

Modelling of Daylight for Computer Graphics

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Abstract

This report represents the collected works of the authors on the modelling of skylight for computer graphics. The first part describes the physical processes of absorption and scattering and presents various atmospheric phenomena. The second part presents several important papers of the last decades. —ed/GZ.

Categories and Subject Descriptors (according to ACM CCS): I.3.7 [Computer Graphics]: Three-Dimensional Graphics and Realism

1. Introduction

The main objective of the first part of this work is to present the physical laws that are responsible for the blue color of the sky. We want to explain and describe them within this work to give an answer to the following question: how would physicists calculate the color of the sky? We also want to explain effects in the atmosphere related to the physical theory, which is responsible for the blue sky.

The second part of this work presents an overview of important papers on clear-sky models, including their geological dependency.

We present the physical processes because before we could understand or even optimize highly advanced methods to model the color of the sky, like [NDKY96, DYN02, PSS99], we need to understand the physical foundations of their models.

Our work ultimately lead us a long way back to the work of Lord Rayleigh [Ray99] about the color of the sky. His work laid the foundation for existing models, like [NDKY96, PSS99].

This work is structured as follows: section 2 covers the basic physical approach to the color of the sky. In Section 3 we will explain in detail Rayleigh's Law for scattering. Section 4 includes an introduction to Mie scattering,

which plays another major role for effects visible in the atmosphere. In Section 5 we will explain several effects of scattering in our atmosphere.

Starting with section 6, we describe options of atmospheric modelling for computer graphics and then give a detailed description of several important papers of the last two decades on the topic.

2. Physical Approach

From a Physicist's point of view the source of the color of the sky as a visible effect of scattering. Therefore scattering is defined as the deflection of fractions of energy of electromagnetic waves, which move through a medium. It occurs when the electromagnetic properties of that medium vary in time or position [Jac02].

In the case of the color of the sky the electromagnetic waves are the light emitted by the sun or other light sources (e.g. nearby cities). The scattering medium is the atmosphere of the earth, composed of small particles, e.g. air molecules, smoke, water or dust. Sunlight in this context means light directly emitted by the sun and not only the light that is perceived on earth's surface. [LL95]

Physics distinguishes between several cases of scattering of electromagnetic waves in matter. Scattering may occur *elastic* or *inelastic*. Elastic scattering means that the wavelength of the electromagnetic wave remains unchanged within the scattering process. In contrast, inelastic scattering means that the wavelength of the electromagnetic wave changes within the process, and we discern fluores-

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cence (immediate re-emission) and phosphorescence (slow re-emission). For considerations of scattering of electromagnetic waves passing through the atmosphere, almost only elastic scattering is important, with the exception of *air-glow*, a weak fluorescence of oxygen in the upper atmosphere. Elastic scattering is further distinguished [HR05, Jac02, ZK98] in:

Mie Scattering: Scattering of waves by spherical particles, where the size of the particle is near the wavelength of the electromagnetic wave. ($d \sim \lambda$) The distribution of the scattered waves has a strong forward component. For example, Mie scattering occurs when light passes through fog. [Jac02, Haf02]

Rayleigh Scattering: Scattering of waves by particles much smaller than the wavelength of the electromagnetic wave. ($d < \lambda/10$) Rayleigh scattering determines the color of the clear sky at any time of the day. The distribution of Rayleigh Scattering is shown in figure 1. [Jac02, Haf02]

Geometric Scattering: If particles are much bigger than the wavelength ($d \gg \lambda$), then scattering is treated as in scalar diffraction theory, which we will not cover in this work. [Haf02]

In this report we will focus on Rayleigh scattering, but we will also present Mie scattering, because it is similar to Rayleigh scattering in its calculation for the sky color. In addition to the phenomena related to elastic scattering, there are other phenomena that occur when sunlight passes through the atmosphere, e.g.: Green Flash, Alpenglow and atmospheric refraction. A good overview is given in [LL95, Min93].

3. Rayleigh Scattering

Rayleigh scattering is the scattering of electromagnetic waves by particles much smaller than their wavelength. This condition is given as

$$d < \frac{\lambda}{10} \quad (1)$$

Lord Rayleigh formulated this type of scattering on the base of the Maxwell equations. He tried to explain the blue color of the sky with knowledge of the Tyndall effect, which is the scattering of light on particles in a hazy medium (e.g.: fine smoke) [Ray99].

Rayleigh's Law was proposed in 1871 by Lord Rayleigh in his work about scattering on small particles. He explained the color of the sky in a work following 1899. Lord Rayleigh stated that the amount of scattered energy is proportional to the 4th power of wavelength (λ^4) of the electromagnetic wave passing through a medium. Rayleigh's Law is generally given as intensity I of the reflected (scattered) light as a function of the wavelength λ and the scattering angle θ . The

function is given [Ray71, Ray99, NSTN93] as:

$$I(\lambda, \theta) = \frac{I_0(\lambda) K \rho F_r(\theta)}{\lambda^4} \quad (2)$$

Here, I_0 is the intensity of the incident light. The scattering angle θ is argument of F_r , which is the scattering phase function. This function indicates the directional characteristics of the scattering and is given [NSTN93] as:

$$F_r(\theta) = \frac{3}{4} \cdot (1 + \cos^2(\theta)) \quad (3)$$

Further, in eq. (2), K is a factor for the standard atmosphere with the molecular density at sea level. K is given [Dit91, NSTN93] as:

$$K = \frac{2\pi^2(n^2 - 1)^2}{3N_s} \quad (4)$$

N_s is the molecular number density of the standard atmosphere and is given in molecules per unit volume in equation 5. The standard atmosphere is defined at a temperature of 288.15K and a pressure of 1013 mbar as conditions for the atmosphere at sea level, and then, according to [PSS99],

$$N_s = 2.545 \cdot 10^{25} m^{-3} \quad (5)$$

n is defined as the refractive index and is given as

$$n \simeq 1.000278 \quad (6)$$

for air in the visible spectrum. [Jac02] The refractive index is a ratio which specifies the slowdown of the phase velocity of an electromagnetic radiation compared to its velocity in vacuum.

ρ is the density ratio for the atmosphere. It is needed because the density of atmosphere changes with increasing height. ρ equals one at sea level and is given as a function of the altitude h and the scale height H_0 . ($H_0 = 7994m$) The scale height is the height of the atmosphere if its density was uniform and varies with the temperature in the atmosphere. ρ is given [NSTN93] as

$$\rho = \exp \frac{-h}{H_0} \quad (7)$$

Remarkable for the scattering phenomena is the factor $1/\lambda^4$ of equation 2. As we stated before, sunlight consists of light of nearly all wavelengths and all polarization planes. Only some minor parts, which are absorbed by the solar atmosphere itself are missing. These parts can be seen in the spectrum of sunlight as missing lines, which are called Fraunhofer lines. [ST94]

The light scattered by Rayleigh scattering is distributed symmetrically, but not isotropically in all directions. This behavior comes from the angular distribution of the scattering, which is given by $1 + \cos^2(\theta)$ and looks as depicted in like figure 1. The scattered light is polarized with a maximum at 90° and no polarization at 0° or 180° [LL95].

