Physically correct surface models

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Abstract

The goal of this paper is to give a short overview on different techniques describing realistic, physical based surface models. At first I am going to give a short introduction to surface models, mentioning the most common definitions and terms used in the paper. Then we take a small look on physical surface model generated by empirical models like the popular Phong or Oren-Nayar. The we proceed to the analytical, physical based models like Cook-Torrance, He or Schlick. We examine anisotropic, heterogeneous and polarized models, as well as the limits on realism when working with surface models. As a result we compare the most popular models to see how they behave under similar conditions.

Keywords: BRDF, surface model, reflectance model, physical correct, survey

1 Introduction

The common known goal of rendering is to create realistic images, which are impossible to distinguish from photographs of real world objects. In order to achieve this goal realistic surface models and representations are needed. Various algorithms exist to create these images, some of them are based on reflectance models. The most common reflectance model is based on the equation 1, which uses the so called *bidirectional reflectance distribution function* or BRDF in order to simulate different surface reflection properties. This function describes the ratio of the reflected radiance for all points of the surface in all every direction.

It is a part of the *Fredholm Integro-differential equation*. This equation defines the amount of radiance that arrives at a distinct point from any other point in the scene. The reflectance function is defined by:

$$I(x,x') = g(x,x')\left[\varepsilon(x,x') + \int_S \rho(x,x',x'')\, I(x',x'')dx''\right] \quad (1)$$

with $I(x,x')$ being the radiance which arrives at point x in the scene from another point x', an emission factor $\varepsilon(x,x')$, responsible for self emission of the material. The most important part of the equation is done by ρ, which is the reflective characterization of the surface. This function is our mentioned BRDF which was introduced first by Nicodemus in [Nicodemus et al. 1977].

In this paper we will focus on the BRDF function, the other parts of the equation 1 are explained at [László 1999] and [Wilkie 2007] in further detail.

1.1 BRDF in detail

The BRDF function is responsible for the creation of realistic surfaces by exactly defining how the light is reflected from any given point in dependence of the incoming light. A general BRDF function can be described as

$$f_r(\omega_i \rightarrow \omega_r) \equiv \frac{L_r(\omega_r)}{L_i(\omega_i)\cos\theta_i d\omega_i} \quad (2)$$

which is a hemispherical function with different stages of complexity. A visualization of the incident and reflected angles can be seen at figure 1.1.

The common parameters for our function are the 4 angles for the incoming and outgoing light. The elevation and azimuth angles ϕ and θ as seen in figure 1.1. In addition the surface reflectance can depend on the wavelength of the light, so we have different reflection schemes for different wavelength of light. In most equations wavelength is not mentioned, but it is contained implicitly. Also the exact location and orientation on the surface can have a significant influence on the reflection. So a BRDF function can have up to 7 dimensions, depending on the current situation and the requirements for the scene and reflectance model.

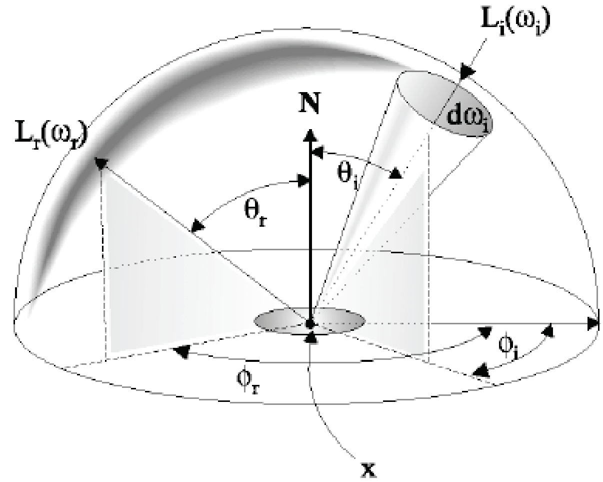

Figure 1: Visualisation of important angles of an BRDF, L_i (or later $\vec{L}$) is the incoming radiance, L_r (or $\vec{V}$) the outgoing radiance on point x on the surface. $\vec{N}$ is the surface normal vector. [Wilkie 2007]

1.2 Properties of BRDFs

Any valid BRDF must fulfill two important, fundamental properties, derived from the physics of light. The first property, the *Energy Conservation Law* describes that the sum of light being reflected from a surface must not be bigger than the incoming light. So a BRDF must not create more output energy than it gained as input, but it can consume as much energy as desired. The second rule, the *Helmholtz Reciprocity Rule* means that a BRDF has to be symmetric. Symmetry in this context is understood that it is possible to exchange the sampling directions of light. As example its

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no difference if you follow the light coming from x', reflected at x and moving on to the eye or another position. For a valid BRDF the opposite way creates the same result.

1.3 Extensions and modifications

An extension to the BRDF is the BSDF (BSSRDF), or *bidirectional scattering distribution function* (*bidirectional subsurface scattering distribution function*) which allows reflections being modified in subsurface areas. With these modifications it is possible to let light travel through the surface instead of being just bounced off. This modification is needed to describe very common effects like the translucency of objects. This effect can be seen on glowing candles or the surface of human skin.

Another extension for textural data is the BTF or *bidirectional texture function* which is again quite similar to the BRDF. It adds positional information to a texture to combine it with individual BRDF datasets.

