

Rendering: Path Tracing I

Bernhard Kerbl

Research Division of Computer Graphics Institute of Visual Computing & Human-Centered Technology TU Wien, Austria



Add the last missing piece, the BSDF (simple version)

- Finally, we will generate some great-looking images by putting together all the things we learned:
 - Light Physics
 - Monte Carlo Integration
 - The Rendering Equation
 - The Path Tracing Algorithm

We will also check out ways to make the procedure fast and stable

Today's Roadmap





Today's Roadmap





The Missing Part of the Rendering Equation

$$L_e(x,v) = E(x,v) + \int_{\Omega} f_r(x,\omega \to v) L_i(x,\omega) \cos(\theta_x) d\omega$$

- Bidirectional Scattering Distribution Function (BSDF)
- Describes the light transport properties of the material
- So far, we avoided this term or replaced it with constant factors
- Can model reflections, refractions, volumetric scattering...



Bidirectional Reflectance Distribution Function (BRDF)



- Considers only the **reflection** of incoming light onto a surface
 - The BRDF is a limited instance of the full BSDF (e.g., no transparency)
 - Good for starting out, complex materials need full BSDF
 - More on that in another lecture
- A BRDF function $f_r(x, \omega_i \to \omega_o)$ with input directions ω_i, ω_o
 - uses convention: ω_i and ω_o are assumed to point away from x
 - How much irradiance from ω_i is reflected as radiance to ω_o at x?





• "How much irradiance from ω_i is reflected as radiance to ω_o at x?"

$$f_{r}(x,\omega_{i} \to \omega_{o}) = \frac{dL_{i}(x,\omega_{o})}{dE_{i}(x,\omega_{i})} = \frac{dL_{i}(x,\omega_{o})}{L_{i}(x,\omega_{i})\cos_{\theta}(\omega_{i})\,d\omega_{i}}$$
$$L_{e}(x,v) = E(x,v) + \int_{\Omega} f_{r}(x,\omega \to v)L_{i}(x,\omega)\cos(\theta_{x})\,d\omega$$

• Helmholtz reciprocity: $f_r(x, \omega_i \to \omega_o) = f_r(x, \omega_o \to \omega_i)$

Conserves energy:
$$\int_{\Omega} f_r(x, \omega \to v) \cos \theta \, d\omega \le 1 \, \forall \, v$$





- Why must the BRDF f_r fulfill $\int_{\Omega} f_r(x, \omega \to v) \cos_{\theta}(\omega) d\omega \le 1$?
- Intuitive interpretation with **reciprocity**: Shine a laser light along -v onto x. We must have $\int_{\Omega} f_r(x, v \to \omega) \cos_{\theta}(\omega) d\omega \leq 1$
- If we find a direction v for which this is not true, it means we would reflect more light than is coming in (furnace test!)







- Why must the BRDF f_r fulfill $\int_{\Omega} f_r(x, \omega \to v) \cos_{\theta}(\omega) d\omega \le 1$?
- Intuitive interpretation with **reciprocity**: Shine a laser light along -v onto x. We must have $\int_{\Omega} f_r(x, v \to \omega) \cos_{\theta}(\omega) d\omega \leq 1$
- If we find a direction v for which this is not true, it means we would reflect more light than is coming in (furnace test!)



BRDF Types



We usually distinguish three basic BRDF types

- Perfectly diffuse (light is scattered equally in/from all directions)
- Perfectly specular (light is reflected in/from exactly one direction)
- Glossy (mixture of the other two, stronger reflectance around r_v)



BRDF Types



We usually distinguish three basic BRDF types

- Perfectly diffuse (light is scattered equally in/from all directions)
- Perfectly specular (light is reflected in/from exactly one direction)
- Glossy (mixture of the other two, stronger reflectance around r_v)





