Critical Points and Visualization

- **Topological analysis** of vector fields
  - searching critical points $x^*$ (1a.): $\mathbf{v}(x^*)=0$
  - analyzing flow behavior near $x^*$ (1b.)

Bsp.:

$$x_1^* = A \quad x_2^* = B \quad x_3^* = C$$

- Linearization around $x^*$:
  $$\mathbf{v}(x^* + \Delta x) = \sum_{i=0}^{\infty} (\Delta x \cdot \nabla) v \bigg|_{x^*} \cdot \Delta x + \Delta x \cdot \nabla \mathbf{v} \bigg|_{x^*}$$
  (Taylor series of $\mathbf{v}$ near $x^*$, $\Delta x$ small, $\mathbf{v}(x^*)=0$)

- Jacobi matrix $\nabla \mathbf{v} \bigg|_{x^*}$ governs the behavior near $x^*$
- Eigenvalue analysis yields classification
- negative $\lambda$ $\Rightarrow$ local attraction
- positive $\lambda$ $\Rightarrow$ local repulsion
- complex $\lambda$ $\Rightarrow$ rotation around $x^*$

Critical Points and Visualization

- **Topological analysis** of vector fields
  - searching critical points $x^*$ (1a.): $\mathbf{v}(x^*)=0$
  - analyzing flow behavior near $x^*$ (1b.)

Bsp.:

- $A$: saddle
- $B$: repellor
- $C$: saddle

Critical Points and Visualization

- **Topological analysis** of vector fields
  - treating critical points $x^*$ (3)...
  - searching higher order critical structures (2a.)
    - cycles $c^*$ through $x^{**}$: $s^{**}(x^{**}, T) = x^{**}$ (period $T>0$)
    - invariant tori $t^*$ ($nD$ with $n\geq 3$ only)
    - etc.
Critical Points and Visualization

- **Topological analysis** of vector fields
  - treating critical points $x^*$ (1.)...
  - searching higher order critical structures (2a.)
  - characterizing higher order crit. structs. (2b.)
    - 2D:
      - attracting or repelling cycles (1D)
    - 3D:
      - attracting, repelling, or saddle cycles (1D)
      - attracting or repelling tori (2D)
    - $n$D:
      - $k$-dim. critical structures with $k \leq n-2$: attr., rep., saddle
      - $(n-1)$-dim. critical structures: attr., rep. only
      - etc. (more research needed)

- **Bsp.**:
  - attracting cycle $D$

- **Topological analysis** of vector fields
  - treating critical points $x^*$ (1.)...
  - connecting crit. points $x^*$: separatrices $\Sigma^*$ (3.)
    - following $(n-1)$-dim. eigen spaces $\{e_i,...\}$
      - yields (characteristic) stream lines in 2D
      - yields (characteristic) stream surfaces in 3D
      - etc.

- **Topological analysis** of vector fields
  - critical points $x^*$ (1.)...
  - + higher order crit. structs. (2.)
  - + separatrices $\Sigma^*$ (3.)

- **Topological skeleton** of vector field $v$
  - crit. pts. A, B, C + cycle D + separatrices

- **Selected direct visualization** (e.g., streamlets)
  - topology-based vis.