Volume Visualization

Part 2 (out of 3)

Review: Slices

Scalar-clipping, combination with 3D

Slices vs. Iso-Surfaces vs. Volume Rendering

- Comparison ozon-data over Antarctica:
  - Slices: selective (z), 2D, color coding
  - Iso-surface: selective (f₀), covers 3D
  - Vol. rendering: transfer function dependent, 
    "(too) sparse -- (too) dense"
Optical Models for Volume Rendering

Display of Semi-Transparent Media

Modelling of Natural Phenomena

- Various models (Examples):
  - Emission only (light particles)
  - Absorption only (dark fog)
  - Emission & absorption (clouds)
  - Single scattering, w/o shadows
  - Multiple scattering

- Two approaches:
  - Analytical model (via differentials)
  - Numerical approximation (via differences)

Emission, Differential Model

- Continuous emission model:
  - Question: how much light (I like intensity) is added along an infinitely short ray segment in the volume
  - Differential \( dI/dt = g(t) \ldots \)
    - volume emits light (corresponding to thickness)
  - Glow factor \( g(t) \)
  - Integration results in: \( I(s) = I_0 + \int_{t=0}^{s} g(t) dt \)
  - Overall emission contrib.: \( G(0,s) = \int_{t=0}^{s} g(t) dt \)
  - Unrealistic, because no absorption
Emission, Numerical Approximation

- **Discrete emission model:**
  - **Question:** how much light (C like color) is added within a small, but finite volume extent
  - $C_i$ ... contribution of vol. extent $i$ (thickness 1)
  - $\Rightarrow$ adding emission of extent $i$ results in $Out_i = In_i + C_i 
  \Rightarrow Out_i = Out_{i-1} + C_i$
  - **Accumulation:**
    - $Out_i = In_i + C_i + \ldots + C_{i-1} + C_i$
    - $Out_i = In_i + \sum_{j=k}^{i} C_k$
  - **Example:**
    - pixel value = background + $\sum_{k\in N} C_{ray(k)}$

Emission Only

- **Differential model:**
  - $I(s) = I_0 + \int_{[0,s]} g(t) dt$
- **Discrete approximation:**
  - $Out_i = In_0 + \sum_{k\in N} C_k$
- **Example:**

Absorption, Differential Model

- **Continuous absorption model:**
  - **Question:** how much light (in % of $I_0$) remains after traversal of ray segment through the volume
  - Differential $dI/dt = -\tau(t) I(t)$ ...
  - light ($I(t)$) is partially absorbed ($\tau$)
  - **Extinction coefficient** $\tau(t)$, e.g., 30%
  - Integration results in: $I(s) = I_0 \cdot \exp\left(-\int_{[0,s]} \tau(t) dt\right)$
  - **Total transparency:** $T(0,s) = \exp\left(-\int_{[0,s]} \tau(t) dt\right)$
  - **Total absorption:** $\alpha(0,s) = 1 - T(0,s)$
Absorption, Numerical Approximation

- Discrete approximation model:
  - Question: how much light (in % of $I_0$) remains after traversal of small, but finite volume extent
  - $\alpha_i$ ... opacity of volume extent $i$ (per unit)
  - $\Rightarrow$ result after traversal of extent $i$
    - $Out_i = In_i \cdot (1 - \alpha_i) \Leftrightarrow Out_i = Out_{i-1} \cdot (1 - \alpha_i)$
  - Akkumulation: $Out_i = In_i \cdot \Pi_{j=i}^{k=1} (1 - \alpha_k)$
  - Unit sampling: unit distance between $\alpha_i$ samples!!

Absorption Only

- Differential model:
  - $I(s) = I_0 \cdot \exp(-\int_{t \in [0,s]} \tau(t) dt)$
- Discrete approximation:
  - $Out_s = In_0 \cdot \Pi_{s \leq k} (1 - \alpha_k)$
- Example:

Emission and Absorption

- Continuous model (no scattering):
  - At each position is given:
    - Emission $g(t)$
    - Extinction coefficient $\tau(t)$
    - Differential $dI/dt = g(t) - \tau(t)I(t)$
  - Emission $g(t)$ attenuated by $T(t,s)$
    - Only Emission: $I_0 + \int_{t \in [0,s]} g(t) dt$
    - With Absorption: $I_0 \cdot T(0,s) + \int_{t \in [0,s]} g(t) \cdot T(t,s) dt$
  - Emission und Absorption:
    - $I_0 \cdot \exp(-\int_{t \in [0,s]} \tau(t) dt) + \int_{t \in [0,s]} g(t) \cdot \exp(-\int_{u \in [0,s]} \tau(u) du) dt$
Numerical Approximation

Discrete model (compositing):
- For each volume extent \( i \):
  - Contribution \( C_i \)
  - Opacity \( \alpha_i \), transparency \( 1 - \alpha_i \)
- \( \text{Out}_i = \text{In}_i \cdot (1 - \alpha_i) + C_i \cdot \alpha_i \) (Std.-compositing)
- Convex combination from background and own contribution
- \( \text{Out}_s = \text{In}_0 \cdot \prod_{k \not\in \mathbb{N}} (1 - \alpha_k) + \sum_{k \in \mathbb{N}} C_k \cdot \alpha_k \cdot \prod_{l > k} (1 - \alpha_l) \)
- Opacity-weighted colors: \( C_i \cdot \alpha_i \) instead of \( C_i \)