Before, we considered the BRDF value and sampling of ω separately

- For implementation, it makes a lot of sense to combine them
 - $f_r(x, \omega \rightarrow v)$ depends only on x, v and next ray direction ω
 - Rendering equation: we can't predict L_i , but $f_r(x, \omega \rightarrow v)$ and $\cos \theta$
 - Our renderings will converge faster if the distribution of ω actually matches the shape of $f_r(x, \omega \to v) \cos \theta$ (importance sampling!)
 - If we put the BRDF in charge of choosing our ω , we can make it sample a distribution that directly matches $f_r(x, \omega \to v) \cos \theta$
 - This actually makes things cleaner in code

Diffuse materials reflect same amount of light in/from all directions

Importance sampling $f_r(x, \omega \to v) \cos \theta \rightarrow use p(\omega) \propto \frac{\rho \cos \theta}{\pi}$

• Making it a valid PDF leads to $p(\omega) = \frac{\cos \theta}{\pi}$

From previous exercise: it's cosine-weighted hemisphere sampling!

Rendering – Path Tracing I





Method sample(v): generate a cosine-weighted sample

Method evaluate(a, b): if
$$a, b \ge n < \frac{\pi}{2}$$
, return $f_r(x, b \to a) = \frac{\rho}{\pi}$

• Method **pdf**(ω) : return the proper $p(\omega)$ for the passed sample

Combine them into unit that takes care of handling diffuse materials

Use terms as before. Abstracts the importance sampling away!



Today's Roadmap





Things get interesting if we look at indirect illumination





Difficult in real-time graphics – comes naturally in path tracing!







Recursive Rendering Equation, Recap





Recursive Rendering Equation, Recap





TU

To get the next bounce, we just evaluate this function recursively



Rendering – Path Tracing I

Implementing the Rendering Equation

```
Li(Scene scene, Ray ray, int depth)
    Color emitted = 0:
    if (!findIntersection(scene, ray)) return 0;
    Intersection its = getIntersection(scene, ray);
    // Take care of emittance
    if (isLightSource(its)) emitted = getRadiance(its);
    if(depth >= maxDepth) return emitted;
                                                                 Recursion limit
    // BRDF should decide on the next ray
    // (It has to, e.g. for specular reflections)
    BRDF brdf = getBRDF(its);
    Ray wo = BRDFsample(brdf, -ray);
                                                                  Diffuse BRDF
    float pdf = BRDFpdf(brdf, wo);
    Color brdfValue = BRDFevaluate(brdf, -ray, wo);
    // Call recursively for indirect lighting
                                                                      Recursion
    Color indirect = Li(scene, wo, depth + 1); <
    return emitted + brdfValue * indirect * cosTheta(its, wo) / pdf;
}
```



One Bounce



```
Li(Scene scene, Ray ray, int depth)
```

```
Color emitted = 0;
```

```
if (!findIntersection(scene, ray)) return 0;
```

```
Intersection its = getIntersection(scene, ray);
```

```
// Take care of emittance
if (isLightSource(its)) emitted = getRadiance(its);
```

```
if(depth >= 1) return emitted;
```

```
// BRDF should decide on the next ray
// (It has to, e.g. for specular reflections)
BRDF brdf = getBRDF(its);
Ray wo = BRDFsample(brdf, -ray);
float pdf = BRDFpdf(brdf, wo);
Color brdfValue = BRDFevaluate(brdf, -ray, wo);
```

```
// Call recursively for indirect lighting
Color indirect = Li(scene, wo, depth + 1);
return emitted + brdfValue * indirect * cosTheta(its, wo) / pdf;
```





Two Bounces



```
Li(Scene scene, Ray ray, int depth)
```

```
Color emitted = 0;
```

```
if (!findIntersection(scene, ray)) return 0;
```

```
Intersection its = getIntersection(scene, ray);
```

```
// Take care of emittance
if (isLightSource(its)) emitted = getRadiance(its);
```

```
if(depth >= 2) return emitted;
```

```
// BRDF should decide on the next ray
// (It has to, e.g. for specular reflections)
BRDF brdf = getBRDF(its);
Ray wo = BRDFsample(brdf, -ray);
float pdf = BRDFpdf(brdf, wo);
Color brdfValue = BRDFevaluate(brdf, -ray, wo);
```

```
// Call recursively for indirect lighting
Color indirect = Li(scene, wo, depth + 1);
return emitted + brdfValue * indirect * cosTheta(its, wo) / pdf;
```





