Hough transform applications in Computer Graphics (with focus on medical visualization)

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Abstract

This paper gives an explanation of the Hough transform (HT) algorithm and an overview of some of the possible applications the HT can be used in. Section 2 is dedicated to the description of the main idea of the HT on the basis of the Line HT (LHT). Then some adaptions of this relatively easy algorithm will be introduced (the Circle HT (CHT), Generalized HT (GHT), Probabilistic HT (PHT), Randomized HT (RHT) and the Connective RHT (CRHT)). In section 3 several real world applications of these algorithms are presented with a main focus on the use in medical visualization.

Keywords: Hough transform, medical visualization, feature extraction

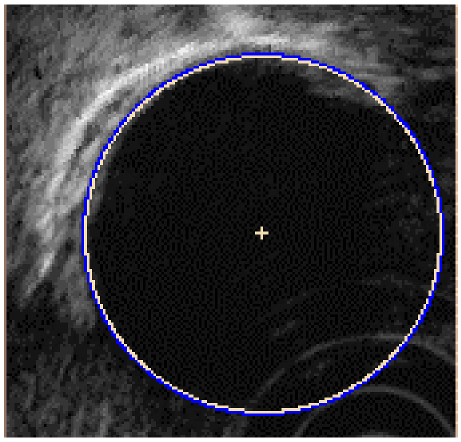

Figure 1.1: Aorta detection in an ultrasound picture

1 Introduction

Feature Extraction from a grayscale or color image is a very often encountered issue in computer graphics. The goal of feature extraction is to locate interesting spots in the input image at a certain precision. When the objects of interest can be described in a parametric form one way to achieve this extraction is the use of the HT, which is a relatively easy method to extract parametrized objects like lines, parabolas, circles or ellipses.

Given, that in medical visualization such features occur very often (e.g. ultrasound-images of blood vessels occur as circles or ellipses, etc.) the HT is a convenient way to achieve feature extraction in this environment (see Figure 1.1). For features, whose boundary cannot be easily described by a parametric curve, adaptions to the original HT have been made.

2 The Hough Transform and its flavors

In general the first step previous to the application of the HT is to detect the edges in the given image. That can be done by a simple edge detection filter (e.g. the canny edge detector [6]). In some cases (e.g. GHT) this step is not necessary, because the algorithm works on the original graylevel image. After this the further steps depend on what object to detect.

2.1 The Standard Hough Transform

The Standard HT (SHT) uses a simple one-to-many scheme to perform a conversion between the original image space and Hough Space. This means that for every point in the source image a curve is drawn in Hough Space. This technique is further explained in the next section on the basis of the Line HT (LHT).

2.1.1 The Line Hough Transform

The simplest object to detect using the HT is the line. Figure 2.1 shows an object, that will be used as an example. There are many ways to describe a line analytically. A convenient equation for describing a set of lines in a parametric form is:

$$x\cos(\theta) + y\sin(\theta) = r$$

where r is the length of the normal from the origin to this line and θ is the orientation of r with respect to the X-axis (see Figure 2.2).

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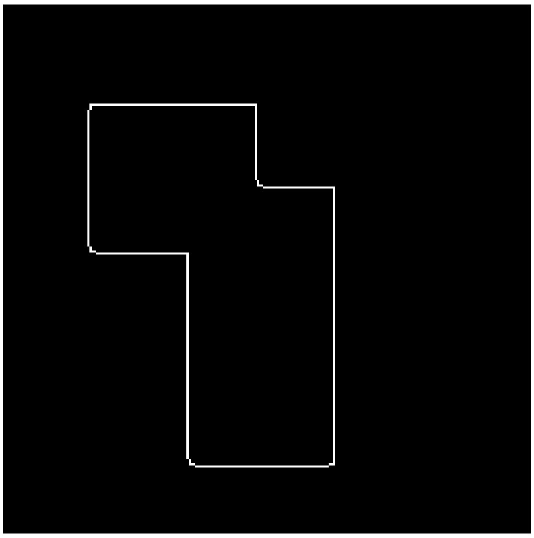

