Critical Points and Visualization
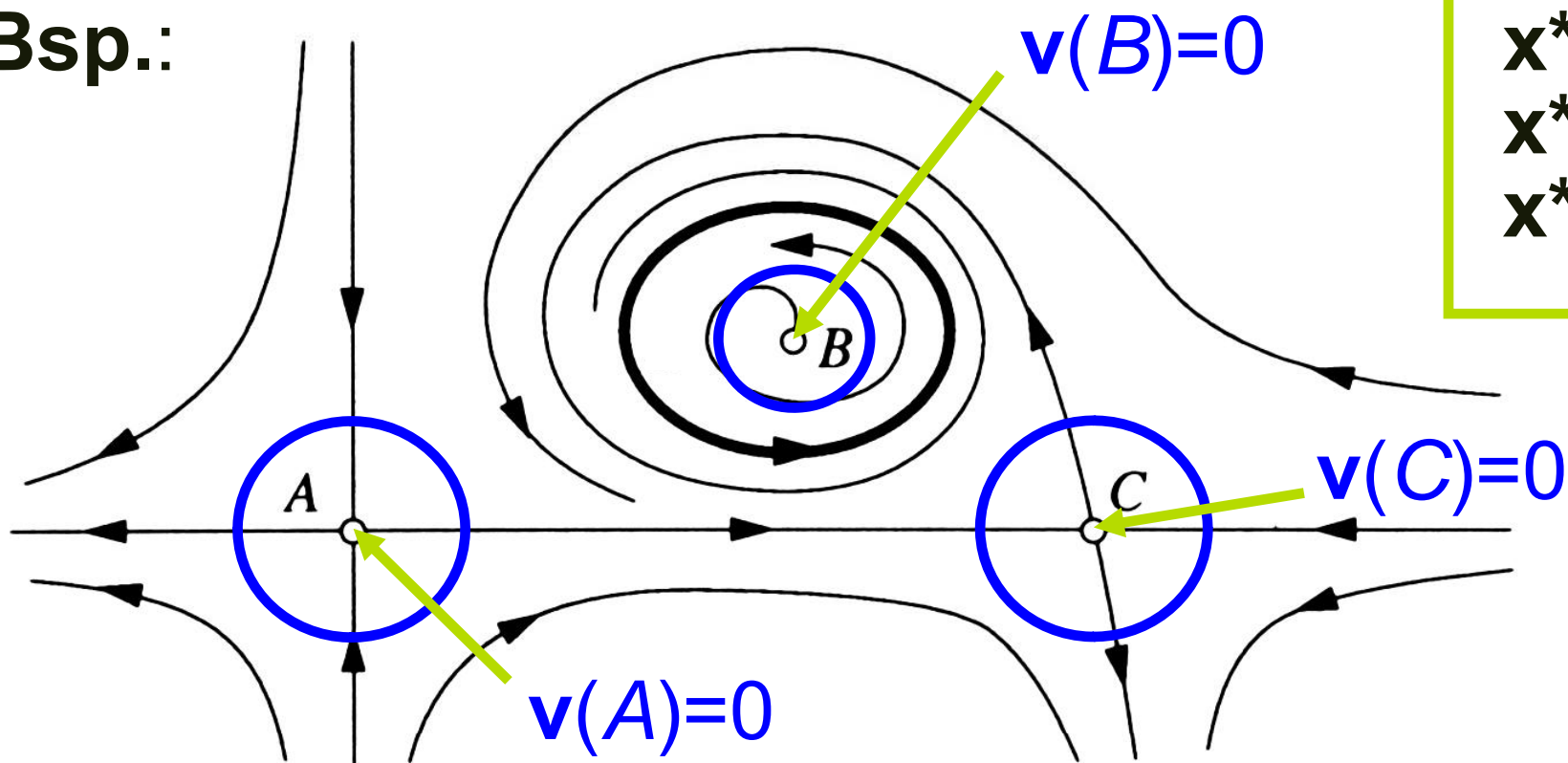
- **Topological analysis** of vector fields
  - searching **critical points x*** (**1a.**): $\mathbf{v}(\mathbf{x}^*)=0$

Bsp.:

