

# Volume Visualization

Part 2 (out of 3)



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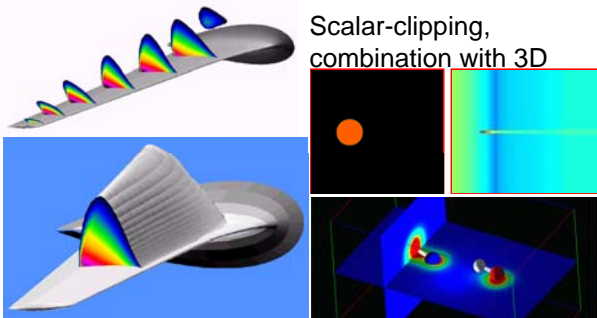
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## Review: Slices



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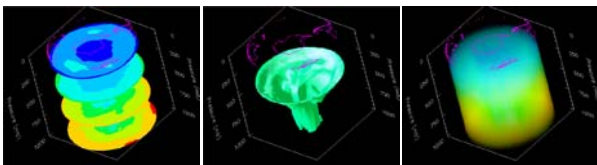
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## Slices vs. Iso-Surfaces. vs. Volume Rendering



- Comparison ozon-data over Antarctica:
  - ◆ Slices: selective ( $z$ ), 2D, color coding
  - ◆ Iso-surface: selective ( $f_0$ ), covers 3D
  - ◆ Vol. rendering: transfer function dependent, “(too) sparse – (too) dense”



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# Optical Models for Volume Rendering

Display of Semi-Transparent Media



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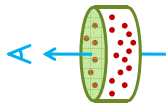
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## Modelling of Natural Phenomena



### Various models (Examples):

- ◆ Emission only (light particles)
- ◆ Absorption only (dark fog)
- ◆ Emission & absorption (clouds)
- ◆ Single scattering, w/o shadows
- ◆ Multiple scattering



### Two approaches:

- ◆ Analytical model (via differentials)
- ◆ Numerical approximation (via differences)



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## Emission, Differential Model



### Continuous emission model:

- ◆ Question: how much light ( $\mathcal{I}$  like intensity) is added along an infinitely short ray segment in the volume
- ◆ Differential  $d\mathcal{I}/dt = g(t) \dots$   
volume emits light (corresponding to thickness)
- ◆ Glow factor  $g(t)$
- ◆ Integration results in:  $\mathcal{I}(s) = \mathcal{I}_0 + \int_{t \in [0,s]} g(t) dt$
- ◆ Overall emission contrib.:  $G(0,s) = \int_{t \in [0,s]} g(t) dt$
- ◆ Unrealistic, because no absorption



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## Emission, Numerical Approximation



### Discrete emission model:

- ◆ **Question:** how much light (C like color) is added within a small, but finite volume extent
- ◆  $C_i$  ... contribution of vol. extent i (thickness 1)  
⇒ adding emission of extent i results in  
 $Out_i = In_i + C_i \Leftrightarrow Out_i = Out_{i-1} + C_i$

### Accumulation:

$$Out_i = In_j + C_j + \dots + C_{i-1} + C_i$$
$$Out_i = In_j + \sum_{j \leq k \leq i} C_k$$

### Example:

$$\text{pixel value} = \text{background} + \sum_{k \in N} C(\text{ray}(k))$$



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## Emission Only



### Differential model:

$$I(s) = I_0 + \int_{t \in [0,s]} g(t) dt$$

### Discrete approximation:

$$Out_s = In_0 + \sum_{s \geq k \in N} C_k$$

### Example:



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## Absorption, Differential Model



### Continuous absorption model:

- ◆ **Question:** how much light (in % of  $I_0$ ) remains after traversal of ray segment through the volume

$$dI/dt = -\tau(t)I(t) \dots$$

light ( $I$ ) is partially absorbed ( $\tau$ )

- ◆ **Extinction coefficient**  $\tau(t)$ , e.g., 30%

$$\text{Integration results in: } I(s) = I_0 \cdot \exp\left(-\int_{t \in [0,s]} \tau(t) dt\right)$$

$$\text{Total transparency: } T(0,s) = \exp\left(-\int_{t \in [0,s]} \tau(t) dt\right)$$

$$\text{Total absorption: } \alpha(0,s) = 1 - T(0,s)$$



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## Absorption, Numerical Approximation



