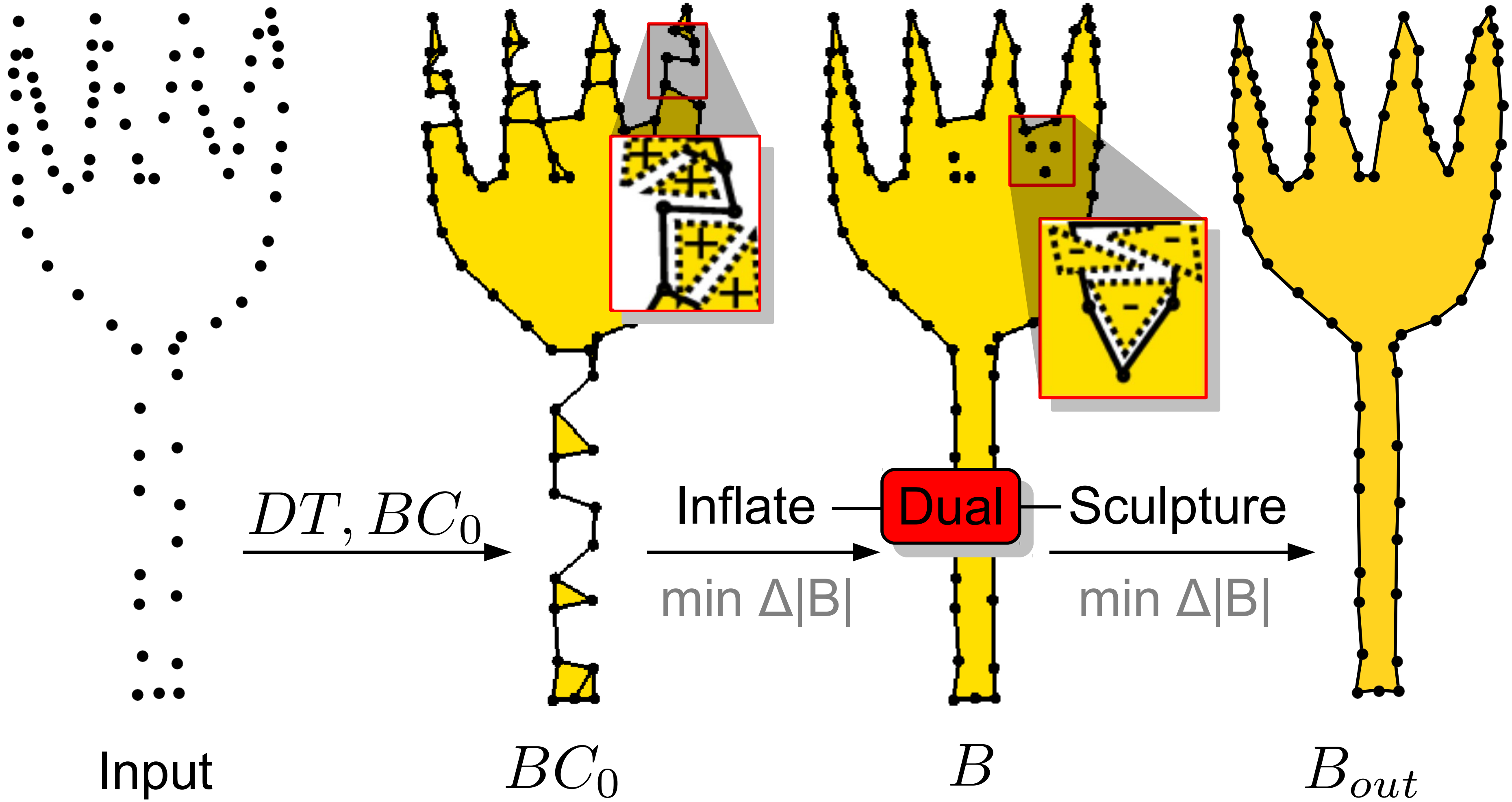
Overview Of Our **'Connect2D'** Algorithm



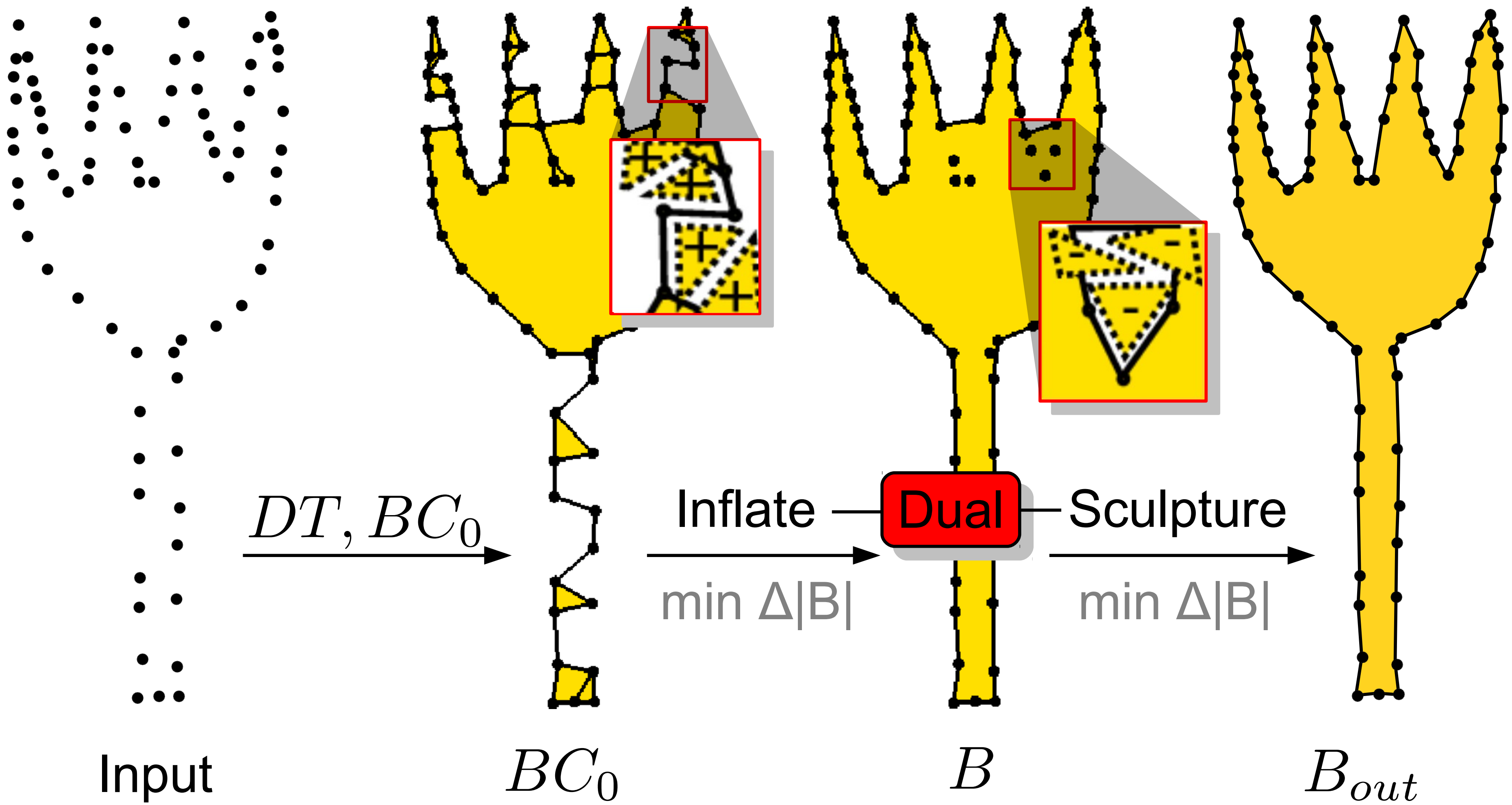
Overview Of Our **'Connect2D'** Algorithm



Overview Of Our 'Connect2D' Algorithm



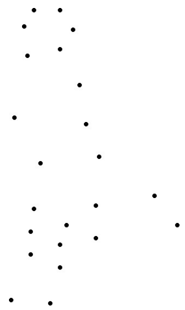
Overview Of Our **'Connect2D'** Algorithm

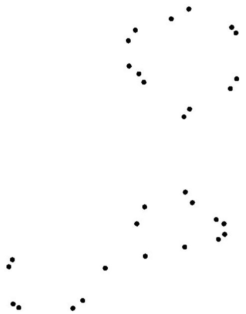


Theorem 1: Our algorithm constructs B_{out} in **O(n log n)** time.

Improved For Sparse Sampling

Points



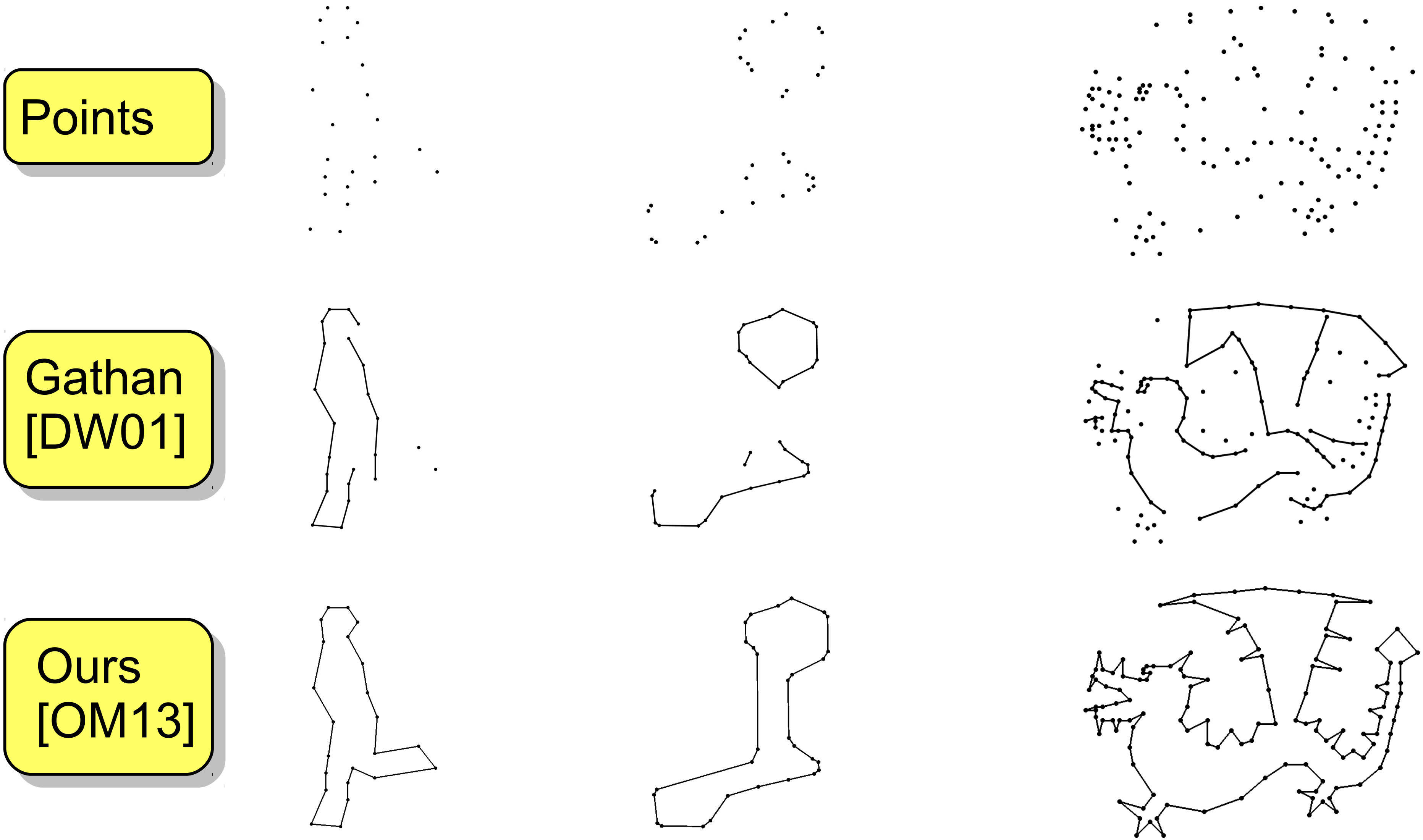


Improved For Sparse Sampling

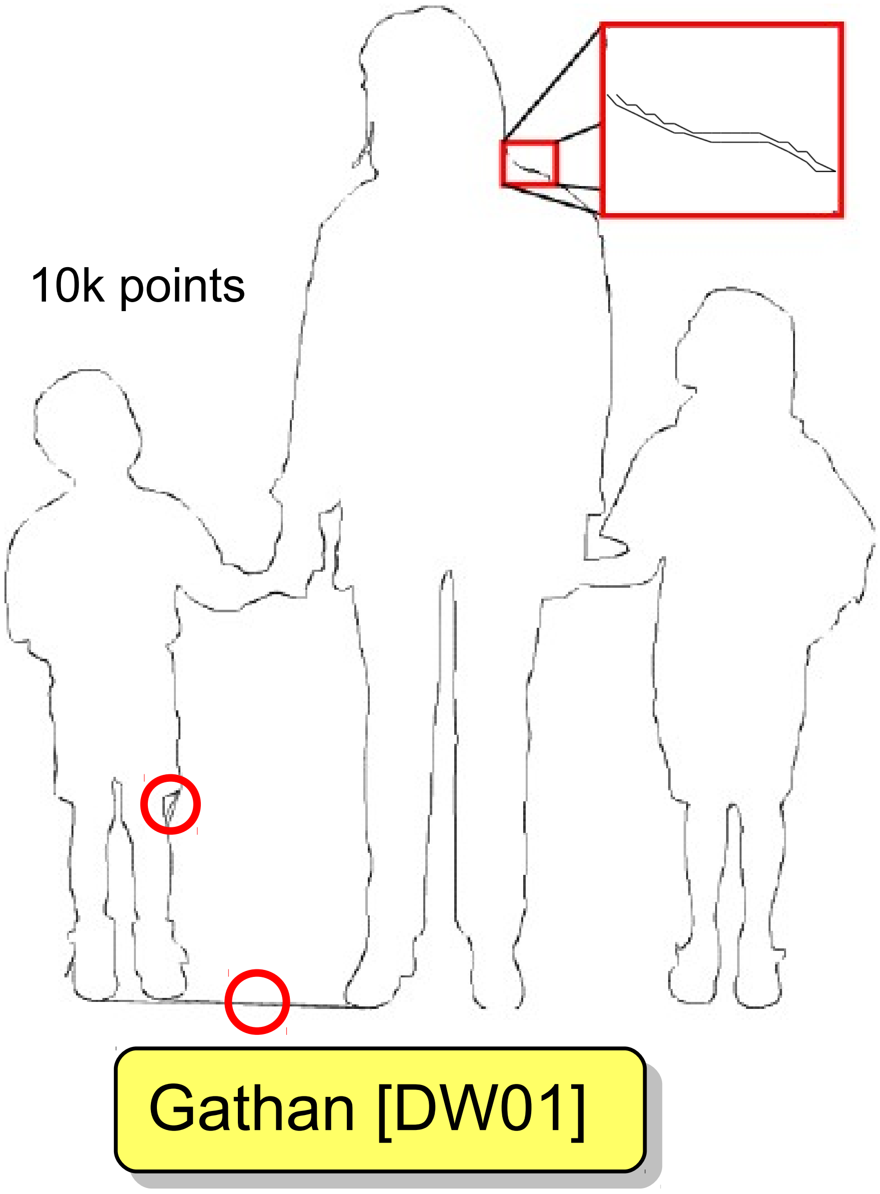
Points

Gathan [DW01]

