

# Accurate Display Gamma Functions based on Human Observation

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## Abstract

This paper describes an accurate method to obtain the Tone Reproduction Curve (TRC) of display devices without using a measurement device. It is an improvement of an existing technique based on human observation, solving its problem of numerical instability and resulting in functions in log-log scale which correspond better to the nature of display devices. We demonstrate the efficiency of our technique on different monitor technologies, comparing it with direct measurements using a spectrophotometer.

**Keywords:** Colour Reproduction, Display Measurement, Human Visual System, Spatial Vision, Generalized Gamma Function

## 1 Introduction

The colorimetric characterization of display devices is performed in two steps. The first step is the definition of the *Tone Reproduction Curve* (TRC) which describes the relationship between the input signal of the monitor and the luminance produced on the screen. The second step consists in the definition of a matrix that describes the additive nature of the display device.

Several methods have been proposed to solve both steps in the recent past, and of particular interest is how to define the TRC without the use of a spectrophotometer. These methods use properties of human perception to achieve the final goal. This paper introduces a method to solve the first step without the necessity of using a spectrophotometer. It is based on the work presented in a previous paper[1], solving its problem of occasional numerical instability and unequal weighting.

The paper is structured as follows. After reviewing related work, we introduce our topic by explaining the principle of a human comparison based measurement system. Following human vision aspects are investigated: the spatial vision system and contrast sensitivity. Then mathematical aspects of human comparison based measurements are explained in detail with the frame

of their application, also with setup of single measurements with the next step and stop criterium, respectively. The chapter about the core mathematical problem discusses 2 different methods: one is defined in a previous paper of the authors [1], describing a smoothness problem in a linear coordinate system. The other method transforms the problem into a log–log scale. The latter method has to face a numerical problem of the enormously different coefficients of the minimum problem, due to the log–log scaling. We show a two-pass method solving the numerical problem and making the algorithm faster and more reliable at the same time. The paper closes with a discussion of the new method and future work.

## 2 Related Work

Several display characterization models with different characteristics have been presented in the past. These models can be classified into two basic categories: *measuring device based* and *human vision based* models.

Many works have been presented for the first category[2, 3, 4, 5, 6, 7], trying to model the real Tone Reproduction Curve (TRC) characteristic of a CRT display device. In many cases these models are still not accurate enough to acquire the real TRC, but just an approximation of it. In addition, even if they reach sufficient accuracy for CRT displays, this accuracy is not achieved for LCD displays. In consequence, the users are not able to gain the high precision required for many applications. On the other hand a model for LCD displays has been proposed[8] which introduces a spline interpolation in order to estimate the TRC.

In any case, these models require a spectrophotometer to get the information necessary to describe the TRC.

The models of the second category are based on interaction with the user and based on human vision and observation. One example is used in commercial software such as Adobe Gamma™, which comes with Adobe Photoshop™.

While acceptable quality can be achieved in the first category, in the second this goal has not been achieved until now for two reasons. First, the current models are not able to estimate or compute the real TRC, but only a simplification of the model used in the first category. Second, the applied mathematical background in these models is typically restricted to describe a kind of simple exponential gamma correction function. In order to solve this problem, a more accurate characterization model of display devices based only on human observation has been presented[1]. In this paper, the human vision is used to compare a series of dithered color patches against interactively changeable homogeneously colored display areas to obtain the TRC of display devices.

## 3 Principle

The main disadvantage of the characterization of a display based only on human perception, as reported in Neumann et al.[1], is implied by its adaptation mechanism making it impossible to define absolute values.

A human observer perceives an arbitrary periodic pattern as a homogeneous field when observed from a sufficient distance: If the spatial frequency of the elementary pattern is above a given value, i.e., the view angle of the elements is below a certain threshold, then it will be perceived as a homogeneous patch, e.g., a black and white chess board of pixels looks gray.

The absolute luminance and contrast are important quantities. We can not distinguish luminance differences or contrasts of two homogeneous patches if the luminance difference is below a given ratio of their absolute average luminance, according to the widely known and simple Weber-Fechner law or to the more accurate color appearance models. This ratio depends on an absolute luminance level (given in  $[cd/m^2]$ ) and whether photopic or scotopic vision is used [9].

In the visual comparisons used in our method, limits of spatial vision in the perception of homogeneous regions will be used. We would like to work with the possible minimal just noticeable luminance difference in order to ensure the highest accuracy. We therefore have to use an optimal

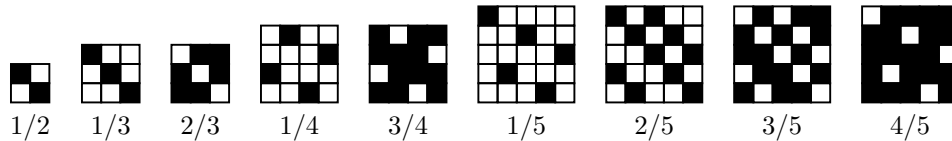


Figure 1: Dither patterns for the “chess board”

balance of conflicting requirements of spatio-chromatic features of observation. In practice, it means appropriate view distances, patterns and luminance levels. These are easily realizable for display devices.

As mentioned above, commercial graphics software sometimes includes applets which allow to find the gamma value of the display based on the same basic principle, so our method can be seen as an extension of them [1].

The inaccuracies of the visual observation based approach are typically invisible for human observers, because the image observation and the measurement observations have the same visible thresholds. Furthermore, our method reduces the errors of individual observations. It is easy to choose more appropriate circumstances to maximize the accuracy of our method. The observation process can happen in a dark environment, and judgements of more observers can be used.

Measurements are to be performed for the 3 color channels separately, so that series of observations will be performed on ‘dithered’ patches embedded in a homogeneous background color. During a single measurement, the user tunes the background luminance level so that visual differences disappear between the central dithered area and the “background”, i.e., the homogeneous surrounding area. Such a measurement step is completed when a background luminance level is accepted. Note that the *relative* error of observed values will be nearly luminance independent, except in cases of very low luminances, especially with an ambient light, when observation may have higher relative error.

A measurement input can be defined as the triplet consisting of the pattern used in the central area (see Fig. 1) and its bright and dark luminance levels. Practically, only a ratio between the number of bright and dark pixels is taken into consideration. An individual measurement results in an equivalent background luminance level. These luminance levels are identified by integer numbers, in practice represented as bytes, as our method does not use any further information of them either.

