Volume Visualization



Introduction to volume visualization

- On volume data
- Voxels vs. cells
- Interpolation
- Gradient
- Classification
- Transfer Functions (TF)
- Slice vs surface vs. volume rendering
- Overview: techniques







Simple methods

- Slicing, multi-planar reconstruction (MPR)
- Direct volume visualization
 - Image-order vs. object-order
 - Raycasting
 - $\diamond \alpha$ -compositing
 - Hardware volume visualization
- Indirect volume visualization
 - Marching cubes





Introduction:

VolVis = visualization of volume data

- Mapping 3D→2D
- Projection (e.g., MIP), slicing, vol. rendering, …

Volume data =

- 3D×1D data
- Scalar data, 3D data space, space filling

User goals:

- Gain insight in 3D data
- Structures of special interest + context





Where do the data come from?

- Medical Application
 - Computed Tomographie (CT)
 - Magnetic Resonance Imaging (MR)
- Materials testing
 - Industrial-CT
- Simulation
 - Finite element methods (FEM)
 - Computational fluid dynamics (CFD)

◆ etc.







How are volume data organized?

- Cartesian resp. regular grid:
 - CT/MR: often dx=dy<dz, e.g. 135 slices (z) á 512² values (as x & y pixels in a slice)
 - Data enhancement: iso-stack-calculation = Interpolation of additional slices, so that $dx=dy=dz \Rightarrow 512^3$ Voxel
 - Data: Cells (cuboid), Corner: Voxel
- Curvi-linear grid resp. unstructured:
 - Data organized as tetrahedra or hexahedra
 - Often: conversion to tetrahedra





Rendering projection,

so much information and so few pixels!

Large data sizes, e.g.

512×512×1024 voxel á 16 bit = 512 Mbytes

Speed,

Interaction is very important, >10 fps!



Voxels vs. Cells



Two ways to interpret the data:

- Data: set of voxel
 - Voxel = abbreviation for volume element (cf. pixel = "picture elem.)
 - Voxel = point sample in 3D
 - Not necessarily interpolated
- Data: set of cells
 - Cell = cube primitive (3D)
 - Corners: 8 voxel (see above)
 - Values in cell: interpolation used







Interpolation





v = S(rnd(x), rnd(y), rnd(z))

Nearest Neighbor

v = (1-x)(1-y)(1-z)S(0,0,0) +(x)(1-y)(1-z)S(1,0,0) +(1-x)(y)(1-z)S(0,1,0) +(x)(y)(1-z)S(1,1,0) +(1-x)(1-y)(z)S(0,0,1) +(x)(1-y)(z)S(1,0,1) +(1-x)(y)(z)S(0,1,1) +(x)(y)(z)S(1,1,1)

Trilinear

Interpolation – Results





Nearest Neighbor Interpolation

Trilinear Interpolation



- If very high quality is needed, more complex reconstruction filters may be required
 - Marschner-Lobb function is a common test signal to evaluate the quality of reconstruction filters [Marschner and Lobb 1994]
 - The signal has a high amount of its energy near its Nyquist frequency
 - Makes it a very demanding test for accurate reconstruction



Higher-Order Reconstruction (2)



Analytical evaluation of the Marschner-Lobb test signal





Higher-Order Reconstruction (3)



Trilinear reconstruction of Marschner-Lobb test signal





Higher-Order Reconstruction (4)



B-Spline reconstruction of Marschner-Lobb test signal





Windowed sinc reconstruction of Marschner-Lobb test signal







- Volume data: $f(\mathbf{x}) \in \mathbb{R}^1$, $\mathbf{x} \in \mathbb{R}^3$
- Gradient ∇f: 3D vector points in direction of largest function change
- Gradient magnitude: length of gradient
- Emphasis of changes:
 - Special interest often in transitional areas
 - Gradients: measure degree of change (like surface normal)
 - ◆ Larger gradient magnitude
 ⇒ larger opacity









- Gradient $\nabla f = (\partial f / \partial x, \partial f / \partial y, \partial f / \partial z)$
- $\nabla f|_{x_0}$ normal vector to iso-surface $f(x_0)=f_0$
- Central difference in x-, y- & z-direction (in voxel):

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$$\nabla f(x,y,z) = \frac{1}{2} \begin{pmatrix} f(x+1)-f(x-1) \\ f(y+1)-f(y-1) \\ f(z+1)-f(z-1) \end{pmatrix}$$

- Then tri-linear interpolation within a cell
 Alternatives:
 - Forward differencing:
 - Backwards differencing:
 - Intermediate differencing:

