# A Reflectance Model for Diffuse Fluorescent Surfaces

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Figure 1: A synthetic image where our proposed BRDF model is used on a fluorescent orange surface that is being illuminated by several collimated monochrome light sources. The scene geometry is similar to that shown in figure 2. Note the colours of the directly viewed bright dots on the material itself, and the in some cases considerably different colours seen in the reflection patterns. It is noteworthy that the blue and green monochrome lights (second and third light from the left), which fall into the main area of the absorption curve shown in figure 4, exhibit the largest colour discrepancies between specular and diffuse reflection.

## Abstract

Fluorescence is an interesting and visually prominent effect, which has not been fully covered by Computer Graphics research so far.

While the physical phenomenon of fluorescence has been addressed in isolation, the actual reflection behaviour of real fluorescent surfaces has never been documented, and no analytical BRDF models for such surfaces have been published yet.

This paper aims to illustrate the reflection properties typical for diffuse fluorescent surfaces, and provides a BRDF model based on a layered microfacet approach that mimics them.

**CR Categories:** I.3.7 [Computer Graphics]: Three-Dimensional Graphics and Realism

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## 1 Introduction

Although they are not always obvious at first glance, fluorescence effects can be found in a large and diverse number of objects

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and substances, such as the natural chlorophyll of algae, seawater [Gutierrez et al. 2005], gemstones and the artificial day-glo pigments found on warning signs and markers.

Unlike normal surfaces that just reflect more or less incident light depending on the viewing and illumination geometry, fluorescent surfaces have the unusual property that they can also change the wavelength of the reflected light through processes at the molecular level of the surface pigments.

While the basic technical issues of integrating fluorescence into rendering systems have already been investigated as far as colour computations are concerned [Glassner 1995; Wilkie et al. 2001], to our knowledge no analytical BRDF models of fluorescent surfaces have been proposed so far.

In this paper, we present a simple experiment that enables us to analyse and discuss the qualitative behavior of diffuse fluorescent surfaces. Based on these observations, we propose a suitable microfacet-based layered BRDF model that exhibits similar characteristics.

## 2 Physical Background

For our work it is important to note that fluorescence is a phenomenon that happens at a purely molecular level. As a consequence of this, practically all reference texts from physics which deal with the actual mechanisms of this wavelength shift concentrate on the quantum mechanical details of the process, and are not applicable to the problem of modelling fluorescence effects at a macroscopic level (i.e. at the level of a surface BRDF).

However, they do document the point that – contrary to the behaviour of "normal" materials – the re-emission of photons that interact with fluorescent matter does not necessarily occur at the same energy level at which they entered. This results in frequency shifts during reflection processes, and ultimately colour shifts that are perceived by human observers [Nassau 1980; Glassner 1995].

Both the case of re-emission at lower energy levels and the case of two lower energy photons being combined into a single higherenergy photon are possible in nature. However, as was already pointed out by Glassner [Glassner 1995] only the first case is of major interest for the purposes of Computer Graphics, since it is the governing phenomenon behind practically all objects which exhibit noticeable fluorescence under normal illumination.

The re-radiation of energy happens within an extremely short time from incidence (typically  $10^{-8}$  seconds) and causes a shift of the colour towards longer wavelengths; see figure 4 for an example.

## 3 Motivation for our Work

To qualitatively document the large variability of the colour shifting effect induced by diffuse fluorescent surfaces (and as a consequence the need for non-trivial BRDF models that capture this effect), we performed a simple experiment that can be easily replicated by any-one wishing to see the phenomenon first-hand.

### 3.1 Reflection Properties of Real Fluorescent Surfaces

What we wanted to demonstrate is the non-trivial correlation between the amount of colour shift and the illumination geometry for a given, fairly typical diffuse fluorescent surface.

We used two types of stock orange cardboard bought at an office supply shop (one of "normal" orange colour, and one fluorescent orange), which we illuminated with a commercially available green laser pointer. The set-up of our experiment, which can be seen in figure 2, allowed us to change the angle of incidence of the green laser light on the cardboard sample, and to observe the reflection pattern on a white diffuse cardboard target.

For the sake of comparison we first pointed the laser beam at a plain, non-fluorescent orange cardboard sample. Figure 3 shows the reflection pattern we obtained: as was to be expected the reflected light was completely green, and there was no color shift.