The “next triplet”, i.e., the “next measurement”, can be defined either depending on, or independent of, the results of the already existing data. To complete the process, a data analysis and optimization step computes the curve in question, which can be specified by combining certain criteria like the minimization of the differences at each point or the overall smoothness of the resulting curve.

The overall evaluation of the measurements as well as the definition of their series can be controlled by different criteria such as minimizing the number of measurements, having a fixed procedure or gaining the most exact characterization of the display possible.

## 4 Human Vision Aspects

### 4.1 Spatial Vision

Based only on human observation, our method cannot determine the absolute contrast, i.e., the ratio of max/min output luminance levels,  $L_c(255)/L_c(0)$  [1]. We recommend a practical approach to calibrate this singular free parameter after applying the mathematical model described below.

The practical range of the perceptual contrast of a typical monitor is 20...100; for a high quality display under special conditions, it can be also be more than 1000, while in the worst case of common CRTs it can fall also below 10 in case of a bright environment.

Why do we speak about perceptual difference? According to the color appearance models [10] we have to take at least the ambient luminance level into consideration for a CRT or LCD screen

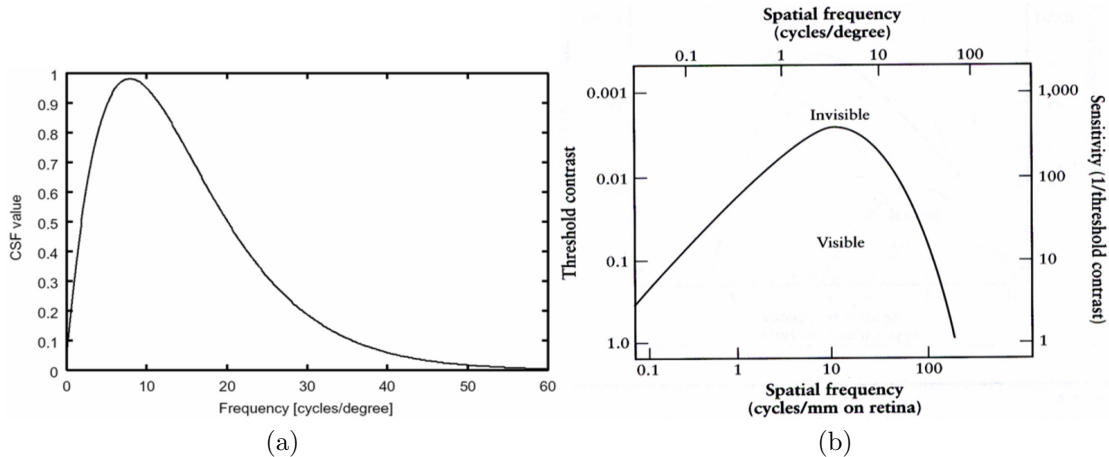


Figure 2: (a) Contrast Sensitivity Function (CSF) – (b) Visible Contrast Threshold

in commonplace environmental settings (e.g., office). Also the rough gamma correction LUT approaches, using a simple gamma value, have to be changed for dark, dim or average adaptation luminance levels: the real gamma value has to be divided approximately by 1.6... 1.7, 1.2 and 1, respectively. Thereby we perceive linear contrast e.g. in darkness only as 60 instead of 1000.

But this perceptual aspect of visible contrast influences only the accuracy of measurement, not the original “pre-perceptive” gamma curve itself, which is not depending on the ambient luminance level, only on the display. We will compute objective emission values of a display for different input signals and for the 3 basic colors. Of course, if we use an arbitrarily measured or human vision based gamma curve to generate color images, we have to additionally take the influence of ambient light and its effect for the gamma LUT into consideration. But this task would already be a problem of correct color appearance or image reproduction.

The core of the comparison process is a single comparison of an area uniformly filled by a single color against another area, the “chess board”, filled by a repeating pixel pattern. The pattern contains only two definite colors or luminance levels (*low* and *high*), assembled according to a simple, fixed ratio, e.g. 1/2 or some other rational number with small denominator (Figure 1). A homogeneous area encloses the chess board, acting as background with luminance *background*.

To be usable, the patterns which are to be observed must appear homogeneous, otherwise the inhomogeneity disturbs our estimation of its equivalence with a homogeneous area. The spatial vision models give us quantitative values for the required viewing distance.

The *contrast sensitivity function* (Figure 2a) and the appropriate *visible contrast threshold function* (Figure 2b) describe the spatial behavior of our vision [11, 12, 13, 14]. This figure corresponds to the case of the highest luminance sensitivity.

Color vision models [9] are working with the opposite color channel models, using achromatic, red-green and yellow-blue channels following the preprocessing in the retina. The color channels are less sensitive, that means that we do not see really sharp in particular in yellow-blue channels, we only have a feeling of sharpness thanks to the achromatic channel and the huge number of rods in the retina.

In order to avoid a complicated model and also to ensure the highest accuracy we selected the curve of the highly sensitive achromatic channel for all colors.

Image detail can be measured in *cycles/degree* (*cpd*). In case of a sinusoidal luminance pattern it is one cycle of the sine function. At a discretized approach it is a pair of a darker and a brighter pixel. We do not see details in images with spatial resolution of more than approximately 60 *cpd*, but a homogeneous field without contrast. For opponent color channels we reach this visibility at 30... 40 *cpd*.

In our case the different “superpixel patterns” of Figure 1 represent different *cpd* resolutions. The required view distance can be estimated with  $5 \times 5$  pixels representing the largest patterns

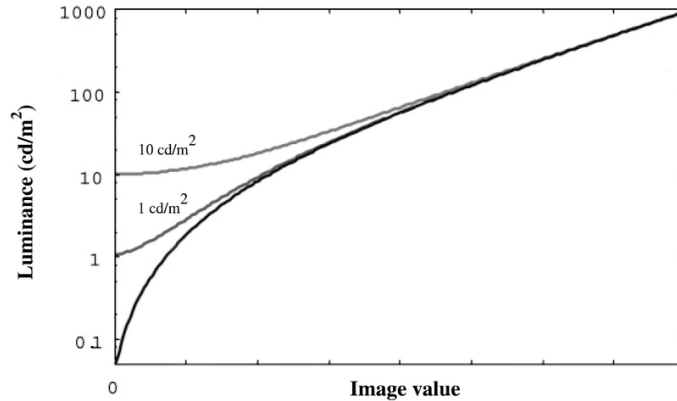


Figure 3: Effect of ambient illumination of  $1\text{cd}/\text{m}^2$  and  $10\text{cd}/\text{m}^2$  vs. dark ambient on image value.

of Figure 1. The resolution of the human eye is about 1 arcminute ( $60\text{ cpd}$ ), so a pattern with  $5 \times 5$  pixels will become invisible for distances farther than about 5m, depending on the screen’s luminances and the pixel size. According to our experience, a distance of 3-4 m is always enough, and for low contrast patterns even 1 m is usually sufficient. Nevertheless, the widely used 40-50 cm distance of gamma calibration tools is not enough for large patterns.