$\mathbf{v}(B)=0$

$\mathbf{v}(C)=0$

$\mathbf{v}(A)=0$

$\mathbf{x}^{*1} = A$
$\mathbf{x}^{*2} = B$
$\mathbf{x}^{*3} = C$

Critical Points and Visualization

- **Topological analysis** of vector fields
  - searching **critical points x*** (**1a.**): **v(x*)=0**
  - analyzing **flow behavior** near **x*** (**1b.**)
    - Linearization around **x***:

$$\mathbf{v}(\mathbf{x^*}+\Delta\mathbf{x}) \;=\; \sum\nolimits_{i\geq 0} (\Delta\mathbf{x}\cdot\nabla)^i\, \mathbf{v}\,\Big|_{\mathbf{x^*}} \;\approx\; \overbrace{\mathbf{v}(\mathbf{x^*})}^{=0} + \Delta\mathbf{x}\cdot\nabla\mathbf{v}\,\Big|_{\mathbf{x^*}}$$

$\Big($Taylor series of **v** near **x***, $\Delta\mathbf{x}$ small, **v(x*)=0**$\Big)$

Critical Points and Visualization

- **Topological analysis** of vector fields
  - searching **critical points x*** (**1a.**): **v(x*)**=0
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$\Big($Taylor series of **v** near **x***, $\Delta\mathbf{x}$ small, **v(x*)**=0$\Big)$

    - Jacobi matrix $\nabla\mathbf{v}\big|_{\mathbf{x^*}}$ governs the behavior near **x***
    - Eigenvalue analysis yields classification
      - negative $\lambda_i \Rightarrow$ local attraction
      - positive $\lambda_i \Rightarrow$ local repulsion
      - complex $\lambda_i \Rightarrow$ rotation around **x***

Critical Points and Visualization

- **Topological analysis** of vector fields
  - searching **critical points x*** (**1a.**): **v(x*)=0**
  - analyzing **flow behavior** near **x*** (**1b.**)

Bsp.:

B: $\lambda_{1,2} = a + bi$
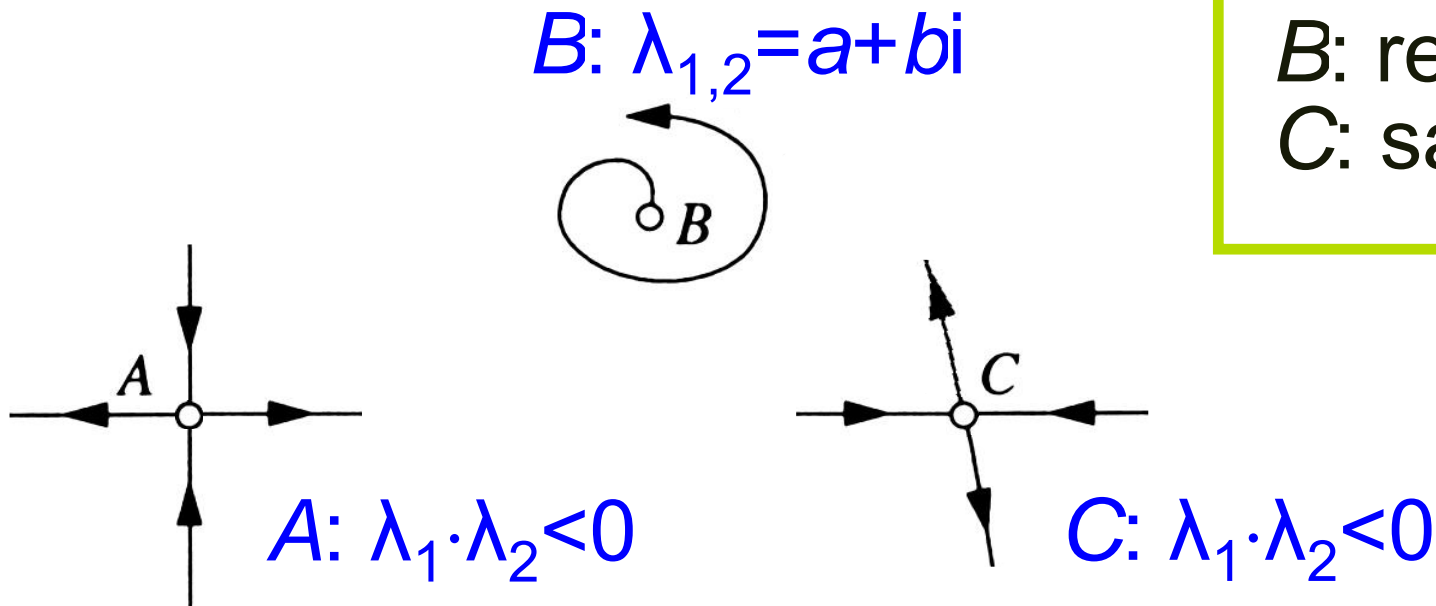
A: saddle
B: repellor
C: saddle