1.4 Classification of BRDF models

Two general types of BRDFs can be classified, a more general anisotropic and a simple isotropic reflection. An anisotropic reflection is a surface reflection scheme which changes its reflected radiance when the surface below is rotated around the normal vector at point x. This is often used to visualize small scratches on the surface which reflect the light differently. In reality nearly all materials are anisotropic reflectors. A simplification of an anisotropic reflectance is now the isotropic reflection model. Here reflections do not change while the object is rotated, so its properties remain constant over the whole surface. The most analytical BRDFs models are isotropic models because they are much easier to calculate and configure. An example for a real, nearly perfect isotropic reflector would be smooth plastic.

1.5 Reflection components

Two types of radiance reflections can be distinguished. The first one is the plain diffuse reflection, where light is distributed equally and independent of the current viewing position. Diffuse reflection can be seen on objects like dull/matte paint. The other type is the specular reflection or mirror like reflection of light. Here the light is reflected nearly into a single (or small) outgoing direction according to the law of reflection. Examples for objects with specular reflections are mirrors, polished metals or coated objects.

2 Surface models

In this section surface models which can be used as a BRDF are described. Beside the two basic requirements of reciprocity and energy conservation analytical models need to be elevated fast, they should be easy to sample (especially when doing stochastic sampling) and should be quite expressive and easy to use. A plot of a BRDF can be viewed in figure 2 where the overall, macro scale behavior of a BRDF is seen, a more detailed milli-scale surface and a nano-scaled facet with its reflection properties.

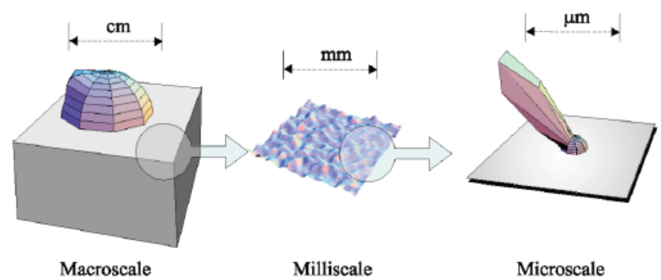

Figure 2: Three different simulations of BRDFs, showing several surface properties. [Wilkie 2007]

2.1 Empirical Models

Empirical models just try to simulate the look of a surface, not to calculate it physically correct. Normally the configuration parameters have no real physic relevance and are just used to configure the models. A main feature of the empirical models is that they can be calculated very fast and are easy to understand and implement. But they can never produce physical correct solutions, although they still look very realistic in most situations. They are mostly used in applications which don't require a lot of realism but need to look quite good and with a main goal of fast computation. Examples for such Applications would be special effects in movies, video art, games or in commercials.

2.1.1 Lambert

The *Lambert model* is the first and simplest diffuse reflection model. It assumes we have a perfectly diffuse reflecting surface and the light leaves this surface in proportion to to the cosine of the incident angle. The general equation of a diffuse reflection is now

$$f_r(\omega_i \to \omega_r) = \frac{k_d}{\pi} \tag{3}$$

with k_d being the diffuse surface constant and π as the normalization factor for correct energy conservation. A lambertian surface is like every diffuse surface view independent, so the brightness of the surfaces remains the same for all angles of view and depends only on the angle of the incoming light.

2.1.2 Oren-Nayar

The Oren-Nayar Model as described in [Oren and Nayar 1994] is a generalization of the Lambert model. It is a microfacet based model which is able to simulate the roughness of a surface. More information about mircofacets is given at section 2.2 where the Cook-Torrance model is explained. For special values of the parameter σ the model is identical to a lambertian surface, for other values it becomes more and more retro-reflective.

2.1.3 Phong

A simple way to add specular highlights to a diffuse model is the combination with the Phong model. It allows fast reflection in a

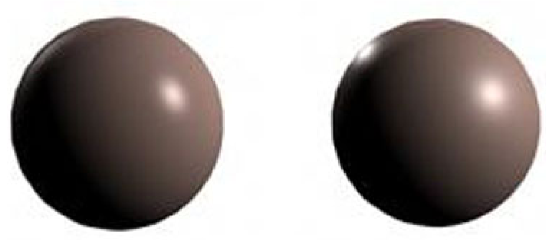

Figure 3: The left sphere is rendered with Phong, the right one with the Blinn-Phong model. [Calkins]

small area. With this model it is possible to to represent various stages of *shininess* on the reflector. The specular Phong reflection is defined by

$$f_r(\vec{L},\vec{V}) = k_s \cdot \frac{\left(\vec{R}\cdot\vec{V}\right)^{n_s}}{\left(\vec{N}\cdot\vec{L}\right)} \tag{4}$$

with k_s being the *specular reflection coefficient*, $\vec{V}$ is the direction to the viewer and $\vec{L}$ the direction from the surface point to the light. $\vec{R}$ defines the direction of the perfect reflection, thats $\vec{L}$ mirrored on the normal vector $\vec{N}$. The parameter n_s defines the shininess of the surface, perfect reflectors would have infinity as value for n_s, complete dull surfaces have 0. A basic problem of the Phong model is that energy conservation depends on the correct choice and combination of the coefficients of the model. Another problem of the model is that the objects often look very plastic, so in the most cases not very realistic. Furthermore the Phong model is quite complex to calculate for an empirical model, specially when used in real time rendering.