We then repeated the same experiment, but this time with the fluorescent day-glo cardboard sample. Figure 8 (b) shows the result we obtained for the same incident angle as in figure 3: there is a very noticeable shift of the monochromatic incident green light towards the orange range of the spectrum.

Since some green light was still evident in the reflection pattern (which suggested the presence of a specular reflection mechanism), we repeated the experiment with two additional angles of incidence; see figure 8 for the results. For a small angle of incidence, a significant amount of light was reflected in unchanged green form, while for a large angle of incidence the reflection colour was largely shifted towards orange hues.

Such a marked dependency of the reflection colour on the incident angle can obviously have a significant impact on the appearance of any scene which contains such surfaces.

BRDF models which use a single, constant re-radiation matrix to describe fluorescence – i.e. all normal analytical BRDF models which are just "retrofitted" to use fluorescence data instead of plain



Figure 2: Overview of the experimental setup. Since our aim was just to provide a qualitative visualisation of the phenomenon we are trying to represent, we used a comparatively simple setup: a green handheld laser pointer (wavelength 532 nm) fixed to a mount, material samples  $4 \times 4$  cm in size, and a small grid printed on diffuse white paper as target for the reflection pattern. The scene was captured with a Nikon 4500 digital camera, which offers the possibility to manually control all relevant parameters such as aperture and shutter speed.

reflectance values, as it has been common practice so far – cannot mimic such behaviour, so a specialised approach is needed.

### 3.2 Limitations of our Experiment

It should be noted that our goal in this experiment was not the acquisition of quantitative BRDF data (a bi-spectral gonio-reflectometer would have been necessary for this), but just the unequivocal documentation of the fact that the frequency shift typical for fluorescence can be highly dependent on exitant angle within a given BRDF.

### 3.3 Explanation of the Observed Effects

In order to derive a usable BRDF model for fluorescent diffuse surfaces we first have to consider the reason for the directional dependency of the reflection colour. As it turns out, a quite simple explanation is sufficient in this case.

For fluorescence effects to happen, a photon that is reflected from the surface has to interact with its colorant molecules. This implies a penetration of the diffuse pigment substrate, and leads to a more or less diffuse distribution of the resulting radiant energy that has been subjected to a frequency shift.

All photons which fail to penetrate the pigment layer retain their original wavelengths, and are only attenuated by the normal, non-fluorescent reflection spectrum of the surface for this type of light–surface interaction. This will typically be the reflection spectrum of the substrate, which may be either clear or opaque, but – more importantly – is usually colourless.

Cases (a) and (b) in figure 5 sketch those two cases of rays: the ones that are specularly reflected do not undergo any color shift, while the diffuse rays which penetrate the fluorescent molecules do.



Figure 3: Reflection pattern on a real plain orange cardboard sample for an incident angle of about 20 degrees. Since no fluorescent surface is involved, both the immediate bright dot on the material and the reflection pattern are monochromatic green with no hints of orange. This photograph is to be compared with those of figure 8 which involve a fluorescent day-glo orange cardboard sample.

The related work presented in the following section will therefore focus those BRDF models that allow a separation of diffuse and specular reflection components.

## 4 Related Work

#### 4.1 Rendering of Fluorescence Effects

Most current rendering systems are not capable of handling fluorescence effects at all, although the technique needed to integrate them on a basic level has been known for quite some time.

Glassner [Glassner 1994] introduced so-called re-radiation matrices – which describe the transfer of energy between different wavelengths – as the basic data structure needed to model fluorescent light-surface interactions in a renderer.

An individual re-radiation matrix encodes a fixed amount of energy transfer between wavelengths, just as a given reflectance spectrum encodes a fixed amount of light absorption.

As a consequence, any conventional surface model which just uses such a matrix in place of a plain reflectance value (e.g. as a dropin replacement for the diffuse attenuation factor  $k_d$  in the Lambert BRDF) will exhibit a fixed amount of wavelength shift, regardless of incident and exitant angle. Which – as we demonstrate in section 3 – is not what happens in real surfaces.