Three Bounces



```
Li(Scene scene, Ray ray, int depth)
```

```
Color emitted = 0;
```

```
if (!findIntersection(scene, ray)) return 0;
```

```
Intersection its = getIntersection(scene, ray);
```

```
// Take care of emittance
if (isLightSource(its)) emitted = getRadiance(its);
```

```
if(depth >= 3) return emitted;
```

```
// BRDF should decide on the next ray
// (It has to, e.g. for specular reflections)
BRDF brdf = getBRDF(its);
Ray wo = BRDFsample(brdf, -ray);
float pdf = BRDFpdf(brdf, wo);
Color brdfValue = BRDFevaluate(brdf, -ray, wo);
```

```
// Call recursively for indirect lighting
Color indirect = Li(scene, wo, depth + 1);
return emitted + brdfValue * indirect * cosTheta(its, wo) / pdf;
```





Today's Roadmap





For purely specular BRDFs (a perfect mirror surface), irradiance from the perfect mirror direction r_v is completely reflected to v

 Irradiance coming from any other direction does not reflect at all towards v



$$f_r(x, \omega \to v) > 0 \Leftrightarrow \omega = r_v$$

Problem: if we pick the next direction ω randomly as before, the chances of ever hitting r_v by accident are infinitely small!



Model specular reflection with the Dirac delta function

Delta function $\delta(x)$ is defined to be 0 everywhere except at x = 0

• Use a shifted version $\delta_v(\omega)$ that is 0 everywhere except at $\omega = r_v$

Per definition, $\int_{\Omega} \delta_{v}(\omega) \, d\omega = 1$ to obtain a valid PDF for sampling

Ponder this for a moment: what value does $\delta_v(r_v)$ have?



Energy-Preserving Specular BRDF



Full energy preservation:
$$\int_{\Omega} f_r(x, \omega \to v) L_i \cos_{\theta}(\omega) d\omega = L_{r_v}$$

If we integrate using $f_r(x, \omega \to v) = \delta_v(\omega)$, we get $L_{r_v} \cos_{\theta}(r_v)$

We lost some light! We compensate:
$$f_r(x, \omega \to v) = \frac{\delta_v(\omega)}{\cos_\theta(r_v)}$$

If we consider the properties of the Dirac delta function, we can try to derive the same methods that we used before for diffuse BRDFs





sample(v**)**: mirror v about n (invert v_x , v_y in *local space*) and return

pdf(
$$\omega$$
): 0 if $\omega \neq r_v$, else: $\delta_v(r_v) = \infty$

But, if $\omega = r_v$, evaluate(v, ω) / pdf(ω) = $\frac{\delta_v(\omega)}{\delta_v(\omega)\cos_\theta(r_v)} = \frac{1}{\cos_\theta(r_v)}$ Rendering - Path Tracing I

How to Implement Diffuse and Specular BRDFs



Specular BRDF: using evaluate/pdf without sample is awkward

Let's make a change to the path tracing routine and BRDF interface

Suggestion: let sample method generate ω and a multiplier for L_i

- Leave application of cos θ and p(ω) to the BRDF (if necessary)
 Diffuse: importance sample ω, apply p(ω), cos θ cancels out
 - Specular: pick $\omega = r_v$, $p(\omega)$ cancels out, $\cos \theta$ cancels out



Revising the Specular BRDF Implementation



- **sample(**v**)**: mirror v about n (invert v_x , v_y in *local space*)
 - Return r_v as generated sample direction
 - Return multiplier for L_i as 1 (full radiance passed on)