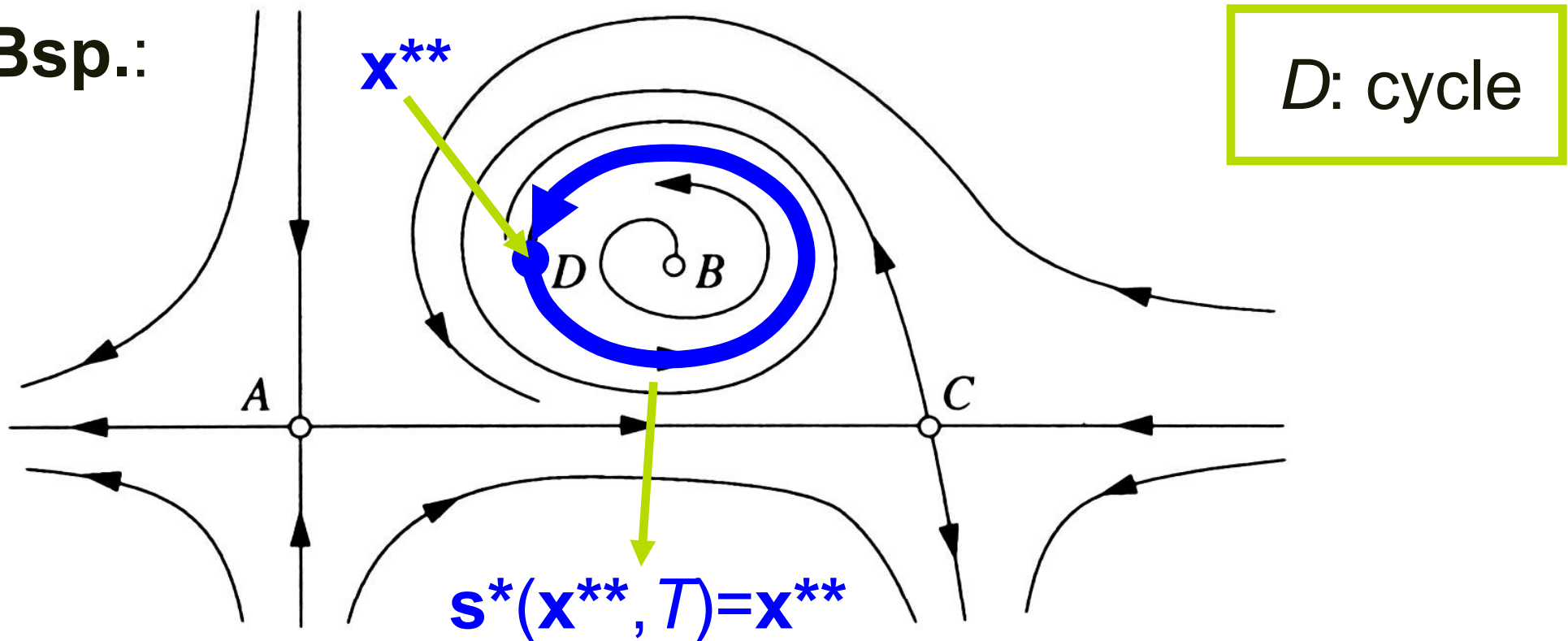A: $\lambda_1 \cdot \lambda_2 < 0$

C: $\lambda_1 \cdot \lambda_2 < 0$

Critical Points and Visualization

- **Topological analysis** of vector fields
  - treating **critical points x*** (1.)…
  - searching **higher order critical structures** (2a.)
    - cycles **s*** through **x****: $s^*(x^{**}, T) = x^{**}$ (period $T > 0$)
    - invariant tori **t*** (nD with $n \geq 3$ only)
    - etc.

Critical Points and Visualization

- **Topological analysis** of vector fields
  - treating **critical points x*** (1.)…
  - searching **higher order critical structures** (2a.)