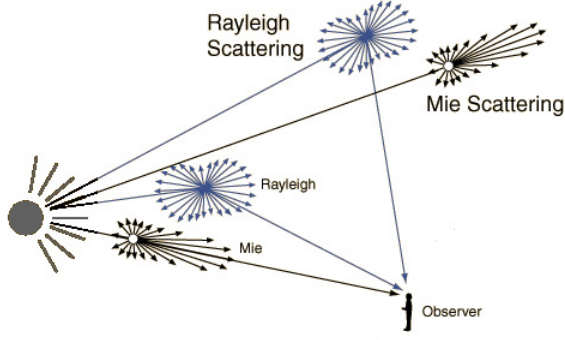


Figure 1: Scattering of sunlight in all directions by Rayleigh and Mie-scattering [Nav06]

color	λ	β	β^{-1}
red	700nm	$4.184 \cdot 10^{-6} m^{-1}$	239km
yellow	570nm	$9.517 \cdot 10^{-6} m^{-1}$	105km
green	540nm	$1.181 \cdot 10^{-5} m^{-1}$	84.6km
blue	470nm	$2.059 \cdot 10^{-5} m^{-1}$	48.5km
violet	400nm	$3.924 \cdot 10^{-5} m^{-1}$	25.5km

Table 1: Values of the attenuation coefficient β and its reciprocal value for different colors of the spectrum (calculated with values for N_s and n of equation 5 and 6)

3.1. Attenuation of direct illumination

To express the light loss by scattering, an attenuation coefficient β is given as

$$\beta = \frac{8\pi^3(n^2 - 1)^2}{3N_s\lambda^4} = \frac{4\pi K}{\lambda^4} \quad (8)$$

This coefficient is a quantity which describes the reduction of an electromagnetic wave when it passes through a medium. A small attenuation coefficient indicates that the medium is relatively transparent. A higher coefficient indicates higher degrees of opacity. The reciprocal value of the coefficient of scattering is called the medium distance an electromagnetic wave travels to be reduced in the ratio $e : 1$ [Ray99]. Some examples for β and its reciprocal value are given in table 1 [Dit91, NSTN93].

The effects of Rayleigh scattering can be seen for example as the blue color of the sky. All other effects that are related to Rayleigh scattering are introduced in section 5.

4. Mie Scattering

Mie scattering is scattering of an electromagnetic wave on a particle, where the size of the particles is nearly equal to the wavelength of the electromagnetic wave [Min93]. This

condition is given as

$$d \sim \lambda. \quad (9)$$

The main difference between Rayleigh and Mie scattering is the strong forward component of Mie scattering, which can be seen in figure 1. This difference is expressed in a different scattering function, shown as

$$F_m(\theta, g) = \frac{(1 - g^2)}{(1 + g^2 - 2g \cos(\theta))^{3/2}} \quad (10)$$

or

$$F_m(\theta, g) = \frac{3(1 - g^2)}{2(2 + g^2)} \frac{(1 + \cos^2 \theta)}{(1 + g^2 - 2g \cos(\theta))^{3/2}} \quad (11)$$

Equation (10) was an approximation by Henyey and Greenstein [HG41], equation (11) is a more recent refinement by Cornette and Shanks [CS92]. The variable g stands for the asymmetry factor and depends on particle size and wavelength. The diagram of scattering as shown as part of figure 1 strongly differs between particle type. [NDKY96].

If g in equation (11) equals zero, then the function is similar to Rayleigh scattering. g is defined in the interval $[-1; +1]$, -1 means total backscattering and $+1$ means total forward scattering [Hil05]. Mie scattering depends like Rayleigh scattering on the density of the atmosphere, shown in equation 7, but with a scale height $H_0 = 1.200m$, because the distribution of larger particles in the atmosphere is different [NDKY96].

For sky color calculation Mie scattering has its strongest influence on sunlight which comes nearly direct to the observer from the sun shown in figure 1 because of its angular distribution. As introduced by Nishita *et al.* [NDKY96], the calculation of intensity and optical depth remain the same. They also demonstrate its application on the calculation of clouds, which follows the same physical principles, where they sum up scattering of different particle size [NDN96]. Effects of Mie scattering can be seen for example as aureole around the sun (section 5.4). Mie scattering is strongest when a lot of particles are in the atmosphere. Other effects involving Mie scattering are introduced in section 5.

5. Effects of Scattering in the atmosphere

Effects of scattering in the atmosphere are often a combination of Mie and Rayleigh scattering. Both types of scattering depend on the existence of particles of different sizes in the atmosphere. If for example the atmosphere is very clear, then no aureole around the sun is visible (section 5.4). Some effects only occur when there exists recent strong volcanic activity, like the effect of the blue moon and blue sun (section 5.6).

5.1. The color of the sky

The color of the sky is determined by the amount of light scattered from the sunlight and the composition of the at-

mosphere. If there was no scattering because no atmosphere existed, the sky would appear black like in space. If more small particles (e.g. dust) are in the atmosphere, then the color becomes dimmer. This composition of particles may lead to some effects like a blue sun, which we will explain later. During the day more violet and blue than other light is scattered out of the sunlight and is then scattered along the whole sky sphere. Because blue and violet is scattered out of the sunlight, the sun appears in a light yellow color. This produces the blue shades of the sky during daytime. During day only single scattering is important, because the distance sunlight travels through the atmosphere is not as big as at sundown. The minimum distance is reached around midday when the sun is near the zenith and grows till sundown to a much greater distance. At sundown much more light is scattered out of the sunlight, because the distance sunlight has to cover in atmosphere increased. So more sunlight is scattered and only a yellow or red sunlight reaches the eyes of the observer. Whether the sun appears yellow or red mainly depends on the existence of particles smaller than $100nm$ in the atmosphere. They scatter the light stronger than air molecules. The more such particles exist the more redder appears the sun [Jac02, FBS90, LL95].

Table 1 shows values of the attenuation coefficient β for different colors of the spectrum. The reciprocal value of β equals the distance the light travels until it is reduced with the ratio $e : 1$. We see that violet has the lowest wavelengths in the visible spectrum, followed by blue. So the highest value of attenuation in the visible spectrum occurs at violet. This means that the sky during day should appear violet instead of blue. This is correct, but the sky is perceived blue, because the human eye perceives blue much better than violet [LL95]. Good overviews over the properties of the human eye are given in [Pér96, FBS90, Ped98].

Figure 2 illustrates the typical colors during sundown. A way to compute the intensity of the colors is suggested by Nishita *et al.* [NDKY96]. We will present their approach in Section 7.5.

Beside the color of the sky, the horizon gains a white color during daytime. That is another effect related to Rayleigh scattering. Contrary to the sunlight near zenith where single scattering predominates and the atmosphere is relatively thin, the light near the horizon is multiply scattered and the atmosphere is relative thick. Multiple scattering is the main reason for the brighter blue color of the sky above the horizon.

The thickness of a medium is described by the optical depth between two points p_0 and p_1 . The optical depth is given as

$$\tau(S, \lambda) = \int_{p_0}^{p_1} \beta(s) \rho(s) ds = \frac{4\pi K}{\lambda^4} \int_{p_0}^{p_1} \rho(s) ds \quad (12)$$

The optical depth τ gives a measure of how opaque a medium is to radiation passing through it. If $\tau < 1$ the



Figure 2: Color of the sky at sundown (photo: T. Kment)

medium is called thin, e.g. glass for visible wavelengths. When $\tau > 1$ the medium is called opaque and absorbs great amounts of radiation passing through it. If $\tau = 1$ then $\approx 37\%$ ($= 1/e$) of the radiation passing through it is not absorbed. This gradient from thin to opaque can be seen during daytime on the sky as the color of dark blue from the top of the sky grows paler and reaches white at the horizon. [LL95] Another way to explain the optical depth is the sum of scatterings an electromagnetic wave has to face on its way to its destination.

