Volumetric Data Registration

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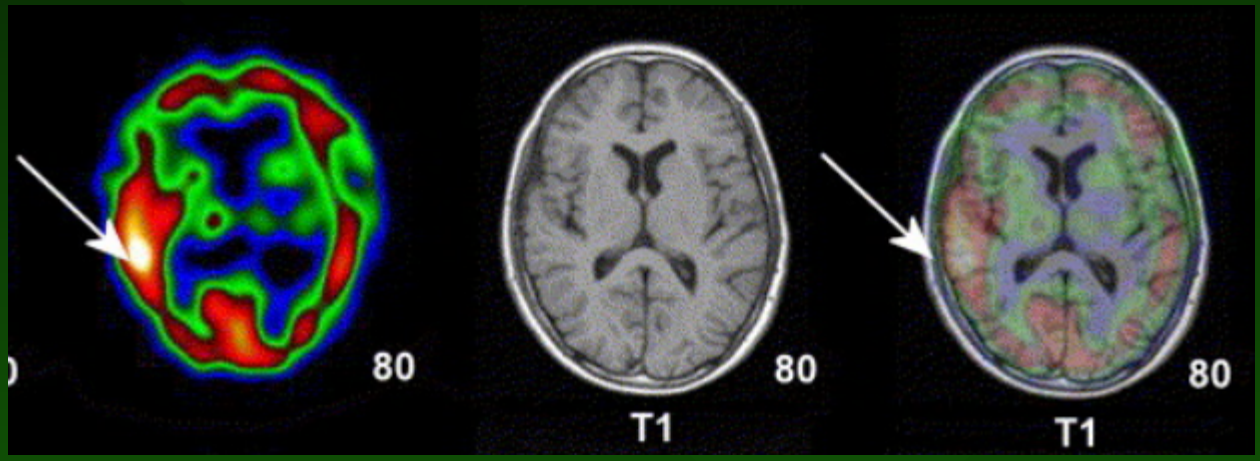
What Is It All About?

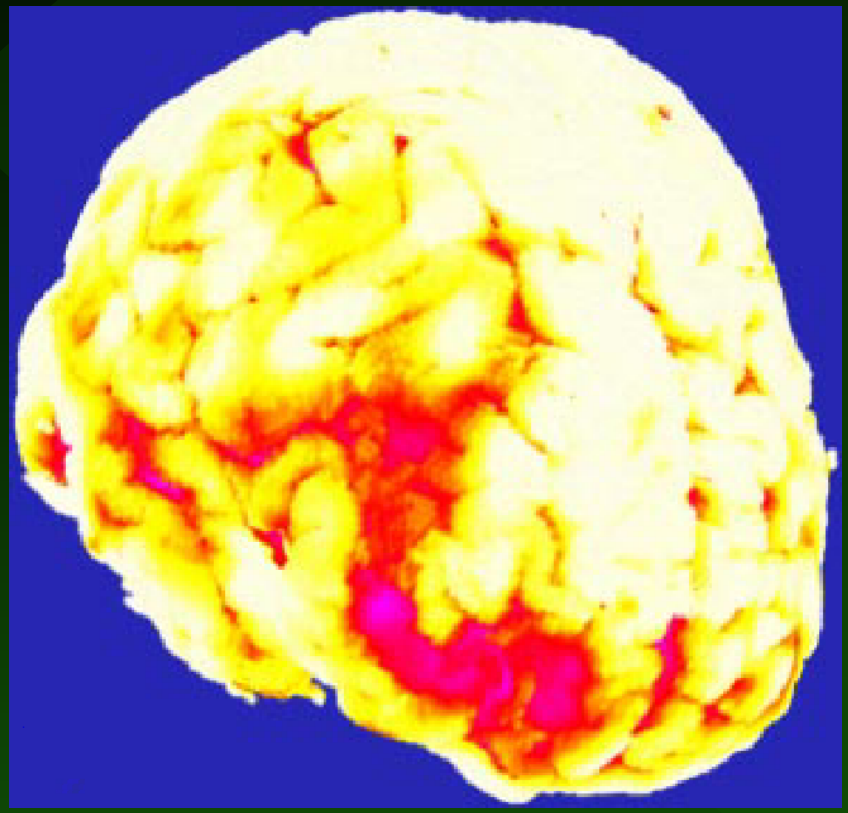
- Different modalities provide us with complementary information.
- Combination of those enhances the possibilities for interpretation.
- There is an ever-increasing need for accuracy and speed.
- Registration is NOT fusion!!!

