Volume Visualization

Part 2 (out of 3)
Review: Slices

Scalar-clipping, combination with 3D
Slices vs. Iso-Surfaces vs. Volume Rendering

- Comparison ozon-data over Antarctica:
  - Slices: selective (z), 2D, color coding
  - Iso-surface: selective ($f_0$), covers 3D
  - Vol. rendering: transfer function dependent, “(too) sparse – (too) dense”
Optical Models for Volume Rendering

Display of Semi-Transparent Media
Modelling of Natural Phenomena

- Various models (Examples):
  - Emission only (light particles)
  - Absorption only (dark fog)
  - Emission & absorption (clouds)
  - Single scattering, w/o shadows
  - Multiple scattering

- Two approaches:
  - Analytical model (via differentials)
  - Numerical approximation (via differences)
Continous emission model:

- **Question**: how much light (I like intensity) is added along an infinitely short ray segment in the volume
- **Differential** \( \frac{dI}{dt} = g(t) \) …
  volume emits light (corresponding to thickness)
- **Glow factor** \( g(t) \)
- **Integration** results in: \( I(s) = I_0 + \int_{t \in [0,s]} g(t) \, dt \)
- **Overall emission contrib.**: \( G(0,s) = \int_{t \in [0,s]} g(t) \, dt \)
- **Unrealistic**, because no absorption
Emission, Numerical Approximation

- **Discrete emission model:**
  
  - **Question:** how much light (C like color) is added within a small, but finite volume extent
  
  - $C_i$ … contribution of vol. extent $i$ (thickness 1)
  
  
  \[ \Rightarrow \text{adding emission of extent } i \text{ results in} \]
  
  \[ \text{Out}_i = \text{In}_i + C_i \iff \text{Out}_i = \text{Out}_{i-1} + C_i \]
  
  - **Accumulation:**
  
  \[ \text{Out}_i = \text{In}_j + C_j + \ldots + C_{i-1} + C_i \]
  
  \[ \text{Out}_i = \text{In}_j + \sum_{j \leq k \leq i} C_k \]
  
  - **Example:**
  
  \[ \text{pixel value} = \text{background} + \sum_{k \in N} C(\text{ray}(k)) \]
Differential model:
\[ I(s) = I_0 + \int_{t \in [0,s]} g(t)dt \]

Discrete approximation:
\[ \text{Out}_s = \text{In}_0 + \sum_{s \geq k \in \mathbb{N}} C_k \]

Example:
Absorption, Differential Model

Continuous absorption model:

- **Question**: how much light (in % of $I_0$) remains after traversal of ray segment through the volume

- **Differential** $\frac{dI}{dt} = -\tau(t)I(t)$ …
  light ($I$) is partially absorbed ($\tau$)

- **Extinction coefficient** $\tau(t)$, e.g., 30%

- **Integration** results in: $I(s) = I_0 \cdot \exp(-\int_{t \in [0,s]} \tau(t)dt)$

- **Total transparency**: $T(0,s) = \exp(-\int_{t \in [0,s]} \tau(t)dt)$

- **Total absorption**: $\alpha(0,s) = 1 - T(0,s)$
Absorption, Numerical Approximation

- **Discrete approximation model:**
  - **Question:** how much light (in % of $I_0$) remains after traversal of small, but finite volume extent
  - $\alpha_i$ ... opacity of volume extent $i$ (per unit)
  - $\Rightarrow$ result after traversal of extent $i$
    \[
    \text{Out}_i = \text{In}_i \cdot (1 - \alpha_i) \iff \text{Out}_i = \text{Out}_{i-1} \cdot (1 - \alpha_i)
    \]
  - **Akkumulation:** $\text{Out}_i = \text{In}_j \cdot (1 - \alpha_j) \cdot \ldots \cdot (1 - \alpha_i)$
    \[
    \text{Out}_i = \text{In}_j \cdot \prod_{j \leq k \leq i} (1 - \alpha_k)
    \]
  - **Unit sampling:** unit distance between $\alpha_i$ samples!!
Absorption Only

- **Differential model:**
  \[ I(s) = I_0 \cdot \exp\left(-\int_{t \in [0,s]} \tau(t)dt\right) \]

- **Discrete approximation:**
  \[ \text{Out}_s = \ln I_0 \cdot \prod_{s \geq k \in \mathbb{N}} (1 - \alpha_k) \]

- **Example:**

Eduard Gröller, Helwig Hauser
Emission and Absorption

Continuous model (no scattering):

- At each position is given:
  - Emission $g(t)$
  - Extinction coefficient $\tau(t)$

- Differential $\frac{dI}{dt} = g(t) - \tau(t)I(t)$

- Emission $g(t)$ attenuated by $T(t,s)$

- Only Emission: $I_0 + \int_{t \in [0,s]} g(t) \, dt$

- With Absorption: $I_0 \cdot T(0,s) + \int_{t \in [0,s]} g(t) \cdot T(t,s) \, dt$

- Emission und Absorption:
  
  $I_0 \cdot \exp\left(-\int_{u \in [0,s]} \tau(u) \, du\right) + \int_{t \in [0,s]} g(t) \cdot \exp\left(-\int_{u \in [t,s]} \tau(u) \, du\right) \, dt$
Numerical Approximation

- **Discrete model (compositing):**
  
  - For each *volume extent* $i$:
    - Contribution $C_i$
    - Opacity $\alpha_i$, transparency $1 - \alpha_i$
  