5.2. Twilight

Twilight is called the effect shortly after sundown and shortly before sunrise. It occurs because the atmosphere multiply scatters the sunlight from behind the horizon. This scattering can be seen as twilight arch, as yellow or red arch at the horizon which can be seen before sunrise and after sundown. Also violet and green shades can be observed. Science differentiates between nautical and astronomical twilight. Nautical twilights ends at a solar altitude of 12° below horizon, astronomical twilight ends at 18° below horizon. The end of astronomical twilight also marks the begin of night. The twilight following sundown is called dusk and the twilight before sunrise is called dawn. Volcanic activity has an influence on the appearance of twilight. [LL95]

5.3. Airlight (Aerial Perspective)

Airlight (also called *Aerial Perspective*) is called the effect of the bluish tint an observer can see, when she looks at mountains. More distant mountains appear in a bluer and brighter tint than mountains close to the observer (Figure 3). The blue tint is called airlight and is sunlight scattered between the observer and the mountain. This effect depends mainly on the distance between the observer and the mountains. The higher the distance the higher is the amount of light from the moun-



Figure 3: Visible Airlight [Sch05]

tains which is scattered out the line of sight by Rayleigh scattering and is replaced by airlight [LL95, Min93].

The effects of Fog, clouds of dust and haze may be confused with airlight. Their main difference to airlight is their high amount of Mie-Scattering, especially on a line of sight towards the sun. [LL95] But instead of blue the effects caused by haze appears gray, white or brown.

5.4. Aureole

The aureole or corona can be seen as bright glare around the sun. It is caused by atmospheric particles, like dust, smoke, pollen or even insects. Each type of particle produces a different effect, but they all are related to Mie scattering. Rayleigh scattering is negligible for this effect, because the strong forward component of Mie scattering, as illustrated in figure 1, is dominating [LL95].

5.5. Bishop's Ring

This effect can be seen as a bluish ring of light with pale narrow red borders around the sun. It is difficult to observe and was first reported after the eruption of the Krakatao in 1883. The effect is related to scattering of dust in the stratosphere, which originates from volcanic eruptions. It is different to an aureole because the size of the particles is restricted around $1\mu\text{m}$, and therefore not all light is scattered. [LL95]

5.6. Blue Sun and Blue Moon

This effect is also related like the Bishop's Ring to volcanic activity. It was also first reported after the eruption of the Krakatoa in 1883. The reason for this effect was the ash of Krakatoa which was blown into the stratosphere. Major parts of these ash clouds consisted of particles of the size of $0.7\mu\text{m}$. Scattering on this particle size scatters out most of the red of the sunlight. The effect only occurs when only this size of particles is present. When the effect occurs, the sun has a lavender or a blue tone and the moon occurs in a blue



Figure 4: Blue moon [If04]

color (Fig. 4). Beside big volcanic eruptions this effect also occurs, when great forest fires may produce particles with the critical size of $0.7\mu\text{m}$. [Phi04]

5.7. Effects of Polarization

Scattered Sunlight is strongly polarized in rings around the sun/anti-sun axis. Near the sun the light is weakly polarized. The strongest polarization occurs at 90° from the sun. This polarization is important in nature. For example, Bees know where the sun stands, even if the sun is covered by thick clouds. The Austrian scientist Karl von Frisch found out that bees are able to recognize the pattern of the sun polarization in the sky in the near ultraviolet spectrum and to use it for navigation. They use this knowledge to tell other Bees where rich sources of food are. For communication they use a movement pattern called waggle dance. With this movement pattern they describe the bearing and the distance of the food source taking into account the position of the sun. Those sensors which are responsible for this recognition are located in the dorsal rim of the eyes of the insect and face upwards [Pye01].

Pye also suggests that the Vikings may have used polarization for navigation. He states that at the time the Vikings colonized Greenland the compass was unknown in Europe. His speculation and interpretation of Viking lore lead him to the assumption that the Vikings may have used a device which they called sunstone. This device could be explained as a short tube with a crystal in it. The crystal has special crystalline properties and works like a polarization filter. As working example, Pye built such a device with a rough, unpolished cordierite crystal and states that it worked as an excellent polariscope for observing sky polarization. It works when cloud cover is not too thick and dark. He says that the sunstone may only be a legend and that the Vikings may have had no navigation device like the sunstone, but he also says that it could be possible [Pye01].

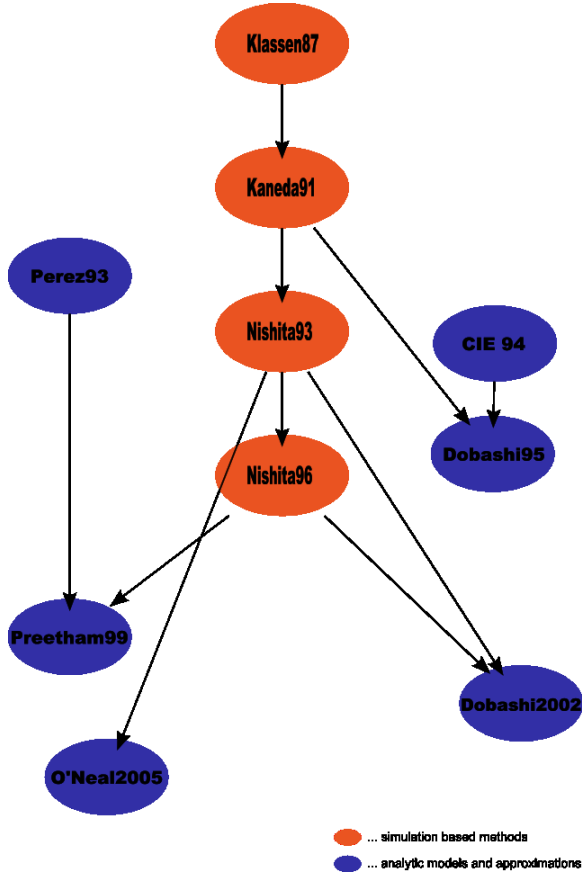


Figure 5: Chronology, genealogy and dependencies of papers investigated in this field of research.

6. Modelling Daylight in Computer Graphics

In this and the following section we want to take a closer look at scientific work about modelling of the daylight in computer graphics. There are two major approaches that need to be distinguished when talking about the modelling of daylight in graphics simulation:

- Simulation Based Methods
- Analytic Models and Approximations

Figure 5 shows an overview of the papers investigated and their respective dependencies in this article.

6.1. Simulation Based Methods

General atmosphere equations as presented in sections 3 and 4 are the foundation for all simulation methods. Variations arise when different models for atmosphere density, scattering coefficients, etc. are used. Computational costs are usually high for simulation based methods.

We will discuss work done by Klassen [Kla87] who uses

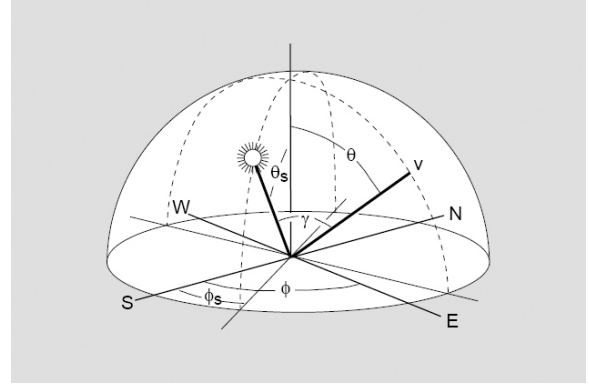


Figure 6: Angles and directions on the sky dome (image from [PSS99])

a simple constant density atmosphere model on a flat earth surface (section 7.1).