## 4.2 Displays and Contrast

As mentioned above, both CRT and LCD have a maximum contrast, which is the ratio of luminance values emitted by the highest and the lowest video signal, e.g., 255 and 0, and is finite instead of infinite, caused by the display technology and also unwanted ambient effects.

A CRT always has unwanted electron backscattering. The reflectivity or BRDF (Bidirectional Reflectance Distribution Function) of the surface of a display is always positive. Reflections can be represented, e.g., by a sum of a specular and a diffuse component. The specular component is often a visible mirror image of the ambient room.

All of these unwanted effects reduce the contrast of a display to a certain finite value. The answer of a zero input video signal will be a positive value at each color channel. We have not mentioned cross effects of color channels as another unwanted factor, e.g., the excitation of phosphors by the “wrong” cathode ray.

Fig. 3 illustrates how the shape of the display response curve changes depending on ambient luminance level.

Unwanted emissions despite zero signals at the different color channels decrease the displayable gamut. Contrasts of color channels influenced by these unwanted emissions can be roughly estimated also by visual observation. Special calibration test images can be used for this observation, which contain equidistant gray, red, green and blue scales. Linear contrast values of the color channels can be estimated from the distinguishable dark patches of these scales.

These contrast values can complete results of gamma functions obtained also by human observation. Although contrast estimation is less accurate than the gamma function, fortunately it is sufficient for tone mapping, because the visual quality of displayed images match the accuracy of measurements based on human observation.

## 5 Setup of measurements

### 5.1 Relative Measurement by Human Observation

Our method deals with isolated color channels, therefore the computation is applied only on one-dimensional problems, independent from each other. We ignore cross effects between the  $r$ ,  $g$  and  $b$

color channels. In fact, for CRTs the cross effect is 1-3%. In lack of other colorimetric information we assume that the CIE xy chromaticity values are according to the sRGB recommendation, the standard ITU-R BT.709:  $r = (0.64, 0.33)$ ,  $g = (0.30, 0.60)$  and  $b = (0.15, 0.06)$ , and white point  $D65 = (0.3127, 0.3290)$ . As mentioned in the previous section, only visual comparisons are used as relative inputs, so the result is also a relative function which describes the relative luminance values of the independent channels, from 0 to 255. The relative luminance values  $l_r, l_g, l_b$  can be converted to absolute values  $L_r, L_g, L_b$  by

$$L_c(val) = L_c(0) + \frac{L_c(255) - L_c(0)}{l_c(255) - l_c(0)} \cdot (l_c(val) - l_c(0)) \quad (1)$$

where  $c = \{r, g, b\}$  and  $val = 0 \dots 255$ , and it is easy to see that any linear transformation of a given series  $l_c(val)(val = 0 \dots 255)$  result in the same absolute values  $L_c(val)(val = 0 \dots 255)$ :

$$\begin{aligned} L_c(0) + \frac{L_c(255) - L_c(0)}{(a \cdot l_c(255) + b) - (a \cdot l_c(0) + b)} \cdot ((a \cdot l_c(val) + b) - (a \cdot l_c(0) + b)) = \\ L_c(0) + \frac{L_c(255) - L_c(0)}{l_c(255) - l_c(0)} \cdot (l_c(val) - l_c(0)) \end{aligned} \quad (2)$$

where the parameters  $a$  and  $b$  are the gain and the offset, respectively. Therefore we can use any linear transformation on the series  $l_c(val)(val = 0 \dots 255)$  without changing the values  $L_c(val)$  obtained. This linear transformation can be defined, e.g., by fixing 2 different values of this series. For the sake of simplicity we will work with  $l_c(0) = 0$  and  $l_c(255) = 1$ , so Eq. (1) is simplified to

$$L_c(val) = L_c(0) + (L_c(255) - L_c(0)) \cdot l_c(val) \quad (3)$$

The measured and computed values  $l_c(val)$  are independent of the values  $L_c(0)$  and  $L_c(255)$ . While these absolute values and their ratio defining the absolute contrast range of the display device in question can be interesting for the overall appearance, finding them is outside the scope of our method.

Showing a dither pattern realized by luminances *low* and *high*, the observer is requested to tune the third, homogeneous luminance level (*background*) until the luminance difference disappears. Now we have 3 values (*low*, *high*, *background*), and a ratio of the number of the *low* luminance level pixels within the pattern,  $ratio = \frac{N_{low}}{N_{low} + N_{high}}$ . The following approximation can be written for the absolute luminance values of channels  $c = \{r, g, b\}$ .

$$L_c(background) \approx L_c(low) \cdot ratio + L_c(high) \cdot (1 - ratio) \quad (4)$$

Using Eq. (1), we see

$$L_c(0) + Q \cdot l_c(background) \approx (L_c(0) + Q \cdot l_c(low)) \cdot ratio + (L_c(0) + Q \cdot l_c(high)) \cdot (1 - ratio) \quad (5)$$

with  $Q = L_c(255) - L_c(0)$  by Eq. (3), and reordering

$$l_c(background) \approx l_c(low) \cdot ratio + l_c(high) \cdot (1 - ratio) \quad (6)$$

shows the independence of the measurements from values of  $L_c(0)$  and  $L_c(255)$ , and also that  $l_c(0)$  and  $l_c(255)$  can be predefined arbitrarily:

$$l_c(0) = 0 \quad \text{and} \quad l_c(255) = 1 \quad (7)$$

A single measurement results in a singular *background* value for the measurement input triplet (*low*, *high*, *ratio*); *background*, *low* and *high* are bytes, and *ratio* is a simple rational number. The goal is to define the function  $l_c$ , that is, 256 separate values for inputs  $0 \dots 255$ . Since values of the function  $l_c$  are real numbers, when the output shall be given in the same format as its domain, they shall be rounded to bytes finally.

We now face a practical and two mathematical problems

1. Conditions of the measurements have to be defined.
2. Having a list of measurements and perhaps a preliminary curve defined by them, either define the next measurement's setup, and/or recommend to stop the process.
3. Having a list of measurements, a curve has to be derived. This curve may be an input of the previous step as well as an output of the whole process.