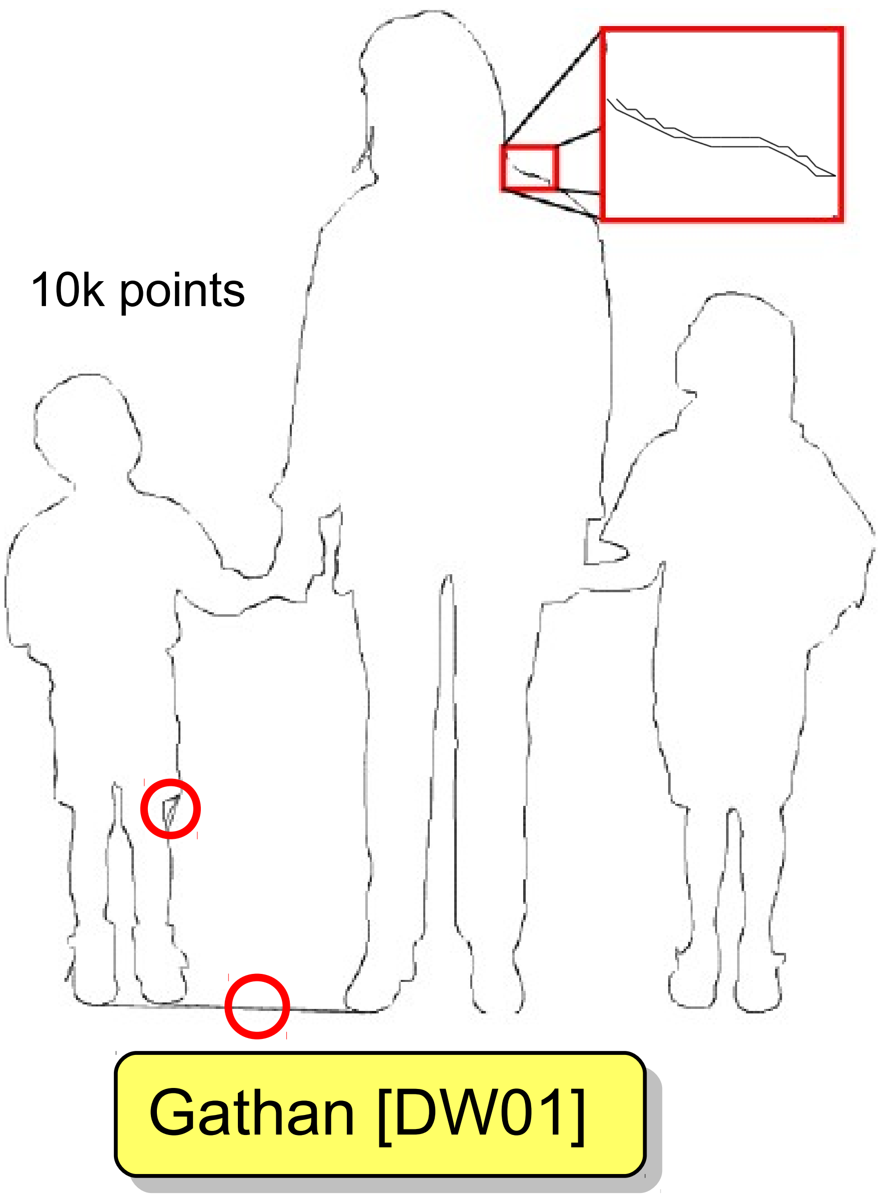
Improved For Sparse Sampling

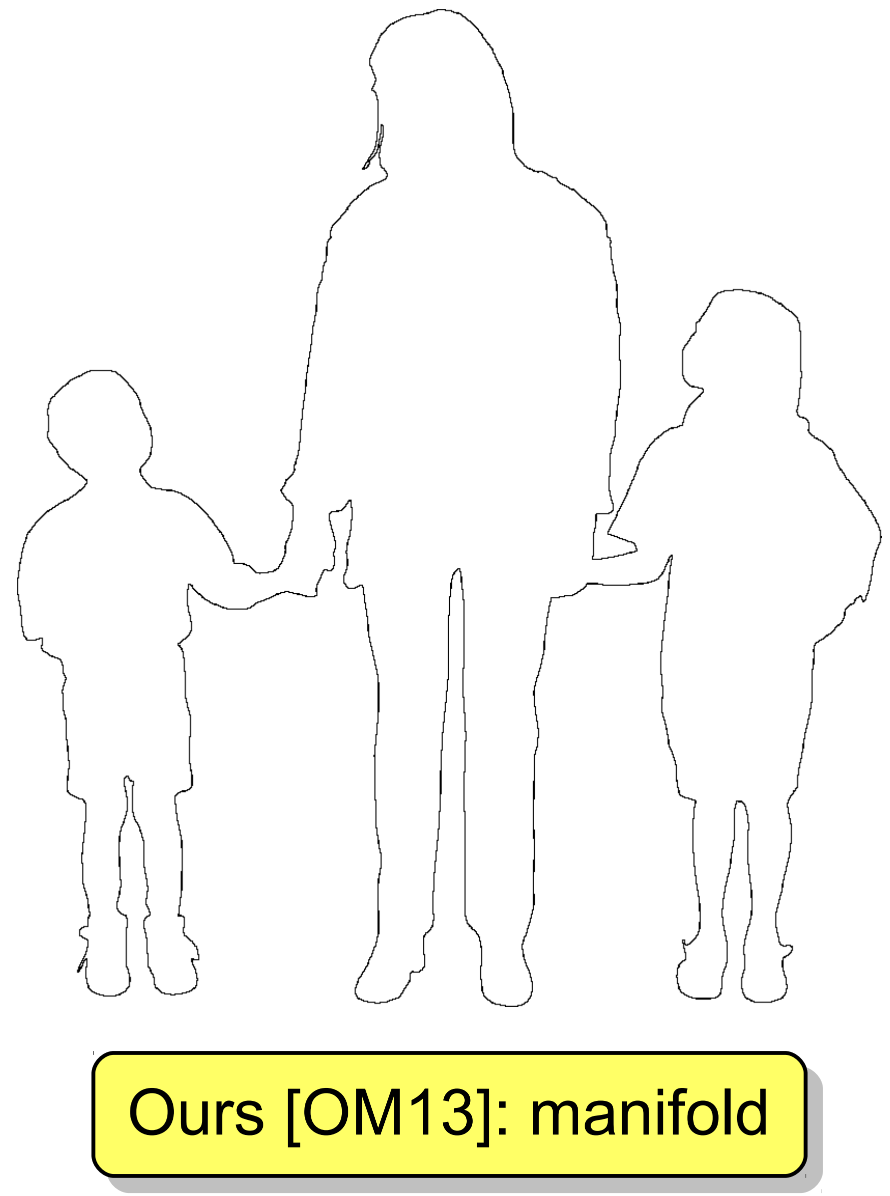


Large Sets: Easy



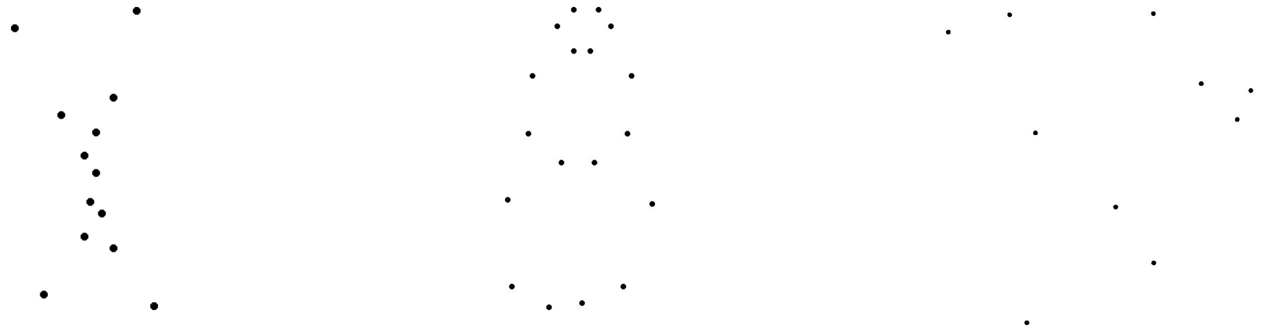
Large Sets: Easy





Challenge: Extremely Sparse Sampling

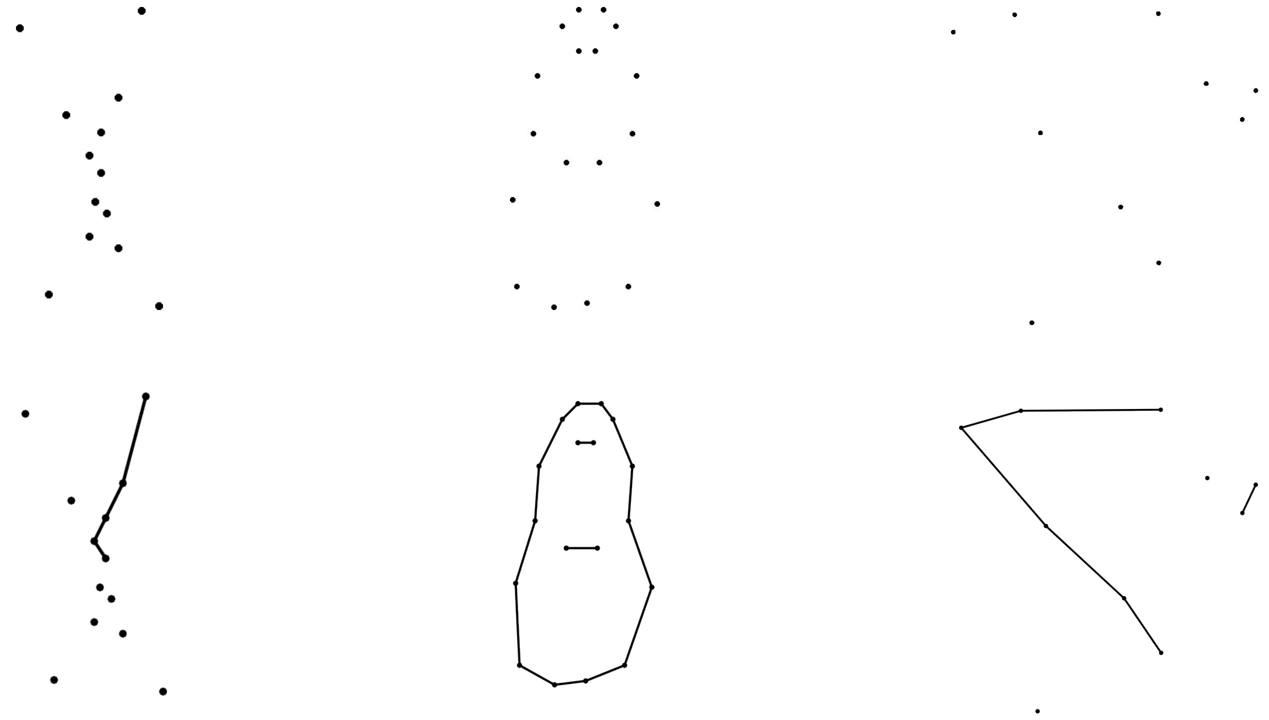
Points



Challenge: Extremely Sparse Sampling

Points

Gathan [DW01]

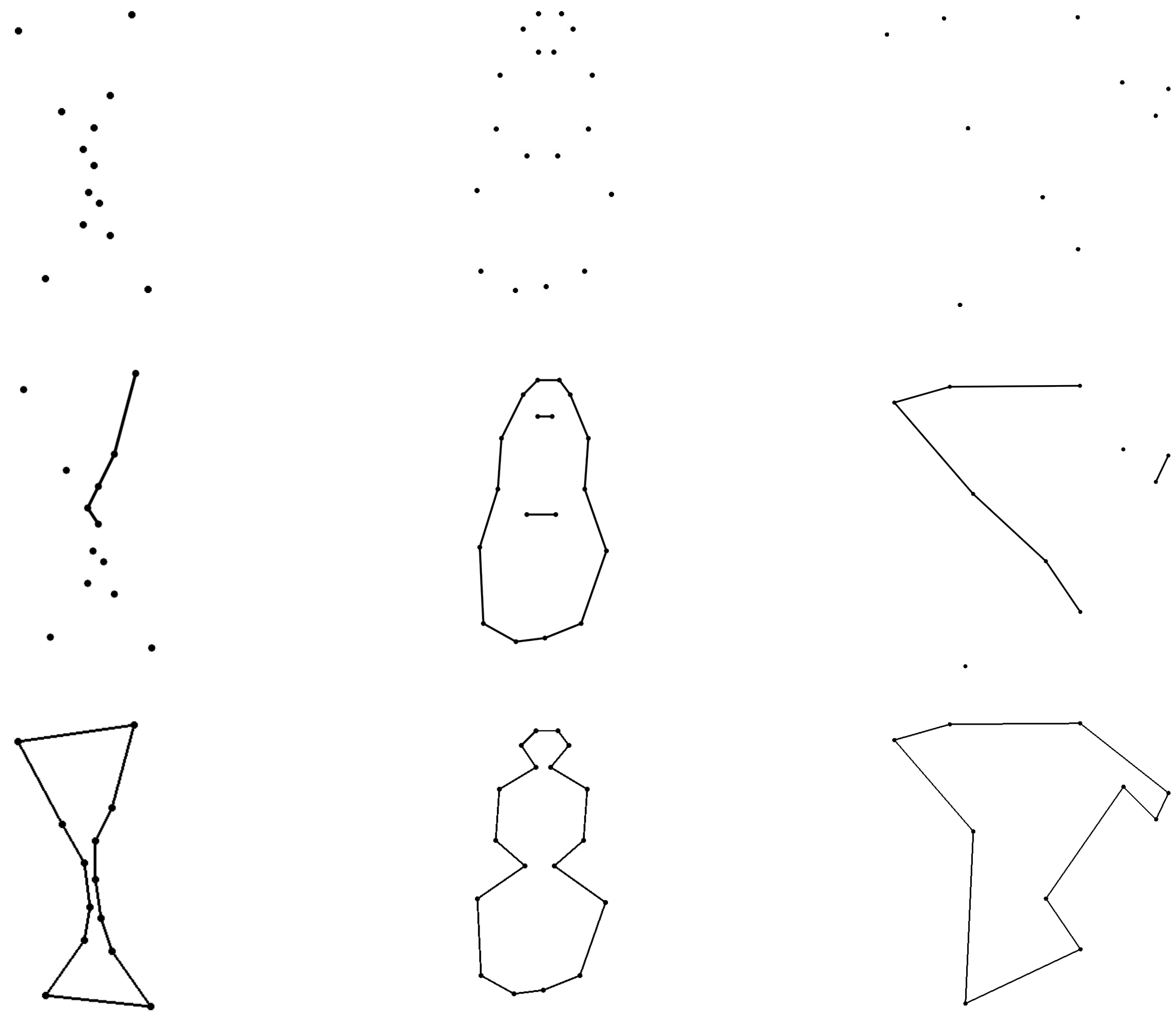


Challenge: Extremely Sparse Sampling

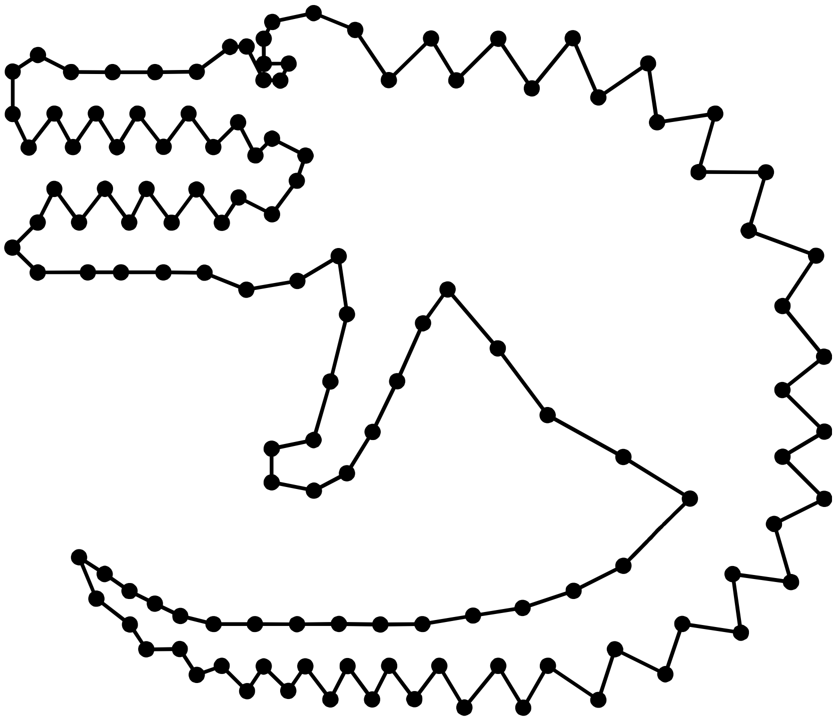
Points

Gathan [DW01]

Ours

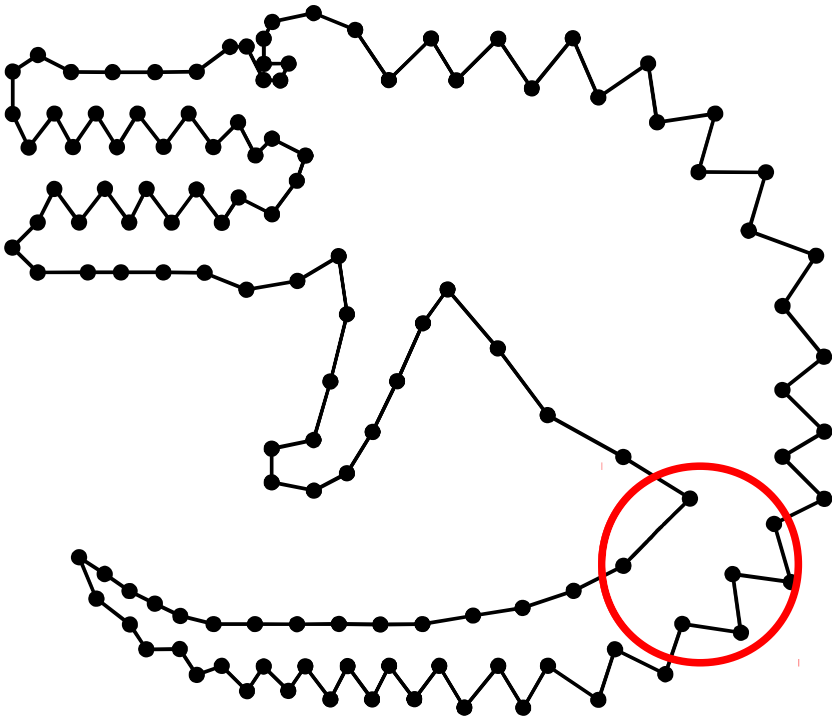


Failure Cases

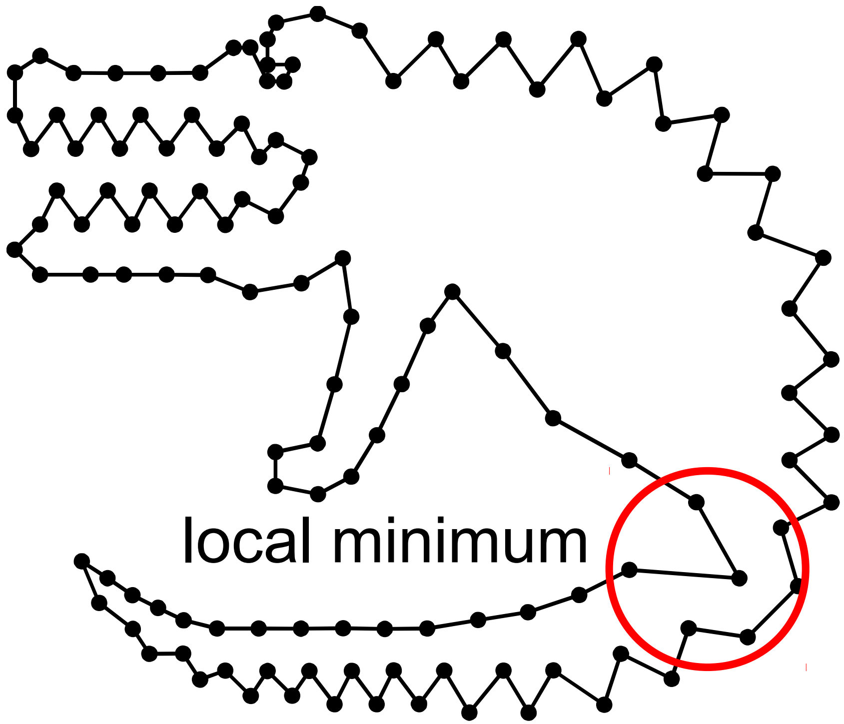


[OM11]

