Critical Points and Visualization

- **Topological analysis** of vector fields
  - searching **critical points** $x^*$ (1a.): $v(x^*) = 0$
  - analyzing **flow behavior** near $x^*$ (1b.)
    - Linearization around $x^*$:
      $$ v(x^* + \Delta x) = \sum_{i=0}^{\infty} (\Delta x \cdot \nabla v) \bigg|_{x^*} \approx v(x^*) + \Delta x \cdot \nabla v \bigg|_{x^*} $$
      (Taylor series of $v$ near $x^*, \Delta x$ small, $v(x^*) = 0$)
  - Jacobi matrix $\nabla v \bigg|_{x^*}$ governs the behavior near $x^*$
  - Eigenvalue analysis yields classification
    - negative $\lambda_i \Rightarrow$ local attraction
    - positive $\lambda_i \Rightarrow$ local repulsion
    - complex $\lambda_i \Rightarrow$ rotation around $x^*$
Critical Points and Visualization

- **Topological analysis** of vector fields
  - searching **critical points** $x^*$ (1a): $v(x^*) = 0$
  - analyzing **flow behavior** near $x^*$ (1b)

Bsp.:  

- $A$: saddle
- $B$: repellor
- $C$: saddle

$\lambda_1, \lambda_2 > 0$

Helwig Hauser: Topology-based FlowVis

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Critical Points and Visualization

- **Topological analysis** of vector fields
  - treating critical points $x^*$ (1a)...
  - searching **higher order critical structures** (2a)
  - cycles $s^*$ through $x^*$: $s^*(x^*, T) = x^*$ (period $T > 0$)
  - invariant tori $t^*$ ($nD$ with $n \geq 3$ only)
  - etc.

Helwig Hauser: Topology-based FlowVis

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Critical Points and Visualization

- **Topological analysis** of vector fields
  - treating critical points $x^*$ (1a)...
  - searching **higher order critical structures** (2a)

Bsp.:  

$D$: cycle

Helwig Hauser: Topology-based FlowVis
Critical Points and Visualization

- **Topological analysis** of vector fields
  - treating critical points $x^*$ (1)
  - searching higher order critical structures (2a)
  - characterizing higher order crit. structs. (2b)
    - 2D: attracting or repelling cycles (1D)
    - 3D: attracting, repelling, or saddle cycles (1D)
    - attracting or repelling tori (2D)
    - $n$D: $k$-dim. critical structures with $k \leq n-2$: attr., rep., saddle
      - $(n-1)$-dim. critical structures: attr., rep. only
      - etc. (more research needed)

Bsp.: $D$: attracting cycle

- connecting crit. points $x^*$: **separatrices** $\Sigma^*$ (3)
  - following $(n-1)$-dim. eigen spaces $\{e_i,\ldots\}$ from $x^*$ per integration
    - yields (characteristic) stream lines in 2D
    - yields (characteristic) stream surfaces in 3D
    - etc.
**Critical Points and Visualization**

- **Topological analysis** of vector fields
  - treating critical points $x^*$ (1.)...
  - treating higher order crit. structs. (2.)
  - connecting crit. points $x^*$: separatrices $\Sigma^*$ (3.)

- **topological skeleton** of vector field $v$
  - critical pts. $A, B, C$
  - + cycle $D$
  - + separatrices

- **selected direct visualization** (e.g., streamlets)

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**Critical Points and Visualization**

- **Topological analysis** of vector fields
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  - + separatrices $\Sigma^*$ (3.)

- **topology-based vis.**

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- **Topological analysis** of vector fields
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- **topology-based vis.**