Rendering: Materials

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Today’s Roadmap

- Adding refractions
  - Snell’s Law
  - Fresnel Reflectance
  - Specular BTDF

- Important concepts
  - Chromatic Aberration
  - Heckbert Notation
  - Caustics
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Reflection Model Sources

- **Physical (wave) optics:**
  - Derived using a detailed model of light
  - Treating it as wave and computing solutions to Maxwell’s equations
  - Computationally expensive, usually not appreciably more accurate

- **Geometric optics:**
  - Requires surface’s low-level scattering and geometric properties
  - Closed-form reflection models derived from these properties
  - More tractable, complex wave effects like polarization are ignored
Specular Reflection (Mirror)

- The angle of exiting light $\theta_o$ is the same as the angle of incidence $\theta_i$

- Incoming light is only transported in a single direction
Specular Reflection and Transmission

- Last time, we assumed that the entire radiance is reflected (mirror)

- This is usually not the case
  - Some light is reflected on the surface
  - Some enters the new material (scattered, absorbed or refracted)
  - Meeting point of two different media is called interface

- When entering a different medium, light often changes direction

- Governed by the materials’ index of refraction and Snell’s law
Specular Reflection and Transmission
Snell’s Law

- Based on the indices of refraction for the two materials
  - $\eta_i$ for the medium that the light ray is currently in
  - $\eta_t$ for the new medium into which light is transmitted

- **Index of refraction**: how fast light travels in medium

- Snell’s law, essentially:
  \[ \eta_i \sin \theta_i = \eta_t \sin \theta_t \]

- Given $\eta_i$, $\theta_i$ and $\eta_t$, we can easily solve for $\theta_t$
Fresnel Reflectance

- How much of the light do we reflect?

- Not constant, but actually depends on the $\theta_i$

- The larger $\theta_i$, the better the chance for reflection

- If $\eta_i > \eta_t$, if incident light exceeds a certain $\theta_i$, all light may be reflected (*total internal reflection*)
Fresnel Reflectance

- Should be handled differently, depending on the materials involved
- Distinguish how material responds to energy transported by light
- We usually consider three major groups:
  - Dielectrics conduct electricity poorly (glass, air...)
  - Conductors (*metals*, reflect a lot, transmitted light quickly absorbed)
  - Semiconductors (complex, but also rare – we can ignore them)
- We will focus on **dielectrics** today
Examples for the Index of Refraction in Dielectrics

- **Gases**: $1 \,–\, 1.0005$ (no-man’s land from $1.05$ to $1.25$)
- **Liquids**: $1.3$ (water) – $1.5$ (olive oil)
- **Solids**: $1.3$ (ice) – $2.5$ (diamond)
Fresnel Reflectance for Dielectrics

- Defined for parallel and perpendicular polarized light ($r_{\parallel}$ and $r_{\perp}$):
  
  \[ r_{\parallel} = \frac{\eta_t \cos \theta_i - \eta_i \cos \theta_t}{\eta_t \cos \theta_i + \eta_i \cos \theta_t}, \quad r_{\perp} = \frac{\eta_i \cos \theta_i - \eta_t \cos \theta_t}{\eta_i \cos \theta_i + \eta_t \cos \theta_t} \]

- Amount of reflected light (unpolarized light, average of squares):
  \[ F_r = \frac{1}{2} (r_{\parallel}^2 + r_{\perp}^2) \]

- Amount of refracted light (conservation of energy): $1 - F_r$
Bidirectional Transmittance Distribution Function (BTDF)

- Refracted light usually changes direction in new medium
- Remember that we work with radiance: \(d\Phi = L_i dA \perp d\omega\)
- Refracted light changes direction \(\rightarrow\) influences radiance!
- Relate incoming to refracted light:

\[
L_o \cos \theta_o dA \sin \theta_o d\theta_o d\phi_o = (1 - F_r)L_i \cos \theta_i dA \sin \theta_i d\theta_i d\phi_i
\]

- Differentiating Snell’s law w.r.t. \(\theta\), we get:

\[
\eta_o \cos \theta_o d\theta_o = \eta_i \cos \theta_i d\theta_i \rightarrow \frac{\cos \theta_o d\theta_o}{\cos \theta_i d\theta_i} = \frac{\eta_i}{\eta_o}
\]
Bidirectional Transmittance Distribution Function (BTDF)