No other function except **sample** should be able to just *guess* r_v

evaluate(a, b): always return 0

```
pdf(\omega): always return 0
```



Implementing the Rendering Equation v2.0

```
Color emitted = 0:
    if (!findIntersection(scene, ray)) return 0;
    Intersection its = getIntersection(scene, ray);
    // Take care of emittance
    if (isLightSource(its)) emitted = getRadiance(its);
    if(depth >= max depth) return emitted;
    // BRDF should decide on the next ray
    // (It has to, e.g. for specular reflections)
    BRDF brdf = getBRDF(its);
    BRDFSample sample;
    sample = BRDFsample(brdf, -ray); +
    // Call recursively for indirect lighting
    Color indirect = Li(scene, sample.wo, depth + 1);
    return emitted + sample.value * indirect;
}
```

New, combined BRDF sample.value contains PDF and cosine factors, if necessary



Li(Scene scene, Ray ray, int depth)

One Bounce

Li(Scene scene, Ray ray, int depth)

```
Color emitted = 0;
```

if (!findIntersection(scene, ray)) return 0;

Intersection its = getIntersection(scene, ray);

```
// Take care of emittance
if (isLightSource(its)) emitted = getRadiance(its);
```

if(depth >= 1) return emitted;

```
// BRDF should decide on the next ray
// (It has to, e.g. for specular reflections)
BRDF brdf = getBRDF(its);
BRDFSample sample;
```

sample = BRDFsample(brdf, -ray);

```
// Call recursively for indirect lighting
Color indirect = Li(scene, sample.wo, depth + 1);
return emitted + sample.value * indirect;
```



Two Bounces



Li(Scene scene, Ray ray, int depth)

```
Color emitted = 0;
```

if (!findIntersection(scene, ray)) return 0;

Intersection its = getIntersection(scene, ray);

```
// Take care of emittance
if (isLightSource(its)) emitted = getRadiance(its);
```

if(depth >= 2) return emitted;

```
// BRDF should decide on the next ray
// (It has to, e.g. for specular reflections)
BRDF brdf = getBRDF(its);
BRDFSample sample;
```

sample = BRDFsample(brdf, -ray);

```
// Call recursively for indirect lighting
Color indirect = Li(scene, sample.wo, depth + 1);
return emitted + sample.value * indirect;
```



Three Bounces



Li(Scene scene, Ray ray, int depth)

```
Color emitted = 0;
```

if (!findIntersection(scene, ray)) return 0;

Intersection its = getIntersection(scene, ray);

```
// Take care of emittance
if (isLightSource(its)) emitted = getRadiance(its);
```

if(depth >= 3) return emitted;

```
// BRDF should decide on the next ray
// (It has to, e.g. for specular reflections)
BRDF brdf = getBRDF(its);
BRDFSample sample;
```

sample = BRDFsample(brdf, -ray);

```
// Call recursively for indirect lighting
Color indirect = Li(scene, sample.wo, depth + 1);
return emitted + sample.value * indirect;
```



}



Remember: if we want to be unbiased, then the probability of each possible path (i.e., journey of a photon) must be non-zero

Photons stop bouncing when they have been entirely absorbed

Problem: no real-world material absorbs 100% of incoming light

■ No matter how many bounces, the probability never goes to zero → you can never stop!



∞ Bounces



Li(Scene scene, Ray ray, int depth)

```
Color emitted = 0;
```

{

}

if (!findIntersection(scene, ray)) return 0;

Intersection its = getIntersection(scene, ray);

```
// Take care of emittance
if (isLightSource(its)) emitted = getRadiance(its);
```

if(false) return emitted;

```
// BRDF should decide on the next ray
// (It has to, e.g. for specular reflections)
BRDF brdf = getBRDF(its);
BRDFSample sample;
```

sample = BRDFsample(brdf, -ray);

// Call recursively for indirect lighting
Color indirect = Li(scene, sample.wo, depth + 1);
return emitted + sample.value * indirect;



Renderer never finishes. What to do?