The first question belongs to the setup problem, covering human vision aspects which have been discussed in section 4, and also to the problem of parameter definition. The second question, concerning similar problems as well as including the definition of the stop criterion, is discussed in the next section 5.2. Then the mathematical core of the method will be presented in section 6.

## 5.2 Definition of the Next Measurement, and Stop Criterion

The measurement shall be accurate, while at the same time the number of measurements should be minimized to achieve good results with low effort for the user. These requirements lead to another optimization problem: defining the next measurement step in an optimal way, and notifying the user when reliability has reached a certain threshold.

The main problem for generating the input for the next measurement is that the expected effect of an additional measurement on the reliability of the function should be evaluated depending on the behavior of the still unknown function itself. How could this function behave between the points (colour values) where we already have some information, and how can we rely on the previous measurements? These questions are connected to the principles of the optimization method. We use some heuristics relying on experiments, in order to get a compromise between simplicity and accuracy, where the observer can still overrule the recommendation to stop.

A “measurement” means a definition of a *background* value by the user, where the *low* and *high* values used in the pattern are given as well as their mixing *ratio*, represented by the pattern. The method restricts the set of the possible *low* and *high* points for the new measurement only to points having taken part in some previous measurement, i.e., 0, 255 and the already defined *background* points. Hereafter they are named also as *support* points.

The process defining the *next measurement* consists of three successive steps, using functions  $f_1$ ,  $f_2$  and  $f_3$  balancing the process properly. The central players of the process are the *Reliability* and its opposite and reciprocal, the *Uncertainty*. Three kinds (layers) of *Reliabilities* will be defined below, differing by their meaning and argumentation, and using each other in their definitions.

1. First the reliabilities of existing *support* points are defined as the sum of all the *Reliabilities* coming from the measurements in which these points appear (as *low*, *high*, or user defined (“measured”) *background* point). Obviously, the points 0 and 255 are assigned absolute *Reliabilities* (i.e., zero *Uncertainty*).

An individual measurement's *Uncertainty*, i.e., the reciprocal of its concerning *Reliability*, is equal to the error of its condition  $M(j)$  to be minimized (see Eq. 12), multiplied by the effect of the pattern's non-homogeneity and the contrast sensitivity belonging to the luminance to be set.

$$\begin{aligned}
 Unc_{supp}(j) &= |M(j)| \\
 &\times f_1(nonhomogeneity(pattern(j))) \\
 &\times f_2(contrastssensitivity(luminance(j)))
 \end{aligned}$$

2. Then, each *regular* (i.e., not *support*) point is assigned the other kind of *Reliability* with its reciprocal *Uncertainty*. For the sake of simplicity, this step will be divided into two substeps. The first substep defines an effect of a *support* point to a *regular* one, resulting in a ‘relative’

*Uncertainty*. Then sum of concerning *Reliabilities* of these *Uncertainties* results in *Reliability* of a regular point.

$$Unc_{rel}(i, j) = Unc_{supp}(j) + f_3((i - j)^2) \quad (8)$$

$$Rel_{reg}(i) = \sum_{i \leftrightarrow j \in \mathcal{M}} Rel_{rel}(i, j) \quad (9)$$

In Eq. (9),  $i \leftrightarrow j \in \mathcal{M}$  means

- $\mathcal{M}$  is a (potentially sparse) subset of  $\{0, \dots, 255\}$ , and
- $i, j \in \{0, \dots, 255\}$
- $j \in \mathcal{M}$
- $i \notin \mathcal{M}$
- no element  $j' \in \mathcal{M}$  is between  $i$  and  $j$ ,

and of course there cannot be more than two such indices  $j$  for any given  $i$ .

$Rel_{reg}(i)$  characterizes an existing set of measurements with respect to  $i$ . Some compound value of  $Unc_{reg}(i)$  over all regular points, actually their maximum, characterizes the obtained approximation at this stage of the process. When it reaches a certain threshold, the process is suggested to be terminated, without performing the next step.

3. In case of having not stopped at the second step, the third step selects the best triplet for the next measurement by a heuristic evaluation of all the possible triplets (*low*, *high*, *ratio*). Each of these investigations consists of four substeps, vis

- definition of the *background* value, belonging to the triplet in question, simply by taking the value of the approximation function, given at the moment.
- definition of a ‘tentative’ *Uncertainty* by the previously defined *background* as ‘tentative’ support point, as

$$Unc_{supptent}(background) = Unc_{supp}(low) + Unc_{supp}(high) \quad (10)$$

- computing *Reliability* at each regular point, in the same way as in Equations (8) and (9), taking the completed Set  $\mathcal{M}$  into consideration.
- similar to the normal case above, the maximum value of *Uncertainties* results in evaluation of the tentative triplet in question.

Finally the best triplet is selected by using their evaluations detailed above.

The third step of the aforementioned process, i.e., definition of the next measurement setup, can be replaced by processing a predefined series of triplets as well. This simplification makes the process more robust, with respect to heuristic components of this step, although it can result in a less optimized solution.

## 6 The Mathematical Problem

Given a list of measurements with input controlled by a predefined or automatically derived series, we wanted to derive a corresponding function.

First of all, properties have to be defined which the function has to fulfill. A set of approximations as in Eq. (6) is obviously not enough to give a definition of the function, simply because of the many degrees of freedom. In order to reduce them, the resulting function should fulfill certain additional conditions, defining its general behaviour. These conditions can restrict the set of the possible functions to a certain class and/or give additional criteria corresponding to the likely nature of the result.



We worked out two different approaches in this paper, both of them assume the function being smooth or, more exactly, having a small second derivative. The first approach works in the original coordinate system while the second one translates its axes, so the problem is defined in another coordinate system.

## 6.1 Problem in Linear Coordinate System

First, we restrict the domain of the function only to the possible inputs, i.e., integers from 0 to 255. The second derivative on a domain consisting of a discrete set of points can be written in the form of finite differences:

$$S(i) = f(i + 1) + f(i - 1) - 2f(i) \quad (i = 1 \dots 254) \quad (11)$$

We transform the approximations (Eq. 6) for the  $N$  measurements ( $j = 1 \dots N$ ):

$$M(j) = f(\text{low}_j) \cdot \text{ratio}_j + f(\text{high}_j) \cdot (1 - \text{ratio}_j) - f(\text{background}_j) \quad (12)$$

Now we have two separate sets of conditions which impact the shape of the desired function: the *smoothness conditions*  $S(i)$  and the *measurement conditions*  $M(j)$ . The latter can be used also by two different modes. One is to take them as hard constraints, i.e.,  $M(j) = 0$ , the other is to minimize them together with the other conditions.

