Volume Visualization

Overview: Volume Visualization (1)

- Introduction to volume visualization
  - On volume data
  - Voxels vs. cells
  - Interpolation
  - Gradient
  - Classification
  - Transfer Functions (TF)
  - Slice vs surface vs. volume rendering
  - Overview: techniques

Overview: Volume Visualization (2)

- Simple methods
  - Slicing, multi-planar reconstruction (MPR)
- Direct volume visualization
  - Image-order vs. object-order
  - Raycasting
  - α-compositing
  - Hardware volume visualization
- Indirect volume visualization
  - Marching cubes

Volume Visualization

- Introduction:
  - VolVis = visualization of volume data
    - Mapping 3D→2D
    - Projection (e.g., MIP), slicing, vol. rendering, …
  - Volume data =
    - 3D×1D data
    - Scalar data, 3D data space, space filling
  - User goals:
    - Gain insight in 3D data
    - Structures of special interest + context

Volume Data

- Where do the data come from?
  - Medical Application
    - Computed Tomographie (CT)
    - Magnetic Resonance Imaging (MR)
  - Materials testing
    - Industrial-CT
  - Simulation
    - Finite element methods (FEM)
    - Computational fluid dynamics (CFD)
  - etc.

3D Data Space

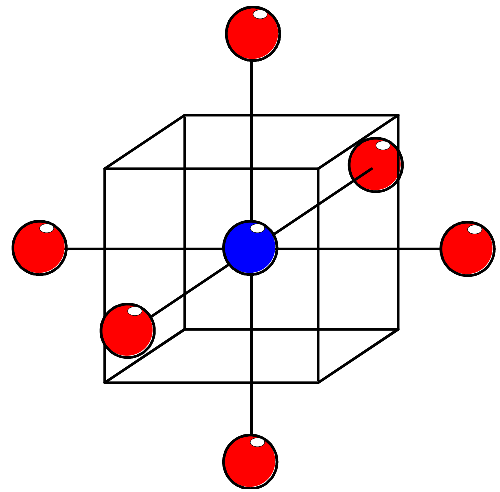
- How are volume data organized?
  - Cartesian resp. regular grid:
    - CT/MR: often dx=dy<dz, e.g. 135 slices (z) á 512^2 values (as x & y pixels in a slice)
    - **Data enhancement**: iso-stack-calculation = Interpolation of additional slices, so that dx=dy=dz $\Rightarrow$ 512^3 Voxel
    - Data: **Cells** (cuboid), Corner: **Voxel**
  - Curvi-linear grid resp. unstructured:
    - Data organized as tetrahedra or hexahedra
    - Often: conversion to tetrahedra

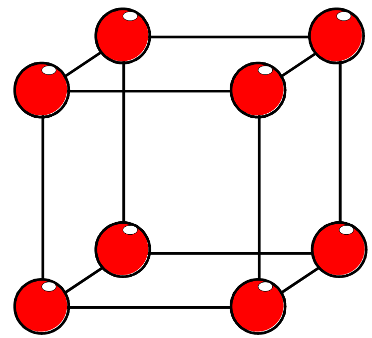
VolVis – Challenges

- Rendering projection,
  so much information and so few pixels!
- Large data sizes, e.g.
  512×512×1024 voxel á 16 bit = 512 Mbytes
- Speed,
  Interaction is very important, >10 fps!

Voxels vs. Cells

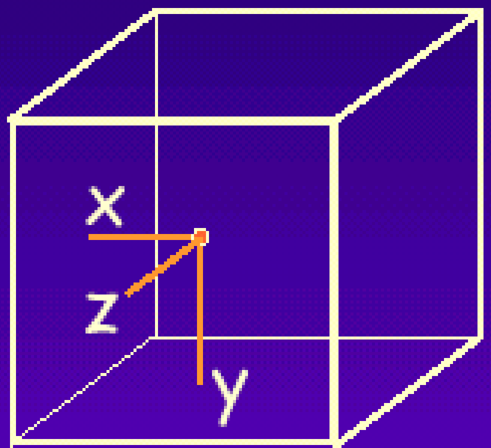
- Two ways to interpret the data:
  - Data: set of voxel
    - **Voxel** = abbreviation for volume element (cf. pixel = "picture elem.)
    - Voxel = point sample in 3D
    - Not necessarily interpolated
  - Data: set of cells
    - Cell = cube primitive (3D)
    - Corners: 8 voxel (see above)
    - Values in cell: interpolation used





Interpolation

(1,1,1)



(0,0,0)

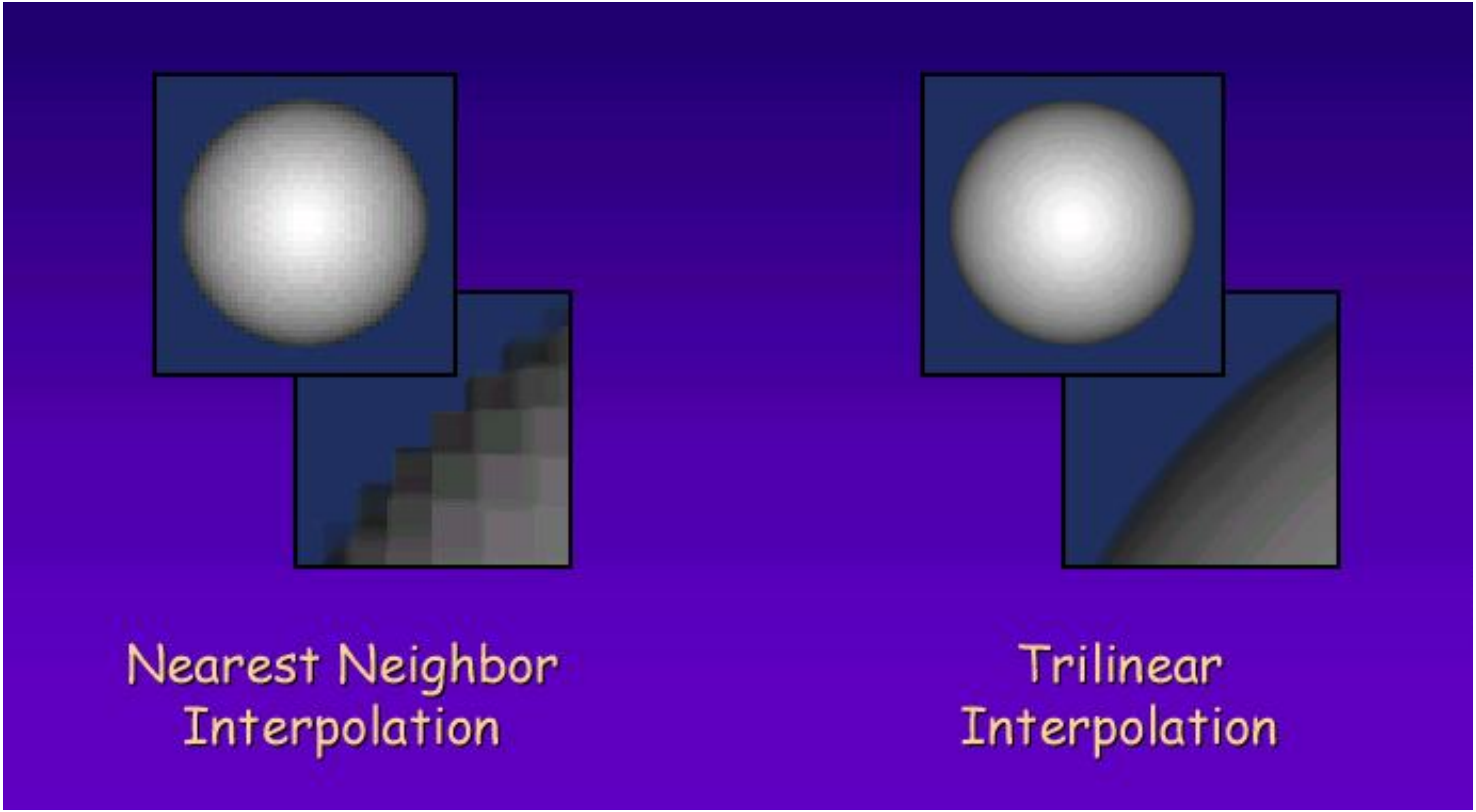
$v = S(rnd(x), rnd(y), rnd(z))$

Nearest Neighbor

$$
\begin{aligned}
v = \; & (1-x)(1-y)(1-z)S(0,0,0) + \\
& (x)(1-y)(1-z)S(1,0,0) + \\
& (1-x)(y)(1-z)S(0,1,0) + \\
& (x)(y)(1-z)S(1,1,0) + \\
& (1-x)(1-y)(z)S(0,0,1) + \\
& (x)(1-y)(z)S(1,0,1) + \\
& (1-x)(y)(z)S(0,1,1) + \\
& (x)(y)(z)S(1,1,1)
\end{aligned}
$$

Trilinear

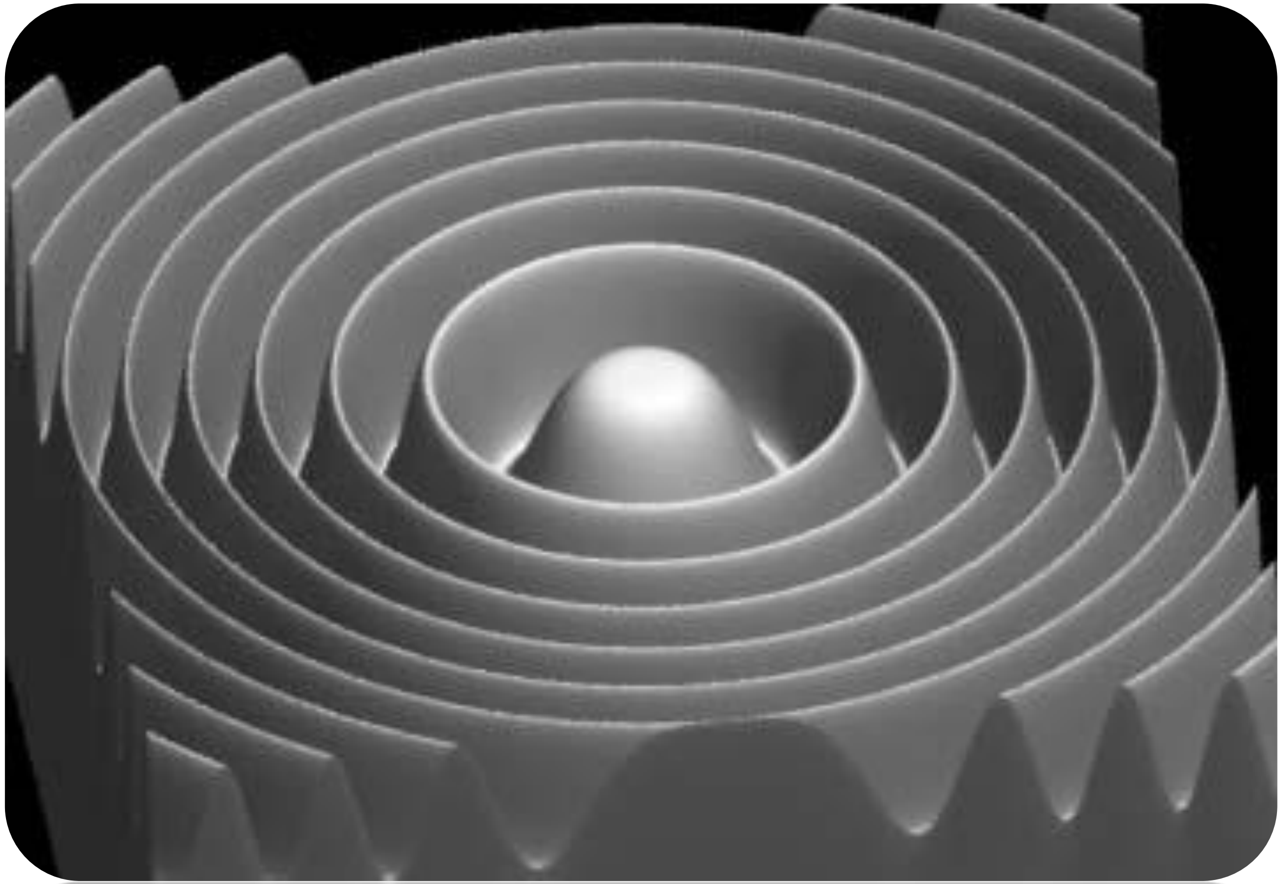
Interpolation – Results



Higher-Order Reconstruction (1)

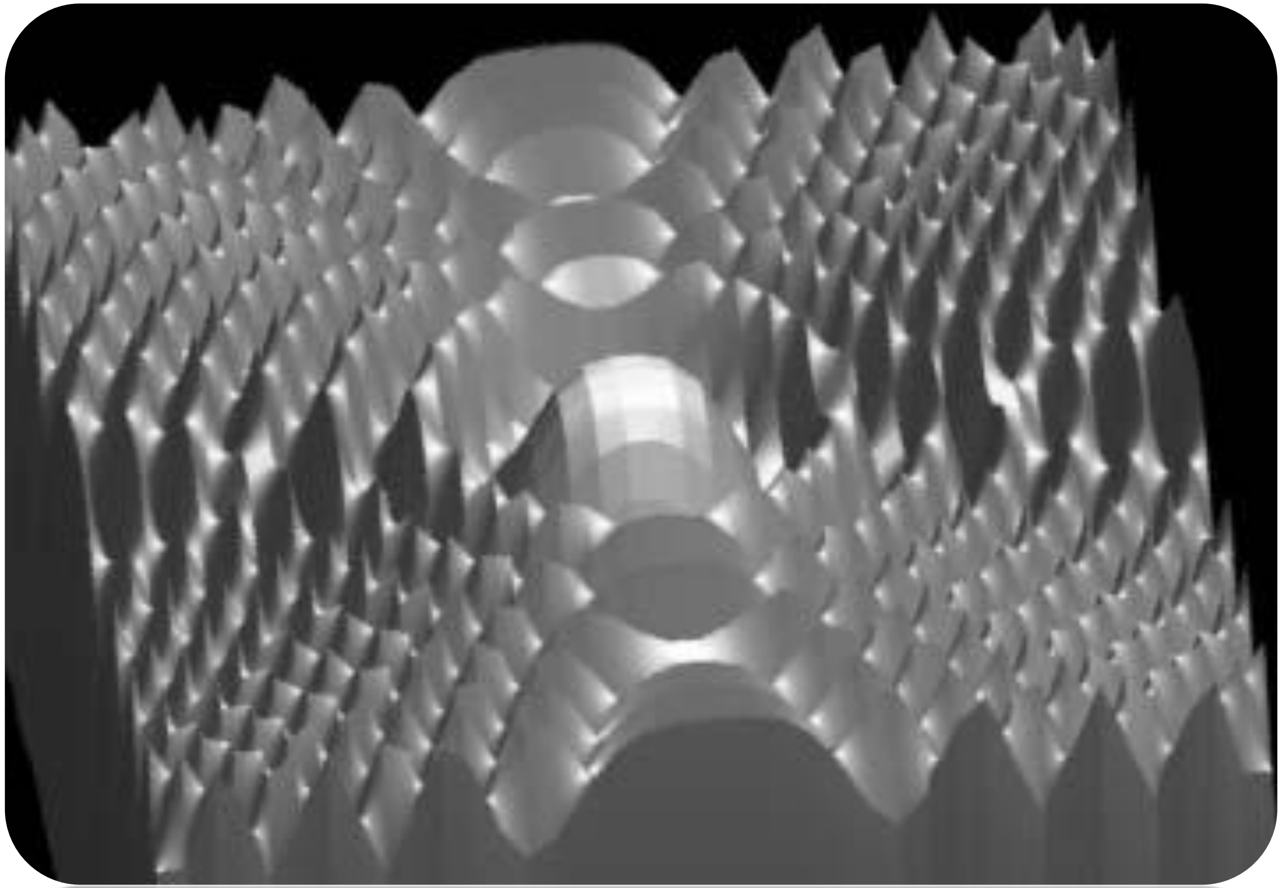
- If very high quality is needed, more complex reconstruction filters may be required
  - Marschner-Lobb function is a common test signal to evaluate the quality of reconstruction filters [Marschner and Lobb 1994]
  - The signal has a high amount of its energy near its Nyquist frequency
  - Makes it a very demanding test for accurate reconstruction

- **Analytical evaluation** of the Marschner-Lobb test signal

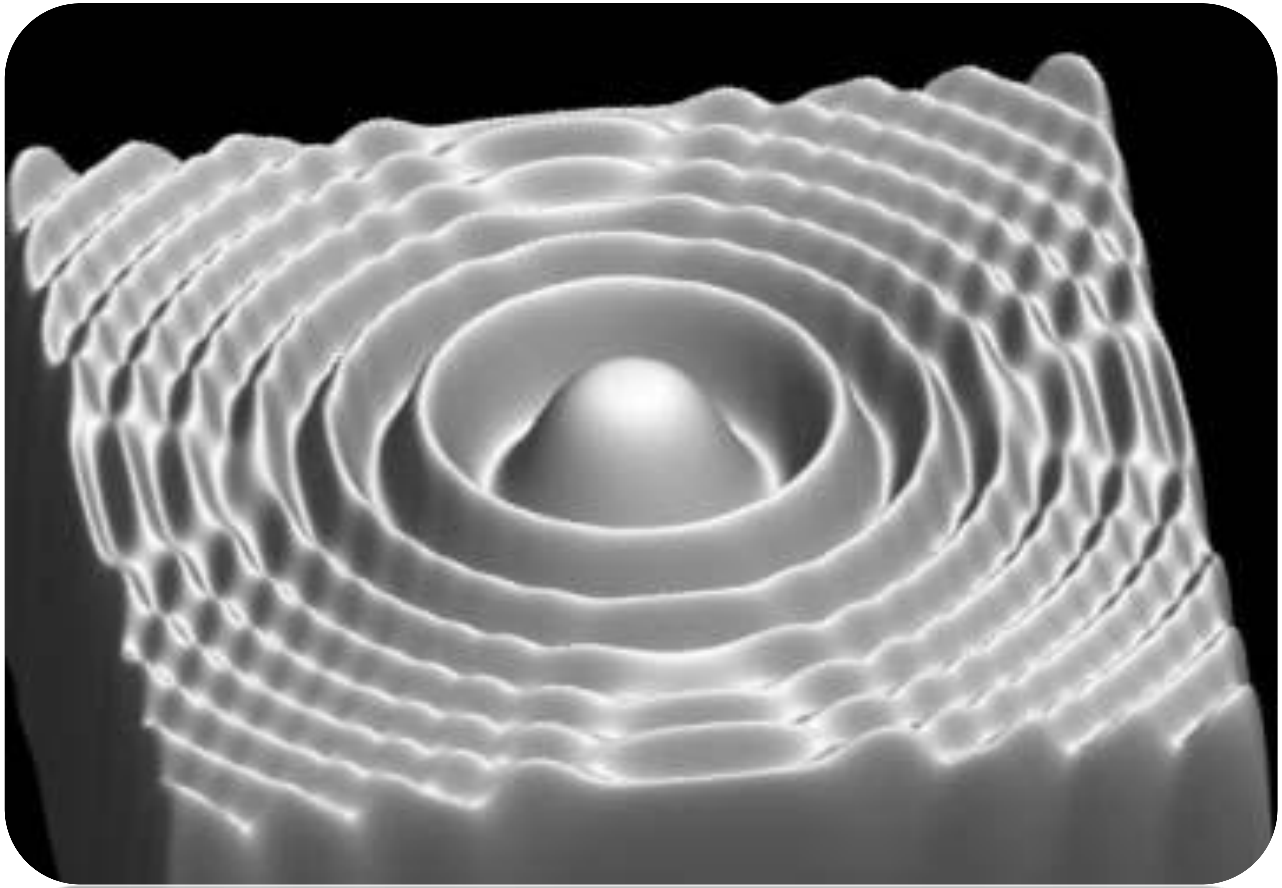


Higher-Order Reconstruction (3)

- **Trilinear** reconstruction of Marschner-Lobb test signal

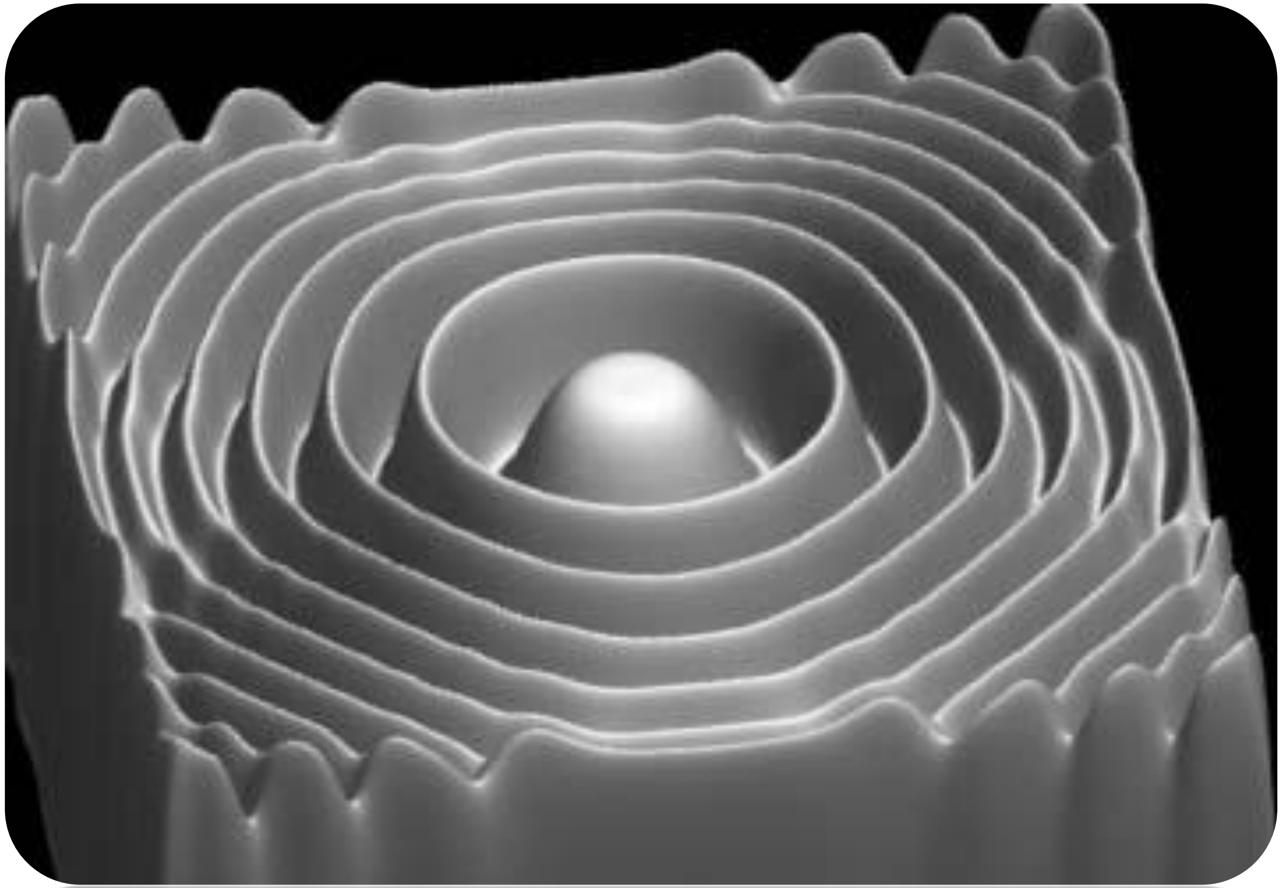


- **B-Spline** reconstruction of Marschner-Lobb test signal



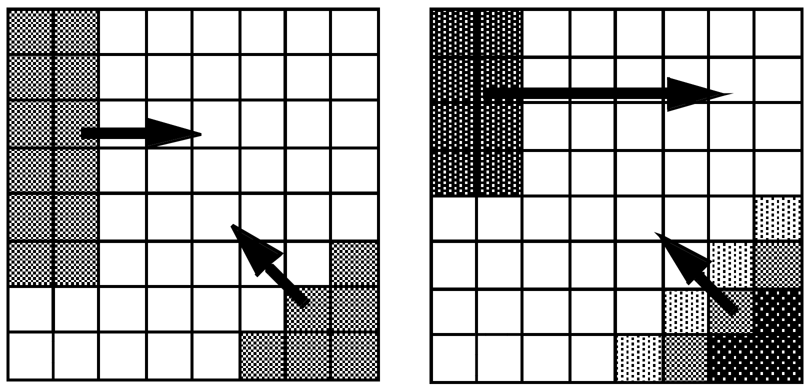
Higher-Order Reconstruction (5)

- **Windowed sinc** reconstruction of Marschner-Lobb test signal



Gradients in Volume Data

- Volume data: $f(\mathbf{x}) \in R^1$, $\mathbf{x} \in R^3$
- Gradient ∇f: 3D vector points in direction of largest function change
- Gradient magnitude: length of gradient
- Emphasis of changes:
  - Special interest often in transitional areas
  - Gradients: measure degree of change (like surface normal)
  - Larger gradient magnitude $\Rightarrow$ larger opacity



Gradients as Normal Vector Replacement

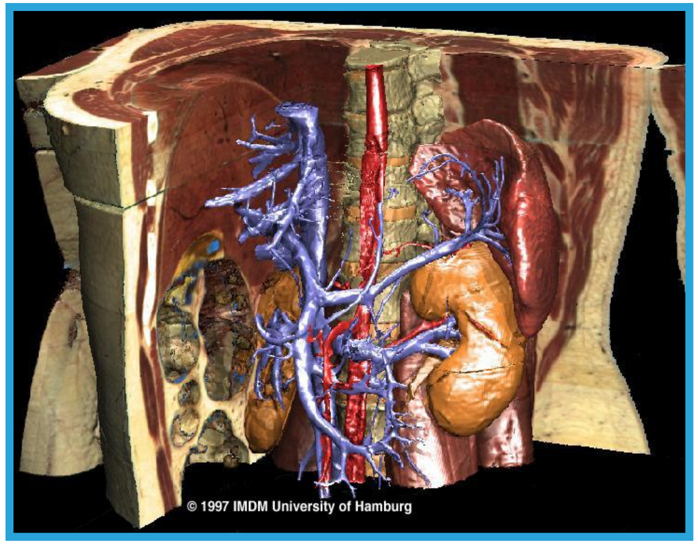
- Gradient $\nabla f = (\partial f/\partial x, \partial f/\partial y, \partial f/\partial z)$
- $\nabla f|_{x_0}$ normal vector to iso-surface $f(x_0)=f_0$
- Central difference in x-, y- & z-direction (in voxel):

$$\nabla f\,(x,y,z) = 1/2 \begin{pmatrix} f(x+1)-f(x-1) \\ f(y+1)-f(y-1) \\ f(z+1)-f(z-1) \end{pmatrix}$$

- Then tri-linear interpolation within a cell
- Alternatives:
  - Forward differencing: $\nabla f\,(x)=f(x+1)-f(x)$
  - Backwards differencing: $\nabla f\,(x)=f(x)-f(x-1)$
  - Intermediate differencing: $\nabla f\,(x+0.5)=f(x+1)-f(x)$