Kaneda *et al.* used a model with a spherical earth and an exponential decay density distribution [KONN91] (Section 7.2). Nishita *et al.* simulate atmospheric scattering to display earth and atmosphere from space [NSTN93] (Section 7.3). Later, he also takes higher order of scattering into account and proposes a fast method for single scattering computations [NDKY96] (Section 7.5).

6.2. Analytic Models and Approximations

These models are widely used in interactive computer graphics and other performance critical applications. These models represent simulated data or real world atmospheric data acquired from measurements.

The CIE Overcast sky luminance model [CIE94] introduces a model for overcast sky luminance:

$$Y_{OC} = Y_z \frac{1 + 2 \cos \theta}{3}, \quad (13)$$

where θ is the angle from the zenith and Y_z is the zenith luminance for overcast skies and can be obtained from analytic formulas adopted by [CIE94].

The CIE Clear sky luminance model [CIE94] is given by:

$$Y_c = Y_z \frac{(0.91 + 10e^{-3\gamma} + 0.45 \cos^2 \gamma)(1 - e^{-0.32/\cos \theta})}{(0.91 + 10e^{-3\theta_s} + 0.45 \cos^2 \theta_s)(1 - e^{-0.32})}, \quad (14)$$

where Y_z is again the zenith luminance and the angles are defined as in Figure 6.

The ASRC-CIE model uses a linear combination of four luminance models - the standard CIE cloudless sky, a high turbidity formulation of the latter, a realistic formulation for intermediate skies proposed by Nakamura and the standard CIE overcast sky. See [Lit94] for more details.

An all-weather sky luminance model was proposed by Perez in [PSI93]. Based on 5 different parameters (darkening or brightening of the horizon, luminance gradient near the horizon, relative intensity of the circumsolar region, width of the circumsolar region and relative backscattered light) this model has been found to be more accurate than the CIE model.

Preetham proposed an analytical model for spectral radiance of sky [PSS99]. Through a turbidity parameter this model is able to model sky conditions from clear to overcast sky. Preetham also takes aerial perspective (section 5.3) into account in his model, which is described in section 7.6.

7. The scientific papers in detail

Now we present important scientific work done in this field of research in more detail.

If you have a working model for calculating color values and color changes during daytime you need to consider how to implement this model in software and/or hardware. The presented papers implement their models in either software or hardware.

7.1. Modelling the Effect of the Atmosphere on Light

In this seminal paper written by R. Victor Klassen [Kla87], a lighting model for sky color that takes into account effects of scattering by suspended particles is presented. This model is suited for sky color and sun color calculations independent of the sun's position that might be any position above the horizon. Further the model supports fog calculation under general lighting conditions.

Klassen presents the basic idea of scattering by modelling the effect as the deflection defined by two angles θ and φ and gives an overview of scattering models starting with the Rayleigh model (see section 3, [Ray71] or [McC76] for details) for small particles, the Mie model (see section 4 or [Mie08]) for spheres and a model by Blinn [Bli82] that provides another approximation for the angular scattering function.

Klassen introduces the idea of a haze-free and haze-filled air and vacuum. He distinguishes between different cases. Both haze-free and haze-filled air may occur (this is the general case). Only haze-filled air may occur for sources within the haze-filled air. If haze and fog are absent only haze-free air will affect calculations. If neither haze-free nor haze-filled air exist the problem reduces to ray tracing in a vacuum. Klassen presents the equations for modelling of light transfer and scattering.

Klassen then investigates sky and sun color. He defines the color as the result of the initial color reduced by the scattering along the light path. Extinction and refraction is investigated. He points to the fact that refraction has a significant

impact only if the sun is near the horizon and bending is even then only a few degrees.

For sky color calculation Klassen simplifies the model by defining a constant color for the sky instead of taking into account light scattered into the line of sight. Only the calculation of color being scattered by haze particles along the line of sight needs to be performed.

Klassen shows how computation of sun and sky colors are achieved with the model. Eventually some performance considerations are done and improvements to reduce computational costs are presented. Klassen also considers lighting in presence of fog and scattering due to fog.

7.2. Photorealistic image synthesis for outdoor scenery under various atmospheric conditions

Kaneda's goal in this work [KONN91] is the displaying of photorealistic images of outdoor scenes by simulating atmospheric scattering under various weather conditions, although after reviewing the results (Fig. 7), the reader might be disappointed of the quality of the results of his simulation. Nevertheless it is an important paper often referred to in related scientific work.

Both Rayleigh and Mie scattering are integrated in Kaneda's model. The molecular density in the model given as an equation dependent on the altitude above the sea level and a constant scale height (see [KONN91] for more details) decreases exponentially with the altitude. Further a fog distribution model is also presented, and Kaneda presents the calculation and rendering of beam and fog effects in his model.

Kaneda also investigates how 3D objects lit by the sun can be rendered taking into account specular reflection.

7.3. Display of The Earth taking into Account Atmospheric Scattering

This paper [NSTN93] - written by the Japanese research group around Tomoyuki Nishita - presents a model for views of the earth from outer space (Figures 8 and 9). It is essential that the model presented in this paper is limited to a viewer's position in outer space and not on earth ground.

The algorithm proposed efficiently calculates optical length and sky light. For the calculation lookup tables are used and further simplifications are made like assuming the earth as a sphere and sun light to be parallel.

A distinction is made between calculation of a spectrum of the earth viewed through the atmosphere, calculation of a spectrum on the surface of the sea taking into account radiative transfer of water molecules as well as calculation of a spectrum of the atmosphere taking into account absorption and scattering.

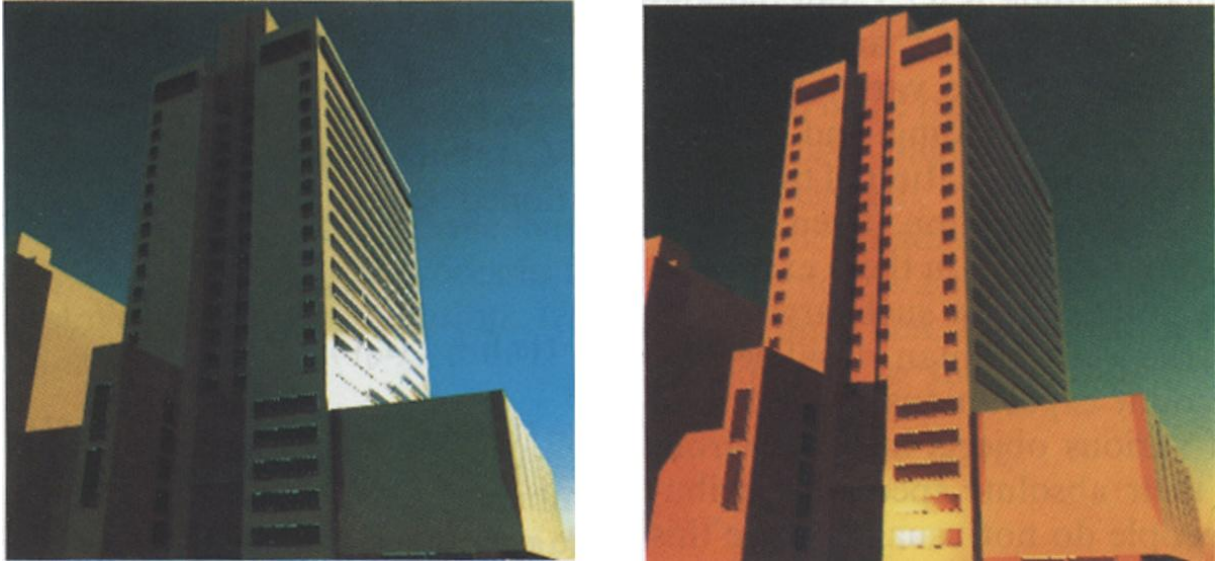


Figure 7: Results from [KONN91]. Note the greenish sunset sky. (Section 7.2)

