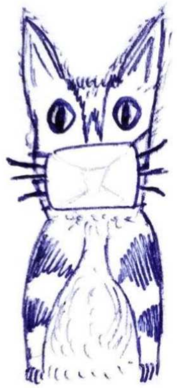
Rendering: Light

Adam Celarek



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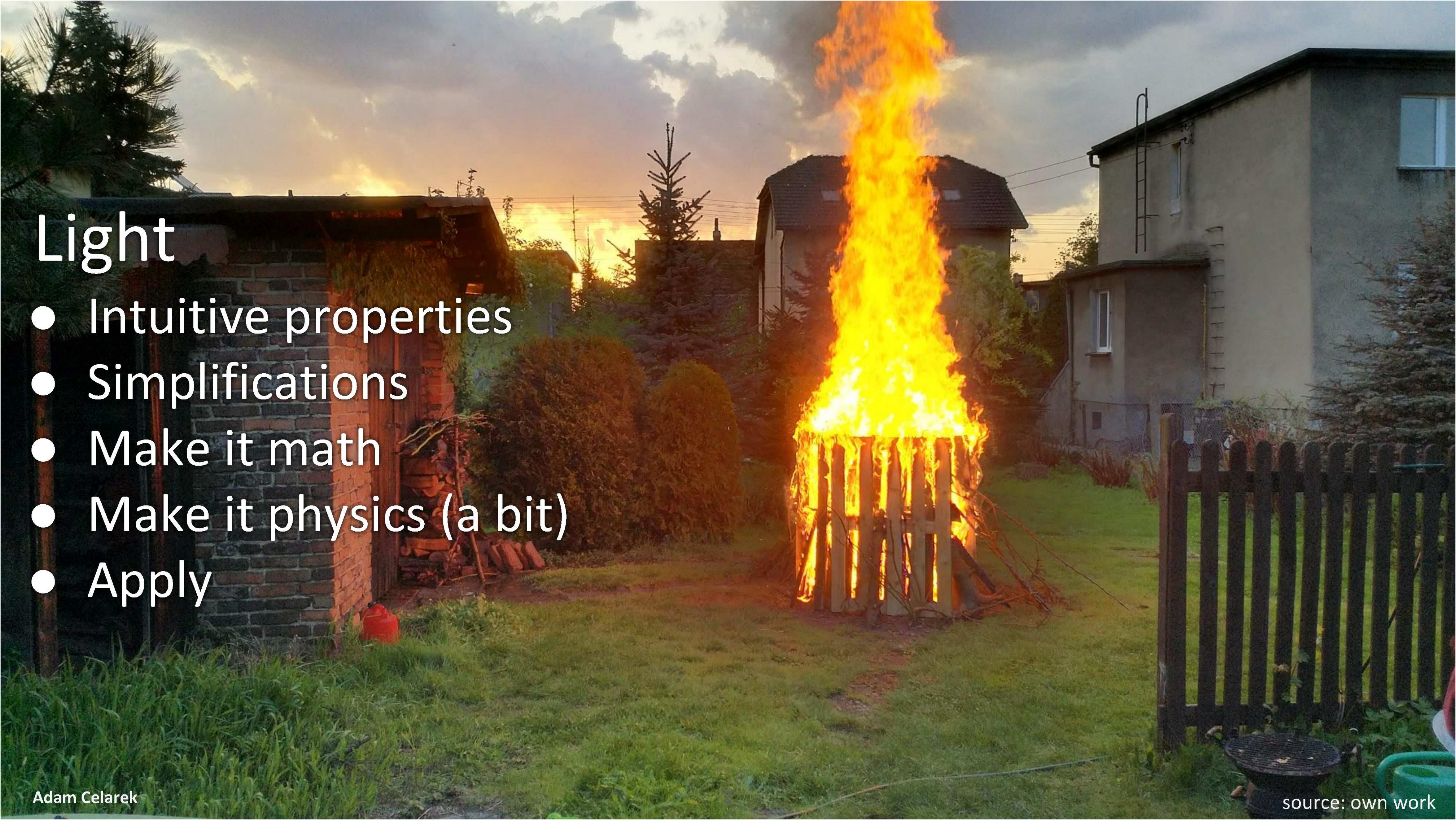
Light

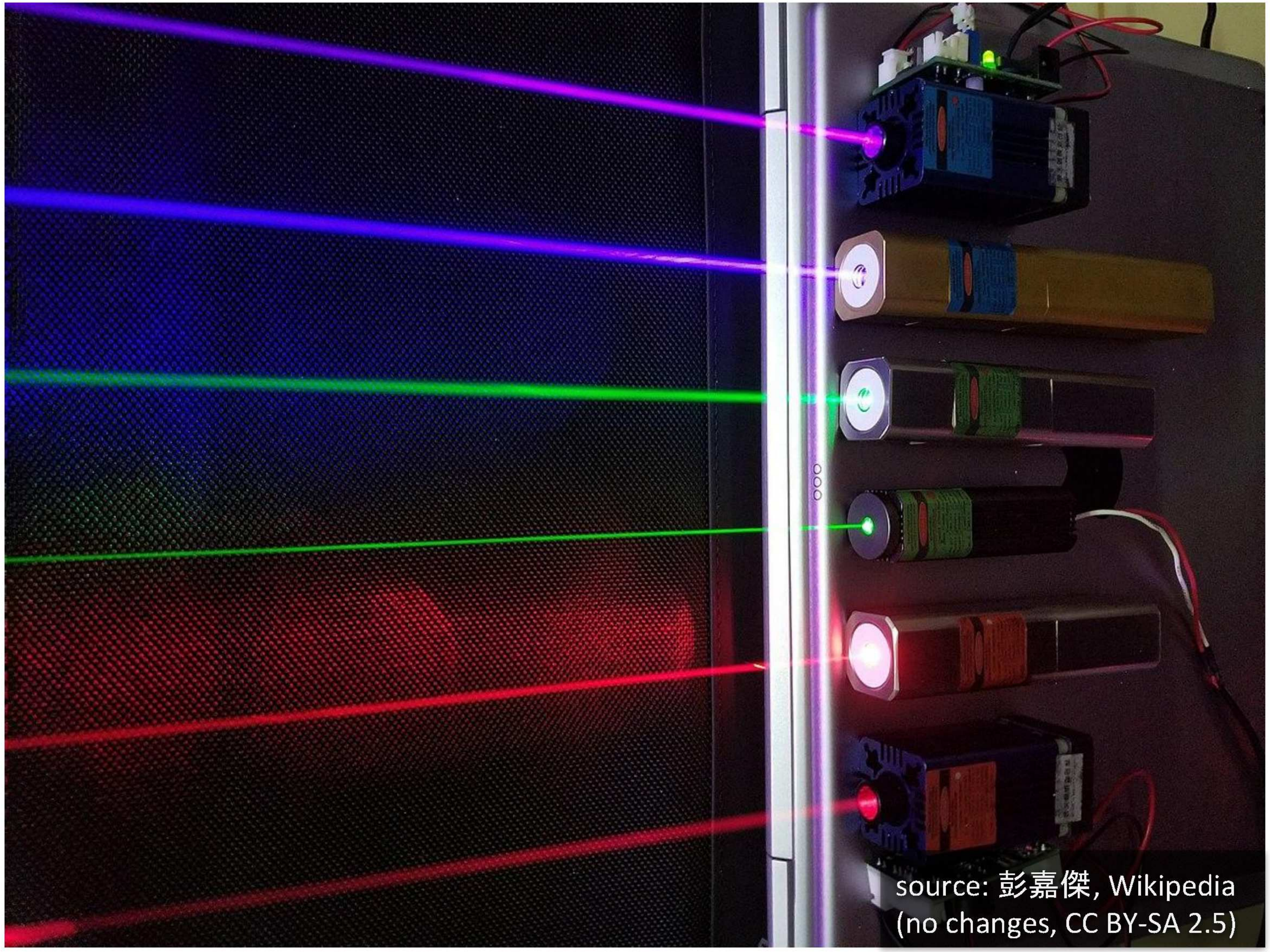
- Intuitive properties
- Simplifications
- Make it math
- Make it physics (a bit)
- Apply



source: own work

Intuitive properties

- It travels in straight lines



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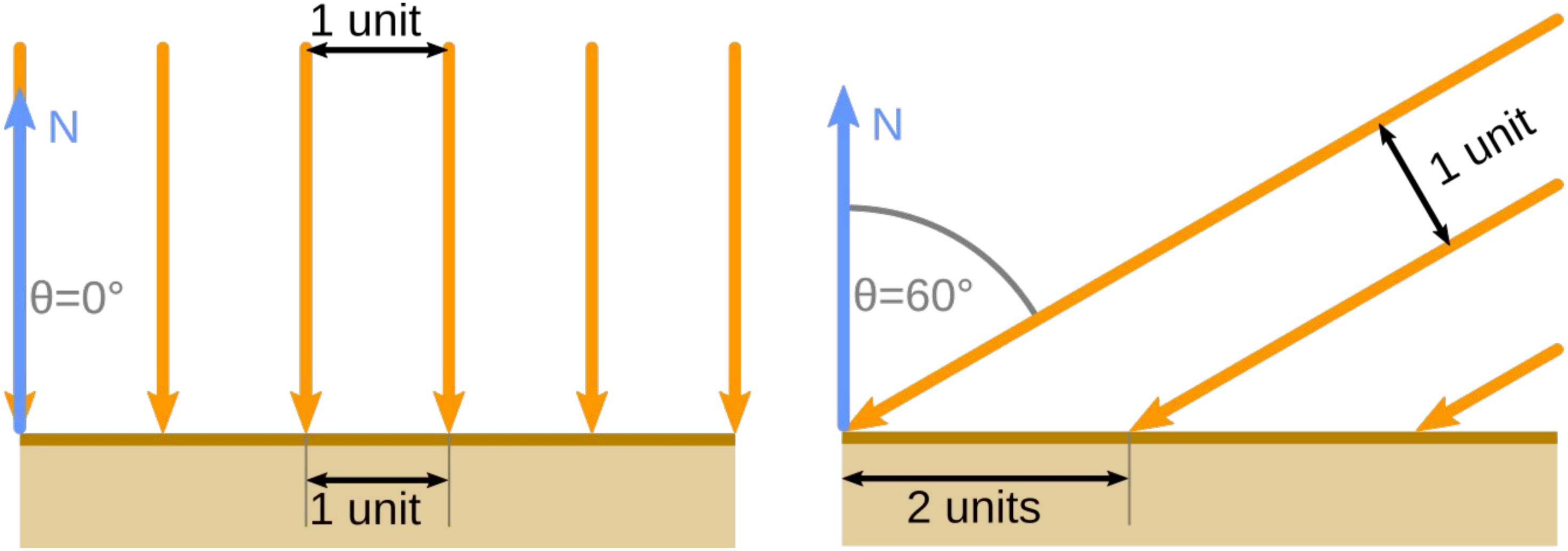
Intuitive properties

- It travels in straight lines
- Angle θ plays a role (cos(θ) rule)



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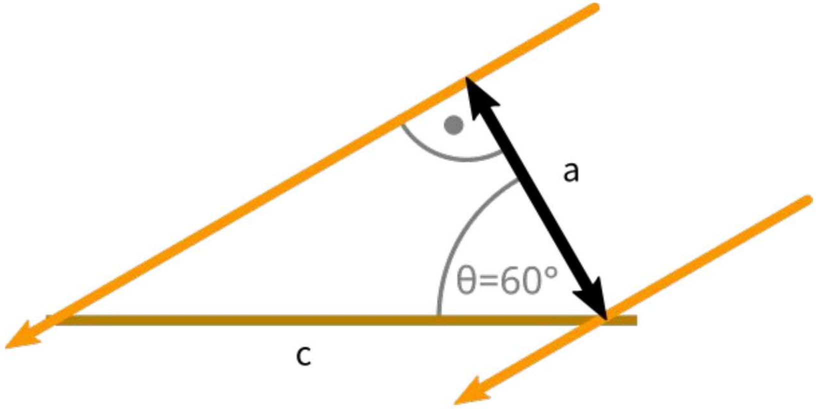
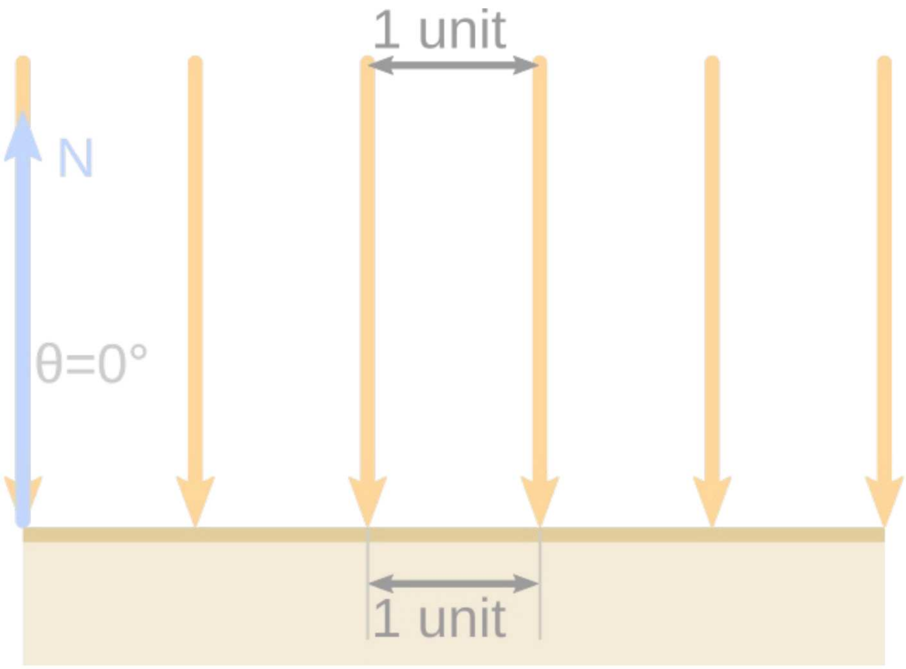
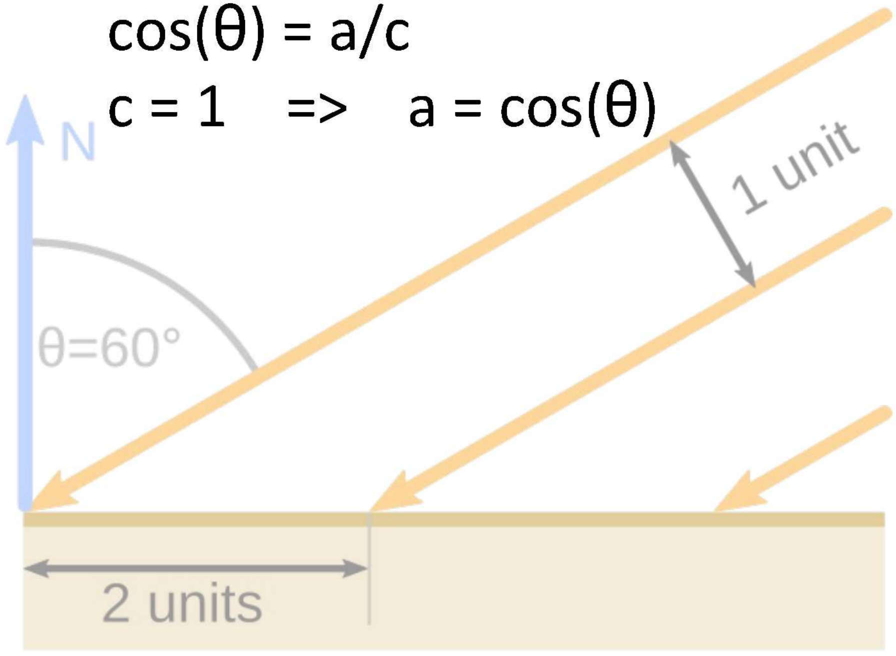


$\cos(\theta) = a/c$

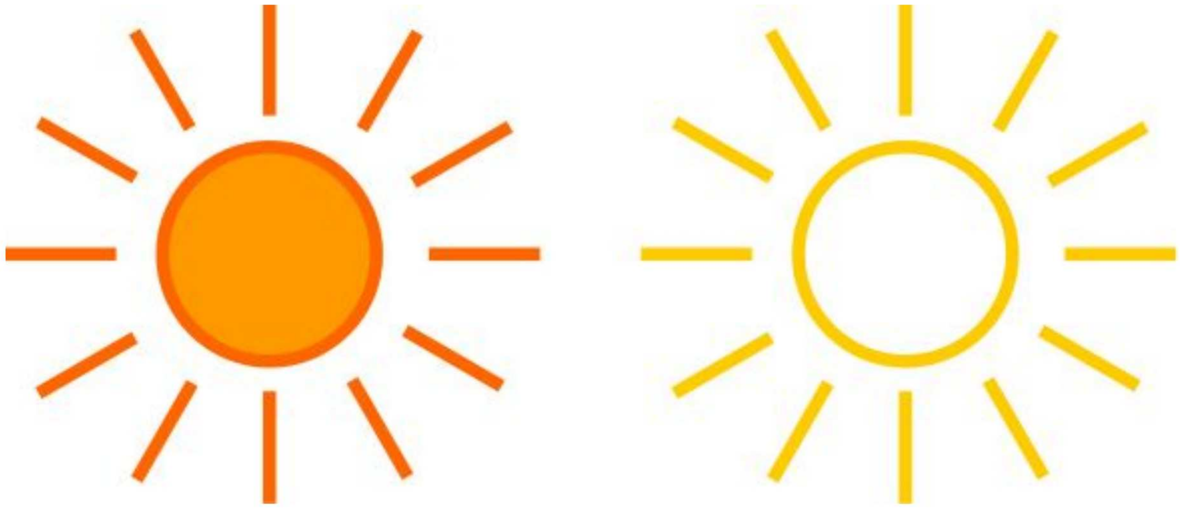
$c = 1 \quad \Rightarrow \quad a = \cos(\theta)$





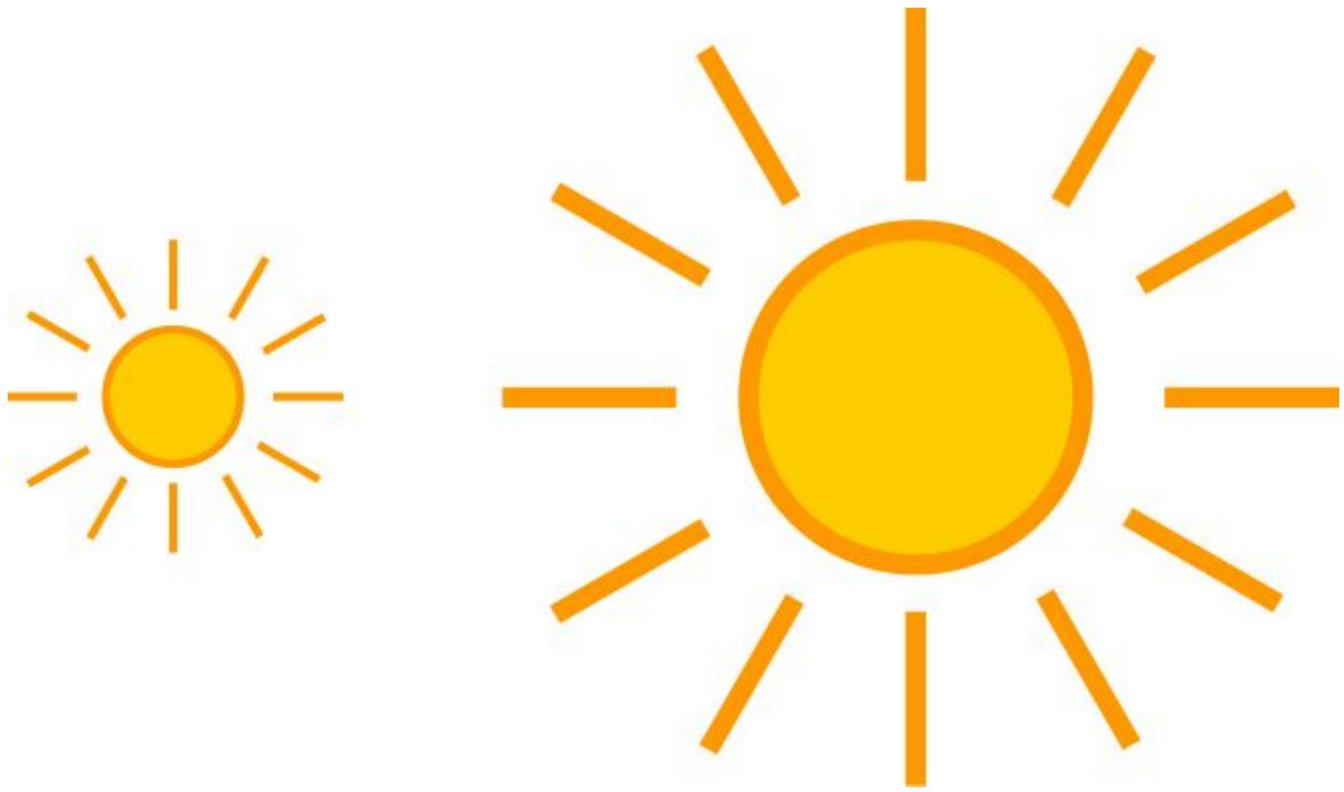
Intuitive properties

- It travels in straight lines
- Angle θ plays a role (cos(θ) rule)
- Intensity is linear (believe me)