Failure Cases



[OM11]



Ours [OM13]

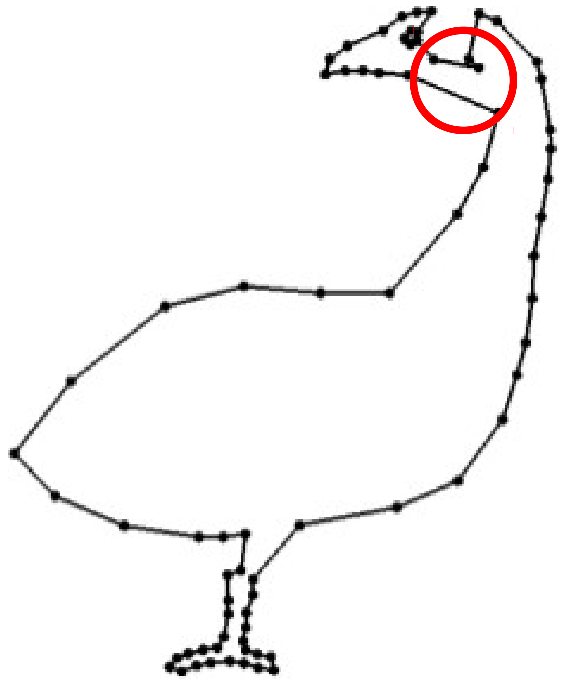
Not Bmin? Insert Points, Manually

Not a solid

Not Bmin? Insert Points, Manually

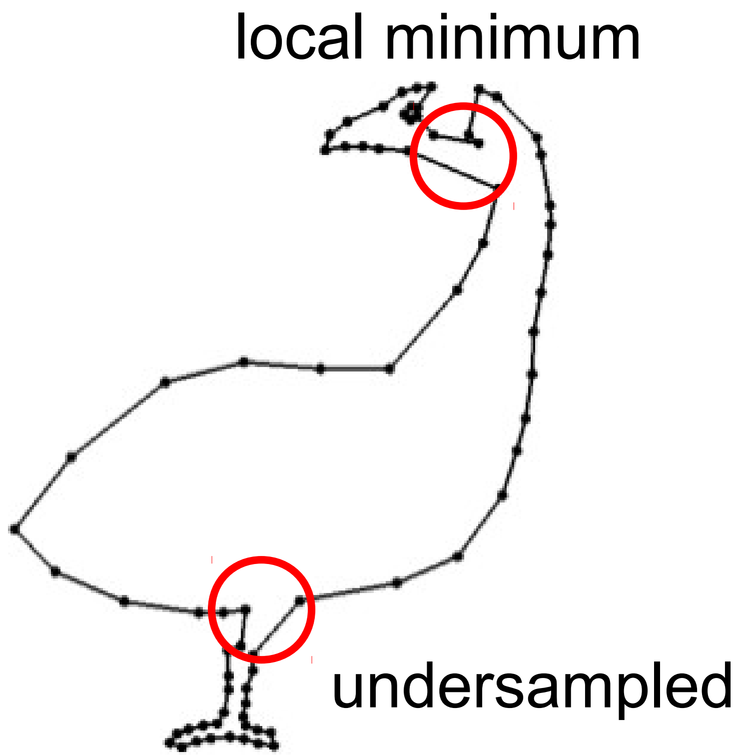
Not a solid

local minimum



$$B_{out} \neq B_{min}$$

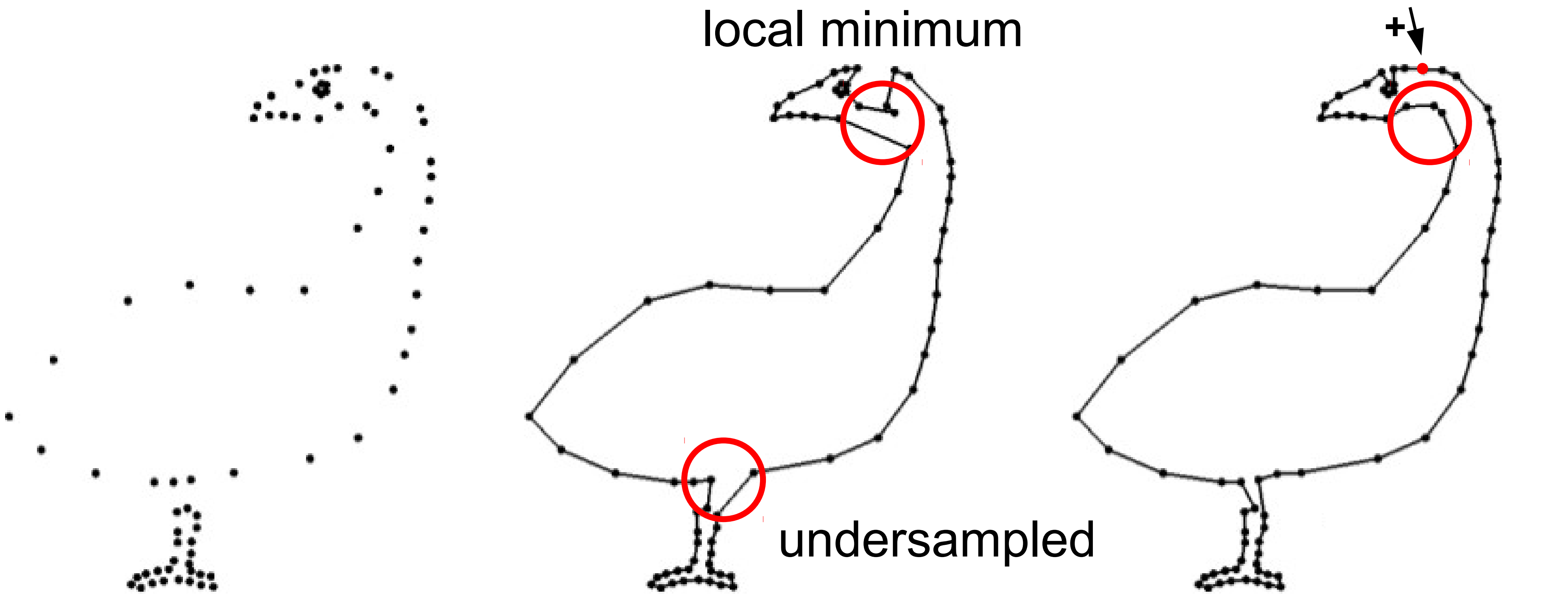
Not Bmin? Insert Points, Manually



Not a solid

$$B_{out} \neq B_{min}$$

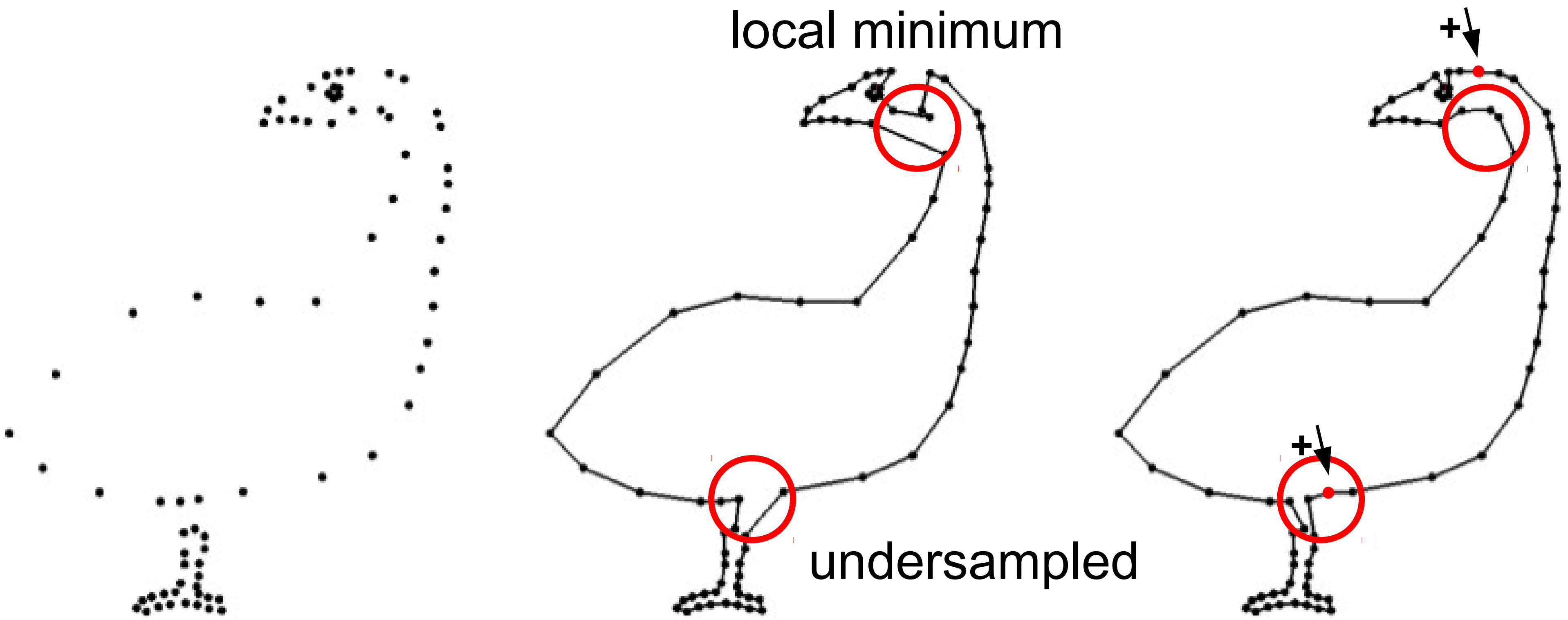
Not Bmin? Insert Points, Manually



Not a solid

$$B_{out} \neq B_{min}$$

Not Bmin? Insert Points, Manually

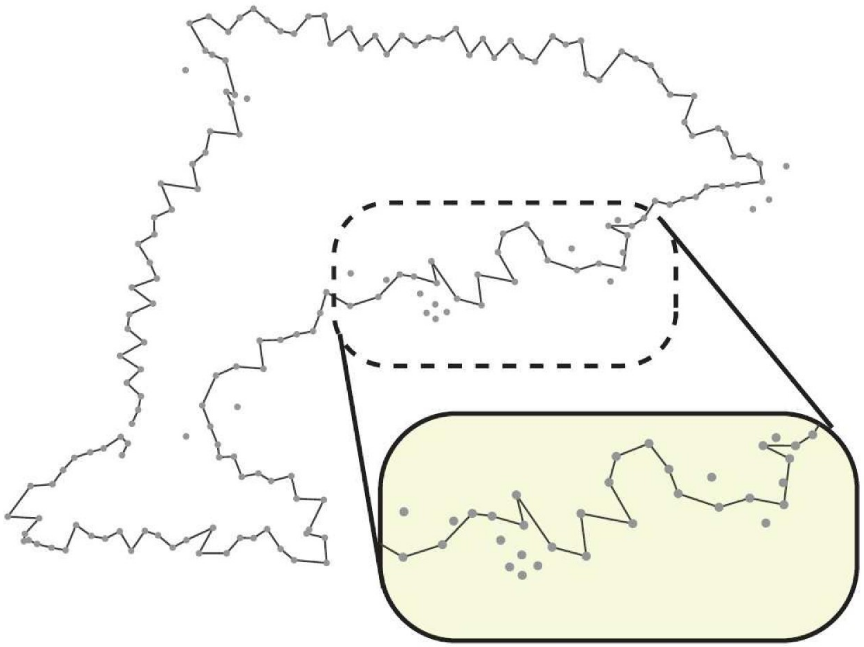


Not a solid

$B_{out} \neq B_{min}$

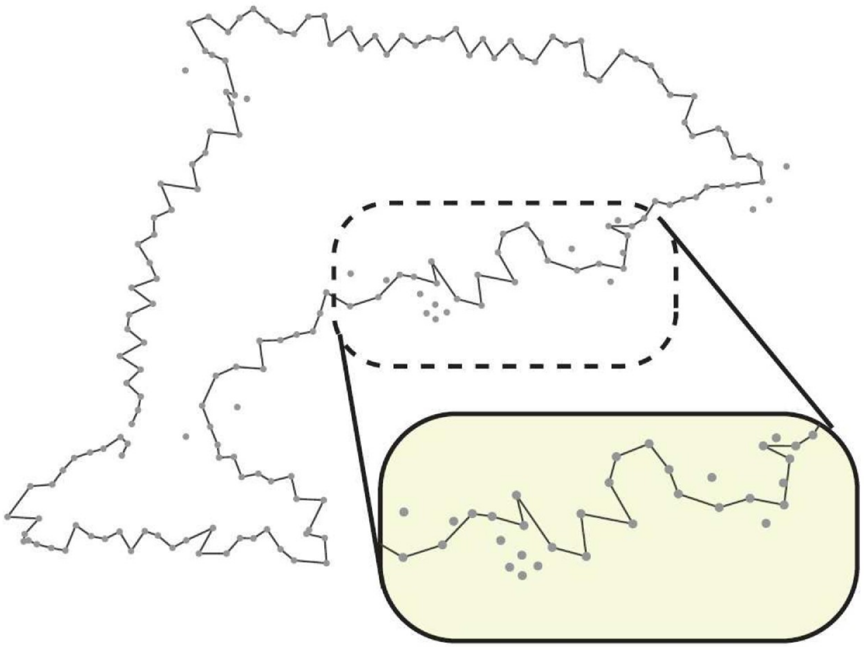
B_{min}

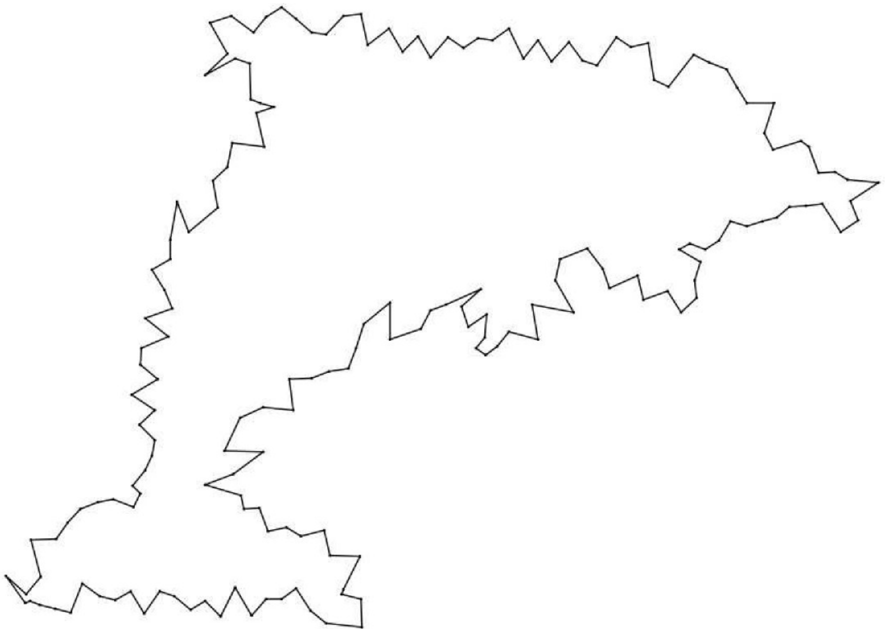
Robust To Noise



[MTSM10]