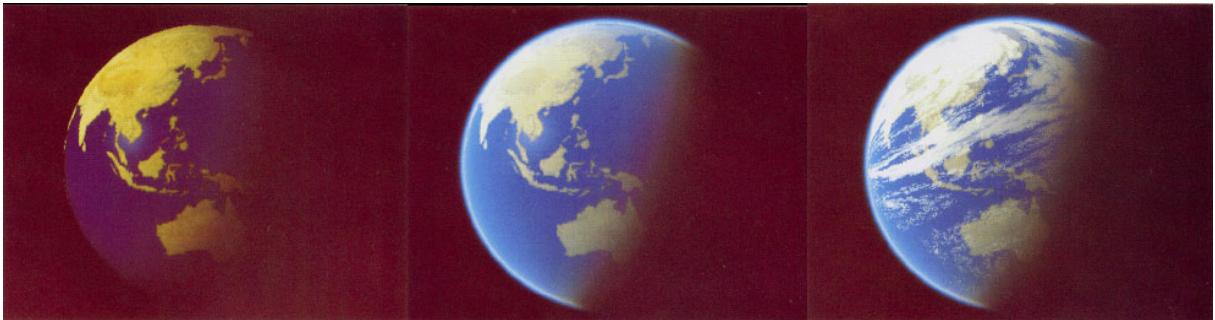


Figure 8: Results of the algorithm for display of the earth viewed from outer space. (left) earth viewed from outer space - the color of the sea, direct sunlight and skylight are taken into account. (center) the second image adds the color of the atmosphere by taking into account atmospheric scattering and absorption, (right) in the third image clouds are added. (images from [NSTN93]; see section 7.3)

Nishita and his fellow researchers need geometric models of the earth, atmosphere, sea, clouds and the spectrum of the sunlight for their algorithm. They describe where different types of color occur in their model as color of the atmosphere, color of the earth's surface, color of the sea and cloud colors.

Their approach is an extension of the model supposed by Klassen [Kla87] that is not limited to rendering sky color viewed from a point on the earth but also from a point in outer space.

Furthermore Nishita assumes some simplifications:

- only single scattering (multiple light scattering between air molecules and aerosols ignored)

- interreflections of light between earth's surface and particles in the air neglected
- it is assumed that light travels in a straight line (actual path is curved)

Nishita takes into account Rayleigh scattering for atmospheric scattering, he presents intensity calculation due to atmospheric scattering for spectrum calculation for only the atmosphere as well as for spectrum calculation for the earth. The color of clouds is investigated and also the calculation of the color of the sea is presented.

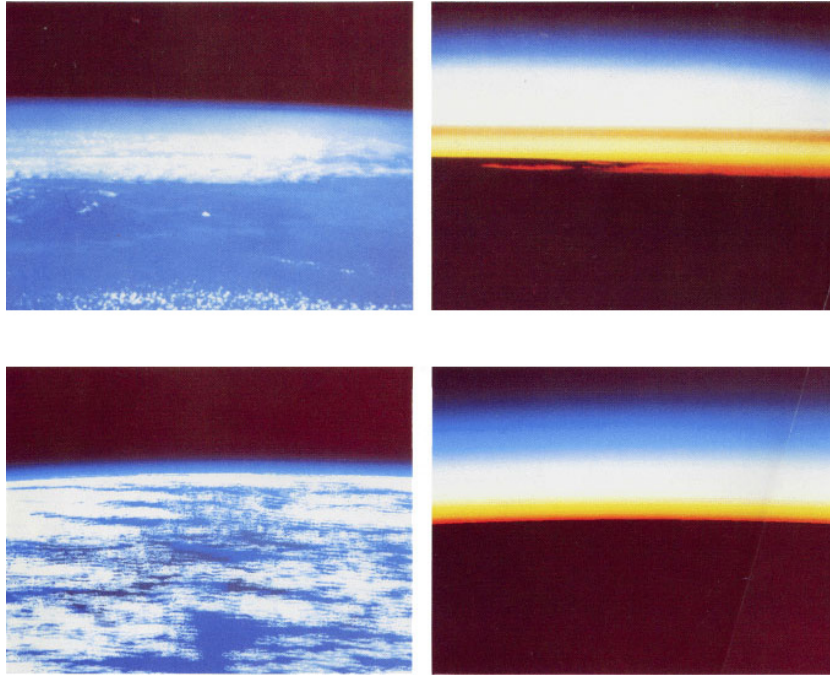


Figure 9: Real photographs of the earth viewed from outer space (top) and results of the algorithm for the same scene (bottom; images used are from [NSTN93]; see section 7.3)

7.4. A Fast Display Method of Sky Color Using Basis Functions

This work [DNKY95] was done by Yoshinori Dobashi, Tomoyuki Nishita, Kazufumi Kaneda and Hideo Yamashita. The topic of investigation is the research of modelling sky color by expressing the intensity distribution of the sky using basis functions even if the sun or camera position is altered. This approach achieves better performance than earlier approaches, but yields similar results as [KONN91]. Dobashi uses cosine functions as basis functions.

The distributions of the sky color for each sun altitude is precalculated by using the basis functions at certain intervals whenever the sun altitude is altered. By using these precalculated distributions it is possible to obtain the color of the sky in the view direction of an arbitrary sun position and display it quickly.

Dobashi considers the sky as a hemisphere with a large radius with the viewpoint as the center of this hemisphere. A view direction can be expressed with an angle from the x axis and a rotation (azimuth angle) and an angle from xy plane (elevation angle). Allowing this angle to take any possible direction together with the sun altitude this spans the intensity distribution for the sky.

This intensity distribution can be considered as a weighted sum of several basis functions. The weights can be calculated as a function of the sun's altitude. By summing the

basis functions with the calculated weights derived from the altitude of the sun the intensity distribution of the sky can be obtained quickly.

As soon as weights for all specific altitudes are calculated you only need to store the basis functions once.

To reduce computational costs and memory consumption it is advised that:

- the intensity distribution of the sky should be expressed with the smallest possible number of basis functions.
- Further any intensity distribution of the sky should be expressed with a linear combination of basis functions.

The intensity distribution of the sky is precalculated for the altitude of the sun at a certain interval. Dobashi presents different ways to partition the hemisphere into intervals, these are:

1. The hemispherical dome is divided into grids. Linear interpolation is used to calculate color values. This method is equivalent to using delta functions as basis functions. (see Figure 10 for more details)
2. Spherical harmonic functions can be used as basis functions with precalculated weights for each spherical harmonic function.
3. The sky dome is divided into slices only in the direction of f (see Figure 11). In the q direction, the intensity distribution of the sky is expressed by cosine functions.