In practice, most contribution comes from the first few bounces



Can we exploit this fact and make long paths possible, but unlikely?

Today's Roadmap







Pick a p > 0. At each bounce, draw a random variable ξ and decide

- $\xi < p$: keep going for another bounce
- $\xi \ge p$: end path

The longer a path goes on, the more likely it is to get terminated

The probability of a ray surviving the N^{th} bounce is p^N

• Whenever a path continues after a bounce, compensate for its (un)likeliness by weighting the color returned from L_i with $\frac{1}{p}$



"...but if the possibility for infinitely long paths remains, doesn't that mean that my renderer may take forever to finish?"

Almost certainly no

In practice, if you choose an adequate p, you are more likely to get struck by lightning while reading this than that ever happening

"Ok, cool, so the lower I choose p, the better, right? Can we just take something really small?" Well, not exactly.

Choosing p = 0.95



Low chance of stopping early

500 samplesper pixel

Runtime: 260s



Choosing p = 0.6



High chance of stopping early

500 samples per pixel

Runtime: 60s

Worse, but faster. More samples?



Choosing p = 0.6



High chance of stopping early

1500 samples per pixel

Runtime: **270s**







p = 0.95, 500 samples, 260s

p = 0.6, 1500 samples, 270s Took longer but looks worse!



Picking the Right Russian Roulette Probability

If p(x) is low but f(x) is not \rightarrow high contribution of rare samples!

Also called "fireflies"

Hard to get rid off!



Choose p at each bounce according to remaining color contribution

 $p_1 = 1, p_N \text{ at } N^{th} \text{ bounce} = \max_{\text{RGB}} \left(\prod_{i=1}^{N-1} \left(\frac{f_r(x_i, \omega_i \to v_i) \cos \theta_i}{\text{pdf}(\omega_i) p_i} \right) \right)$

Picking the Right Russian Roulette Probability



- Some materials absorb barely any incoming light (mirrors!)
 - Imagine two mirrors opposite of each other
 - Ray may bounce between them forever
 - Bad: limit bounces to a strict maximum
 - Better: clamp RR p to a value < 1, e.g. 0.99</p>



- Use a minimal depth before allowing Russian Roulette to take effect
 - Preserve a minimal path length for indirect illumination
 - Make sure to exclude guaranteed bounces from path weights



Path Tracing + Russian Roulette





It works. But what about all that noise?



Rendering – Path Tracing I



- A path is defined by the random values that you draw along it
- Path of length N can be seen as a multi-dimensional random variable, e.g.: $(\xi_1, \xi_2, \dots, \xi_{2N})^T$ (need at least θ, ϕ per bounce)
- The more bounces we make, the more dimensions we add
- Monte Carlo is fine with handling infinite-dimensional integrals

We pay the price for additional dimensions with additional noise

We already know some of them

- Random sample positions inside pixel (2)
- Constructing a new ray after each bounce (2*N*)
- Choosing a specific strategy for MIS (1)

Other possible choices we have not yet considered^[1]

- Lens coordinates (for depth-of-field) (2)
- Time (for motion blur) (1)

50





Depth-of-Field

Simulate depth-of-field for focal length $f^{[2]}$

- Create ray r through pixel as before
- Find focal point *f* along *r* at distance *f*
- Pick random location x, y on lens (disk)
- Actually shoot ray from x, y through f











For motion blur, we make geometry a function of time t

- Draw a random t, follow path as before
- Check which triangles ray intersects at t
- Acceleration structure must support parameterization with t!





Niabot, "Two animations rotating around a figure, with motion blur (left) and without", Wikipedia, "Motion Blur", horizontally flipped, <u>CC BY-SA 3.0</u>



Higher-dimensional path tracing is particularly prone to noise

How can we fix it?

We already saw some solutions – and they still apply

- More samples (brute force)
- Importance sampling whenever we can (we already do it for BRDFs)
- Light source sampling, recursively? → Next Event Estimation (NEE)
 - Building on NEE: recursive multiple importance sampling



Today's Roadmap







Builds on light source sampling. Think: where can light come from?