Robust To Noise

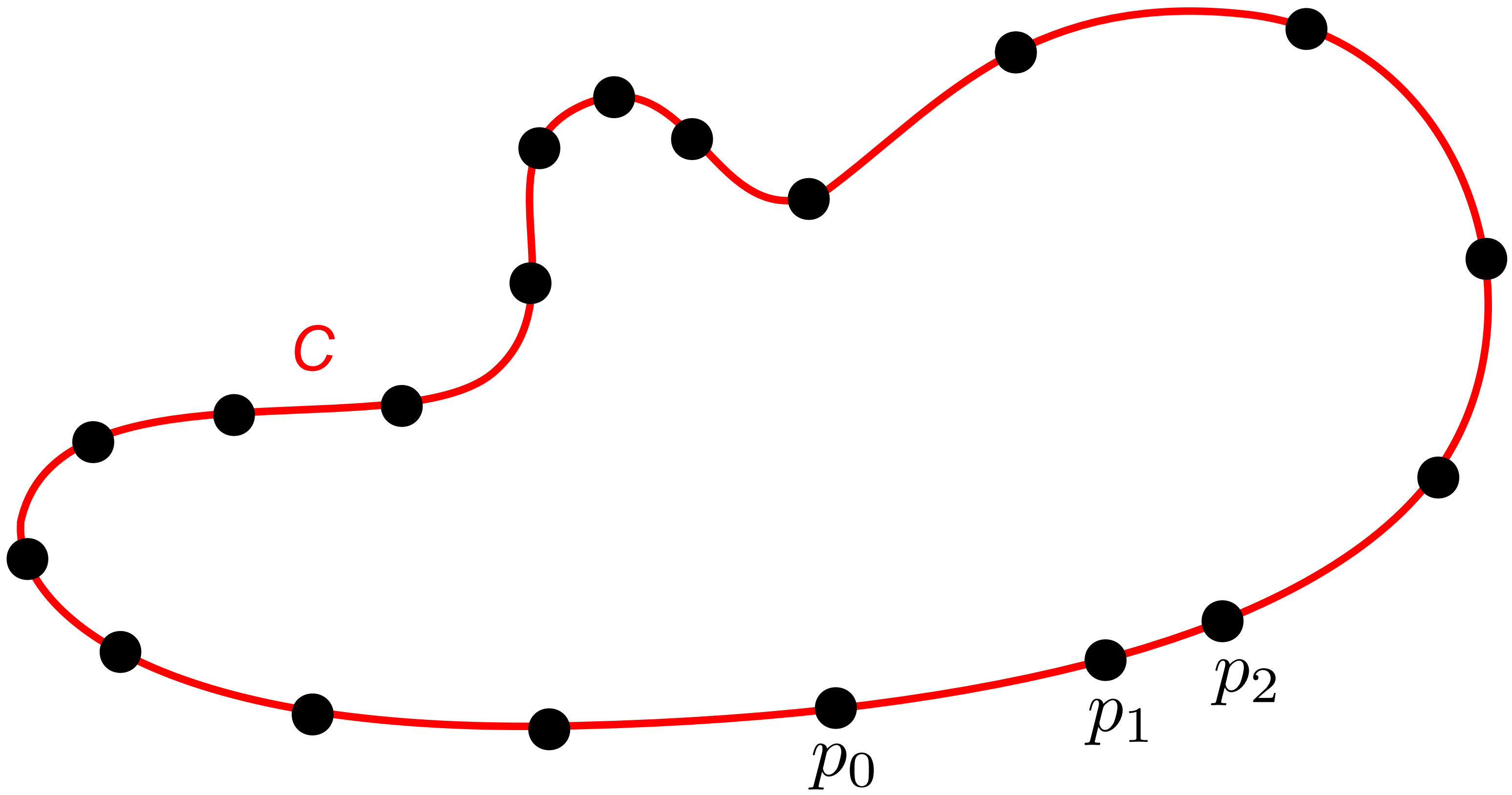




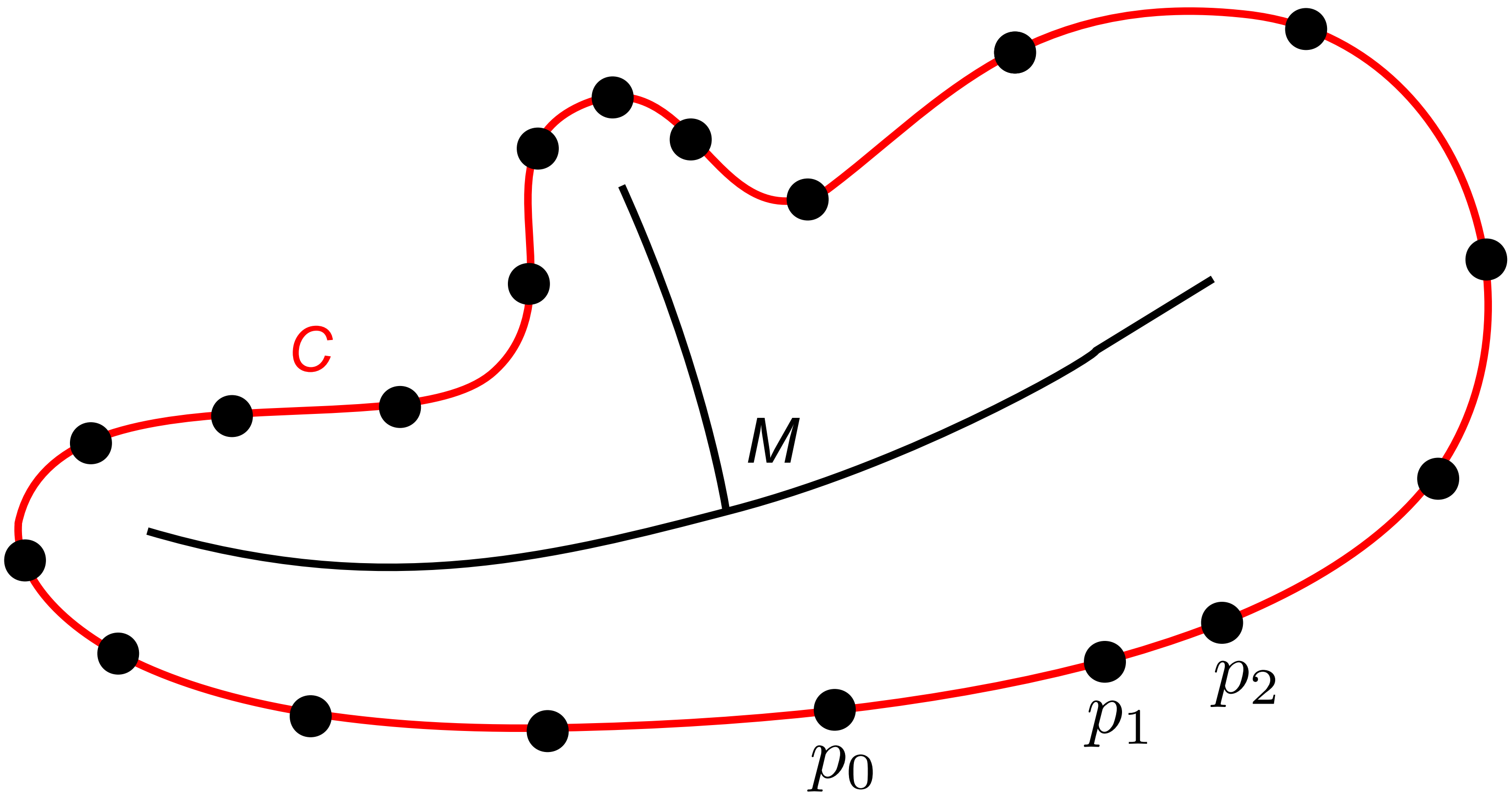
[MTSM10]

Ours [OM13]: manifold+interpolating

Class Of Point Sets

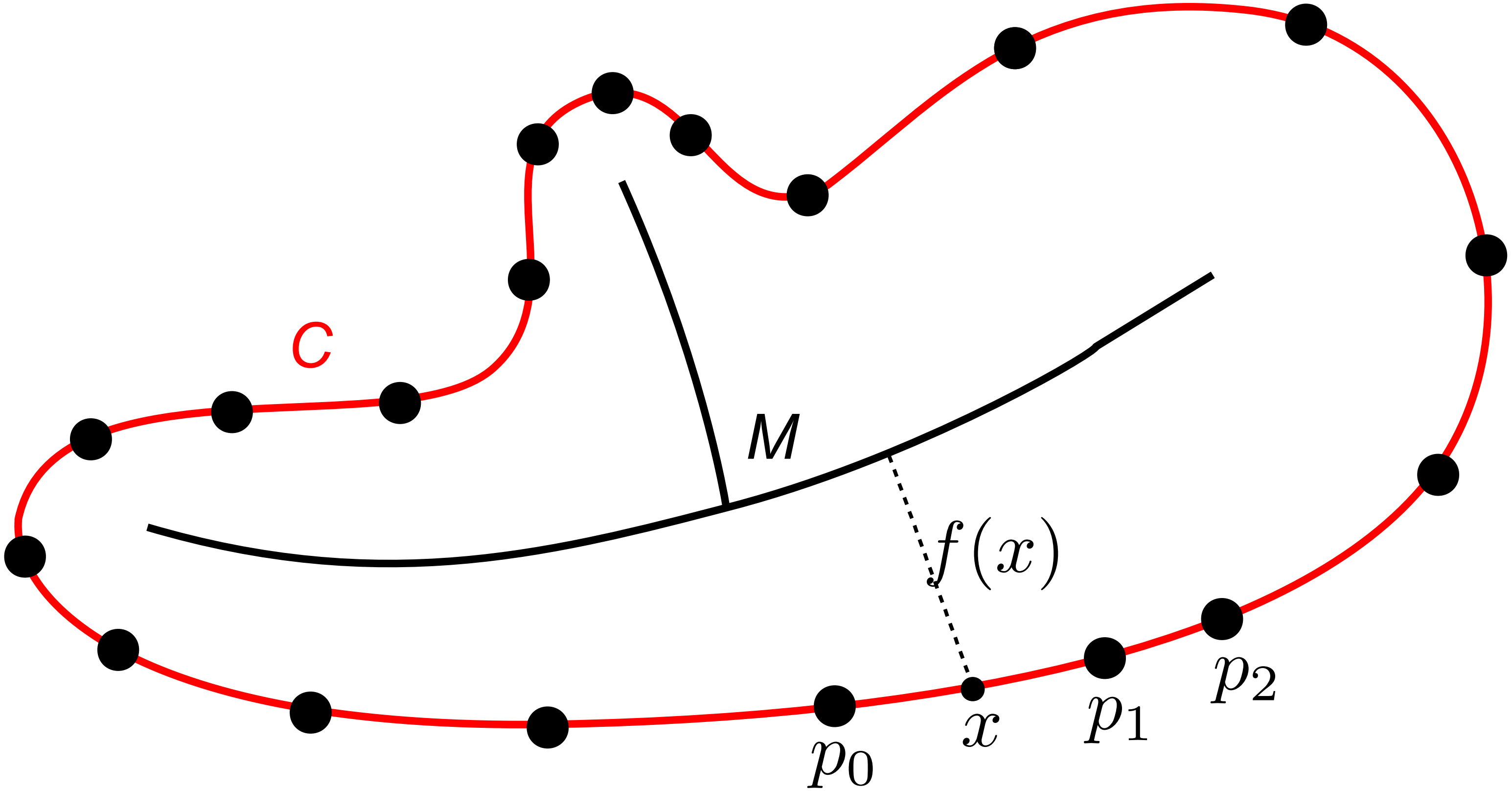


Class Of Point Sets



Medial axis M

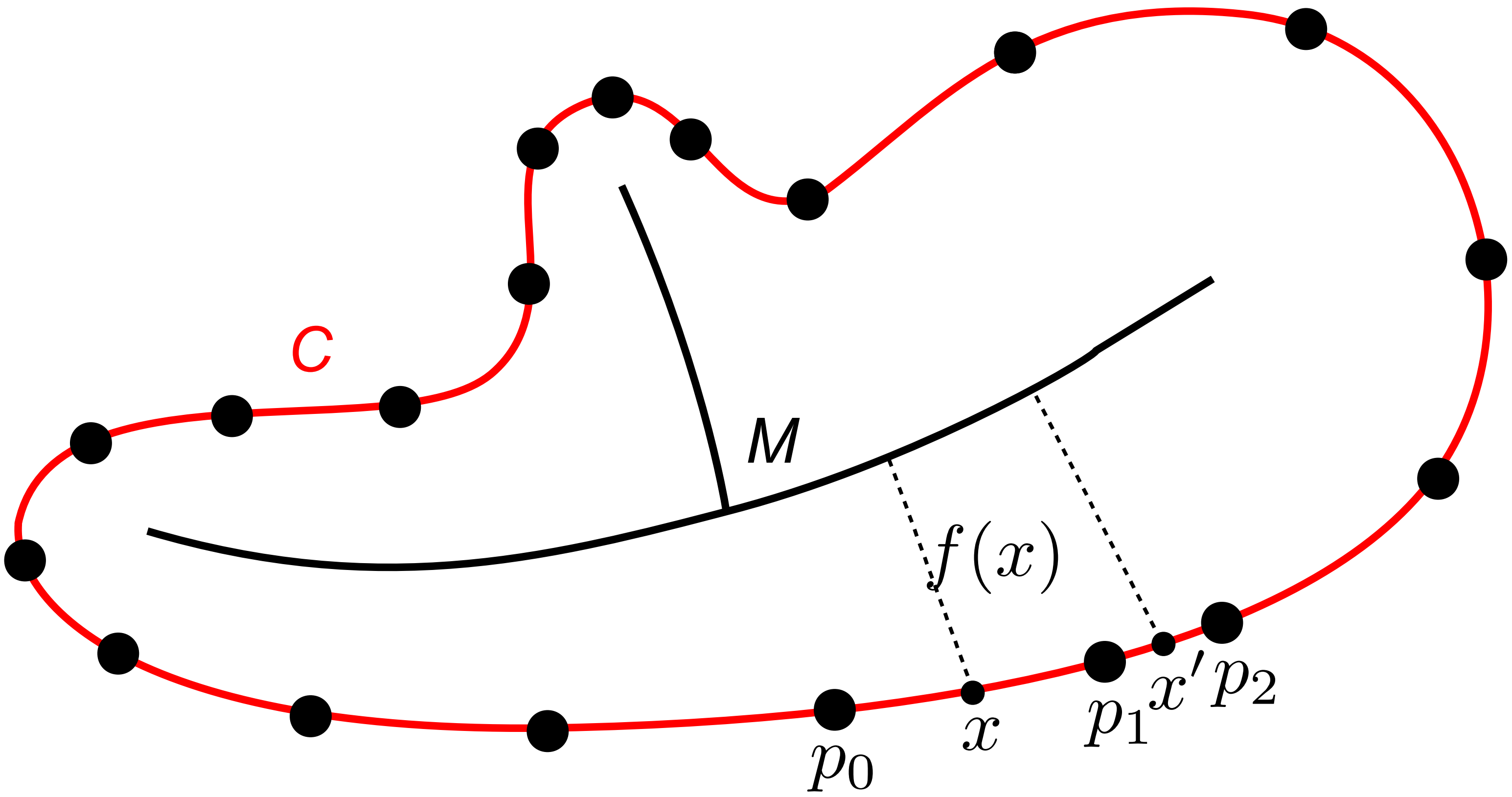
Class Of Point Sets



Medial axis M

Local feature size f(x)=||x, M||

Class Of Point Sets



Medial axis M

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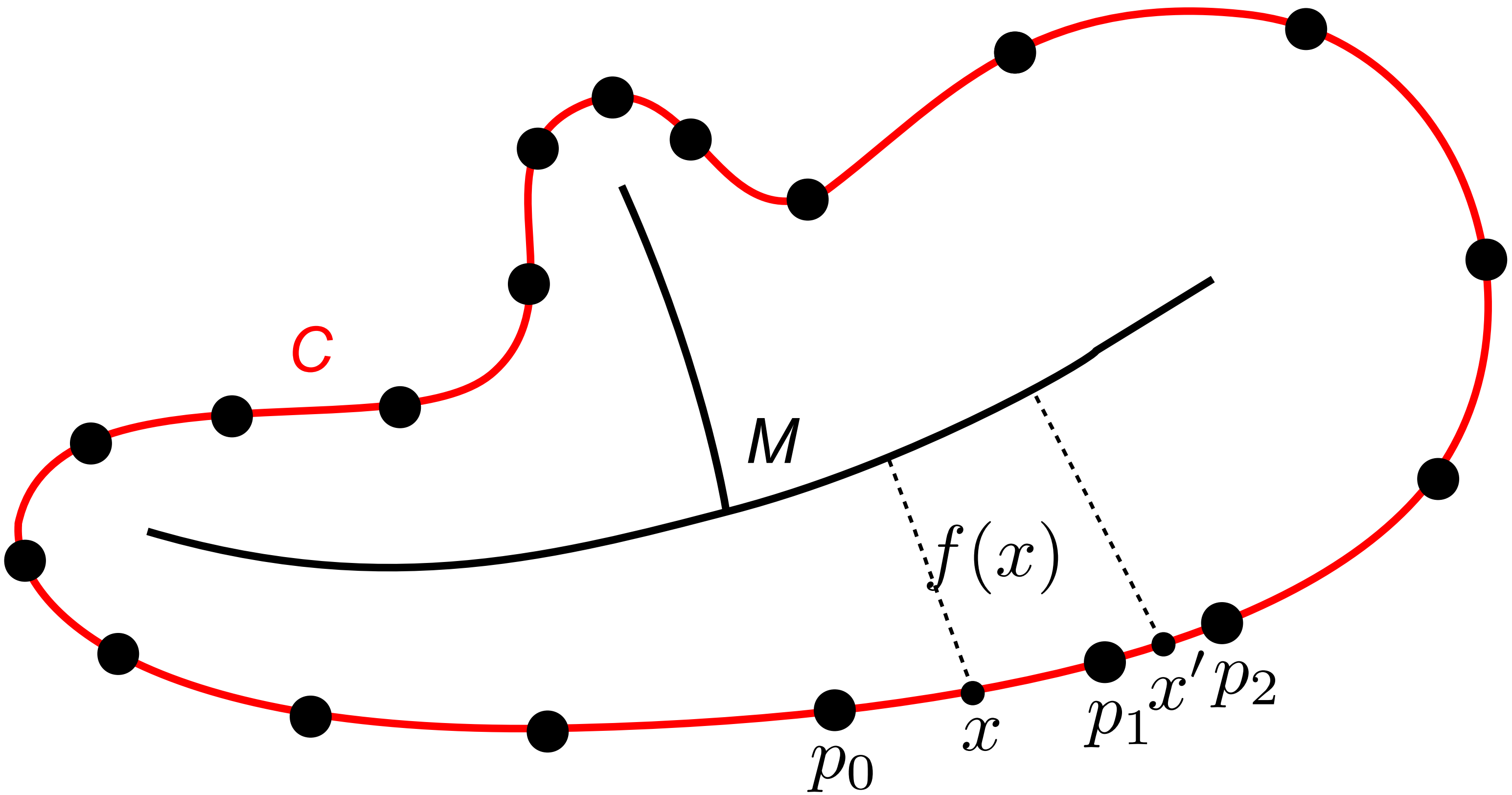
Class Of Point Sets