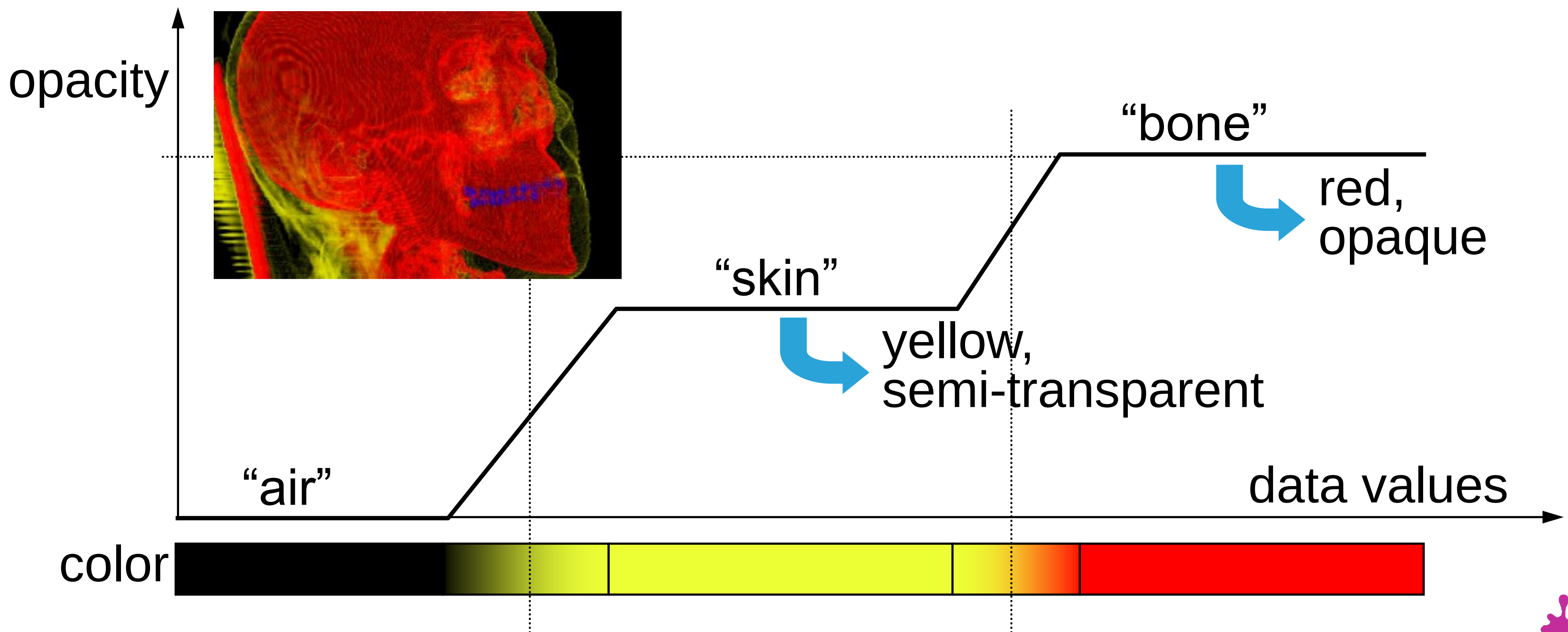
Classification

- Assignment data ⇒ semantics:
  - Assignment to objects, e.g., bone, skin, muscle, etc.
  - Usage of data values, gradient, curvature
  - Goal: segmentation
  - Often: semi-automatic resp. manual
  - Automatic approximation: transfer functions (TF)

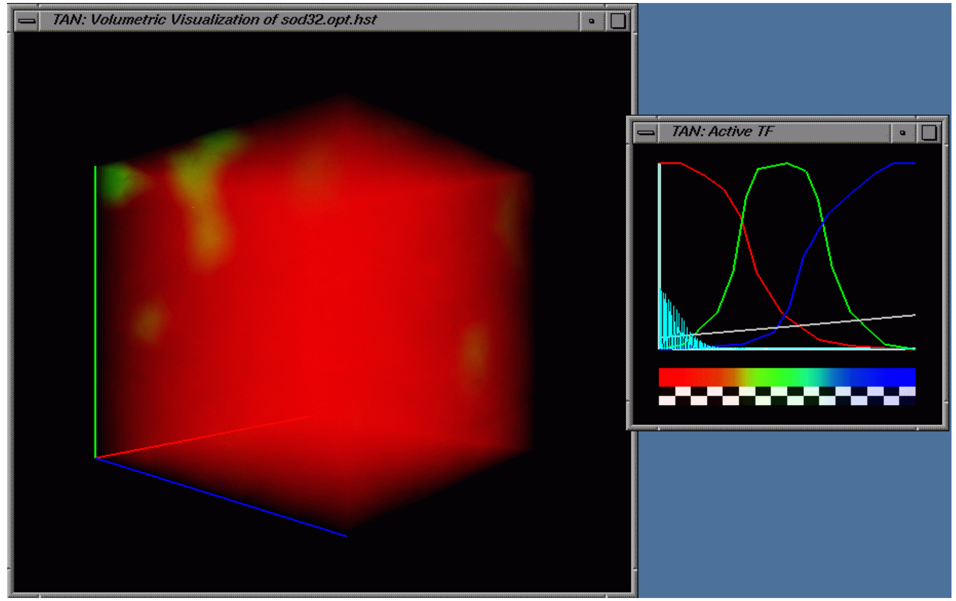


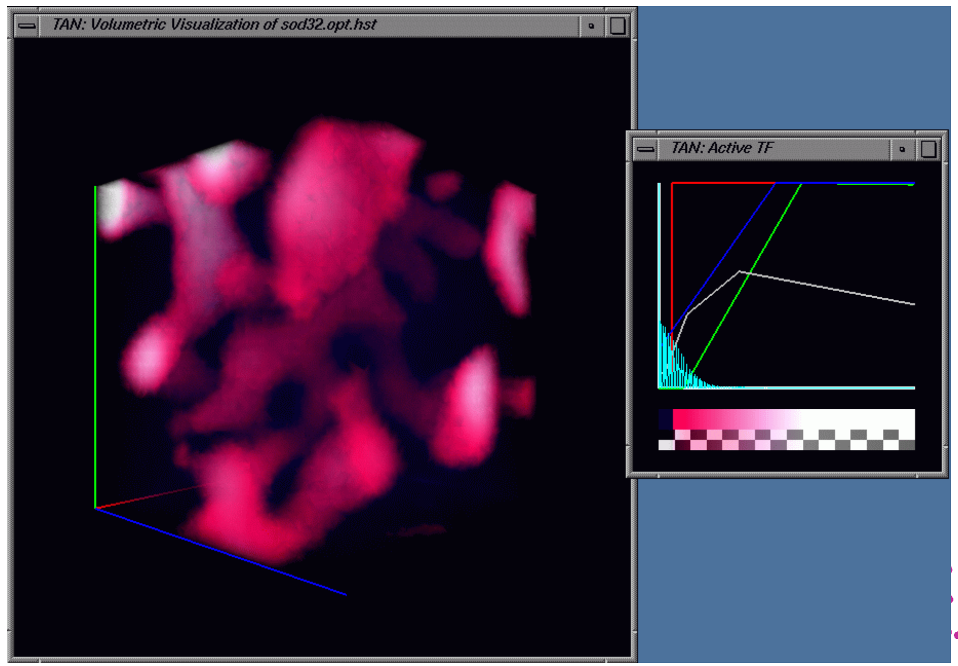
Transfer Functions (TF)

- Mapping data → ”renderable quantities”:
  - 1.) data→color (f(i)→C(i))
  - 2.) data→opacity (non-transparency) (f(i)→α(i))



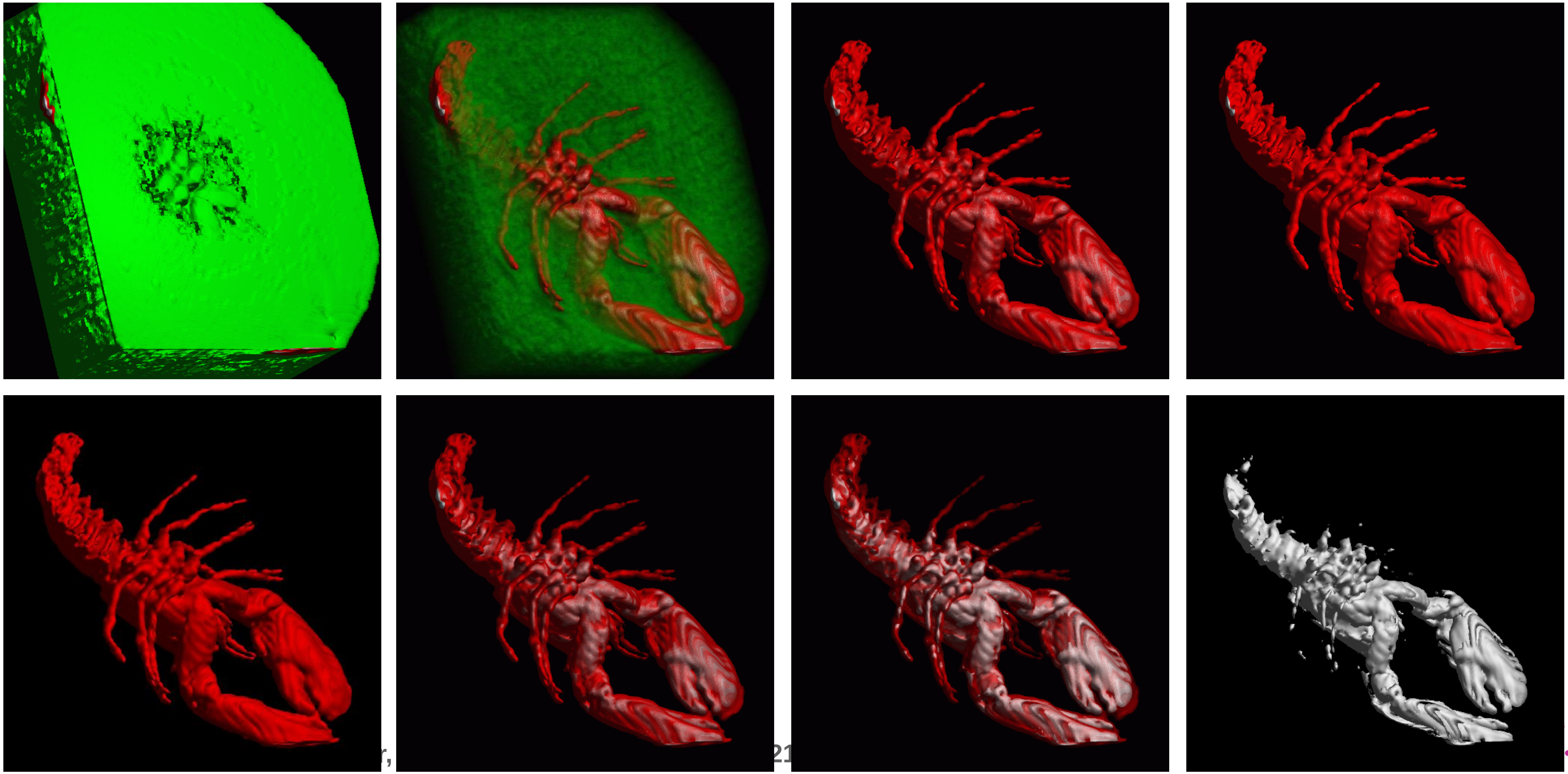
- Image results:
  - Strong dependence on transfer functions
  - Non-trivial specification
  - Limited segmentation possibilities



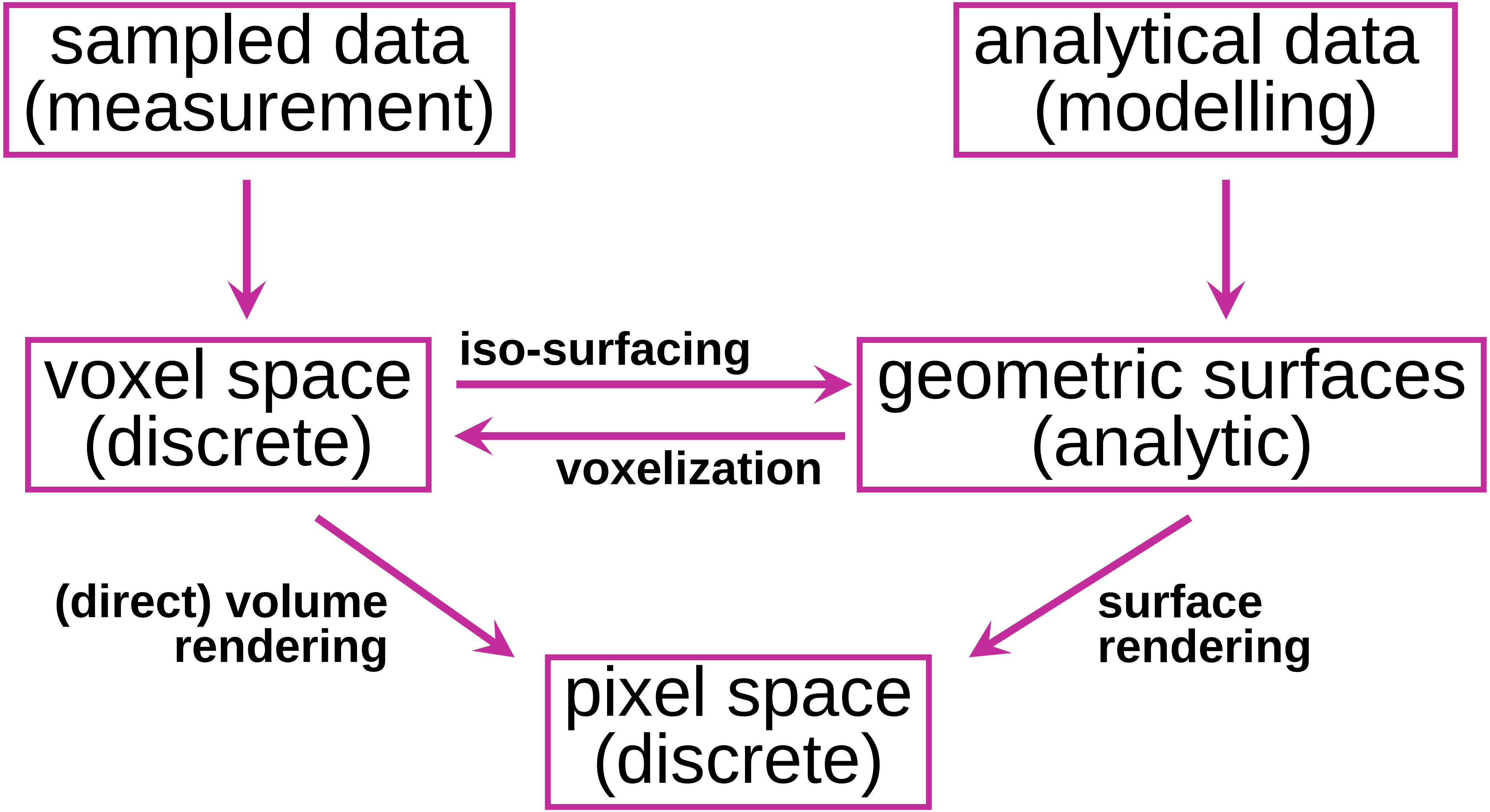


Lobster – Different Transfer Functions

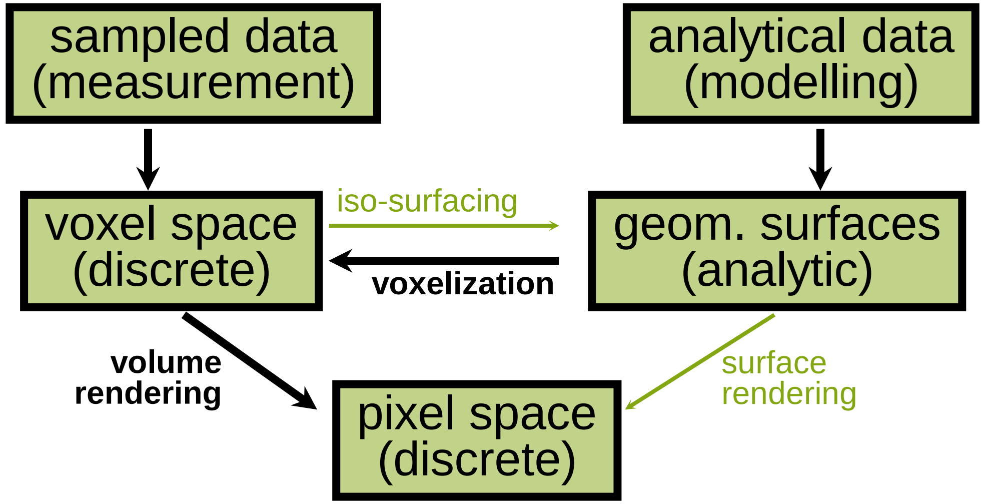
- Three objects: media, shell, flesh

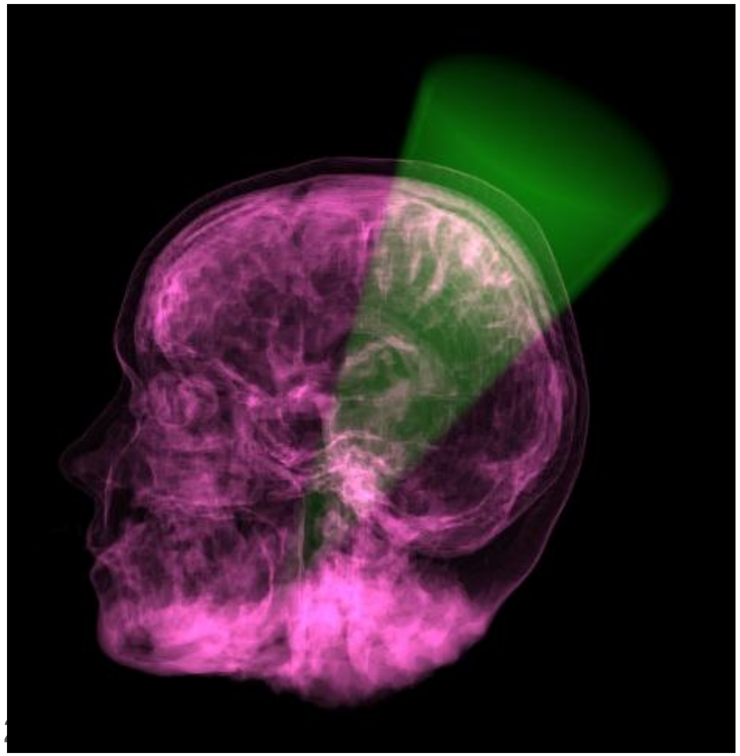


Concepts and Terms



- Example
  - X-Ray Modelling
  - Surface-definition
  - Sampling (voxelization), combination
  - Direct volume rendering

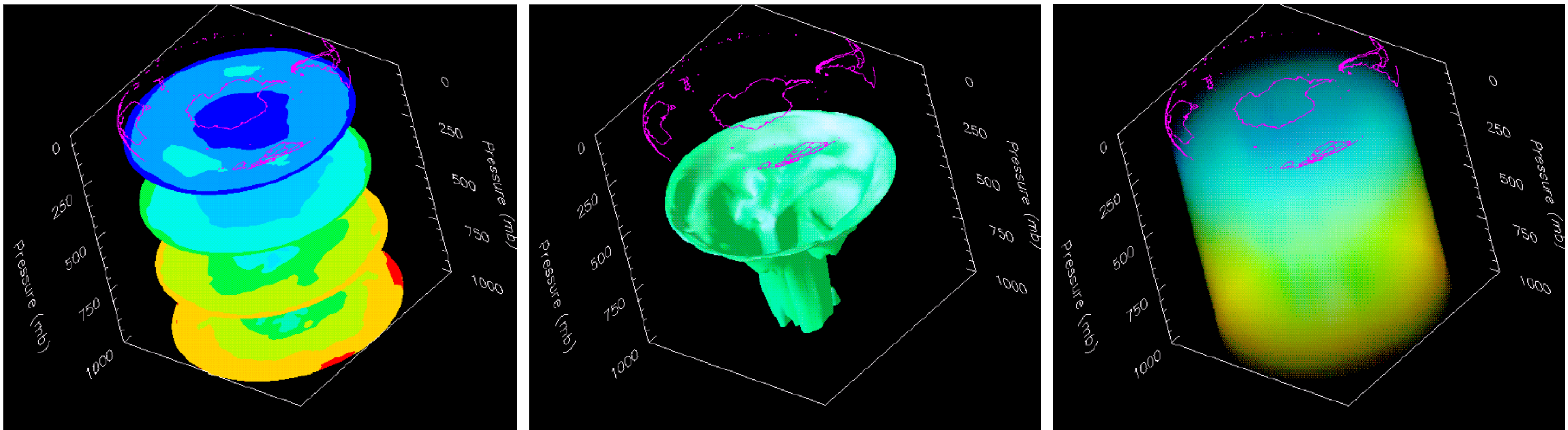




Slice vs. Surface vs. Volume Rendering

- Slice rendering
  - 2D cross-section from 3D volume data
- Surface rendering:
  - **Indirect** volume visualization
  - Intermediate representation: iso-surface, “3D”
  - Pros: Shading→Shape!, HW-rendering
- Volume rendering:
  - **Direct** volume visualization
  - Usage of transfer functions
  - Pros: illustrate the interior, semi-transparency

- Comparison ozon-data over Antarctica:
  - Slices: selective (z), 2D, color coding
  - Iso-surface: selective (f_0), covers 3D
  - Vol. rendering: transfer function dependent, “(too) sparse – (too) dense”



VolVis-Techniques – Overview

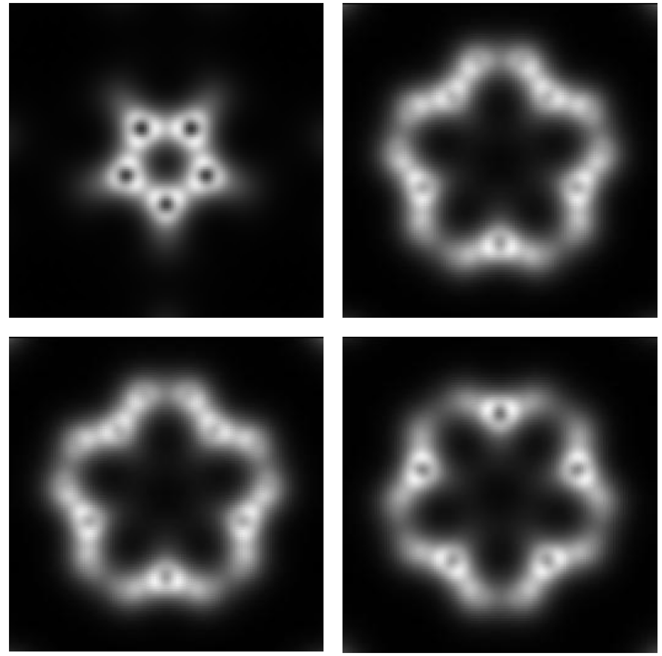
- Simple methods:
  - Slicing, MPR (multi-planar reconstruction)
- Direct volume visualization:
  - Ray casting
  - Shear-warp factorization
  - Splatting
  - 3D texture mapping
  - Fourier volume rendering
- Surface-fitting methods:
  - Marching cubes (marching tetrahedra)

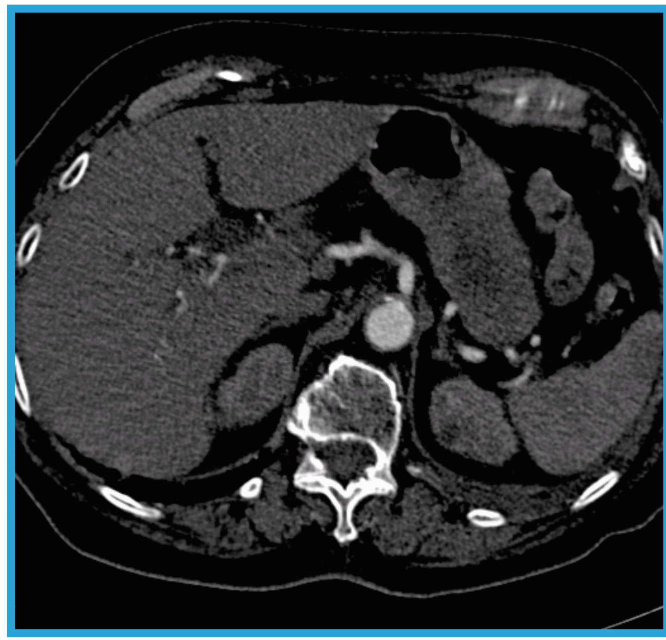
Simple Methods

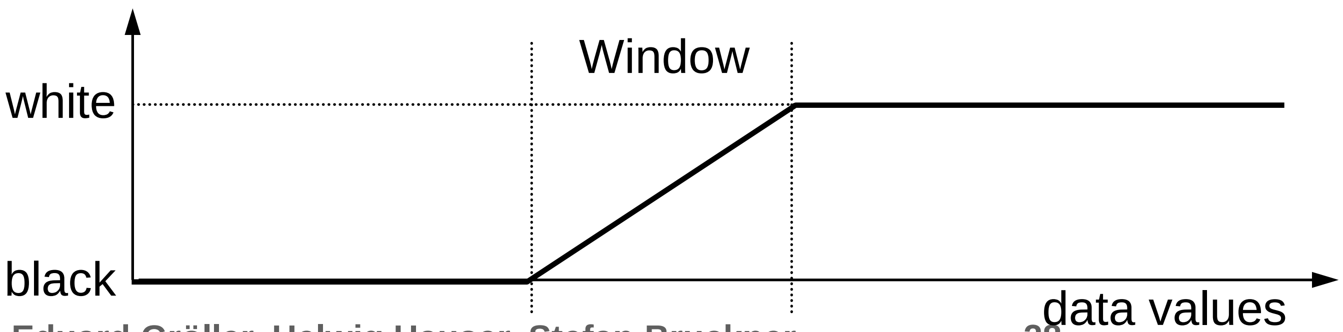
Slicing, etc.

Slicing

- Slicing:
  - Axes-parallel slices
  - Regular grids: simple
  - Without transfer function no color
  - Windowing: adjust contrast
- General grid, arbitrary slicing direction



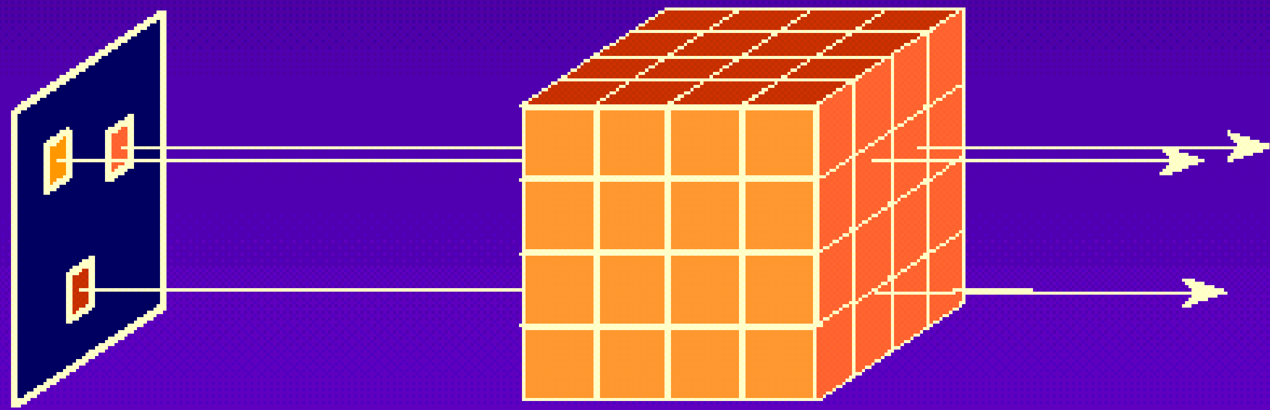




Direct Volume Visualization

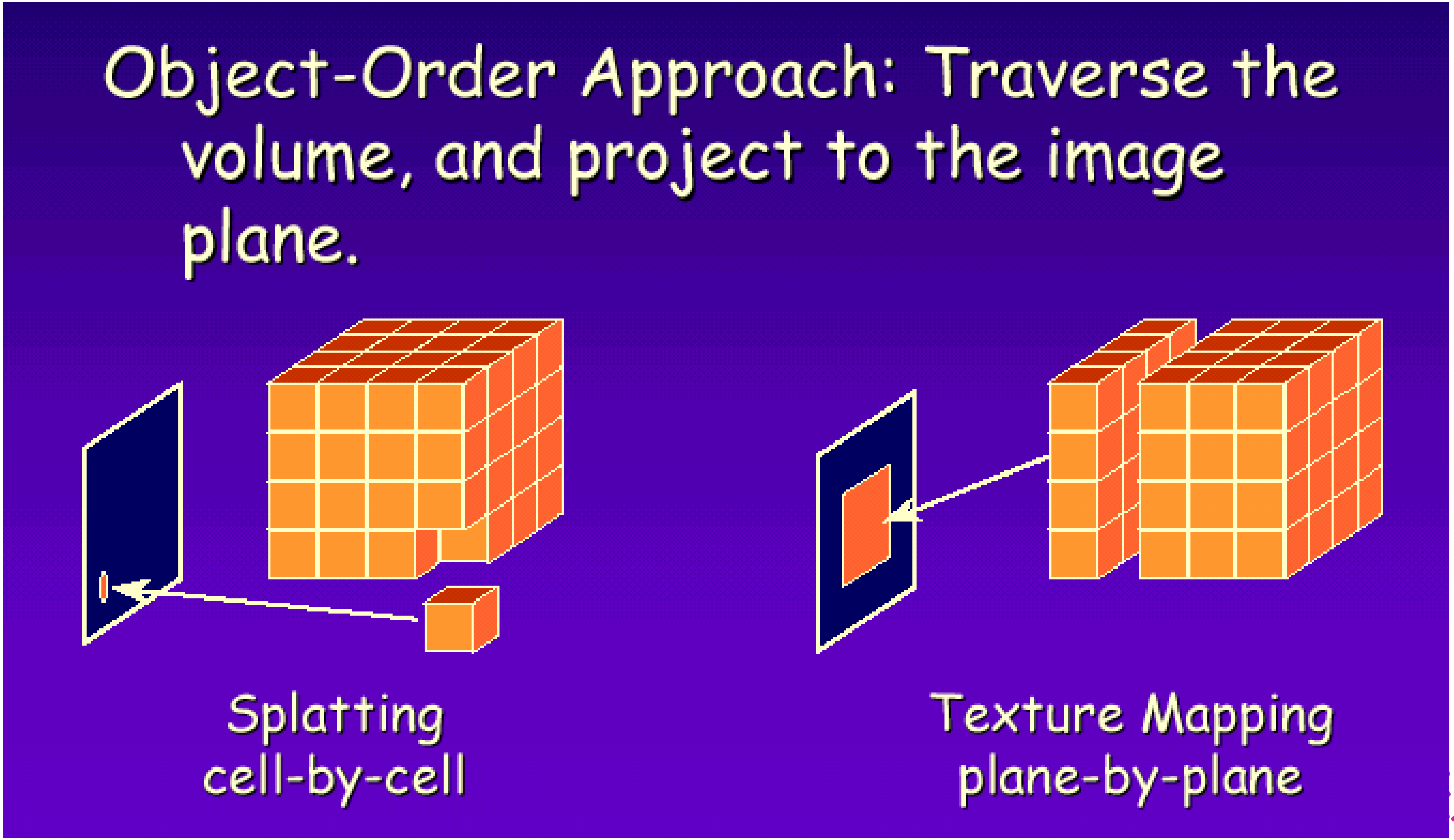
Image-Order vs. Object-Order

Image-Order Approach: Traverse the image pixel-by-pixel and sample the volume.