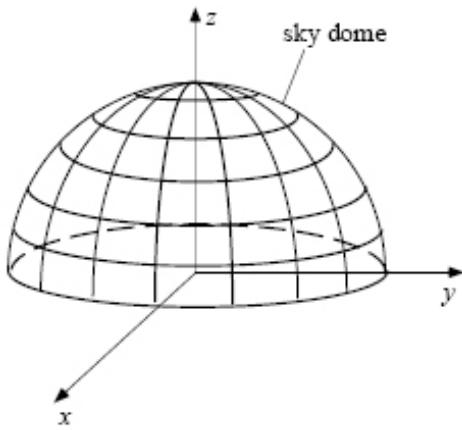


Figure 10: The sky dome divided into grids. (from [DNKY95])

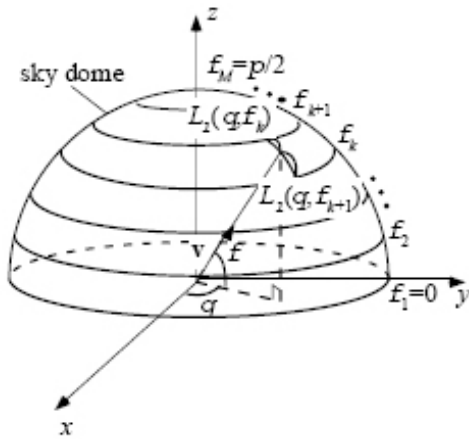


Figure 11: The sky dome divided in the direction of f . (from [DNKY95])

The first method is memory intensive the second method takes disproportional calculation time. Dobashi proposes the third method that contains advantages of both the first and second method. He also proposes to divide the sky dome in the q direction instead of the f direction. The intensity distribution in each divided area can then be expressed by a Fourier series.

Furthermore Dobashi presents an algorithm for determining the minimum number of cosine functions. Error evaluation is performed (see Figure 13) as well as experimental results are presented like the relation between the number of cosine functions and the altitude of the sun (see Figure 12).

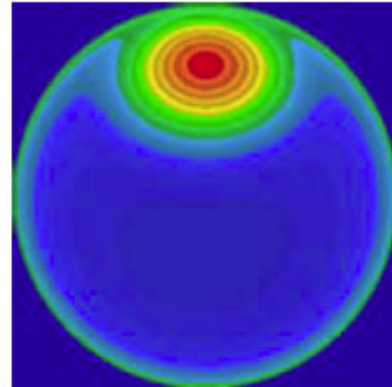


Figure 12: The intensity distribution of the sky. (from [DNKY95])

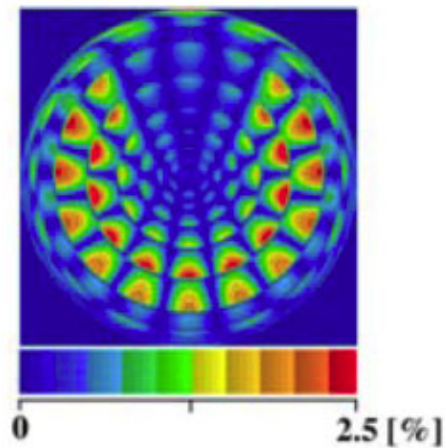


Figure 13: Relative error distribution. (from [DNKY95])

7.5. Display Method of the Sky Color Taking into Account Multiple Scattering

Nishita *et al.* [NDKY96] proposed a model to display sky color taking into account multiple scattering. Their model implements Rayleigh-Scattering. It also covers Mie-scattering and multiple scattering.

The model ignores scattering by clouds. Scattering by the ozone layer in the visible spectrum is negligible and also ignored in their model. Figure 15 shows the main optical paths for scattering if a viewer is looking from point P to point P_v . The paths are:

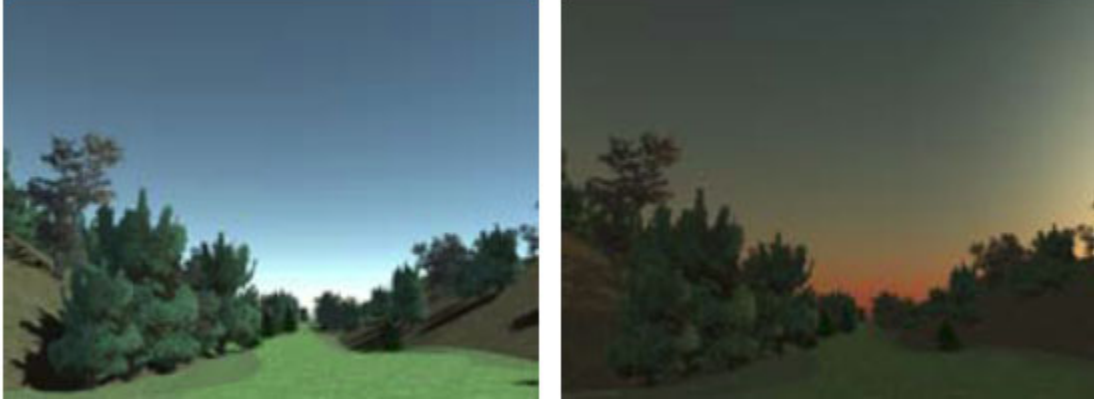


Figure 14: examples of a natural scene with different altitudes of the sun [DNKY95] (section 7.4)

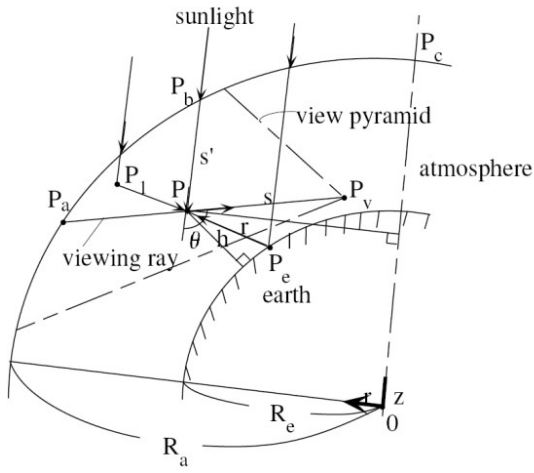


Figure 15: Optical paths for calculation of sky color [NDKY96]

1. Sunlight travels through the atmosphere and arrives at P after traveling on the path P_b, P
2. Light arrives at point P after being multiply scattered in the atmosphere like on the path P_1, P
3. Sunlight travels through the atmosphere and arrives at P after being reflected of earth's ground P_e, P

Nishita *et al.* [NDKY96] see the sky as *sky sphere* and assume the calculation point P as its center. Their sky sphere is divided and approximated by several directions, with the aim of aligning all calculation points. This alignment reduces the calculation effort.

For the calculation of single scattering the attenuated intensity of light arriving at point P has to be computed. The attenuation is determined by the optical depth between the

atmosphere (P_b in Figure 15) and point P . This dependence comes from Rayleigh's law, shown in equation 2.

The effort for this calculation is very high, because optical depth for every sample point on the viewing ray has to be calculated. This effort is reduced by using geometrical properties of the earth and the distribution of the density of the atmosphere. Therefore Nishita *et al.* [NDKY96] placed a cylindrical coordinate system with its axis passing through the center of earth and parallel to sunlight. They introduced two variables, r for the radius and z for the distance from the center of earth. The distribution of density is symmetrical within this cylindrical space and axis symmetrical to the sunlight with the axis O, P_c . This coordinate system is also shown in figure 15.

For example if the intensity at the viewpoint P_v is calculated as the integration of the intensity of all light scattered along the path from point P_a to P_v . This is shown in figure 15 and given by equation 15 and 16, where I_s is the intensity of sunlight, s the variable for the integral, s' the distance between point P and the top of the atmosphere. $K(\lambda)$ is given by equation 4, the scattering function $F_r(\theta)$ by equation 3 and the optical depth $\tau(s, \lambda)$ by equation 12.