Intuitive properties

- It travels in straight lines
- Angle θ plays a role (cos(θ) rule)
- Intensity is linear (believe me)
- Size of the light source



Intuitive properties

- It travels in straight lines
- Angle θ plays a role (cos(θ) rule)
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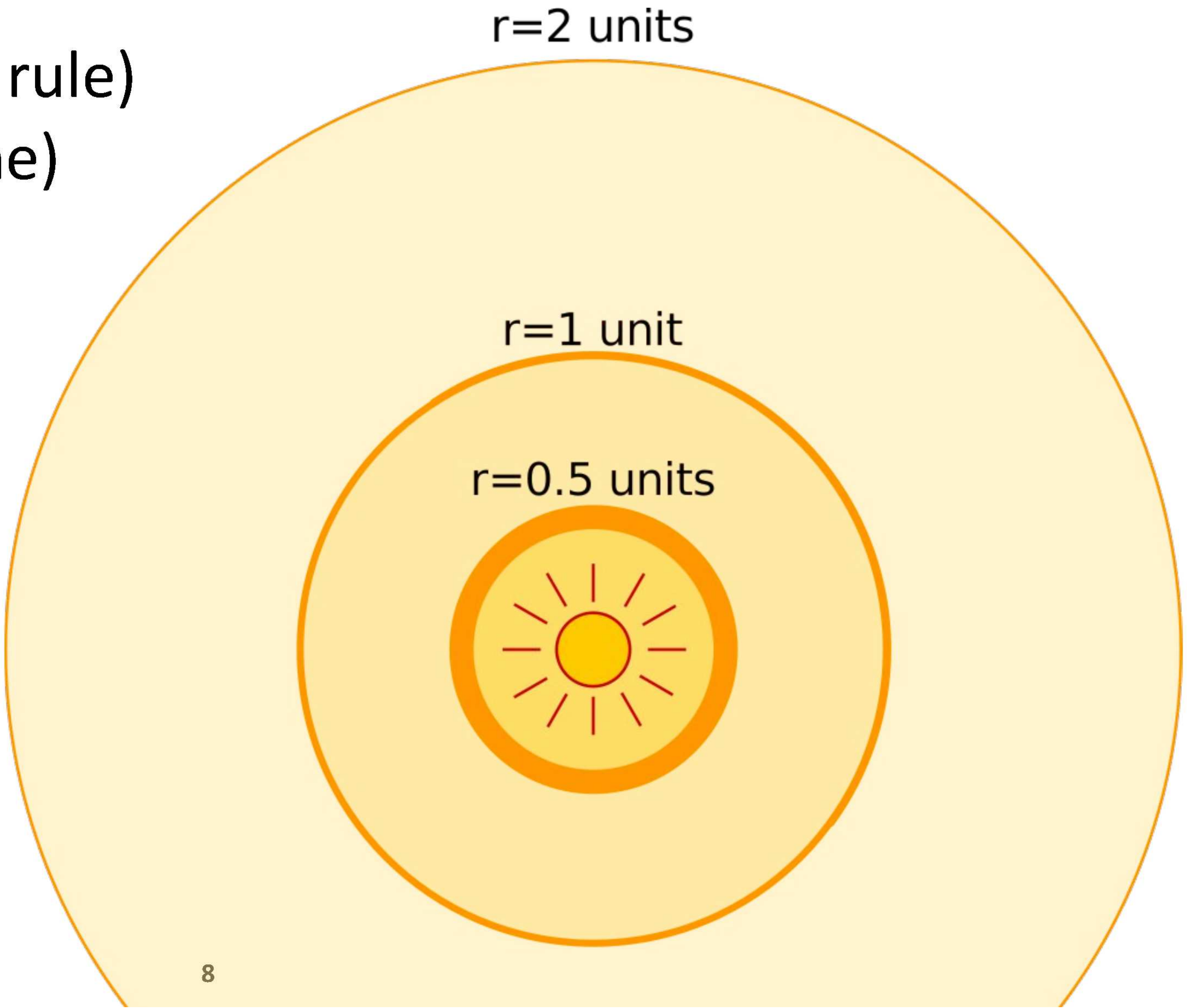
- Size of the light source
- Distance to light source



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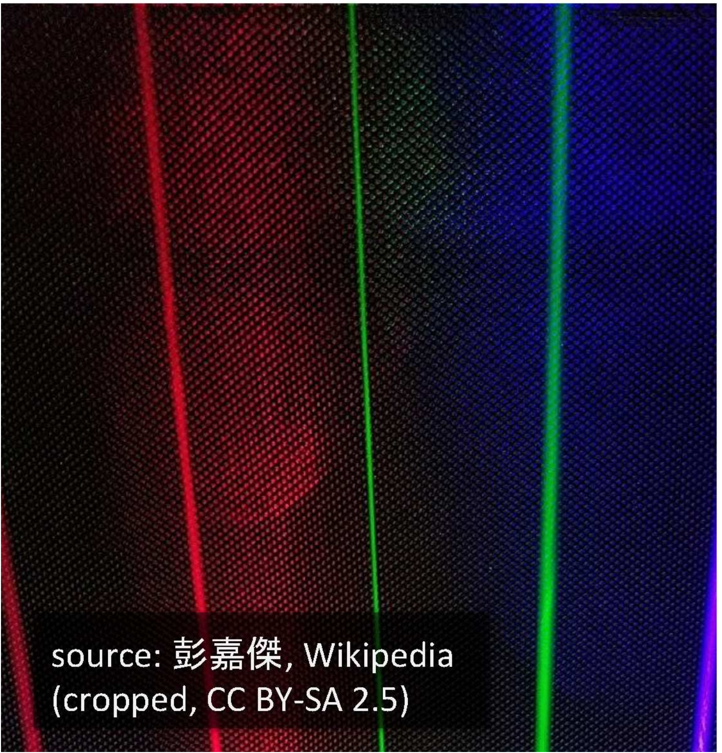

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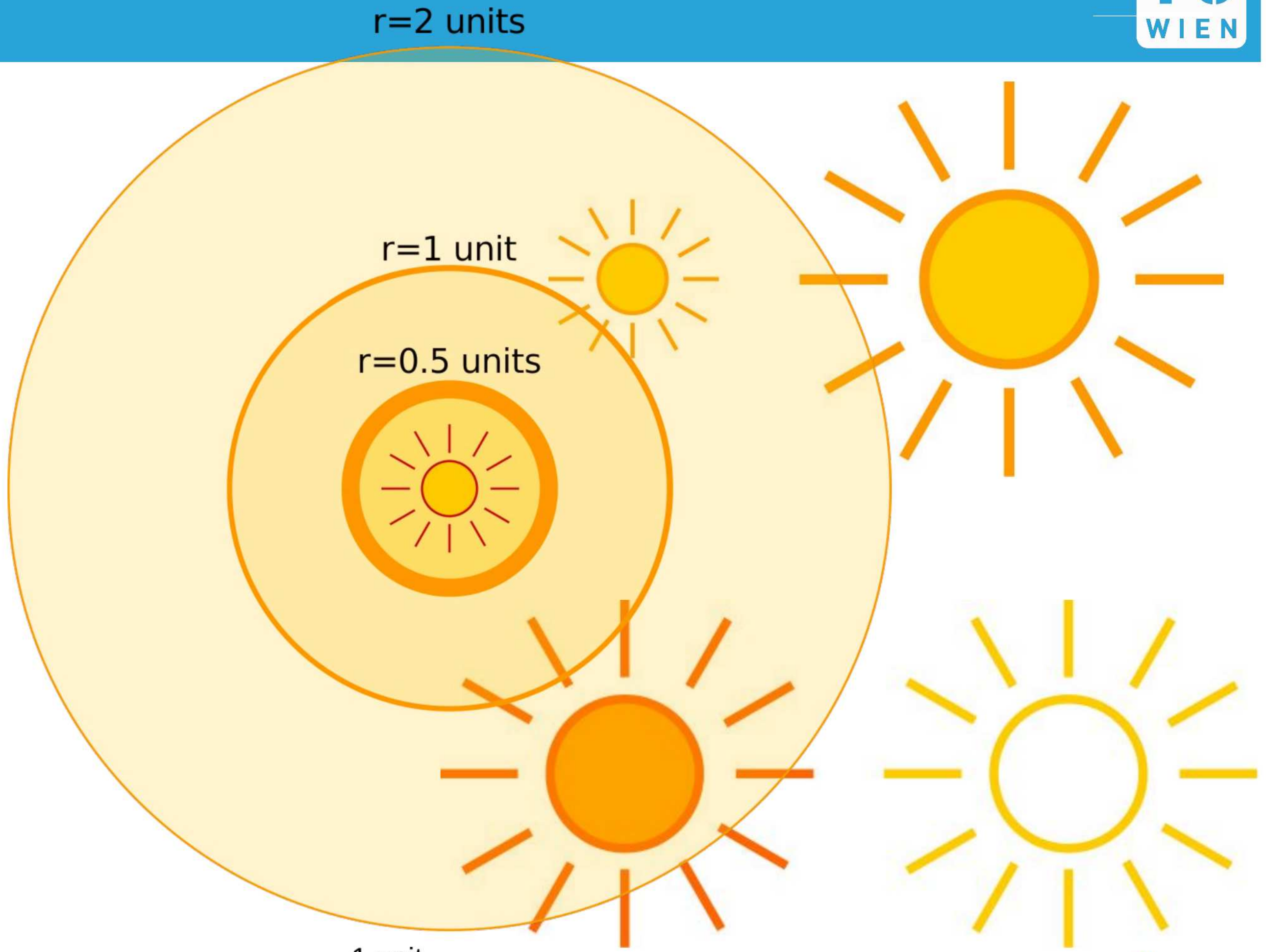
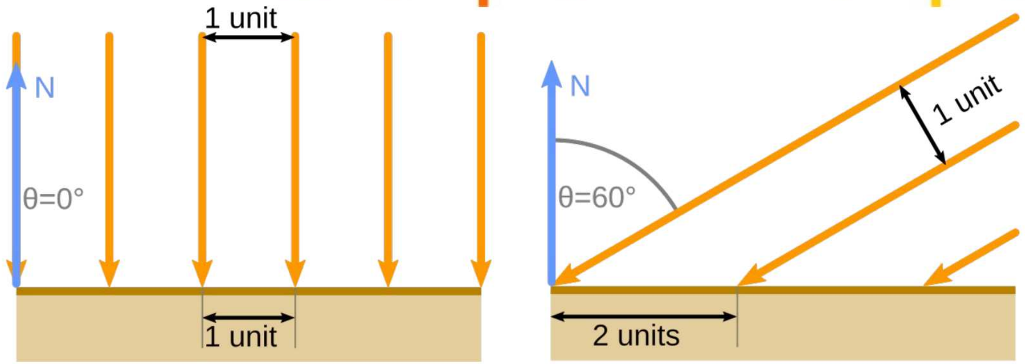


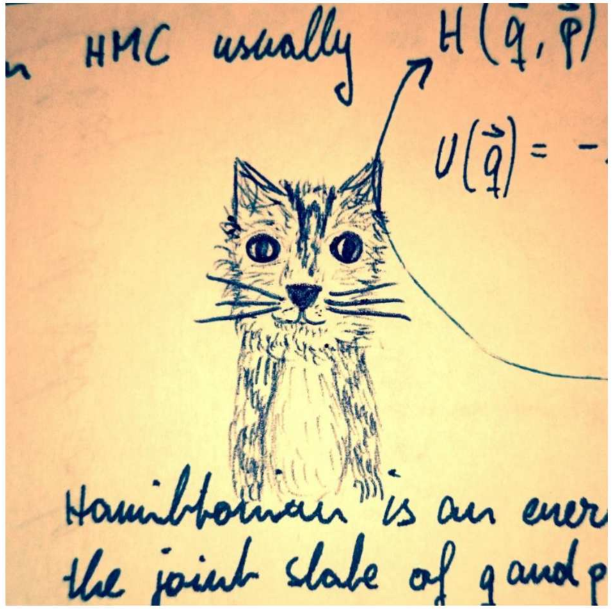


Intuitive properties

- How “bright” something is doesn’t directly tell you how brightly it *illuminates* something
  - The lamp appears just as bright from across the room and when you stick your nose to it (“intensity does not attenuate”)
  - Also, the lamp’s apparent brightness does not change much with the angle of exitance

Intuitive properties

- How “bright” something is doesn’t directly tell you how brightly it *illuminates* something
  - The lamp appears just as bright from across the room and when you stick your nose to it (“intensity does not attenuate”)
  - Also, the lamp’s apparent brightness does not change much with the angle of exitance
- **However**:
  - If you take the receiving surface further away, it will reflect less light and appear darker
  - If you tilt the receiving surface,it will reflect less light and appear darker



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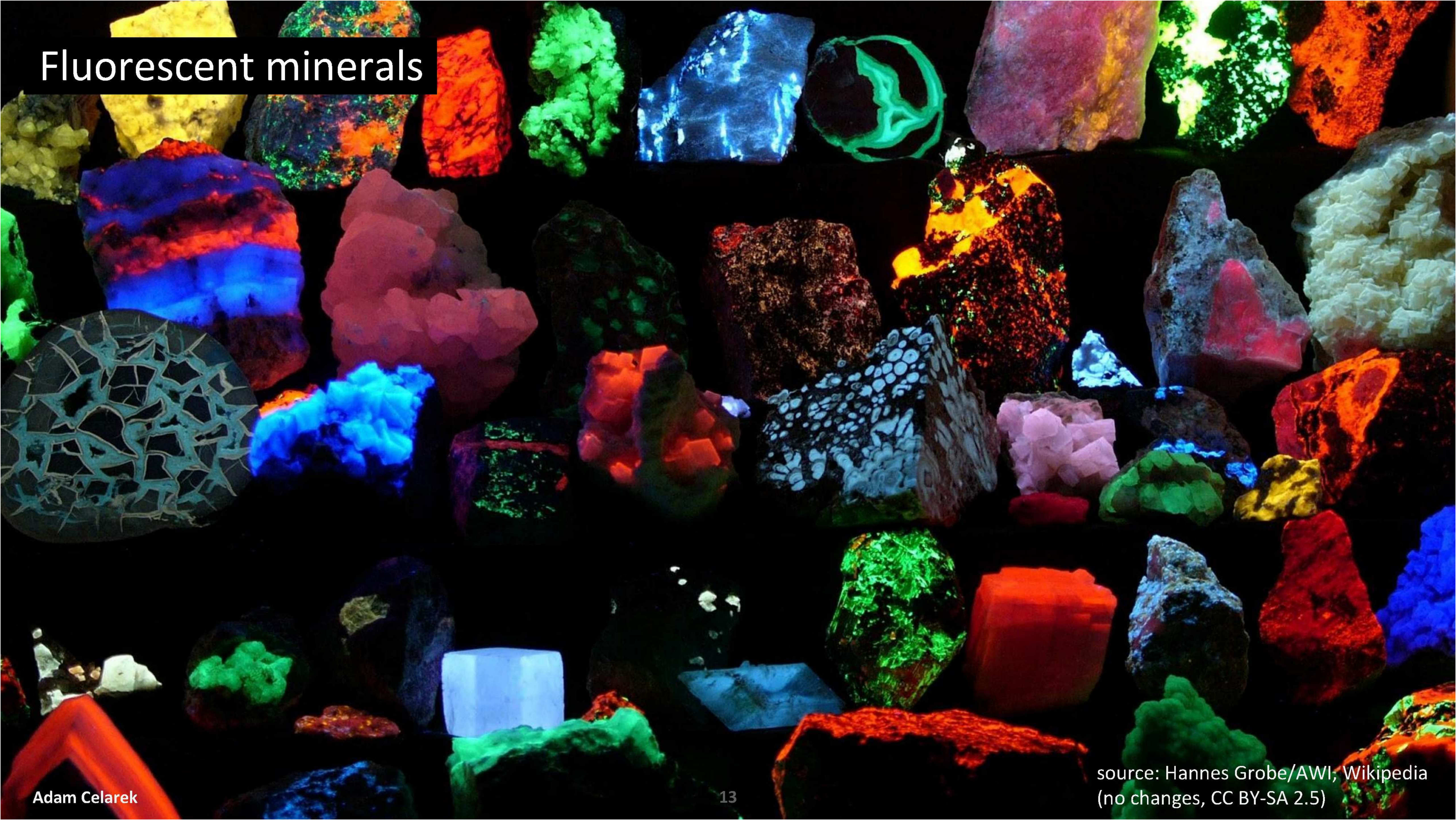
Light