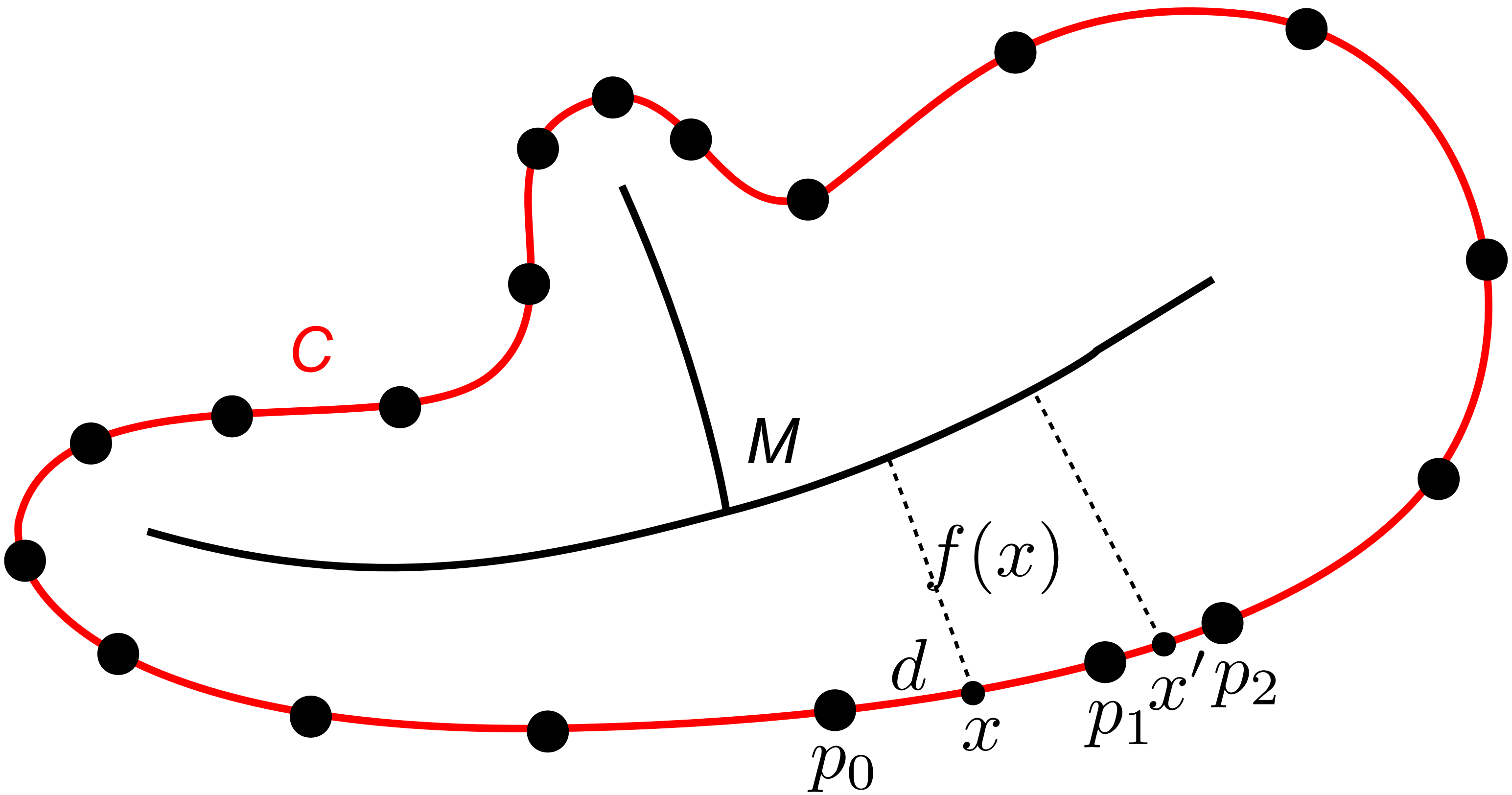


Medial axis M

Local feature size f(x)=||x, M||

ε-sampling: $\forall x \in C, \exists p \in P, \;\; ||x, p|| \leq \epsilon f(x)$

Class Of Point Sets



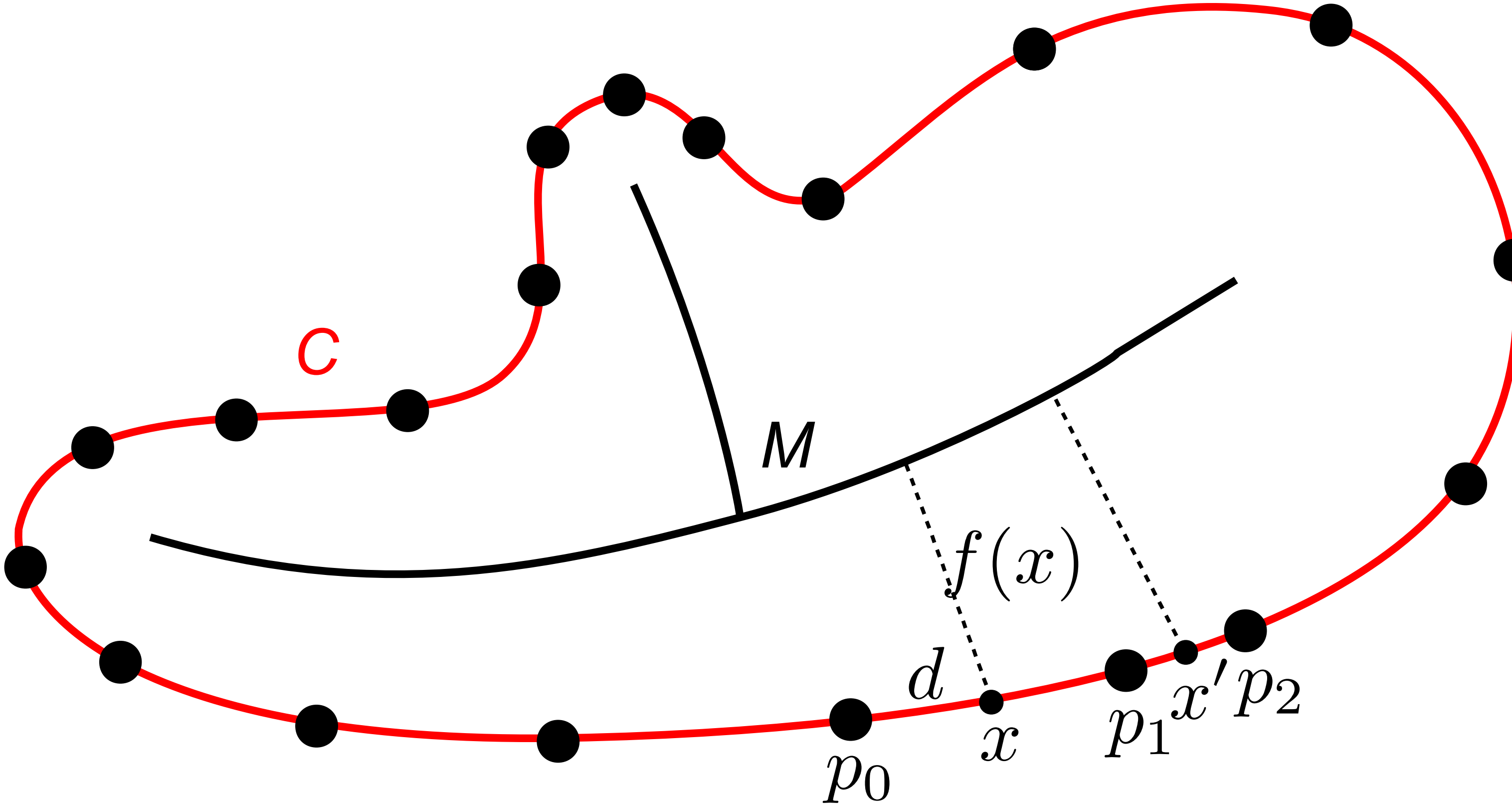
$$\epsilon(x) = \frac{d}{f(x)}$$

Medial axis M

Local feature size f(x)=||x, M||

ε-sampling: $\forall x \in C, \exists p \in P, \; ||x, p|| \leq \epsilon f(x)$

Class Of Point Sets



$$u = \frac{e_{\text{longer}}}{e_{\text{shorter}}}$$

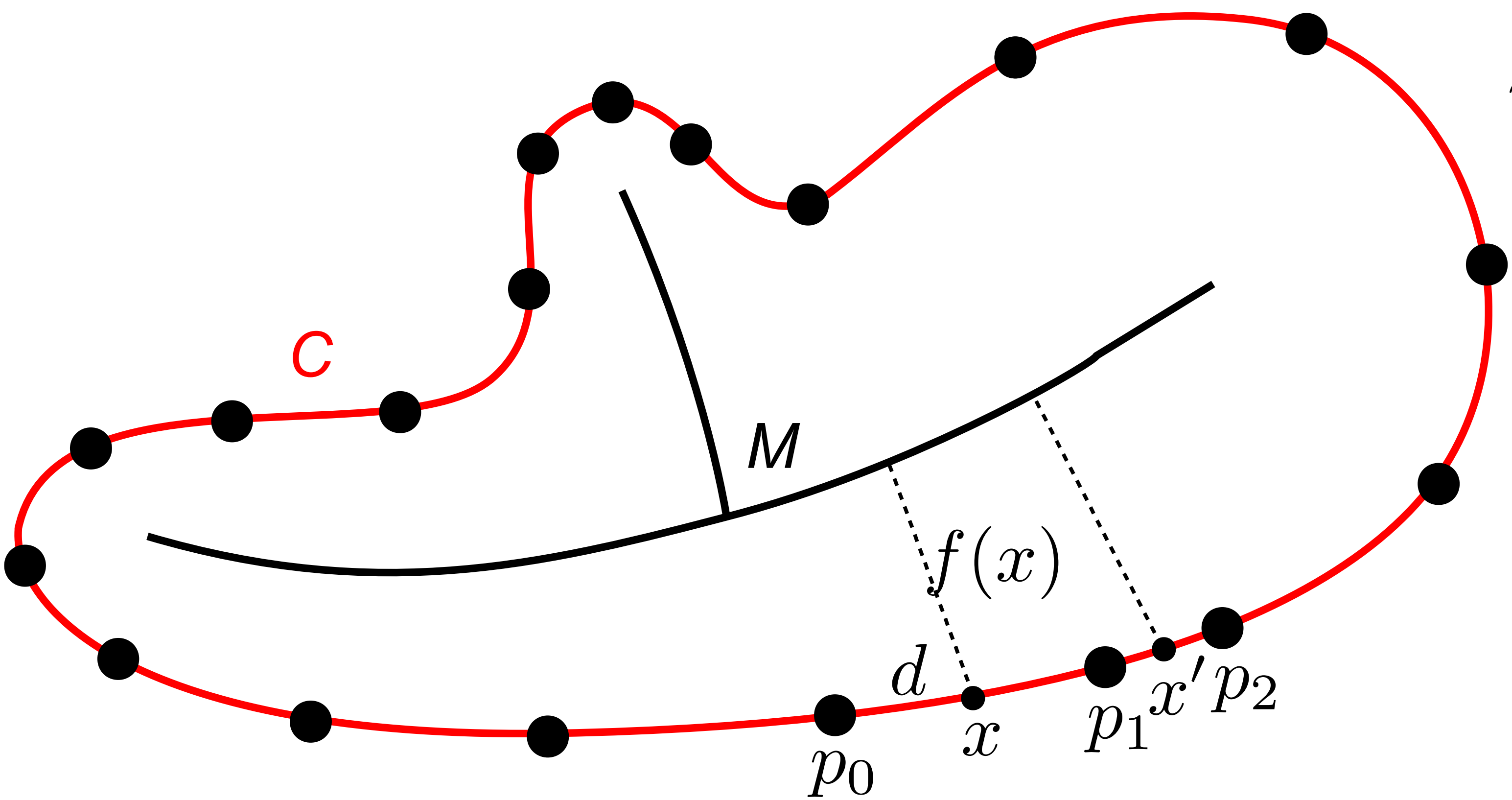
$$\epsilon(x) = \frac{d}{f(x)}$$

Medial axis M

Local feature size f(x)=||x, M||

ε-sampling: $\forall x \in C, \exists p \in P, \; ||x, p|| \leq \epsilon f(x)$

Class Of Point Sets



$$u = \frac{e_{\text{longer}}}{e_{\text{shorter}}}$$

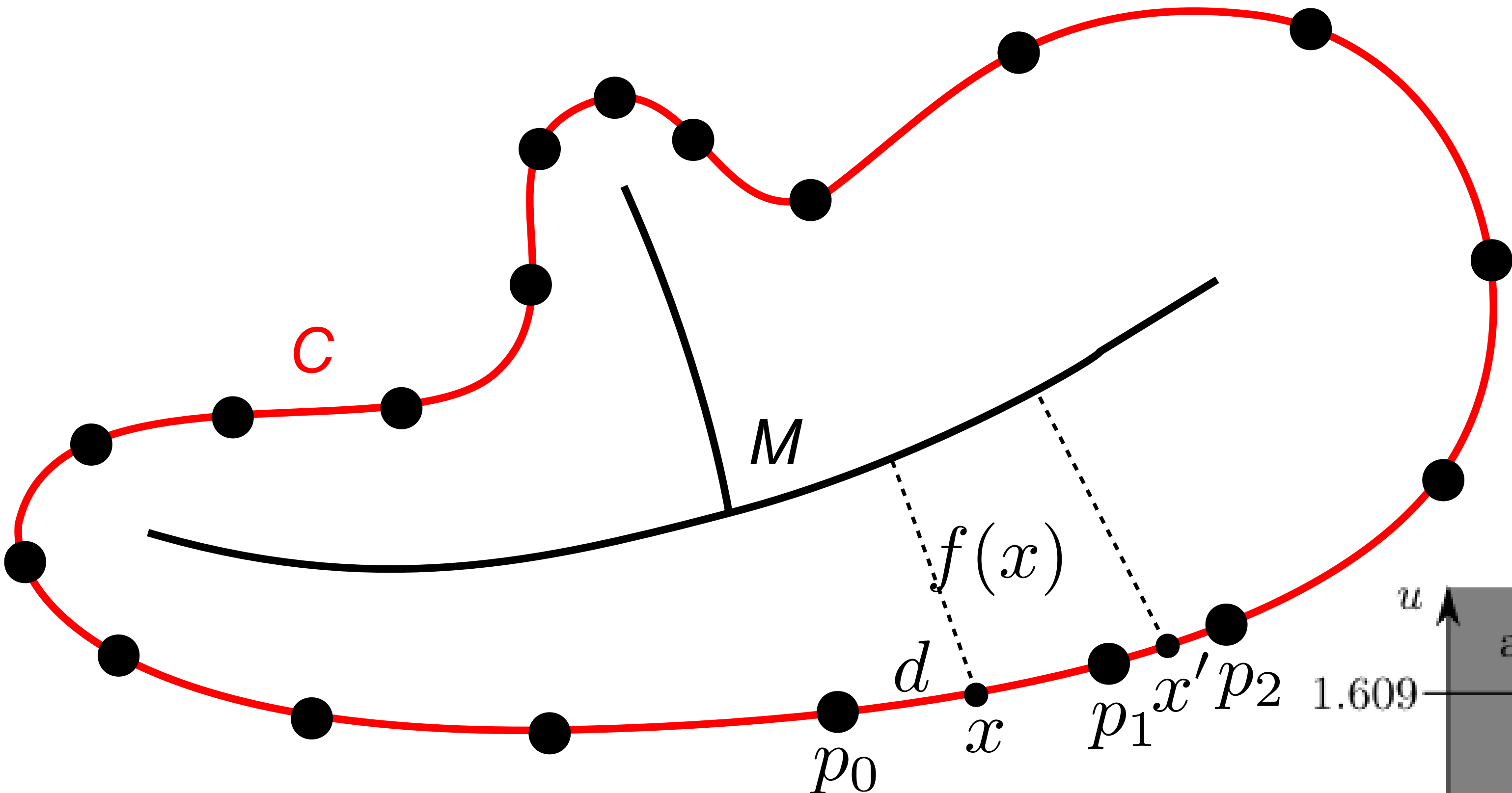
$$\epsilon(x) = \frac{d}{f(x)}$$

Theorem 2

BC₀ reconstructs ε-sampled *C* from *P* with

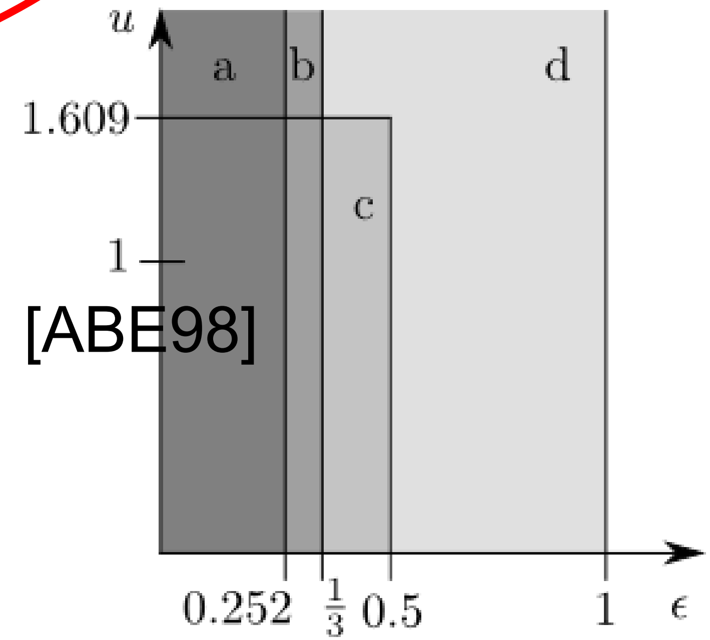
ε < 0.5 and a local non-uniformity *u* < 1.609.