2.1.4 Blinn-Phong

An extension to the Phong model is the Blinn-Phong model, as described in [Blinn 1977]. It is a much faster modification of the classical Phong model. It uses the so called halfway vector for the reflectance calculation, a combination between the view and light vector. This vector is used in many other models mentioned later.

$$\vec{H} = \frac{\vec{L}+\vec{V}}{\left|\vec{L}+\vec{V}\right|} \tag{5}$$

The big advantage of this model is that for cases where viewer and light are at infinity the halfway vector is independent of position and surface curvature. So the vector will only to be calculated once per light and frame and not for each pixel as in the original model. Because of this optimization we have a huge performance increase while rendering.

A common issue for both specular models is the fact that they act not very accurate when they are viewed from grazing angles. The reflections are either to sharp or far to blurred compared to a real image. These problems can not be handled by empirical models so more complicated reflectance models are needed.

2.2 Cook-Torrance

Cook and Torrance [Cook and Torrance 1982] introduced a general physically plausible reflectance model which is able to correctly predict the directional and spectral composition of reflected light. The model is based on a work from Torrance and Sparrow [Torrance and Sparrow 1967] and Blinn [Blinn 1977]. Its basic assumption is that the surface of an objects consists of many small parts, or microfacets which are all perfect reflecting mirrors. The microfacets are evenly distributed on the surface. The model itself works only correct if the wavelength of the light is smaller than the mean roughness of the surface patches. Each facet has a random orientation but they are aligned along a specified distribution around the mean surface normal N with a maximum slope difference angle α. The Gaussian distribution for the microfacets used in the Torrance-Sparrow model is now replaced by the Beckmann-distribution function:

$$D = \frac{1}{m^2\cos^4\alpha} e^{-\left(\frac{\tan\alpha}{m^2}\right)^2} \tag{6}$$

with m being the root of the mean slope of microfacets and α the maximum slope difference angle. Small values of m create gentle, smooth slopes, while bigger values create a much bigger distribution. Beside this distribution others are possible, depending on the current situation. Surfaces with different types of roughness can use a weighted sum of different distribution functions.

The calculation of the reflected light is now done by two terms. The first one, the *Fresnel-reflection-coefficient*, has three parameters, the index of refection n, the surface extinction coefficient k and the angle of illumination θ. The values of n and k vary with the wavelength so their exact values are often unknown, except if they were measured in an experiment. If they are unknown there is a way to calculate the missing values. For non-metals $k = 0$, for metals we can set it to zero to get an effective value for the refraction coefficient n. The Fresnel equation gives us the angular dependence of F, it is only weakly dependent on the coefficient k. With the assumption $k = 0$ the Fresnel coefficient can be written as:

$$\begin{aligned} F &= \frac{1}{2}\frac{(g-c)^2}{(g+c)^2}\left(1+\frac{(c(c+g)-1)^2}{(c(c-g)-1)^2}\right) \qquad (7)\\ c &= cos\theta = \vec{V}\cdot\vec{H} \\ g^2 &= n^2+c^2-1 \end{aligned}$$

For a given angle, as example $\theta = 0$ we can quantify F and with it we can estimate n:

$$n = \frac{1+\sqrt{F_0}}{1-\sqrt{F_0}}$$

The second term in the calculation of the reflectance is the *geometrical attenuation factor*. With this factor the self shadowing and masking of the single facets is simulated. Self shadowing means that light coming to a facet can be blocked by other facets due to irregularities on the surface or simply shadowing of the neighbor facets. Masking is the same effect only that now the already reflected light from a facet is blocked by other facets. An example for masking and self-shadowing can be viewed in figure 2.2. The attenuation factor describing these effects is now defined as

$$\begin{aligned} G\left(\vec{N},\vec{V},\vec{L}\right) &= \min(1, G_{mask}, G_{shadow}) \qquad (8)\\ G_{mask} &= \frac{2\left(\vec{N}\cdot\vec{H}\right)\left(\vec{N}\cdot\vec{V}\right)}{\left(\vec{V}\cdot\vec{H}\right)} \\ G_{shadow} &= \frac{2\left(\vec{N}\cdot\vec{H}\right)\left(\vec{N}\cdot\vec{L}\right)}{\left(\vec{L}\cdot\vec{H}\right)} \end{aligned}$$

with $\vec{N}$ being the average surface normal, $\vec{V}$ the vector facing the viewer and $\vec{L}$ the vector facing the light from the current point.

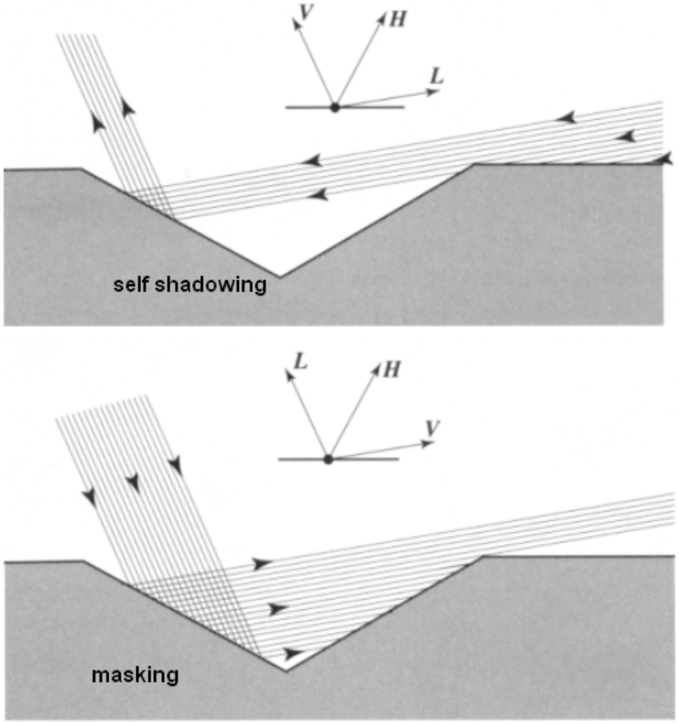

Figure 4: The two figures show situations with self shadowing and masking. [Wilkie 2007]