- Discrete approximation model:
  - ◆ **Question:** how much light (in % of  $I_0$ ) remains after traversal of small, but finite volume extent
  - ◆  $\alpha_i$  ... opacity of volume extent  $i$  (per unit)  
 $\Rightarrow$  result after traversal of extent  $i$   
 $Out_i = In_i \cdot (1 - \alpha_i) \Leftrightarrow Out_i = Out_{i-1} \cdot (1 - \alpha_i)$
  - ◆ **Akkumulation:**  $Out_i = In_j \cdot (1 - \alpha_i) \cdot \dots \cdot (1 - \alpha_i)$   
 $Out_i = In_j \cdot \prod_{j \leq k \leq i} (1 - \alpha_k)$
  - ◆ **Unit sampling:** unit distance between  $\alpha_i$  samples!!




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## Absorption Only



- Differential model:
  - ◆  $I(s) = I_0 \cdot \exp(-\int_{t \in [0, s]} \tau(t) dt)$
- Discrete approximation:
  - ◆  $Out_s = In_0 \cdot \prod_{s \geq k \in N} (1 - \alpha_k)$
- Example:




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## Emission and Absorption



- Continuous model (no scattering):
  - ◆ **At each position** is given:
    - Emission  $g(t)$
    - Extinction coefficient  $\tau(t)$
  - ◆ **Differential**  $dI/dt = g(t) - \tau(t)I(t)$
  - ◆ Emission  $g(t)$  **attenuated** by  $T(t, s)$
  - ◆ **Only Emission:**  $I_0 + \int_{t \in [0, s]} g(t) dt$
  - ◆ **With Absorption:**  $I_0 \cdot T(0, s) + \int_{t \in [0, s]} g(t) \cdot T(t, s) dt$
  - ◆ **Emission und Absorption:**  
 $I_0 \cdot \exp(-\int_{u \in [0, s]} \tau(u) du) + \int_{t \in [0, s]} g(t) \cdot \exp(-\int_{u \in [t, s]} \tau(u) du) dt$




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## Numerical Approximation



- Discrete model (compositing):
  - ◆ For each **volume extent**  $i$ :
    - Contribution  $C_i$
    - Opacity  $\alpha_i$ , transparency  $1-\alpha_i$
  - ◆  $Out_i = In_i \cdot (1-\alpha_i) + C_i \cdot \alpha_i$  (Std.-compositing)
  - ◆ **Convex combination** from background and own contribution
  - ◆  $Out_s = In_0 \cdot \prod_{s \geq k \in N} (1-\alpha_k) + \sum_{s \geq k \in N} C_k \cdot \alpha_k \cdot \prod_{s \geq l > k} (1-\alpha_l)$
  - ◆ **Opacity-weighted colors**:  $C_i \cdot \alpha_i$  instead of  $C_i$




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## Emission and Absorption



- Differential model:
  - ◆  $I(s) = I_0 \cdot T(0,s) + \int_{t \in [0,s]} g(t) \cdot T(t,s) dt$
  - ◆  $I(s) = I_0 \cdot \exp(-\int_{u \in [0,s]} \tau(u) du) + \int_{t \in [0,s]} g(t) \cdot \exp(-\int_{u \in [t,s]} \tau(u) du) dt$
- Discrete Approximation:
  - ◆  $Out_i = In_i \cdot (1-\alpha_i) + C_i \cdot \alpha_i$  (Compositing)
  - ◆  $Out_s = In_0 \cdot \prod_{s \geq k \in N} (1-\alpha_k) + \sum_{s \geq k \in N} C_k \cdot \alpha_k \cdot \prod_{s \geq l > k} (1-\alpha_l)$




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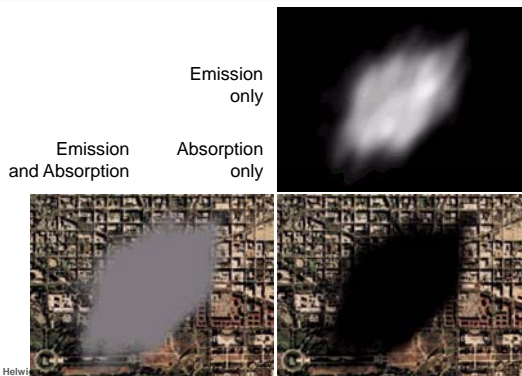
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## Emission or/and Absorption




