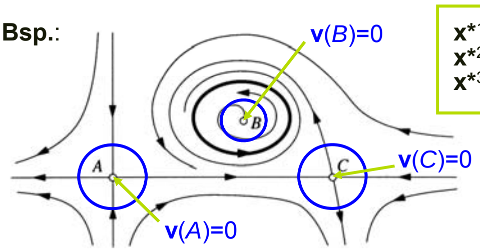
Critical Points and Visualization

- **Topological analysis** of vector fields
  - searching **critical points x*** (**1a**.): **v**(**x***)=0

Bsp.:



$\mathbf{x}^{*1} = A$
$\mathbf{x}^{*2} = B$
$\mathbf{x}^{*3} = C$

Critical Points and Visualization

- **Topological analysis** of vector fields
  - searching **critical points x*** (**1a**.): **v**(**x***)=0
  - analyzing **flow behavior** near **x*** (**1b**.)
    - Linearization around **x***:

$$\mathbf{v}(\mathbf{x}^*+\Delta\mathbf{x}) = \sum_{i\geq 0} (\Delta\mathbf{x}\cdot\nabla)^i\, \mathbf{v}\big|_{\mathbf{x}^*} \approx \overbrace{\mathbf{v}(\mathbf{x}^*)}^{=0} + \Delta\mathbf{x}\cdot\nabla\mathbf{v}\big|_{\mathbf{x}^*}$$

(Taylor series of **v** near **x***, Δ**x** small, **v**(**x***)=0)

Critical Points and Visualization

- **Topological analysis** of vector fields
  - searching **critical points x*** (**1a**.): **v**(**x***)=0
  - analyzing **flow behavior** near **x*** (**1b**.)
    - Linearization around **x***:

$$\mathbf{v}(\mathbf{x}^*+\Delta\mathbf{x}) = \sum_{i\geq 0} (\Delta\mathbf{x}\cdot\nabla)^i\, \mathbf{v}\big|_{\mathbf{x}^*} \approx \overbrace{\mathbf{v}(\mathbf{x}^*)}^{=0} + \Delta\mathbf{x}\cdot\nabla\mathbf{v}\big|_{\mathbf{x}^*}$$

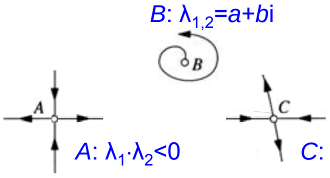
(Taylor series of **v** near **x***, Δ**x** small, **v**(**x***)=0)

    - Jacobi matrix $\nabla\mathbf{v}\big|_{\mathbf{x}^*}$ governs the behavior near **x***
    - Eigenvalue analysis yields classification
      - negative λ_i ⇒ local attraction
      - positive λ_i ⇒ local repulsion
      - complex λ_i ⇒ rotation around **x***

Critical Points and Visualization

- **Topological analysis** of vector fields
  - searching **critical points x*** (**1a**.): **v**(**x***)=0
  - analyzing **flow behavior** near **x*** (**1b**.)

Bsp.:



A: saddle
B: repellor
C: saddle

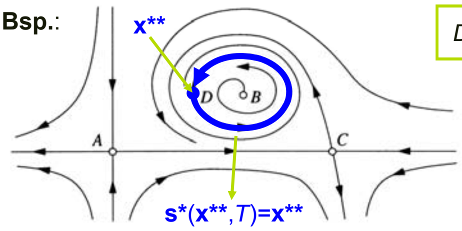
Critical Points and Visualization

- **Topological analysis** of vector fields
  - treating **critical points x*** (1.)…
  - searching **higher order critical structures** (**2a**.)
    - cycles **s*** through **x****: **s***(**x****,*T*) = **x**** (period *T*>0)
    - invariant tori **t*** (*n*D with *n*≥3 only)
    - etc.

Critical Points and Visualization

- **Topological analysis** of vector fields
  - treating **critical points x*** (1.)…
  - searching **higher order critical structures** (**2a**.)

Bsp.:



D: cycle

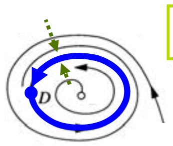
Critical Points and Visualization

- **Topological analysis** of vector fields
  - treating **critical points x*** (1.)…
  - searching **higher order critical structures** (**2a**.)
  - characterizing **higher order crit. structs.** (**2b**.)
    - 2D:
      - attracting or repelling cycles (1D)
    - 3D:
      - attracting, repelling, *or saddle* cycles (1D)
      - attracting or repelling tori (2D)
    - *n*D:
      - k-dim. critical structures with $k \leq n-2$: attr., rep., *saddle*
      - $(n-1)$-dim. critical structures: attr., rep. only
      - etc. (more research needed)

Critical Points and Visualization

- **Topological analysis** of vector fields
  - treating **critical points x*** (1.)…
  - searching **higher order critical structures** (**2a**.)
  - characterizing **higher order crit. structs.** (**2b**.)

Bsp.:



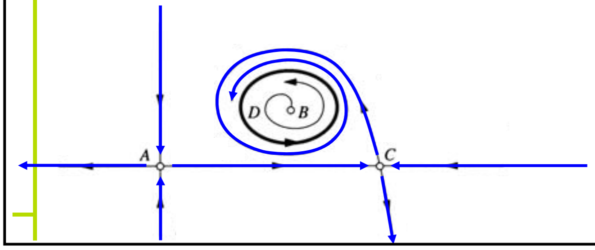
D: attracting cycle

Critical Points and Visualization

- **Topological analysis** of vector fields
  - treating **critical points x*** (1.)…
  - treating **higher order crit. structs.** (2.)
  - connecting crit. points **x***: **separatrices Σ*** (**3**.)
    - following $(n-1)$-dim. eigen spaces $\{\mathbf{e}_i, \ldots\}$ from **x*** per integration
      - yields (characteristic) stream lines in 2D
      - yields (characteristic) stream surfaces in 3D
      - etc.

Critical Points and Visualization

- **Topological analysis** of vector fields
  - treating **critical points x*** (1.)…
  - treating **higher order crit. structs.** (2.)
  - connecting crit. points **x***: **separatrices Σ*** (3.)

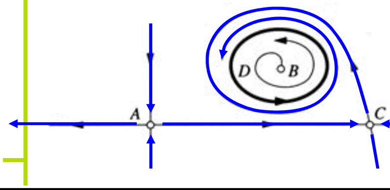


Critical Points and Visualization

- **Topological analysis** of vector fields
  - **critical points x*** (1.)…
  - + **higher order crit. structs.** (2.)
  - + **separatrices Σ*** (3.)

- **topological skeleton** of vector field v

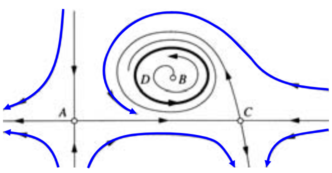
crit. pts. A, B, C
+ cycle D
+ separatrices

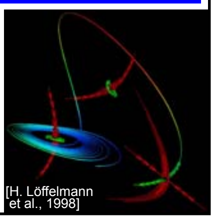


Critical Points and Visualization

- **Topological analysis** of vector fields
  - **critical points x*** (1.)…
  - + **higher order crit. structs.** (2.)
  - + **separatrices Σ*** (3.)
  - + **selected direct visualization** (e.g., streamlets)

- **topology-based vis.**





[H. Löffelmann et al., 1998]