Ray Casting

Image-Order vs. Object-Order



Ray Casting

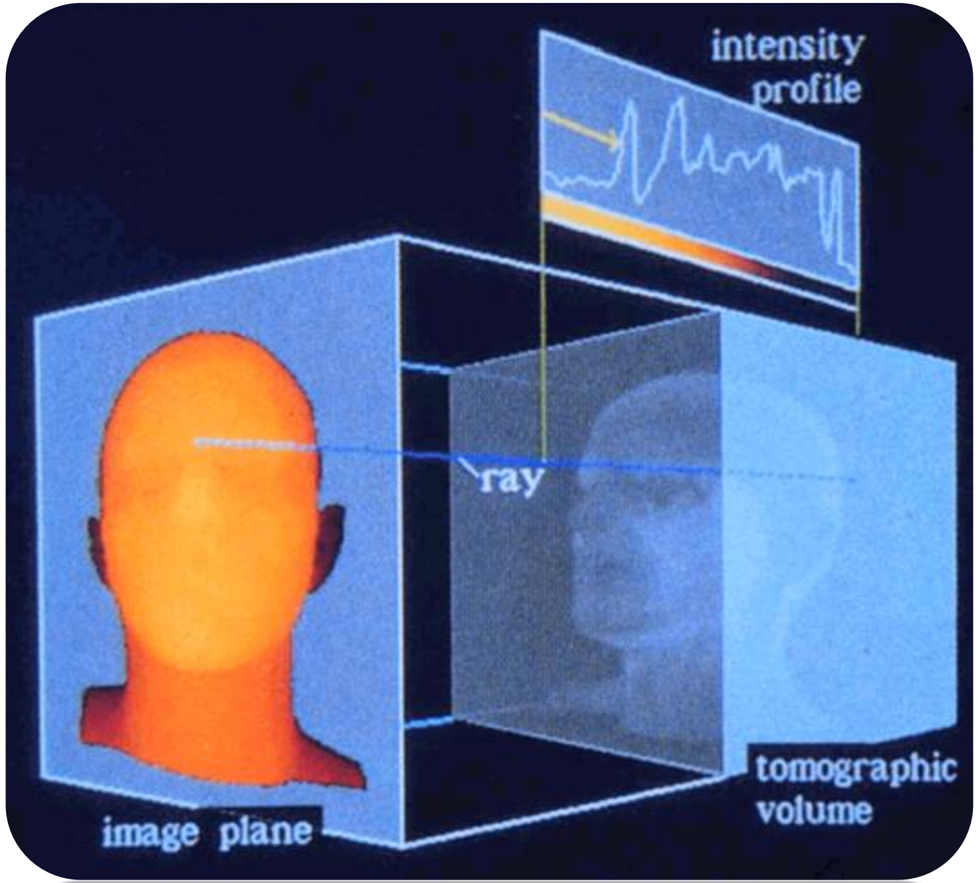
Image-Order Method

Ray Tracing vs. Ray Casting

- Ray Tracing: method from image generation
- In volume rendering: only viewing rays
  $\Rightarrow$ therefore Ray Casting
- Classical image-order method
- Ray Tracing: ray – object intersection
  Ray Casting: no objects, density values in 3D
- In theory: take all data values into account!
  In practice: traverse volume step by step
- Interpolation necessary for each step!

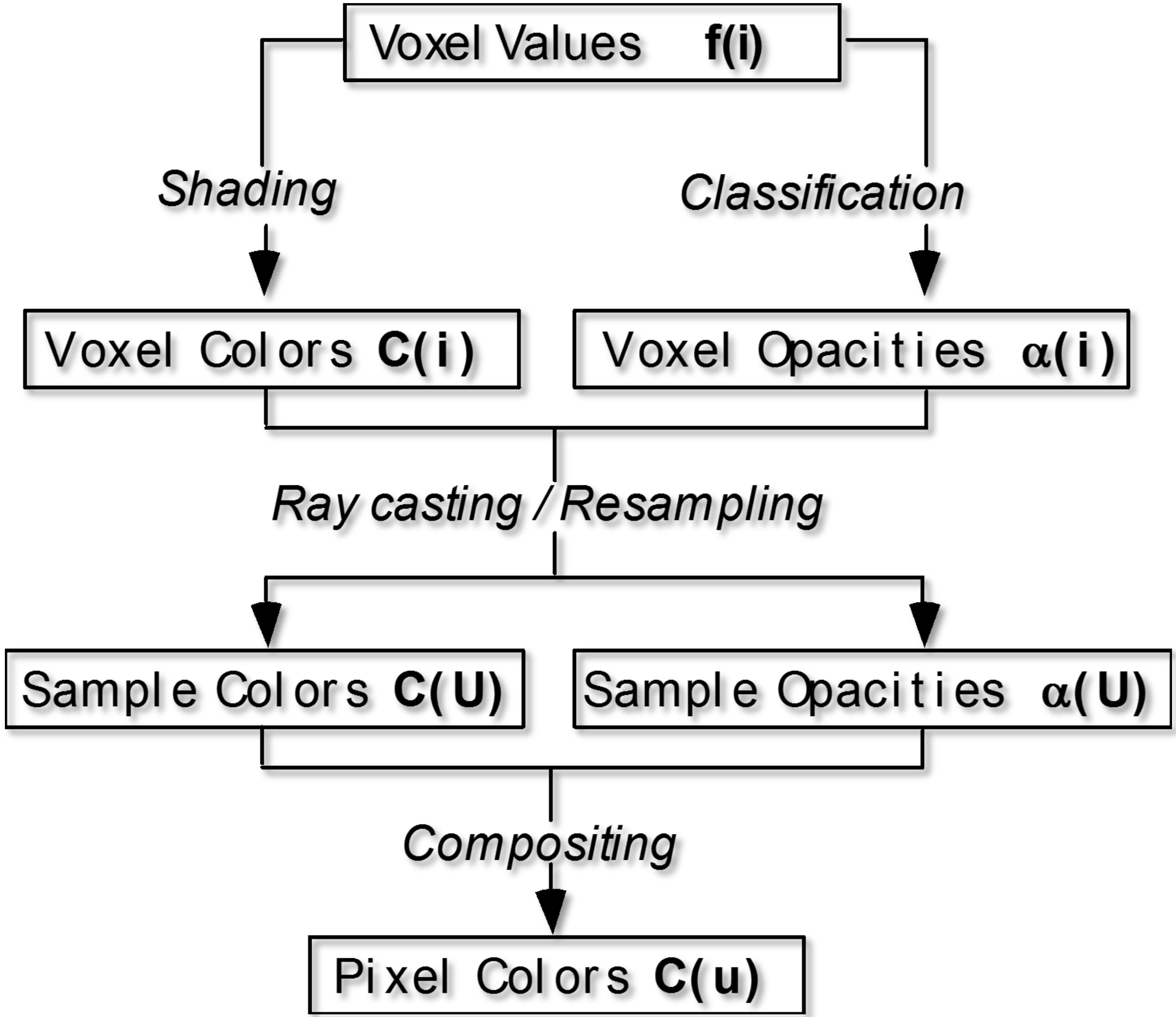
Ray Traversal through Volume Data

- Context:
  - Volume data: 1D value defined in 3D – $f(\mathbf{x}) \in R^1$, $\mathbf{x} \in R^3$
  - Ray defined as half-line: $\mathbf{r}(t) \in R^3$, $t \in R^1 > 0$
  - Values along Ray: $f(\mathbf{r}(t)) \in R^1$, $t \in R^1 > 0$ (intensity profile)



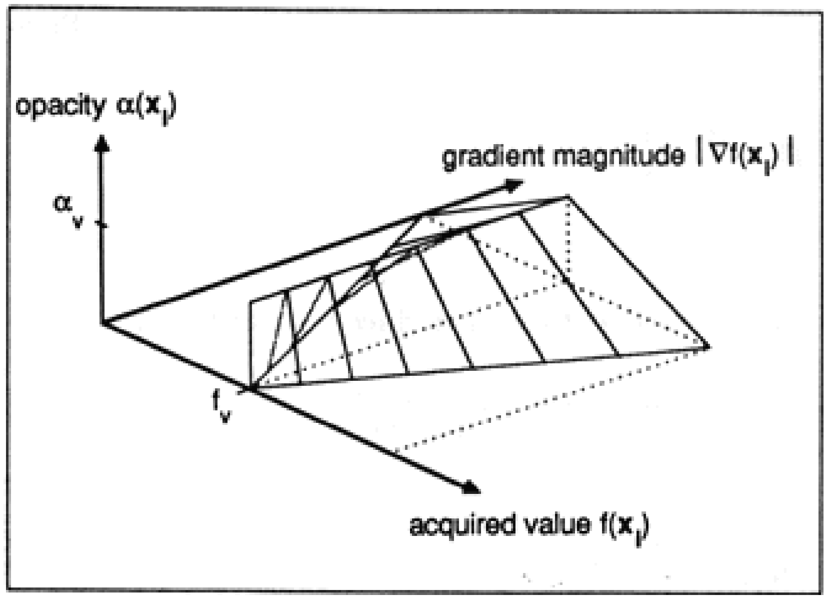
Standard Ray Casting

- Levoy '88:
- 1. C(i), α(i) (from TF)
- 2. Ray casting, interpolation
- 3. Compositing (or combinations)

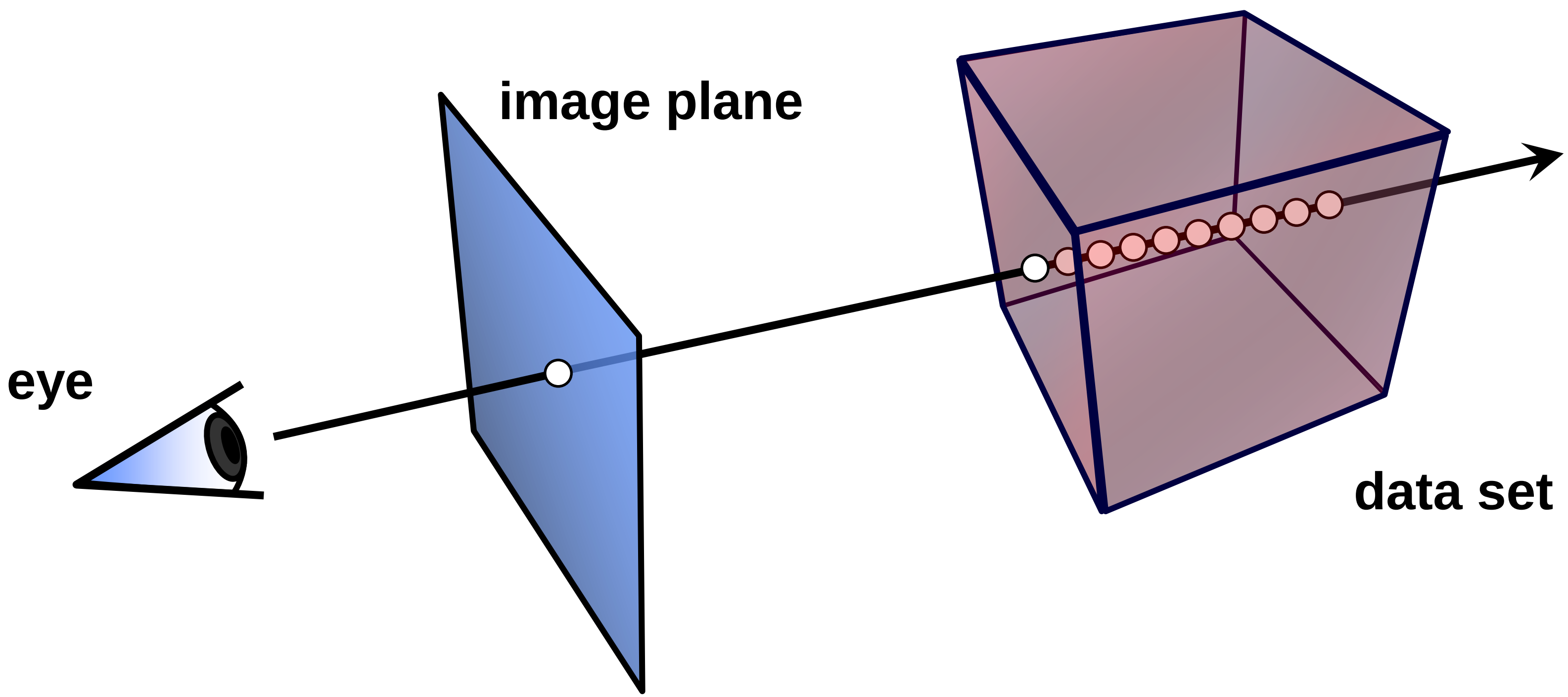


1. Shading, Classification

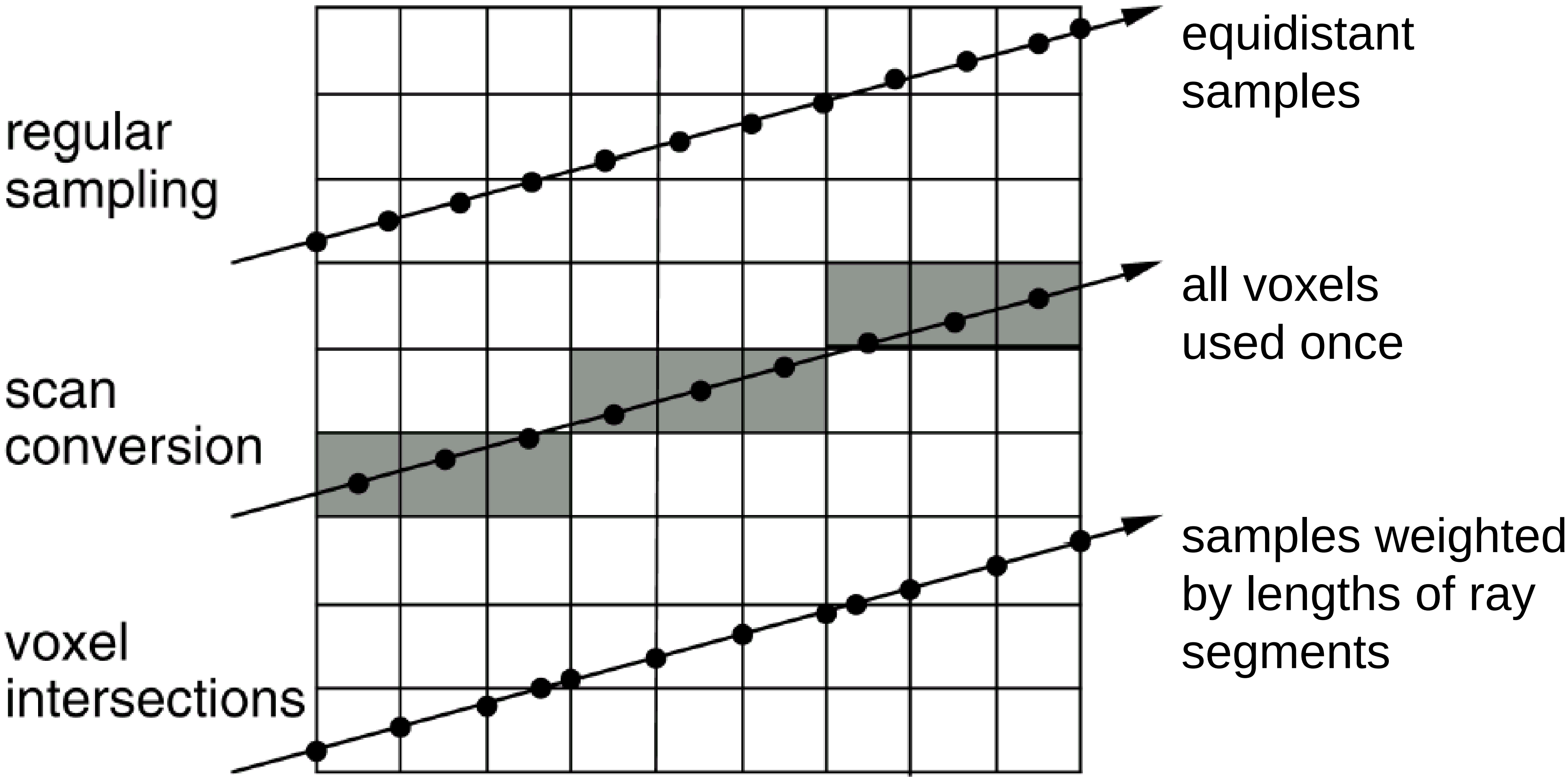
- 1. Step:
  - Shading, f(i)→C(i):
    - Apply transfer function
    - diffuse illumination (Phong), gradient ≈ normal
  - Classification, f(i)→α(i):
    - Levoy '88, gradient enhanced
    - Emphasizes transitions
  - Nowadays: shading/classification after ray-casting/resampling



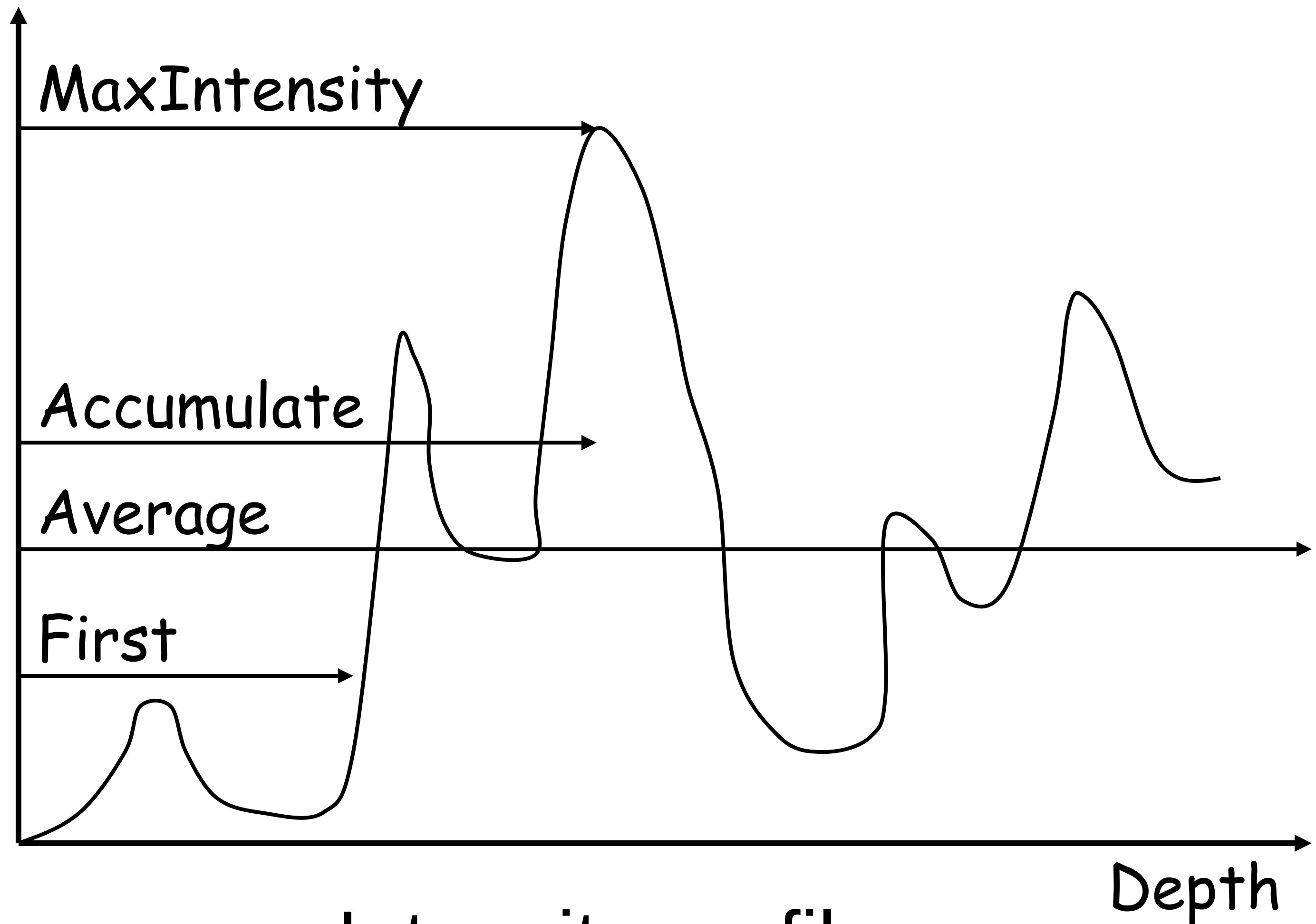
- Cast ray through the volume and perform sampling at discrete positions



2. Ray Traversal – Three Approaches



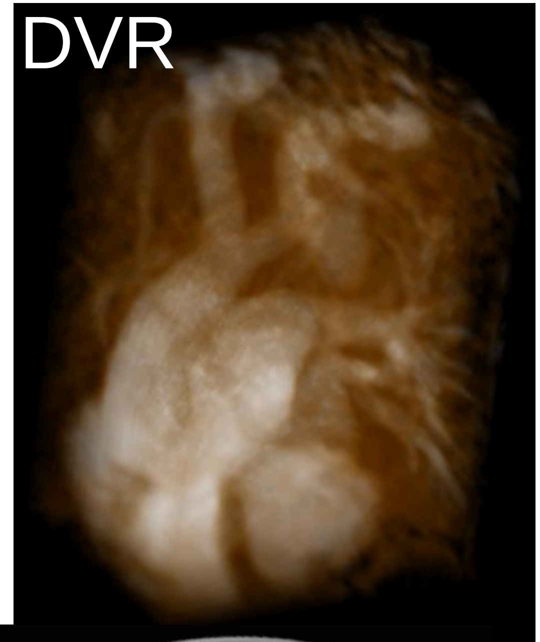
- Overview:
  - MIP ⇒
  - Compositing ⇒
  - X-Ray ⇒
  - First hit ⇒

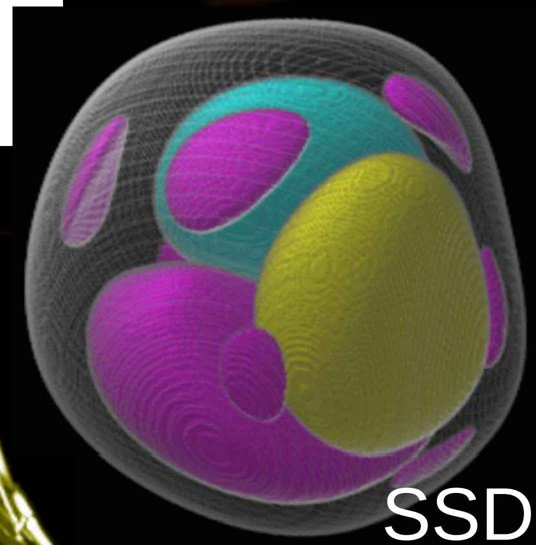


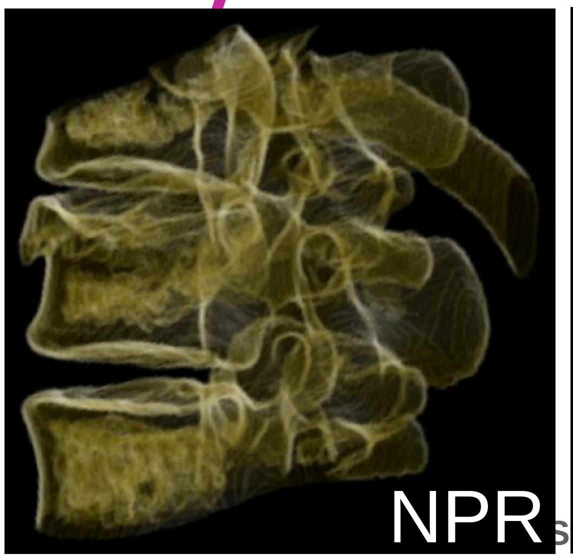
Intensity profile

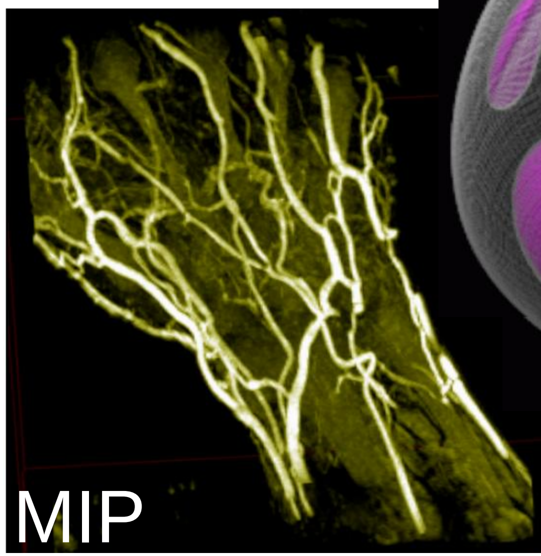
Types of Combinations

- Possibilities:
  - α-compositing
  - Shaded surface display (first hit)
  - Maximum-intensity projection (MIP)
  - X-ray simulation
  - Contour rendering









