Rendering: Materials

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Today's Roadmap

- Adding refractions
  - Snell's Law
  - Fresnel Reflectance
  - Specular BTDF

- Important concepts
  - Chromatic Aberration
  - Heckbert Notation
  - Caustics

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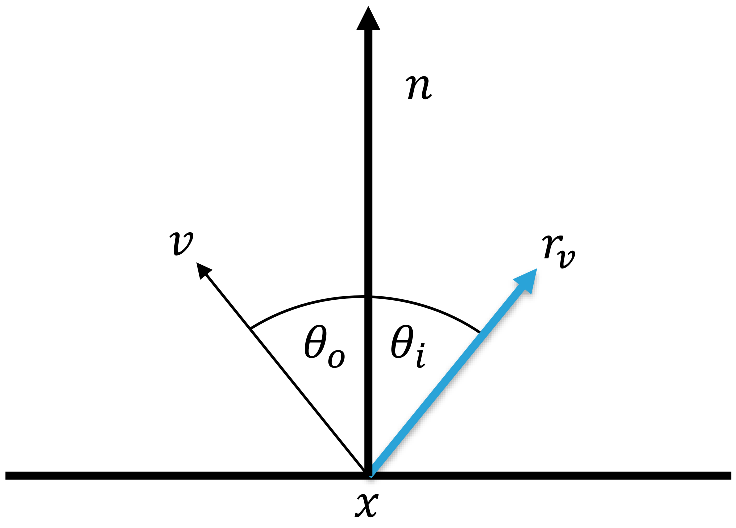
Reflection Model Sources

- *Physical (wave) optics:*
  - Derived using a detailed model of light
  - Treating it as wave and computing solutions to Maxwell's equations
  - Computationally expensive, usually not appreciably more accurate
- ***Geometric optics**:*
  - Requires surface's low-level scattering and geometric properties
  - Closed-form reflection models derived from these properties
  - More tractable, complex wave effects like polarization are ignored

Specular Reflection (Mirror)