travels in straight lines
cos rule
distance
intensity
size

Next: Less intuitive effects

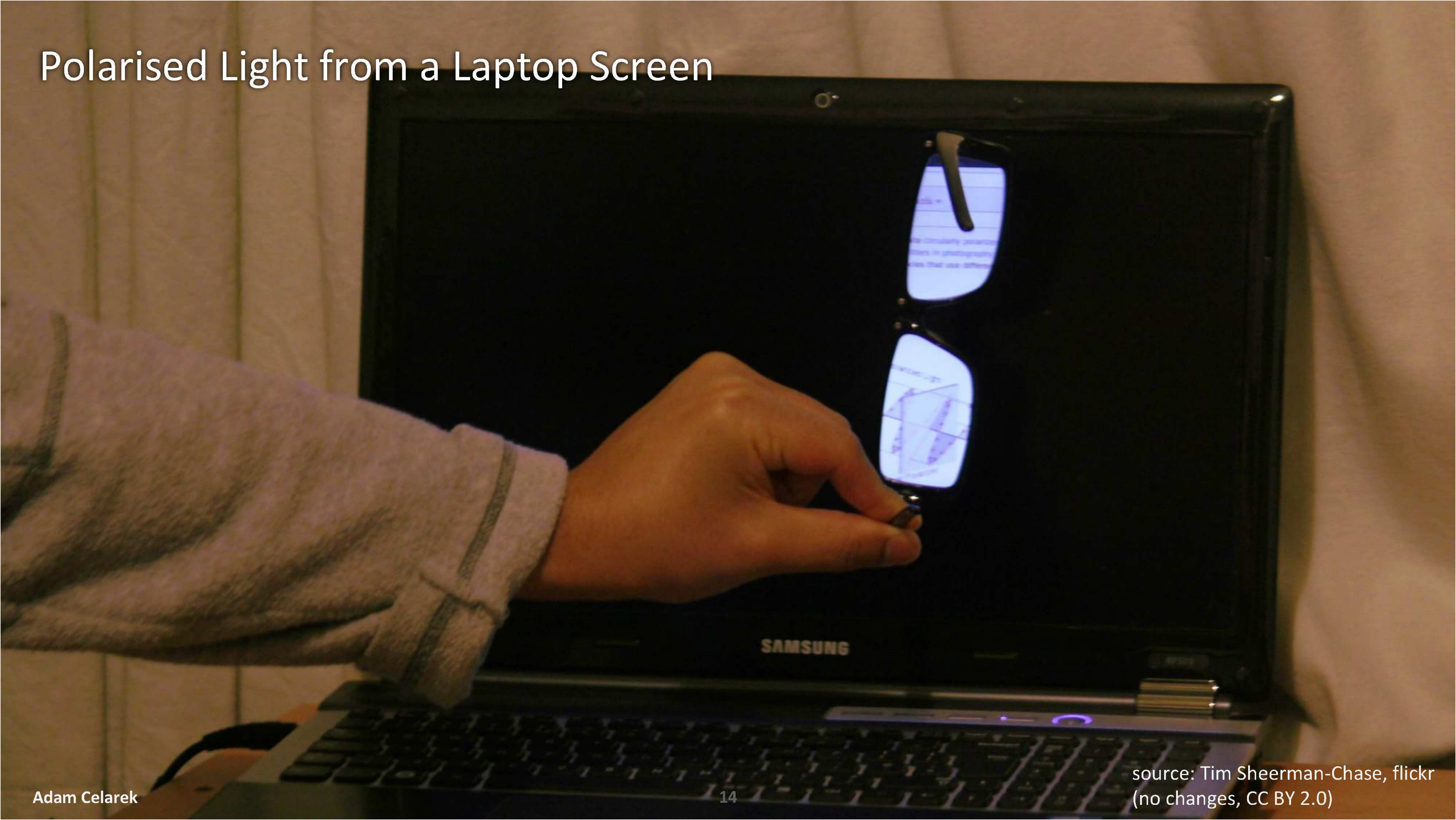
source: own work

Fluorescent minerals

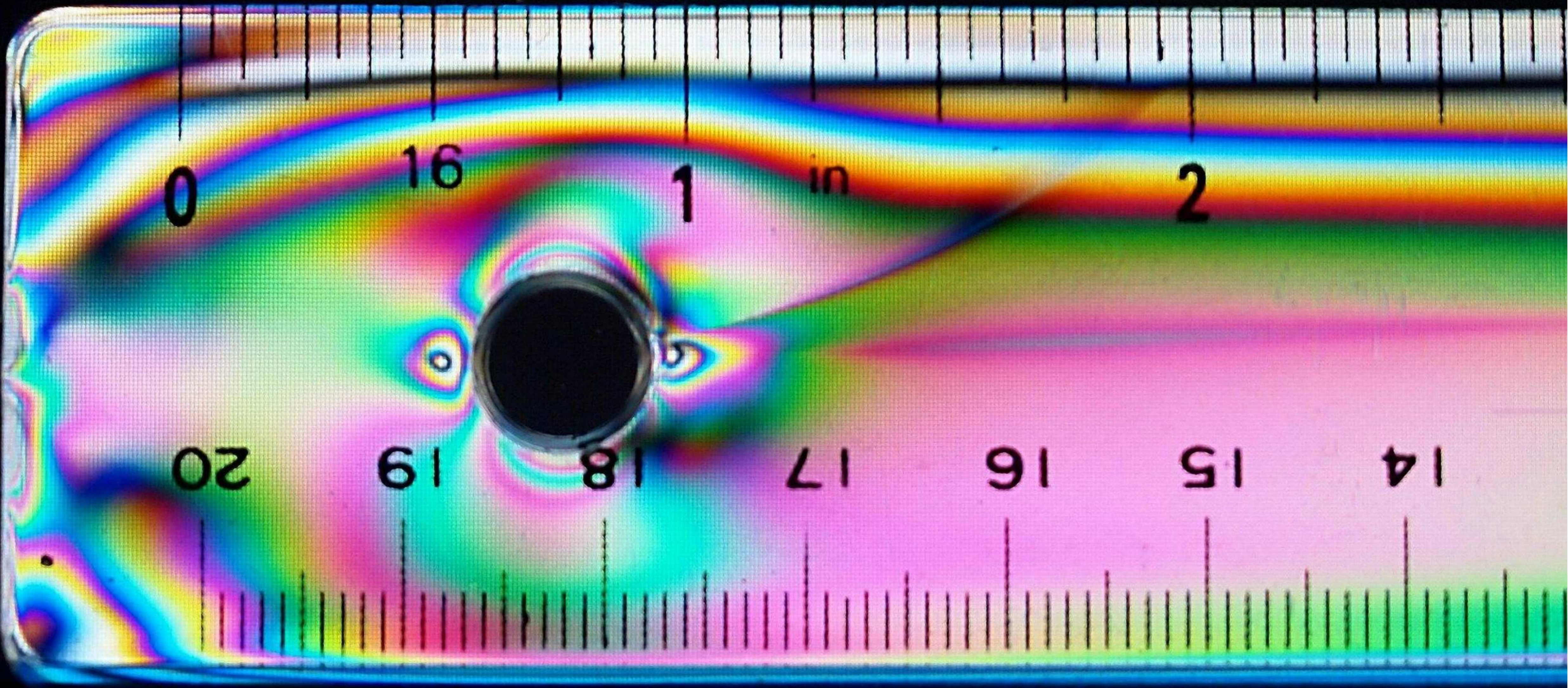


source: Hannes Grobe/AWI, Wikipedia

Polarised Light from a Laptop Screen

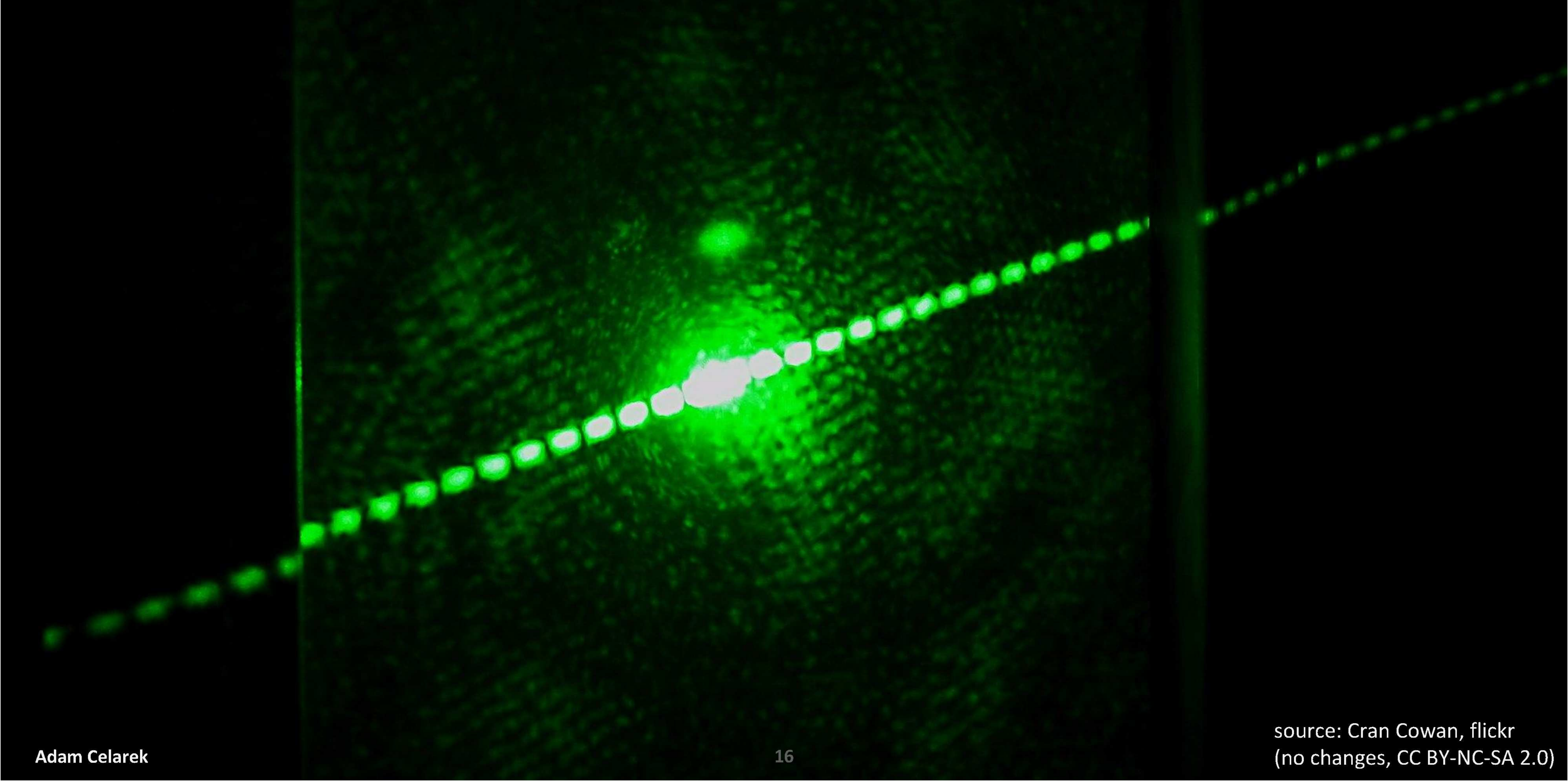


Stress Induced Birefringence:
Photoelasticity - perpendicular polarization



source: Cran Cowan, flickr

Quantum Entanglement: Self-interference of Photons



source: Cran Cowan, flickr

Simplifications (things that we will not do)

- We use ray optics (also called geometrical optics)
  - Doesn't account for phenomena like diffraction or interference (rendering optical discs is hard)
- No energy transfer between frequencies (fluorescence)
- In this course we disregard the spectrum and just compute RGB separately (though production renderers often simulate a spectrum)
- And we will ignore polarisation.

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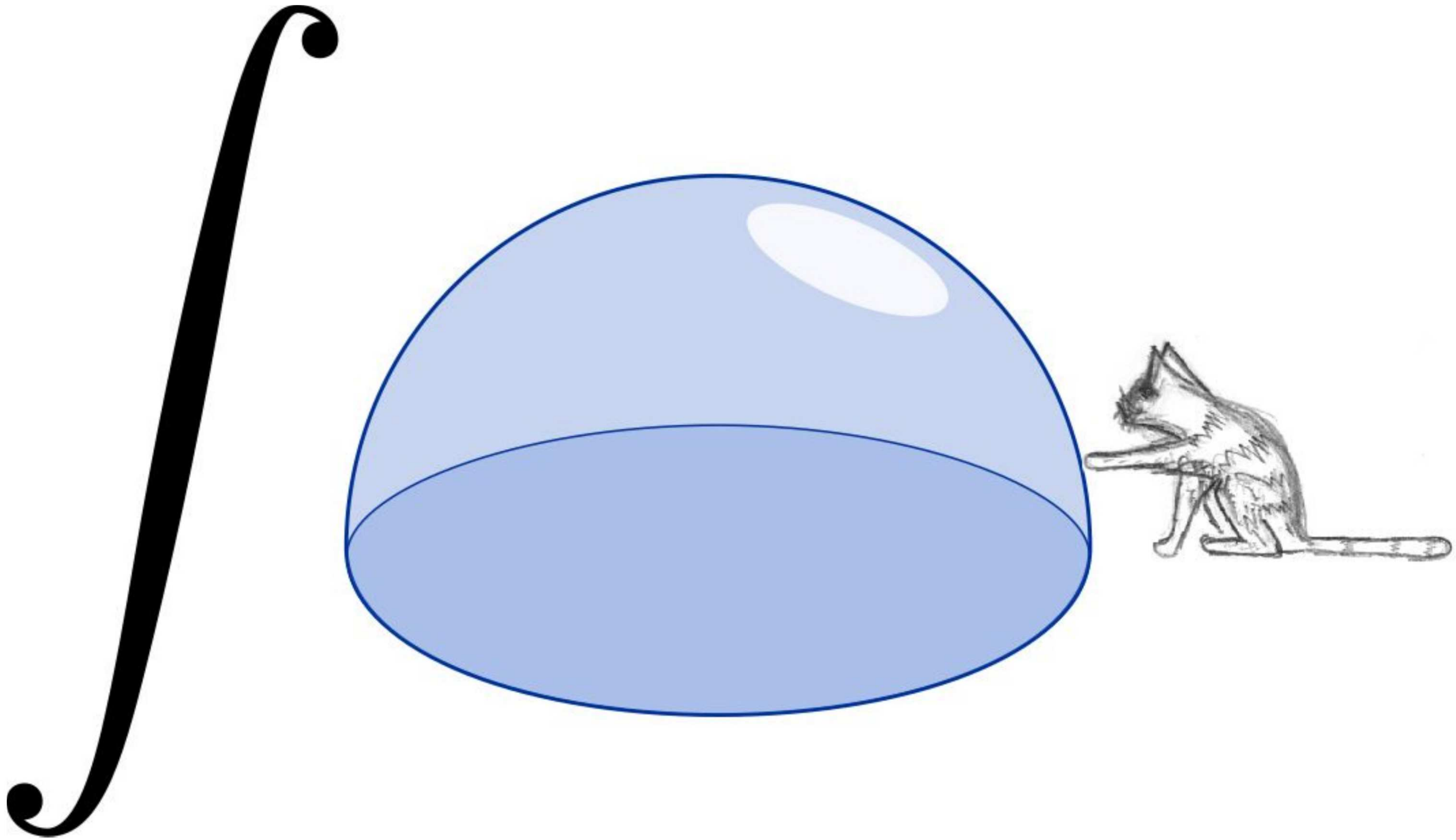
Make it math



Make it math

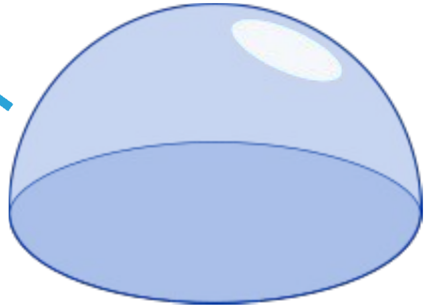
- Don’t be afraid of integrals, we’ll learn how to compute them later
- Basically, look into all directions and sum up all incoming light

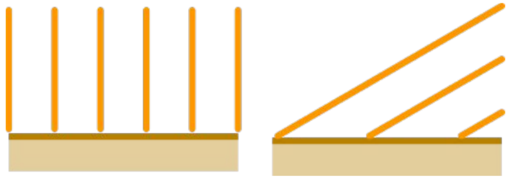
Light arriving at point x

Light from direction ω

Solid angle (next)

$$L_i(x) = \int_\Omega L_i(x, \omega) \cos(\theta_x) \, \mathrm{d}\omega$$





(not useful for rendering yet)

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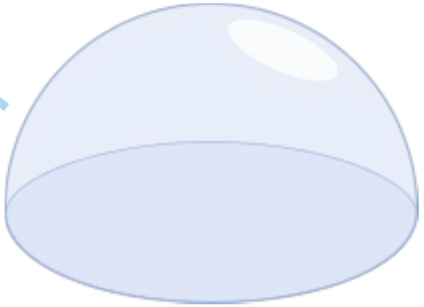

compare to a 1d integral from basic calculus

$$A = \int_a^b f(x)\, \mathrm{d}x$$

Make it math

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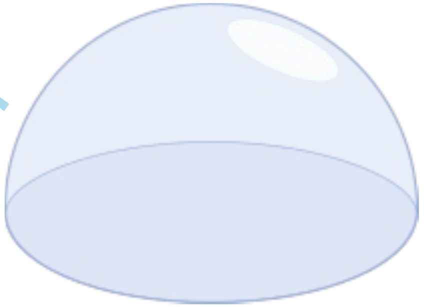
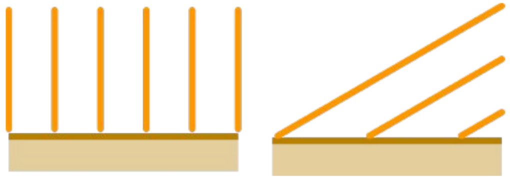
Light arriving at point x

Light from direction ω

Solid angle (next)

$$L_i(x) = \int_\Omega L_i(x, \omega)(\omega \cdot n)\,\mathrm{d}\omega$$





(not useful for rendering yet)

Make it math

- What's going on with that object size, distance etc?

- "Illumination power" is determined by the solid angle subtended by the light source (simple, how big something looks).

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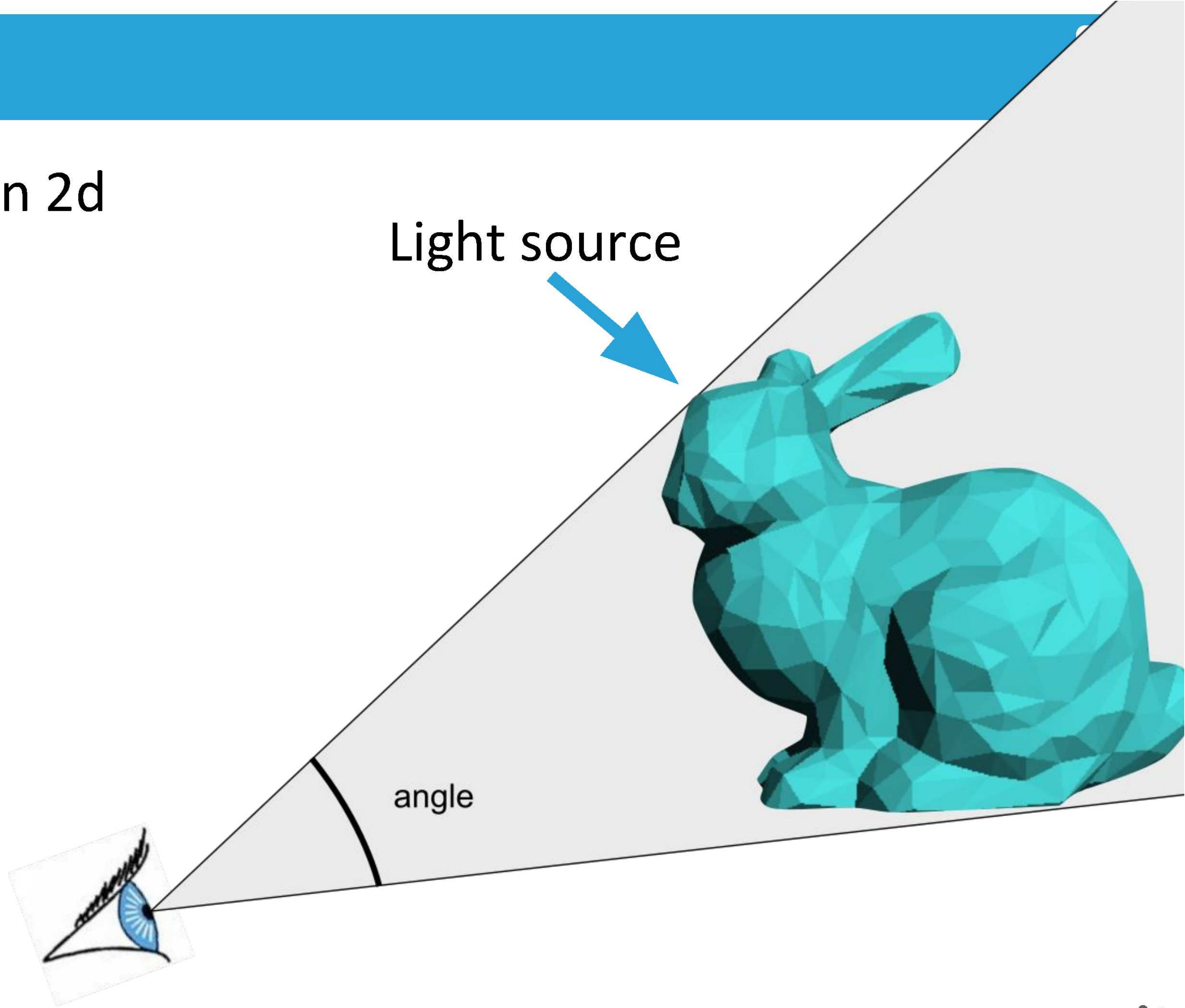
Make it math

- How big something looks in 2d



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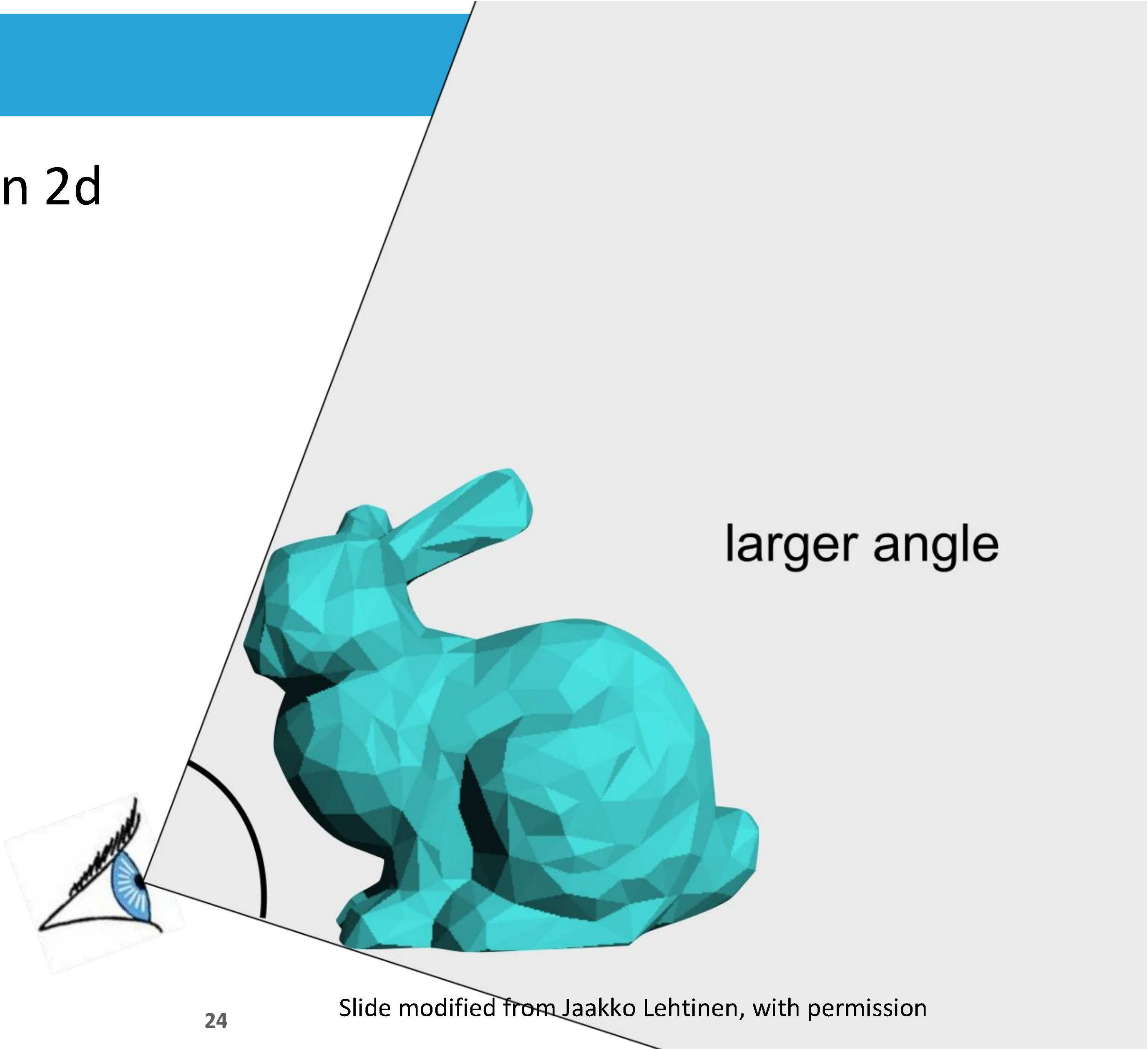
Make it math