Class Of Point Sets



$$u = \frac{e_{\text{longer}}}{e_{\text{shorter}}}$$

$$\epsilon(x) = \frac{d}{f(x)}$$

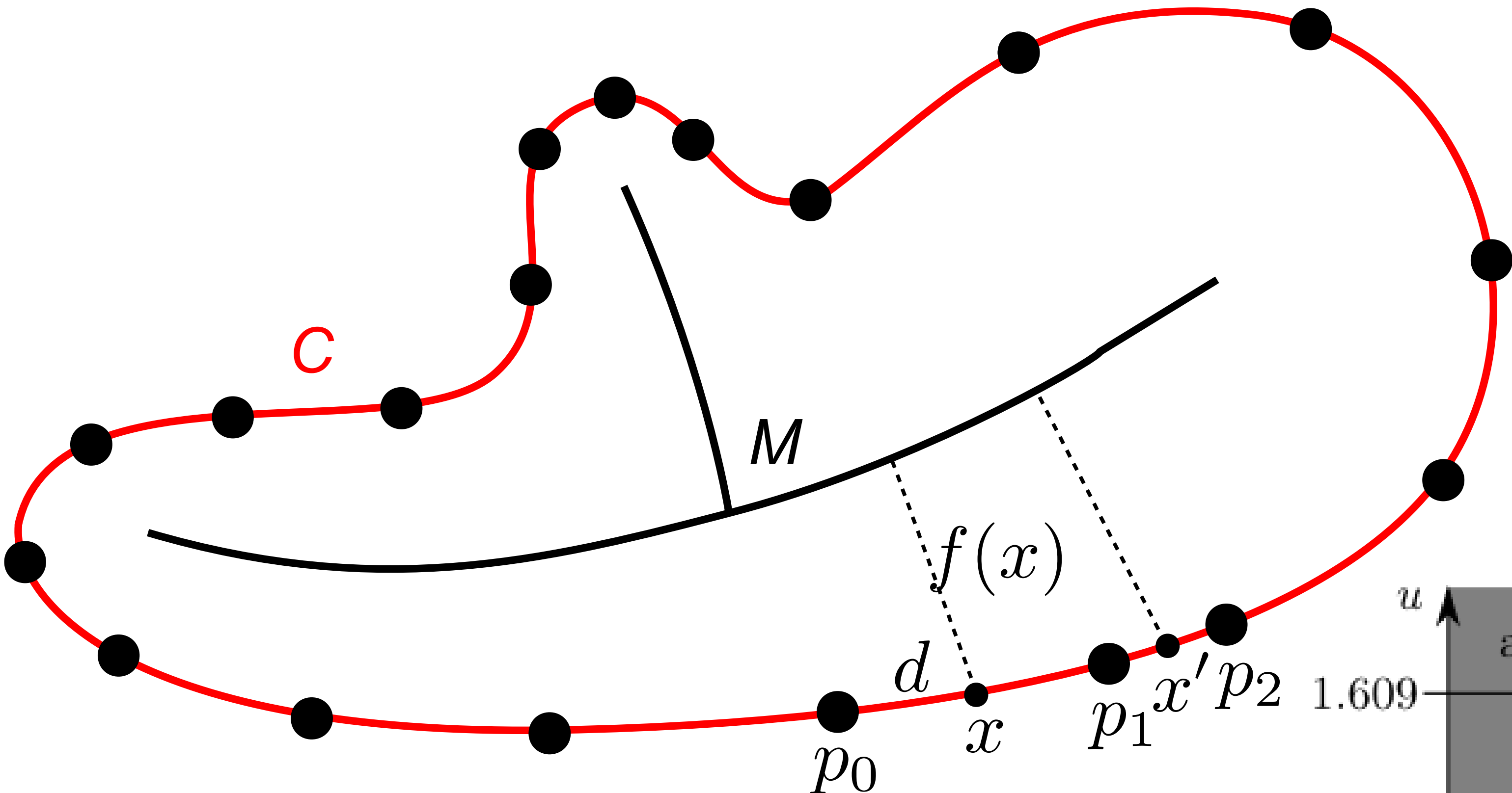


[ABE98]

Theorem 2

BC_0 reconstructs ε-sampled C from P with $\varepsilon < 0.5$ and a local non-uniformity $u < 1.609$.

Class Of Point Sets



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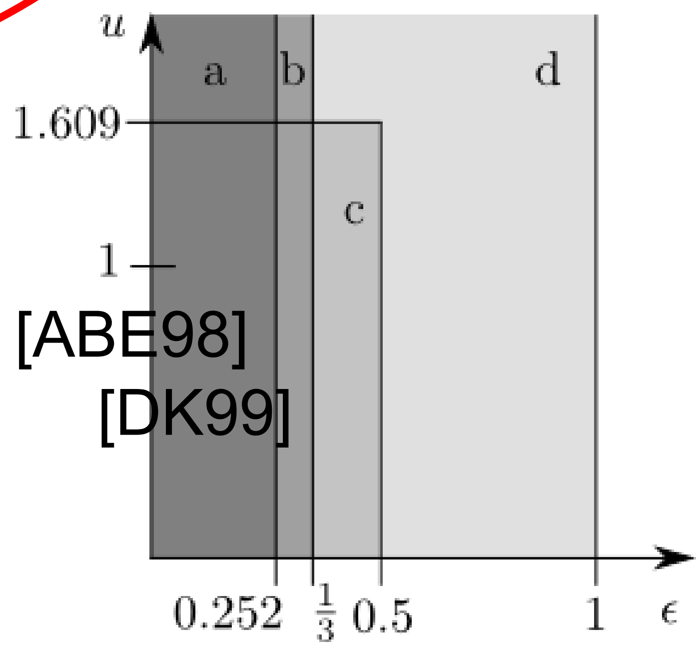

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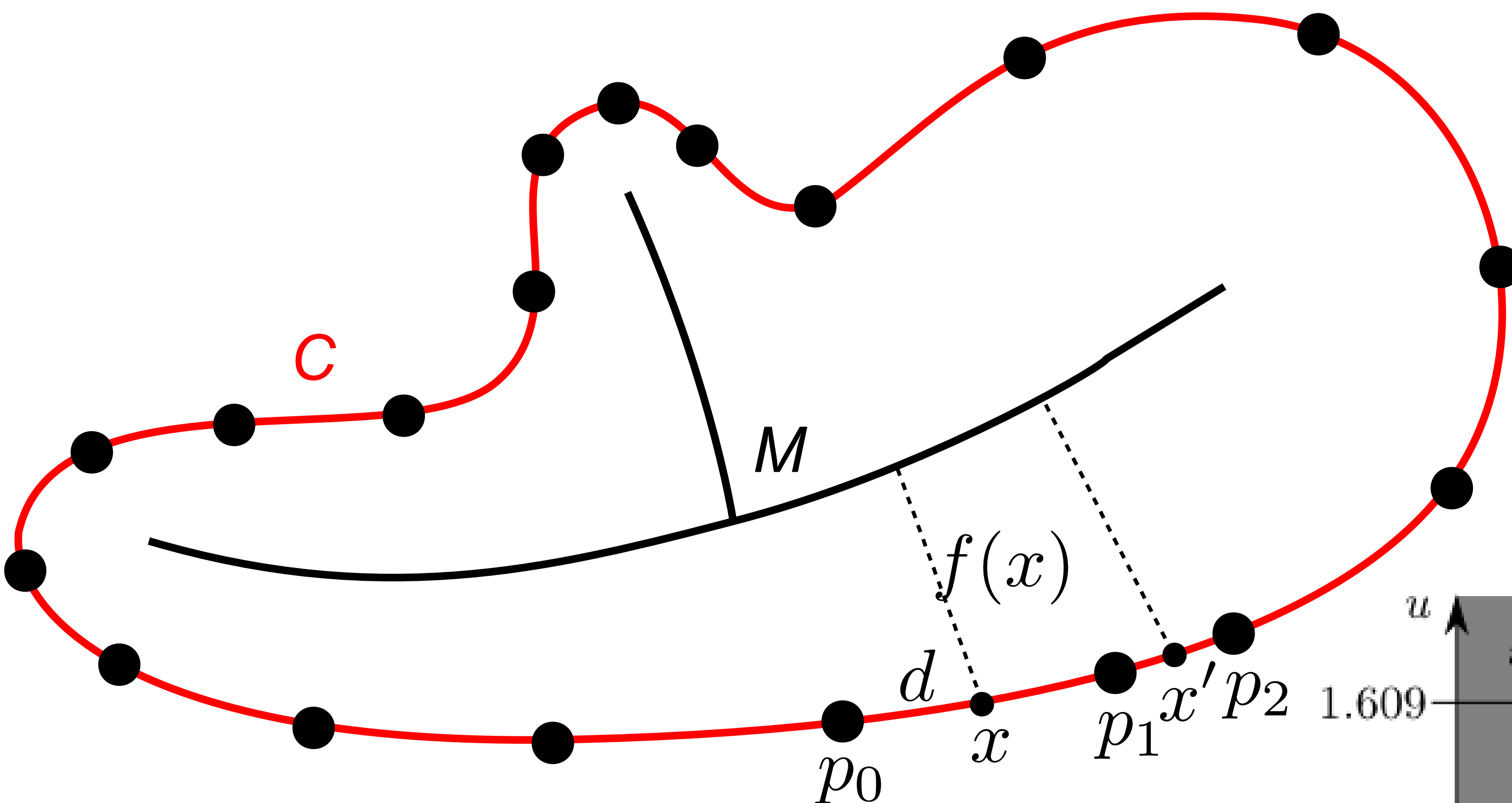
Class Of Point Sets



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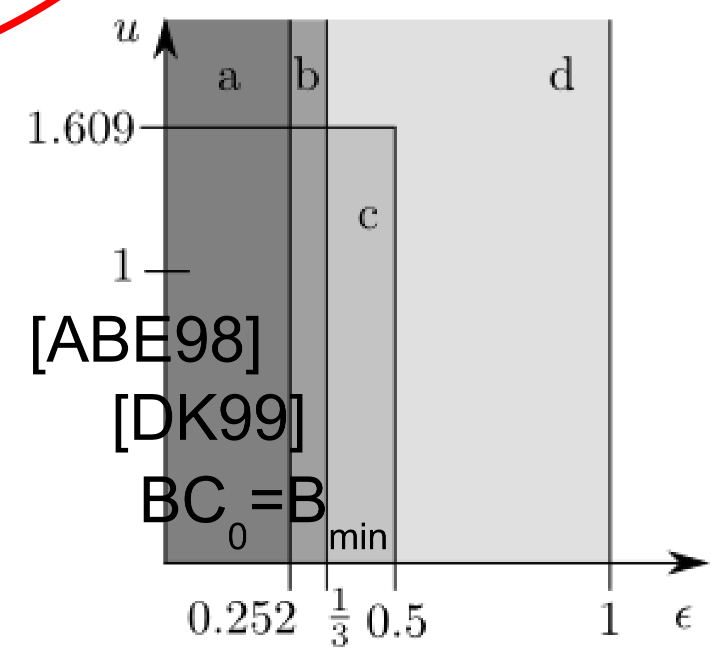

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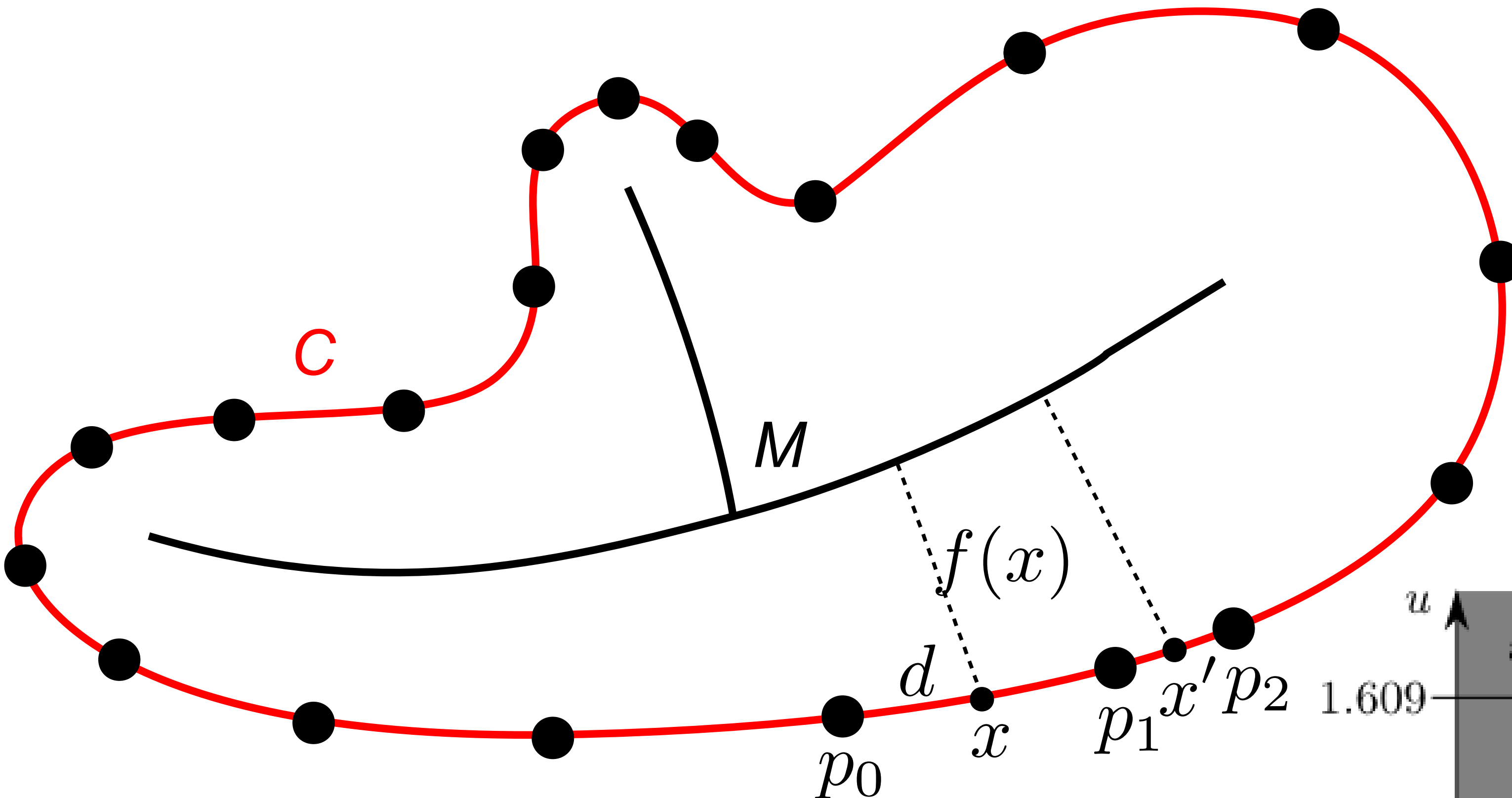
Class Of Point Sets



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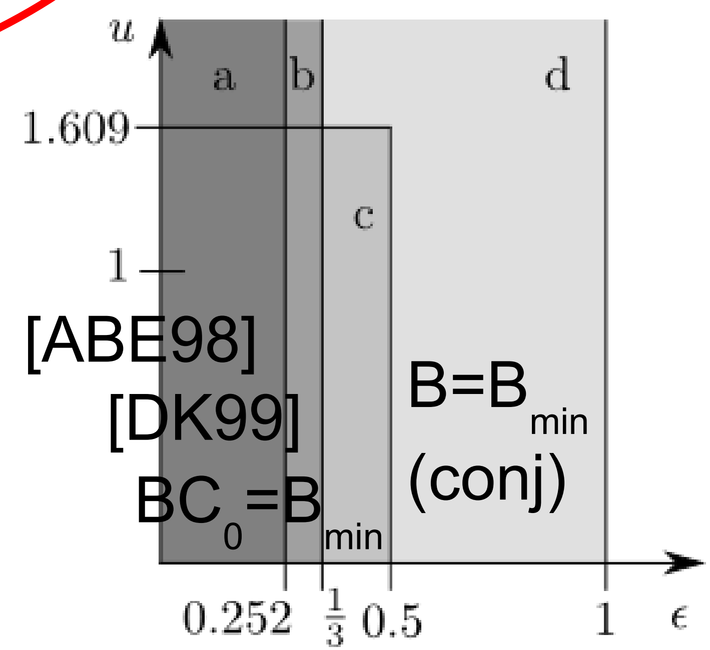

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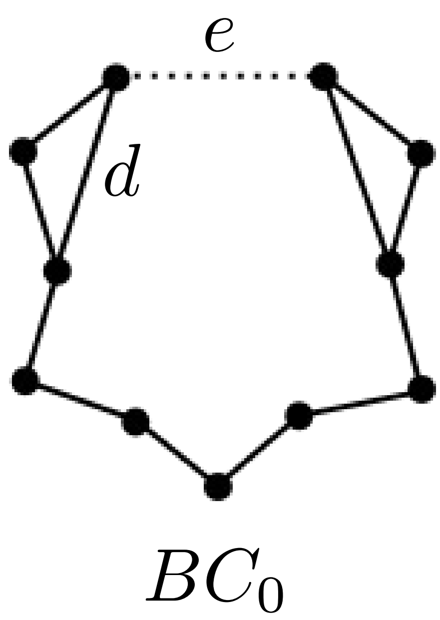
Why Limited Non-Uniformity?