- The angle of exiting light θ_o is the same as the angle of incidence θ_i

- Incoming light is only transported in a single direction

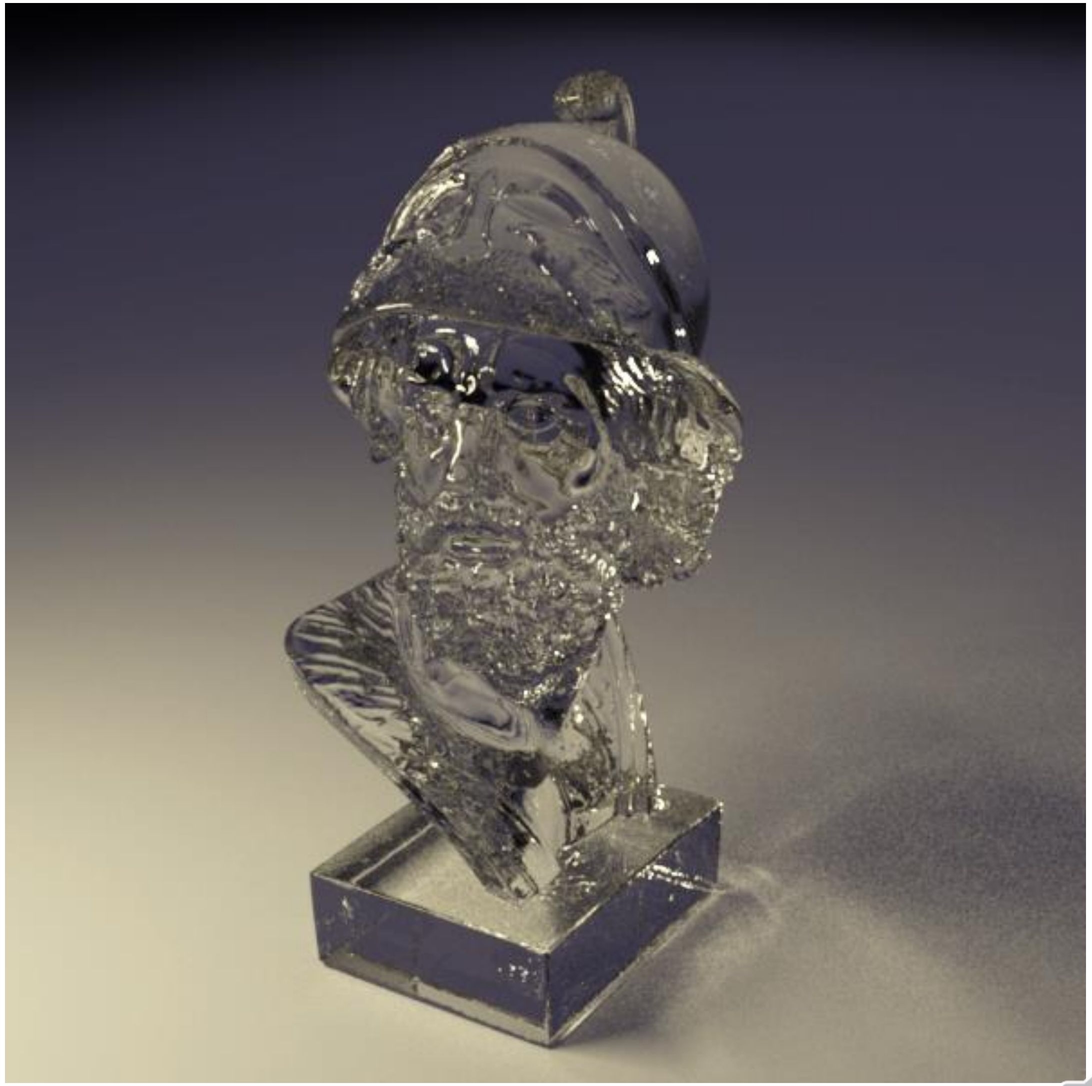


Specular

Specular Reflection and Transmission

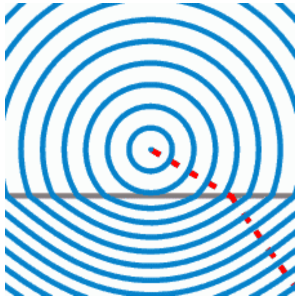
- Last time, we assumed that the entire radiance is reflected (mirror)
- This is usually not the case
  - Some light is reflected on the surface
  - Some enters the new material (scattered, absorbed or **refracted**)
  - Meeting point of two different media is called **interface**
- When entering a different medium, light often changes direction
- Governed by the materials' index of refraction and *Snell's law*

Specular Reflection and Transmission



Snell's Law

- Based on the indices of refraction for the two materials
  - η_i for the medium that the light ray is currently in
  - η_t for the new medium into which light is transmitted

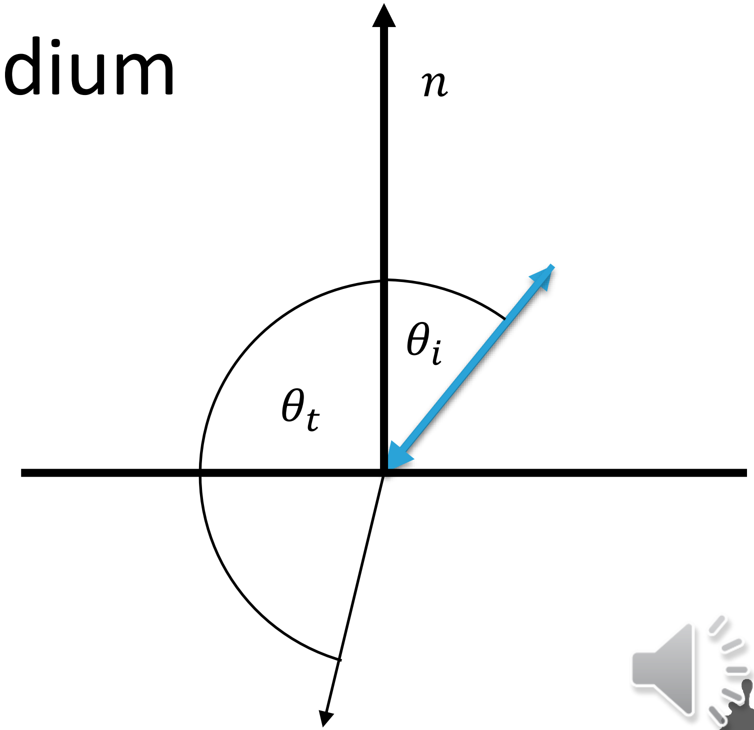


Public domain, Oleg Alexandrov, Snell's law wavefrons, Wikipedia, "Snell's law"