Now putting all the parts from equation 6, 7 and 8 together we get the following expression for specular reflectance when using the Cook-Torrance model:

$$f_r(\vec{L},\vec{V}) = \frac{F}{\pi}\frac{DG}{(\vec{N}\cdot\vec{L})(\vec{N}\cdot\vec{V})} \tag{9}$$

To get the complete BRDF we need a diffuse part, which is normally just a lambertian reflection. It needs to be combined with the Cook-Torrance model by simply adding them.

The big advantages of this model is the physical plausibility which creates excellent results. Besides that it has a generic approach, various parts of the model can be replaced according to the current requirements, like other Fresnel reflection terms, different slope distribution functions or other attenuation factors. On the other side it is quite difficult to obtain correct material constants, it is difficult to code an often not so easy to sample. Sample pictures of different materials can be seen at 10.

2.3 He-Sillion-Torrance-Greenberg

The model introduced by He, Torrance, Sillion and Greenberg in 1991 [He et al. 1991] is one of the most comprehensive BRDF models. It is able to handle previously ignored features like anisotropic surfaces, polarization or subsurface scattering. This rich set of features is very expensive in terms of computation time. In other papers this model is just called He or He-Torrance model.
The models splits the BRDF in three parts:

$$f_r = f_{r,sp} + f_{r,dd} + f_{r,ud} \tag{10}$$

with the specular $f_{r,sp}$, uniform-diffuse $f_{r,ud}$ and directional diffuse $f_{r,dd}$ reflection. The specular and directional diffuse reflection re-

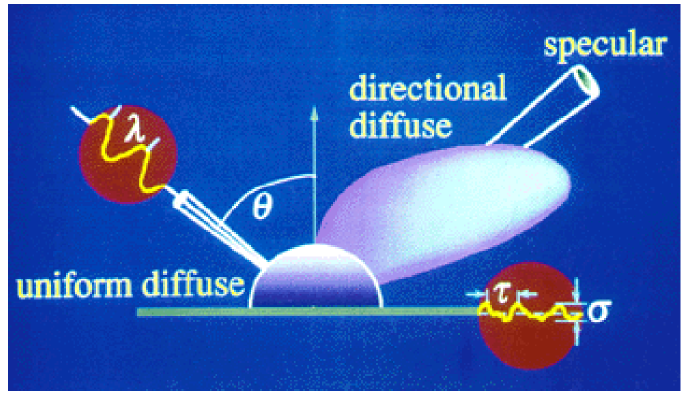

Figure 5: A visualization of the three different light reflection parts sp-dd-ud, the roughness parameter σ and the autocorrelation factor τ from the He reflection model. [Greenberg et al. 1997]

sult from an ordinary first surface reflection like in previous models. The third component is now responsible for multiple surface reflections and the subsurface scattering.
For the uniform diffuse component only a light wave dependent constant $a(\lambda)$ is defined. Values for $a()$ can be estimated either theoretical by using known V-grooves and subsurface scattering parameters. Or they can be calculated via a Genio-reflecto-meter to get the reflection response over a hemisphere. Without any known parameters the value can also be estimated as long as it is valid to the energy conservation law.
The specular reflection term is defined as:

$$f_{r,sp} = \frac{k_s}{\cos\theta_i d\omega_i}\cdot\Delta = \frac{|F|^2\cdot e^{-g}\cdot S}{\cos\theta_i d\omega_i}\cdot\Delta \tag{11}$$

with k_s being the specular reflectivity of the surface, $|F|^2$ is the Fresnel reflectivity depending on the index of refraction $n(\lambda)$. Δ is a masking function for a specular term, its value is one if the reflection is in the specular cone, otherwise it is zero.
The variables mentioned in the next parts can be viewed in figure 6.
g is a function for the effective surface roughness:

$$g = \left[\left(\frac{2\pi\sigma}{\lambda}\right)(\cos\theta_i + \cos\theta_r)\right]^2 \tag{12}$$

with σ being the mean difference of the surface height.
S is the shadow function defining which part of the surface is viewed an illuminated.

$$\begin{aligned} S &= S_i(\theta_i)\cdot S_r(\theta_r) \\ S_i(\theta_i) &= \frac{1-\frac{1}{2}erfc\left(\frac{\tau\cot\theta_i}{2\sigma_0}\right)}{\Lambda(\cot\theta_i)+1} \\ S_r(\theta_r) &= \frac{1-\frac{1}{2}erfc\left(\frac{\tau\cot\theta_r}{2\sigma_0}\right)}{\Lambda(\cot\theta_r)+1} \\ \Lambda(\cot\theta) &= \frac{1}{2}\left(\frac{2}{\pi^{1/2}}\cdot\frac{\sigma_0}{\tau\cot\theta} - erfc(\frac{\tau\cot\theta}{2\sigma_0})\right) \end{aligned} \tag{13}$$