Bsp.:

D: cycle

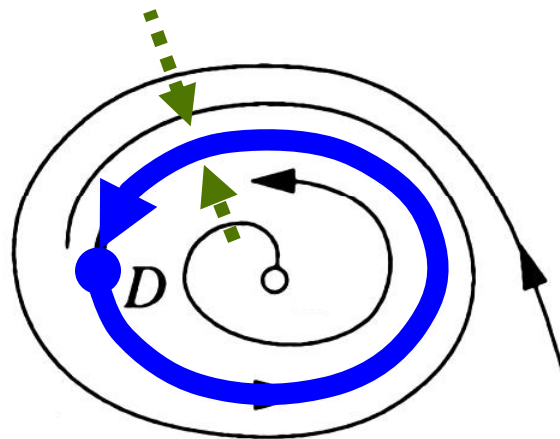$$\mathbf{s}^*(\mathbf{x}^{**}, T) = \mathbf{x}^{**}$$

Critical Points and Visualization

- **Topological analysis** of vector fields
  - treating **critical points x*** (**1.**)…
  - searching **higher order critical structures** (**2a.**)
  - characterizing **higher order crit. structs.** (**2b.**)
    - 2D:
      - attracting or repelling cycles (1D)
    - 3D:
      - attracting, repelling, *or saddle* cycles (1D)
      - attracting or repelling tori (2D)
    - nD:
      - k-dim. critical structures with $k \le n{-}2$: attr., rep., *saddle*
      - $(n{-}1)$-dim. critical structures: attr., rep. only
      - etc. (more research needed)

Critical Points and Visualization

- **Topological analysis** of vector fields
  - treating **critical points x*** (1.)…
  - searching **higher order critical structures** (2a.)
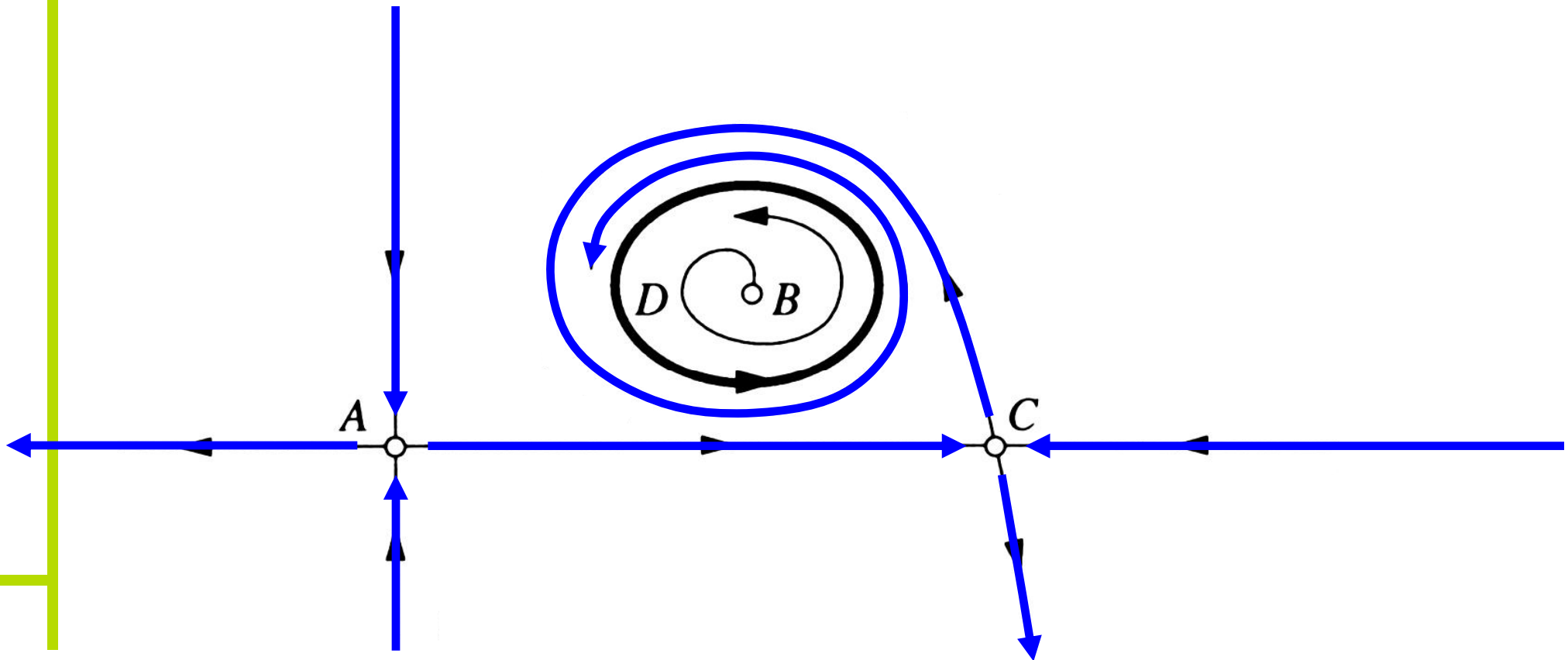 - characterizing **higher order crit. structs.** (2b.)