Figure 2.1: gradients of the example object

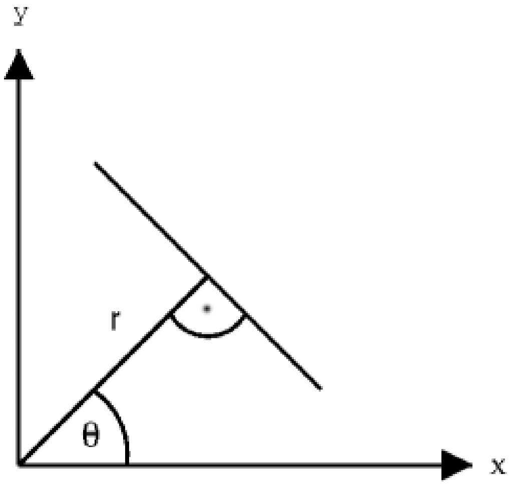

Figure 2.2: parametric view of a line

When analyzing an image, the (x, y) parameters are well known (they are the detected edge pixels of the image to be analyzed). Pairs of θ and r that are solvations of this equation are entered into an accumulator array. Another way to think about it is that all lines that go through (x, y) are converted into (θ, r) space and the according cell (θ, r) is increased by one. Every point in this accumulator array represents one particular line in the original image. Because of the discrete nature of this array, it only contains a subset of all possible lines in $\mathbb{R}^2$. This leads to a set of sinusoids that intersect in certain points. These intersection points appear as white areas in Figure 2.3.

Those intersection points that reach a high value in the accumulator array indicate, that many of the detected edge pixels lie on the same line. Through thresholding, or another method to filter maximums in an image, these points can be obtained.

In order to visualize the results of the HT, the lines according to the peaks in the accumulator array are inserted into the original image. This technique is called de-houghing.

A speciality of this algorithm is its relative robustness against noise and gaps in the original image, which is very important for the use in real world applications(e.g. an ultrasound image always contains some noise).

Figure 2.3: houghed image

2.1.2 The Circle Hough Transform

In order to use the HT for circle detection, we have to find an appropriate parametric representation for a circle. The equation:

$$(x - ax)^2 + (y - ay)^2 = R^2$$

describes any circle, where (ax, ay) is the center of the circle and R its radius. As we can see there are now three parameters, that are needed to describe a circle. This leads to a 3D-Accumulator array in case the radius is not known. In the special case that the radius is known the accumulator dimension reduces to 2D space.

The transform itself is very similar to the Line HT, the only difference is, that every set pixel in the edge picture now votes for every circle (ax, ay, R) it can be part of. If there are many pixel that vote for the same circle (or in other words, lie on the same circle) a peak in the accumulator array will arise, that can also be detected by thresholding.

This detection scheme can be extended to any object that can be described in parametric form. With increasing complexity of the detected objects the original algorithm although becomes problematic, because the dimensionality of the accumulator array is linear to the number of parameters the object needs to be described (e.g. An ellipse already needs five parameters).

This leads to an exponential size increase of the array that can cause resource problems and makes the algorithm slow. One solution to this problem is explained in section 2.4.

2.2 The Generalized Hough Transform

When the shape of the detected object cannot be described in a parametric form, the generalized HT (GHT) can be used for detection. Instead of using a parametric description of the object to be searched, as it would be in the SHT, a look-up table defines the relationship between the boundary positions and orientations and the Hough parameters(see Figure 2.4).