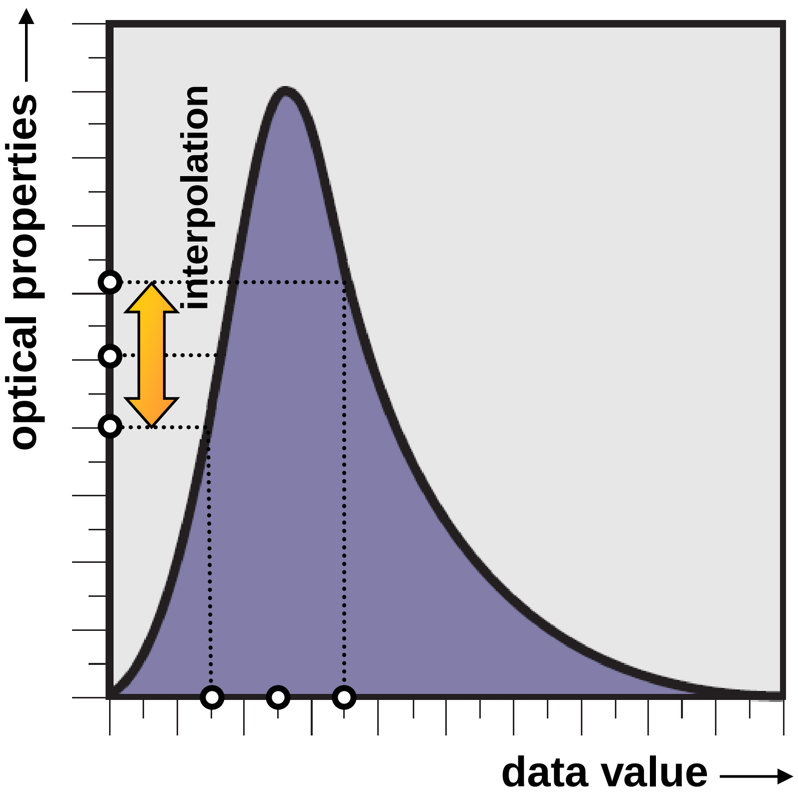
Classification Order (1)

- Shading/classification can occur before or after ray traversal

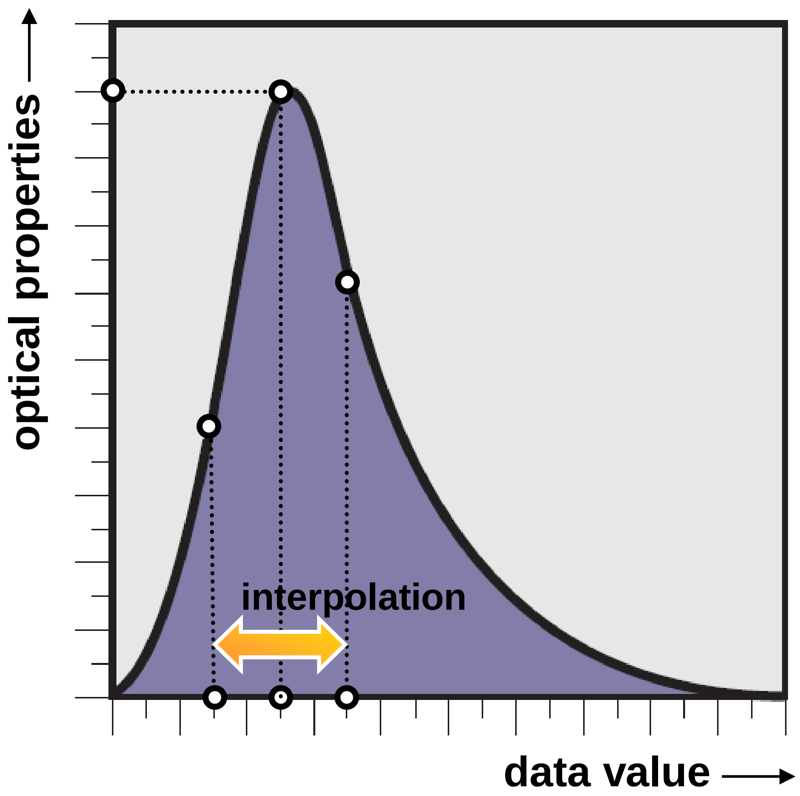
  - **Pre-interpolative:** classify all data values and then interpolate between RGBA-tuples

  - **Post-interpolative:** interpolate between scalar data values and then classify the result

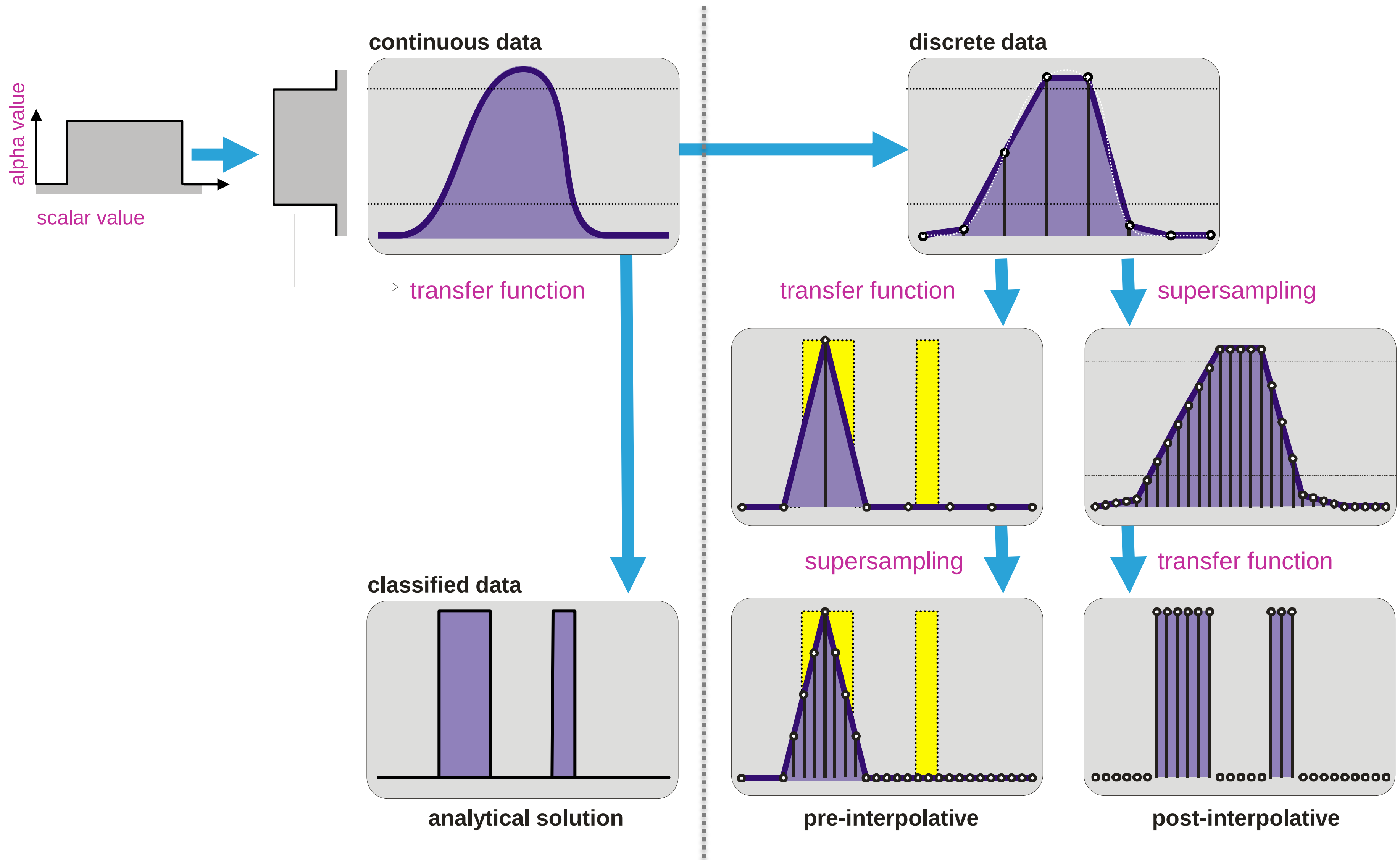
PRE-INTERPOLATIVE



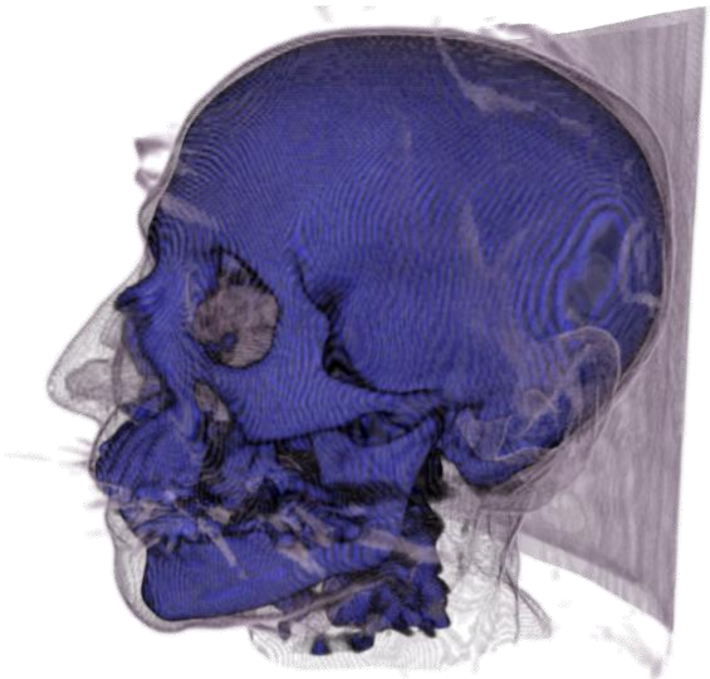
POST-INTERPOLATIVE



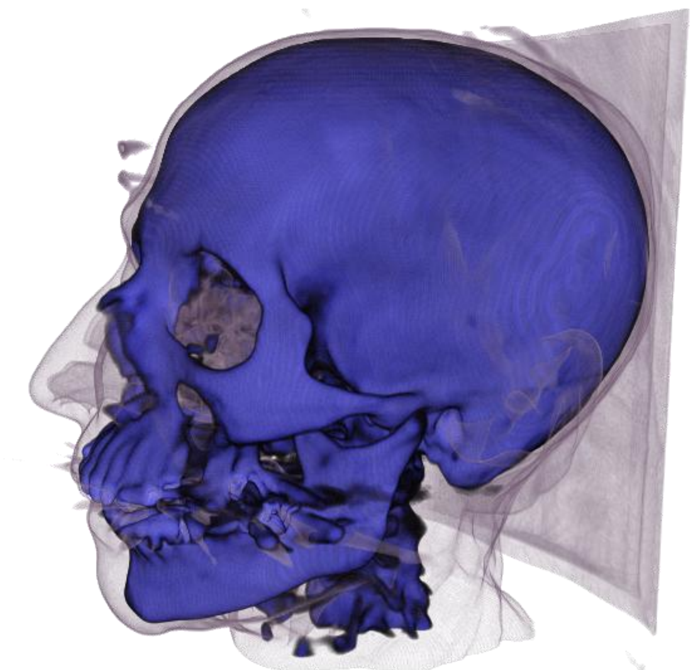
Classification Order (3)



Classification Order: Example 1



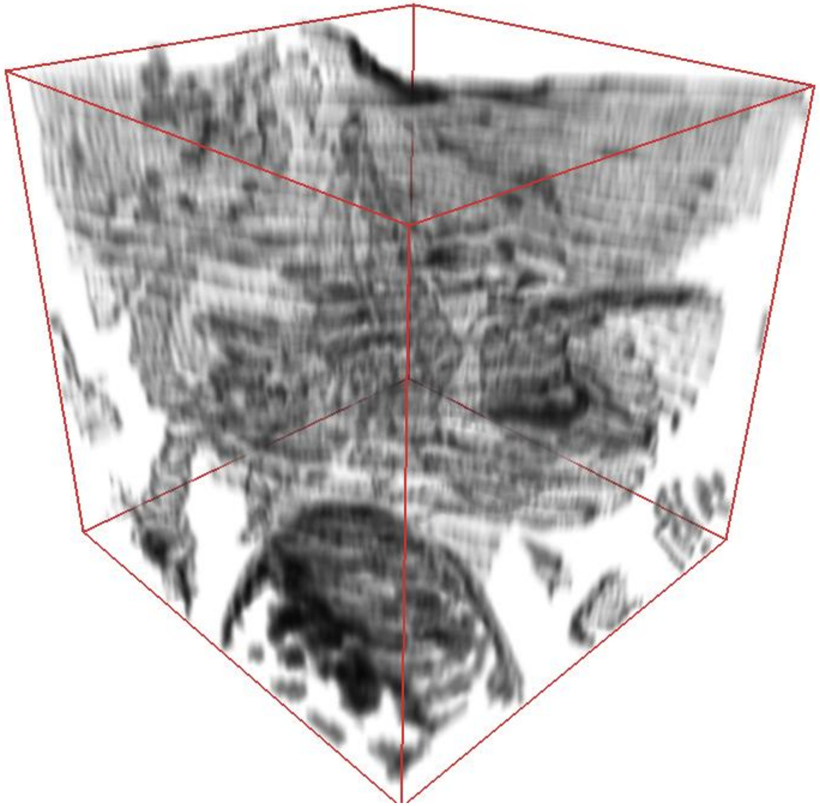
pre-interpolative

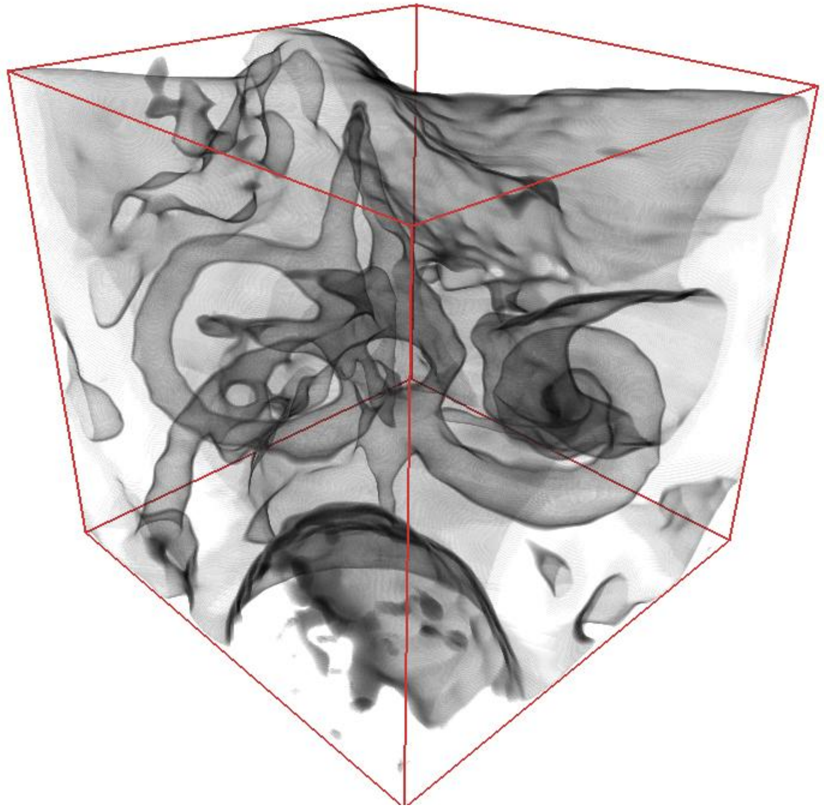


post-interpolative

same transfer function, resolution, and sampling rate

Classification Order: Example 2





pre-interpolative **post-interpolative**

same transfer function, resolution, and sampling rate

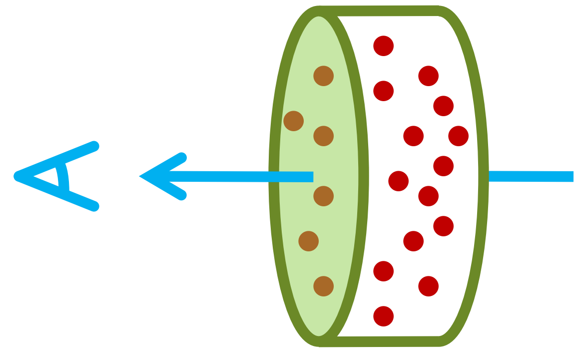
α-Compositing –

a Specific Optical Model for Volume Rendering

Display of

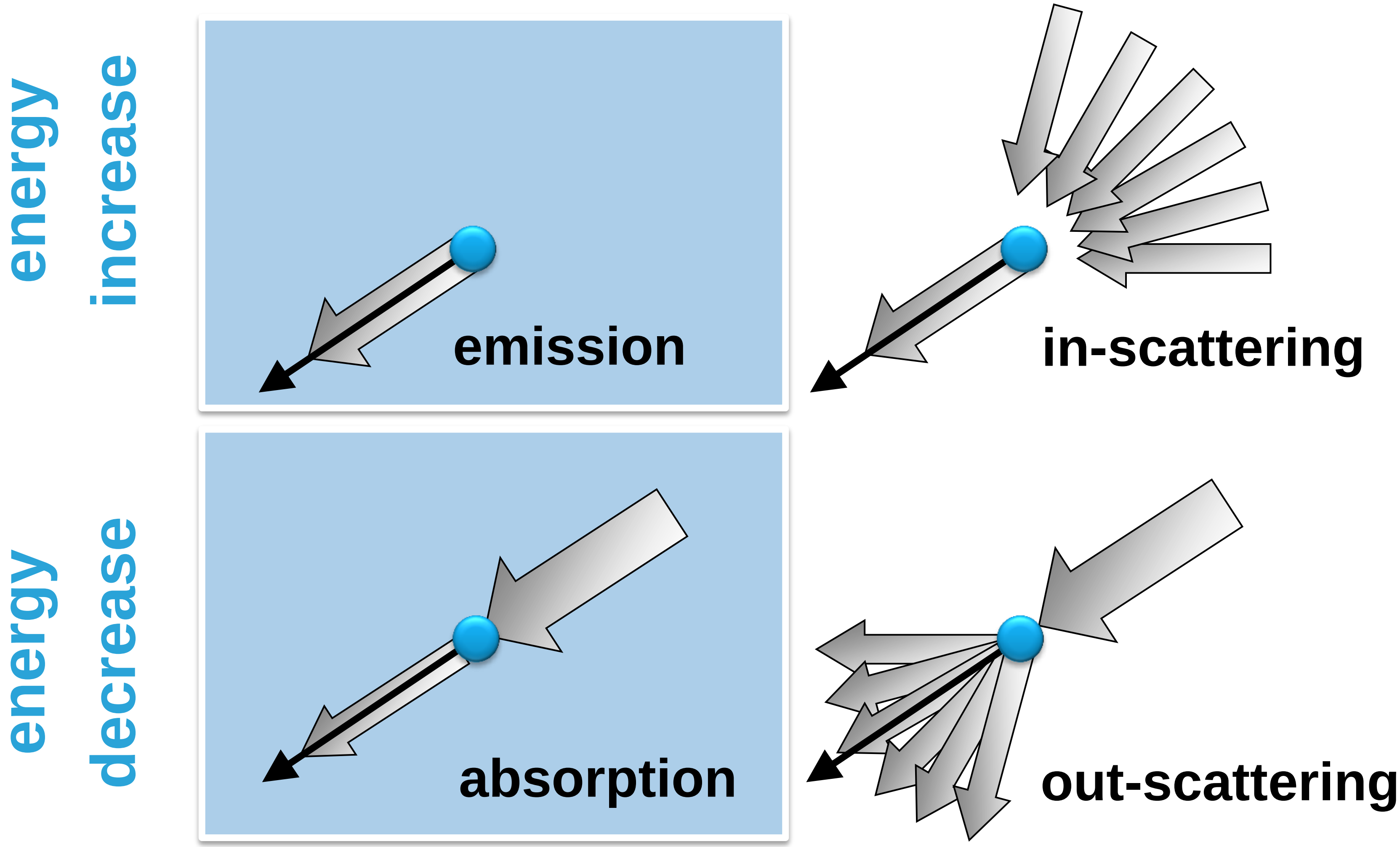
Semi-Transparent Media

- Various models (Examples):
  - Emission only (light particles)
  - Absorption only (dark fog)
  - Emission & absorption (clouds)
  - Single scattering, w/o shadows
  - Multiple scattering

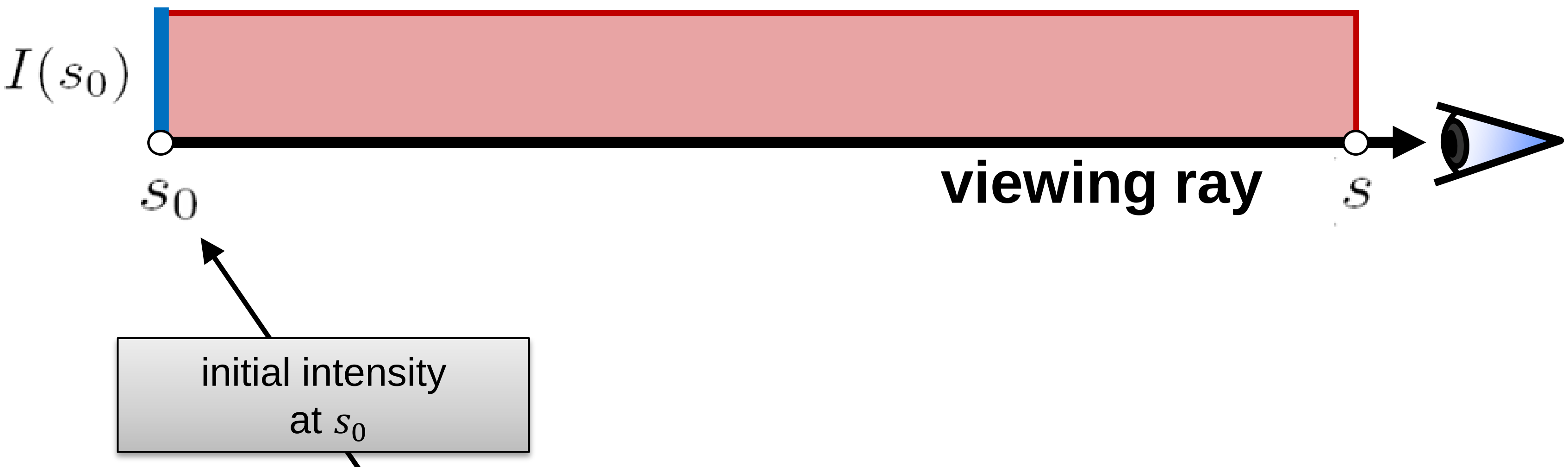


- Two approaches:
  - Analytical model (via differentials)
  - Numerical approximation (via differences)

Physical Model of Radiative Transfer



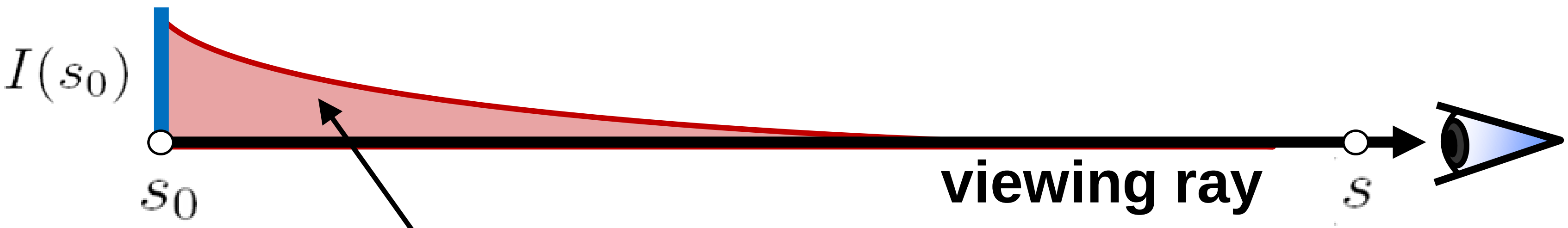
Analytical Model (1)



$$I(s) = \boxed{I(s_0)}$$

without absorption all the initial radiant energy would reach the point s

Analytical Model (2)



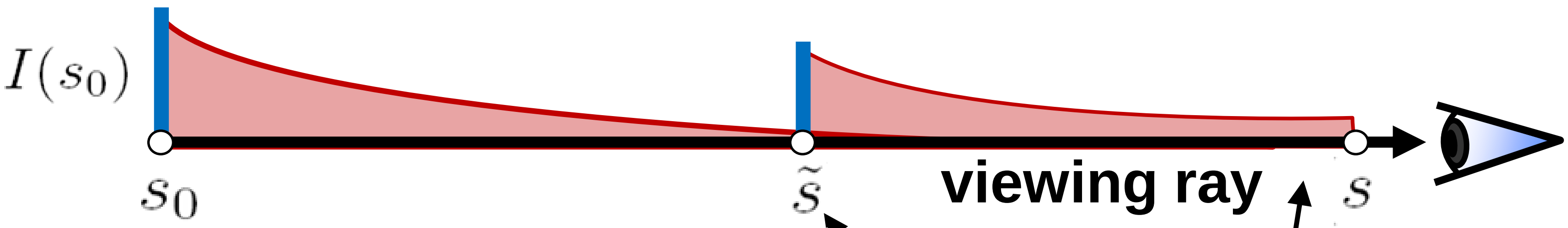
absorption between s_0 and s

$$I(s) = I(s_0)\, e^{-\tau(s_0, s)}$$

Extinction τ
Absorption κ

$$\tau(s_1, s_2) = \int_{s_1}^{s_2} \kappa(s)\, ds.$$

Analytical Model (3)



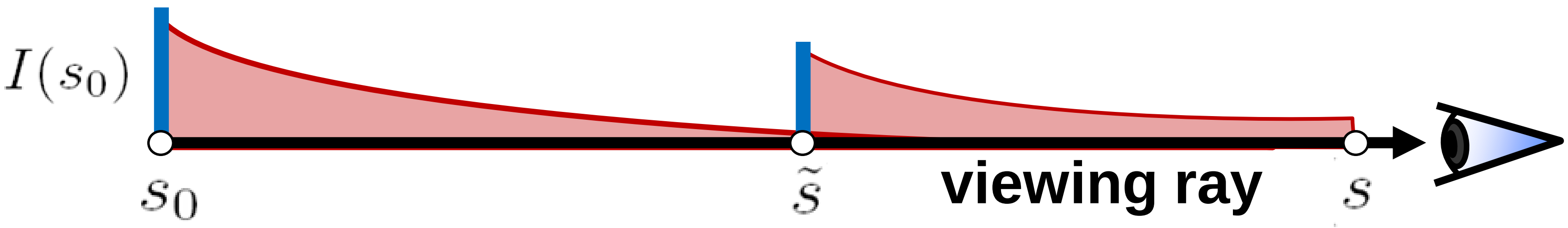
one point $\tilde{s}$ along the viewing ray emits additional radiant energy

active emission at point $\tilde{s}$

absorption between $\tilde{s}$ and s

$$I(s) = I(s_0)\, e^{-\tau(s_0, s)} + q(\tilde{s})\, e^{-\tau(\tilde{s}, s)}$$

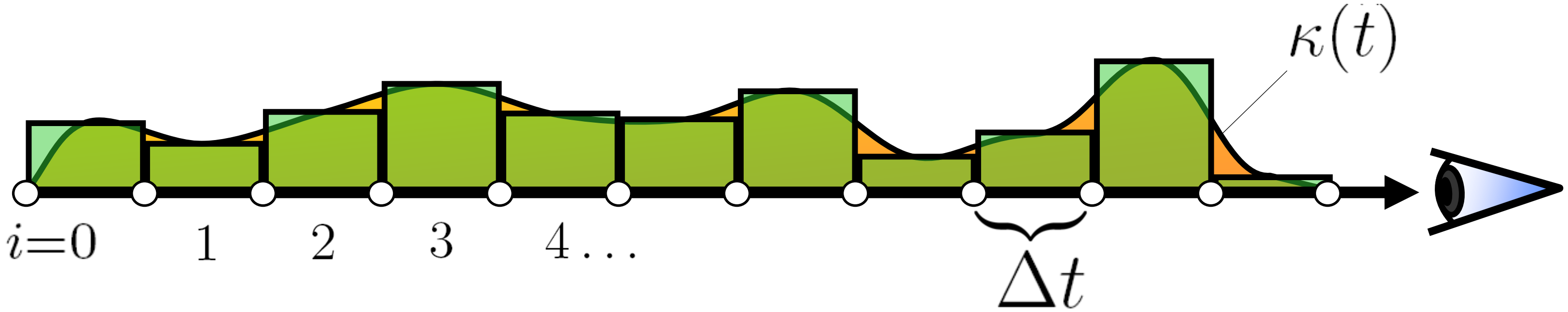
Analytical Model (4)



every point $\tilde{s}$ along the viewing ray emits additional radiant energy

$$I(s) \;=\; I(s_0)\, e^{-\tau(s_0,s)} \;+\; \int_{s_0}^{s} q(\tilde{s})\, e^{-\tau(\tilde{s},s)}\, d\tilde{s}$$