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$$\nabla f(x) = f(x) - f(x-1)$$

 $\nabla f(x+0.5) = f(x+1) - f(x)$

 $\nabla f(x) = f(x+1) - f(x)$



• Assignment data \Rightarrow semantics:

- Assignment to objects, e.g., bone, skin, muscle, etc.
- Usage of data values, gradient, curvature
- Goal: segmentation



- Often: semi-automatic resp. manual
- Automatic approximation: transfer functions (TF)





Mapping data \rightarrow "renderable quantities":

- ◆ 1.) data→color (f(i)→C(i))
- ◆ 2.) data→opacity (non-transparency) (f(i)→ α (i))



Different Transfer Functions



Image results:

- Strong dependence on transfer functions
- Non-trivial specification
- Limited segmentation possibilities



Lobster – Different Transfer Functions



Three objects: media, shell, flesh









Concepts and Terms



Example

- X-Ray Modelling
- Surfacedefinition
- Sampling (voxelization), combination
- Direct volume rendering







Slice rendering

- 2D cross-section from 3D volume data
- Surface rendering:
 - Indirect volume visualization
 - Intermediate representation: iso-surface, "3D"
 - ♦ Pros: Shading→Shape!, HW-rendering
- Volume rendering:
 - Direct volume visualization
 - Usage of transfer functions
 - Pros: illustrate the interior, semi-transparency





Comparison ozon-data over Antarctica:

- Slices: selective (z), 2D, color coding
- Iso-surface: selective (f_0), covers 3D
- Vol. rendering: transfer function dependent, "(too) sparse – (too) dense"



Simple methods:

- Slicing, MPR (multi-planar reconstruction)
- Direct volume visualization:
 - Ray casting
 - Shear-warp factorization
 - Splatting
 - 3D texture mapping
 - Fourier volume rendering
- Surface-fitting methods:
 - Marching cubes (marching tetrahedra)



Simple Methods

Slicing, etc.



Slicing

Slicing:

- Axes-parallel slices
- Regular grids: simple
- Without transfer function no color
- Windowing: adjust contrast
 General grid, arbitrary slicing
 direction









Direct Volume Visualization



Image-Order Approach: Traverse the image pixel-by-pixel and sample the volume.



Ray Casting



Object-Order Approach: Traverse the volume, and project to the image plane.





Splatting cell-by-cell

Texture Mapping plane-by-plane

Ray Casting

Image-Order Method





- Ray Tracing: method from image generation
- In volume rendering: only viewing rays
 - \Rightarrow therefore Ray Casting
- Classical image-order method
- Ray Tracing: ray object intersection
 Ray Casting: no objects, density values in 3D
- In theory: take all data values into account! In practice: traverse volume step by step
 - Interpolation necessary for each step!





Context:

- Volume data: 1D value defined in 3D –
 f(x)∈R¹, x∈R³
- Ray defined as half-line:
 r(t)∈R³, t∈R¹>0
- Values along Ray: f(r(t))∈R¹, t∈R¹>0 (intensity profile)







1. Shading, Classification

- 1. Step:
 - Shading, $f(i) \rightarrow C(i)$:
 - Apply transfer function
 - diffuse illumination (Phong), gradient ≈ normal
 - Classification, $f(i) \rightarrow \alpha(i)$:
 - Levoy '88, gradient enhanced
 - Emphasizes transitions
 - Nowadays: shading/classification after ray-casting/resampling








Cast ray through the volume and perform sampling at discrete positions





2. Ray Traversal – Three Approaches







TU

Overview:



Types of Combinations

TU

DVR

Possibilities:

- α -compositing
- Shaded surface display (first hit)
- Maximum-intensity projection (MIP)
- X-ray simulation
- Contour rendering











Shading/classification can occur before or after ray traversal

 Pre-interpolative: classify all data values and then interpolate between RGBA-tuples

 Post-interpolative: interpolate between scalar data values and then classify the result







POST-INTERPOLATIVE



Classification Order (3)







Classification Order: Example 1







pre-interpolative

post-interpolative

same transfer function, resolution, and sampling rate



Classification Order: Example 2





pre-interpolative

post-interpolative

same transfer function, resolution, and sampling rate



α -Compositing –

a Specific Optical Model for Volume Rendering

Display of Semi-Transparent Media



- Various models (Examples):
 - Emission only (light particles)
 - Absorption only (dark fog)
 - Emission & absorption (clouds)
 - Single scattering, w/o shadows
 - Multiple scattering
- Two approaches:
 - Analytical model (via differentials)
 - Numerical approximation (via differences)