Emission and Absorption

Differential model:
- \( I(s) = I_0 \cdot T(0,s) + \int_{t \in [0,s]} g(t) \cdot T(t,s) \, dt \)
- \( I(s) = I_0 \cdot \exp\left(-\int_{u \in [0,s]} \tau(u) \, du\right) + \int_{t \in [0,s]} g(t) \cdot \exp\left(-\int_{u \in [0,t]} \tau(u) \, du\right) \, dt \)

Discrete Approximation:
- \( \text{Out}_i = \text{In}_i \cdot (1 - \alpha_i) + C_i \cdot \alpha_i \) (Compositing)
- \( \text{Out}_s = \text{In}_0 \cdot \prod_{k \not\in \mathbb{N}} (1 - \alpha_k) + \sum_{k \in \mathbb{N}} C_k \cdot \alpha_k \cdot \prod_{l > k} (1 - \alpha_l) \)

Emission or/and Absorption

Emission only
Absorption only

Emission and Absorption
Scattering

- **Scattering**: particles deviate light at a position
  - BRDF (bidirectional reflectance distribution function)
  - Single scattering
    - Too little light in the interior
  - Single scattering with shadows
  - Multiple Scattering
    - Radiosity techniques
    - Very realistic, very costly

Literature

- Paper (more details):

Ray Casting / Compositing

- Classical Image-Order Methods
Ray Tracing vs. Ray Casting

- **Ray Tracing**: method from image generation
- **In volume rendering**: only viewing rays
  - therefore Ray Casting
- **Classical image-order method**
- **Ray Tracing**: ray – object intersection
- **Ray Casting**: no objects, density values in 3D
- **In theory**: take all density values into account!
- **In practice**: traverse volume step by step
- **Interpolation** necessary for each step!

Ray Traversal through Volume Data

- **Context**:
  - **Volume data**: 1D value defined in 3D – \( f(x) \in \mathbb{R}^1, x \in \mathbb{R}^3 \)
  - **Ray** defined as half-line: \( r(t) \in \mathbb{R}^3, t \in \mathbb{R}^+ \)
  - **Values along Ray**: \( f(r(t)) \in \mathbb{R}^1, t \in \mathbb{R}^+ \) (intensity profile)

Standard Ray Casting

- **Levoy ’88**:
  1. \( C(i), \alpha(i) \) (from TF)
  2. **Ray casting**, interpolation
  3. **Compositing**
1. Shading, Classification

1. Step:
- Shading, \( f(i) \rightarrow C(i) \):
  - Apply transfer function
  - Diffuse illumination (Phong), gradient \( \approx \) normal
- Classification, \( f(i) \rightarrow \alpha(i) \):
  - Levoy '88, gradient enhanced
  - Emphasizes transitions

2. Ray Traversal – Three Approaches

- Voxel-based vs. cell-based traversal
- Tri-linear (interpolation within a cell) vs. bi-linear (interpolation within a cell face)
- Tri-linear:
  - First 4* in z-direction (interpolated square),
  - Then 2* in y-direction (interpolated line),
  - Then 1* in x-direction (interpolated value)
- Unit sampling vs. variable sample distances – compositing different!!
Compositing: F2B vs. B2F

- **Back-to-Front (B2F):**
  - \( \text{Out}_i = \text{In}_i \cdot (1 - \alpha_i) + C_i \cdot \alpha_i, \text{In}_{i+1} = \text{Out}_i \ldots \)
  - Depending on local transparency \((1 - \alpha_i) \Rightarrow \) convex combination of old \( \text{In}_i \) & new \( C_i \)
  - Example:
    - Voxel i: \( C_i = \text{red}, \alpha_i = 30\%; \) so far: \( \text{In}_i = \text{white} \)
    - Result of compositing: 70% white + 30% red

- **Front-to-Back (F2B):**
  - \( \text{Col} = \text{Col} + (1 - \alpha_{akk}) \cdot C_i \cdot \alpha_i \ldots \) accumulated color
  - \( \alpha_{akk} = \alpha_{akk} + (1 - \alpha_{akk}) \cdot \alpha_i \ldots \) accumulated opacity

Front-to-Back Compositing

Ray Casting – Examples

- **CT scan of human hand** (244x124x257, 16 bit)
**Acceleration - Progressive Refinement**

- First render every $2^n \times 2^n$-th pixel, then render the $2^{n-1} \times 2^{n-1}$-th pixel inbetween, aso. (until interruption or completion)

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**Literature**

- Paper (more details):

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**Acknowledgement**

- For material for this lecture unit:
  - Nelson Max (LLNL), Marc Levoy (Stanford)
  - Hans-Georg Pagendarm (DLR, Göttingen)
  - Lloyd Treinish (IBM)
  - Roberto Scopigno, Claudio Montani (CNR, Pisa)
  - Roger Crawfis (Ohio State Univ.)
  - Michael Meißner (GRIS, Tübingen)
  - Torsten Möller
  - etc.