Nishita *et al.* calculated Mie and Rayleigh-scattering in the same step, which is given in equation 15 by R , which is given in equation 16, where the index r stands for Rayleigh-scattering and the index m for Mie-scattering.

$$I_v(\lambda) = \int_{P_v}^{P_a} I_s(\lambda) R(\lambda, s, \theta) \exp(-\tau(s, \lambda) - \tau(s', \lambda)) ds \quad (15)$$

$$R(\lambda, s, \theta) = K_r(\lambda) \rho_r(s) F_r(\theta) + K_m(\lambda) \rho_m(s) F_m(\theta) \quad (16)$$

Nishita *et al.* [NDKY96] pointed out that equation 15 cannot be solved analytically and should therefore be discretized and calculated by numerical integration methods. They also introduced a way to calculate multiple scattering and a way

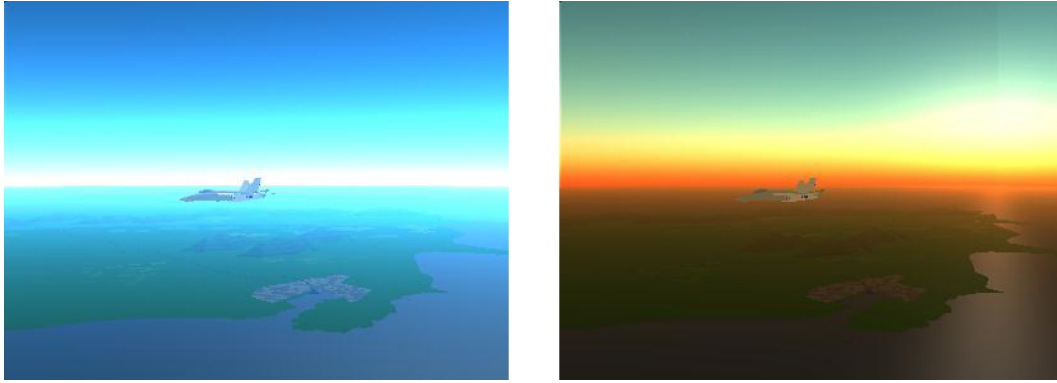


Figure 16: Examples of the multiple scattering algorithm by Nishita from [NDKY96] (section 7.5)

to optimize the whole calculation. Some of the authors introduced 2002 a more advanced version of their calculation model [DYN02] (see also section 7.7).

7.6. A Practical Analytic Model for Daylight

Preetham *et al.* developed an inexpensive analytic model that approximates full spectrum daylight for various atmospheric conditions and incorporated the effects of aerial perspective (section 5.3) in their work [PSS99]. With either measured or estimated terms these conditions are parameterized. All formulas presented are for clear and overcast skies only.

Preetham's goal is to present a formula that returns the spectral radiance for an input direction, viewer's position, date and time under certain conditions. The sun's position can be computed from latitude, longitude, time and date with the formulas described in the paper.

The sun light that reaches the viewer is calculated from the sun's spectral radiance outside the earth's atmosphere. A fraction gets removed by scattering and absorption in the atmosphere. Different kinds of scattering and absorption are incorporated like scattering by molecular and dust particles and absorption by ozone, mixed gases and water vapor. Because of the multiplicative and thus commutative nature of attenuation the order of attenuation does not matter. The total attenuation coefficient can be calculated if the accumulated densities along the illumination path are known. To get the sun's spectral radiance at the surface of the earth the spectral radiance of the sun in outer space is multiplied with the spectral attenuation.

For the skylight model Preetham computed the sky spectral radiance function for a variety of sun positions and turbidities and then fitted a parametric function. The sky spectral radiance function is computed with the sky light model of Nishita [NDKY96] (see section 7.5).

For the parametric formula for luminance Preetham uses the approach by Perez [PSI93].

Aerial perspective cannot be precomputed for a given rendering. A slightly simpler atmospheric model than the model used for skylight modelling is used. The density of particles is approximated as exponential with respect to height. The aerial perspective equations can be approximated accurately then. A further assumption is made by modelling a flat earth, which is reasonable for viewers on the ground.

Rendering examples of this model can be seen in Figure 17.

A further extension to this paper is made in [HP02]. Absorption calculations and Rayleigh and Mie scattering calculations are performed on the vertex and fragment shaders of graphics hardware to achieve real time rendering (Figure 18).

7.7. Interactive Rendering of Atmospheric Scattering Effects Using Graphics Hardware

In this paper [DYN02] Dobashi presents a hardware-accelerated rendering system for atmospheric scattering.

The shape of the earth is assumed to be a sphere and the density of particles is expected to decrease exponentially according to their height above the ground. Multiple scattering is approximated by taking a constant ambient term into account.

A hardware-accelerated volume rendering technique is applied that places planes in front of the screen and subdivide them into a mesh. Light scattered on a certain point on a plane is obtained by using look-up tables computed in a preprocess bound as textures and mapped onto the sampling planes. Sub-planes are used for accurate sampling of shadows and the luminous intensity distribution of the light source (Figure 19).

The intensity of scattered light at a point depends on the light source position, the viewing direction and a few more parameters.

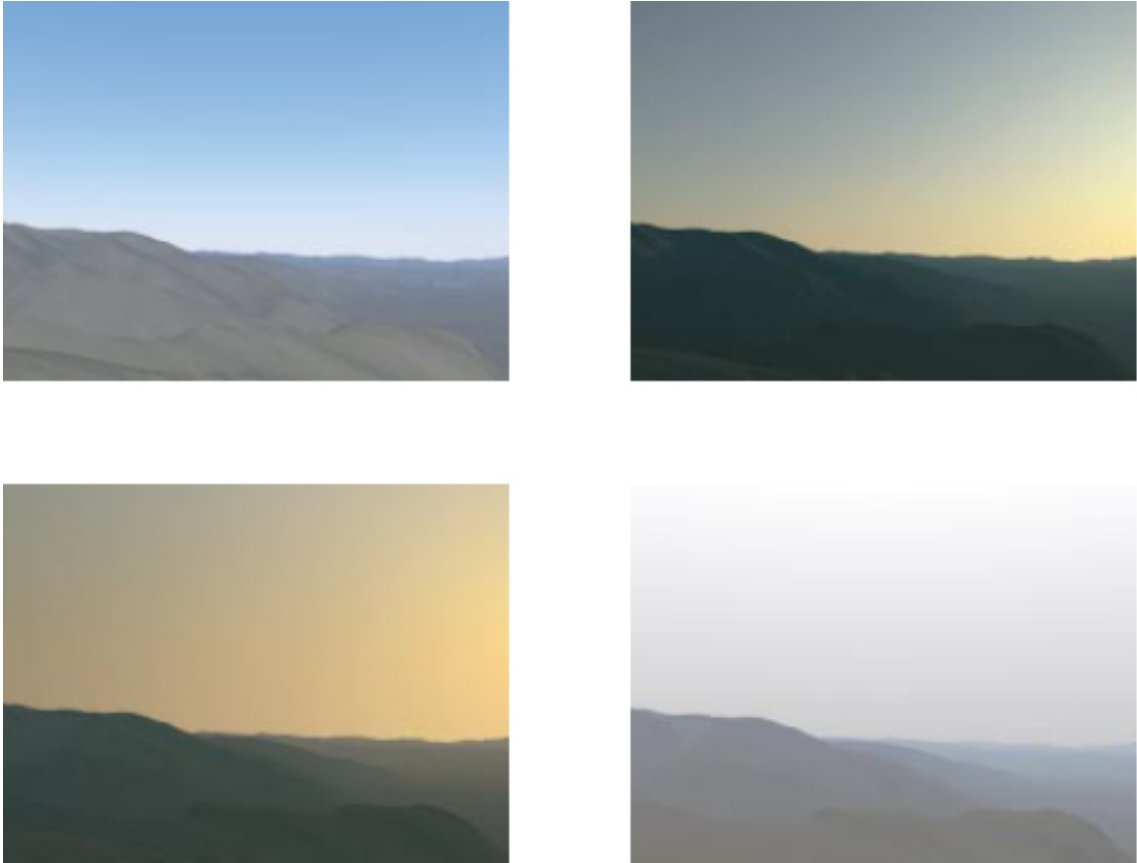


Figure 17: A scene at different times/turbidities rendered with Preetham's model. (image from [[PSS99](#)])