After specifiying a reference point $(xref, yref)$ in the shape, the lookup table (also called R-Table) can be filled with the distance from the reference point to the edge points and the angle β of that distance, indexed by the gradient angle ω. The Hough transform space is defined as possible locations of the reference point location according to the lookup table. The Hough transform space is now defined in terms of the possible positions of the shape in the image, i.e. the possible ranges of $(xref, yref)$. In other words, the transformation is defined by:

$$xref = x - r\cos(\beta)$$

$$yref = y - r\sin(\beta)$$

The r and β values are taken from the R-table for the particular known angle ω, which is obtained through calculating the gradient vector of the local area around the point (x, y). The angle of this vector then serves as an index into the lookup table, resulting in the possible locations of the reference point.

If the orientation of the desired feature is unknown, this procedure is complicated by the fact that we must extend the accumulator by incorporating an extra parameter to account for changes in orientation.

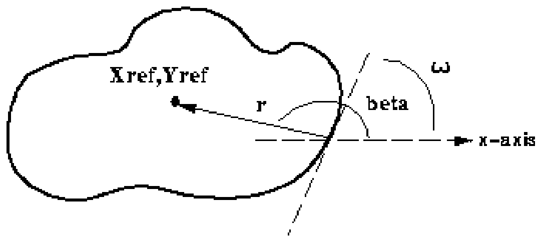

Figure 2.4: Description of R-Table components

2.3 The Probabilistic Hough Transform

The Probabilistic HT (PHT) as such was first introduced by N. Kiryati , Y. Eldar and A.M. Bruckshtein in 1990 [9].

The PHT asserts the following: to detect objects it is sufficient to compute the HT of only a proportion $(0\% < \alpha \leq 100\%)$ of the pixels in the original edge image. These pixels are randomly chosen from the original edge pixel image.

The resulting image, that contains only a part of the original edge information is then used for the conversion from image to Hough space, as proposed in the last two sections. Through the reduced input data to the space conversion computation expanse can be reduced.

The authors found that the α value can be set to $5-15\%$ depending on the application. A smaller value than that results in a fast increase of false feature detections.

2.4 The Randomized Hough Transform

The Randomized HT (RHT) has been invented by Lei Xu, Erkki Oja and Pekka Kultanen in 1989 [1]. It uses a different mechanism for generating values in the histogram which is defined over the parameter space. In the SHT, a pixel in the image corresponds to a curve in the parameter space and this is discretised and recorded in the accumulator array. This is called a one-to-many scheme. The RHT uses a many-to-one scheme, as explained in the next paragraph on the basis of the LHT.

In the RHT, a pair of pixels (the amount of pixels depends on the dimensionality of the parameter space) is randomly chosen and the parameters of the unique line passing through these pixels is computed. This line is recorded as a single entry in the array(many-to-one). This is iterated a preset number of times, where the number of iterations is much less than the number of pixel pairs in the image. In this way, entries are accumulated in the parameter space. This algorithm is iterated to detect line segments one at a time. Thus the global maximum of the parameter space histogram is found, and the equation of the corresponding line computed. The pixels on this line segment are then removed, leaving a simpler image to analyze.

The algorithm is then repeated to find the next line. The algorithm halts when no lines are detected for a number of iterations. To generalize the RHT to circle detection, triplets of pixels are randomly chosen. The unique circle passing through each triplet is computed and recorded as an entry in the 3D parameter space.

For ellipse detection, triplets of pixels are again chosen, but an estimate is made of the tangent at each pixel. The parameters of the unique ellipse passing through the triplet of pixels and satisfying the estimated tangents is computed. This ellipse is recorded as an entry in the 5-D parameter space.

For every extracted object a simple postprocessing step is applied. The number of pixels lying near the object (given some preset tolerance) are counted and divided by the number of pixels expected (the line length or circle, ellipse perimeter). If this proportion is higher than a predefined threshold, the object is decided to exist, and its parameters are entered into the accumulator.