### 4.2 Models of Surface Reflectance

Computer Graphics uses two distinct approaches to deliver realistic surface reflectance behaviour. The first one is the measurement of real surfaces, and the subsequent use of the gathered reflectance data in the rendering process. The second one is the derivation and use of analytical models of varying realism.



Figure 4: The effects of bispectral reflectivity in the case of a sample day-glo type fluorescent orange surface similar to the one used in our experiment. This graph shows **a**) the non-fluorescent reflection spectrum (the main diagonal of the re-radiation matrix), **b**) the energy absorbed at longer wavelengths, **c**) the energy re-radiated at shorter wavelengths and **d**) the resulting "reflection" spectrum for an equal-energy illumination of 1.0. Note that the resulting spectrum is well over 1.0 in some areas. Fluorescence data courtesy of Labsphere Inc.

#### 4.2.1 BRDF Measurements

In this paper we focus on the analytical approach, mainly because we do not have access to measurement equipment with bi-spectral capabilities that could be used to accurately capture the behaviour of fluorescent surfaces. We are also not aware of any work where such measurements were undertaken in a form that is usable for Computer Graphics purposes.

It has to be noted that normal, non-fluorescent BRDF measurements and their use in image synthesis applications are already very demanding [Foo 1997; Cornell University BRDF measurement archive ], and using equipment with bi-spectral capabilities would significantly increase the already significant storage requirements and complexity of such an effort.

#### 4.2.2 Analytical BRDF Models

Due to the reflectance properties of fluorescent surfaces shown in section 3, we are here primarily interested in those analytical BRDF models which allow a separation of diffuse and specular reflection components. Such models are usually compound approaches, where two or more basic BRDFs are combined to yield a more complex and realistic surface model.

Perhaps the oldest and best-known compound BRDF model in Computer Graphics is the combination of the diffuse Lambert BRDF [Lambert orig. 1760, reprint with translation and commentary 2001] with the Phong model [Phong 1998]. This approach yields surfaces with (usually) white specular highlights, which roughly corresponds to the appearance of coloured plastic. It has certain well-known deficiencies as far as physical plausibility



Figure 5: The two different mechanisms at work on fluorescent surfaces: **a**) rays that are directly reflected off the substrate are not subjected to colour shifts while **b**) rays that penetrate the substrate and interact with the wavelength-shifting colorant molecules are.

is concerned, but is still frequently used due to its simplicity and robustness.

The compound BRDF technique introduced by Lafortune et al. [Lafortune et al. 1997] formalised and generalised this type of approach, and provided a framework to model almost arbitrary BRDFs that could be easily sampled through the component functions they are assembled from.

More sophisticated reflectance models – such as those by Cook and Torrance [Cook and Torrance 1981] or by Oren and Nayar [Oren and Nayar 1994] – are usually based on a microfacet-based mathematical approach, and cannot be easily split into diffuse and specular components.

Ashikhmin et al. [Ashikhmin et al. 2000; Ashikhmin and Shirley 2000] improved the microfacet-based approach, and even though they extended them beyond the capabilites of the original results through inclusion of arbitrary microfacet characteristics, the end result is still an integrated BRDF which cannot be readily split into diffuse and specular components.

## 5 Diffuse Fluorescent Surface BRDFs

As a consequence of the mechanism discussed in section 3.3, we claim that a satisfactory approximative representation of real-world diffuse fluorescent surfaces with their small but significant specular component can be achieved through a separation of the BRDF into two distinct parts: a non-fluorescent semi-gloss specular component, and a perfectly diffuse component which accounts for all wavelength-changing interactions.

We use a standard re-radiation matrix to describe the wavelength shift that occurs in the perfectly diffuse part of the reflection. This data structure (a detailed description of which can be found in [Glassner 1994]) encodes the energy transfer between different wavelengths for a perfectly diffuse version of the colorant. Figure 4 shows a visualization of the bispectral reflectance dataset for orange day-glo pigment, which closely matches that of the orange fluorescent cardboard we used, and which we used in our simulations.

For the specular component several options are available, such as the Phong model, or a microfacet-based technique.

The open technical question is how one combines these two components. We investigated two possible approaches to this problem:

1. We first implemented a simple combination of specular Phong lobes with a fluorescent Lambertian component. This ap-

proach is – at least from a Computer Graphics standpoint – obvious and easy to implement. Somewhat surprisingly, however, it yields significantly sub-optimal results, and should probably not be used.