Numerical Approximation (1)

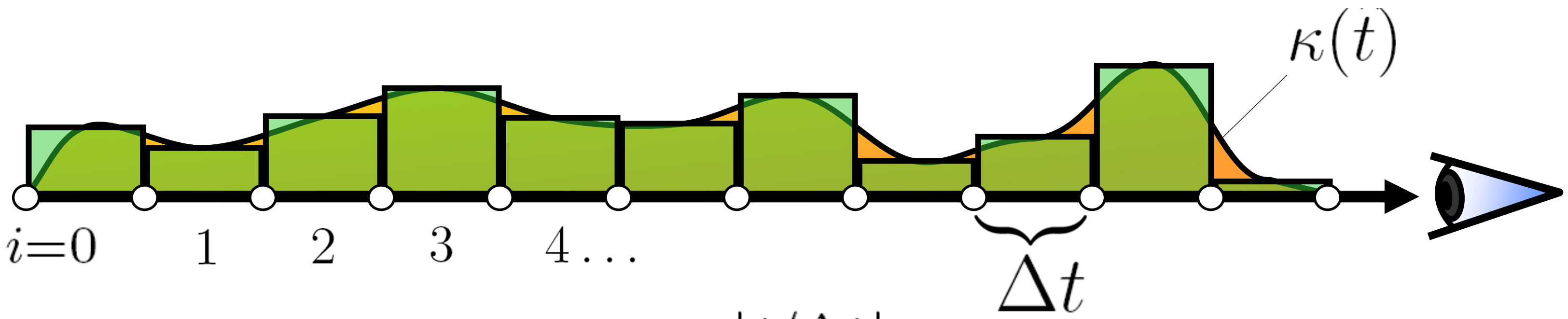


Extinction: $$\tau(0,t) \;=\; \int_0^t \kappa(\hat{t})\, d\hat{t}$$

approximate integral by Riemann sum:

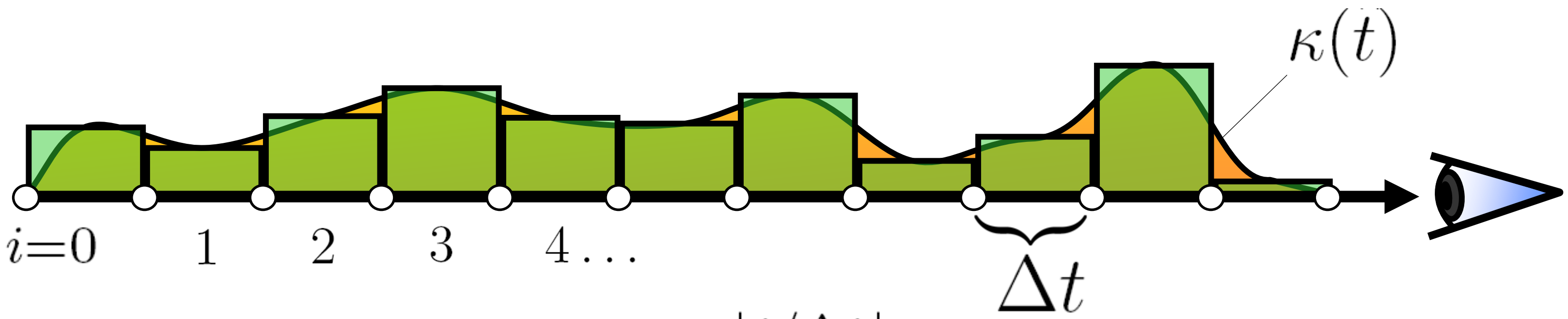
$$\tau(0,t) \;\approx\; \sum_{i=0}^{\lfloor t/\Delta t \rfloor} \kappa(i \cdot \Delta t)\, \Delta t$$

Numerical Approximation (2)



$$\tau(0,t) \approx \tilde{\tau}(0,t) = \sum_{i=0}^{\lfloor t/\Delta t \rfloor} \kappa(i \cdot \Delta t)\, \Delta t$$

$$e^{-\tilde{\tau}(0,t)} = e^{-\sum_{i=0}^{\lfloor t/\Delta t \rfloor} \kappa(i \cdot \Delta t)\, \Delta t}$$



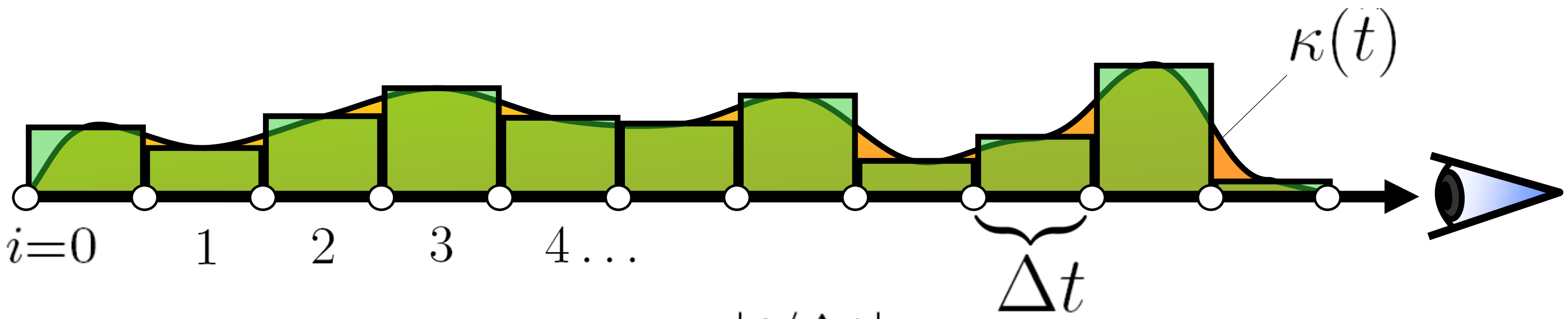
$$\tau(0,t) \approx \tilde{\tau}(0,t) = \sum_{i=0}^{\lfloor t/\Delta t \rfloor} \kappa(i \cdot \Delta t)\,\Delta t$$

$$e^{-\tilde{\tau}(0,t)} = \prod_{i=0}^{\lfloor t/\Delta t \rfloor} e^{-\kappa(i \cdot \Delta t)\,\Delta t}$$

now we introduce opacity:

$$A_i = 1 - e^{-\kappa(i \cdot \Delta t)\,\Delta t}$$

Numerical Approximation (4)



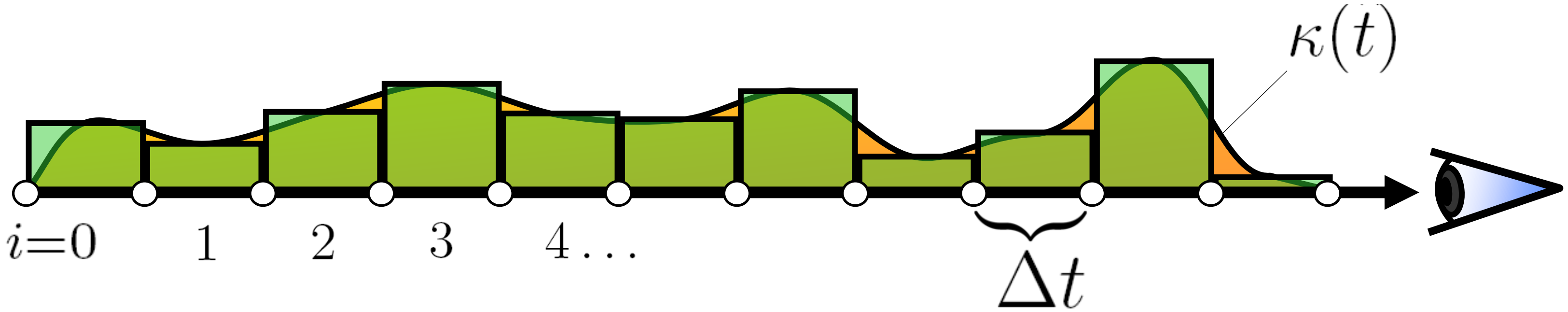
$$\tau(0,t) \approx \tilde{\tau}(0,t) = \sum_{i=0}^{\lfloor t/\Delta t \rfloor} \kappa(i \cdot \Delta t)\, \Delta t$$

$$e^{-\tilde{\tau}(0,t)} = \prod_{i=0}^{\lfloor t/\Delta t \rfloor} e^{-\kappa(i \cdot \Delta t)\, \Delta t}$$

now we introduce opacity:

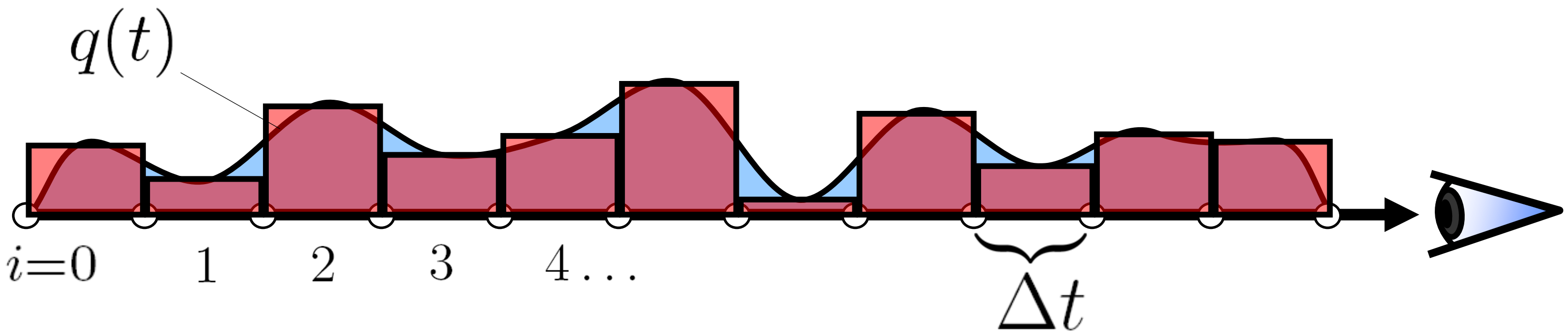
$$(1 - A_i) = e^{-\kappa(i \cdot \Delta t)\, \Delta t}$$

Numerical Approximation (5)



$$e^{-\tilde{\tau}(0,t)} = \prod_{i=0}^{\lfloor t/\Delta t \rfloor} (1 - A_i)$$

Numerical Approximation (6)

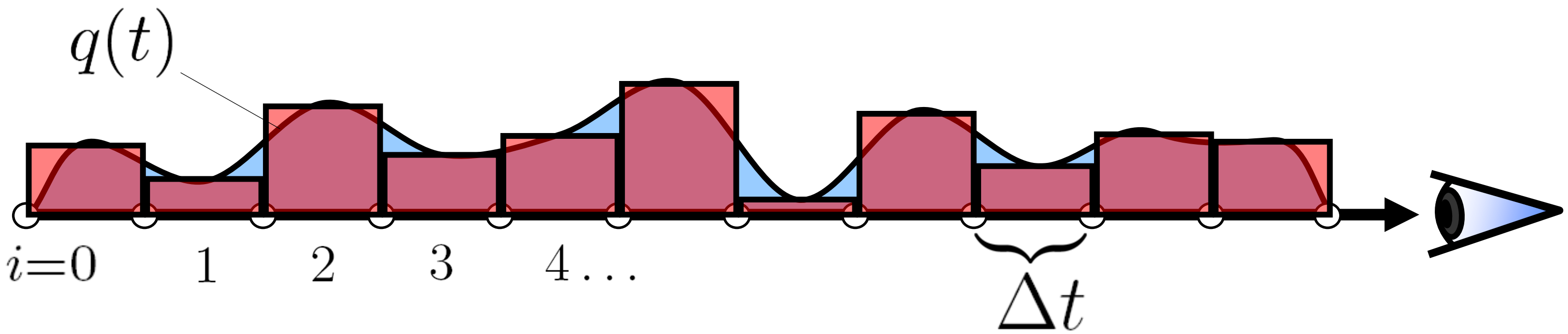


$$e^{-\tilde{\tau}(0,t)} = \prod_{i=0}^{\lfloor t/\Delta t \rfloor} (1 - A_i)$$

$$q(t) \approx C_i = c(i \cdot \Delta t)\,\Delta t$$

$$\tilde{C} = \sum_{i=0}^{\lfloor T/\Delta t \rfloor} C_i \, e^{-\tilde{\tau}(0,t)}$$

Numerical Approximation (7)

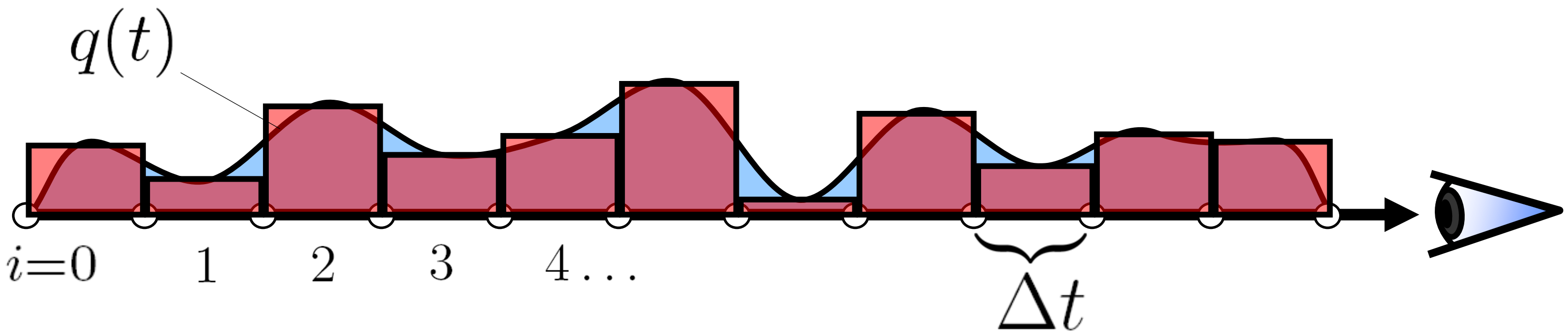


$$e^{-\tilde{\tau}(0,t)} = \prod_{i=0}^{\lfloor t/\Delta t \rfloor} (1 - A_i)$$

$$q(t) \approx C_i = c(i \cdot \Delta t)\,\Delta t$$

$$\tilde{C} = \sum_{i=0}^{\lfloor T/\Delta t \rfloor} C_i \prod_{j=0}^{i-1} (1 - A_j)$$

Numerical Approximation (8)



$$\tilde{C} = \sum_{i=0}^{\lfloor T/\Delta t \rfloor} C_i \prod_{j=0}^{i-1} (1 - A_j)$$

can be computed recursively:

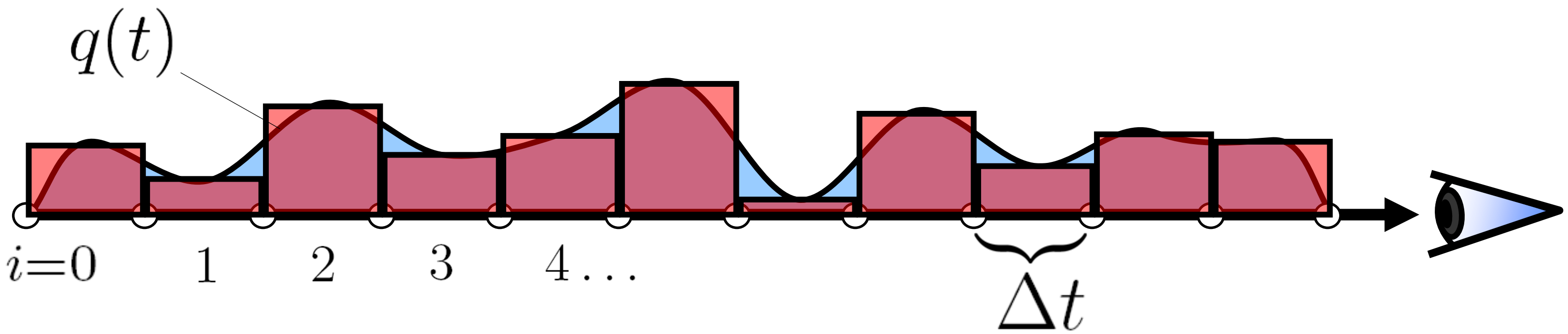
$$C'_i = C_i + (1 - A_i) C'_{i-1}$$

radiant energy observed at position i

radiant energy emitted at position i

absorption at position i

radiant energy observed at position $i-1$



back-to-front compositing

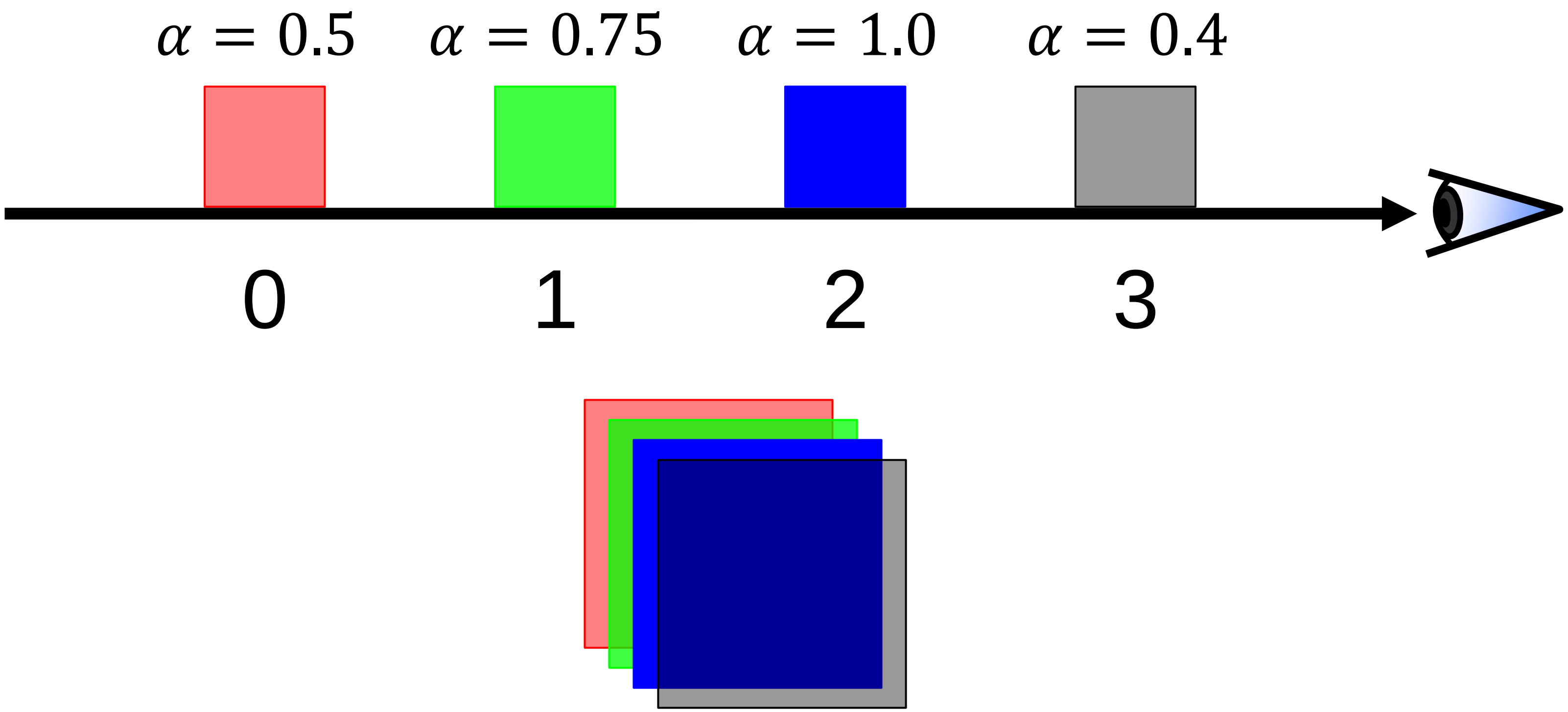
$$C_i' = C_i + (1 - A_i)C_{i-1}'$$

front-to-back compositing

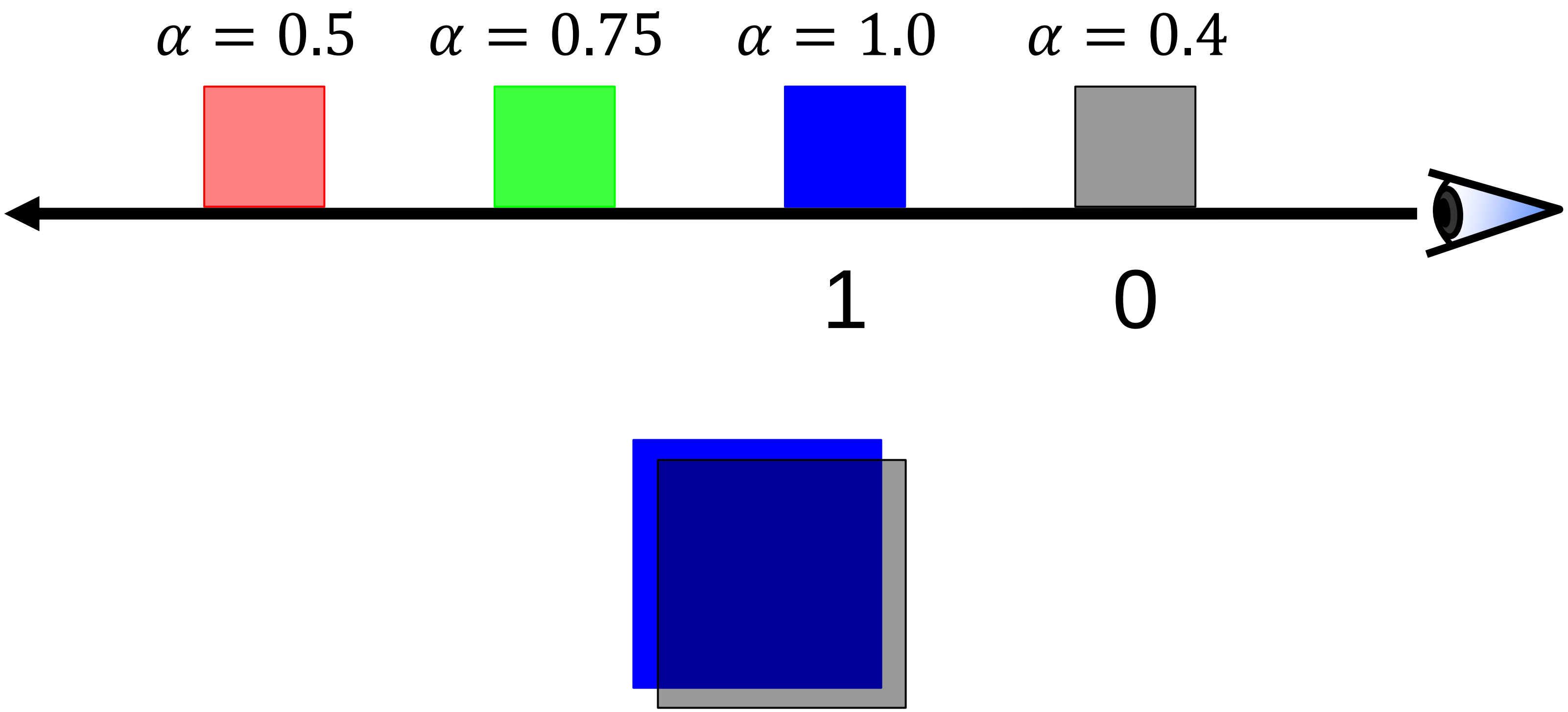
$$C_i' = C_{i+1}' + (1 - A_{i+1}')C_i$$
$$A_i' = A_{i+1}' + (1 - A_{i+1}')A_i$$

early ray termination: stop the calculation when $A_i' \approx 1$

Back-to-Front Compositing: Example



Front-to-Back Compositing: Example



Summary

- Emission Absorption Model

$$I(s) = I(s_0)\, e^{-\tau(s_0,s)} + \int_{s_0}^{s} q(\tilde{s})\, e^{-\tau(\tilde{s},s)}\, \mathrm{d}\tilde{s}$$

- Numerical Solutions **[pre-multiplied alpha assumed]**

back-to-front iteration

$$C'_i = C_i + (1 - A_i)C'_{i-1}$$

front-to-back iteration

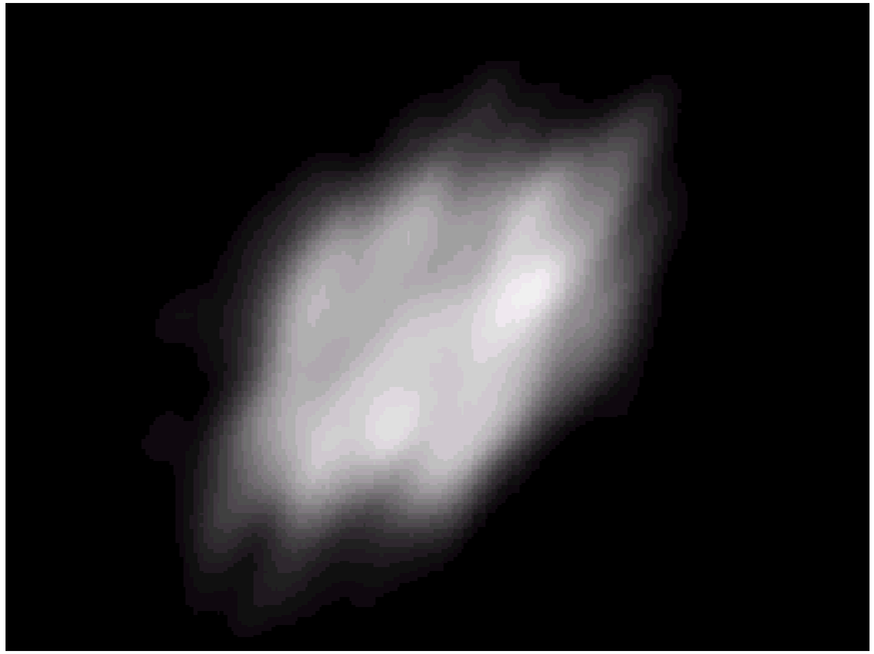
$$C'_i = C'_{i+1} + (1 - A'_{i+1})C_i$$
$$A'_i = A'_{i+1} + (1 - A'_{i+1})A_i$$

Pre-Multiplied Alpha

- Color values are stored pre-multiplied by their opacity: $(\alpha r, \alpha g, \alpha b)$
- Consequence: transparent red is the same as transparent black, etc.
- Simplifies blending: color and alpha values are treated equally
- Can result in loss of precision