- How big something looks in 2d



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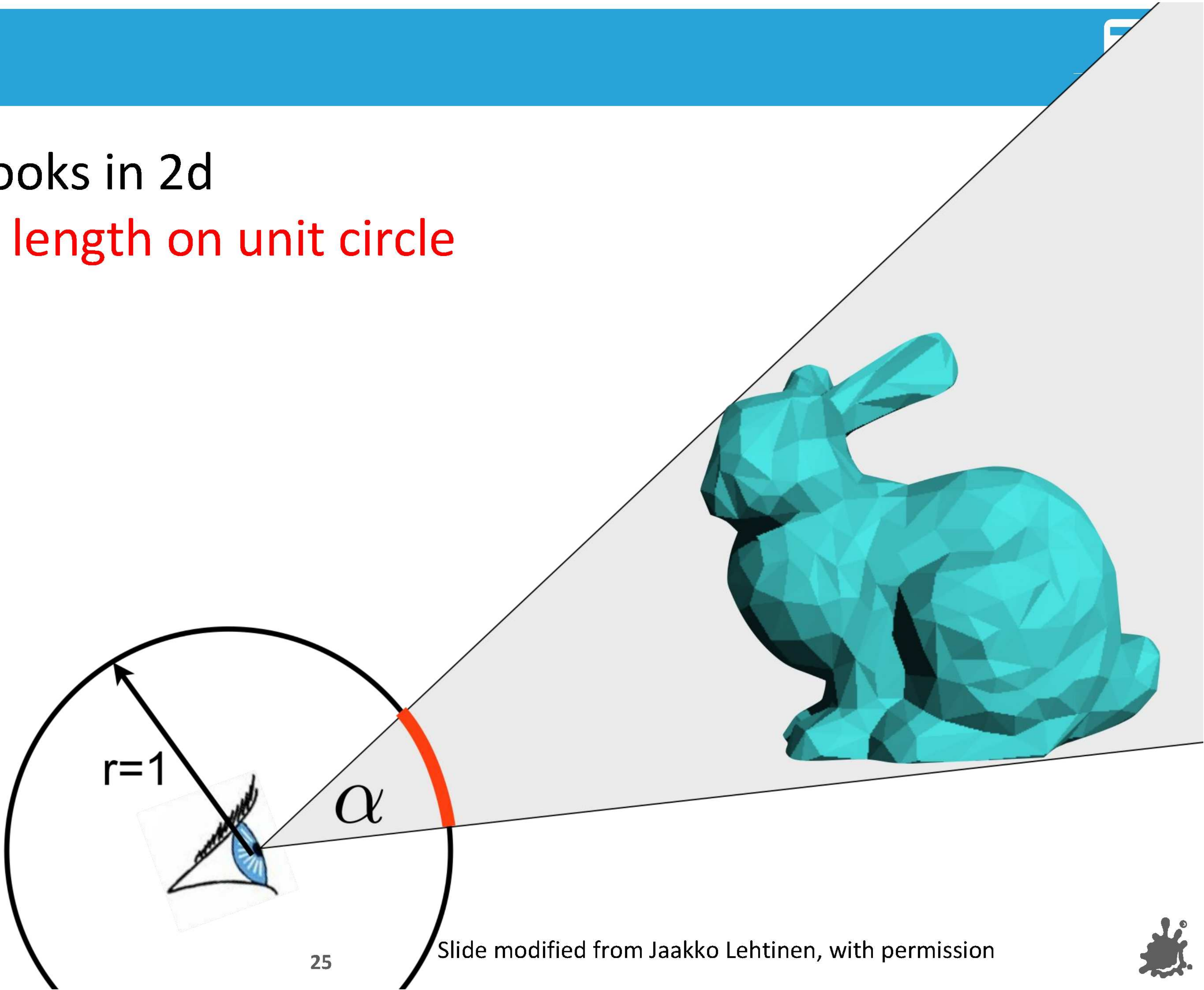
Make it math

- How big something looks in 2d
- Angle α in radians ⇔ length on unit circle
- Full circle is 2π



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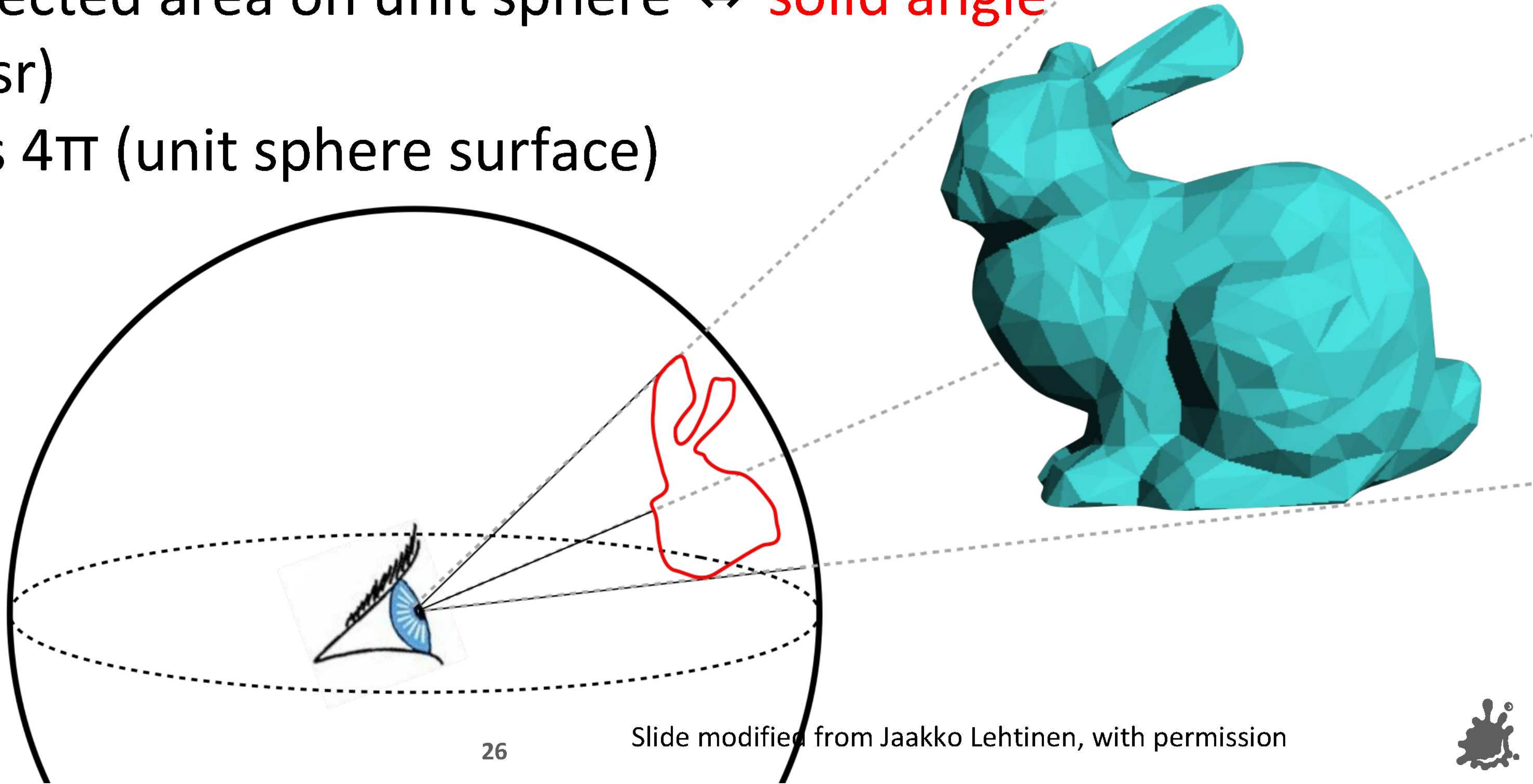
Make it math

- How big something looks in **3d**
- replace unit circle with unit sphere
- Same thing: projected area on unit sphere ⇔ solid angle
- Unit: steradian (sr)
- Full solid angle is 4π (unit sphere surface)



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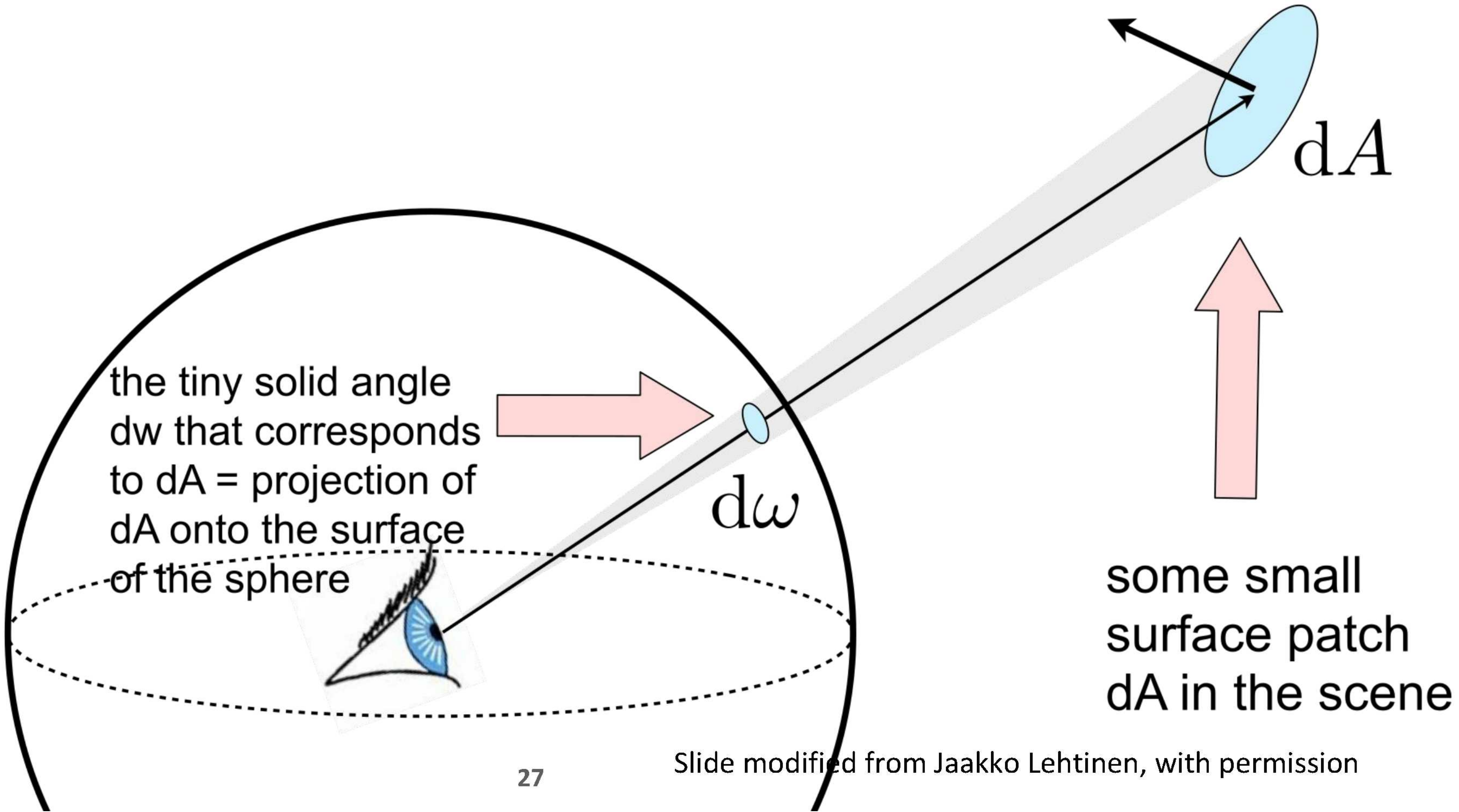
Relationship between a surface patch and the solid angle

=> what determines the area of the projected patch (solid angle)



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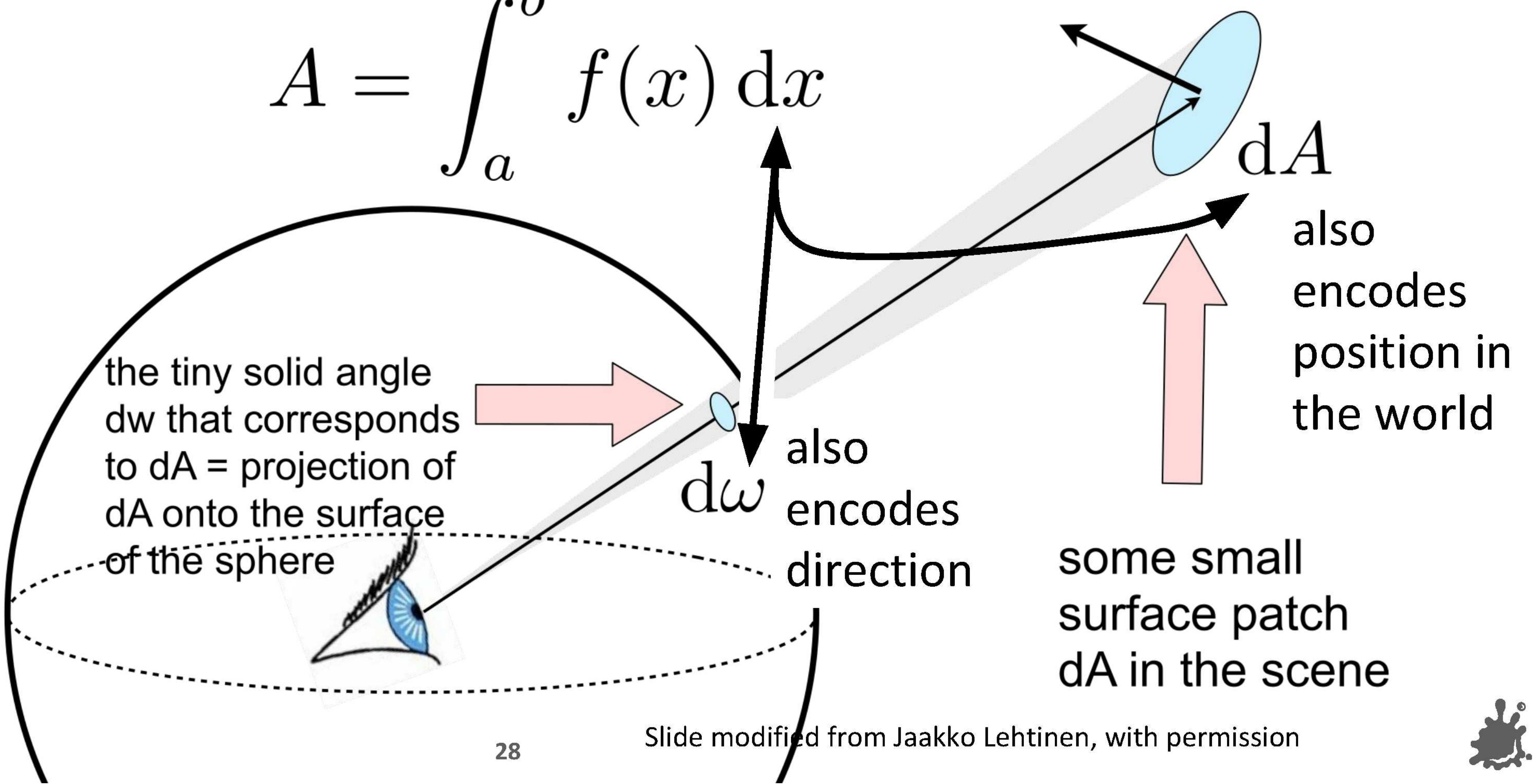

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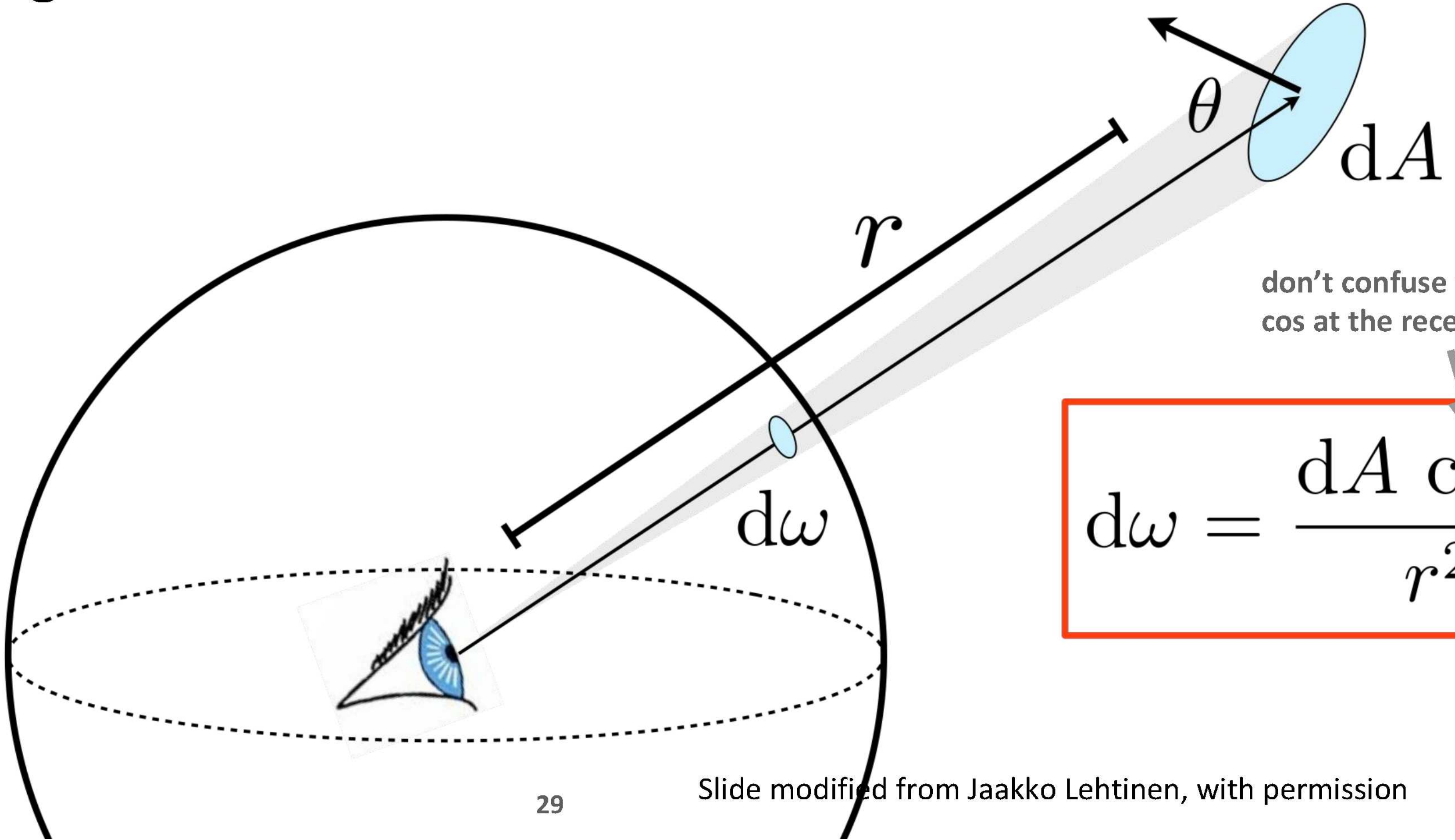
Relationship between a surface patch and the solid angle

It holds for infinitesimally small surface patches dA and the corresponding differential solid angles dω



$$\mathrm{d}\omega = \frac{\mathrm{d}A\,\cos\theta}{r^2}$$

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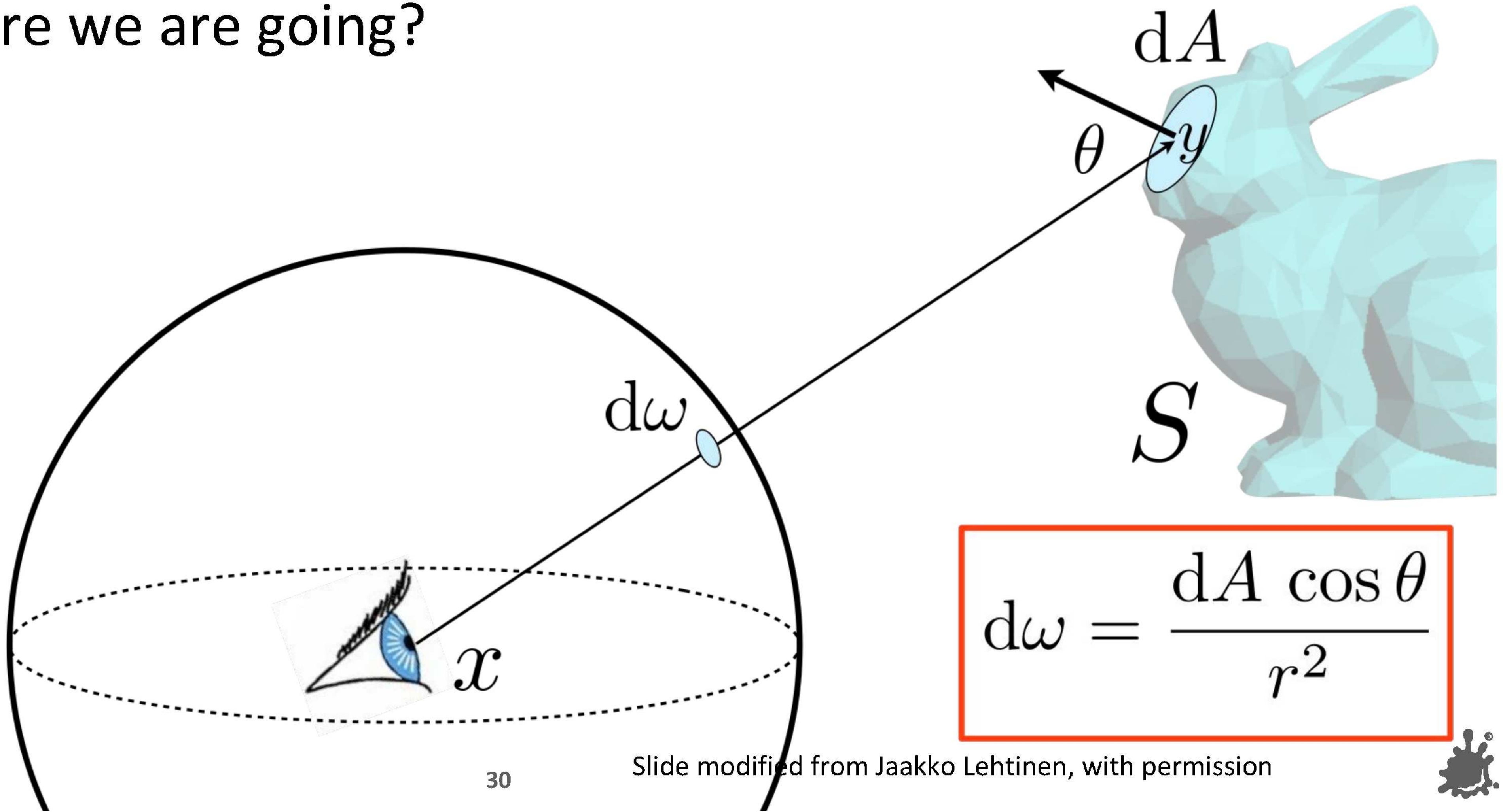
Larger Surfaces