2. Our second, more sophisticated approach was to use a layered rough microfacet surface model with a Blinn distribution and a simplified sub-surface scattering simulation; here the results were satisfactory.

In the remainder of this section the two approaches are discussed in more detail.

#### 5.1 Phong plus Lambert

Combining these two reflection models is straightforward; the only real degree of freedom is the ratio between specular and diffuse BRDF component. In our case this relationship was determined heuristically. A very weak and small specular lobe proved sufficient to re-create a BRDF that provided a reasonable first approximation to the behaviour of fluorescent cardboard; figure 6 shows a sketch of this arrangement. The lobes were only assigned the main diagonal of the fluorescence re-radiation matrix, and the diffuse term used the entire data.



Figure 6: A 2D cut through the simple, faintly specular compound Lambert-Phong BRDF in the plane of incidence (red). It consists of a large diffuse component (blue – this component uses the full re-radiation matrix) which is augmented by a weak specular lobe made up from 3 separate Phong lobes (green – this component only uses the main diagonal reflection spectrum).

As one can see in figure 9, the resulting BRDF is capable of reproducing two-coloured reflection patterns, but lacks two important properties: the specular reflection lobe is not longitudinal but round, and the relationship between specular and diffuse component does not change with incident angle.

#### 5.2 Layered Microfacet Model

As a more realistic approach we modelled the specular reflection using a microfacet-based layered rough varnish surface, with a distribution according to a Blinn probability density function; figure 7 shows a sketch of the arrangement we used.

The simpler case of a smooth layered varnish surface was partially investigated by Neumann<sup>2</sup> [Neumann and Neumann 1989]; we extended this model so that it became usable in a path tracer, and generalised it to be applicable to rough varnish surfaces, which proved to be quite good initial models of the reflectance behaviour of diffuse fluorescent cardboard.



Figure 7: Concept of the rough varnish microfacet model we used as the more advanced diffuse fluorescent BRDF. A rough layer of transparent Fresnel facets is assumed to cover a thin layer of transparent varnish on top of a fluorescent Lambertian base layer. Fluorescence only takes place for those rays that come in contact with the base layer.

As we were not interested in macroscopic translucency effects – such as the subsurface light transport discussed by Wann Jensen et al. [Jensen et al. 2001] – we did not implement a full sub-surface scattering simulation. We instead assumed that the energy leaves at the same location where it enters the surface. Since the layers of varnish on an object are usually very thin, such a simplified implementation of sub-surface light transport is a valid assumption in this particular case.

The microfacet distribution function we used for the transparent varnish surface is a normalised Blinn distribution, with a single slope angle parameter *m* that controls its roughness for a given halfway vector  $\omega_h$ :

$$D(\boldsymbol{\omega}_h) = \frac{m+2}{2\pi} (\boldsymbol{\omega}_h \cdot \boldsymbol{n})^m \tag{1}$$

We did not use any of the more recent microfacet models (such as the work of Ashikhmin et al. [Ashikhmin et al. 2000; Ashikhmin and Shirley 2000]) because the characteristic we are mainly interested in – the separability of diffuse and specular term through the layered approach – is possible for all of these models (i.e. there would not have been any advantage in using the more recent models), and we therefore opted for using the older, simpler model as base for our experiments.

### 6 Results

We implemented both proposed BRDFs in our spectral rendering research system, which is capable of performing fluorescence calculations. With these BRDFs we then attempted to re-create the test scene shown in figure 2. We also rendered an overview scene – shown in figure 1 – where the effect of various other monochrome lights on the orange fluorescent surface we used for our tests is demonstrated.

The images of figures 9 and 10 were obtained with a bi-directional path tracer [Lafortune and Willems 1993; Veach and Guibas 1995].

### 6.1 Accuracy of Our Results

Our goal was to demonstrate that the BRDFs proposed in section 5 can indeed capture the basic properties of fluorescent BRDFs we observed in our practical experiment; as figures 9 and 10 demonstrate we achieve this with the more complicated model. The match

between the simulation and reality is not perfect, but the qualitative similarity of the results generated with the layered varnish surface is apparent.