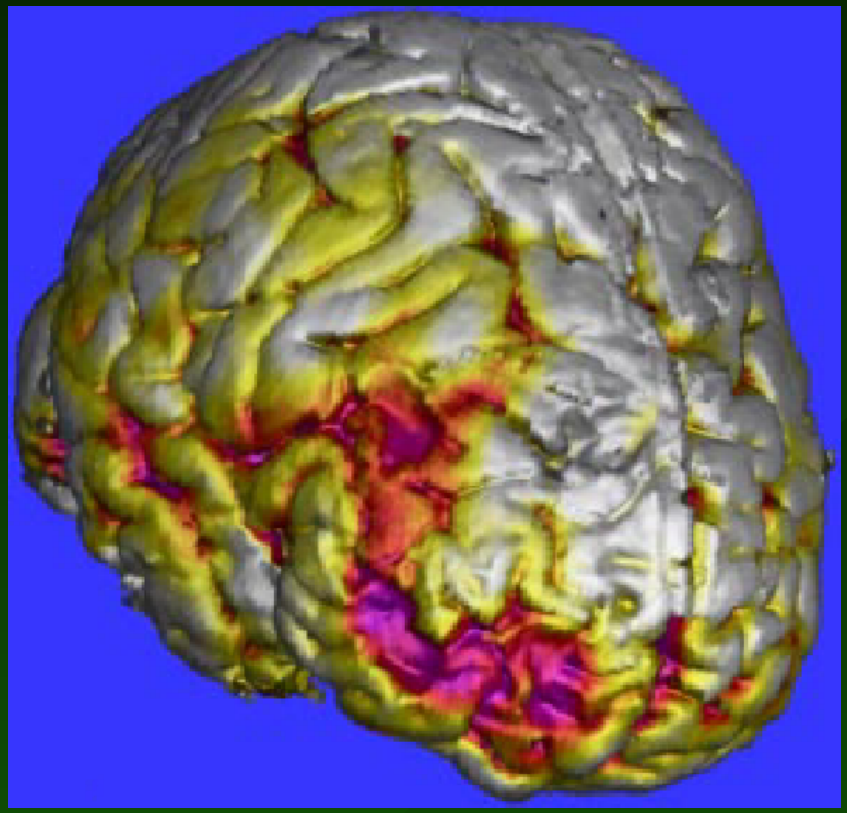
Image Fusion

- Simultaneous display of two images
- Registered images are assumed



3D Example: MRI+SPECT





Informal Statement of the Registration Problem

Develop a method which transforms geometrically the point samples of one data set to the point samples of another one in such a way that they fit together optimally.

Overview of Registration Methods

- Classification according to different criteria:
  - What is to be registered?
  - How is it to be registered?
  - Why is it to be registered?

What?

- Inter- vs. Intra-modality
- Inter- vs. Intra-subject
- Image-to-atlas
- 2D vs. 3D

How?

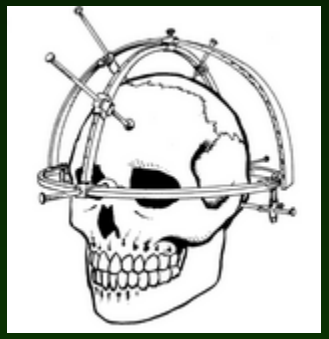
- Rigid – affine – non-linear
- Points – lines – surfaces – voxels
- Interactive – semi-automatic – automatic

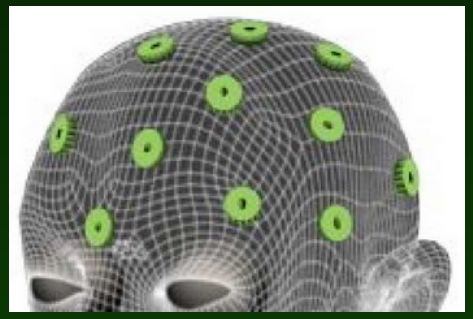
Why?

- Diagnostics
- Treatment planning
- Image-guided surgery
- Treatment evaluation

Popular Methods

- Stereotactic frame
- Fiducial marks
- Principal axes
- Atlas-oriented
- Surface similarity
- Voxel-based





Formal Statement of the Problem

- Definitions:

Let

$$M = \{ f_m \mid \text{is localizable model feature} \}$$

and

$$O = \{ f_o \mid \text{is localizable object feature} \}$$

be model resp. object.