Bsp.:

D: attracting cycle

Critical Points and Visualization

- **Topological analysis** of vector fields
  - treating **critical points x*** (1.)…
  - treating **higher order crit. structs.** (2.)
  - connecting crit. points **x***: **separatrices Σ*** (3.)
    - following (n–1)-dim. eigen spaces $\{e_i,…\}$ from **x*** per integration
      - yields (characteristic) stream lines in 2D
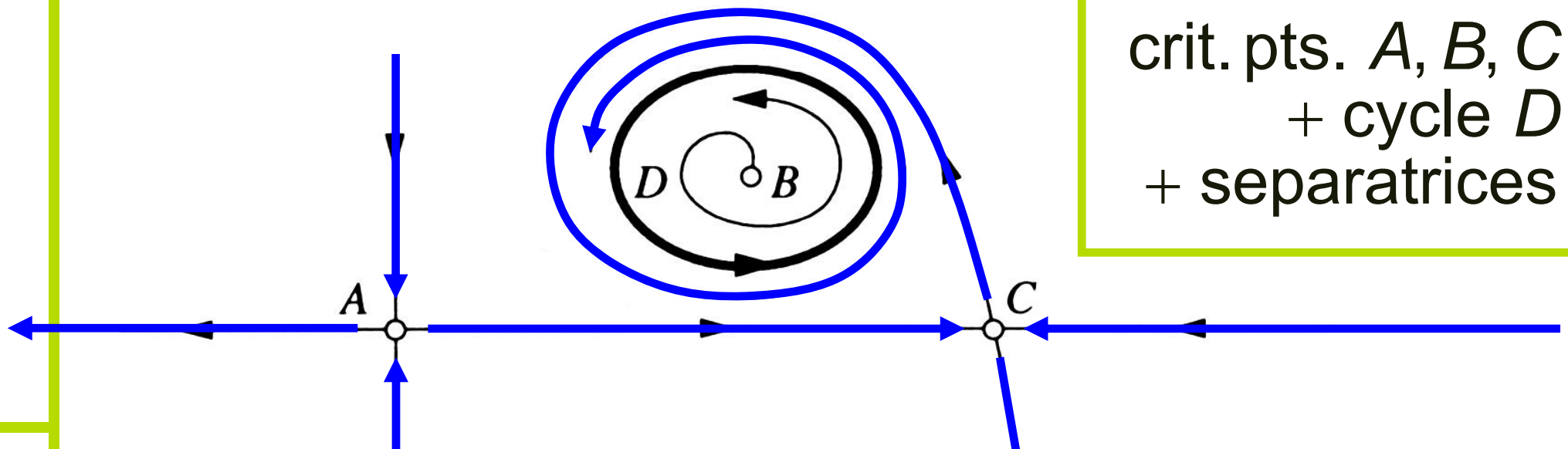   - yields (characteristic) stream surfaces in 3D
      - etc.

Critical Points and Visualization

- **Topological analysis** of vector fields
    - treating **critical points x*** (1.)…
    - treating **higher order crit. structs.** (2.)
    - connecting crit. points **x***: **separatrices Σ*** (3.)

Critical Points and Visualization

- **Topological analysis** of vector fields
  - **critical points x* (1.)…**
  - **+ higher order crit. structs. (2.)**
  - **+ separatrices Σ* (3.)**

  - **topological skeleton** of vector field v

crit. pts. *A, B, C*
+ cycle *D*
+ separatrices

Critical Points and Visualization

- **Topological analysis** of vector fields
  - **critical points x*** (1.)…
  - **+ higher order crit. structs.** (2.)
  - **+ separatrices Σ*** (3.)
  - **+ selected direct visualization** (e.g., streamlets)

  - **topology-based vis.**

[H. Löffelmann et al., 1998]