Physical Model of Radiative Transfer















Analytical Model (2)

















every point *s* along the viewing ray emits additional radiant energy

$$I(s) = I(s_0) e^{-\tau(s_0,s)} + \int_{s_0}^{s} q(\tilde{s}) e^{-\tau(\tilde{s},s)} d\tilde{s}$$



Numerical Approximation (1)





approximate integral by Riemann sum:

$$\tau(0,t) \approx \sum_{i=0}^{\lfloor t/\Delta t \rfloor} \kappa(i \cdot \Delta t) \,\Delta t$$



Numerical Approximation (2)







Numerical Approximation (3)





now we introduce opacity:

$$A_i = 1 - e^{-\kappa(i \cdot \Delta t) \,\Delta t}$$



Numerical Approximation (4)





now we introduce opacity:

$$(1 - A_i) = e^{-\kappa(i \cdot \Delta t) \,\Delta t}$$



Numerical Approximation (5)





$$e^{-\tilde{\tau}(0,t)} = \prod_{i=0}^{\lfloor t/\Delta t \rfloor} (1 - A_i)$$



Numerical Approximation (6)







Numerical Approximation (7)







Numerical Approximation (8)







Numerical Approximation (9)





back-to-front $C_i' = C_i + (1 - A_i)C_{i-1}'$

front-to-back compositing

$$C'_{i} = C'_{i+1} + (1 - A'_{i+1})C_{i}$$

$$A'_{i} = A'_{i+1} + (1 - A'_{i+1})A_{i}$$

early ray termination: stop the calculation when $A'_i \approx 1$



Back-to-Front Compositing: Example





VIENNA

Front-to-Back Compositing: Example











Emission Absorption Model

$$I(s) = I(s_0) e^{-\tau(s_0,s)} + \int_{s_0}^{s} q(\tilde{s}) e^{-\tau(\tilde{s},s)} d\tilde{s}$$

Numerical Solutions [pre-multiplied alpha assumed]

back-to-front iteration

front-to-back iteration

$$C'_i = C_i + (1 - A_i)C'_{i-1}$$

$$C'_{i} = C'_{i+1} + (1 - A'_{i+1})C_{i}$$

$$A'_{i} = A'_{i+1} + (1 - A'_{i+1})A_{i}$$





- Color values are stored pre-multiplied by their opacity: (*αr*, *αg*, *αb*)
- Consequence: transparent red is the same as transparent black, etc.
- Simplifies blending: color and alpha values are treated equally
- Can result in loss of precision



Emission or/and Absorption





Eduard Gröller, Helwig

Ray Casting – Examples



CT scan of human hand (244x124x257, 16 bit)





Ray Casting – Examples





Ray Casting – Further Examples



Tornado Visualization:







Ray Casting – Further Examples



Molecular data:









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Hardware-Volume Visualization

Faster with Hardware?!





3D-textures:

Volume data stored in 3D-texture

- Proxy geometry (slices) parallel to image plane, are interpolated tri-linearly
- Back-to-front compositing

2D-textures:

- 3 stacks of slices (x-, y- & z-axis), slices are interpolated bi-linearly
- Select stack (most "parallel" to image plane)
- Back-to-front compositing


Variation of View Point



3D-textures:

 Number of slices varies

2D-textures:

 Stack change: discontinuity





Indirect Volume Visualization

Iso-Surface-Display



Concepts and Terms



Example

- CT measurement
- Iso-stackconversion
- Iso-surfacecalculation (marching cubes)
- Surface rendering (OpenGL)







Iso-Surfaces



- Intermediate representationAspects:
 - Preconditions:
 - expressive Iso-value, Iso-value separates materials
 - Interest: in transitions
 - Very selective (binary selection / omission)
 - ◆ Uses traditional hardware
 ◆ Shading ⇒ 3D-impression!