Actual surfaces consist of infinitely many tiny patches dA

-- do you see where we are going?



$$\boxed{\mathrm{d}\omega = \frac{\mathrm{d}A \cos\theta}{r^2}}$$

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Make it math

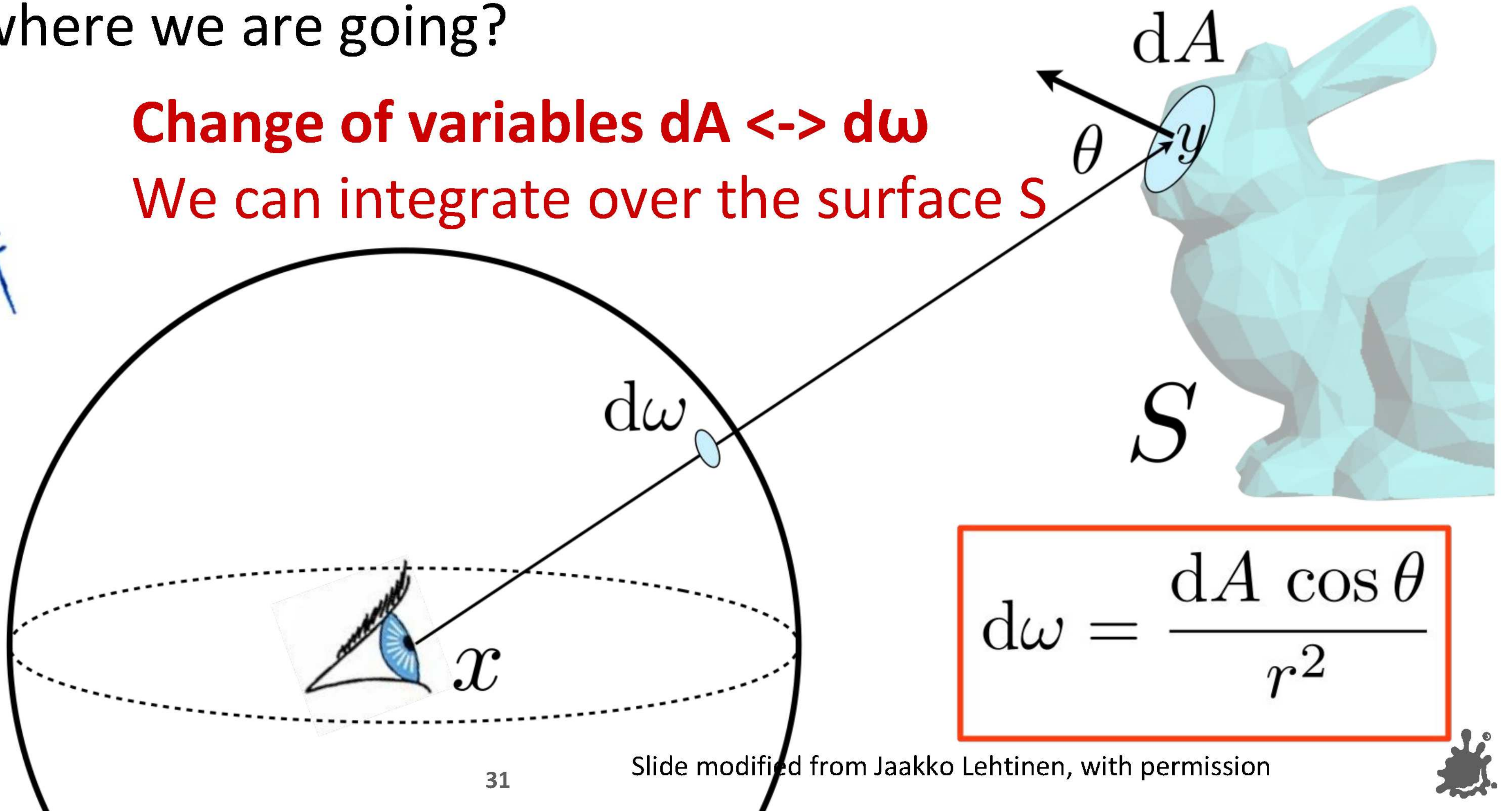
Larger Surfaces

Actual surfaces consist of infinitely many tiny patches dA

------- do you see where we are going?

Change of variables dA <-> dω

We can integrate over the surface S



$$\boxed{\mathrm{d}\omega = \frac{\mathrm{d}A \cos\theta}{r^2}}$$

Make it math

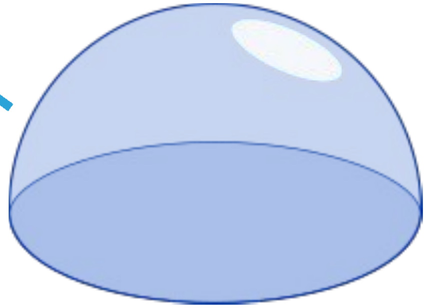
We have seen this before, but now we want to integrate over a single light surface. How do we need to change the formula?

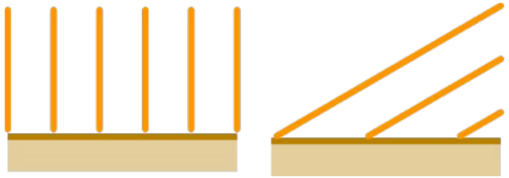
Light arriving at point x

Light from direction ω

Solid angle (just before)

$$L_i(x) = \int_\Omega L_i(x, \omega) \cos(\theta_x) \, \mathrm{d}\omega$$



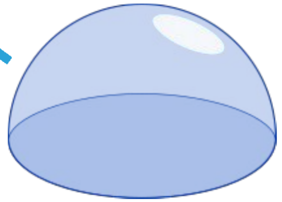


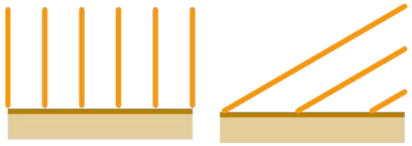
(not useful for rendering yet)

Light arriving at point x

Light from direction ω

$$L_i(x) = \int_\Omega L_i(x, \omega) \cos(\theta_x) \, d\omega$$





Solid angle (just before)

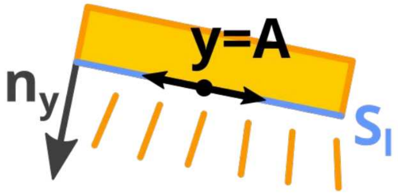
Light from source [l] arriving at point x

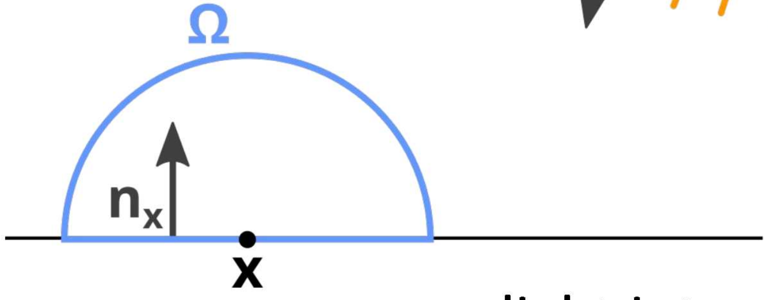
$$L_i^{[l]}(x) = \int_{S_l} L_e^{[l]}(y) \cos(\theta_x) \boxed{\frac{\cos(\theta_y)}{r^2} \, dA_y}$$

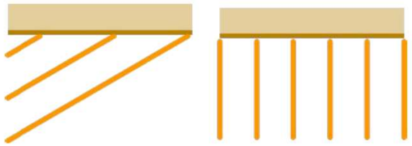
light intensity at position y on the surface

Make it math

situation





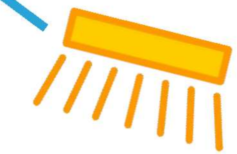


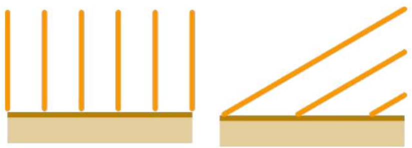
emitter cos(θ)

light intensity at position y
on the surface

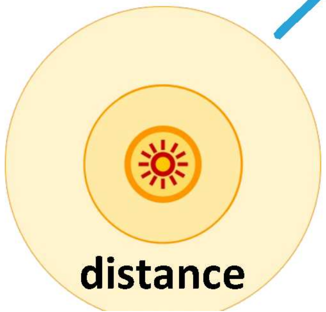
$$L_i^{[l]}(x) = \int_{S_l} L_e^{[l]}(y)\left(\frac{y-x}{|y-x|}\cdot n_x\right)\frac{\left(\frac{x-y}{|x-y|}\cdot n_y\right)}{|x-y|^2}\,\mathrm{d}A_y$$

(not useful for rendering yet)





receiver cos(θ)



Light integral

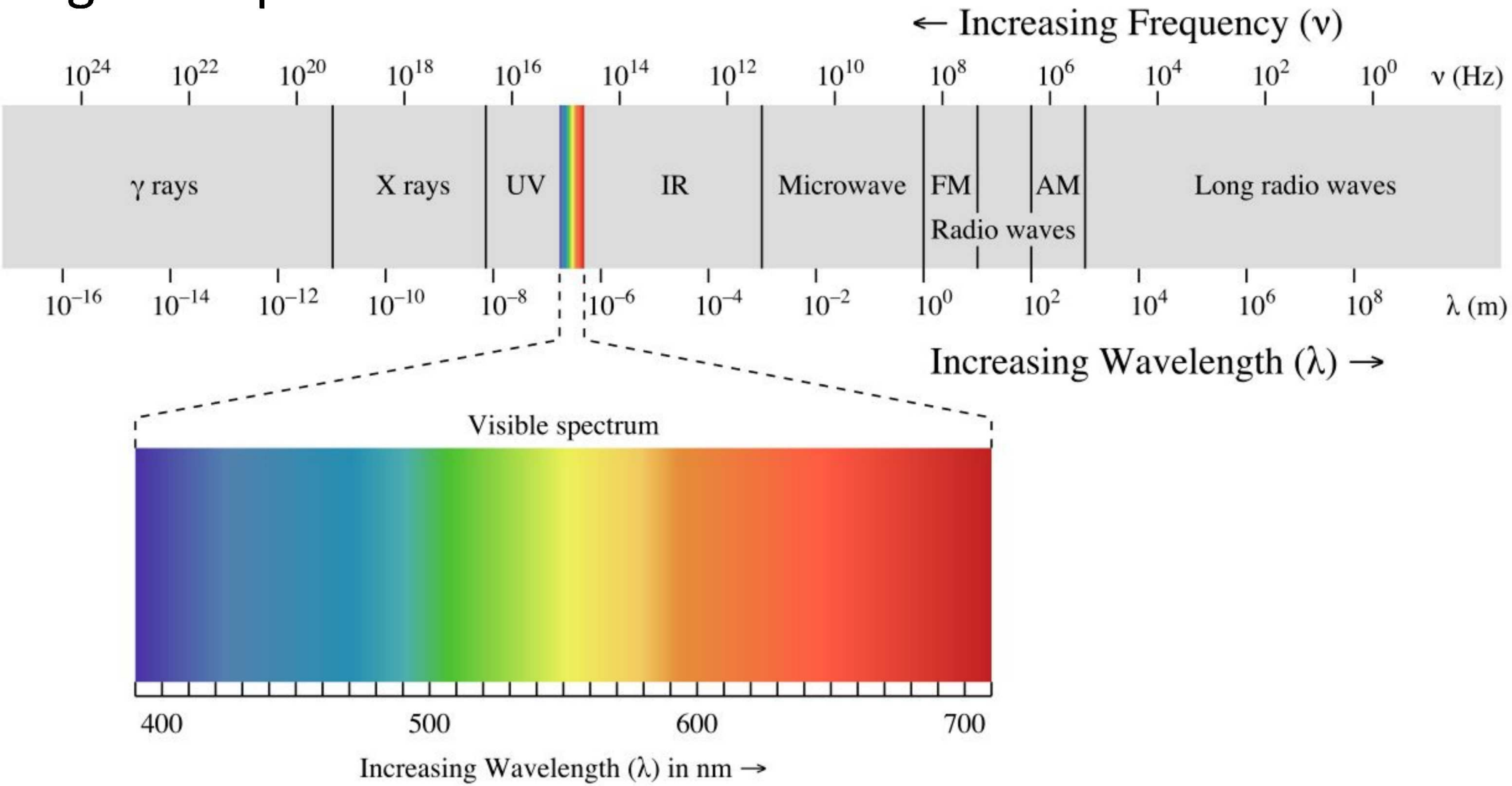
How to compute the amount
of light that reaches a certain point?

Next: Physics

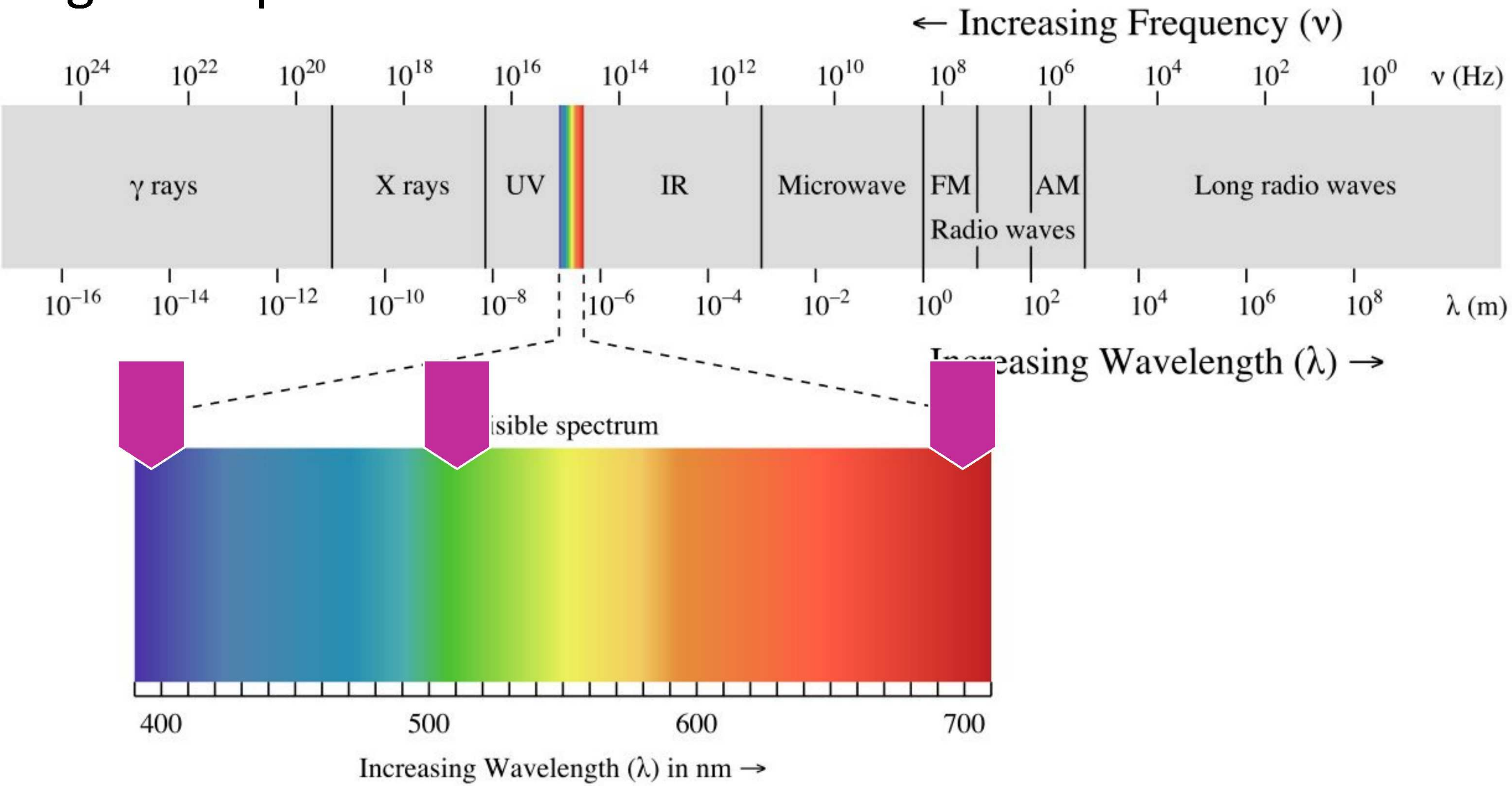
Make it physics (a bit)

- Electromagnetic spectrum
- Radiometry and photometry
  - Units and naming
  - How is that stuff perceived in the human eye
- Radiance (constant along straight lines)
- Rendering
  - Irradiance
  - Materials
  - White furnace test (energy conservation)

Electromagnetic spectrum



Electromagnetic spectrum



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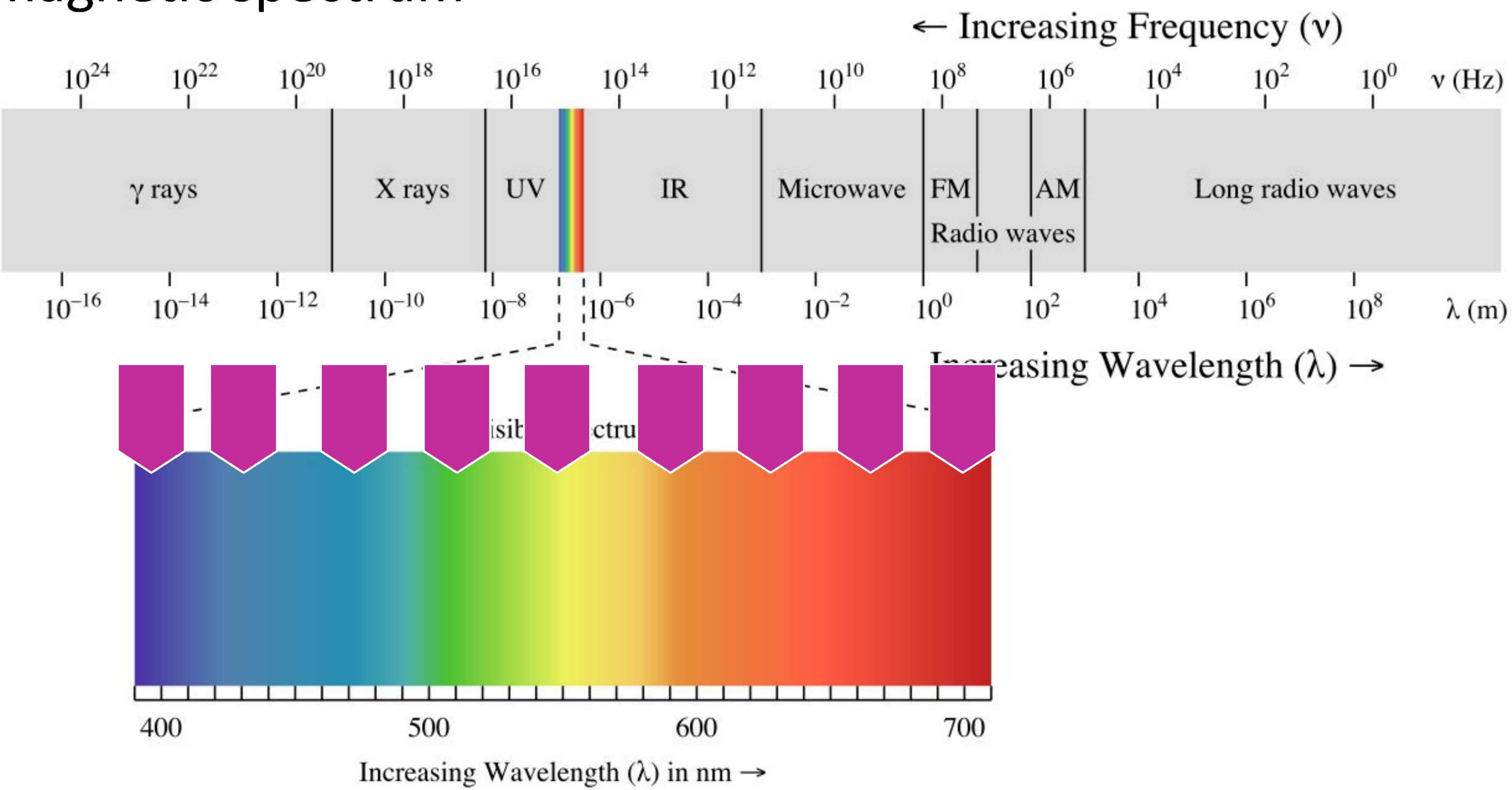
Electromagnetic spectrum