Find a *matching transformation*

$$\tau : R^3 \rightarrow R^3$$

specified by a *parameter vector*

$$v \in N \subset R^k$$

where

$$N = \left\{ v \in R^k : r_{0\min} \le v_0 \le r_{0\max}, \cdots, r_{k\min} \le v_k \le r_{k\max} \right\}$$

is the *feasible region*

such that a *cost function*

$$C : R^k \rightarrow R$$

takes its *optimum* in

$$v_{opt} = \operatorname{argmin}\{C(v) \mid v \in N\}$$

Matching Transformations

- Rigid-body transformations
  - Translations
  - Rotations
  - Scalings
- Affine transformations
  - Reflexion
  - Stretch
  - Skew
- Non-linear transformations

Cost Functions

- Basic types:
  - Surface-based
    - Employ spatial information
  - Density- (value-) based
    - Employ histogram information
- Basic property
  - Global minimum determines anatomical optimum

Surface-based Cost Functions

- Features are surface points
- Euclidean distance of a point P to the model M:

$$d(P) = \min_{R \in M} \|P - R\|$$

- Surface similarity measure:

$$C(v) = \sum_{i=1}^{N} d^2(\tau_v(P_i))$$

Density-Based Similarity Functions

- Cross-correlation
  - Multiplicative
  - Subtractive
- Scatter-plot based
  - Histogram moments
  - Information entropy
  - Mutual information

Multiplicative Cross-correlation

- Defined as

$$C(v)=\frac{\sum_{i=1}^{N} g_m(\tau_v(P_i))\cdot g_o(P_i)}{\sqrt{\sum_{i=1}^{N} g_m^2(\tau_v(P_i))}\cdot\sqrt{\sum_{i=1}^{N} g_o^2(P_i)}}$$

- where g_m, g_o are model resp. object densities.

Subtractive Cross-correlation

- Absolute differences instead of products:

$$C(v)=\frac{\sum_{i=1}^{N}\left|g_m(\tau_v(P_i))-g_o(P_i)\right|}{\sqrt{\sum_{i=1}^{N}g_m^2(\tau_v(P_i))}\cdot\sqrt{\sum_{i=1}^{N}g_o^2(P_i)}}$$

Feature-Space Histogram or Scatter-Plot

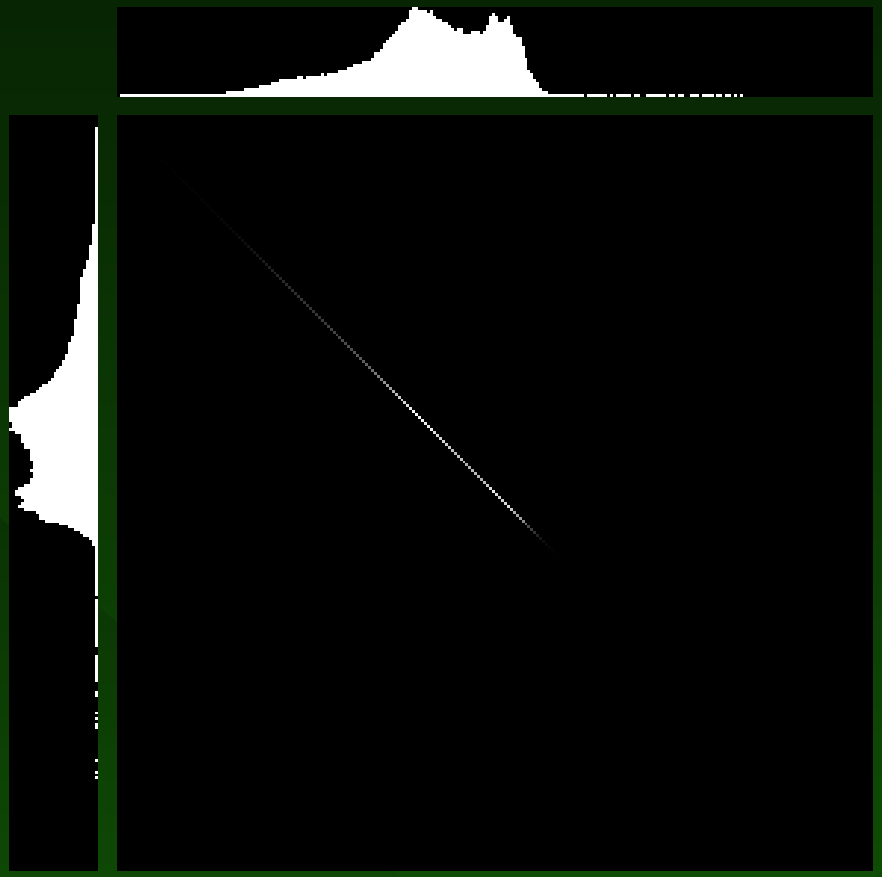
- Maps the set of ordered density pairs into a set of counts:

$$S : G_o \times G_m \rightarrow G_s \subset N$$

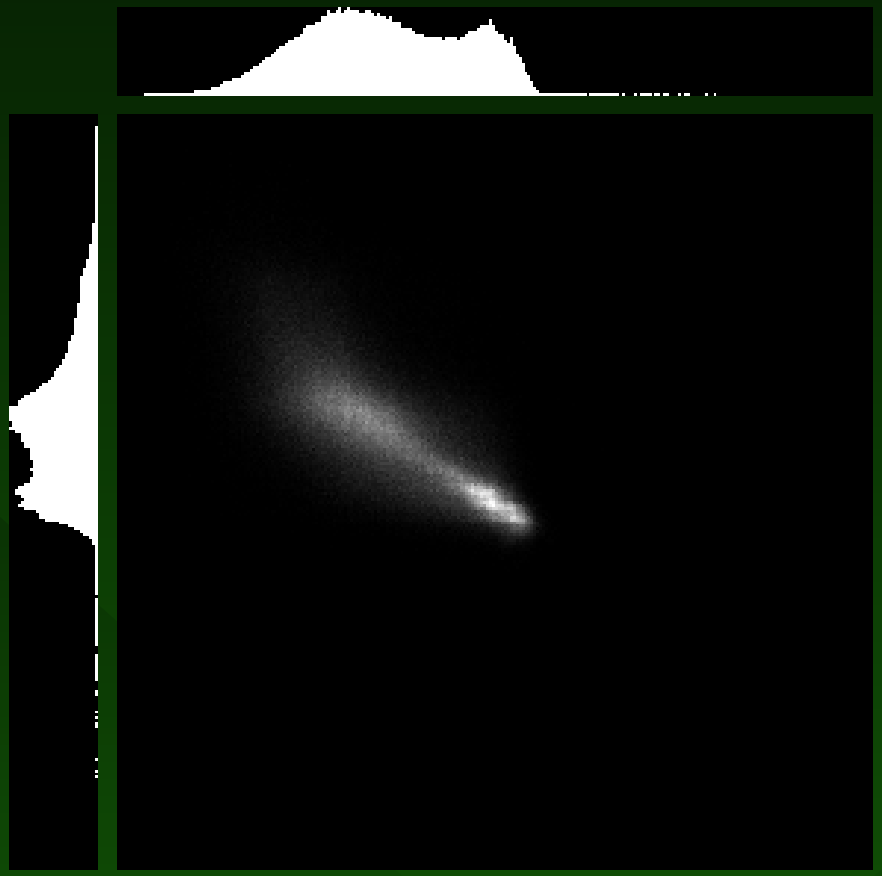
- Each value (scatter-plot pixel) represents the number of such ordered pairs:

$$S_\tau(g_o, g_m) = \left| \left\{ \left(g_o(P), g_m(\tau(P)) \right) \right\} \right|$$

Scatter-plot: Perfect Alignment



Scatter-plot: Misalignment



Inter modality registration

- Values cannot be directly compared.
- Used:
  - Joint probability
  - Mutual information
  - Histogram entropy
  - …