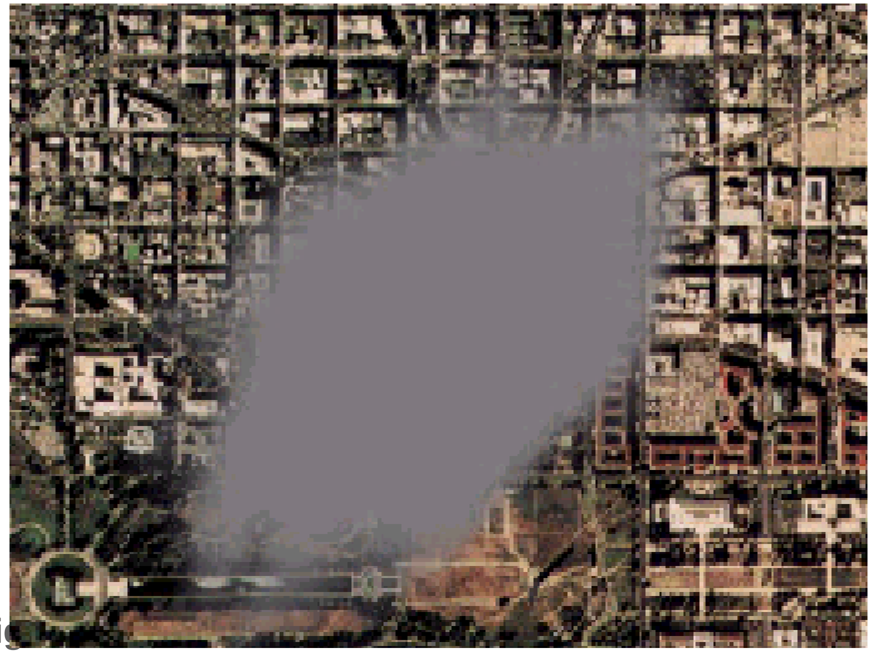
Emission or/and Absorption

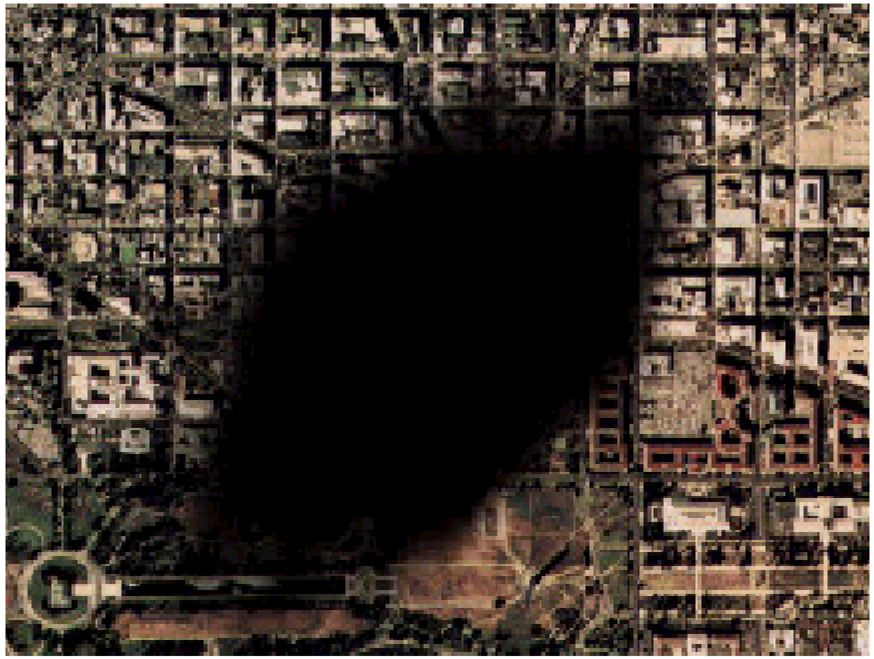
Emission
only



Emission
and Absorption

Absorption
only

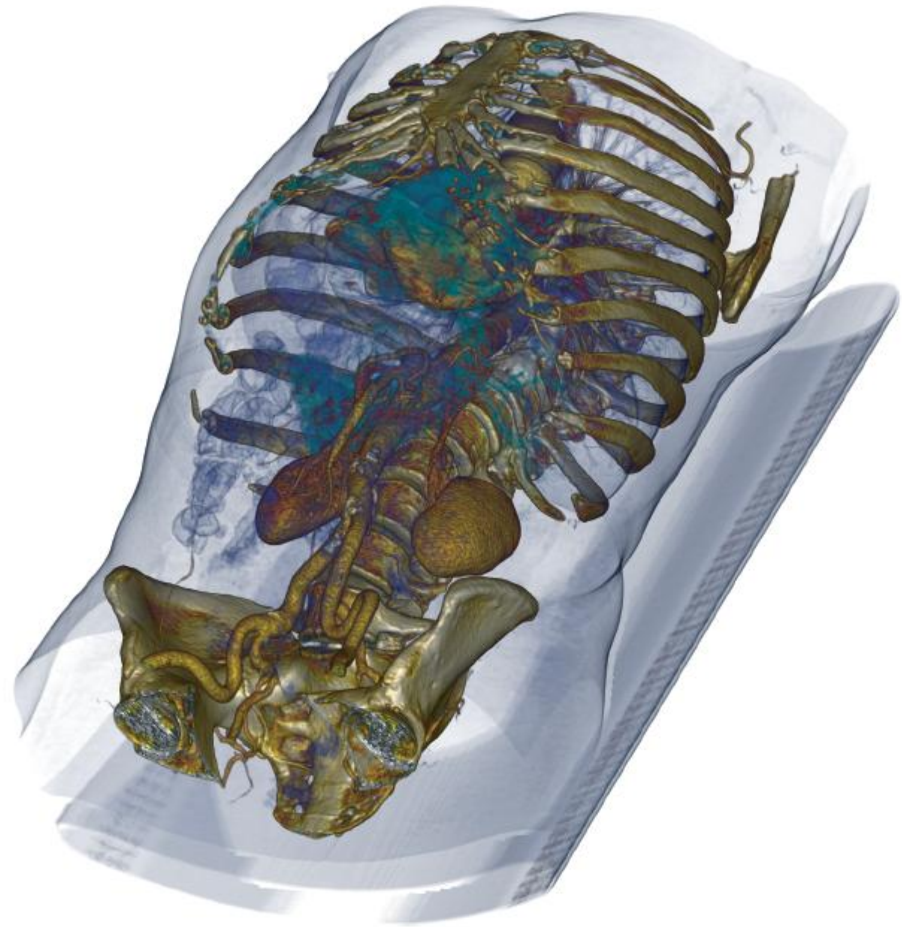


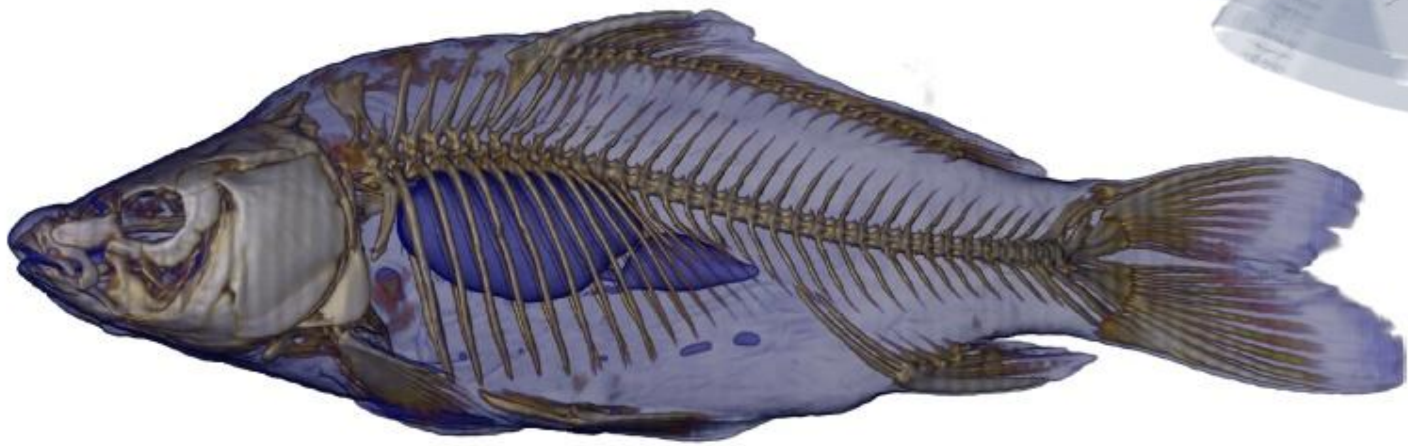


Ray Casting – Examples

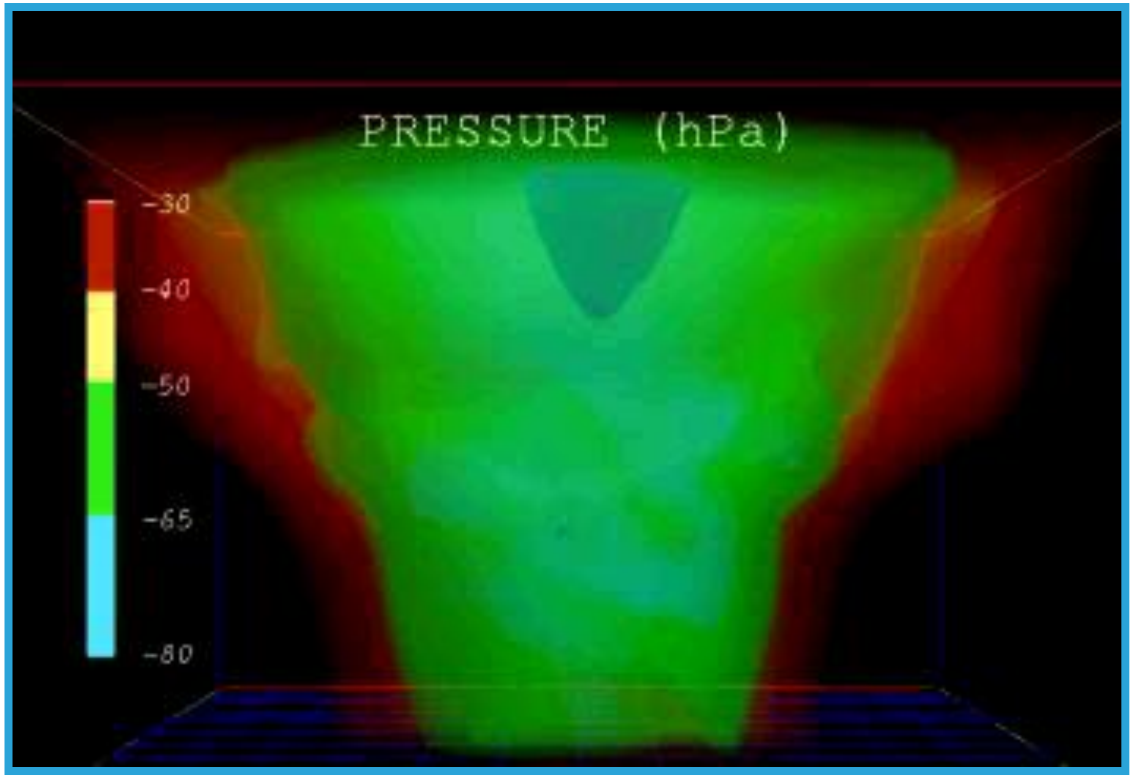
- CT scan of human hand (244x124x257, 16 bit)

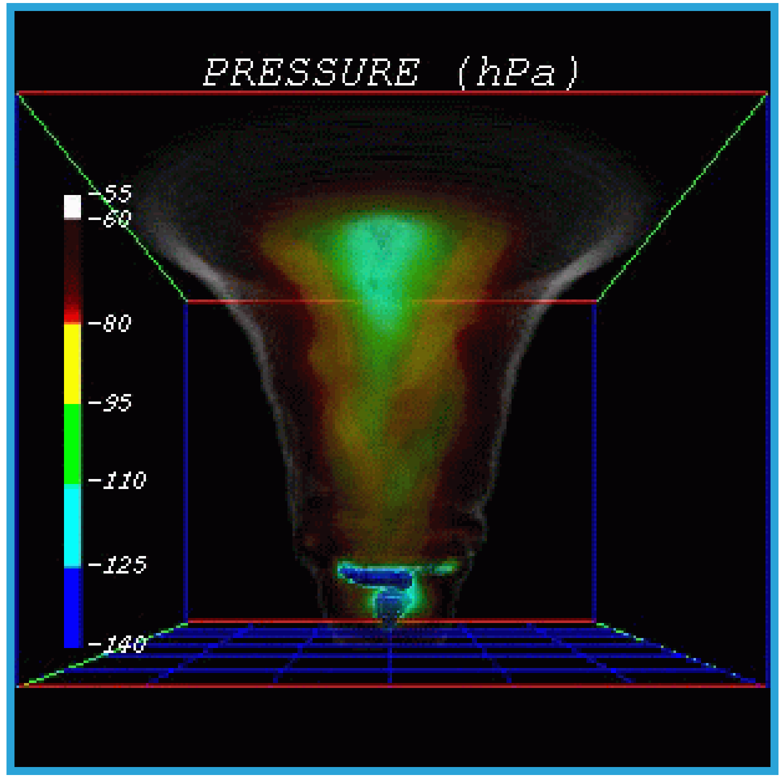
Ray Casting – Examples





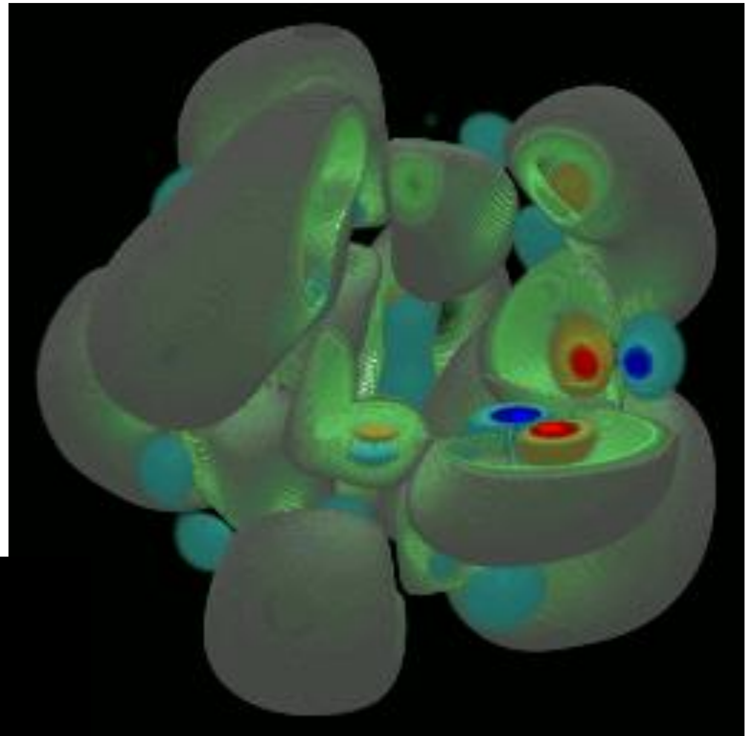
- Tornado Visualization:

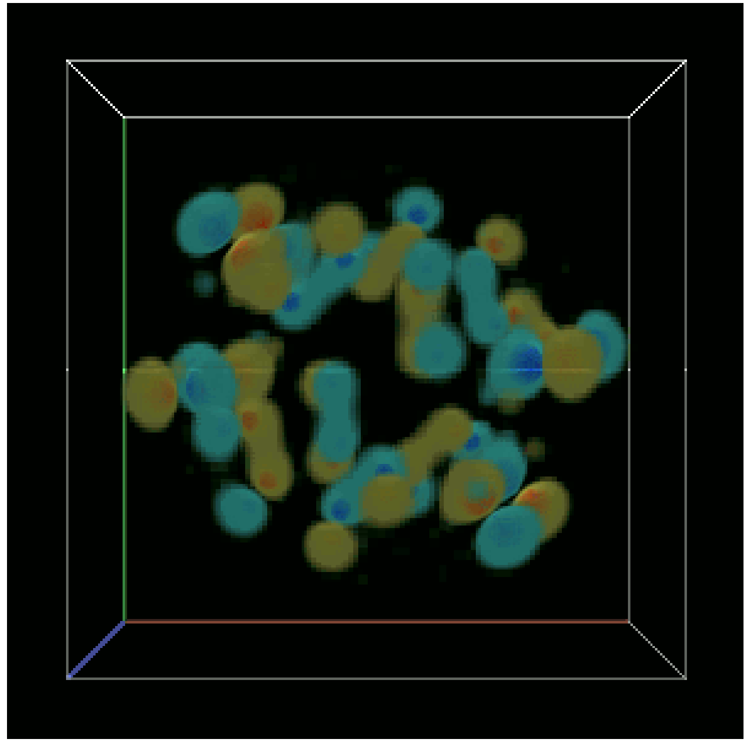


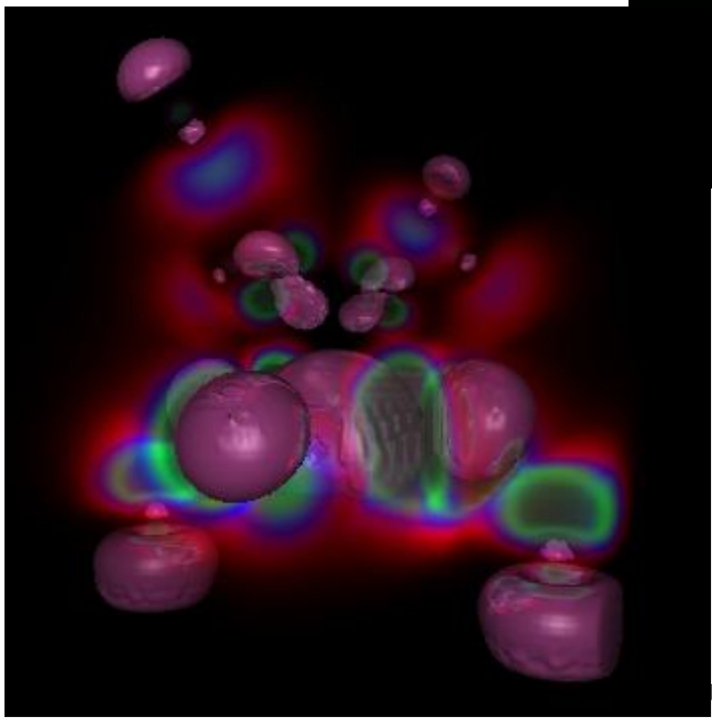


Ray Casting – Further Examples

- Molecular data:







Hardware-Volume Visualization

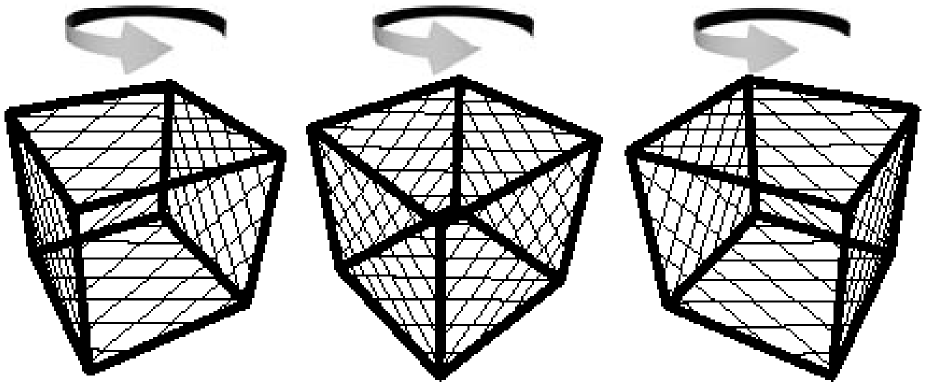
Faster with Hardware?!

Two Approaches

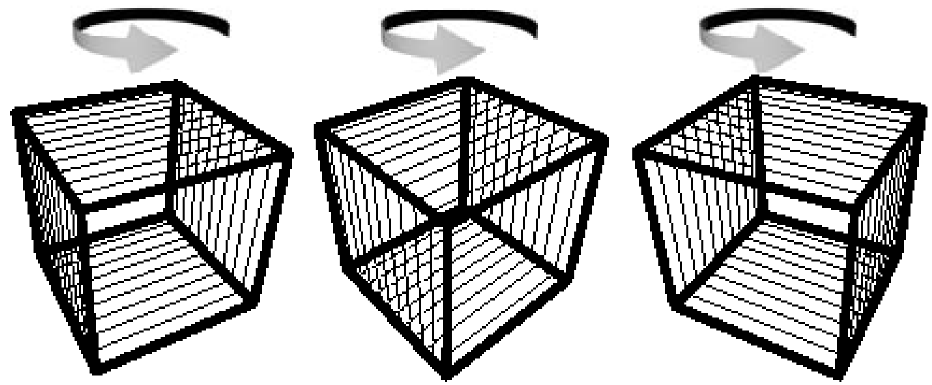
- 3D-textures:
  - Volume data stored in 3D-texture
  - Proxy geometry (slices) parallel to image plane, are interpolated tri-linearly
  - Back-to-front compositing
- 2D-textures:
  - 3 stacks of slices (x-, y- & z-axis), slices are interpolated bi-linearly
  - Select stack (most “parallel” to image plane)
  - Back-to-front compositing

Variation of View Point

- 3D-textures:
  - Number of slices varies
- 2D-textures:
  - Stack change: discontinuity



Viewport-Aligned Slices



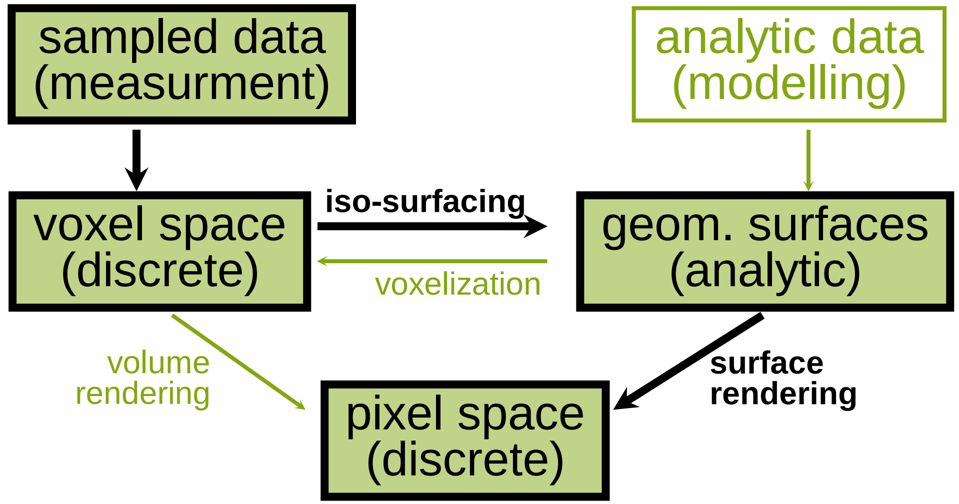
Object-Aligned Slices

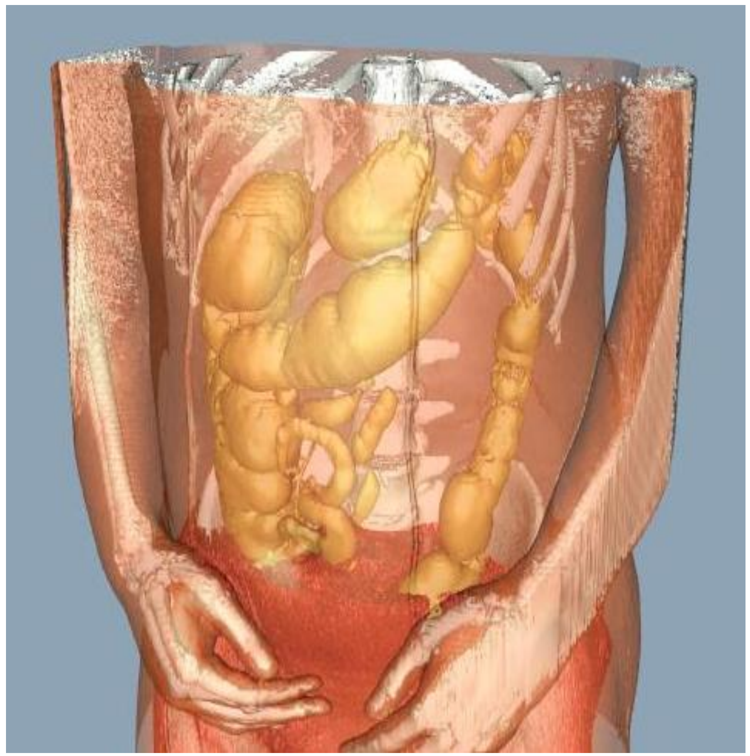
Indirect Volume Visualization

Iso-Surface-Display

Concepts and Terms

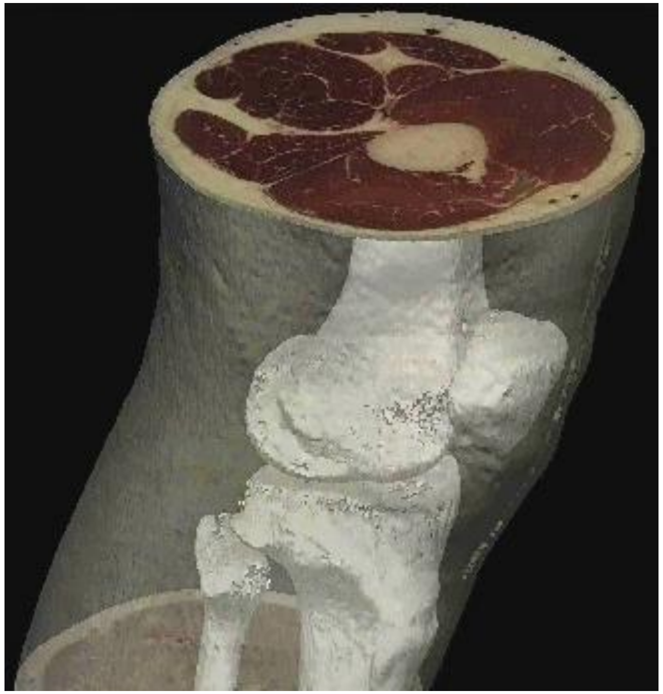
- Example
  - CT measurement
  - Iso-stack-conversion
  - Iso-surface-calculation (marching cubes)
  - Surface rendering (OpenGL)

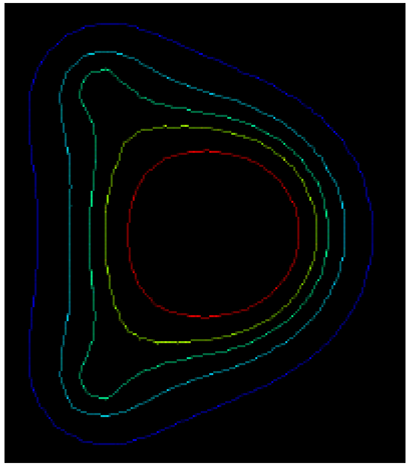


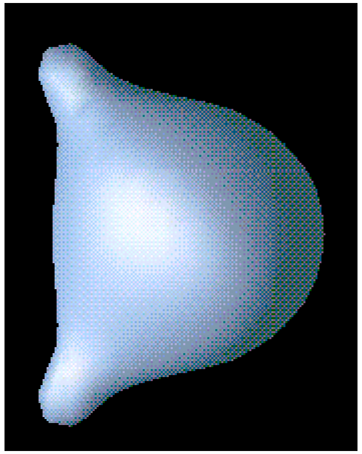


Iso-Surfaces

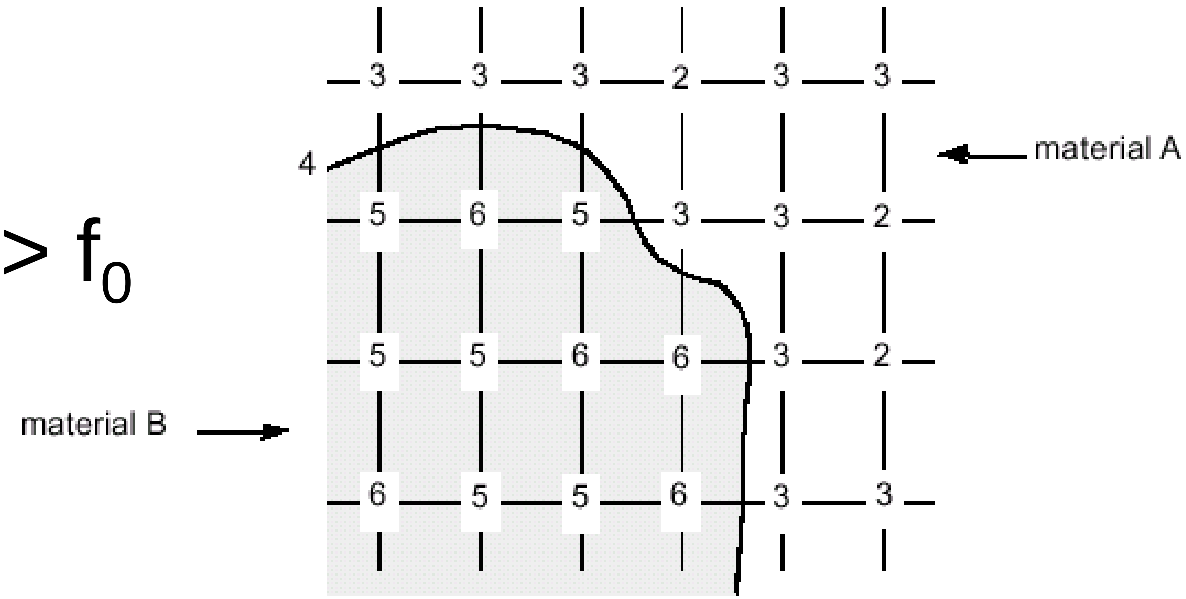
- Intermediate representation
- Aspects:
  - Preconditions:
    - expressive Iso-value, Iso-value separates materials
    - Interest: in transitions
  - Very selective (binary selection / omission)
  - Uses traditional hardware
  - Shading ⇒ 3D-impression!