An extension of the RHT , called connective RHT was proposed by Kalvianen and Hirvonen [10] in order to improve the RHT for complex and noisy pictures. In the CRHT a $w \times w$ window is first randomly picked with center one of the edge pixels. Then the CRHT does a connective component search of the windowed points. Only those points of the window are used that are connected to the starting point of the 8-path search. After that a curve is fitted with the connected points. The curve fitting can be done for example by the least square method. Only the curves with their parameters satisfying a certain goodness of the fitting are accepted to update accumulator space.

3 The Hough Transform in medical visualization

After showing some of the possible objects that can be detected by using the hough transform in the last chapter, this chapter will mainly focus on real world applications of the aforementioned techniques with a main focus on medical visualization.

As already mentioned, medical visualization offers a lot of interesting applications where the HT can be useful. Some of these will be introduced here.

3.1 Valid region recognition in X-ray images

The first example of use for the HT deals with a problem often encountered in so called PACS (picture archive and communication systems). PACS can be described as a mixture between Medical Imaging, Information Storage, and the web technology used for the transmission of the images.

An important part of such systems is the digital processing of medical X-ray images. Although the output of X-ray systems is mostly a rectangular image, the valid region slightly differs from device to device. Valid region recognition is important for reduction of storage space and operation quantity, as well as the image quality improvement for further image analysis.

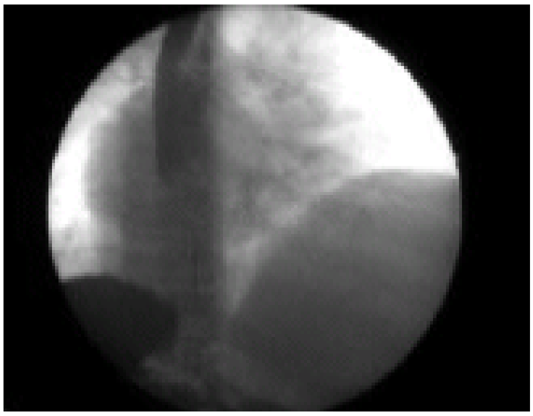

Figure 3.1: a raw medical X-ray image

Figure 3.2: Image after seed and sobel operators

Figure 3.1 shows an input image to the introduced algorithm, which uses a slightly modified circle HT. The first step in this algorithm is to distinguish, approximately, the valid from the invalid region through using a traditional seed-growing algorithm. This leads to a bilevel image shown in Figure 3.2. After that a sobel operator [7]is used to determine the edges of the bilevel image.

This leads to a nearly perfect input image for the CHT. Because of the threedimensional nature of the accumulator array (parameterspace is (x, y, r)) the computational expanse would be very high, this is where the aformentioned modification comes in.

Instead of searching the full range of the radius the circle could have, an estimation is done before the use of the HT. After counting the set pixels in the seeded image, the equation

$$A = r^2\pi$$

for the circle plane can be used to estimate the radius of the circle. Based on this value a radius range is used in the HT, which minimizes the computational expanse dramatically.

The effect on further image processing is shown in Figure 3.3. It shows the differences that occur between the processed and non-processed image in a histogram equalization.

For further information on this algorithm and some experimental results that have been made, refer to [2].