- Index of refraction: how fast light travels in medium

- Snell's law, essentially:

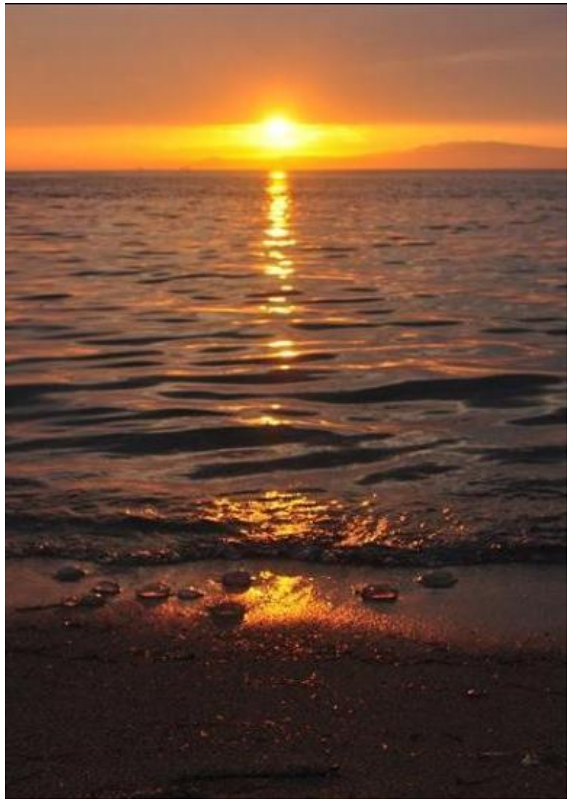
$$\eta_i \sin\theta_i = \eta_t \sin\theta_t$$



- Given η_i, θ_i and η_t, we can easily solve for θ_t

Fresnel Reflectance

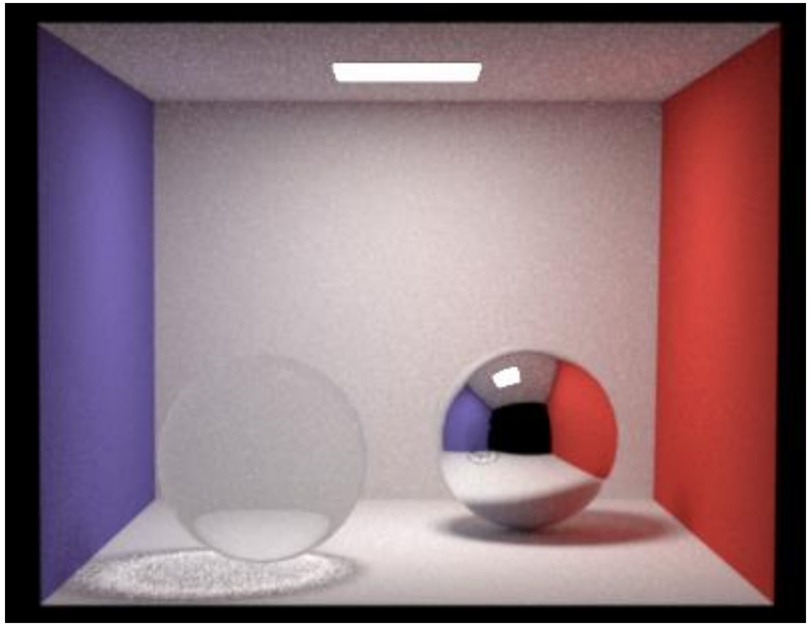
- How much of the light do we reflect?
- Not constant, but actually depends on the θ_i
- The larger θ_i, the better the chance for reflection
- If $\eta_i > \eta_t$, if incident light exceeds a certain θ_i, all light may be reflected (*total internal reflection*)