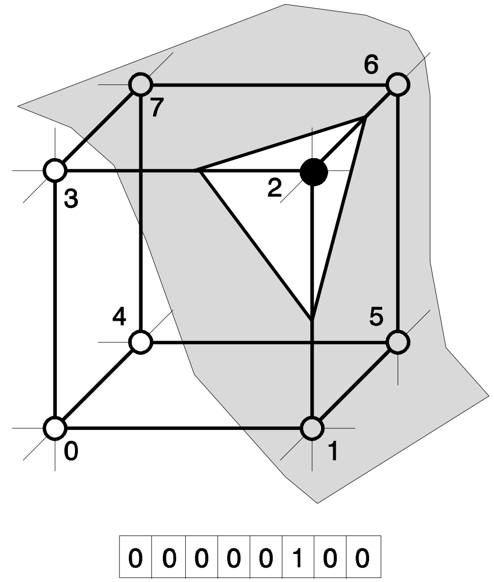
- Iso-Surface:
  - Iso-value f_0
  - Separates values $> f_0$ from values $\leq f_0$
  - Often not known →
  - Can only be approximated from samples!
  - Shape / position dependent on type of reconstruction

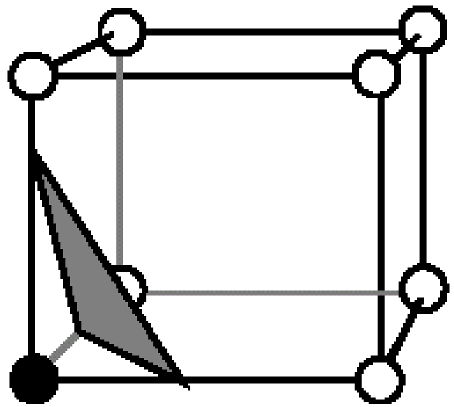


Marching Cubes (MC)

Iso-Surface-Display

- Approach:
  - Iso-Surface intersects data volume = set of all cells
- Idea:
  - Parts of iso-surface represented on a(n intersected) cell basis
  - As simple as possible: Usage of triangles

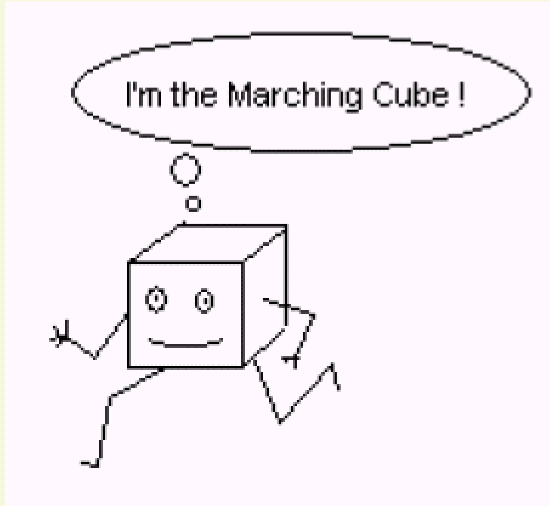




Marching Cubes

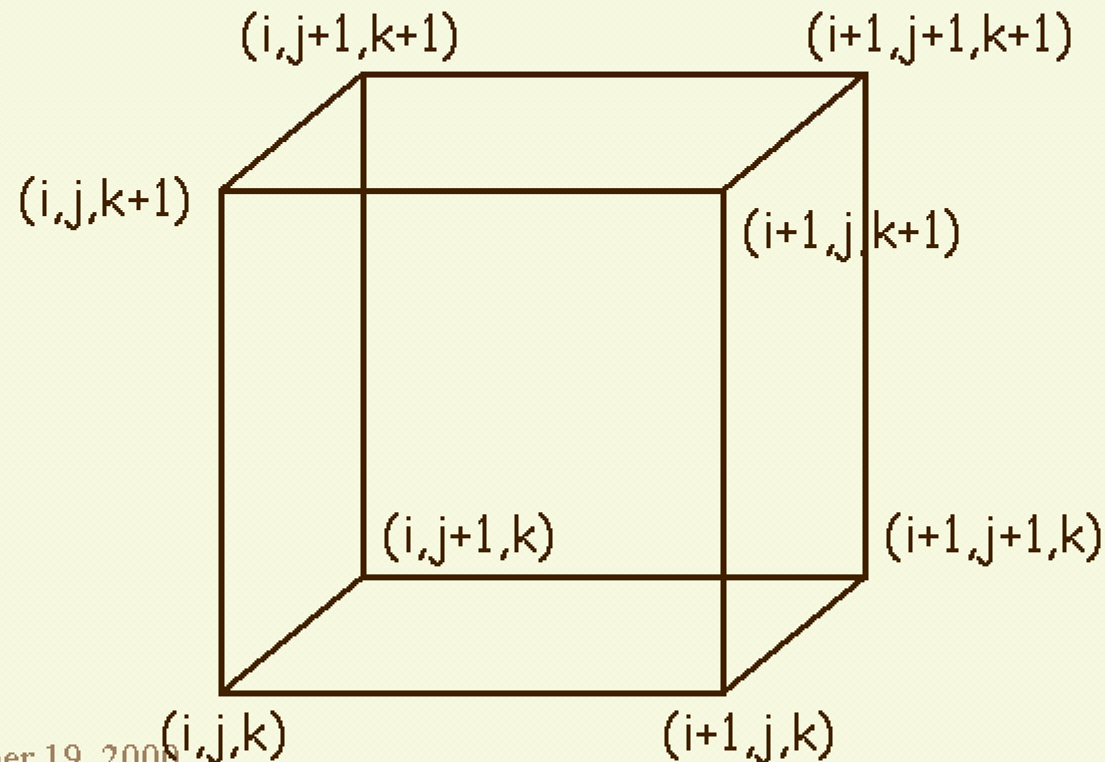
- ✓ Cell consists of 4(8) pixel (voxel) values: (i+[01], j+[01], k+[01])

1. Consider a Cell
2. Classify each vertex as inside or outside
3. Build an index
4. Get edge list from table[index]
5. Interpolate the edge location
6. Go to next cell



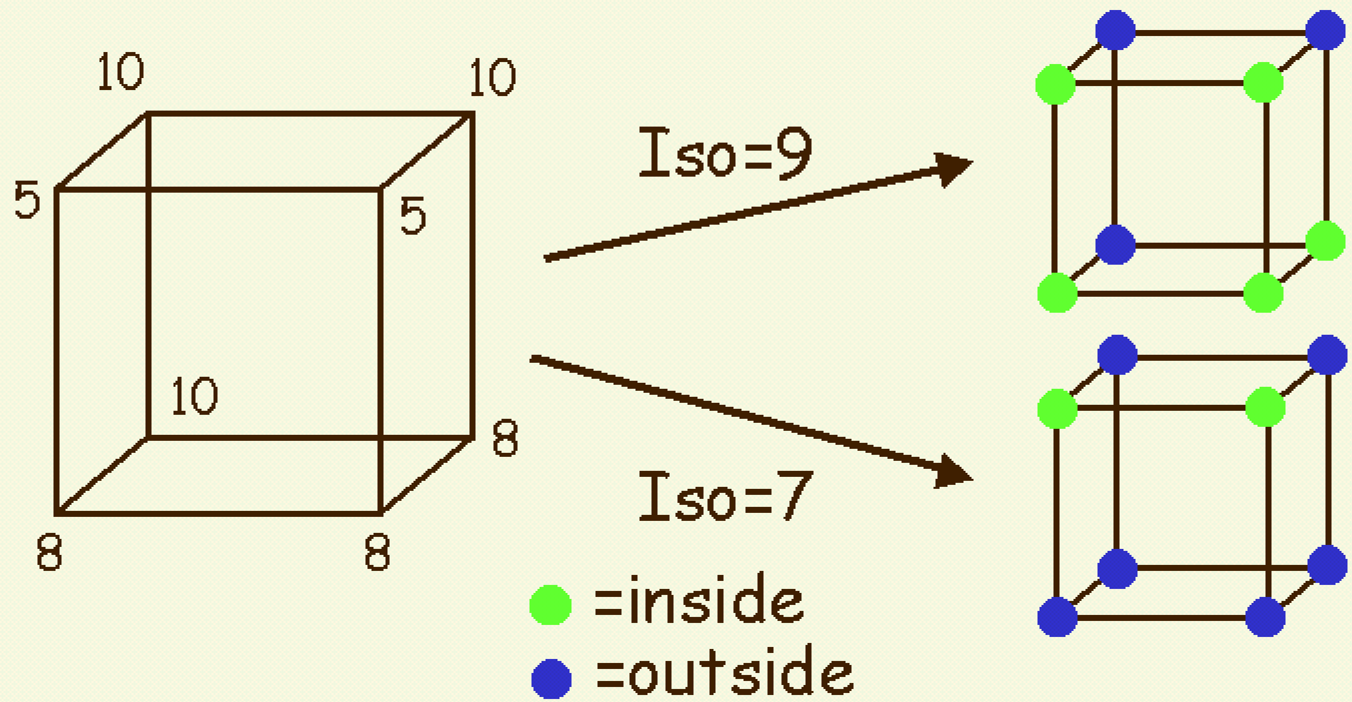
MC 1: Create a Cube

- ✓ Consider a Cube defined by eight data values:



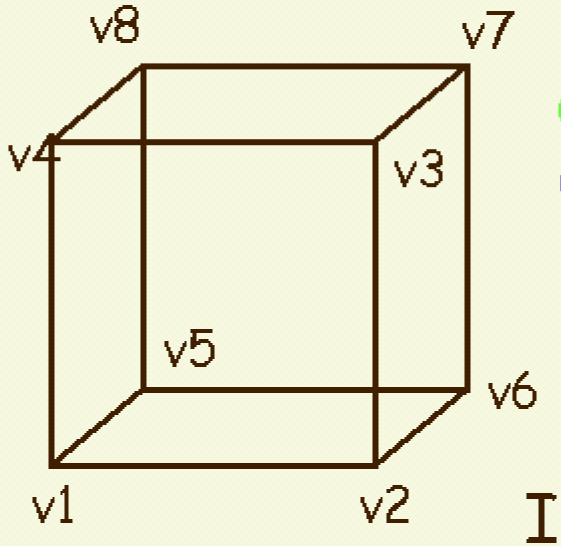
MC 2: Classify Each Voxel

- ✓ Classify each voxel according to whether it lies outside the surface (value > iso-surface value) inside the surface (value <= iso-surface value)

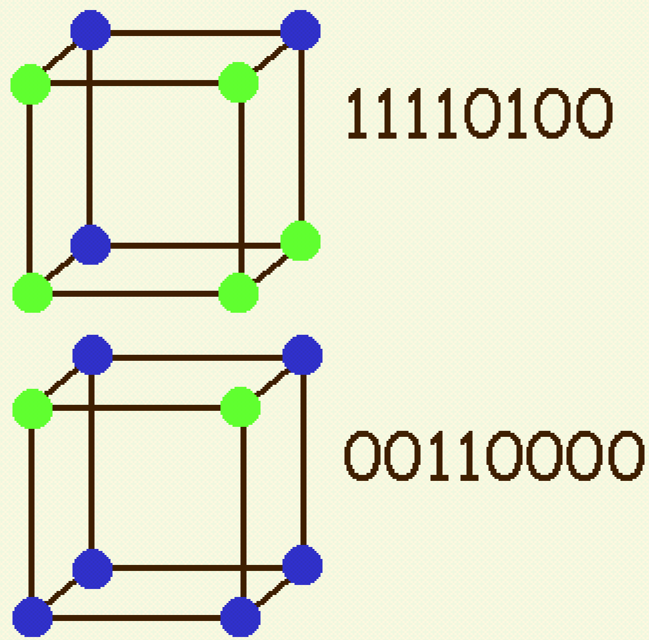


MC 3: Build An Index

- ✓ Use the binary labeling of each voxel to create an index



- inside =1 (green)
- outside=0 (blue)

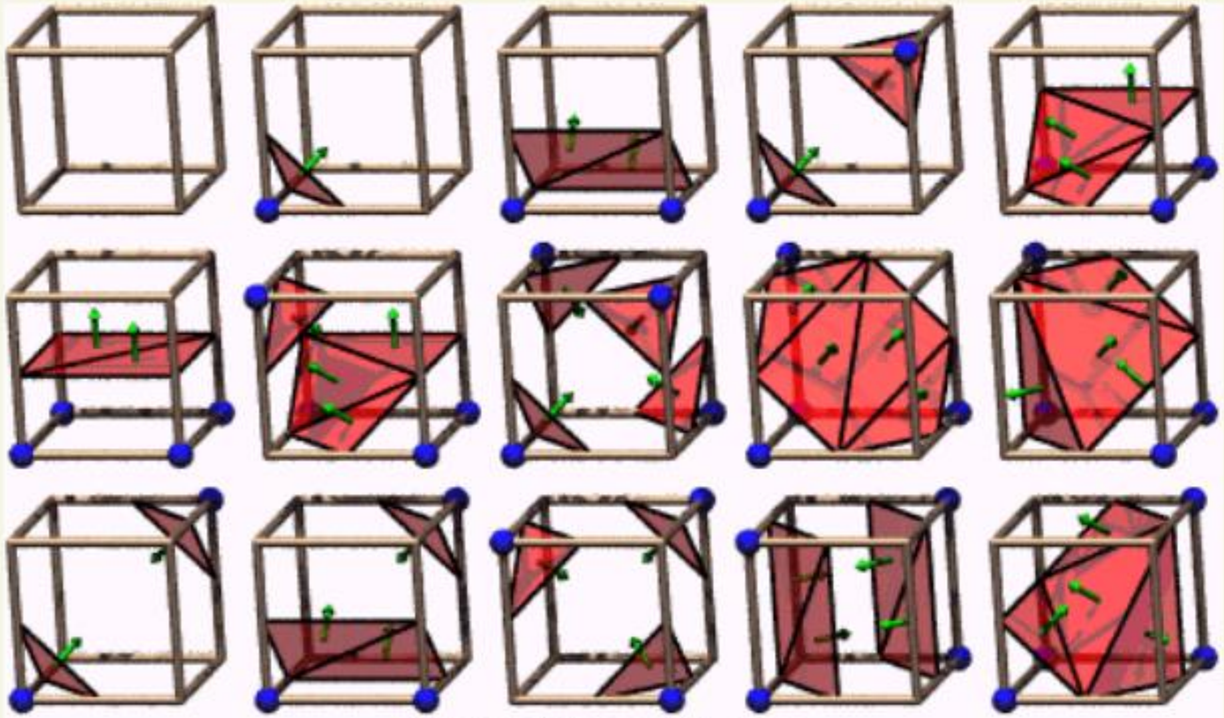


Index:

v1	v2	v3	v4	v5	v6	v7	v8

MC 4: Lookup Edge List

- For a given index, access an array storing a list of edges

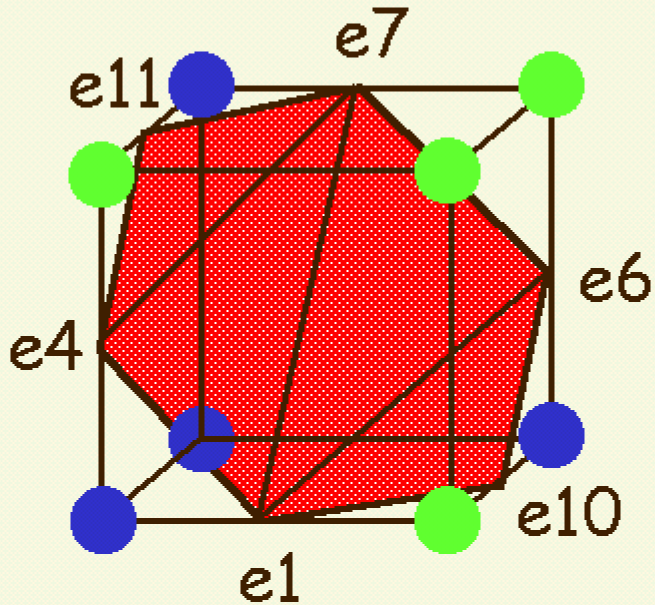


The 15 Cube Combinations

- all 256 cases can be derived from 15 base cases

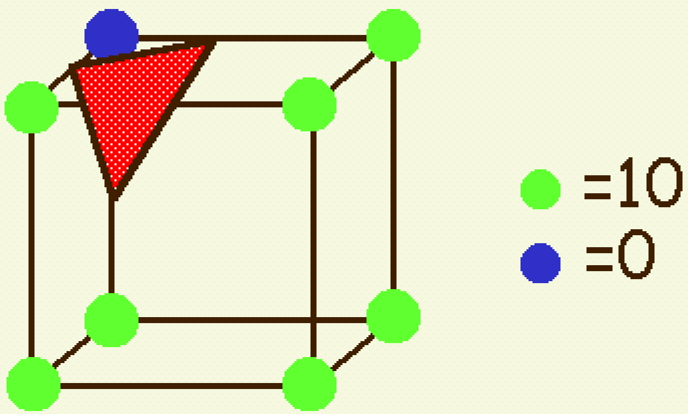
MC 5: Example

- ✓ Index = 10110001
- ✓ triangle 1 = e4,e7,e11
- ✓ triangle 2 = e1, e7, e4
- ✓ triangle 3 = e1, e6, e7
- ✓ triangle 4 = e1, e10, e6

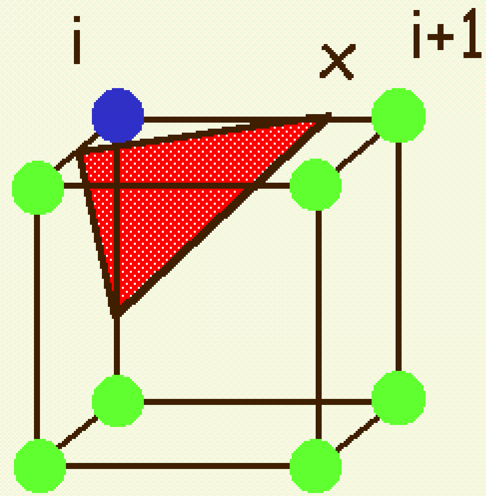


MC 6: Interp. Triangle Vertex

- ✓ For each triangle edge, find the vertex location along the edge using linear interpolation of the voxel values



T=5



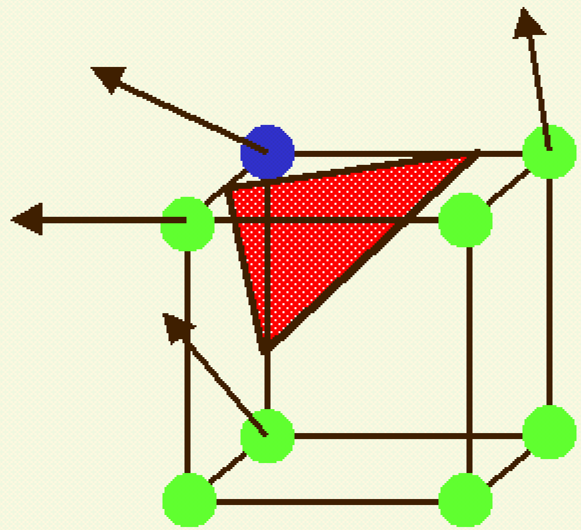
T=8

$$x = i + \left(\frac{T - v[i]}{v[i+1] - v[i]} \right)$$

MC 7: Compute Normals

✓ Calculate the normal at each cube vertex

$$G_x = V_{x-1,y,z} - V_{x+1,y,z}$$
$$G_y = V_{x,y-1,z} - V_{x,y+1,z}$$
$$G_z = V_{x,y,z-1} - V_{x,y,z+1}$$

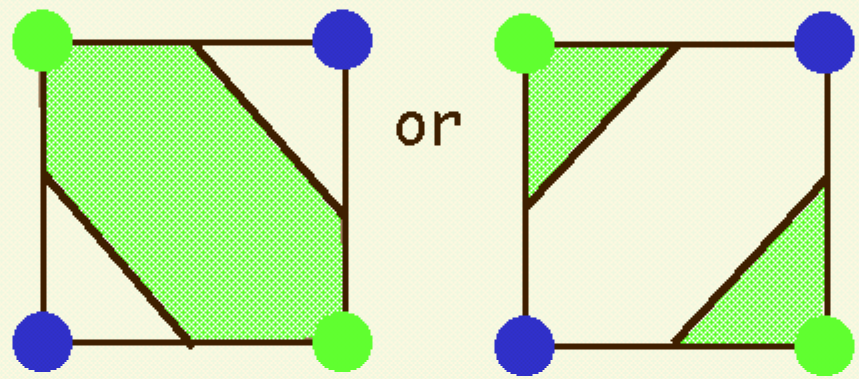


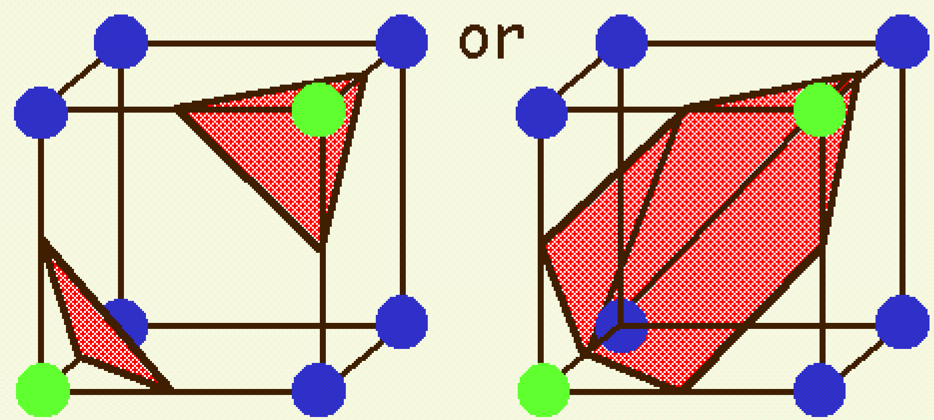
$$\vec{N} = \frac{\vec{G}}{|\vec{G}|}$$

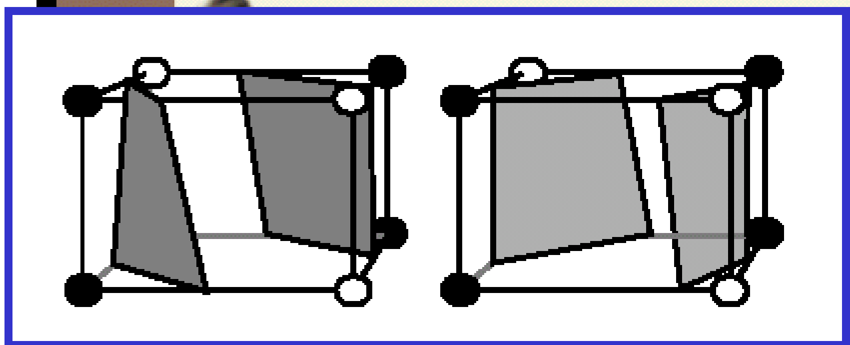
✓ Use linear interpolation to compute the polygon vertex normal

MC 8: Ambiguous Cases

- ✓ Ambiguous cases: 3, 6, 7, 10, 12, 13
- ✓ Adjacent vertices: different states
- ✓ Diagonal vertices: same state
- ✓ Resolution: decide for one case

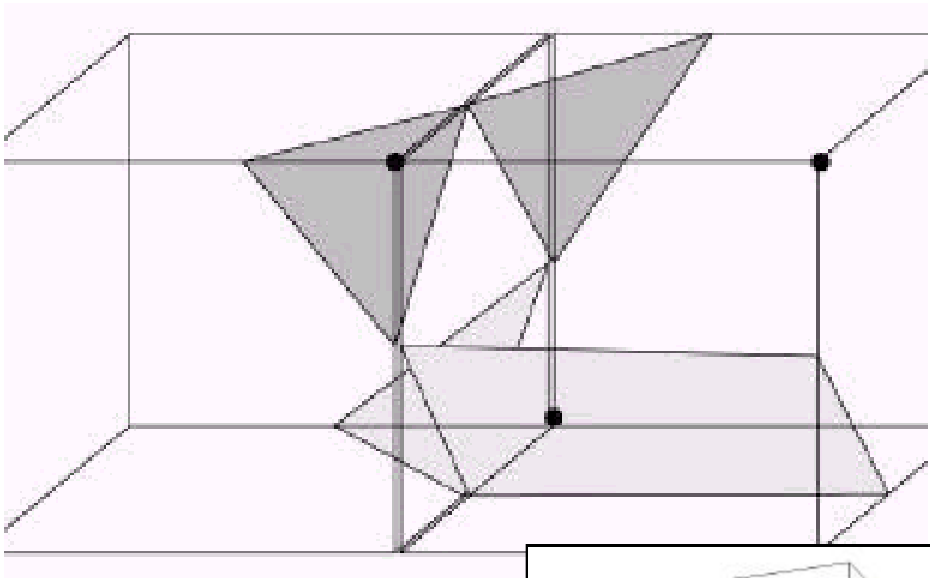


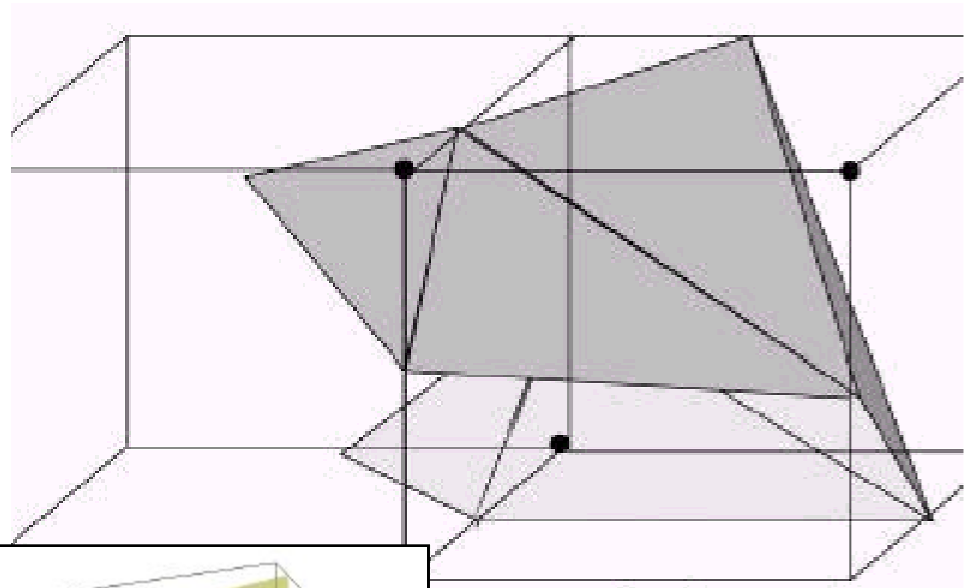




Danger: Holes!

- Wrong vs. correct classification!