The parameter σ_0 and τ are both material depended values for the mean difference from the surface and the correlation length coefficient.
On smooth surfaces the wavelength of incident light λ is relative large compared to the surface roughness the specular term is not attenuated because $g \to 0$ and $S \to 1$ because on smooth surfaces

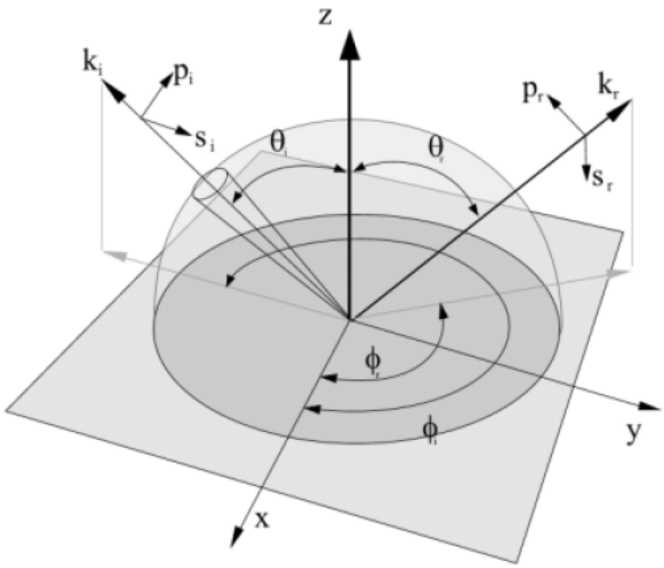

Figure 6: Input parameters for the He model. k_i is the incident light, k_r the reflected, p_i, s_i / p_r, s_r the polarization vectors for the incident/reflected light, z is the surface normal vector. [Gebhardt 2003]

there are not many shadows. So the term is reduced to the common from:

$$f_{r,sp,smooth} = \frac{|F|^2}{\cos\theta_i d\omega_i} \tag{14}$$

The last factor of equation 10 is the directional diffuse reflection. With a small wavelength compared to the size of the surface elements the first reflection introduces several diffraction and interference effects. It is again dependent on the roughness σ and the correlation τ, with a smoother surface the effect of the directional reflection is disappearing.

$$f_{r,dd} = \frac{F(\vec{n_b},\vec{n_b},p)\cdot S}{\cos\theta_i\cdot\cos\theta_r}\cdot\frac{\tau^2}{16\pi}\cdot\sum_{m=1}^{\infty}\frac{g^m e^{-g}}{m!\cdot m}\cdot\exp(-\frac{v_{xy}^2\tau^2}{4m}) \tag{15}$$

p is describing the polarization state vector of the incident light defined via the the polarizations coefficients c_s and c_p and the unit polarization vectors $\vec{s_i}$, $\vec{p_i}$ of the incident plane formed by $(\vec{k_i},z)$. The vectors are defined by

$$\begin{aligned} s_i &= \frac{k_i\times z}{|k_i\times z|} \\ p_i &= s_i\times k_i \\ p &= c_s s_i + c_p p_i \end{aligned}$$

F calculates the Fresnel coefficients and takes the polarization vectors and the unit vector $\vec{n_b}$ into account.

$$n_b = \frac{k_r - k_i}{|k_r - k_i|}$$

v_{xy} is a function depending on the incident light and reflection angle and describing the change of the wavelength with the factor k.

$$\begin{aligned} v_{xy} &= \sqrt{v_x^2 + v_y^2} \\ v &= k(k_r - k_i) \end{aligned}$$

With the He model it is possible to simulate many different materials and reflections properties, but it uses a lot of different equations what results in a very long computation time. A teapot rendered with the He model is shown in figure 11, cylinders with anisotropic reflectance is shown in figure 9.
Ward mentioned in his paper [Ward 1992] that this model has some lacks concerning energy conservation, because the ambient term is added without regard to the overall reflectance of the material. Also He did not make absolute BRDF measurements to compare them with his model, he only scaled existing data to match his function. Overall Ward states that He did not treat normalization adequately enough in his derivation.

2.4 Ward

The model from Ward [Ward 1992] can be uses specially for modeling anisotropic surfaces and reflections. One main goal of the model was to combine physical correctness and computation efficiency.
To realize his goal Ward needed a method to measure the reflectance properties of a surface easily, cheap and fast. He developed an imaging Gonioreflectometer with a simple CCD camera and a fish eye lens. This device was able to measure the reflectance of anisotropic surfaces much quicker and cheaper than other devices. From the measurements Ward extracted the reflection coefficients and surface roughness parameters.With these values Ward introduced now his isotropic elliptical Gaussian model, similar to other models like Torrance-Sparrow. But in his model he eliminated the Fresnel coefficient and the geometrical attenuation factor because they are rather difficult to integrate. He replaced them with a single normalization factor that insures the distribution integrates easily and predictable over the hemisphere. The isotropic reflectance is now defined as:

$$f_{r,iso} = \frac{k_d}{\pi} + k_s\frac{1}{\sqrt{\cos\theta_i\cos\theta_r}}\cdot\frac{e^{-\tan^2\delta/\alpha^2}}{4\pi\alpha^2} \tag{16}$$

k_d and k_s are the normal diffuse and specular surface coefficients, δ is the angle between the normal vector $\vec{N}$ and the halfway vector $\vec{H}$. α is the standard deviation of the surface slope. Values for k_s and k_d can vary on demand as long as $k_d + k_s < 1$. The normalization factor

$$\frac{1}{4\pi\alpha^2}$$

is accurate as long as α is not much greater than 0.2, when the surface becomes very diffuse.
The isotropic model is simple to extend to the anisotropic elliptical Gaussian model. It has two perpendicular, uncorrelated slope distributions α_x and α_y:

$$f_{r,aniso} = \frac{k_d}{\pi} + k_s\frac{1}{\sqrt{\cos\theta_i\cos\theta_r}}\cdot\frac{e^{-\tan^2(\cos^2\phi/\alpha_x^2+\sin^2\phi/\alpha_y^2)}}{4\pi\alpha_x\alpha_y} \tag{17}$$