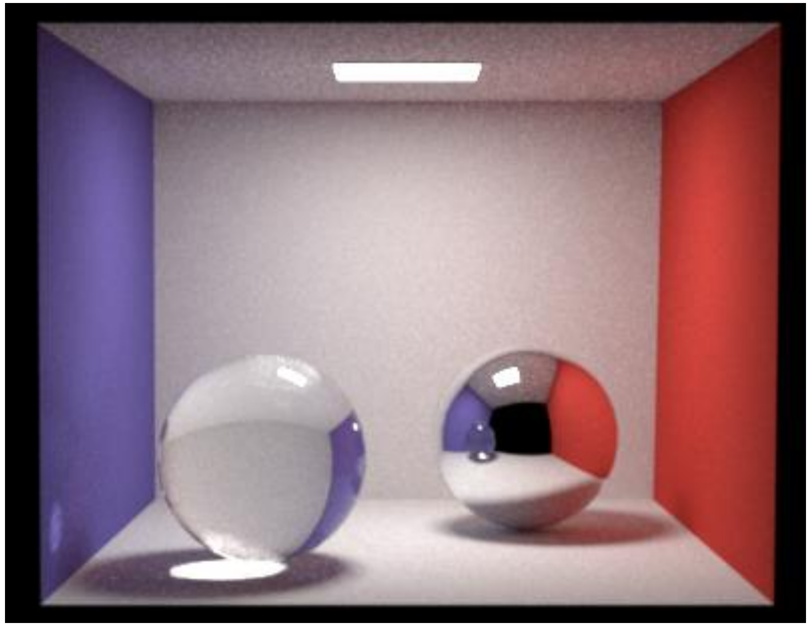
Fresnel Reflectance

- Should be handled differently, depending on the materials involved
- Distinguish how material responds to energy transported by light
- We usually consider three major groups:
  - Dielectrics conduct electricity poorly (glass, air…)
  - Conductors (*metals*, reflect a lot, transmitted light quickly absorbed)
  - Semiconductors (complex, but also rare – we can ignore them)
- We will focus on **dielectrics** today

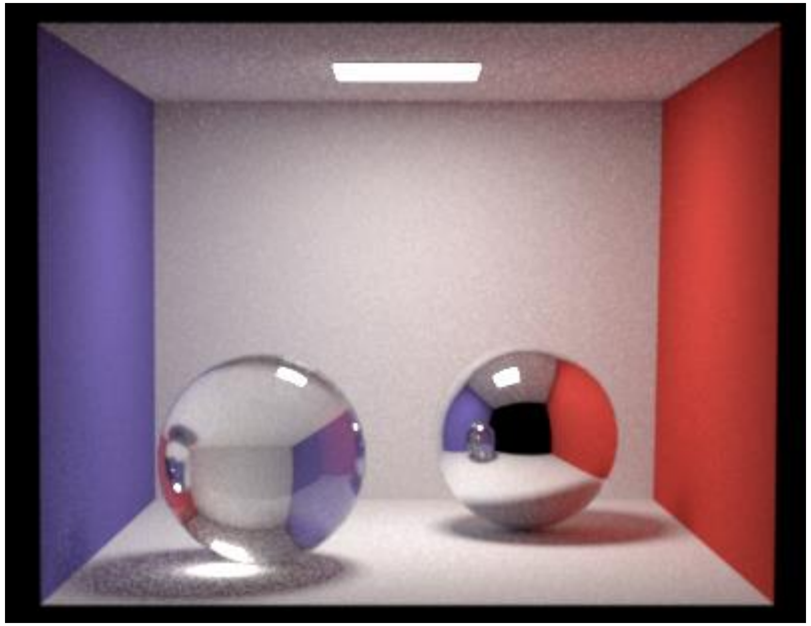
Examples for the Index of Refraction in Dielectrics



$\eta_t = 1.025$ (liquid helium)



$\eta_t = 1.5$ (glass)



$\eta_t = 2.5$ (diamond)

- Gases: 1 – 1.0005 (no-man‘s land from 1.05 to 1.25)
- Liquids: 1.3 (water) – 1.5 (olive oil)
- Solids: 1.3 (ice) – 2.5 (diamond)

Fresnel Reflectance for Dielectrics

- Defined for parallel and perpendicular polarized light ($r_{\parallel}$ and $r_{\perp}$):

$$r_{\parallel} = \frac{\eta_t \cos\theta_i - \eta_i \cos\theta_t}{\eta_t \cos\theta_i + \eta_i \cos\theta_t}, r_{\perp} = \frac{\eta_i \cos\theta_i - \eta_t \cos\theta_t}{\eta_i \cos\theta_i + \eta_t \cos\theta_t}$$