It can be argued that there is no exact measurement, at least because setting their values should give an exact real number, because the measurements can only be chosen from a discrete set of numbers actually. On the other hand, the user can introduce errors by his/her estimation as well, so in addition there can even be more or less contradictory conditions. The problem is solved as compound minimum problem of the smoothness and measurement conditions, and their importances are accounted for by weight factors  $s_i$  and  $m_j$ . The optimal result would have all of the expressions  $S(i)$  and  $M(j)$  equal to zero ( $i = 1 \dots 254, j = 1 \dots N$ ), so we have to minimize the expression

$$F = \sum_{i=1}^{254} s_i \cdot S(i)^2 + \sum_{j=1}^N m_j \cdot M(j)^2, \quad (13)$$

where, from Eq. (7),  $f(0)$  and  $f(1)$  are constant, and  $F$  is a 254-variable function. As a result we get a smooth function conforming well to the measurements, as expected. All in all there are  $256 - 2 + N$  minimum criteria and 2 constraints despite the original 256 variables. These obviously cannot be made zero at the same time, so the solution will be a compromise depending on the weights  $s_i, m_j$  and the content of  $M(j)$ .

There are several efficient methods to solve the quadratic minimum problem, two of which are mentioned here. One is solving  $F$  using a system of 254 linear equations with a sparse matrix. The other is directly solving the minimum problem by an appropriate descent method [15].

We have chosen a conjugate gradient method, which is a modification of the steepest descent method, and in our case is faster by order of one magnitude than the ordinary steepest descent (gradient) method. An optimization needs 20–50 elementary steps, each of them consisting of an evaluation of  $F$  and its derivative.

A problem not yet mentioned is the definition of the weights  $s_i$  and  $m_j$ . Considering the equal importance of the different smoothness conditions and the different measurement conditions respectively, we assume all  $s_i = s$  and  $m_j = m$ . Multiplying the whole problem by a constant we can set  $s = 1$ , so it is enough to define  $m$ .

To define this value, let us consider the overall behaviour of the solution. Optimizing the total smoothness leads to spline-like curves, where the magnitude of the average value of the second derivative is  $1/255^2$ , and its overall sum is  $1/255$ . Minimizing its distribution, we get a slowly changing second derivative, i.e., a curve behaving locally similar to a polynomial of about 3rd degree. A sudden jump causes an  $O(1)$  constant anomaly, so if the magnitude of  $m$  is between 1 and  $1/255$ , we have a locally and also globally well behaving function. The method is described in more details by Neumann et al. [1].

## 6.2 Solution in log-log Coordinate System

Considering that the optimization introduced above tries to reach maximal smoothness which leads to functions behaving locally like 3rd order polynomials, and also considering that the display characteristics used to be approximately a power function which differs from this one, another approach was investigated.

We transform the coordinate system, the domain as well as the range, to a log-log scale so that

$$\log y = g(\log(x/255)) \quad \text{where} \quad y = f(x) \quad (14)$$

Linear functions of this coordinate system are corresponding to  $c \cdot x^p$  power functions in the original system, and the minimum problem results in the possibly smoothest functions, that is, the functions must be similar to linear functions, so this transformation looks like the appropriate way to get power-like functions in the original coordinate system.

### 6.2.1 Problem Statement

Taking the new variables, the coefficients  $s_i$  of the smoothness conditions  $S(i)$  will change, and also the measurement conditions  $M(j)$  shall be rewritten with exponential functions of the new variables, since the formula is applicable to the original values. All in all another minimum problem is to be solved by the conditions:

$$S(i) = s_{i,i+1} \cdot f(\log(i+1)) + s_{i,i-1} \cdot f(\log(i-1)) + s_{i,i} \cdot f(\log i) \quad (15)$$

with ( $i = 2 \dots 254$ ), where, from the finite difference formulas,

$$\begin{aligned} s_{i,i-1} &= \frac{2}{(\log i - \log(i-1)) \cdot (\log(i+1) - \log(i-1))} \\ s_{i,i} &= \frac{2}{(\log(i+1) - \log i) \cdot (\log i - \log(i-1))} \\ s_{i,i+1} &= \frac{2}{(\log(i+1) - \log i) \cdot (\log(i+1) - \log(i-1))} \end{aligned}$$

$S(1)$  has been omitted, since the variable  $y_0 = -\infty$  is omitted anyway, which means that no smoothness condition could be defined using this variable. This is neither a real problem in this coordinate system, nor in its retransformed linear version. (Behind this little but strange point the characteristics of the power-like functions can be found.) All in all, the problem is simpler by one variable, which gives one more degree of freedom. This additional freedom lies behind the phenomenon that an arbitrary power function can be defined by one measurement, as it can be done by e.g. the simple gamma applets.

The measurement conditions change into

$$\begin{aligned} M(j) &= \exp\left(f(\log(\text{low}_j))\right) \cdot \text{ratio}_j \\ &+ \exp\left(f(\log(\text{high}_j))\right) \cdot (1 - \text{ratio}_j) \\ &- \exp\left(f(\log(\text{background}_j))\right) \end{aligned} \quad (16)$$

where ( $j = 1 \dots N$ ).

The minimum problem (Eq. 13) can be written with the expressions above in order to obtain the form of the directly transformed measurement conditions, but in this case we face two new problems. One is the convexity of the measurement conditions, corresponding to Eq. (16). The square of these expressions will not be convex, which leads to some algorithmical difficulties, especially when considering the original dimensionality of the problem, which has 256 variables.

The other problem is much more crucial. Conditions in Eq. (15) can be weighted arbitrarily, their weights expressing their individual importance. If different weights are used, they could

distort the overall smoothness depending how unbalanced they are, that is, the function would be smoother in one part and less smooth in another. In a drastic case it destroys the original expectation, that is, the function will not behave like a power function, which was the argument to apply the axis transformation in the first place. Unfortunately, if the weights are equal, the magnitudes of the coefficients will be enormously different, their maximum value will be

$$\begin{aligned} s_{2,2}/s_{254,254} &= \frac{(\log 255 - \log 254) \cdot (\log 254 - \log 253)}{(\log 3 - \log 2) \cdot (\log 2 - \log 1)} \\ &\approx \frac{1/254.5 \cdot 1/253.5}{1/2.5 \cdot 1/1.5} \\ &\approx 2^{-14} \end{aligned}$$

which leads to unsolvable numerical instabilities when minimizing them, especially by taking their squares the magnitude will be squared as well ( $2^{-28}$ ). So it is obvious that the log-log transformation with the form of the minimum problem cannot work because of numerical reasons.