The equation 17 can be approximated with unit vector from the surface plane which enables a much faster computation. The exact formulas can be viewed in Wards paper [Ward 1992].
Ward achieved its goal to create a model where all parameters have a physical meaning and they can be set independently. In addition its model can be evaluated fast and it can be used with Monte-Carlo sampling. A sample picture of a chair rendered with the Ward model can be viewedin figure 7.

2.5 Schlick

Schlick introduced a model which is like the Ward model a hybrid between empirical and theoretical models [Schlick 1994]. A goal of the model was to keep it simple and efficient, and just use a small set of parameters with physical relevance.
Schlicks model breaks with the common used method that the light is split into ambient, diffuse and specular reflection part which is combined via a linear combination. Schlick says that this combination is incorrect in the most cases because the diffuse and specular components are normally not constants but functions of the incident angle. Beside these *heterogeneous* materials there are a lot of homogeneous materials for which this distinction is completely unnecessary, like metals. So the distinction in diffuse and specular terms is never really satisfying.
Another criticized point is the geometrical attenuation factor which is normally just used as a multiplicative factor for the light that is not obstructed. The obstructed light itself does not disappear as the most models assume, it is reflected in other, overall random directions.
The last unsatisfactory point for Schlick is the fact that the most empirical BRDFs are not physically correct but fast to calculate. The physical models on the other hand are to detailed and are to complex compared with the errors in other parts of the rendering pipeline, so that this increased complexity is often useless because their advantages disappear in the errors of the other stages.

Schlicks solution is now the approximation of complex equations with a method he called *rational fraction approximation*. For terms like the Fresnel equation in 7 Schlick tries to find intrinsic characteristic kernel conditions. Conditions like a value at a given point of the function or of its derivatives or integrals. With this kernel conditions a fraction is formed that approximates the original function. With this method it is possible to minimize the Fresnel term to the simple form of

$$\begin{aligned} F_y(u) &= f_y + (1-f_y)(1-u)^5 \\ u &= \vec{V}\cdot\vec{L} \end{aligned} \tag{18}$$

which is optimized and has very similar conditions as the real function. The function is now only dependent on the vector u and the spectral distribution f_y. The refraction factor and the surface extinction coefficient were eliminated because they have to less influence on the function. This optimized formula can be computed up to 32 times faster with an error less then 1% compared to the orignial Fresnel formula.
Schlick uses a geometrical attenuation factor which is invariant by rotation around the normal vector and dependent from the surface roughness instead of the attenuation factors mentioned in the previous chapters.

$$\begin{aligned} G(v,v') &= G(v)G(v') \\ G(v) &= \frac{g}{g+1} \\ g &= \sqrt{h\pi}(2-erfc\sqrt{h}) \\ h &= \frac{v^2}{2m^2(1-v^2)} \end{aligned} \tag{19}$$

with v, v' being the incident and reflected angles and m as the root mean square slope of the microfacets. Normally $m \in [0, 0.5]$ for real surfaces. The approximation has now the following form:

$$\begin{aligned} G(v) &= \frac{v}{v-kv+k} \\ k &= \sqrt{\frac{2m^2}{\pi}} \end{aligned} \tag{20}$$

With a precomputed k and 1 - k the function only needs a few operations during runtime and is again much faster than the original version.
The last approximation is a different formulation for the slope distribution function. Schlick uses like Cook-Torrance in section 2.2 the Beckmann-distribution, as described in equation 6. Schlick managed to reduce the function to

$$\begin{aligned} D(t) &= \frac{m^3x}{t(mx^2-x^2+m^2)^2} \\ x &= t+m-1 \end{aligned} \tag{21}$$

with $t = \cos\alpha$.
If we put the parts from equation 19, 21 and 22 together we get a very fast, even in hardware realizable Cook-Torrance model. But Schlick has chosen a different way, instead a split up in diffuse, ambient and specular he divided the BRDF in heterogeneous and homogeneous materials. Homogeneous material consists of material with a single optical property and reflection scheme, objects like glas, or metal. Heterogeneous material has two layers of different properties, each of them are homogeneous materials. Normally the first layer is transparent and the second opaque. Popular examples are the human skin, plastic or painted/varnished surfaces.
Each material is characterize by a set of parameters. The first one is C_λ, the reflection factor at wavelength λ, it is similar to the factor used in 19. The second parameter is the roughness factor r with r = 0 as perfect specular and r = 1 as a perfect diffuse surface. r is releated to the RMS slope m of the surface in equation 22. The last parameter is the isotropy factor p, with perfect anisotropy at 0 and perfect isotropy at 1.
How we have two different BRDF functions, one for each material type.