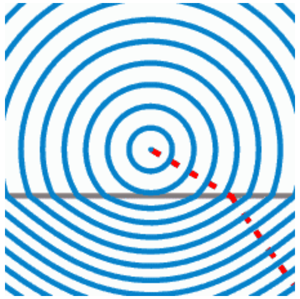
- Amount of **reflected** light (unpolarized light, average of squares):

$$F_r = \frac{1}{2}(r_{\parallel}^2 + r_{\perp}^2)$$

- Amount of **refracted** light (conservation of energy): $1 - F_r$

Bidirectional Transmittance Distribution Function (BTDF)

- Refracted light usually changes direction in new medium
  - Remember that we work with radiance: $d\Phi = L_i dA_\perp d\omega$
  - Refracted light changes direction → influences radiance!
  - Relate incoming to refracted light:



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$$L_o \cos\theta_o \, dA \sin\theta_o \, d\theta_o \, d\phi_o = (1 - F_r) L_i \cos\theta_i \, dA \sin\theta_i \, d\theta_i \, d\phi_i$$

  - Differentiating Snell's law w.r.t. θ, we get:

$$\eta_o \cos\theta_o \, d\theta_o = \eta_i \cos\theta_i \, d\theta_i \rightarrow \frac{\cos\theta_o d\theta_o}{\cos\theta_i d\theta_i} = \frac{\eta_i}{\eta_o}$$

Bidirectional Transmittance Distribution Function (BTDF)

- Substituting, we get:

$$L_o \eta_i^2 d\phi_o = (1 - F_r) L_i \eta_o^2 \, d\phi_i \rightarrow L_o = (1 - F_r) \frac{\eta_o^2}{\eta_i^2} L_i$$

- We have all the required information for the specular BTDF!
  - Use $T(\omega, n)$ to compute direction of ω when refracted at interface
  - Like specular BRDF, light only goes in a single direction
  - Can reuse BRDF $\delta(\omega)$ and normalization (similar implementation!)

$$f_r(x, \omega_i \rightarrow \omega_o) = \frac{\eta_o^2}{\eta_i^2} (1 - F_r) \frac{\delta(\omega_i - T(\omega_o, n))}{|\cos \theta_i|}$$

Bidirectional Transmittance Distribution Function (BTDF)

- When light refracts into a material with a higher η, the energy is compressed into a smaller set of angles

- For the BTDF, $f_r(x, \omega_i \to \omega_o) = f_r(x, \omega_o \to \omega_i)$ is not guaranteed

- No reciprocity, but $\eta_i^2 f_r(x, \omega_i \to \omega_o) = \eta_o^2 f_r(x, \omega_o \to \omega_i)$ holds!

- If you follow a **view ray**, do the same computations as above, just:
  - Make sure you choose η_i for medium ray comes from
  - Make sure you choose η_t for medium ray goes to

Dielectrics Implementation

- Just continue one path, use Fresnel to decide → reflect or refract?
- View ray behaves exactly like **incident light** in the above equations
- You may find it easier to flip the normal if light **exits** a medium
  - Light that enters e.g. a glass body must also exit at some point
  - I.e., the incoming light ray is not in same hemisphere as n
  - Consistent with using η_i and η_t for current/new medium
- Solving for θ_t, you may get "$\sin\theta_t > 1$" → **total internal reflection**

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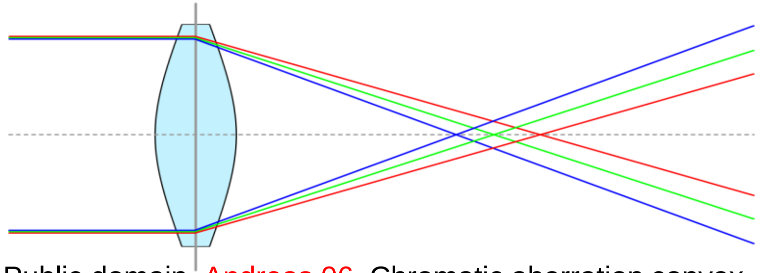
Chromatic Aberration

- Physically speaking, the change in direction is wavelength-dependent
- For proper simulation, would have to at least bend R/G/B differently
- Would spawn two additional rays!
- Can of course be done, but is often ignored (tiny effect on most images)



Public domain, Andreas 06, Chromatic aberration convex, Wikipedia, “Chromatic aberration”

CC BY-SA 3.0, Stan Zurek, Chromatic aberration (comparison), Wikipedia, “Chromatic aberration”

