Sampling Aspects of Volume Rendering

Miloš Šrámek and Leonid Dimitrov
Volume Rendering
Ray-Casting

- Shoot rays from each pixel
- Define a sequence of samples
- Accumulate color along each ray
Compositing

- Approximate light attenuation
  e.g. Front-to-Back compositing:

\[
I_m = I_{m-1} + (1 - \beta_{m-1}) C_m \\
\beta_m = \beta_{m-1} + (1 - \beta_{m-1}) \alpha_m
\]

- Segment opacity

\[
\alpha_i = 1 - e^{-\int_{t_i}^{t_{i+1}} \rho(u) \, du}
\]
VR: Different Techniques

Shaded (gradients evaluated)

Unshaded (reprojection, no gradient)

MIP
VR Questions

- Which is the correct sampling density along a ray
- Which is the influence of gradient on the correct sampling density
- Which is the correct sampling density in perspective rendering
Frequency Properties of Sampled Data

- Maximum representable frequency:
  \[ f_{\text{max}} = \frac{1}{2 \, VU} \]

- Optimal sampling (ideal): 1 sample/voxel
- Optimal sampling (real): more than 1 sample/voxel. It depends on
  - spectral properties of data
  - used visualization technique (gradients!)
  - Used reconstruction filters
Low vs. High Sampling Density

- Rendered images at
  - 10 samples along a ray / voxel
  - Samples on voxel boundaries (0.7 / voxel)

- Results:

\[
SNR\ (dB) = 10 \log \frac{\sum f_i^2}{\sum |t_i - f_i|^2}
\]

<table>
<thead>
<tr>
<th>Technique</th>
<th>Image SNR [dB]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shaded</td>
<td>63</td>
</tr>
<tr>
<td>Unshaded</td>
<td>120</td>
</tr>
<tr>
<td>MIP</td>
<td>87</td>
</tr>
</tbody>
</table>
Rules of Thumb

- **Unshaded** (reprojection)
  - 1 sample / voxel is OK
- **MIP**
  - 1 sample / voxel is not that bad
- **Shaded**
  - 1 sample / voxel is definitely bad
  - (2, 4, 8 samples?)
Perspective Rendering

Problems in perspective rendering

- Incorrect spatially variable sampling:
  - Oversampling (long time)
  - Undersampling (low image quality)
Motivation (Image Quality)
Motivation (Image Quality)
Motivation (Image Quality)
Motivation (Rendering Speed)
A Correct Solution

- VR is a resampling process:
  - Reconstruction of a continuous field
  - Antialiasing by low-pass filtering
  - Sampling
  - Compositing

Can be combined in one filter:

Size and sampling density depend on distance between rays
A Correct Solution
Practical Solutions

- 3D-mip map
  - *multum in parvo*, much in a small space
    [Levoy & Whitaker `90]
- Adaptive sampling [Novins et al. `90]
  - Splitting a ray in 4 rays when necessary
- Exponential Regions Perspective
  [Kreeger et al. `98]
3D Mip Maps

- Build a hierarchy of volumes by downsampling:
  - Low pass filtering
  - Take every second sample
- Sample a pair of volumes simultaneously:
  - 2 x trilinear + linear interpolation
ER Perspective

- One sample per voxel
- Regular pattern of merge/split
- Exponentially growing regions ⇒ uniform divergence
ER Perspective
Ray Density Resampling

\[ A = \frac{0}{4}a + \frac{1}{4}b + \frac{2}{4}c + \frac{1}{4}d + \frac{0}{4}e \]
ER Perspective - Results

Undersampling  ER - Perspective  Oversampling
ER Perspective – Results
ER Perspective - Results
The End