  - $\text{Out}_i = \text{In}_i \cdot (1 - \alpha_i) + C_i \cdot \alpha_i$ (Std.-compositing)
  
  - Convex combination from background and own contribution
  
  - $\text{Out}_s = \text{In}_0 \cdot \prod_{s \geq k \in N} (1 - \alpha_k)$
    
    $$+ \sum_{s \geq k \in N} C_k \cdot \alpha_k \cdot \prod_{s \geq l > k} (1 - \alpha_l)$$
  
  - Opacity-weighted colors: $C_i \cdot \alpha_i$ instead of $C_i$
Emission and Absorption

- Differential model:
  - \( I(s) = I_0 \cdot T(0, s) + \int_{t \in [0, s]} g(t) \cdot T(t, s) \, dt \)
  - \( I(s) = I_0 \cdot \exp(-\int_{u \in [0, s]} \tau(u) \, du) \)
    \[ + \int_{t \in [0, s]} g(t) \cdot \exp(-\int_{u \in [t, s]} \tau(u) \, du) \, dt \]

- Discrete Approximation:
  - \( \text{Out}_i = \text{In}_i \cdot (1 - \alpha_i) + C_i \cdot \alpha_i \) (Compositing)
  - \( \text{Out}_s = \text{In}_0 \cdot \prod_{s \geq k \in N} (1 - \alpha_k) \)
    \[ + \sum_{s \geq k \in N} C_k \cdot \alpha_k \cdot \prod_{s \geq l > k} (1 - \alpha_l) \]
Emission or/and Absorption

- Emission only
- Absorption only
- Emission and Absorption only
Scattering

- **Scattering**: particles deviate light at a position
  - BRDF (bidirectional reflectance distribution function)
  - Single scattering
    - Too little light in the interior
  - Single scattering with shadows
  - Multiple Scattering
    - Radiosity techniques
    - Very realistic, very costly
Paper (more details):

Ray Casting / Compositing

Classical Image-Order Methods
Ray Tracing vs. Ray Casting

- **Ray Tracing**: method from image generation
- In volume rendering: *only viewing rays*
  ⇒ therefore Ray Casting
- Classical **image-order** method
- **Ray Tracing**: ray – object intersection
  **Ray Casting**: no objects, density values in 3D
- **In theory**: take all density values into account!
  **In practice**: traverse volume step by step
- **Interpolation** necessary for each step!
**Context:**

- **Volume data:** 1D value defined in 3D –
  \[ f(\mathbf{x}) \in \mathbb{R}^1, \mathbf{x} \in \mathbb{R}^3 \]
- **Ray** defined as half-line:
  \[ \mathbf{r}(t) \in \mathbb{R}^3, t \in \mathbb{R}^1 > 0 \]
- **Values along Ray:**
  \[ f(\mathbf{r}(t)) \in \mathbb{R}^1, t \in \mathbb{R}^1 > 0 \]
  (intensity profile)
Standard Ray Casting

- Levoy ’88:
  1. C(i), α(i) (from TF)
  2. Ray casting, interpolation
  3. Compositing
1. Shading, Classification

1. Step:

- Shading, \( f(i) \rightarrow C(i) \):
  - Apply transfer function
  - diffuse illumination (Phong), gradient \( \approx \) normal

- Classification, \( f(i) \rightarrow \alpha(i) \):
  - Levoy ’88, gradient enhanced
  - Emphasizes transitions
2. Ray Traversal – Three Approaches

- Regular sampling
- Scan conversion
- Voxel intersections

- Equidistant samples
- All voxels used once
- Samples weighted by lengths of ray segments
2. Ray Traversal, Interpolation

- Voxel-based vs. cell-based traversal
- Tri-linear (interpolation within a cell) vs. bi-linear (interpolation within a cell face)
- Tri-linear:
  - first 4* in z-direction (interpolated square),
  - then 2* in y-direction (interpolated line),
  - then 1* in x-direction (interpolated value)
- Unit sampling vs. variable sample distances – compositing different!!
Compositing: F2B vs. B2F

**Back-to-Front (B2F):**
- \( \text{Out}_i = \text{In}_i \cdot (1 - \alpha_i) + C_i \cdot \alpha_i, \ \text{In}_{i+1} = \text{Out}_i \ldots \)
- Depending on local transparency \((1 - \alpha_i) \Rightarrow\) convex combination of old \(\text{In}_i\) & new \(C_i\)
- Example:
  - Voxel i: \(C_i = \text{red}, \ \alpha_i = 30\%; \ \text{so far: In}_i = \text{white}\)
  - Result of compositing: 70% white + 30% red

**Front-to-Back (F2B):**
- \( \text{Col} = \text{Col} + (1 - \alpha_{akk}) \cdot C_i \cdot \alpha_i \ldots\) accumulated color
- \(\alpha_{akk} = \alpha_{akk} + (1 - \alpha_{akk}) \cdot \alpha_i \ldots\) accumulated opacity
Interpolation Kernels

volumetric compositing

object \text{ (color, opacity) }

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R. Crawford, Ohio State Univ.
Ray Casting – Examples

- CT scan of human hand (244x124x257, 16 bit)
Ray Casting – Further Examples

- Tornado Visualization:
Ray Casting – Further Examples

- Molecular data:
Acceleration - Progressive Refinement

- First render every $2^n \times 2^n$-th pixel, then render the $2^{n-1} \times 2^{n-1}$-th pixel inbetween, aso. (until interruption or completion)
Paper (more details):

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