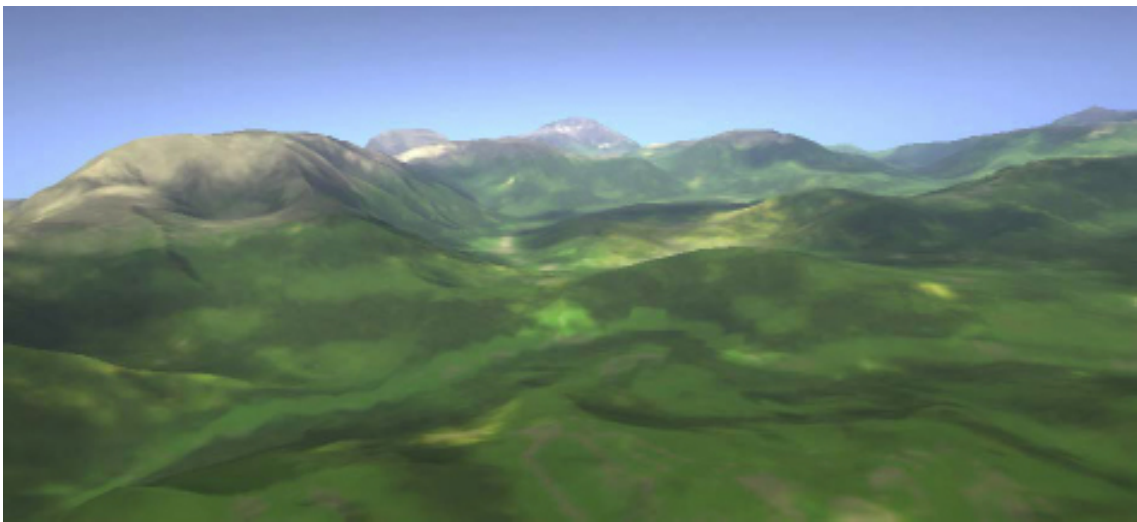


Figure 18: Real time atmospheric scattering [[HP02](#)]. (from [[Pre03](#)])



Figure 19: Interactive atmospheric scattering (from [DYN02])

This leads to high dimensional look-up tables and requires a large amount of memory. Several graphics hardware specific problems are mentioned:

- At the time of the writing of this paper precision problems occurred on graphics hardware of that generation. Today's graphics hardware offers higher precision.
- The problem of clamping of the texture values in graphics hardware is addressed in this paper. Also this problem can be solved with today's graphics hardware.

7.8. Accurate Atmospheric Scattering

Sean O'Neil presents a real-time atmospheric scattering algorithm running entirely on the graphics processing unit [O'N05]. This algorithm is based on the methods described in [NSTN93] (section 7.3) making use of HDR rendering. (Figure 20)

The phase function used for calculation atmospheric scattering is an adaption of the Henyey-Greenstein function used in [NSTN93] suitable to simulate both Rayleigh and Mie scattering. The out-scattering and in-scattering equation for calculating the optical depth for both Rayleigh and Mie scattering are presented depending on the density defined by the height of a given sample point in a given ray. A scale height (the height at which the atmosphere's average density is found) is used to calculate the density for an arbitrary height (eq. (7)). O'Neal also takes surface scattering into account.

O'Neal presents ideas how to implement the algorithm. First he demonstrates how 2D lookup tables can be used to encode the information needed for the scattering equations. The rows of the lookup table can be used to represent the altitudes the columns encode the viewing angle. Different

channels of the lookup table can be used to hold information about Rayleigh and Mie scattering each having two separate channels for their corresponding density values and the optical depth. These lookup tables can reduce computation per vertex by a factor of 50.

O'Neal seeks to prevent using Shader Model 3.0 when implementing the algorithm on a graphics processor leading to the fact that no textures can be used for lookup tables in vertex shaders. So O'Neal searches the lookup tables for any dependencies and coherences between the parameters. By normalizing the data, O'Neal manages to eliminate the lookup in the lookup table for one axis and to use a polynomial equation to approximate the other dimension. This makes the use of lookup tables - more precisely - texture lookups in the vertex shader obsolete.

A simple HDR rendering approach is pursued to deal with the fact that result values of the equations do not fall in the interval 0 to 1 by default. Exponential scaling is used to bring the values into the right interval.

8. Conclusion

This report fulfilled a double purpose. First we described the physical processes of atmospheric daylight. We found out that the main reason for the blue color of the sky is Rayleigh's law of scattering of electromagnetic waves on particles smaller than their wavelength.

We introduced Mie scattering and presented several effects related to scattering in atmosphere. We presented Rayleigh's law and calculated some basic values with it within this paper and showed an approach for the calculations of the sky by Nishita *et al.* [NDKY96].

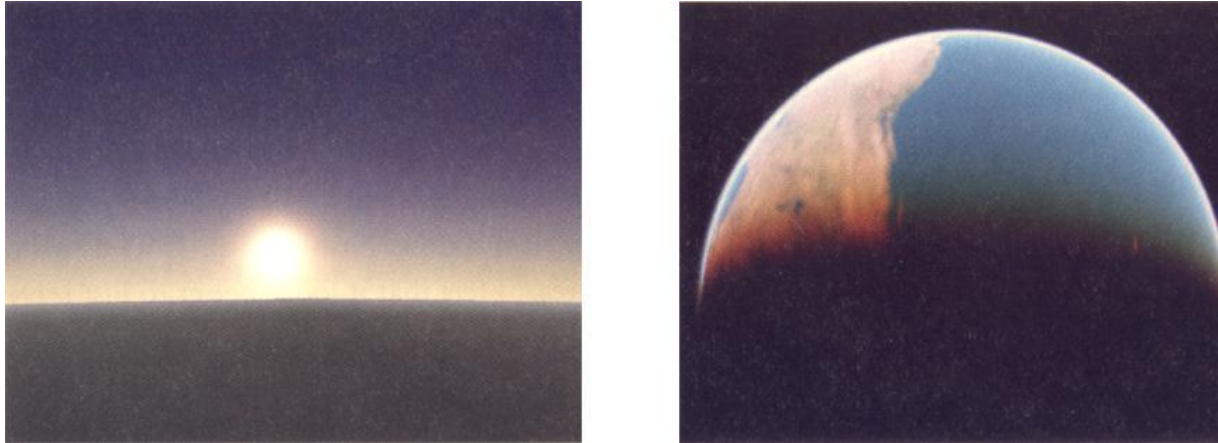


Figure 20: Accurate atmospheric scattering in real time (image from [O'N05])

We can say that the way how to calculate Rayleigh scattering is clearly documented by literature of physics. Mie scattering seems to be some more complex, because of the different scattering functions for different particle sizes.

The second purpose of this report was to give an overview of important papers on clear sky models. We showed a chronological *genealogical diagram* which also differentiated simulation based from analytic models to better understand the various dependencies between the most frequently quoted papers on skylight models, and then gave descriptions of these papers.

With this report, we hope to have provided a good primer to gain a rapid overview of the topic of clear-sky models for computer graphics.

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