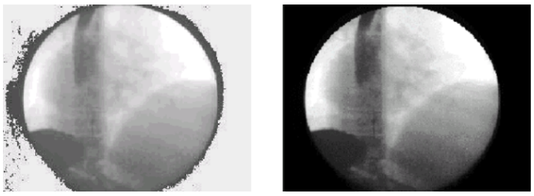

Figure 3.3: histogram eualization with and without valid region

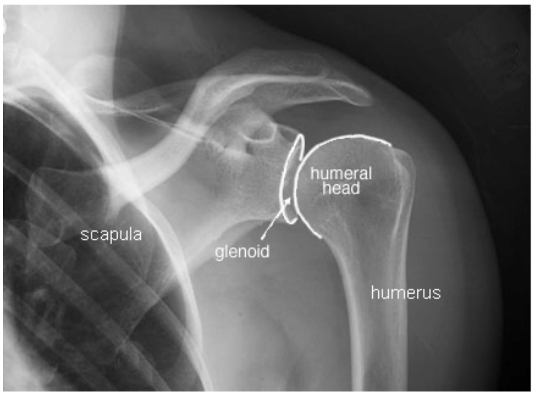

Figure 3.4: a human shoulder and its components

3.2 Automatic determination of the center of rotation of the glenohumeral joint

Rheumatic arthritis is an inflammation of the joints that affects the articular surfaces. The replacement of the affected joint by a prosthesis can reduce its effects, that can result in pain and loss of function. The successrate for the replacement of joints, such as the hip has reached a level as high as 90%. Shoulder replacements unfortunately do not reach these levels, which is mainly caused by the complex anatomy of the shoulder.

Most important for a successful surgery is that the position of the center of rotation of the glenohumeral joint is maintained. This depends on the curvature of both the humeral head and the glenoid. For a description of the human shoulder components see Figure 3.4.

Possible input images come from a CT (computer tomography) or MRI (magnetic resonance imaging) as shown in Figure 3.5. It is professed, that the 3D center of rotation, that is the output of the introduced algorithm can be used for pre-operative planning of shoulder replacements. It will help the surgeon to determine the best position for the prosthesis and can be used for fast visualization of bone surfaces during surgery.

A general parametric description of a sphere in threedimensional space consists of the four parameters (x, y, z, r), where x, y, z refers to the center of the sphere and r refers to the sphere radius. Similar to the implementation for circles in [5], sphere detection is organized into two stages:

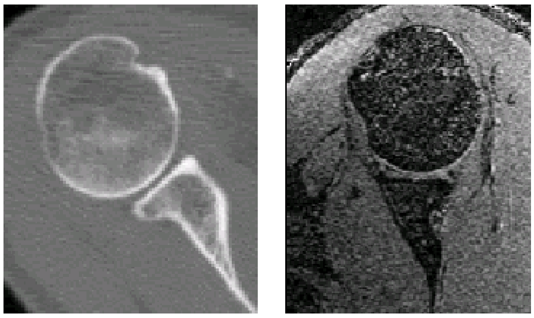

Figure 3.5: Slices of CT (left) and MRI (right) 3D scans of the shoulder

- find the sphere center
- find the radius of the sphere

The sphere detection algorithm uses a modified HT with a threedimensional parameter space (x, y, z). It takes advantage of the fact, that for all voxels on the surface of a sphere, the center will lie on the normal to any local isosurface. The normal of any isosurface through a voxel is represented by the gradient vector.

These normals are entered into the (x, y, z) parameter space by the use of a threedimensional Bresenham algorithm , and vote for any sphere whose center lies on this normal. The Bresenham algorithm is a special line drawing algorithm that only uses additions. More on this algorithm can be found in [8]. The sphere center is determined by the voxel with the highest count in the parameter space.

From the sphere center, the radius is determined by the use of a radial histogram. In this histogram the number of voxels within a range of grey values is counted as a function of the distance to the sphere center. The radius of the sphere is shown as a maximum in the histogram.

The method that is introduced in [3] can be used to accurately determine the center and radius of a sphere in a 3D grey-value image, as long as a minium fraction of 20% of the sphere surface is visible in the image. The method is robust to noise and can also be used for anisotropic voxels.

3.3 Automatic lumbar vertebrae segmentation

The main motivation for the development of the modified HT, which is introduced in this paper, is the study of low back pain and its causes. Chronic low back pain results in 225000 to 300000 lumbar surgeries and indirect medical cost of 75 to 100 billion dollars in the U.S.

In spinal motion study, it is essential to locate landmarks, which can be used to determine the positions of the vertebral bodies. Several automatic approaches to this problem have been developed. The approach of this paper uses a method in which a genetic algorithm is combined with the HT. By using this genetic algorithm to search the Hough spaces, multiple objects can be found within the

Figure 3.6: DVF image of lumbar spine

same frame simultaneously and false peaks are avoided by considering the relationship between these vertebrae.