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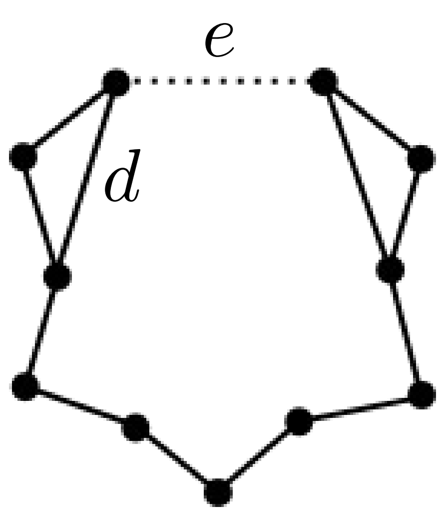
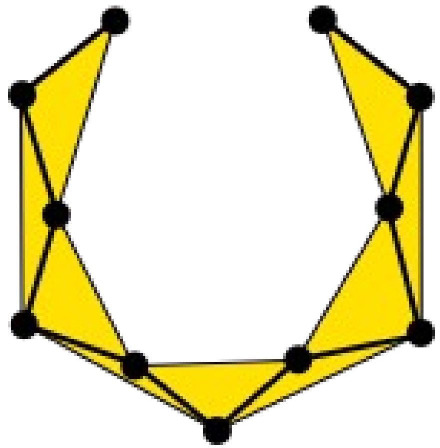
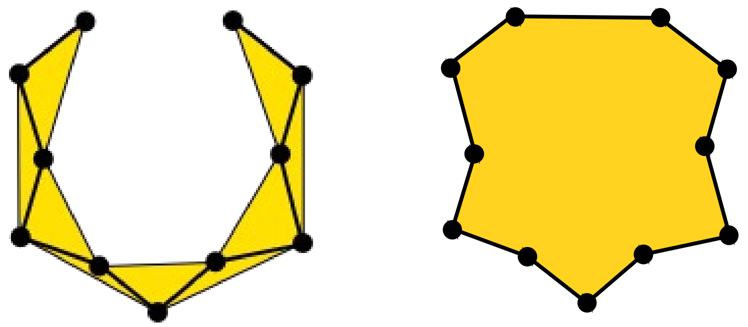


BC_0



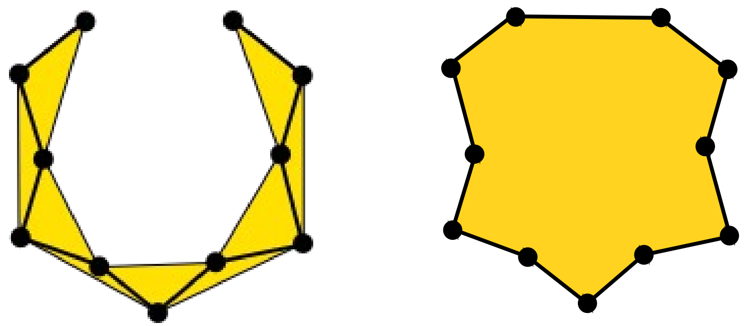
$B \neq B_{min}$

Future Work (2D)



Local minimum → fill hole

Future Work (2D)



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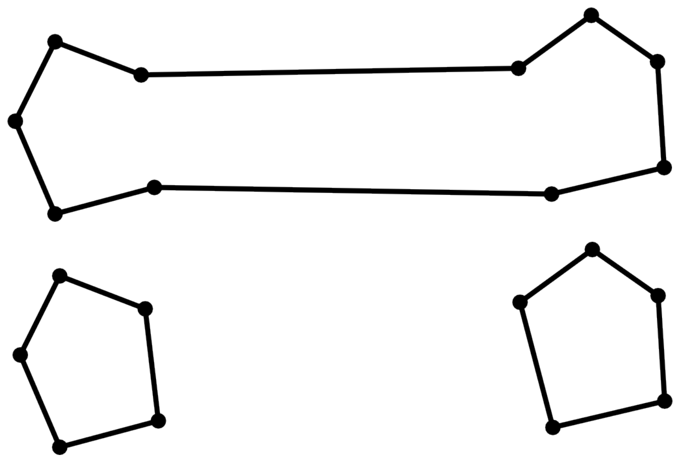
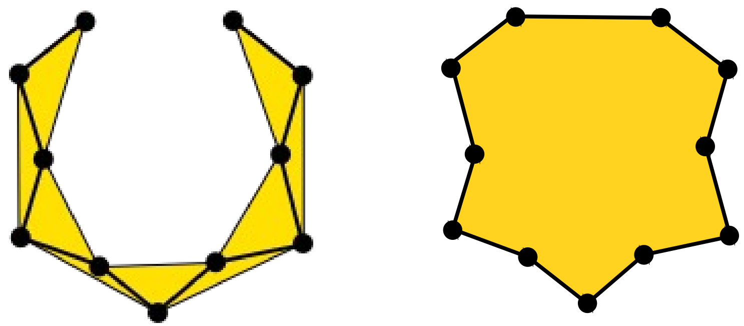


Multiply connected components

Future Work (2D)



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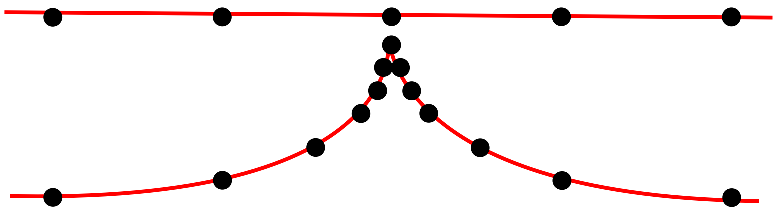
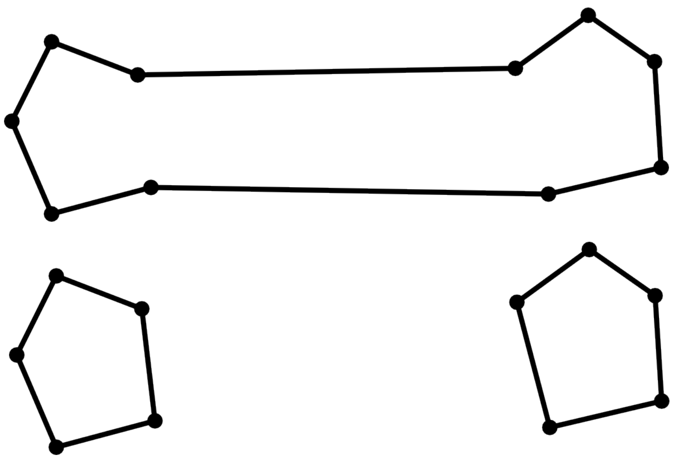


Prove B_{min} for $\varepsilon<1$



Multiply connected components

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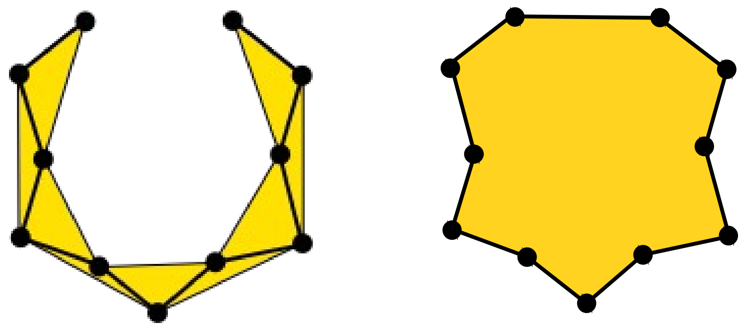
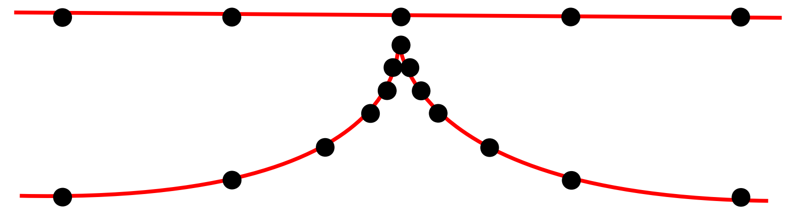


Local minimum → fill hole



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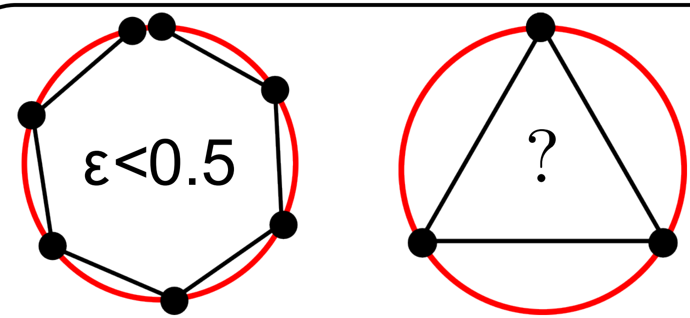
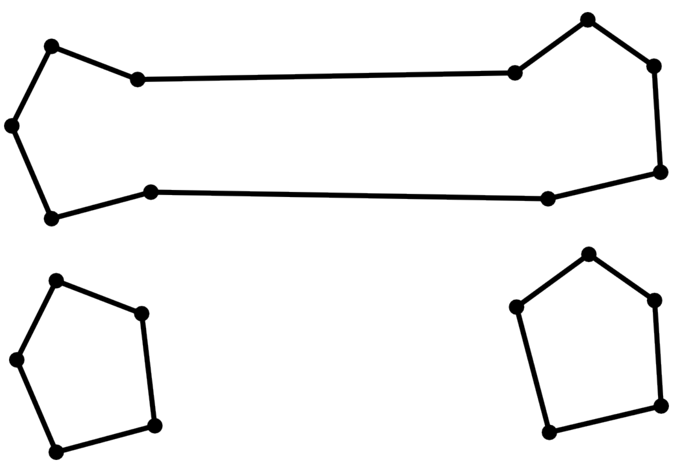


Sampling: tighter bound



Multiply connected components

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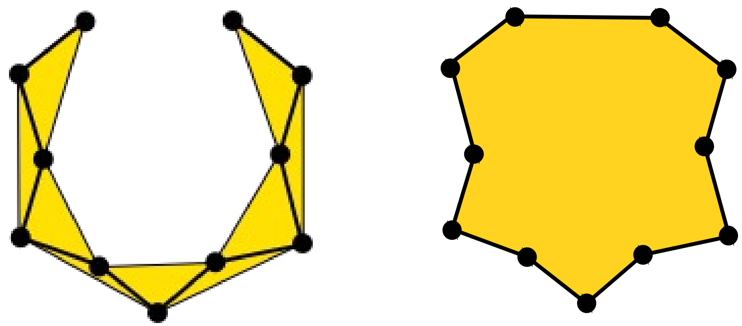
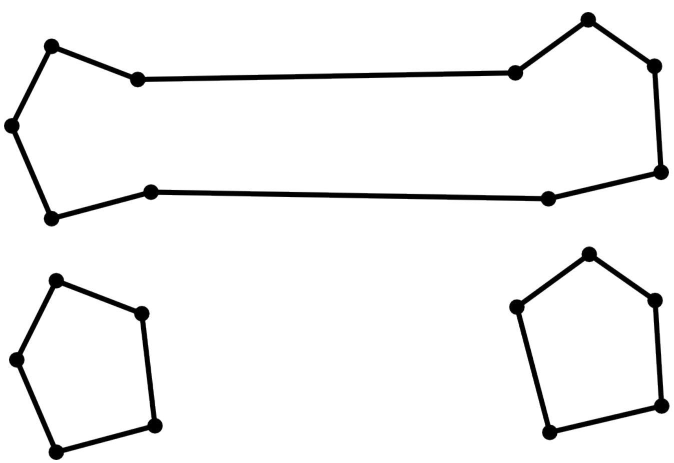
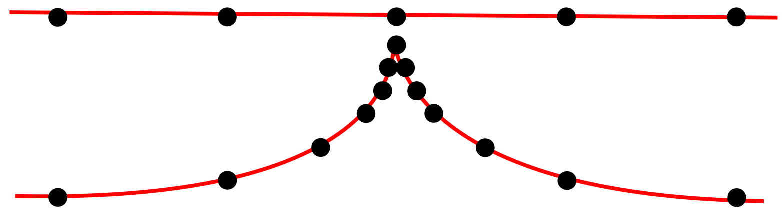


Local minimum → fill hole



Multiply connected components



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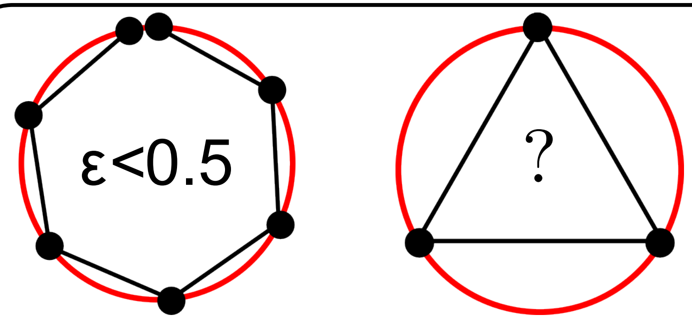

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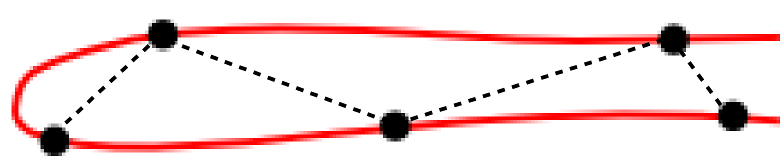
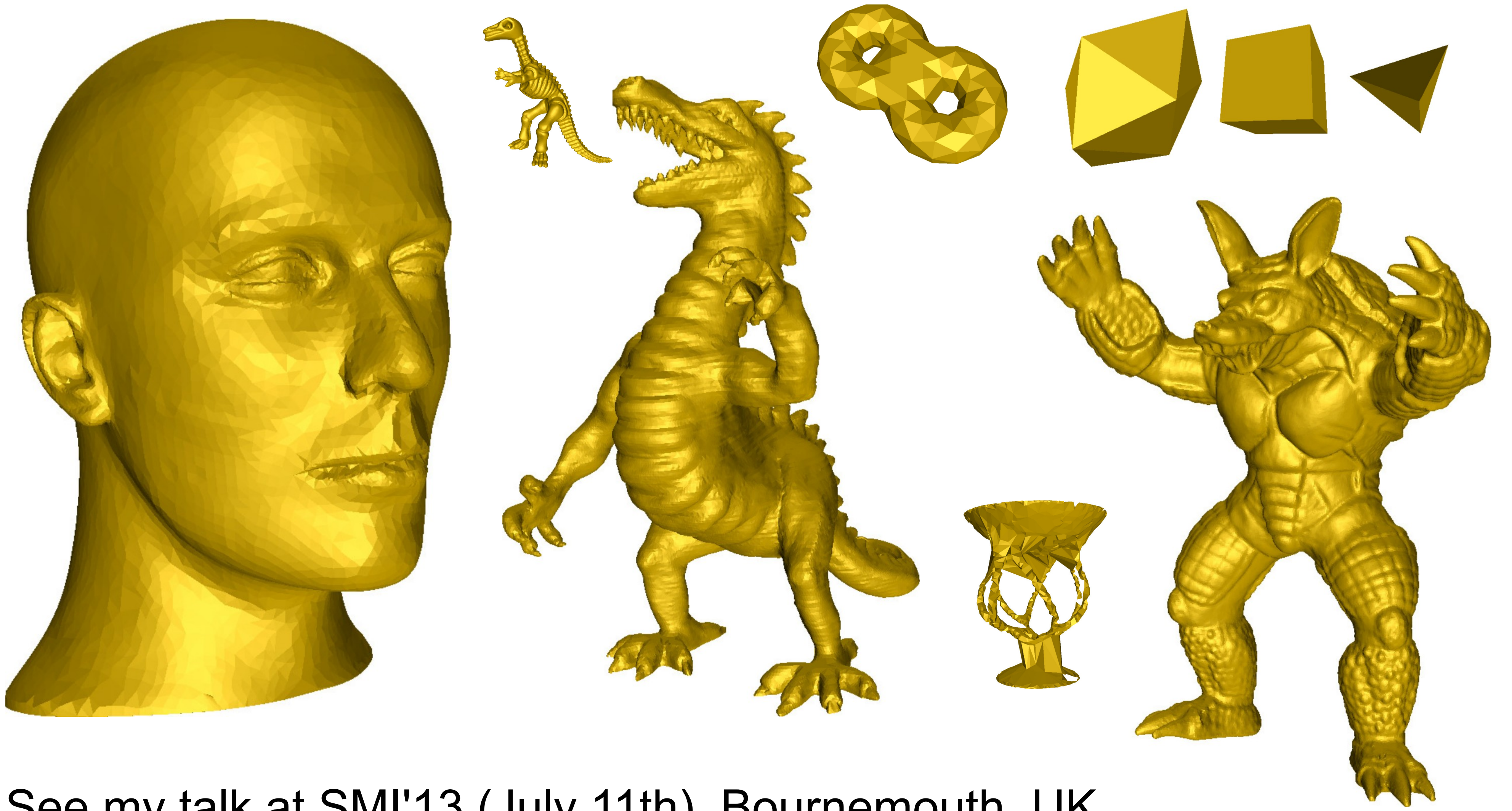