There are some visual discrepancies between the rendered images and the photographs; in the remainder of this subsection we briefly discuss the main causes of these.

#### 6.1.1 Laser Speckle

A noticeable – but due to general limitations of ray-based image synthesis inevitable – difference to the photographs is the absence of laser speckle effects (i.e. the grainy pattern in the real reflection); these would only be tractable in a renderer capable of wave optics.

#### 6.1.2 Shape of the Green Reflection Pattern

For the simple model the big discrepancies between the photographs and the rendering are the round shape of the simulated green highlight, which is caused by the round cosine lobe of the Phong model we use, and the fact that the intensity of the green lobe does not vary with incident angle.

Both of these deficiencies are largely alleviated by the use of the layered microfacet model, and we assume that a further improvement of the already good correspondence between the reflection patterns and the photographs could be achieved through further fine-tuning of the parameters used for the varnish model.

### 6.2 Performance Considerations

Apart from a slight runtime penalty for the inclusion of the more costly matrix evaluations used in the Lambertian term the rendering times were unaffected by this, since all we did was to assign different reflection spectra to the components of the compound BRDF of a single surface.

Arguably the evaluation of the compound BRDF would actually be even slower if the – inaccurate – approach of indiscriminately using the full re-radiation matrix in all components were to be used.

## 7 Conclusion

We presented a case for using specialised BRDFs when rendering diffuse fluorescent surfaces, and presented a technique based on a layered microfacet approach which falls into this category.

The diffuse fluorescent BRDF described in this paper should also be applicable to the modelling of specular fluorescent paint-work – such as that used on ambulances or SAR vehicles – through modifications to the roughness parameter. Establishing a match between specular paint-work samples and rendered images will be part of our future work in this direction.

We also plan to investigate the separation between the diffuse and fluorescent case more closely, and attempt to find better analytical expressions for it than the rather simple "hard" split between diffuse and specular terms we propose in this paper. Effective as it is, it would be naïve to assume that it is a fully accurate model of the underlying processes. Predictive rendering will eventually demand a model which has an even better rooting in theory than the layered varnish approach.



Figure 8: Reflection Patterns on a real diffuse day-glo orange cardboard sample: **a**) incident angle of less than 5 degrees; **b**) incident angle of about 20 degrees; **c**) incident angle of about 60 degrees. Note that while the bright dot where the laser hits the surface – and also parts of the reflection pattern – are a bright orange caused by the frequency shift typical for this kind of fluorescent colorant, other parts of the reflection – essentially the specular component – are still the native green of the incident laser beam. **a**) is a nice visualisation of the well-known effect that even seemingly very diffuse surfaces get more specular at grazing angles. In case **c**), almost no green specular component is evident; practically the entire reflection is orange and has been modified by the fluorescence effects in the colorant molecules.



Figure 9: A rendering of a scene similar to the setting of our real-world example shown in figure 8, for similar incident angles, using the simple approach of combining Phong lobes with a Lambert model: **a**) angle of incidence of 5 degrees; **b**) angle of incidence of 20 degrees; **c**) angle of incidence of 60 degrees. Compare this to figures 8 (photographs) and 10 (layered microfacet model). The main deficiencies of the Phong compound model are that the green reflection patterns are not realistic (being round, and not oblong as they should be), and that the intensity of the green reflection does not change with incident angle like it does in reality.



Figure 10: A rendering of a scene similar to the setting of our real-world example shown in figure 8, for similar incident angles, using the layered varnish model: **a**) angle of incidence of 5 degrees; **b**) angle of incidence of 20 degrees; **c**) angle of incidence of 60 degrees. The parameter m of the microfacet distribution was set to a value which corresponds to a mean slope angle of 17 degrees. Note that some of the visual differences to the images in figure 8 are due to the differences between the tone reproduction process used for the synthetic images, and the characteristics of the digital camera.

In order to conduct an investigation into the exact nature of the split we would like to perform accurate spectral reflectance measurements with a gonioreflectometer and/or calibrated CCD, and partly base our further strategy on an analysis of the data obtained through such a setup.

We also plan to quantify the impact of the phenomenon for the case of polychromatic illumination; we expect the influence of the fluorescence on the shape of the BRDF to be significantly less in most cases, although it probably still contributes noticeable effects.

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