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## Scattering



### ■ Scattering: particles deviate light at a position

- ◆ BRDF (bidirectional reflectance distribution function)
- ◆ Single scattering
  - Too little light in the interior
- ◆ Single scattering with shadows
- ◆ Multiple Scattering
  - Radiosity techniques
  - Very realistic, very costly



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## Literature



### ■ Paper (more details):

- ◆ Nelson Max: "Optical Models for Direct Volume Rendering" in *IEEE Transactions on Visualization and Computer Graphics*, Vol. 1, No. 2, June 1995



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## Ray Casting / Compositing

Classical  
Image-Order Methods



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## Ray Tracing vs. Ray Casting



- **Ray Tracing**: method from image generation
- In volume rendering: **only viewing rays**  
⇒ therefore Ray Casting
- Classical **image-order** method
- **Ray Tracing**: ray – object intersection  
**Ray Casting**: no objects, density values in 3D
- In **theory**: take all density values into account!  
In **practice**: traverse volume step by step
- **Interpolation** necessary for each step!




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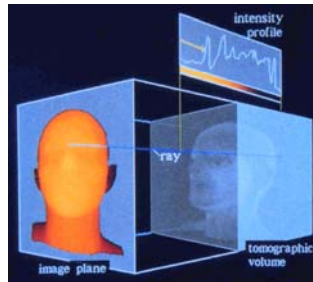
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## Ray Traversal through Volume Data



### Context:

- ◆ **Volume data**: 1D value defined in 3D –  
 $f(\mathbf{x}) \in \mathbb{R}^1, \mathbf{x} \in \mathbb{R}^3$
- ◆ **Ray** defined as half-line:  
 $\mathbf{r}(t) \in \mathbb{R}^3, t \in \mathbb{R}^1 > 0$
- ◆ **Values along Ray**:  
 $f(\mathbf{r}(t)) \in \mathbb{R}^1, t \in \mathbb{R}^1 > 0$   
(intensity profile)




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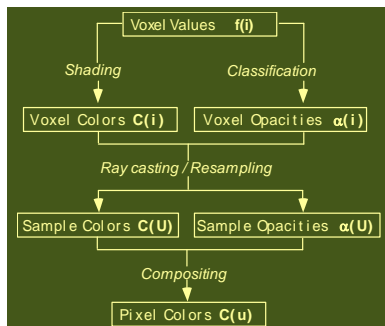
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## Standard Ray Casting



- Levoy '88:
- 1. **C(i), α(i)**  
(from TF)
- 2. **Ray casting, interpolation**
- 3. **Compositing**




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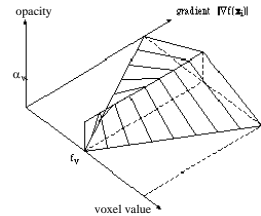
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## 1. Shading, Classification



### 1. Step:

- ◆ Shading,  $f(i) \rightarrow C(i)$ :
  - Apply transfer function
  - diffuse illumination (Phong), gradient  $\approx$  normal
- ◆ Classification,  $f(i) \rightarrow \alpha(i)$ :
  - Levoy '88, gradient enhanced
  - Emphasizes transitions




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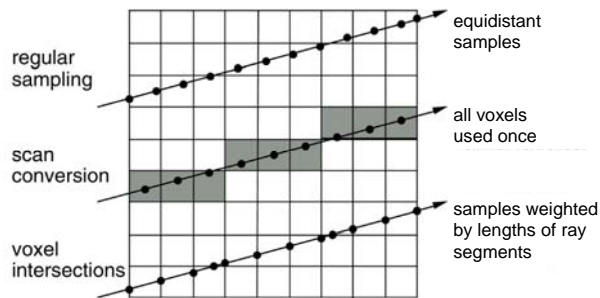
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## 2. Ray Traversal – Three Approaches




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## 2. Ray Traversal, Interpolation



- Voxel-based vs. cell-based traversal
- Tri-linear (interpolation within a cell) vs. bi-linear (interpolation within a cell face)
- Tri-linear:
  - ◆ first 4\* in z-direction (interpolated square),
  - ◆ then 2\* in y-direction (interpolated line),
  - ◆ then 1\* in x-direction (interpolated value)
- Unit sampling vs. variable sample distances – compositing different!!