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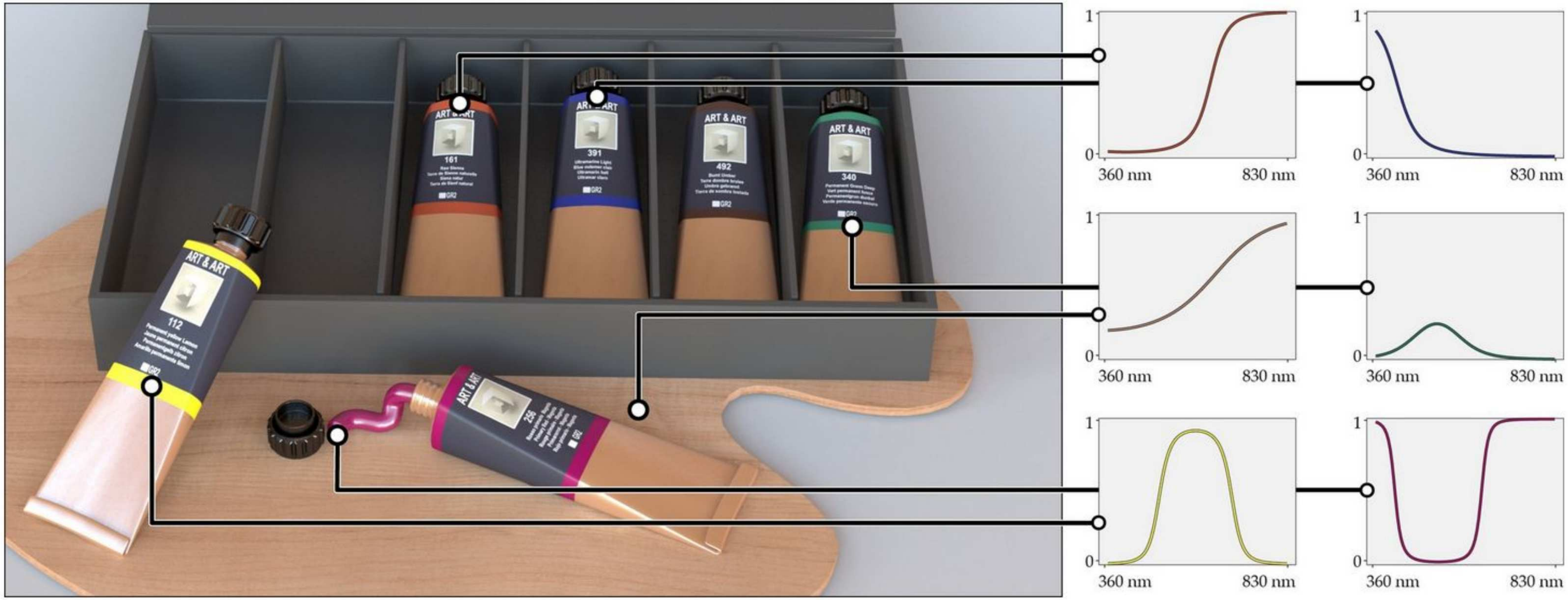
Make it physics (a bit)

A Low-Dimensional Function Space for Efficient Spectral Upsampling

Wenzel Jakob Johannes Hanika

In *Computer Graphics Forum (Proceedings of Eurographics 2019)*



Left. *A spectral rendering performed using the proposed technique. This scene uses a variety of RGB textures that have been converted into reflectance spectra.* ***Right***. *Plots of highlighted surface regions over the visible range.*

Make it physics (a bit)

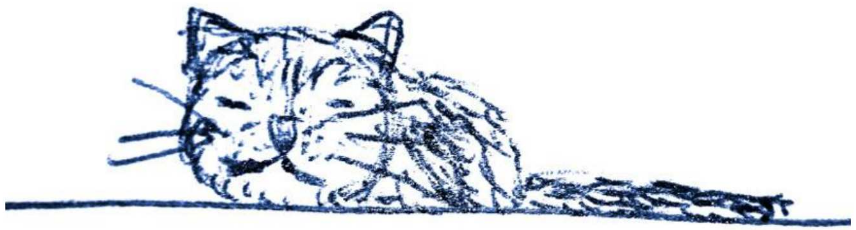
Radiometry

- Units and naming
  - Radiant energy Q_e [J] (Joule)
  - Radiant flux / power Θ_e [W=Js] (Watt = Joule seconds)
  - Radiant intensity $I_e(\omega)$ [W/sr] (Watt / steradians = solid angle)
  - Irradiance $E_e(x)$ [W/m^2] (incident flux per unit area, think of photons, integral from before)
  - Radiant exitance $M_e(x)$ [W/m^2] (emitted flux per unit area, i.e. light source)
  - Radiosity $J_e(x)$ [W/m^2] (flux per unit area emitted + reflected)
  - Radiance $L_e(x, \omega)$ [$W/(m^2 sr)$] (flux per unit area per solid angle)
  - Radiometric quantity per wavelength $L_{e,\lambda}(x, \omega)$ [$W/(m^2\ sr\ nm)$] (erm..)

Make it physics (a bit)

Photometry

- Measurement of perceived brightness
- The human eye has a different sensitivity to different wavelengths (colours), sometimes we have to account for that
- Radiance -> Luminance
- There are also units and names



Radiometry and Photometry

Radiometric quantity	Symbol	Unit	Photometric quantity	Symbol	Unit
Radiant energy	Q_e	[J] *joule*	Luminous energy	Q_v	[lm s] *talbot*
Radiant flux	Φ_e	[W] *watt*	Luminous flux	Φ_v	[lm] *lumen*
Radiant intensity	I_e	[W sr^{-1}]	Luminous intensity	I_v	[cd] *candela*
Radiance	L_e	[W sr^{-1} m^{-1}]	Luminance	L_v	[cd m^{-2}] *nit*
Irradiance	E_e	[W m^{-2}]	Illuminance	E_v	[lx] *lux*
Radiant exitance	M_e	[W m^{-2}]	Luminous emittance	M_v	[lx]
Radiosity	J_e	[W m^{-2}]	Luminosity	J_v	[lx]

source: previous year's lecture (Auzinger and Zsolnai)

Make it physics (a bit)

- **Radiance** is the fundamental quantity that simultaneously explains effects of both light source size and receiver orientation
- Let's consider a tiny almost-collimated beam of cross-section $dA^{\perp} = dA \cos(\theta)$ where the directions are all within a differential angle $d\omega$ of each other

dA and dω are differentials. check out 3blue1brown, if you want a really good explanation

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Make it physics (a bit)

Radiance **L =**
flux per unit projected area per unit solid angle

$$L = \frac{\mathrm{d}\Phi}{\mathrm{d}A^{\perp}\,\mathrm{d}\omega}$$

dA, dω and dΦ are differentials. check out 3blue1brown, if you want a really good explanation

$$[L] = \left[\frac{W}{m^2\ sr}\right]$$

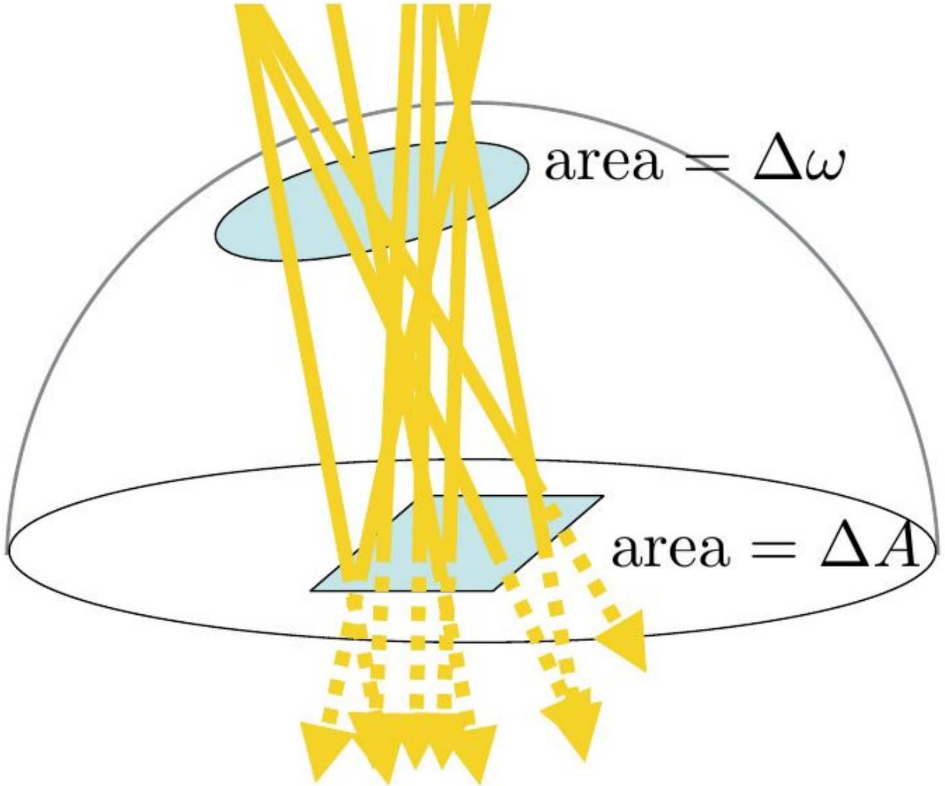
Make it physics (a bit)

Radiance, intuitively

Let's count energy packets, each ray carries the same ΔΦ (dΦ)

$$L = \frac{\mathrm{d}\Phi}{\mathrm{d}A^{\perp}\,\mathrm{d}\omega}$$

$$[L] = \left[\frac{W}{m^2\ sr}\right]$$



dA, dω and dΦ are differentials. check out 3blue1brown, if you want a really good explanation

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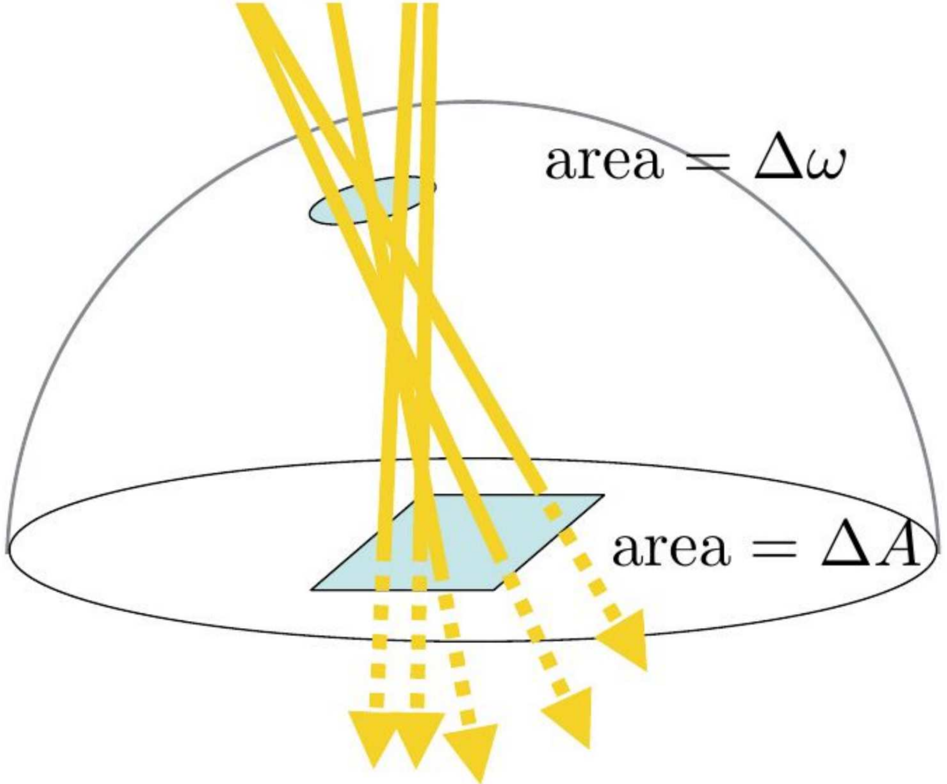
Make it physics (a bit)

Radiance, intuitively

Smaller solid angle

=> fewer rays => less energy



$$L = \frac{\mathrm{d}\Phi}{\mathrm{d}A^{\perp}\,\mathrm{d}\omega}$$

$$[L] = \left[\frac{W}{m^2\ sr}\right]$$

dA, dω and dΦ are differentials. check out 3blue1brown, if you want a really good explanation

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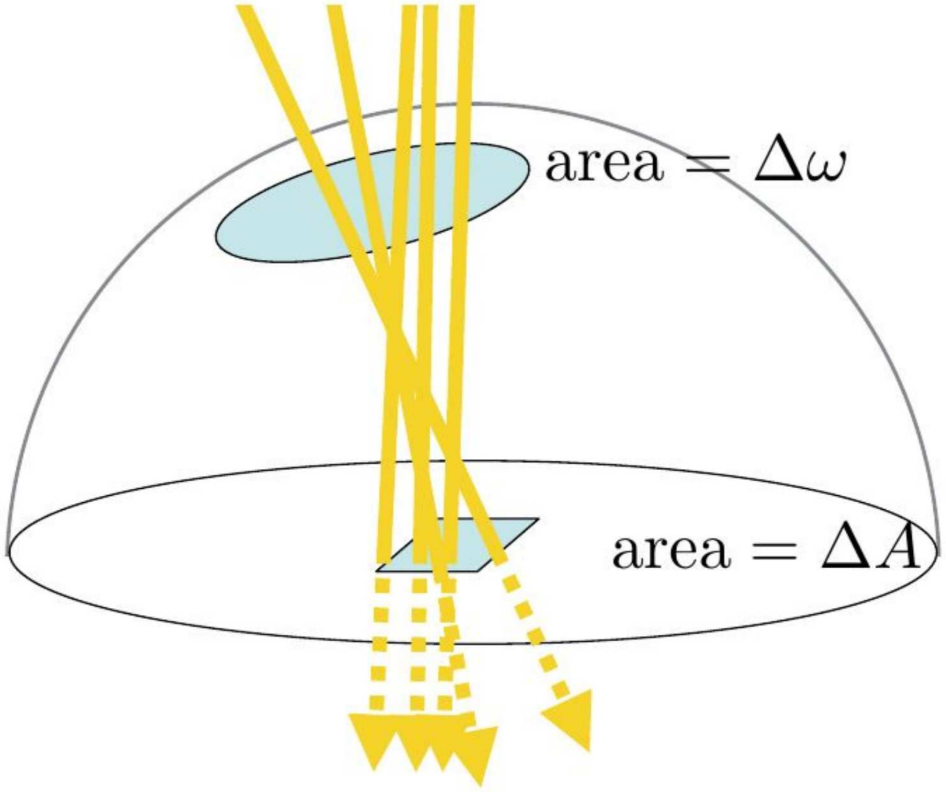
Make it physics (a bit)

Radiance, intuitively

Smaller projected surface area

=> fewer rays => less energy



$$L = \frac{\mathrm{d}\Phi}{\mathrm{d}A^{\perp}\,\mathrm{d}\omega}$$

$$[L] = \left[\frac{W}{m^2\ sr}\right]$$

dA, dω and dΦ are differentials. check out 3blue1brown, if you want a really good explanation

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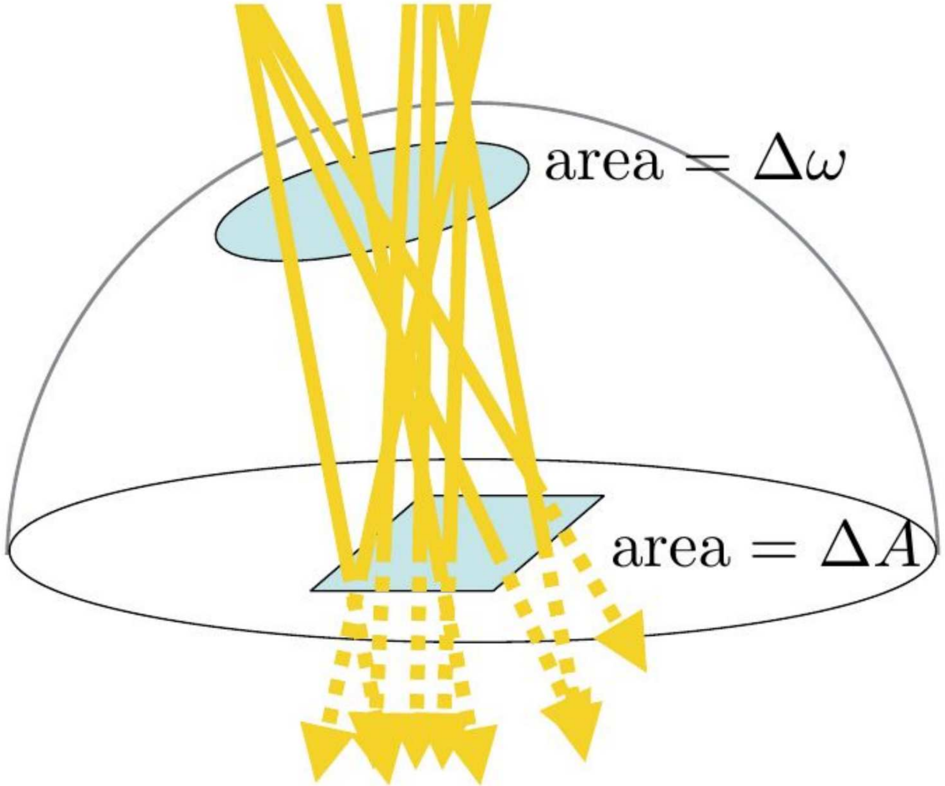
Make it physics (a bit)

Radiance, intuitively
I.e., radiance is a density over both space and angle



$$\boxed{L = \frac{\mathrm{d}\Phi}{\mathrm{d}A^{\perp}\,\mathrm{d}\omega}}$$

$$[L] = \left[\frac{W}{m^2\ sr}\right]$$

dA, dω and dΦ are differentials. check out 3blue1brown, if you want a really good explanation

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Make it physics (a bit)

Radiance

- **Sensors are sensitive to radiance**
  - It’s what you assign to pixels
  - The fundamental quantity in image synthesis
- “Intensity does not attenuate with distance”
  $\Leftrightarrow$ **radiance stays constant along straight lines***
- All relevant quantities (irradiance, etc.) can be derived from radiance

* unless the medium is participating, e.g. smoke, fog, wax, water, air..