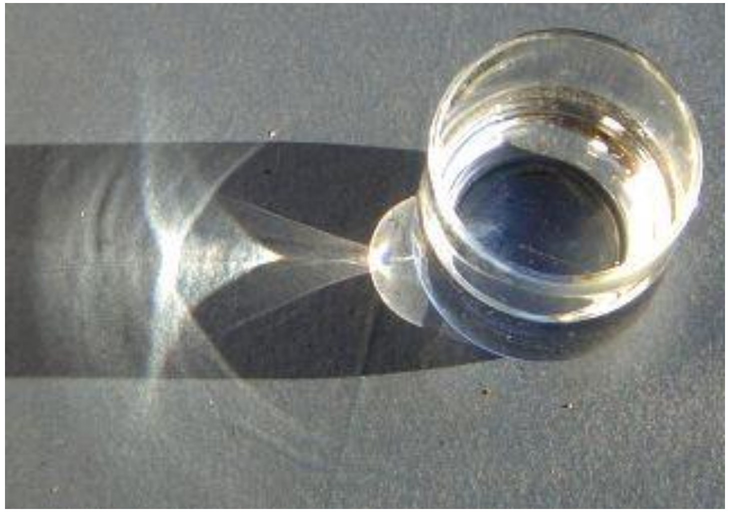
Heckbert Path Notation

- Assign a letter to every interaction of a light path from light to eye
  - L – light
  - D – diffuse surface
  - S – specular surface
  - E – eye

- Use regex to describe specific (e.g., very challenging) path types
  - LE: direct path from light to eye
  - L(D|S)*E: any path from light to eye
  - LDS+E: a path with one diffuse bounce, followed by specular bounces

A Quick Word on Caustics

- General: focused light from interacting with curved, specular surface



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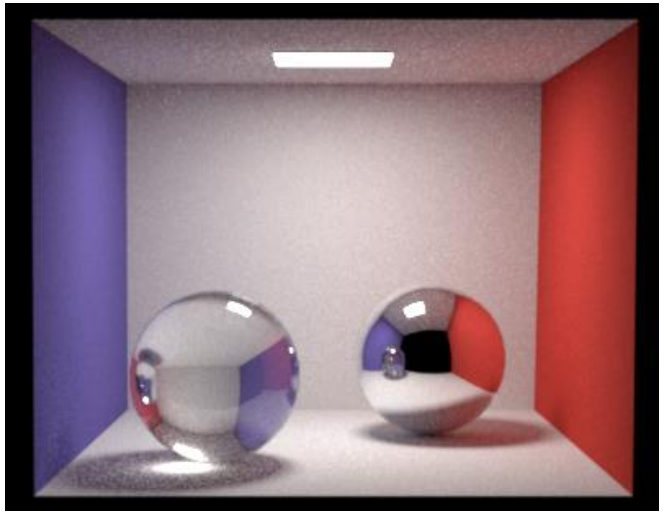
CC BY-SA 4.0, Markus Selmke, Computer rendering of a wine glass caustic, Wikipedia, “Caustic (optics)”



- For us, who are concerned with rendering and path tracing: LS+DE

- Usually challenging to render (takes extremely long to converge)

That’s it from us!

- Neither Adam nor I are experts on materials (yet!) and we ran out of time due to some other obligations…
- We would have liked to talk about:
  - Glossy BSDFs (microfacets) and physics
  - Participating media
  - …

- We will put videos of people that are experts on the topic into the playlist. You‘ll probably learn more than what you could from us :)
- There will be links to reading material as well
- These topics will not be covered in the exam!

Video Suggestions

- SIGGRAPH University - Introduction to "Physically Based Shading in Theory and Practice" by Naty Hoffman (!!!)

- SIGGRAPH University - Recent Advances in Physically Based Shading by Naty Hoffman
(advanced, in the same video there are also some other talks)

References and Further Reading

- *Material for Dielectrics largely based on "Physically Based Rendering" book, chapter 8: Reflection Models*

- [1] Physically Based Rendering (course book, chapters 8 and 9 for materials, chapter 11 for volume rendering)
- [2] Background: Physics and Math of Shading by Naty Hoffman
- [3] Wojciech Jarosz, "Efficient Monte Carlo Methods for Light Transport in Scattering Media", PhD Thesis, https://cs.dartmouth.edu/~wjarosz/publications/dissertation/
- [4] Production Volume Rendering (SIGGRAPH 2017 Course)
- [5] Monte Carlo methods for physically based volume rendering (SIGGRAPH 2018 Course)