### 6.2.2 A Two Pass Solution for the log-log Problem

There are two sets of points on the transformed independent axis, which play a special role in our problems.  $\mathcal{S}$  consists of all points involved in the smoothness conditions (Eq. 15). The other, set  $\mathcal{M}$ , is a subset of the 256-element set  $\mathcal{S}$ .  $\mathcal{M}$  consists of all points playing any role in the measurement conditions (Eq. 16). Let us consider the next smoothness conditions written for the triplets of the sparser set  $\mathcal{M}$ .

$$\begin{aligned} S_{\mathcal{M}}(k) &= s_{\mathcal{M}_{k,j}} \cdot f(\log j) \\ &\quad + s_{\mathcal{M}_{k,k}} \cdot f(\log k) \\ &\quad + s_{\mathcal{M}_{k,l}} \cdot f(\log l) \end{aligned} \tag{17}$$

where  $(\log j, \log k, \log l)$  is a successive triplet of  $\mathcal{M}$ , and, from the finite difference formulas,

$$\begin{aligned} s_{\mathcal{M}_{k,j}} &= \frac{2}{(\log k - \log j) \cdot (\log l - \log j)} \\ s_{\mathcal{M}_{k,k}} &= \frac{2}{(\log l - \log k) \cdot (\log k - \log j)} \\ s_{\mathcal{M}_{k,l}} &= \frac{2}{(\log l - \log k) \cdot (\log l - \log j)} \end{aligned}$$

It can be seen that solving the minimum problem of the smoothness criteria (Eq. 15) over  $\mathcal{S}$ , and taking its values only over the subset  $\mathcal{M}$ , this restricted solution can be taken as a good quasi-minimum for the problem of the smoothness criteria (Eq. 17) defined directly over  $\mathcal{M}$ . And vice versa, solving the problem just over  $\mathcal{M}$  and fixing its results, then solving the previous problem by these constraints, the obtained overall solution will also be a good quasi-minimum for the non-constrained original problem.

It can be seen as well that the situation is similar if other, new conditions are added to the original problem. As its main example, the measurement conditions of Eq. (16) can be added to both versions of the smoothness problem above, that is, to the original condition system of Eq. (15), and also to its extension by Eq. (17) over  $\mathcal{M}$ , and their solutions will approximate each other well.

Let us suppose that the minimum problem of this extended condition system has already been solved. Now, let values  $f(\log j)$  be fixed for each  $\log j$  from  $\mathcal{M}$ , and the minimum problem be solved again with this additional constraints. Let us recognise that the solution of this second problem cannot differ from the non-constrained one, since this solution could be a better solution also for the first problem, contradicting its minimality! Therefore the same solution must be obtained for both of them.

After this recognition let us consider the series of problems:

- A – Smoothness condition (15) over  $\mathcal{S}$ 
  - Measurement condition (16) over  $\mathcal{M}$
  - no constraints
- B – Smoothness condition (15) over  $\mathcal{S}$ 
  - Smoothness condition (17) over  $\mathcal{M}$
  - Measurement condition (16) over  $\mathcal{M}$
  - no constraints
- C – Smoothness condition (17) over  $\mathcal{M}$ 
  - Measurement condition (16) over  $\mathcal{M}$
  - no constraints
- D – Smoothness condition (15) over  $\mathcal{S}$ 
  - Smoothness condition (17) over  $\mathcal{M}$
  - Measurement condition (16) over  $\mathcal{M}$
  - constraints of solution C over  $\mathcal{M}$
- E – Smoothness condition (15) over  $\mathcal{S}$ 
  - constraints of solution C over  $\mathcal{M}$

$A$  is the original problem, it gives an equivalent solution with  $B$ , and  $B$  gives also an equivalent solution over  $\mathcal{M}$  with  $C$ .  $D$  gives also an equivalent solution with  $C$ , so that they are equivalent over  $\mathcal{M}$ . Finally, as proven above, the solution of  $E$  is identical to the solution of  $D$ .

This implies a two pass method resulting in a well-approximating solution of the original minimum problem.

First, problem  $C$  shall be solved, which is a minimum problem in few variables, containing also slightly non-convex components, but its technical problems can be overcome with moderate effort.

Second, problem  $E$  shall be solved, which includes all the numerical instabilities, but in this form it is a pure smoothness problem. This problem is identical with defining a spline across a finite set of given points. Independent from the number of given points, it leads to a 2-dimensional optimum problem defining interval-wise 3rd order polynomials. These polynomials are defined by their coefficients instead of a set of points of the domain that would promise numerical difficulties.

For the last step a reversion shall be performed, transforming the  $\log y_i = f(\log i)$  that is  $\log y_i = f_i$  values back to  $y_i = \exp f_i$ . The obtained curve is a piecewise transition of power-like base functions, so it coalesces the behaviour of the power function and changes one power function to another in the smoothest way in the sense of the log-log coordinate system.

### 6.2.3 Discussion of the Methods

The original linear problem is a 256 variable,  $256 + N$  condition minimum problem, with  $N \ll 256$ . It is solved in practice by a steepest descent method, which is a stable and fast algorithm.

The behaviour of the approximation highly depends on the balance between the smoothness conditions and measurement conditions. The solution of a compound quadratic optimum problem has been selected in order to compensate for the possible errors of the measurements, which are actually human perception based, but errors would come into account in any sort of measurements. In case of absolute weights of measurements, that is, assuming their absolute accuracy, a spline problem is implied, similar to the second pass of the two pass method. This case shows the nature of the obtained curve better, which is an interval-wise 3rd-order polynomial in a general case too. Interval-wise 3rd-order polynomials of a spline realize the smoothest transition between their supporting points or, in other words, the smoothest transformation of linear segments in order to

link them together smoothly. This character is watermarked also in a softer weighting similar to the weights given in the linear coordinate system.

The second method using the log-log coordinate transformation also keeps the smoothness, since the transformation of the coordinate system has been introduced just in order to conform the preferred shapes to the displays. It is only a practical question, but just because of that, it can result in a better practical solution.

After the coordinate transformation, a two pass separation of the algorithm has been applied in order to overcome the numerical instabilities. In fact this method could have been applied also for the original “lin-lin” coordinate system, and this could reveal that the solution is also a spline in any case because of the equality of the solutions of the constrained and non-constrained problems, as it has been proven in the previous section. However, it is clear that the quasi-equivalences of the previous section are not strict equivalences, so this solution is not the theoretical optimum.