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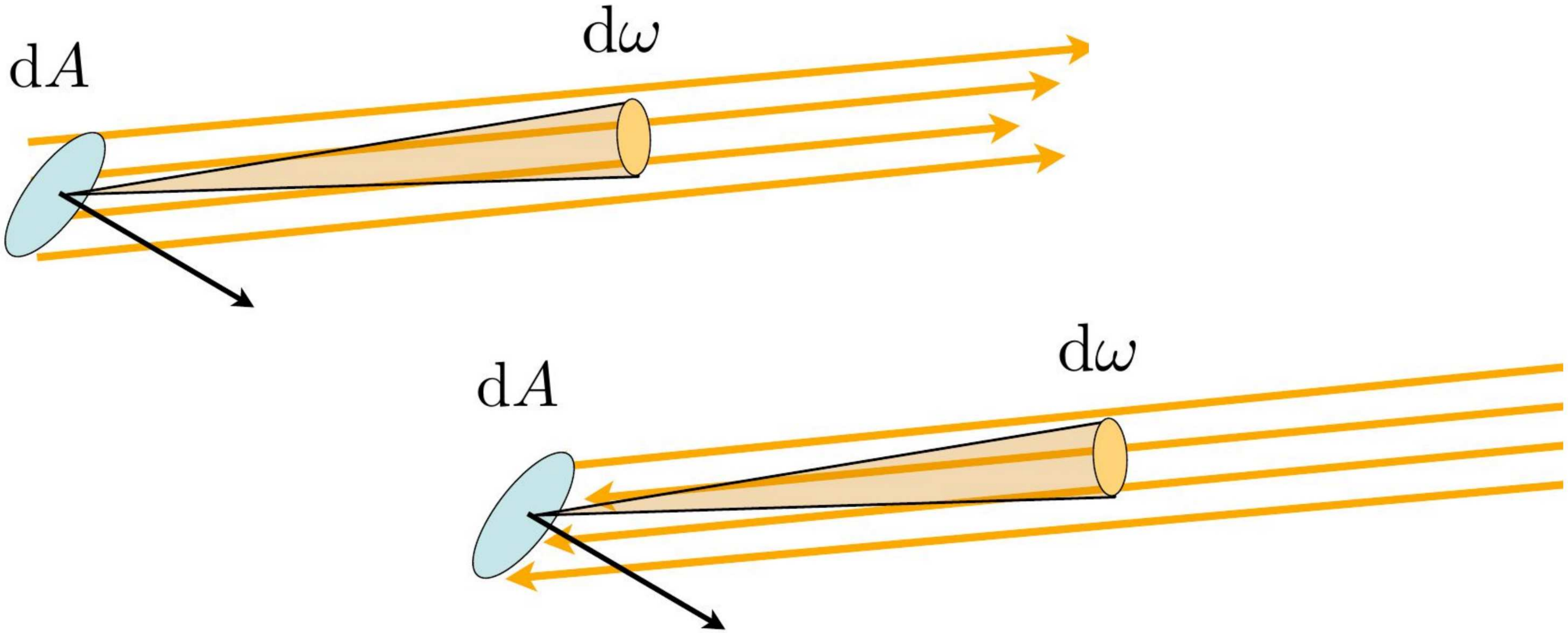
Make it physics (a bit)

Radiance characterises

- Light that leaves a surface patch dA to a given direction
- Light that arrives at a surface patch dA from a given direction

(just flip the direction)



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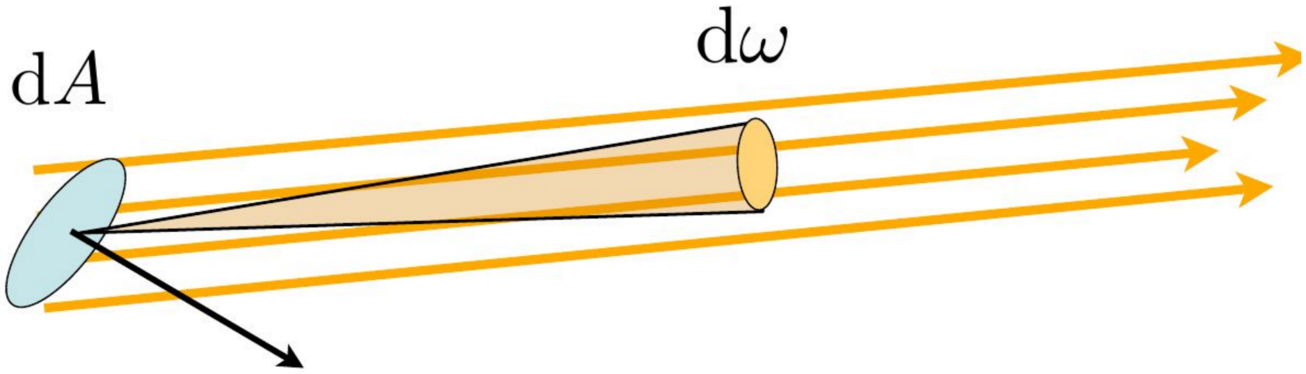
Make it physics (a bit)

Radiance also exists in empty space, away from surfaces

- Radiance L(x,ω), when taken as a 5d function of position (3d) and direction (2d) completely nails down the light flow in a scene
- Sometimes called the “plenoptic function”



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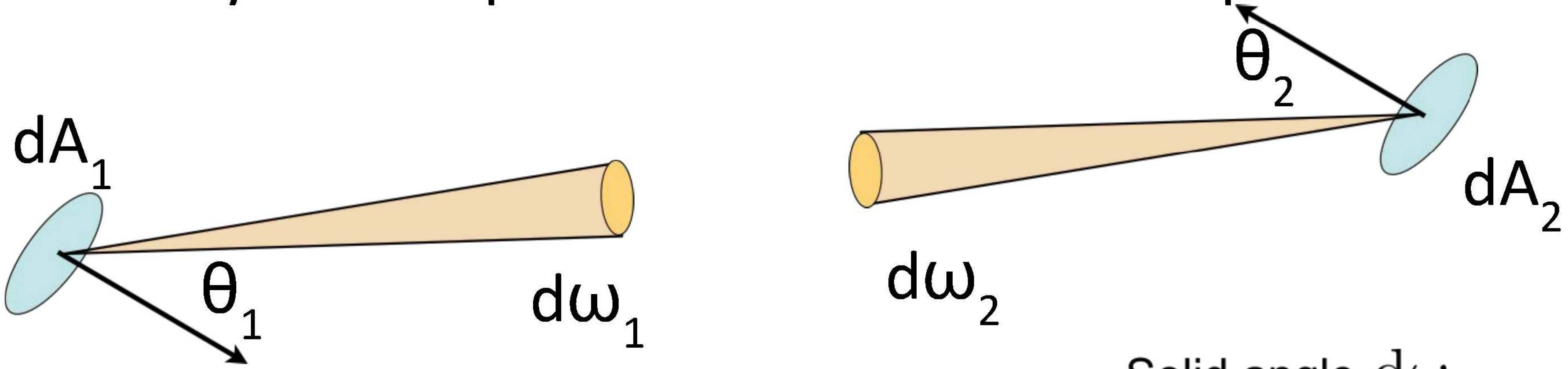
Make it physics (a bit)

Constancy along straight lines

Let’s look at the flux sent by a small patch onto another small patch



$$\boxed{L = \frac{\mathrm{d}\Phi}{\mathrm{d}A^{\perp}\,\mathrm{d}\omega}}$$

$$\mathrm{d}\Phi = L(x_1 \leftarrow \omega_1)\,\overbrace{\cos\theta_1 \mathrm{d}A_1}^{\mathrm{d}A_1^{\perp}}\;\overbrace{\frac{\mathrm{d}A_2\,\cos\theta_2}{r^2}}^{\text{Solid angle } \mathrm{d}\omega_1 \text{ subtended by } dA_2 \text{ as seen from } dA_1}$$

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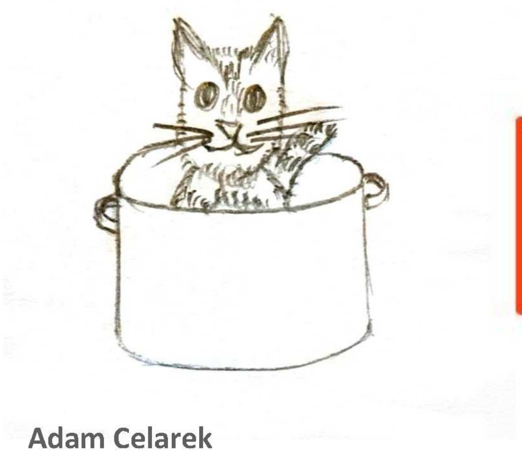
Constancy along straight lines

Eureka

$$\mathrm{d}\Phi = L(x_2 \rightarrow \omega_2)\ \cos\theta_2 \mathrm{d}A_2\ \frac{\mathrm{d}A_1\ \cos\theta_1}{r^2}$$

$$\mathrm{d}\Phi = L(x_1 \leftarrow \omega_1)\ \cos\theta_1 \mathrm{d}A_1\ \frac{\mathrm{d}A_2\ \cos\theta_2}{r^2}$$



$$\boxed{\Rightarrow L(x_1 \leftarrow \omega_1) = L(x_2 \rightarrow \omega_2)}$$

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Make it physics (a bit)

- Electromagnetic spectrum
- Radiometry and photometry
  - Units and naming
  - How is that stuff perceived in the human eye
- Radiance (constant along straight lines)
- Rendering
  - Irradiance
  - Materials
  - White furnace test (energy conservation)

Make it physics (a bit)

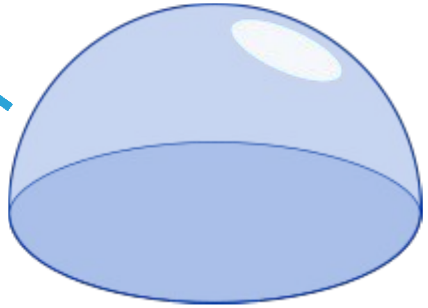
We have seen this before, this is **irradiance** (incoming light).

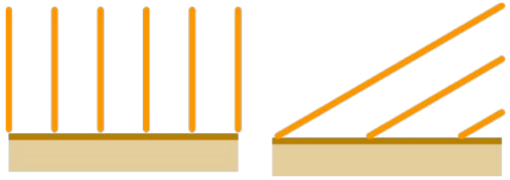
Light arriving at point x

Light from direction ω

Solid angle

$$L_i(x) = \int_\Omega L_i(x, \omega) \cos(\theta_x) \, \mathrm{d}\omega$$

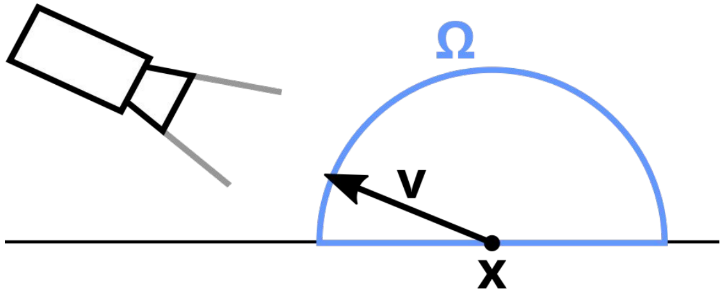




(not useful for rendering yet)

Make it physics (a bit)

Now we want to know how much light is going to the camera.



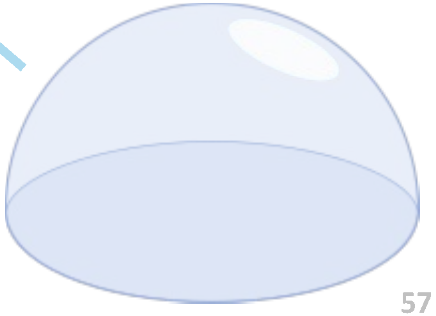
Material, modelled by the BRDF

Light from direction ω

Solid angle

$$L_e(x, v) = \int_\Omega \boxed{f_r(x, \omega \rightarrow v)} L_i(x, \omega) \cos(\theta_x) \, \mathrm{d}\omega$$

Light going in direction v



Make it physics (a bit)

Material (BRDF = Bidirectional reflectance distribution function)

- How much light is reflected from a given direction into another given direction at a given position, and in which wavelengths
- The colour
- You probably already implemented simple BRDFs in “*Übung Computergraphik (186.831)*”
- More in a later lecture

Make it physics (a bit)

White furnace test (energy conservation)

- A material can not create light, otherwise it would be a light source
- It can only absorb light, turn it into another form of energy or radiation
- We can make unit tests
- Set L_i to 1 and check $L_e \leq 1$

Material, modelled by the BRDF

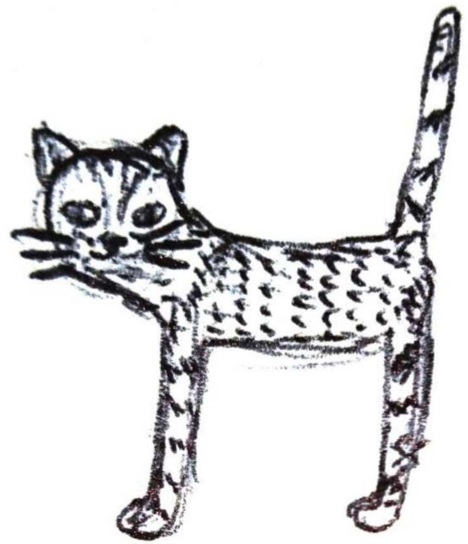
Light from direction ω

$$L_e(x, v) = \int_\Omega \boxed{f_r(x, \omega \rightarrow v)} L_i(x, \omega) \cos(\theta_x) \, \mathrm{d}\omega$$

Make it physics (a bit)

White furnace test (energy conservation)

- Ok cat, set L_i to 1
- Assume a white diffuse material (all light is reflected)
- And check $L_e \leq 1$



Material, modelled by the BRDF

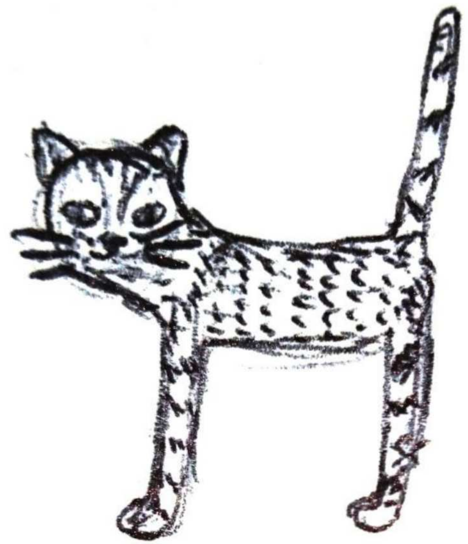
Light from direction ω

$$L_e(x, v) = \int_\Omega \boxed{1} \quad 1 \quad \cos(\theta_x)\, \mathrm{d}\omega$$

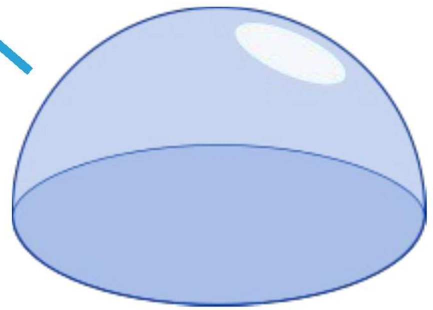
Make it physics (a bit)

White furnace test (energy conservation)

- Ok cat, how can I integrate that half sphere
- -> change of variables!



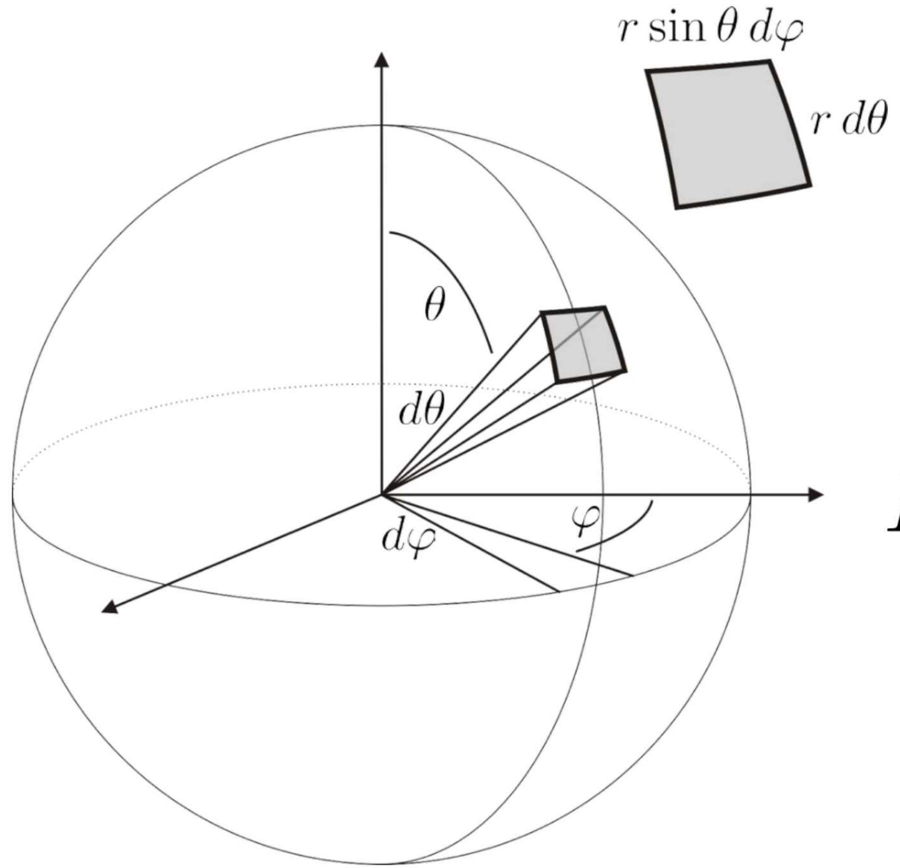
$$L_e(x,v) = \int_\Omega \cos(\theta)\,\mathrm{d}\omega$$



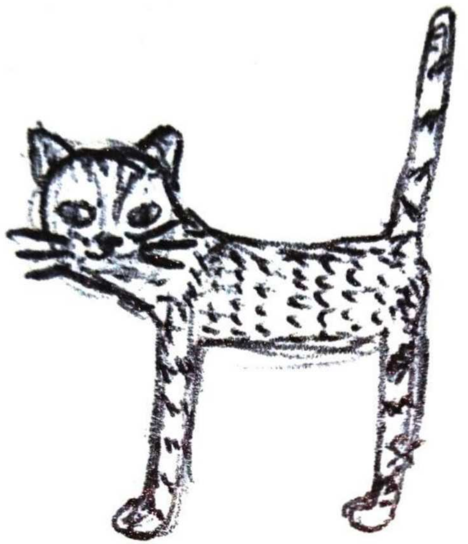
Make it physics (a bit)

White furnace test (energy conservation)

Change of variable



$$L_e(x, v) = \int_0^{2\pi} \int_0^{\pi/2} \cos(\theta) \sin(\theta) \, \mathrm{d}\theta \, \mathrm{d}\phi$$

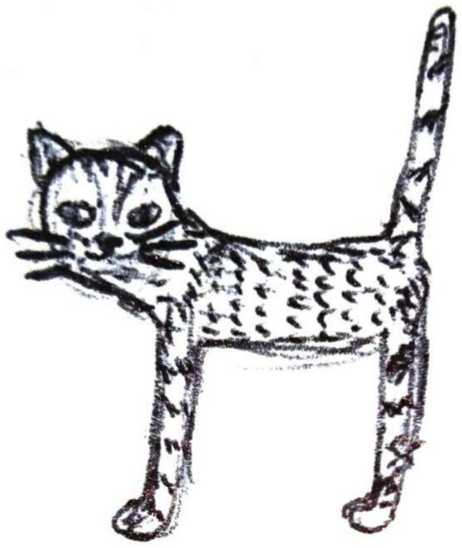


[WolframAlpha]

source: previous year's lecture (Auzinger and Zsolnai)

Make it physics (a bit)

White furnace test (energy conservation)



$$L_e(x, v) = \int_0^{2\pi} \int_0^{\pi/2} \cos(\theta) \sin(\theta) \, \mathrm{d}\theta \, \mathrm{d}\phi$$

WolframAlpha: π own work: π > 1

Make it physics (a bit)

White furnace test (energy conservation)

Failed

$$L_e(x, v) = \int_0^{2\pi} \int_0^{\pi/2} \cos(\theta)\sin(\theta)\,\mathrm{d}\theta\,\mathrm{d}\phi$$

WolframAlpha: π own work: π > 1

Make it physics (a bit)

White furnace test (energy conservation)

- A material can not create light, otherwise it would be a light source
- It can only absorb light, turn it into another form of energy or radiation
- **f_r for a white diffuse material is 1/π,**
  for a general diffuse material it is ρ/π, where ρ is the colour

Material, modelled by the BRDF

Light from direction ω

$$L_e(x, v) = \int_\Omega \boxed{f_r(x, \omega \to v)} L_i(x, \omega) \cos(\theta_x)\, \mathrm{d}\omega$$

Physics

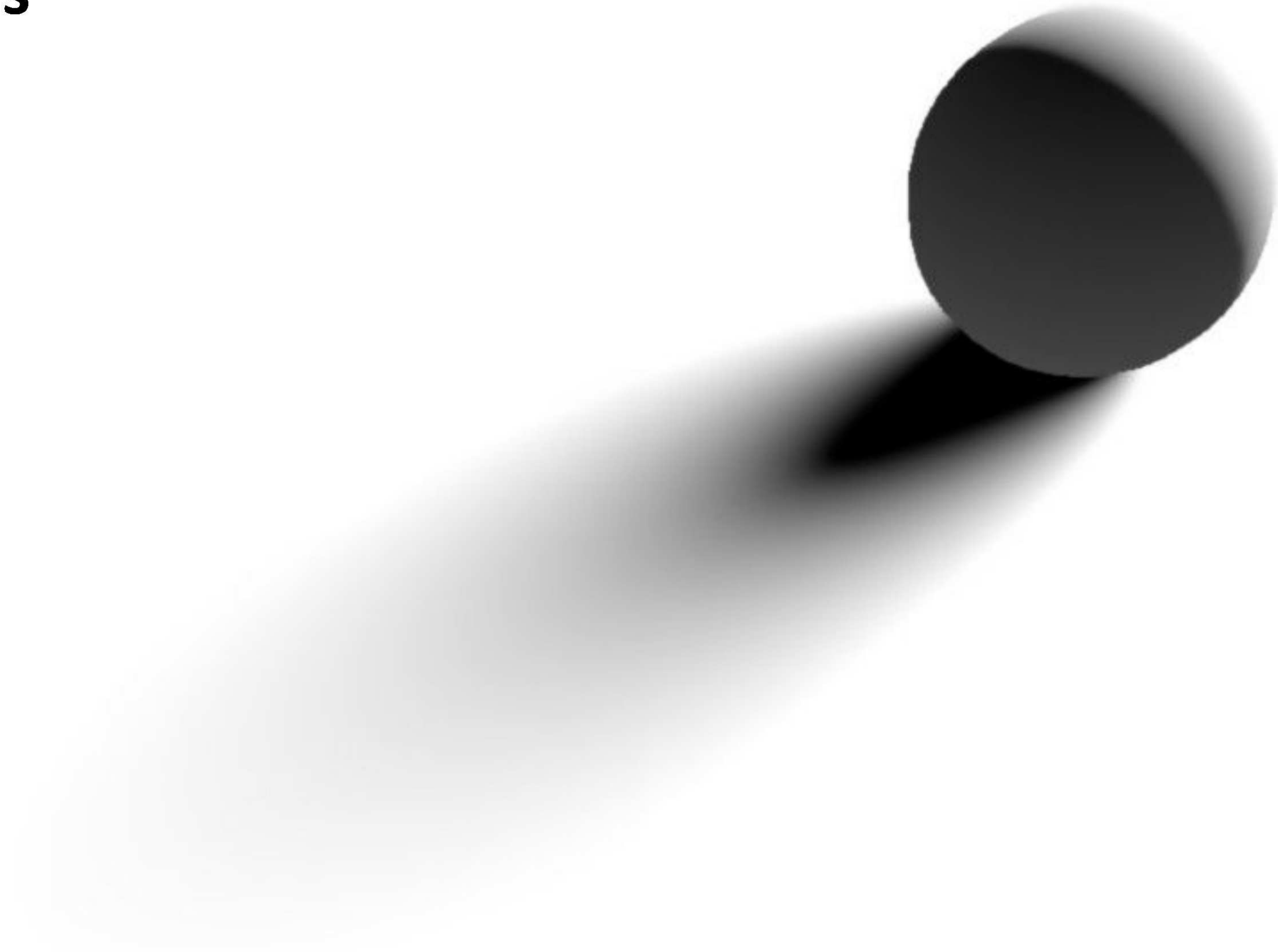
Quantities and units
Materials
White furnace test