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## Compositing: F2B vs. B2F



### Back-to-Front (B2F):

- ◆  $Out_i = In_i \cdot (1 - \alpha_i) + C_i \cdot \alpha_i$ ,  $In_{i+1} = Out_i \dots$
- ◆ Depending on local transparency  $(1 - \alpha_i) \Rightarrow$  convex combination of old  $In_i$  & new  $C_i$
- ◆ Example:
  - Voxel  $i$ :  $C_i = \text{red}$ ,  $\alpha_i = 30\%$ ; so far:  $In_i = \text{white}$
  - Result of compositing: 70% white + 30% red

### Front-to-Back (F2B):

- ◆  $Col = Col + (1 - \alpha_{akk}) \cdot C_i \cdot \alpha_i \dots$  accumulated color
- ◆  $\alpha_{akk} = \alpha_{akk} + (1 - \alpha_{akk}) \cdot \alpha_i \dots$  accumulated opacity



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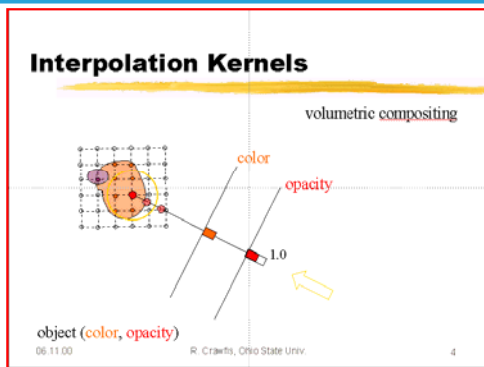
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## Front-to-Back Compositing



### Interpolation Kernels



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## Ray Casting – Examples



- CT scan of human hand (244x124x257, 16 bit)



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## Ray Casting – Examples



Eduard Gröller, Helwig Hauser

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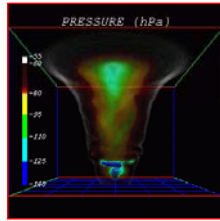
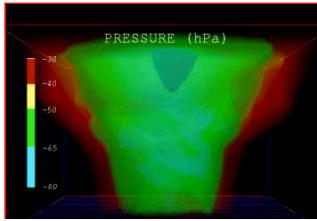
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## Ray Casting – Further Examples



### ■ Tornado Visualization:



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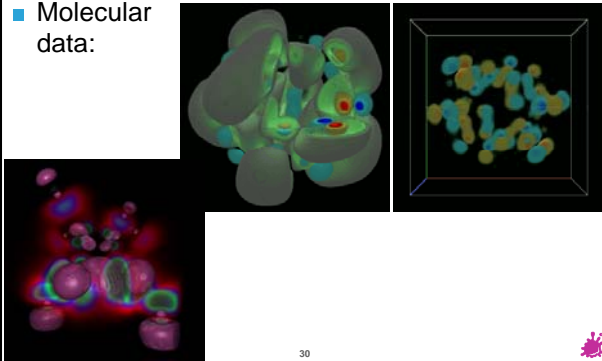
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## Ray Casting – Further Examples



### ■ Molecular data:



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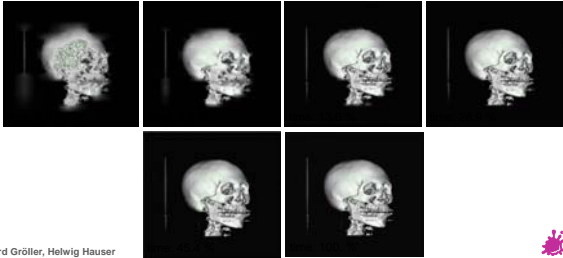
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## Acceleration - Progressive Refinement



- First render every  $2^n \times 2^n$ -th pixel, then render the  $2^{n-1} \times 2^{n-1}$ -th pixel inbetween, aso. (until interruption or completion)



Eduard Gröller, Helwig Hauser



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## Literature



- Paper (more details):
  - ◆ **Marc Levoy**: "Display of Surfaces from Volume Data" in *IEEE Computer Graphics & Applications*, Vol. 8, No. 3, June 1988

Eduard Gröller, Helwig Hauser

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  - ◆ Michael Meißner (GRIS, Tübingen)
  - ◆ Torsten Möller
  - ◆ etc.

Eduard Gröller, Helwig Hauser

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