- Substituting, we get:

\[ L_o \eta_i^2 d\phi_o = (1 - F_r) L_i \eta_o^2 d\phi_i \rightarrow L_o = (1 - F_r) \frac{\eta_o^2}{\eta_i^2} L_i \]

- We have all the required information for the specular BTDF!
  - Use \( T(\omega, n) \) to compute direction of \( \omega \) when refracted at interface
  - Like specular BRDF, light only goes in a single direction
  - Can reuse BRDF \( \delta(\omega) \) and normalization (similar implementation!)

\[ f_r(x, \omega_i \rightarrow \omega_o) = \frac{\eta_o^2}{\eta_i^2} (1 - F_r) \frac{\delta(\omega_i - T(\omega_o, n))}{|\cos \theta_i|} \]
Bidirectional Transmittance Distribution Function (BTDF)

- When light refracts into a material with a higher $\eta$, the energy is compressed into a smaller set of angles.

- For the BTDF, $f_r(x, \omega_i \rightarrow \omega_o) = f_r(x, \omega_o \rightarrow \omega_i)$ is not guaranteed.

- No reciprocity, but $\eta_i^2 f_r(x, \omega_i \rightarrow \omega_o) = \eta_o^2 f_r(x, \omega_o \rightarrow \omega_i)$ holds!

- If you follow a view ray, do the same computations as above, just:
  - Make sure you choose $\eta_i$ for medium ray comes from
  - Make sure you choose $\eta_t$ for medium ray goes to
Dielectrics Implementation

- Just continue one path, use Fresnel to decide $\rightarrow$ reflect or refract?

- View ray behaves exactly like **incident light** in the above equations

- You may find it easier to flip the normal if light **exits** a medium
  - Light that enters e.g. a glass body must also exit at some point
  - I.e., the incoming light ray is not in same hemisphere as $n$
  - Consistent with using $\eta_i$ and $\eta_t$ for current/new medium

- Solving for $\theta_t$, you may get “$\sin \theta_t > 1$” $\rightarrow$ **total internal reflection**
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Physically speaking, the change in direction is wavelength-dependent.

For proper simulation, would have to at least bend R/G/B differently.

Would spawn two additional rays!

Can of course be done, but is often ignored (tiny effect on most images).
Heckbert Path Notation

- Assign a letter to every interaction of a light path from light to eye
  - L – light
  - D – diffuse surface
  - S – specular surface
  - E – eye

- Use regex to describe specific (e.g., very challenging) path types
  - LE: direct path from light to eye
  - L(D|S)*E: any path from light to eye
  - LDS+E: a path with one diffuse bounce, followed by specular bounces
A Quick Word on Caustics

- General: focused light from interacting with curved, specular surface

  ![Image of caustics in wine glass](CC BY-SA 3.0, Heiner Otterstedt, Kaustik, Wikipedia, “Caustic (optics)"

- For us, who are concerned with rendering and path tracing: LS+DE

  ![Computer rendering of a wine glass caustic](CC BY-SA 4.0, Markus Selmke, Computer rendering of a wine glass caustic, Wikipedia, “Caustic (optics)”)

- Usually challenging to render (takes extremely long to converge)
Neither Adam nor I are experts on materials (yet!) and we ran out of time due to some other obligations...

We would have liked to talk about:
- Glossy BSDFs (microfacets) and physics
- Participating media
- ...

We will put videos of people that are experts on the topic into the playlist. You’ll probably learn more than what you could from us :) 

There will be links to reading material as well

These topics will not be covered in the exam!
SIGGRAPH University - Introduction to "Physically Based Shading in Theory and Practice" by Naty Hoffman (!!!)

SIGGRAPH University - Recent Advances in Physically Based Shading by Naty Hoffman (advanced, in the same video there are also some other talks)
References and Further Reading

- Material for Dielectrics largely based on “Physically Based Rendering” book, chapter 8: Reflection Models

- [1] Physically Based Rendering (course book, chapters 8 and 9 for materials, chapter 11 for volume rendering)


- [4] Production Volume Rendering (SIGGRAPH 2017 Course)