Volume Data ⇔ Iso-Surfaces



Iso-Surface: • Iso-value f_0 • Separates values > f_0 from values $\leq f_0$ • Often not known \rightarrow

Can only be approximated from samples!
 Shape / position dependent on type of reconstruction



Marching Cubes (MC) Iso-Surface-Display



Approximation of Iso-Surface

Approach:

Iso-Surface intersects data volume = set of all cells

Idea:

- Parts of iso-surface represented on a(n intersected) cell basis
- As simple as possible:
 Usage of triangles









Marching Cubes

- Cell consists of 4(8) pixel (voxel) values: (i+[01], j+[01], k+[01])
- 1. Consider a Cell
- 2. Classify each vertex as inside or outside
- 3. Build an index
- Get edge list from table[index]
- 5. Interpolate the edge location
- 6. Go to next cell





MC 1: Create a Cube

Consider a Cube defined by eight data values:



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MC 2: Classify Each Voxel

Classify each voxel according to whether it lies outside the surface (value > iso-surface value) inside the surface (value <= iso-surface value)</p>





MC 3: Build An Index

Use the binary labeling of each voxel to create an index







MC 5: Example

✓ Index = 10110001

✓ triangle 1 = e4,e7,e11

✓ triangle 2 = e1, e7, e4

✓ triangle 3 = e1, e6, e7

🗸 triangle 4 = e1, e10, e6



MC 6: Interp. Triangle Vertex

 For each triangle edge, find the vertex location along the edge using linear interpolation of the voxel values



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MC 7: Compute Normals

Calculate the normal at each cube vertex





 $\overline{N} = \overline{G}$

Use linear interpolation to compute the polygon vertex normal

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MC 8: Ambiguous Cases



- Adjacent vertices: different states
 - Diagonal vertices: same state
 - Resolution: decide for one case





Danger: Holes!



Wrong vs. correct classification!



MC 9: Asymptotic Decider

- Assume bilinear interpolation within a face
- 🗸 hence iso-surface is a hyperbola
 - compute the point p where the asymptotes meet
 - sign of S(p) decides the connectedness



Marching Cubes - Summary 1

🗸 256 Cases

- reduce to 15 cases by symmetry
- Complementary cases -(swap in- and outside)
- Ambiguity resides in cases 3, 6, 7, 10, 12, 13
 - Causes holes if arbitrary choices are made.





(a) Volume data

(b) Isosurface S = f(x,y,z)



(c) Polygonal Apploximation

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Marching Cubes - Summary 2

- Up to 4 triangles per cube
- ✓ Dataset of 512³ voxels can result in several million triangles (many Mbytes!!!)
- Iso-surface does not represent an object!!!
- Vo depth information
- Semi-transparent representation --> sorting
 - Optimization:
 - Reuse intermediate results
 - Prevent vertex replication
 - Mesh simplification





MC Examples

1 Iso-surface







3 Iso-surfaces

2 Iso-surfaces

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Further Examples







Eduard Gröller, Helwi

Even Further Examples















LiveFetoscopic Visualization of 4D Ultrasound Data Joint project with Kretztechnik (GE)





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SKYCAST Overhead this month in parts of the world

Early October Comet ISON visible

 October 18
 Penumbral lunar eclipse



Baby Pictures "The number

one thing parents want to see is if babies have ten fingers and ten toes," says engineer Karl-Heinz Lumpi. His team developed software that shows the digits in full-color 3-D. Beyond allaying parents' curiosity, the more exact image of what's going on in the womb may play a role in diagnostics. Doctors who were formerly resigned to a blurred heartbeat can now see inside that organ's chambers. It's all in the lighting. The image starts out like a traditional 3-D ultrasound's. Then a computer program adds virtual illumination, mimicking how light plays across human skin—reflecting, casting shadows, and giving shape. As in regular photography, the light source is movable. Plus the image is rotatable, so wriggling fingers, or a floating umbilical cord like this eight-month fetus's, likely won't hinder a thorough exam. *—Johnna Rizzo*





Conclusion Volume Visualization General Remarks



Surface vs. Volume Rendering



Surface Rendering:

- Indirect representation / display
- Conveys surface impression
- Hardware supported rendering (fast?!)
- Iso-value-definition

- Volume Rendering:
 - Direct representation / display
 - Conveys volume impression
 - Often realized in software (slow?!)
 - Transfer functions



Eduard Gröller, Helwig Hauser, Stefan Bruckner

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- For material for this lecture unit
 - Roberto Scopigno, Claudio Montani (CNR, Pisa)
 - Hans-Georg Pagendarm (DLR, Göttingen)
 - Michael Meißner (GRIS, Tübingen)
 - Torsten Möller
 - Gordon Kindlmann
 - Joe Kniss
 - Nelson Max (LLNL), Marc Levoy (Stanford)
 - Lloyd Treinish (IBM)
 - Roger Crawfis (Ohio State Univ.)
 - Hanspeter Pfister (MERL)
 - Dirk Bartz
 - Markus Hadwiger
 - Christof Rezk Salama