Nevertheless, applying this two-step process to our case we have decreased the problem of the weights, which is addressed in both steps to the same domain  $\mathcal{M}$  where they are naturally commensurable. On the other hand, the second pass is also obvious, and both are fast. In addition, one can introduce additional freedom into the spline computation, allowing the values over  $\mathcal{M}$  not to be exact, but the function being smoother. This leads to a similar question as it has appeared with the lin-lin method’s weighting, but in a much simpler form and in a faster algorithm. All in all, it means that the theoretical exactness has been sacrificed for getting a faster, simpler result which is still extendable to a similarly flexible solution.

In addition it can be seen that the applied methods and forms are secondary compared to the original problem, which is not exactly formalized, but a matter of decision.

## 7 Results

To evaluate the ability of our method to capture the real TRC curve of a monitor, we compare it with results from direct measurements of the TRC curve using a GretagMacbeth Spectrolino spectrophotometer. In order to demonstrate the general usability of the method, the experiments are performed on two different monitor technologies: Cathode Ray Tube (CRT, Nokia 446Pro) and Liquid Crystal Display (LCD, EIZO FlexScan L985EX). The Spectrolino measured 52 color values per channel, with color values 0 to 255 increasing in equal steps of 5.

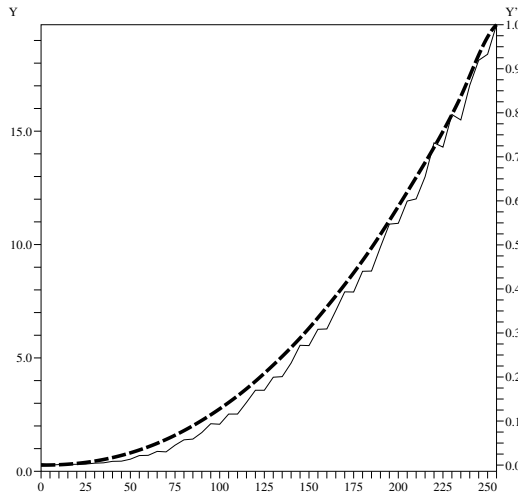
Figure 4 shows the results for the CRT Monitor in the left half, for the LCD monitor in the right half, for all three channels. In this figure the TRCs obtained as result of the spectrophotometer measurements are taken as reference (solid line). The TRCs obtained as result of our method (dashed line) are practically able to reproduce the reference curve, or a close approximation. This is valid for both monitors used in the experiments. Note the jaggedness in the Spectrolino CRT curves caused by the different colour depth setting.

## 8 Conclusion and Future Work

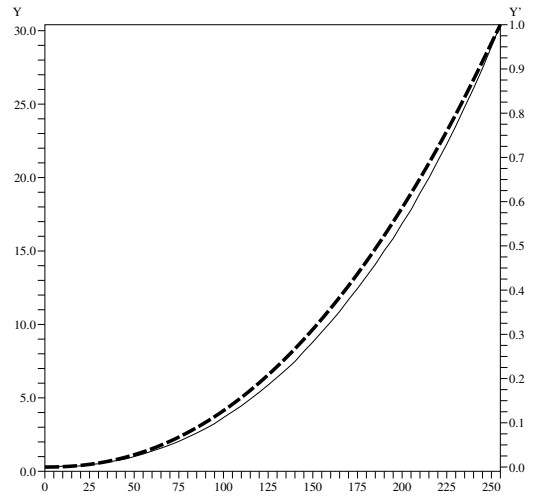
A significant improvement of our model for gamma display characterization [1] has been presented in this paper which is able to overcome the limitations of the previous version, in particular, solving the problems of instability and unequal weighting. This method is simplified and made more reliable by changing the simple smoothness principle to a log-log scale based one. A trade-off between the smoothness and observation conditions is analyzed, improving the reliability of the method’s results.

The flexibility of the model allows it to be used in many applications without the necessity to use a spectrophotometer. Also a fast and simple (re)characterization is possible, just using an interactive process with the end user. The main benefit of this approach is that it is a cheap and simple solution, either to define a relative TRC of a display or to verify a given one.

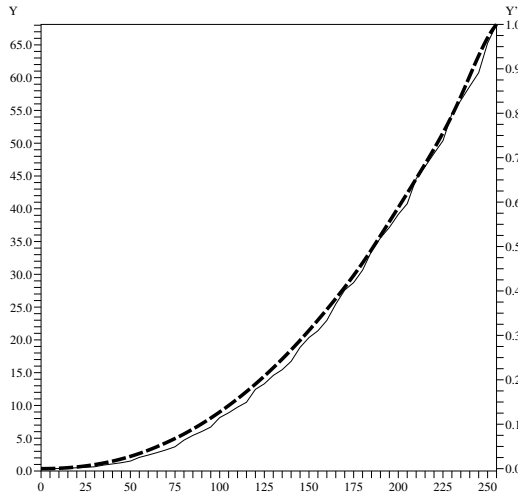
There are a couple of open possibilities in the method. On one hand its usability could be further improved by taking information about the absolute values, that is, about the contrast