Open curves (vs. sparse)

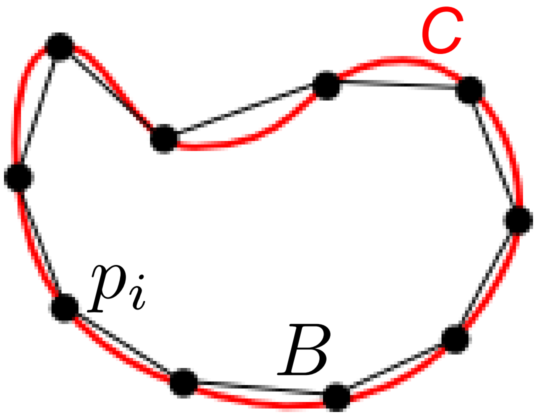
Future Work (3D)



See my talk at SMI'13 (July 11th), Bournemouth, UK

Contributions

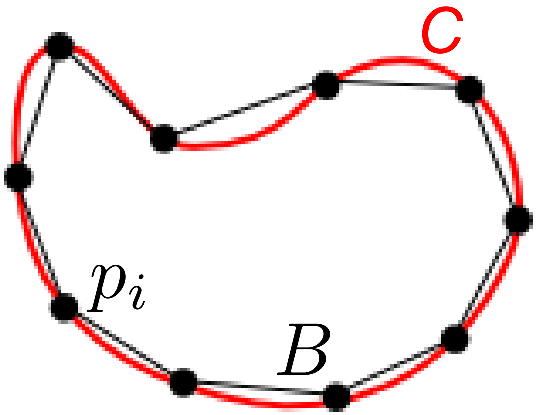
Source: http://sf.net/p/connect2dlib/



$$\underset{B_i \in DT}{\operatorname{argmin}} ||B_i||$$

Contributions

Source: http://sf.net/p/connect2dlib/



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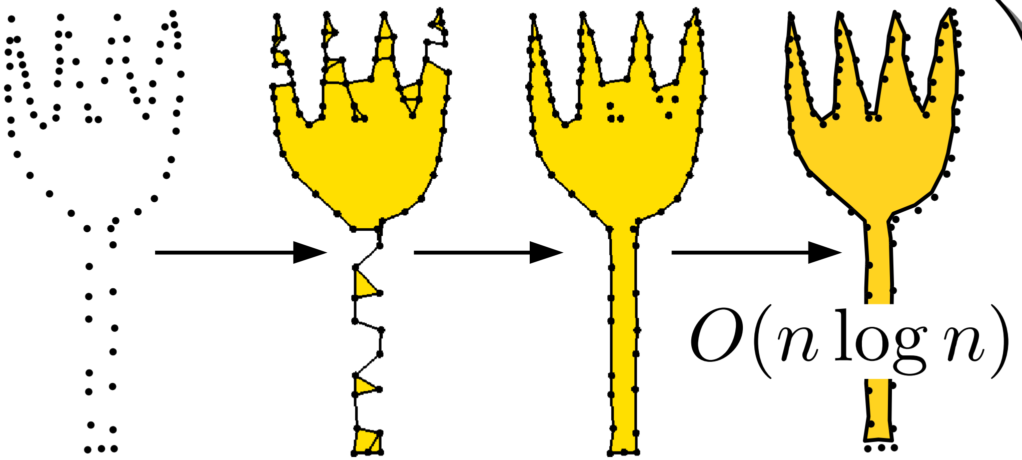

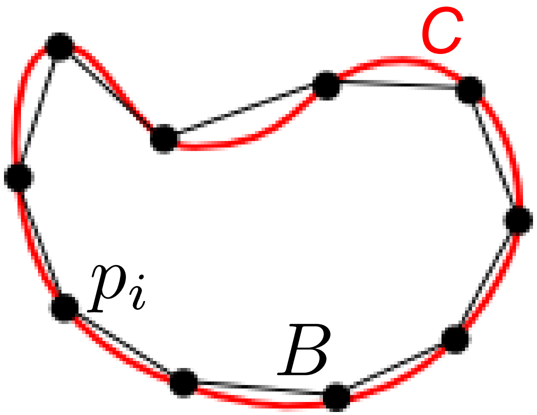

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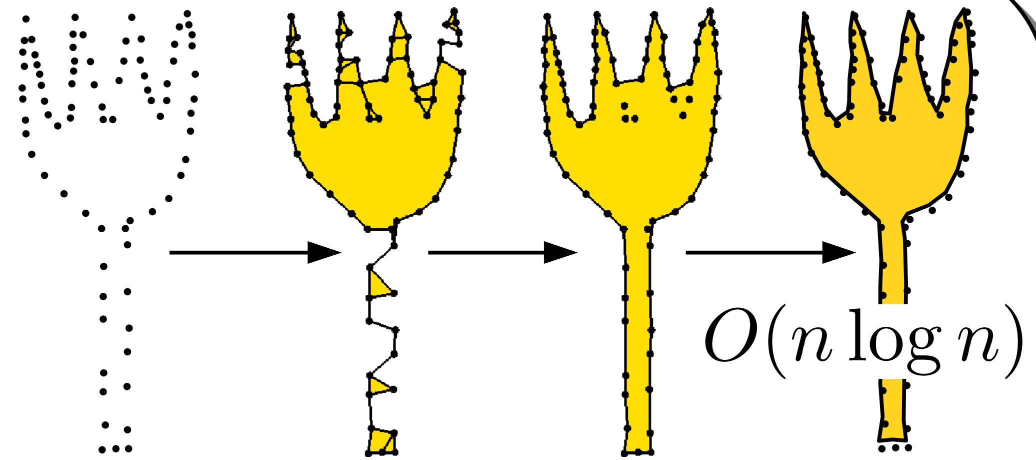

MST $c \geq 1$

BC_0 $c \geq 2$

Contributions

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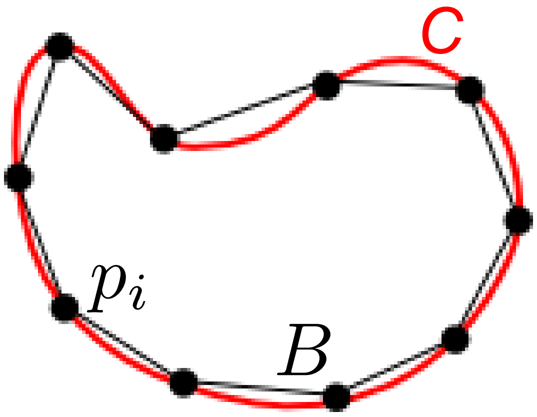

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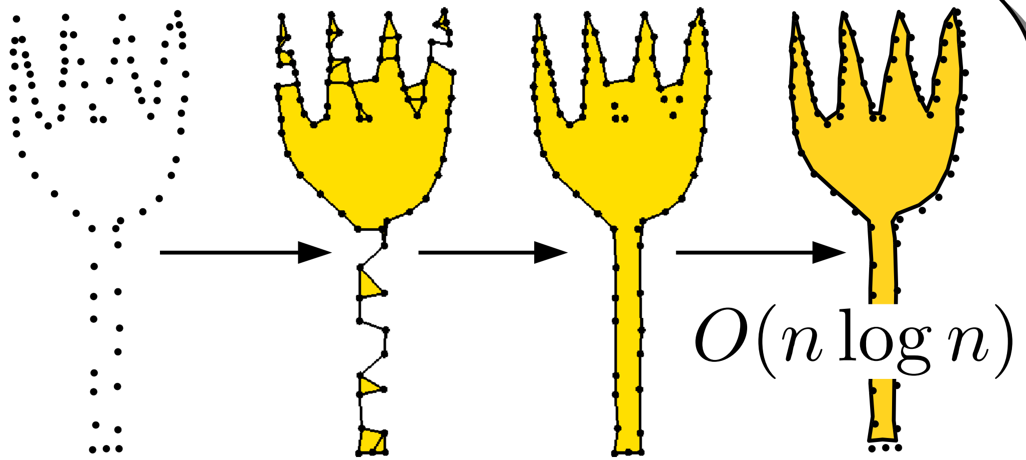

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Inflate as Sculpture

Dual

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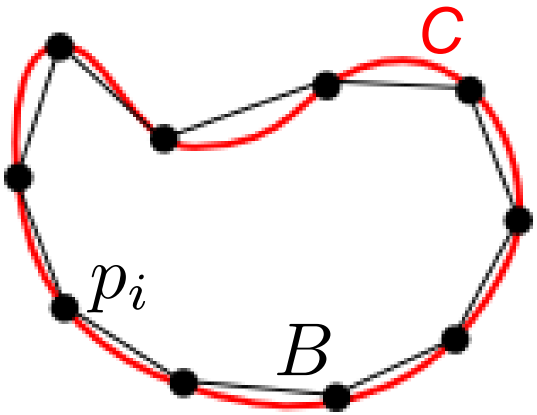

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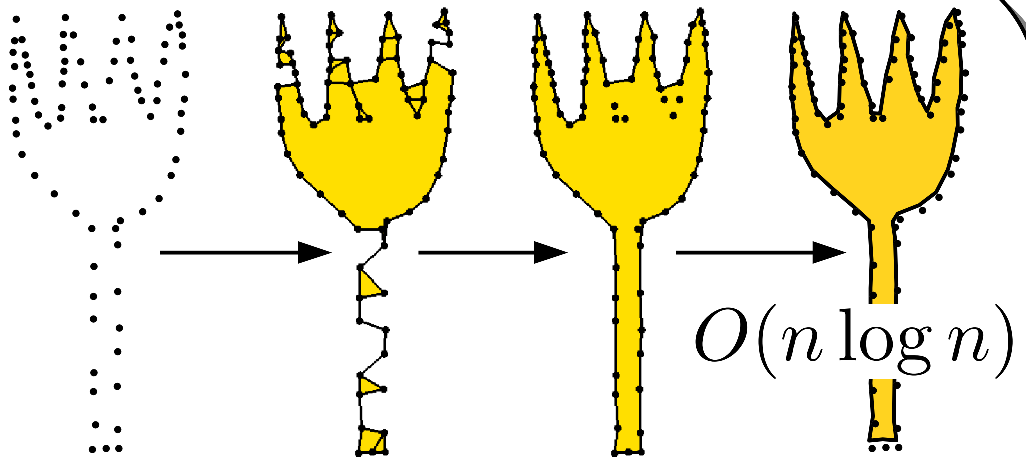

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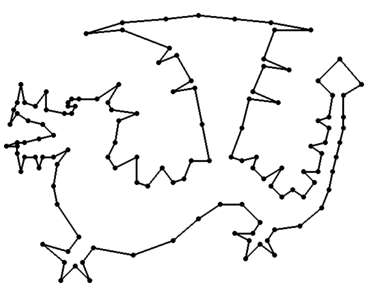

Theorem 2

$\varepsilon < 0.5$, $u < 1.609$

Experimental evidence + Proof

Contributions

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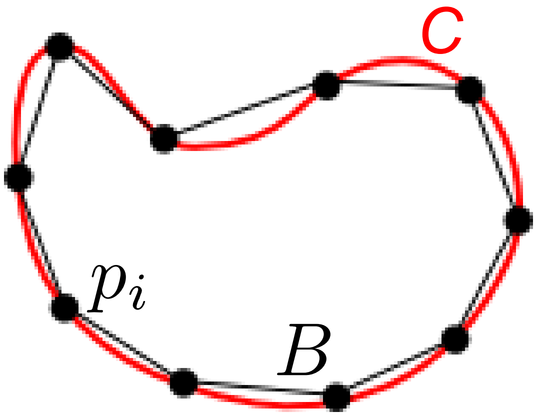

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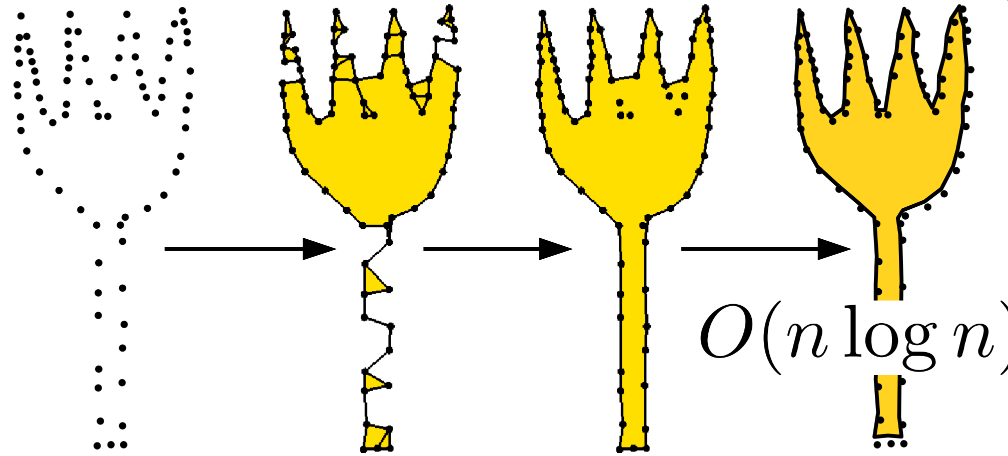

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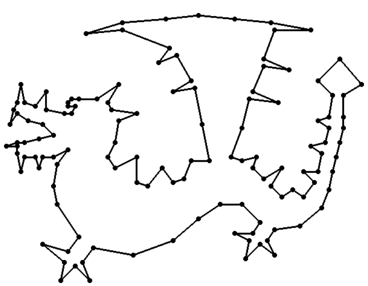

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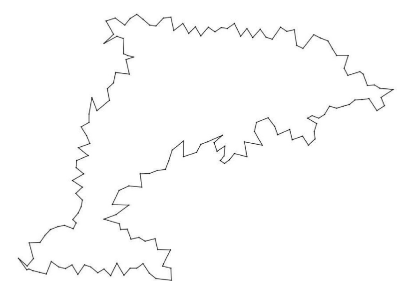

Noise robust

Contributions

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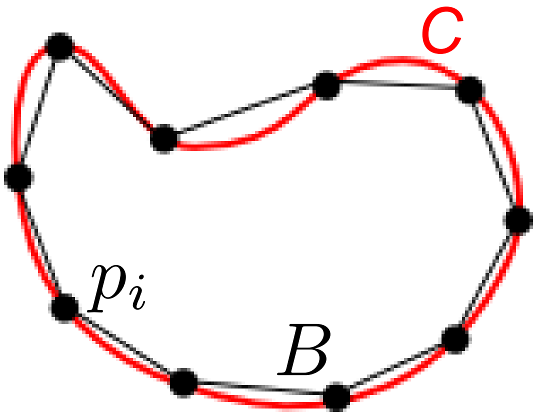

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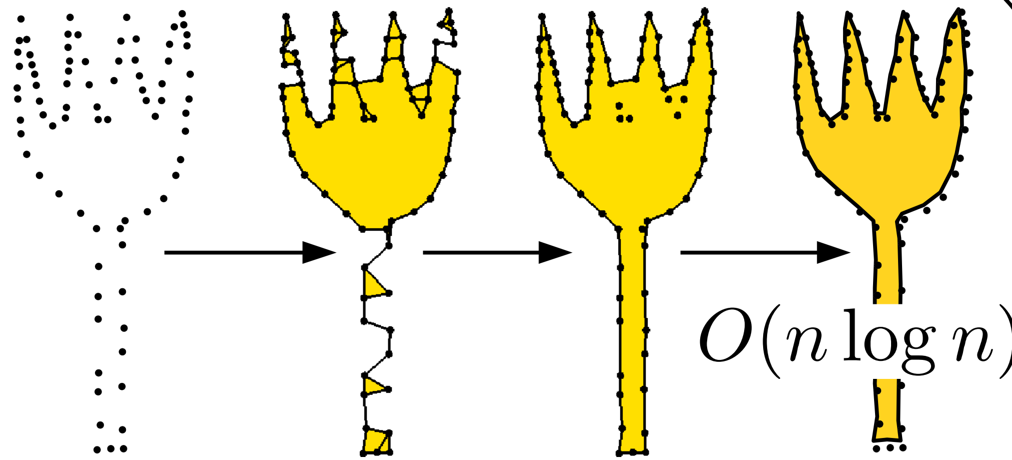

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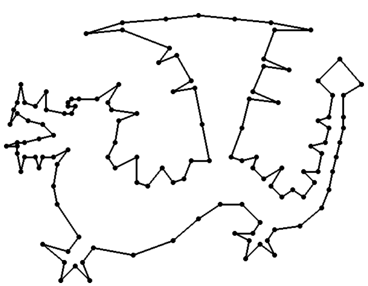

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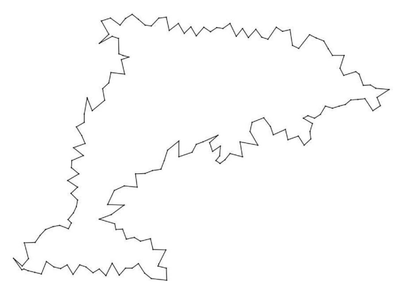

Noise robust

See SMI'13