Next: Apply

source: own work

Apply

Soft shadows



source: Martin Kraus, Wikipedia
(no changes, CC BY-SA 3.0)

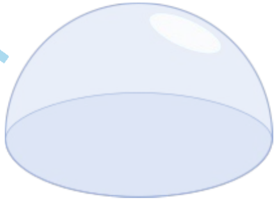
Apply

(from the math chapter)

Light from direction ω

$$L_i(x) = \int_\Omega L_i(x,\omega)\cos(\theta_x)\,\mathrm{d}\omega$$

Light arriving at point x



Solid angle

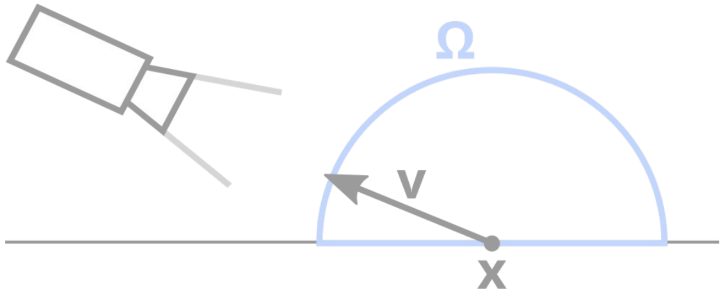
Light from source [l] arriving at point x

$$L_i^{[l]}(x) = \int_{S_l} L_e^{[l]}(y)\cos(\theta_x)\boxed{\frac{\cos(\theta_y)}{r^2}\,\mathrm{d}A_y}$$

light intensity at position y on the surface

Apply

(from the physics chapter)



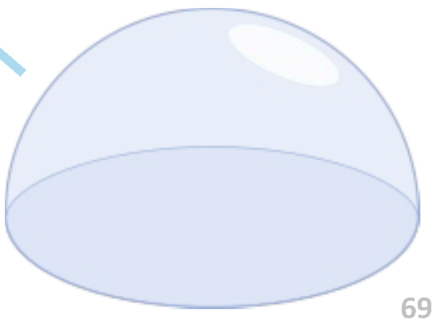
Material, modelled by the BRDF

Light from direction ω

Solid angle

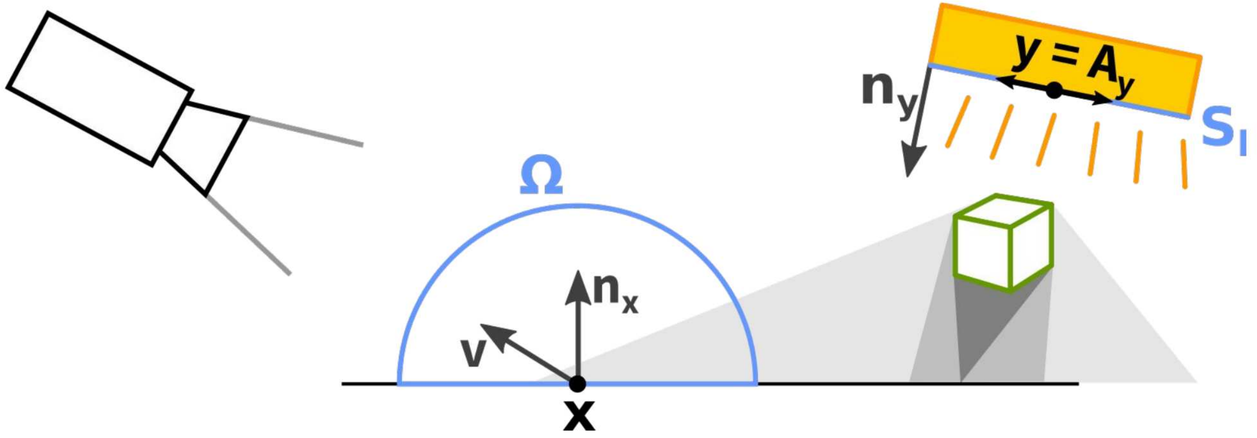
$$L_e(x, v) = \int_\Omega f_r(x, \omega \to v) L_i(x, \omega) \cos(\theta_x)\, \mathrm{d}\omega$$

Light going in direction v



Apply

Soft shadows (something is missing)



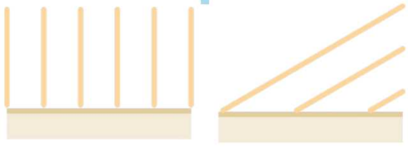
light intensity at position y on the surface

emitter cos(θ)

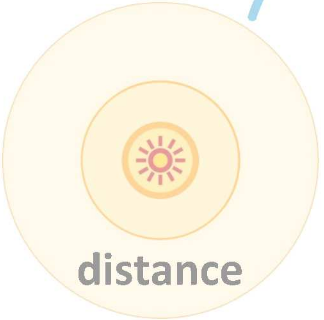
$$L_e^{[l]}(x) = \int_{S_l} f_r(x, y \to v) L_e^{[l]}(y) \cos(\theta_x) \frac{\cos(\theta_y)}{r^2} \, \mathrm{d}A_y$$

Light going in direction v

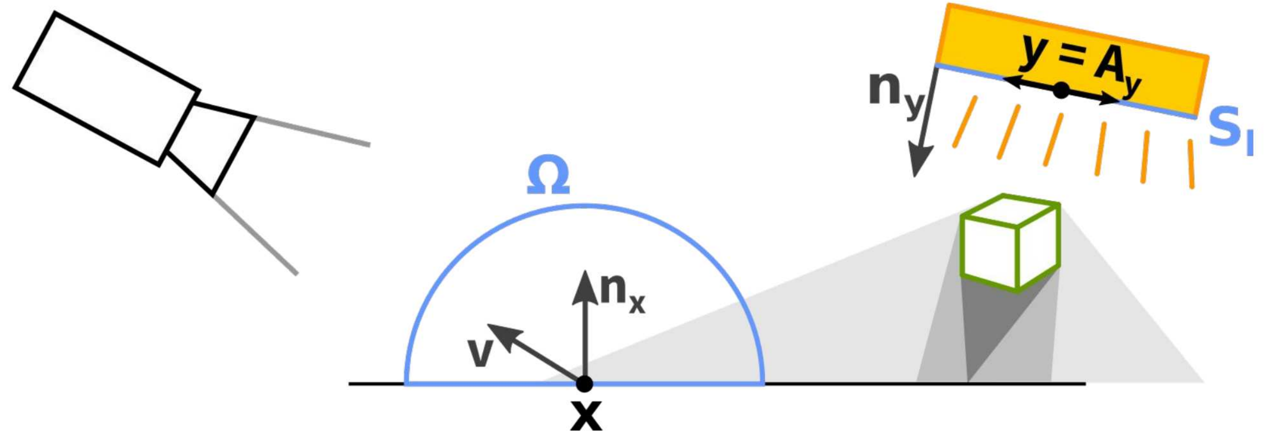
Material, modelled by the BRDF



receiver cos(θ)



Soft shadows (usable for rendering)



light intensity at position y on the surface

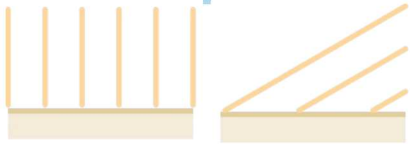
emitter cos(θ)

visibility (new, ray tracing)

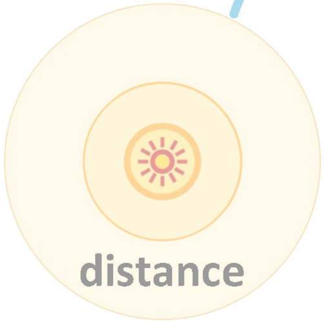
$$L_e^{[l]}(x) = \int_{S_l} f_r(x, y \to v) L_e^{[l]}(y) V(x,y) \cos(\theta_x) \frac{\cos(\theta_y)}{r^2} \, \mathrm{d}A_y$$

Light going in direction v

Material, modelled by the BRDF

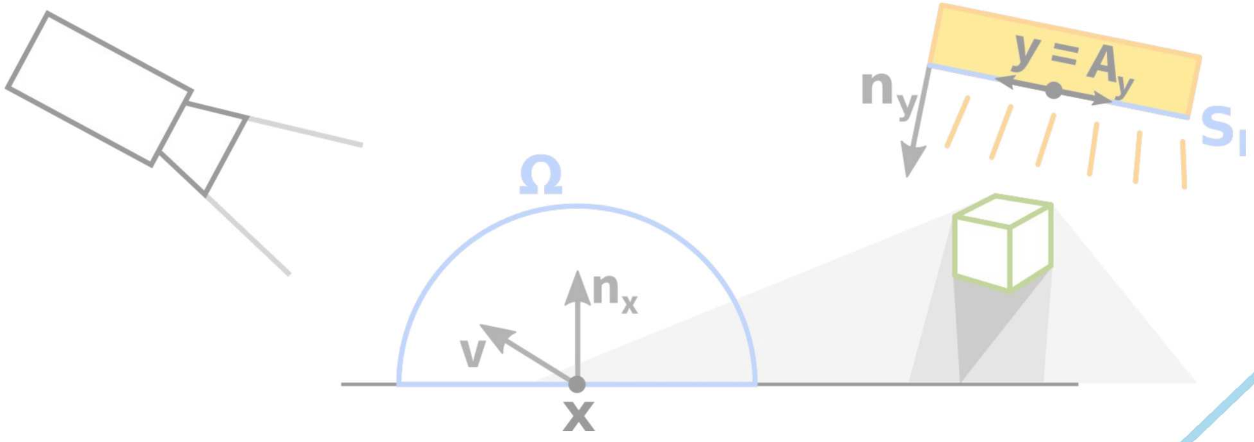


receiver cos(θ)



Apply

Soft shadows (the same, but more explicit)



light intensity at position y on the surface

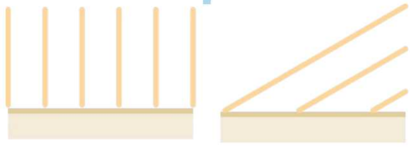
visibility (new, ray tracing)

emitter cos(θ)

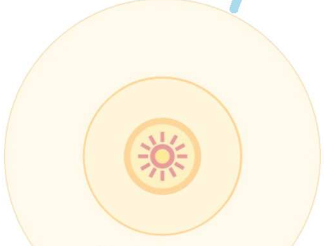
$$L_e^{[l]}(x) = \int_{S_l} f_r(x, y \to v) L_e^{[l]}(y) V(x,y) \left(\frac{y-x}{|y-x|} \cdot n_x\right) \frac{\left(\frac{x-y}{|x-y|} \cdot n_y\right)}{|x-y|^2} \mathrm{d}A_y$$

Light going in direction v

Material, modelled by the BRDF



receiver cos(θ)



distance

Apply

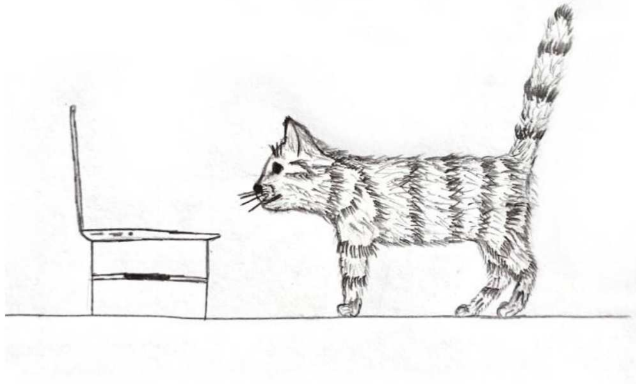
How to build a direct lighting renderer out of these two friends?

$$L_e^{[l]}(x) = \int_{S_l} f_r(x, y \to v) L_e^{[l]}(y) V(x, y) \cos(\theta_x) \frac{\cos(\theta_y)}{r^2} \, \mathrm{d}A_y$$

$$L_e^{[l]}(x) = \int_{S_l} f_r(x, y \to v) L_e^{[l]}(y) V(x, y) \left(\frac{y - x}{|y - x|} \cdot n_x\right) \frac{\left(\frac{x-y}{|x-y|} \cdot n_y\right)}{|x - y|^2} \, \mathrm{d}A_y$$

ρ(x)/π

E(y)



How to build a direct lighting renderer?

$$L_{\text{out}} = \frac{\rho(x)}{\pi} \int_{\text{light}} E(y)\, V(x,y)\, \frac{\cos\theta_y}{r^2}\, \cos\theta\, \mathrm{d}A_y$$

```
for each visible point x
 Generate N random points y_i on light source, store
 probabilities p_i as well (uniform: p_i == 1/A)
 est = 0
 for each y_i, i=1,...,N
  Cast shadow ray to evaluate V(x,y_i)
  if visible
   est = est + E(y_i)cos(theta_yi)cos(theta)/r^2/p_i
  endif
 endfor
 L_out(x) = 1/N * est * rho(x)/pi
endfor
```

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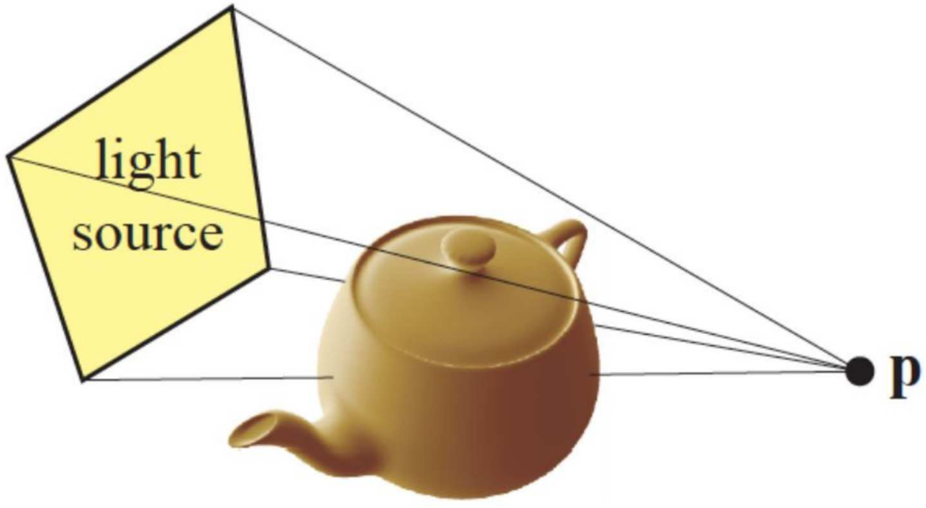
Intuitive Picture



(a)

(b)

Laine et al. SIGGRAPH 2005

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Apply

I’ve skipped ahead of our lecture

- Note the use of random numbers
  - We are performing Monte Carlo integration
  - We’ll come to that very soon
- **BUT**: Why not write an area light renderer as an extra for your first programming assignment?
  - After writing code to place the light where you want, you can pretty much translate the pseudocode into actual C++
- Also, note that we haven’t talked about non-diffuse surfaces or indirect illumination, yet.

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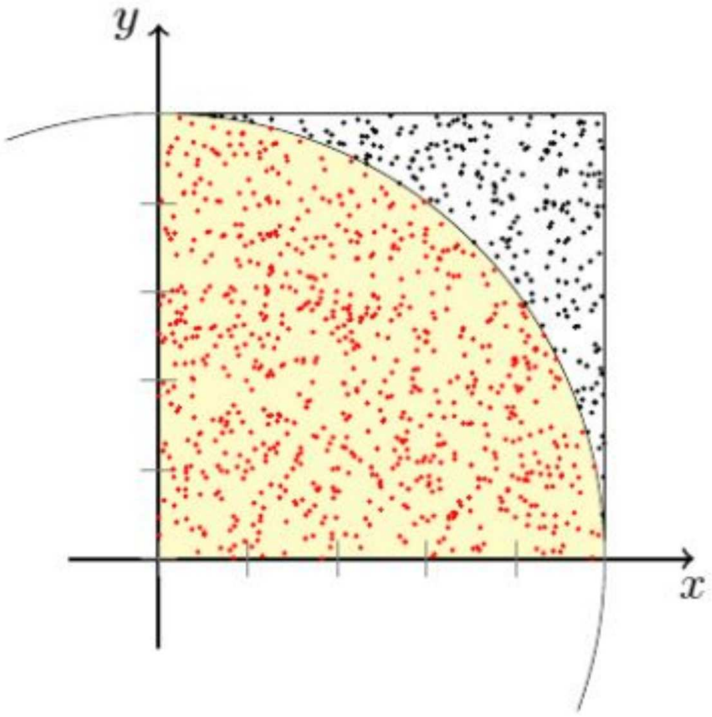
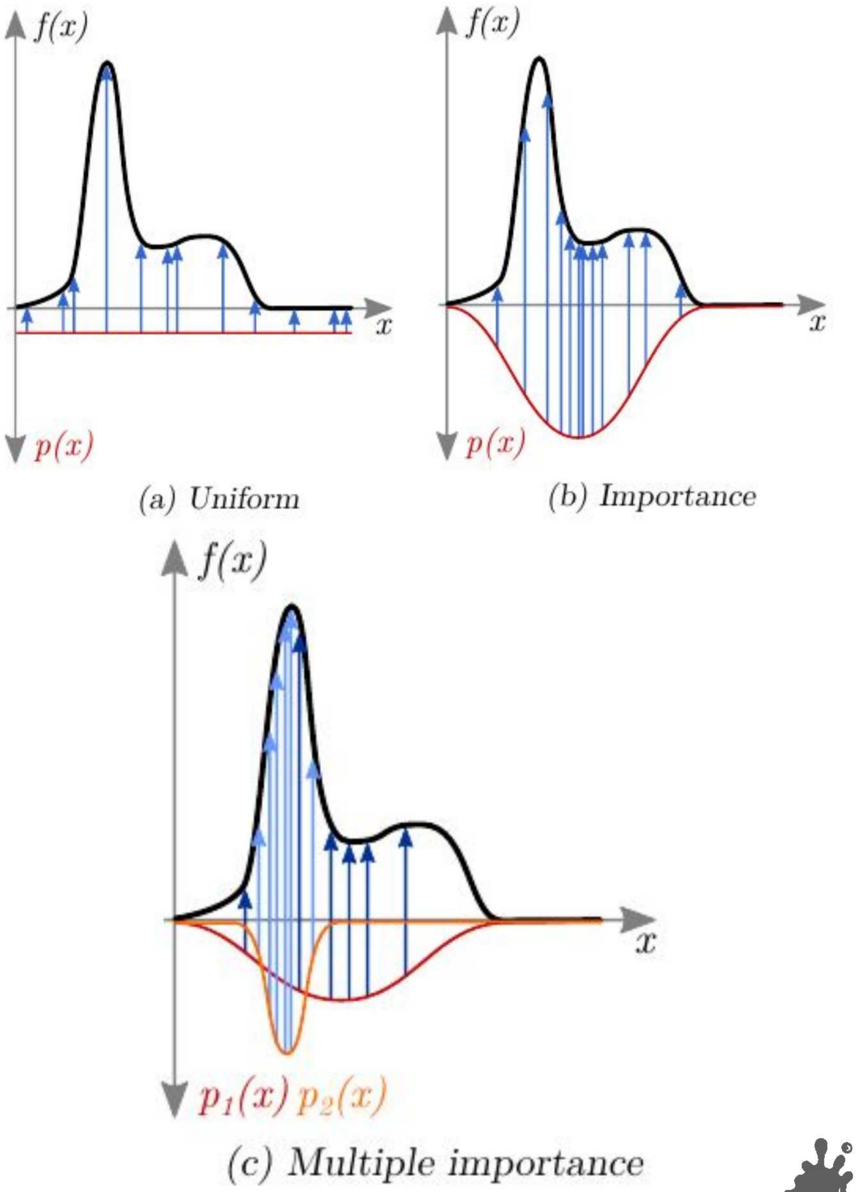
Next time

MC integration





Direct light (soft shadows)

Change of variables
Monte Carlo sneak peak

Next lecture: Monte Carlo

There are some reading links on the next page, in case you feel bored :)

source: own work

Useful reading (links)

- Change of variables
- Monte Carlo Integration
- Jaakko Lehtinens slides (I borrowed a lot from lecture 2, but there is more on point lights, intuition, links..)
- Last years slides (more on history, physics, different approach on solid angle etc.)
- Last years lecture (recordings)