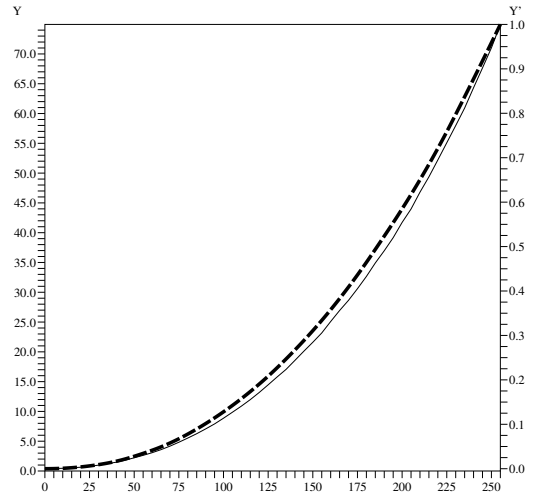
(a) CRT, red channel



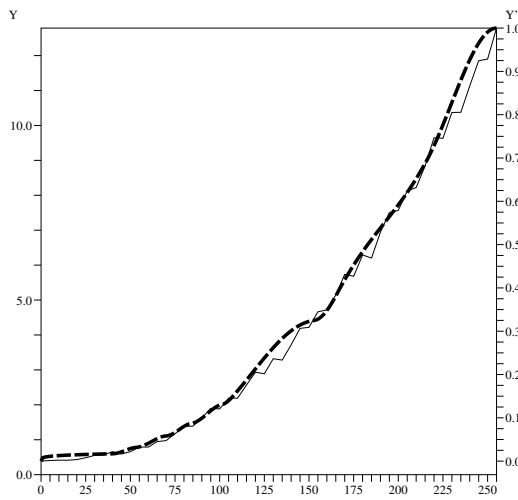
(d) LCD, red channel



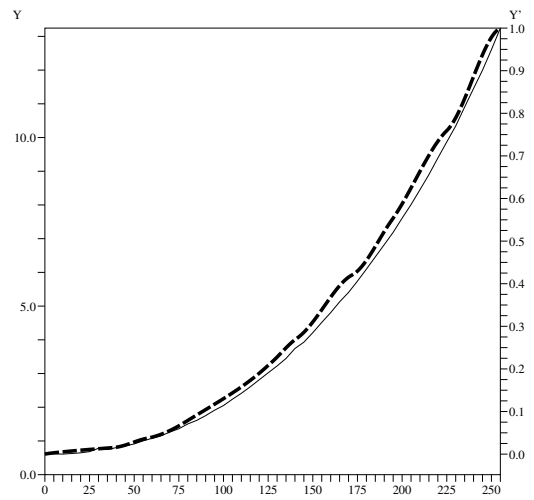
(b) CRT, green channel



(e) LCD, green channel



(c) CRT, blue channel



(f) LCD, blue channel

Figure 4: Comparison of TRC acquired with the new method (thick dashed line) and Spectrolino measurements (thin solid line)

value and the smallest (or the largest) value of the absolute luminance, either staying with human perception based input or using other methods.

On the other hand, it should be possible to solve the phenomenon of the cross effect between the colour channels, and the method may be extended for a multidisplay system application.

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## References

- [1] A. Neumann, A. Artusi, G. Zotti, L. Neumann, and W. Purgathofer. An Interactive Perception Based Model for Characterization of Display Devices. In *Proc. SPIE Conf. Color Imaging IX: Processing, Hardcopy, and Applications IX*, pages 232–241, January 2004.
- [2] R. S. Berns, R. J. Motta, and M. E. Gorzynski. CRT colorimetry – Part I: Theory and practice. *Color. Res. Appl.*, 8(5):299–314, Oct. 1993.
- [3] R. S. Berns, R. J. Motta, and M. E. Gorzynski. CRT colorimetry – Part II: Metrology. *Color. Res. Appl.*, 8(5):315–325, Oct. 1993.
- [4] M. D. Fairchild and D. Wyble. Colorimetric characterization of the apple studio display (flat panel LCD). Technical report, Munsell Color Science Lab., Rochester Institute of Technology, Rochester, NY, July 1998.
- [5] E. Day. Colorimetric Characterization of a Computer Controlled (SGI) CRT Display. Technical report, Munsell Color Science Lab., Rochester Institute of Technology, Rochester, NY, April 2002.
- [6] N. Kato and T. Deguchi. Reconsideration of CRT Monitor Characteristics. In *Color Imaging: Device Independent Color, Color Hardcopy, and Graphics Arts III*, pages 33–40, 1998.
- [7] D. L. Post and C. S. Calhoun. An Evaluation of Methods for Producing Desired Colors on CRT Monitors. *Color. Res. Appl.*, 14(4), Aug. 1989.
- [8] N. Tamura, N. Tsumura, and Y. Miyake. Masking Model for Accurate Colorimetric Characterization of LCD. *Journal of the SID*, 11(2):1–7, 2003.
- [9] R.W.G. Hunt, editor. *Measuring Color*. Ellis Horwood, 2nd edition, 1992.
- [10] M. D. Fairchild, editor. *Color Appearance Models*. Addison-Wesley, 1998.
- [11] Russell L. De Valois and Karen K. De Valois. *Spatial Vision*. Number 14 in Oxford Psychology Series. Oxford Univ Press, September 1990.
- [12] J.A. Schirillo and S.K. Shevell. Brightness Contrast from Inhomogeneous Surrounds. *Vision Research*, 36(12):1783–1796, 1996.
- [13] K. Tiippana and R. Nasanen. Spatial-frequency bandwidth of perceived contrast. *Vision Research*, 39(20):3399–3403, 1999.
- [14] A. Watson and C. Ramirez. A Standard Observer for Spatial Vision. [http://vision.arc.nasa.gov/publications/watson\\_arvo2000.pdf](http://vision.arc.nasa.gov/publications/watson_arvo2000.pdf).
- [15] R. Fletcher. *Practical Methods of Optimization*. John Wiley & Sons, 2nd edition, 2001.

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Attila Neumann received his M.Sc. degree in Mathematics from the ELTE University of Science at Budapest in 1982 and a Ph.D. in Computer Graphics from Technical University of Vienna in 2001. After dealing with several mathematical problems his interest has turned to computer graphics in the beginning of the nineties. His main interest in this area is concerning Bidirectional Reflectance Distribution Functions (BRDFs), global illumination, image processing and color science. He is a member of the EG and HunGraph.

### Alessandro Artusi

Alessandro Artusi is an Assistant Professor in Computer Science, holds an M.Sc. degree in Computer Science from University of Milan (Italy) and a PhD degree in Computer Graphics from the Vienna University of Technology, Austria (Institute of Computer Graphics and Algorithms). In 1998 he joined the CNR Italy where he was working on a National project founded by the MURST. In 2000 he joined the Institute of Computer Graphics and Algorithms of the Vienna University of Technology, where he participated in several international and national research projects, international collaborations and teaching as well. In January 2005 he joined Intercollege, and he is currently an Assistant Professor at Intercollege, Cyprus.

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### László Neumann

László Neumann received his M.Sc. degree in Engineering and Mathematics from the Technical University of Budapest in 1978 and a Ph.D. in the field of operation research also from TU Budapest in 1982. He is currently research professor of ICREA (Institució Catalana de Recerca i Estudis Avançats, Barcelona). After dealing with economical optimization, digital terrain modeling, stochastic problems as head of a software department at the Mining Research Institute in Budapest, he turned to problems of different fields of imaging from realistic image synthesis, to medical image processing, visualization and color theory/color dynamics. He is visiting lecturer since 1995 at TU Vienna, member of the EG, the HunGraph and the Color Committee of Hungarian Academy of Science.

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