Builds on light source sampling. Think: where can light come from?





We can map out the full hemisphere and distinguish direct/indirect







At each bounce, use light source sampling to get direct illumination
 Use BRDF sample to generate new direction to collect indirect light







At each bounce, use light source sampling to get direct illumination
 Use BRDF sample to generate new direction to collect indirect light





At each bounce, use light source sampling to get direct illumination
 Use BRDF sample to generate new direction to collect indirect light





Light source sampling for direct light

BRDF sampling for finding indirect light



- Add them together to cover the hemisphere
 - Light source sampling to project light source onto hemisphere
 - Importance sampling of the hemisphere via BRDF to generate next direction to collect potential indirect light from next hit point





Problem: what happens if the indirect sample actually hits the light?

Indirect sample accidentally direct, light is added twice in one bounce!

We did not restrict BRDF directions (and we actually don't want to)



Idea: actually ignore emittance completely! We don't need it, because what emittance did, light source sampling now does for us

First Attempt at Next Event Estimation



Color emitted = 0;

```
[...]
```

```
// DON'T take care of emittance
// if (isLightSource(its)) emitted = getRadiance(its);
```

[...] // Stop at some point based on Russian Roulette probability

```
BRDF brdf = getBRDF(its);
```

```
// Get direct sample on a light source with light surface sampling
LightSourceSample sampleLS = sampleLightSurface(its);
// Light source direction is not generated by the BRDF, so we evaluate rendering equation the old way
// Note: sampleLS.radiance already includes light source cosTheta(y), 1/r^2, 1/dA
float direct = BRDFevaluate(brdf, -ray, sampleLS.dir) * cosTheta(its, sampleLS.dir) * sampleLS.radiance;
```

```
// BRDF should decide on the next indirect sample
BRDFSample sampleBRDF = BRDFsample(brdf, -ray);
// Call recursively for indirect lighting
Color indirect = Li(scene, sampleBRDF.wo, depth + 1);
return (emitted + direct + sampleBRDF.value * indirect) / RR_probability;
```



A First Test Run of Next Event Estimation



The noise is mostly gone now!

But some information lost:

- Specular reflections of lights
- Light sources themselves

Caustics

It seems eliminating emittance altogether was too much...





- TU
- At the first bounce, there was no previous bounce for which we computed the direct lighting (i.e., no next event estimation)

With specular materials, we know that the BRDF allows reflection only from a single direction, thus light source sampling will fail

- Idea: actually ignore emittance most of the time, except if
 - The current hit point is the first hit after leaving the camera
 - The last material was fully specular (light source sampling denied)



Path Tracing + Russian Roulette + Next Event Estimation





Rendering – Path Tracing I

- TU
- Most objects are actually neither completely diffuse nor completely specular. We never talked about glossy BRDFs...

Also, we only looked at *reflections* (BRDFs). What about other light scattering or transparency, the full BSDF?

We will handle those soon...



References and Further Reading

- TU
- [1] Toshiya Hachisuka, Wojciech Jarosz, Richard Peter Weistroffer, Kevin Dale, Greg Humphreys, Matthias Zwicker, and Henrik Wann Jensen. 2008. Multidimensional adaptive sampling and reconstruction for ray tracing. ACM Trans. Graph. 27, 3 (August 2008)
- [2] Depth-of-Field Implementation in a Path Tracer: <u>https://medium.com/@elope139/depth-of-field-in-path-tracing-e61180417027</u>
- [3] Ryan Overbeck, Craig Donner, and Ravi Ramamoorthi. Adaptive Wavelet Rendering. ACM Transactions on Graphics (SIGGRAPH ASIA 09), 28(5), December 2009.
- [4] Johannes Hanika, Marc Droske, and Luca Fascione. 2015. Manifold Next Event Estimation. Comput. Graph.
 Forum 34, 4 (July 2015), 87–97.