$$f_{r,homo}(t,u,v,v',\omega) = S_\lambda(u)D(t,v,v',\omega) \tag{22}$$

$$f_{r,het}(t,u,v,v',\omega) = S_\lambda(u)D(t,v,v',\omega) + [1-S_\lambda(u)]\,S'_\lambda(u)D'(t,v,v',\omega) \tag{23}$$

Now it is possible to select different functions for the S_λ and D.
For the spectral factor S_λ it is possible to consider it as a constant function and set it to reflection factor C_λ. But its easier to replace it with the Fresnel approximation from 19.
The directional factor D has an extra term for each angle, expressed by two factors Z() and A().

$$\begin{aligned} D(t,v,v',\omega) &= \frac{1}{4\pi vv'}\cdot Z(t)A(\omega) \\ Z(t) &= \frac{1}{(1+rt^2-t^2)^2} \\ A(\omega) &= \sqrt{\frac{p}{p^2+p^2\omega^2+\omega^2}} \end{aligned} \tag{24}$$

With a r = 0 Z() is a constant function and simulating a lambert surface, with r = 1 it changes to a perfect specular. p acts similar for isotropy. If the geometric attenuation factor is needed it can be included:

$$D(t,u,v,v',\omega) = \frac{G(v)G(v')}{4\pi vv'}\cdot Z(t)A(\omega) + \frac{1-G(v)G(v')}{4\pi vv'} \tag{25}$$

The last term is responsible for the reemission of the self obstructed light at a random direction so the light is not absorbed but reflected. A third method for D is possible, because 25 has not a complete transition from perfect diffuse to perfect specular.

$$D(t,v,v',\omega) = \frac{a}{\pi} + \frac{b}{4\pi vv'}B(t,v,v',\omega) + \frac{c}{v'd\omega}\Delta \tag{26}$$

With the values a,b,c being calculated from the roughness factor r. f r ¡ 0.5 we have b = 4r(1-r); a = 0; c = 1 - b i or else b = 4r(1 - r);

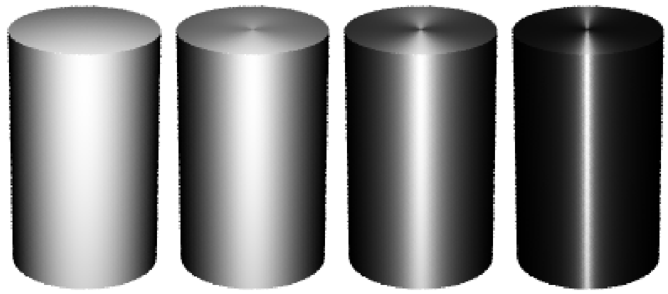

Figure 8: Schlick model: A continuum between isotropic and anisotropic reflection. r = 1.0, $p = \{1.0, 0.5, 0.2, 0.05\}$. [Schlick 1994]

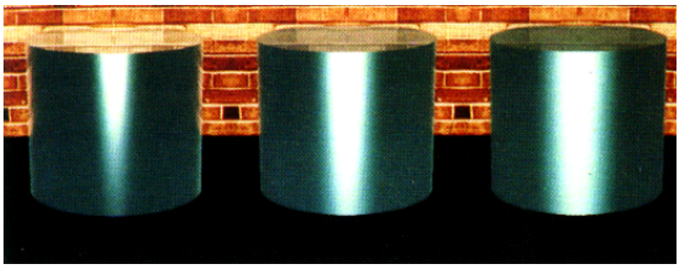

Figure 9: Aluminum cylinders with surface roughness $\sigma_0 = \{0.18, 0.28, 0.38\}\,\mu m, \tau = 3.0\mu m$. The nearly total reflection at a very flat angle can be seen especially good on the first object. [He et al. 1991]

c = 0; a = 1 - b. B() is a Beckmann like factor from equation 24 or 25, Δ is the Dirac function.
It is possible to use the Schlick model with Monte-Carlo rendering techniques and importance sampling. Overall this model is a very flexible approach, it offers many possibilities to add customized functions needed for actual scenes. A cylinder drawn with different stages of anisotropy is shown in figure 8.

3 Conclusion

This paper gave a short overview on the most important and popular surface models used for physical rendering. It started with the common definition of the surface function BRDF and its properties. It introduced the basic, and widely used empirical models and proceeded then to the more complicated physical models.
We looked at the somehow basic Cook-Torrance model from which many parts are used in other, later introduced models. Then we proceeded to the complex but comprehensive He model with its nearly perfect physical simulation. Then we learned two other approaches, one by Ward, were we relied on measurement data to gain the required parameters of our model. At the end we mentioned the Schlick model, a model created to be evaluated fast but still being very accurate. The last models show that BRDF does not need to be very complex in terms of calculation, some of them could even be implemented in hardware!
Beside these models there are many more, all with different properties and other fields of applications. In the next years they will also get more and more popular in consumer near field of applications enhancing the realism.