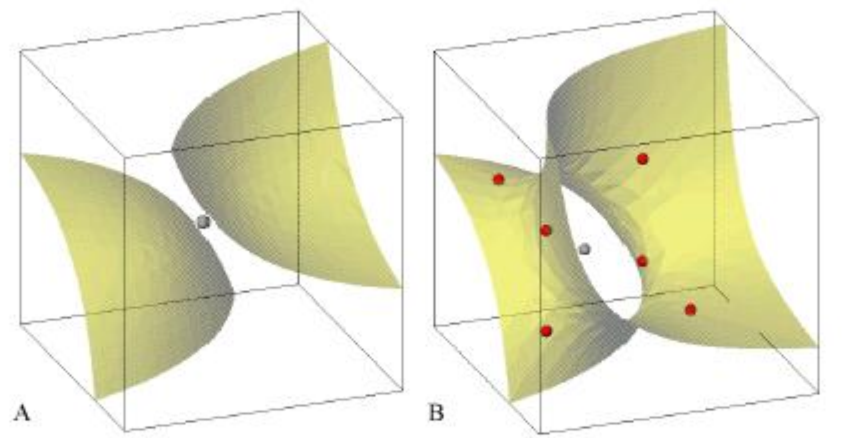
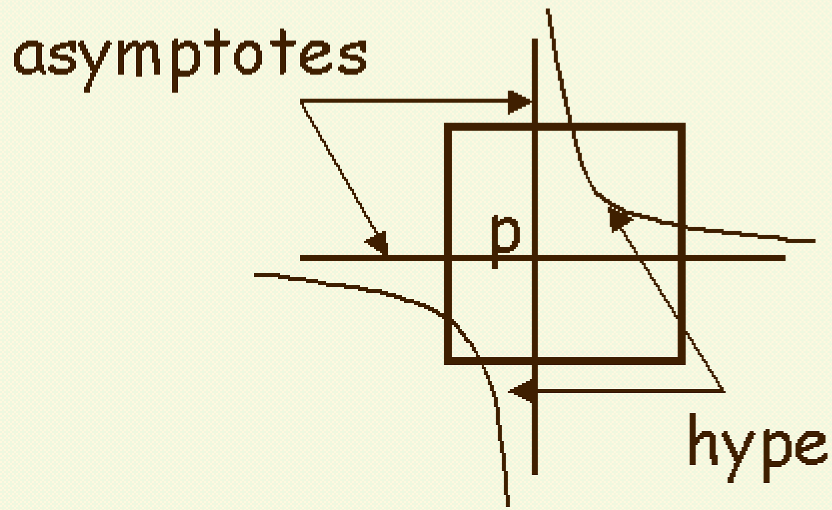


Figure 4: *Two internal configurations for the Marching Cubes configuration 5*

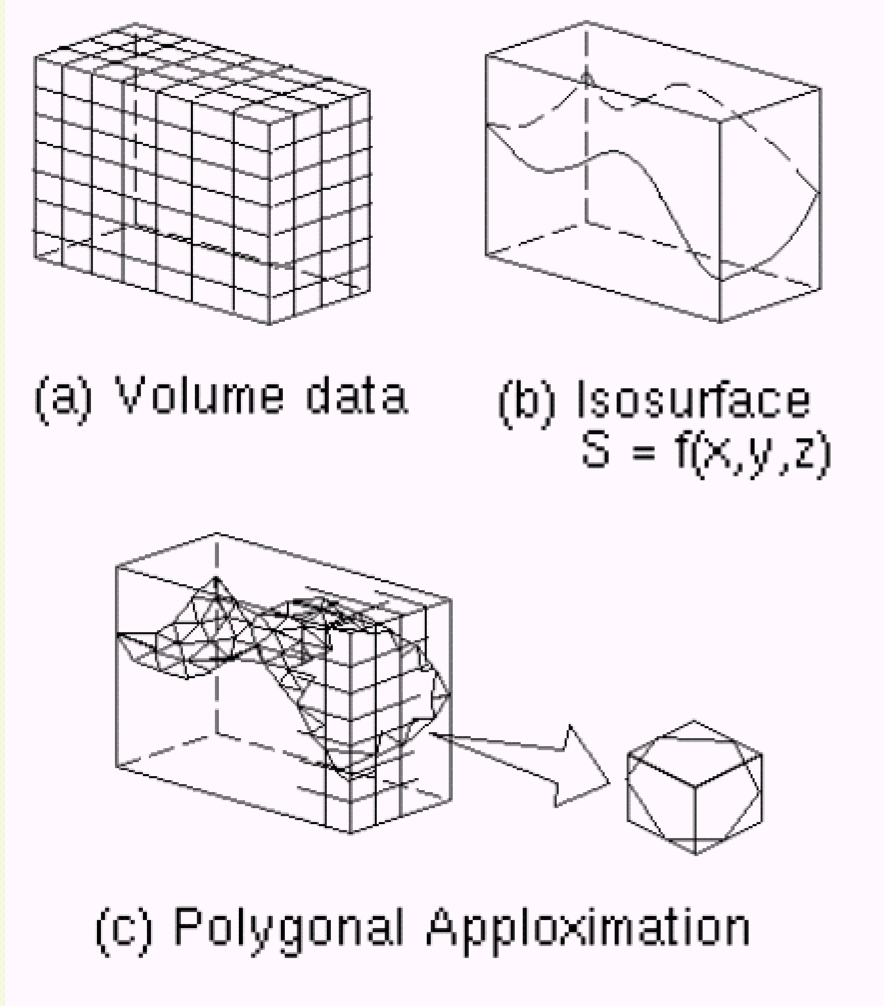
MC 9: Asymptotic Decider

- ✓ Assume bilinear interpolation within a face
- ✓ hence iso-surface is a hyperbola
- ✓ compute the point p where the asymptotes meet
- ✓ sign of S(p) decides the connectedness



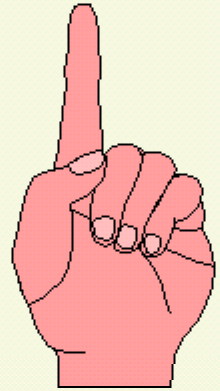
Marching Cubes - Summary 1

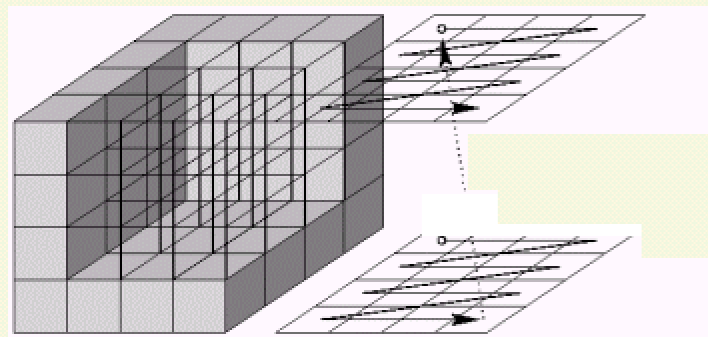
- ✓ 256 Cases
- ✓ reduce to 15 cases by symmetry
- ✓ Complementary cases - (swap in- and outside)
- ✓ Ambiguity resides in cases 3, 6, 7, 10, 12, 13
- ✓ Causes holes if arbitrary choices are made.



Marching Cubes - Summary 2

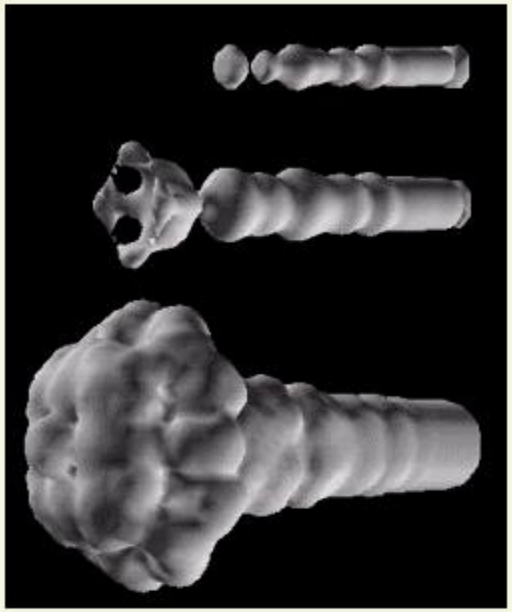
- ✓ Up to 4 triangles per cube
- ✓ Dataset of 512^3 voxels can result in several million triangles (many Mbytes!!!)
- ✓ Iso-surface does not represent an object!!!
- ✓ No depth information
- ✓ Semi-transparent representation --> sorting
- ✓ Optimization:
  - Reuse intermediate results
  - Prevent vertex replication
  - Mesh simplification



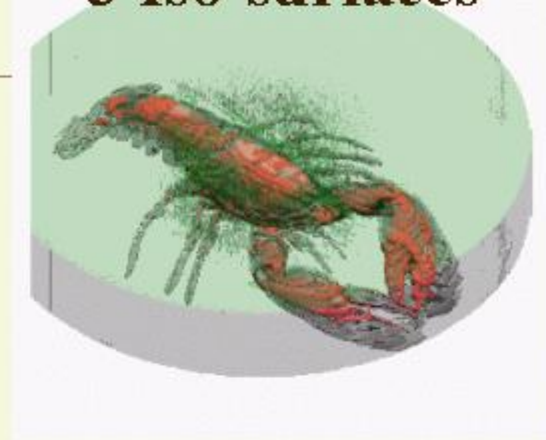


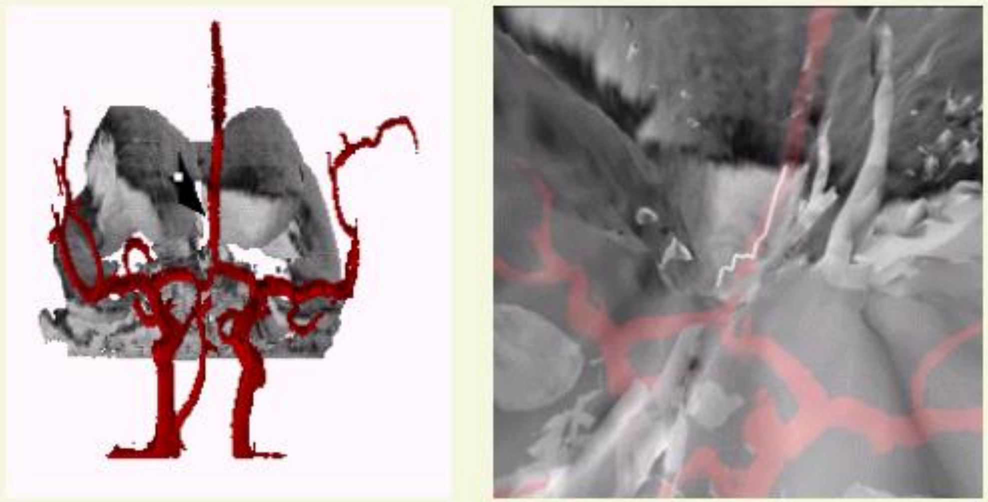
MC Examples

1 Iso-surface



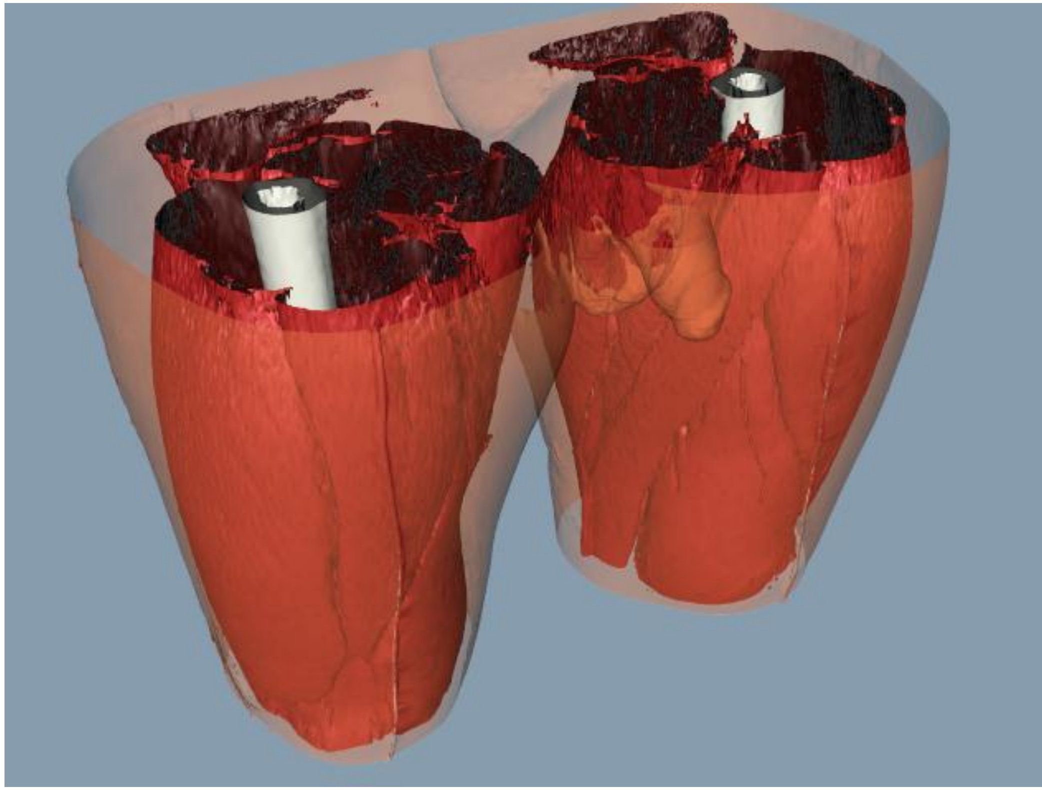
3 Iso-surfaces

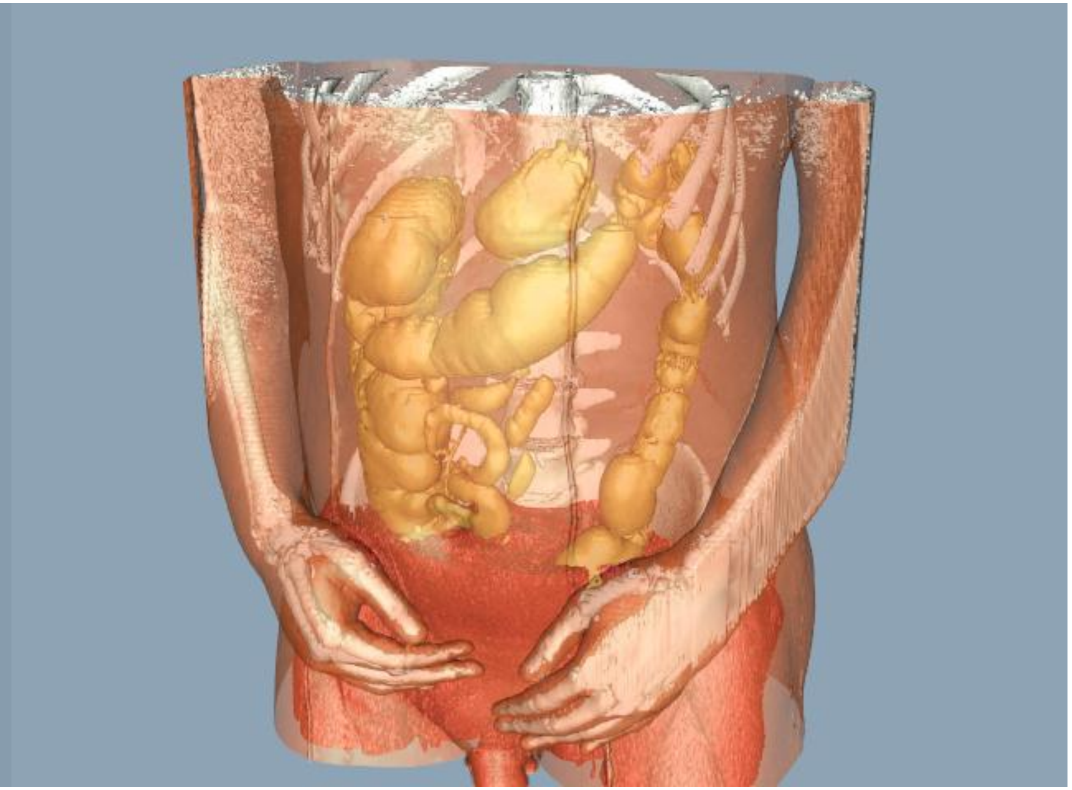


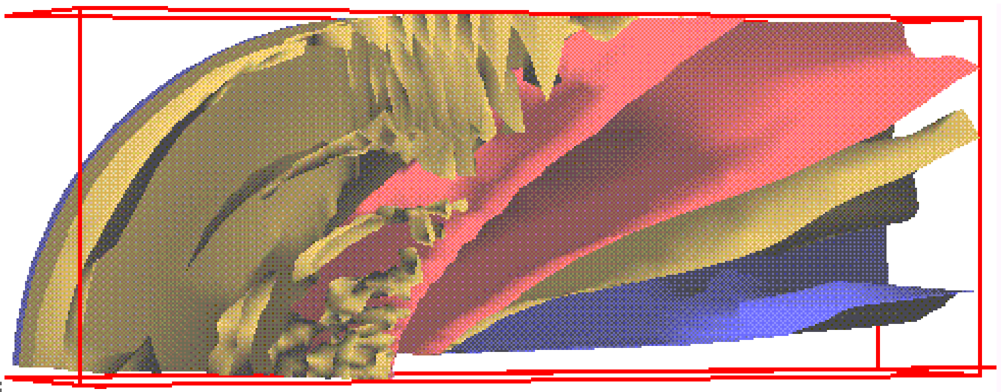


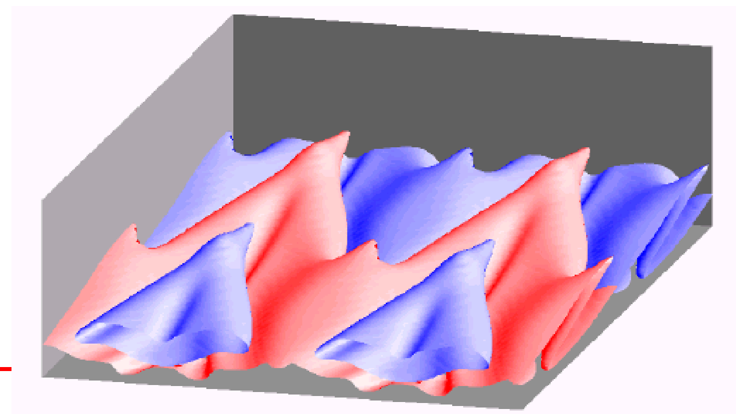
2 Iso-surfaces

Further Examples

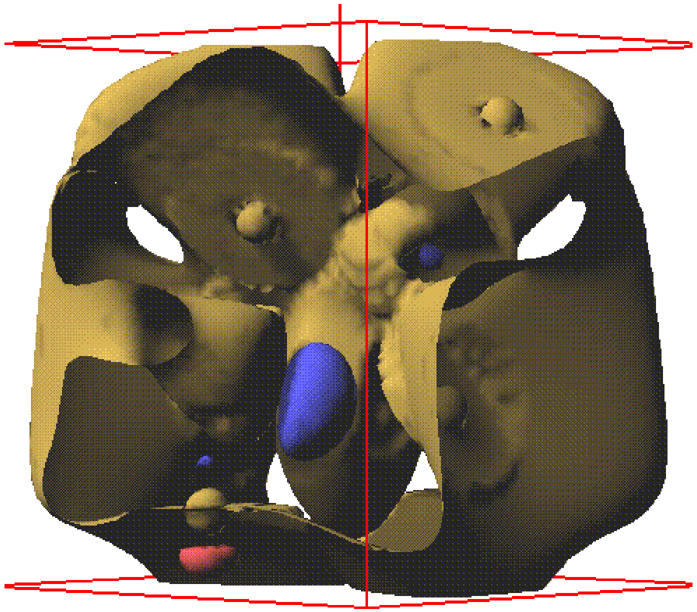


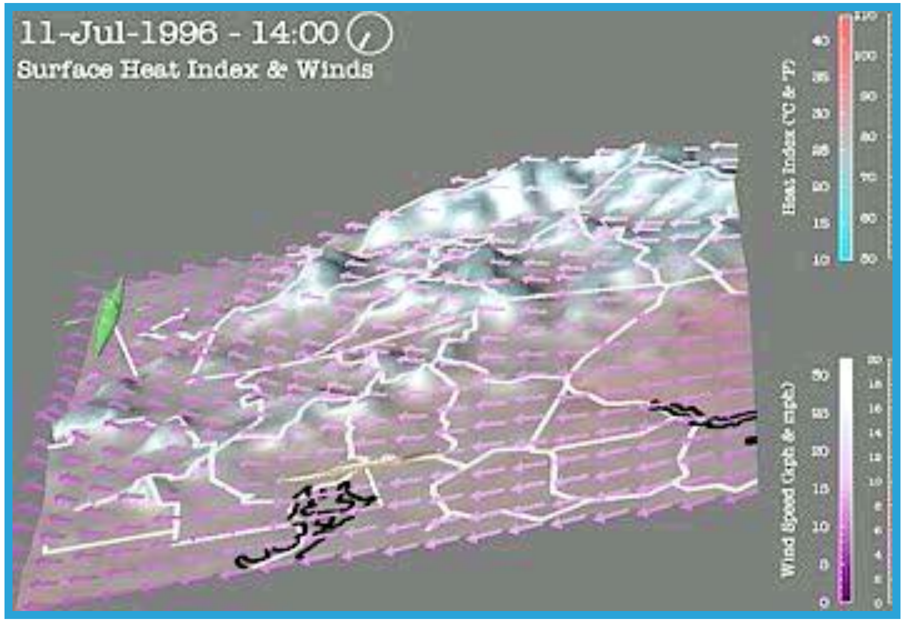


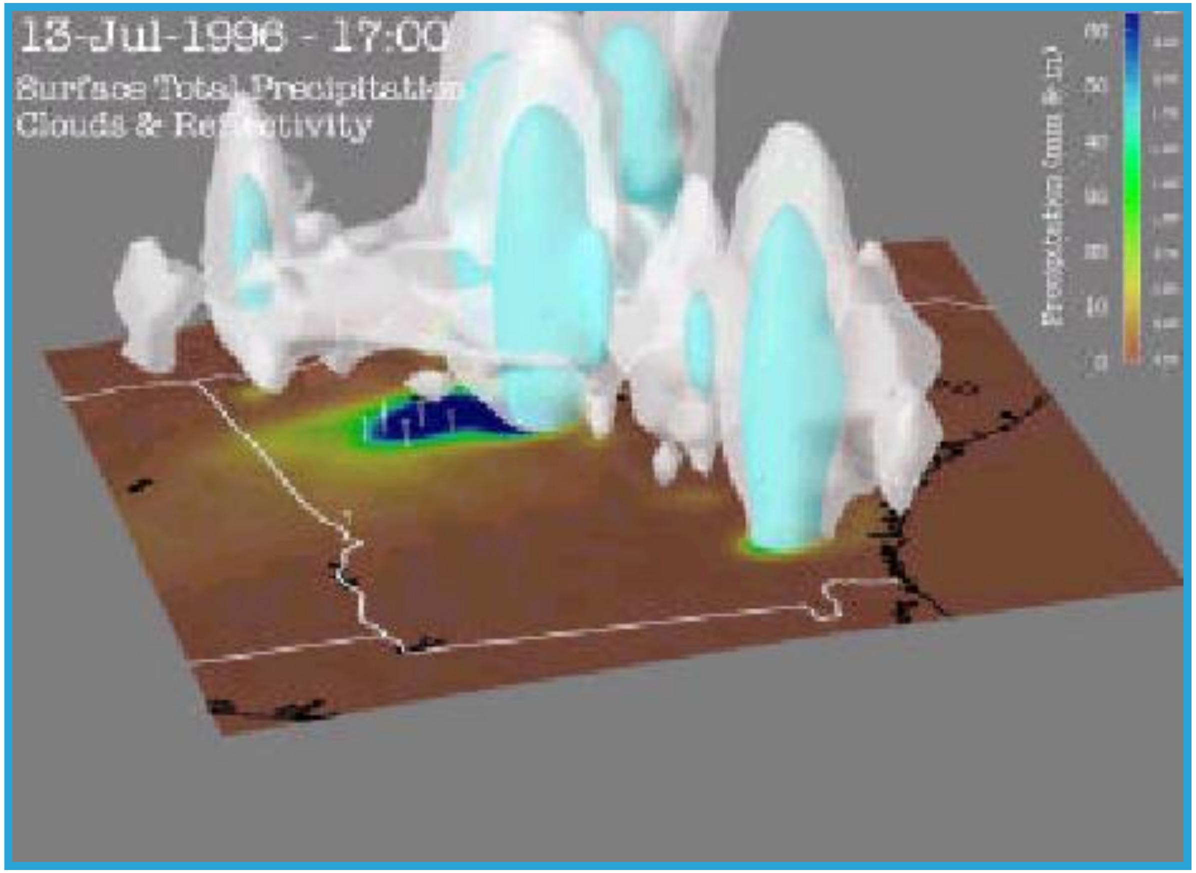




Even Further Examples





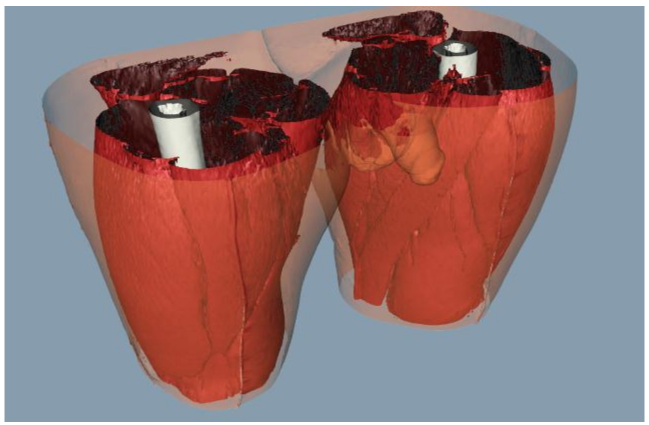


Conclusion
Volume Visualization

General Remarks

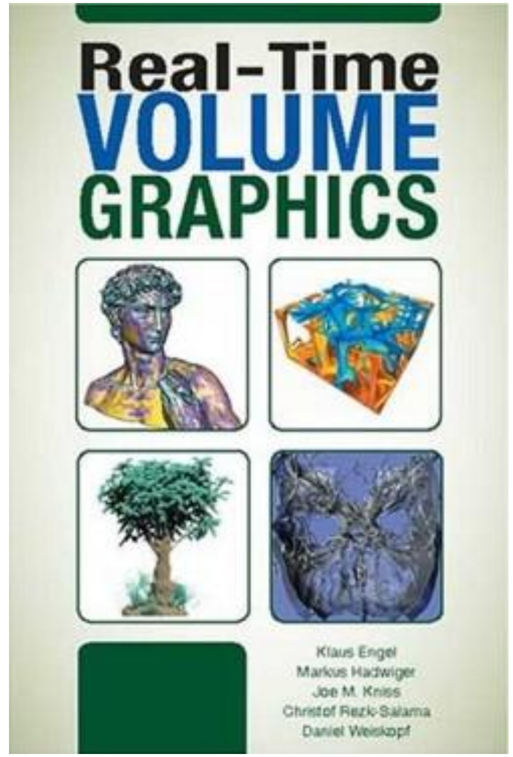
Surface vs. Volume Rendering

- Surface Rendering:
  - Indirect representation / display
  - Conveys surface impression
  - Hardware supported rendering (fast?!)
  - Iso-value-definition
- Volume Rendering:
  - Direct representation / display
  - Conveys volume impression
  - Often realized in software (slow?!)
  - Transfer functions



Literature, References

- Marc Levoy: “**Display of Surfaces from Volume Data**” in *IEEE Computer Graphics & Applications*, Vol. 8, No. 3, June 1988
- Nelson Max: “**Optical Models for Direct Volume Rendering**” in *IEEE Transactions on Visualization and Computer Graphics*, Vol. 1, No. 2, June 1995
- W. Lorensen & H. Cline: “**Marching Cubes: A High Resolution 3D Surface Construction Algorithm**” in *Proceedings of ACM SIGRRAPH ’87 = Computer Graphics*, Vol. 21, No. 24, July 1987
- K. Engel, M. Hadwiger et al. “**Real-Time Volume Graphics**” http://www.real-time-volume-graphics.org/



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