The input images to this algorithm come from a Digital videofluorescency device. It avoids the problems with high doses of radiation, that CT images have. It is also, in contrast to MRI, fast enough in image acquisition for motion analysis. An example input image is shown in Figure 3.6.

Edge information is, in contrast to other applications of the HT, gained by the use of Phase Congruency[11]. Different from Canny and other gradient-based methods it utilizes the fact that the feature points are perceived at points in an image where the fourier components are maximally in phase. Phase congruency is a dimensionless quantity that is invariant to changes in image brightness or contrast; hence, it provides an absolute measure of the significance of feature points, thus allowing the use of universal threshold values that can be applied over wide classes of images. Figure 3.7. shows the different results of Canny and Phase Congruency applied to the same source image.

Here a modified GHT is used. Instead of the aforementioned R-table it represents the vertebrae through Fourier descriptors.

The core of the HT is how to form the hough space. In this study only rotation and translations in the x and y directions are considered. Thus the Hough space for each vertebra is threedimensional. Each edge point will vote in this array, and the parameters can be determined by locating the maximum in this array.

Figure 3.8 shows a frame overlay of the result of the feature detection.

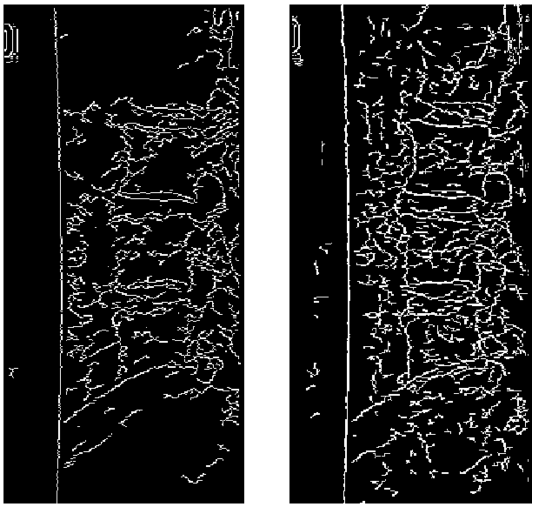

Figure 3.7: Canny and Phase Congruency applied to the same image

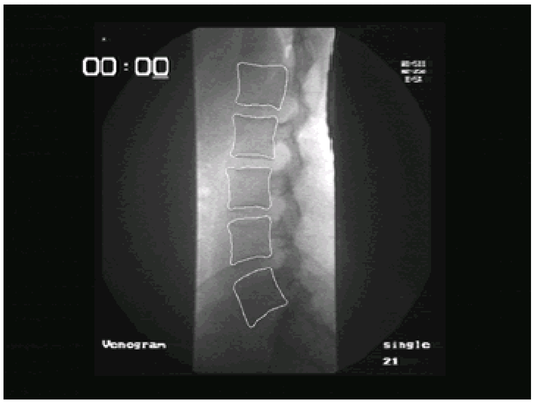

Figure 3.8: Results of the vertebrae detection

4 Conclusion

Based on the information that is shown in this paper the reader should have a main idea of the function and the possible applications of the HT. The SHT, that takes advantage of the fact that each pixel belonging to a certain object in the source image maps to a single point in parameter space, is introduced on the basis of the LHT. Several extensions to this approach are explained, that improve storage space and computational cost which gets very high with an increasing number of parameters. The GHT is used to detect arbitrary shapes in graylevel images. It uses a lookup table to parametrize shapes that can not be described analytically. The PHT improves the SHT through using only a proportion of the original source image. It is shown that this technique has a vast potential of detecting features of nearly any shape in image processing. This fact makes it a good and widely used technique in medical visualization.

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