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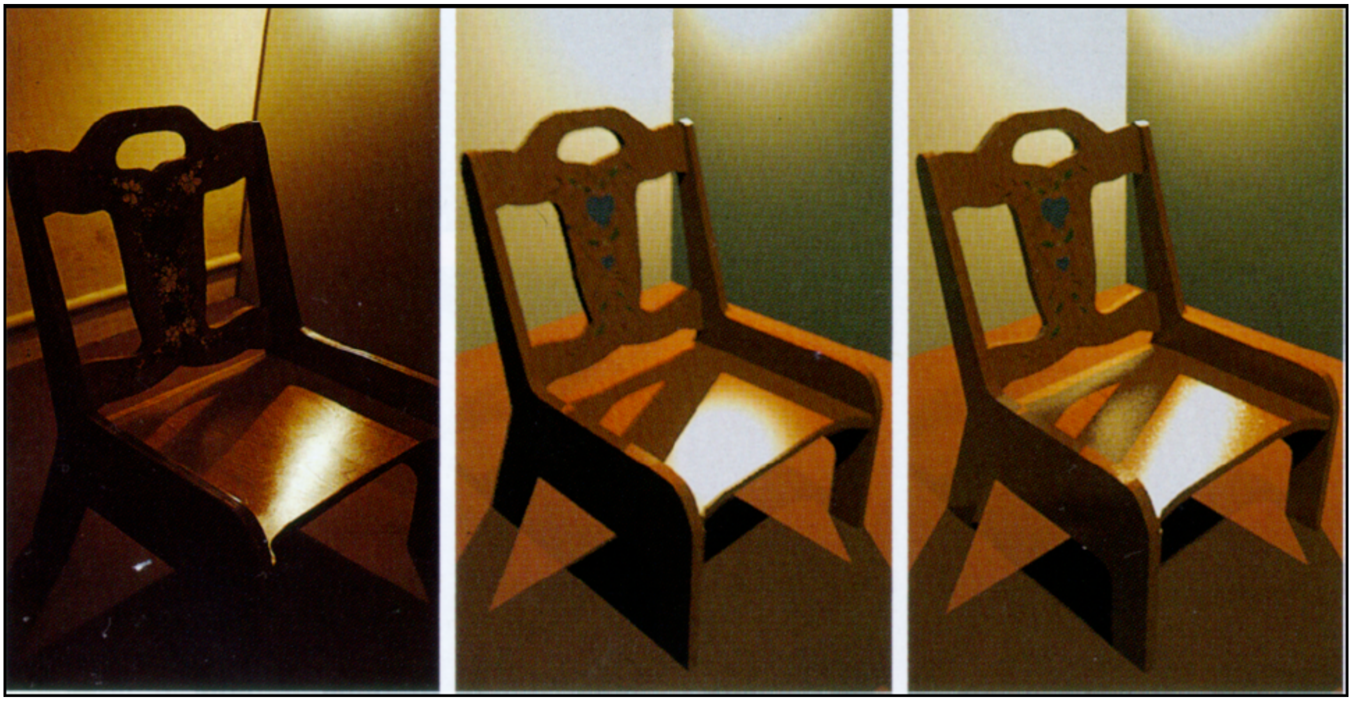

Figure 7: 1. picture: photograph of a chair with two lights (one above, one behind). 2. picture isotropic reflection. 3. picture anisotropic gaussian. [Ward 1992]

Figure 10: Different materials with reflection types shown on the same object. [Cook and Torrance 1982]

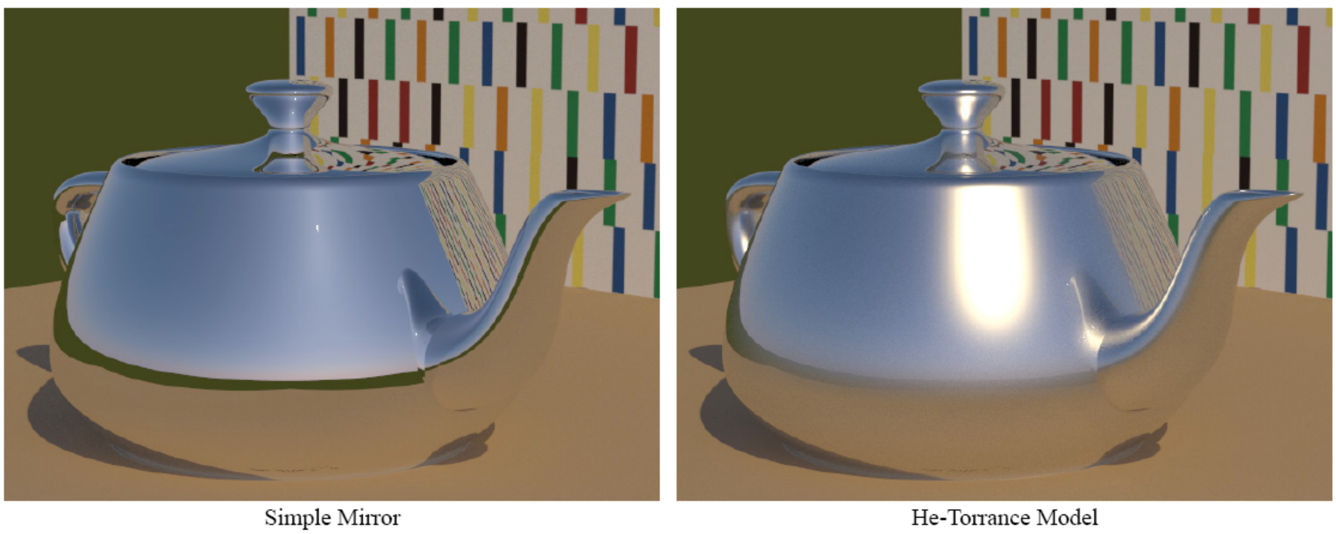

Figure 11: Rendering of a smooth metal teapot. The mirror like reflectance has only a very small highlight from the sun. The He-Torrance model shows the teapot rendered with the Fresnel reflectance. [Westin et al. 2004]