Similarity Functions Summary

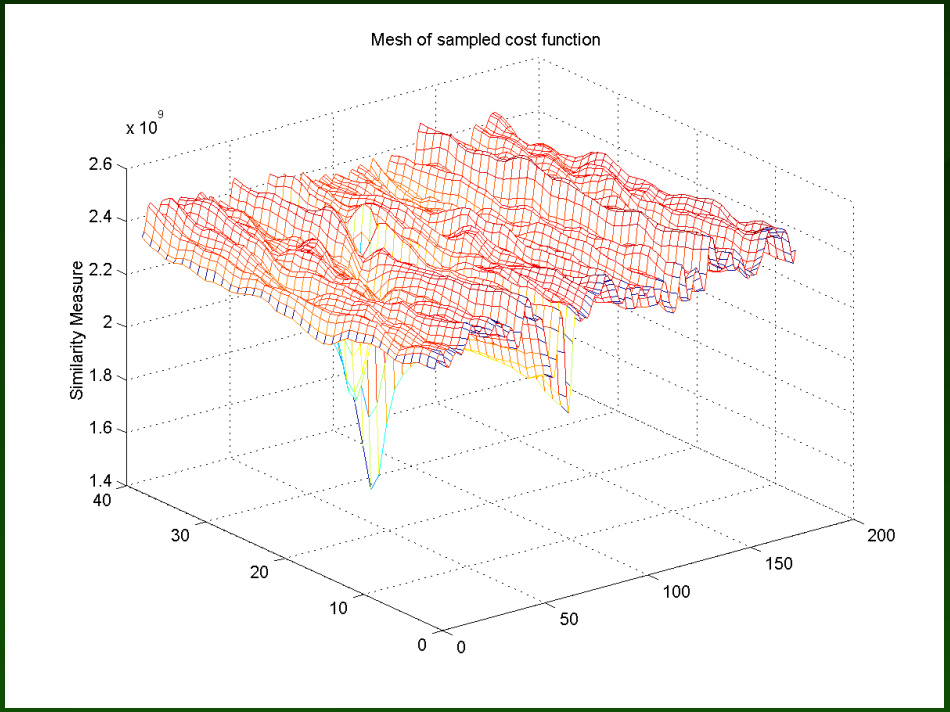
- The quality of registration is measured by similarity functions
- The registration process searches a function parameter space for an optimal solution
- There is a great diversity of them
- They are subject of active research

Optimization Task

- The problem is
  - Multivariate
  - Continuous
  - Non-linear
  - Constrained
- Solution: numerical algorithms

Optimization Methods

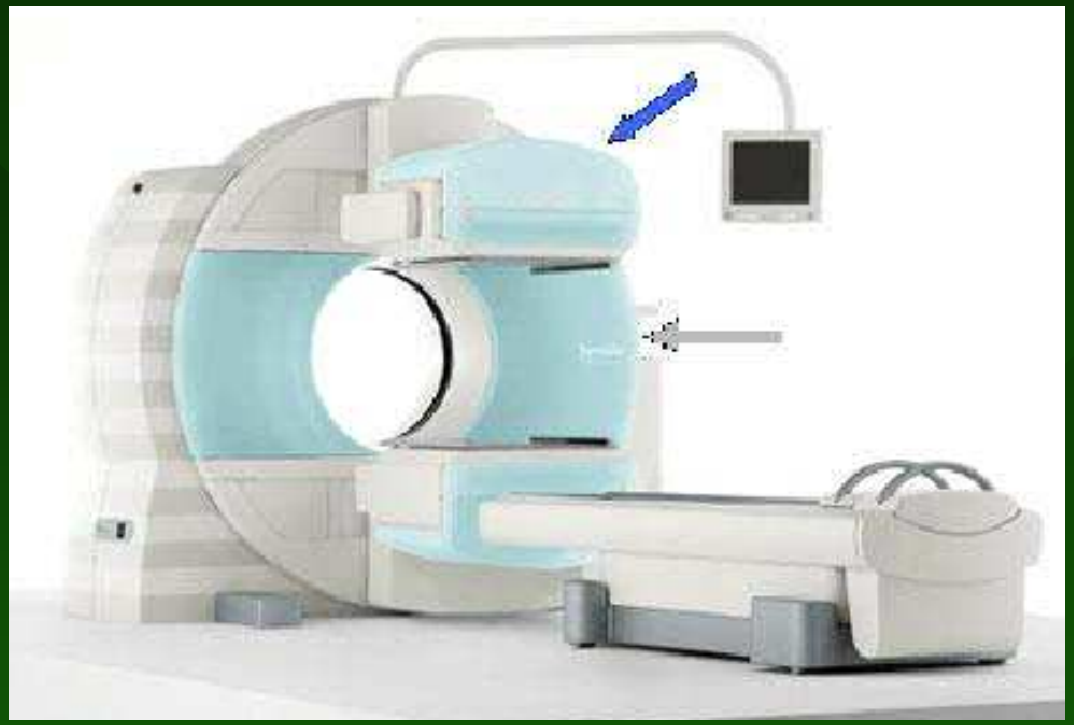
- Exhaustive search
- Gradient-based
- Simulated annealing
- Genetic algorithms



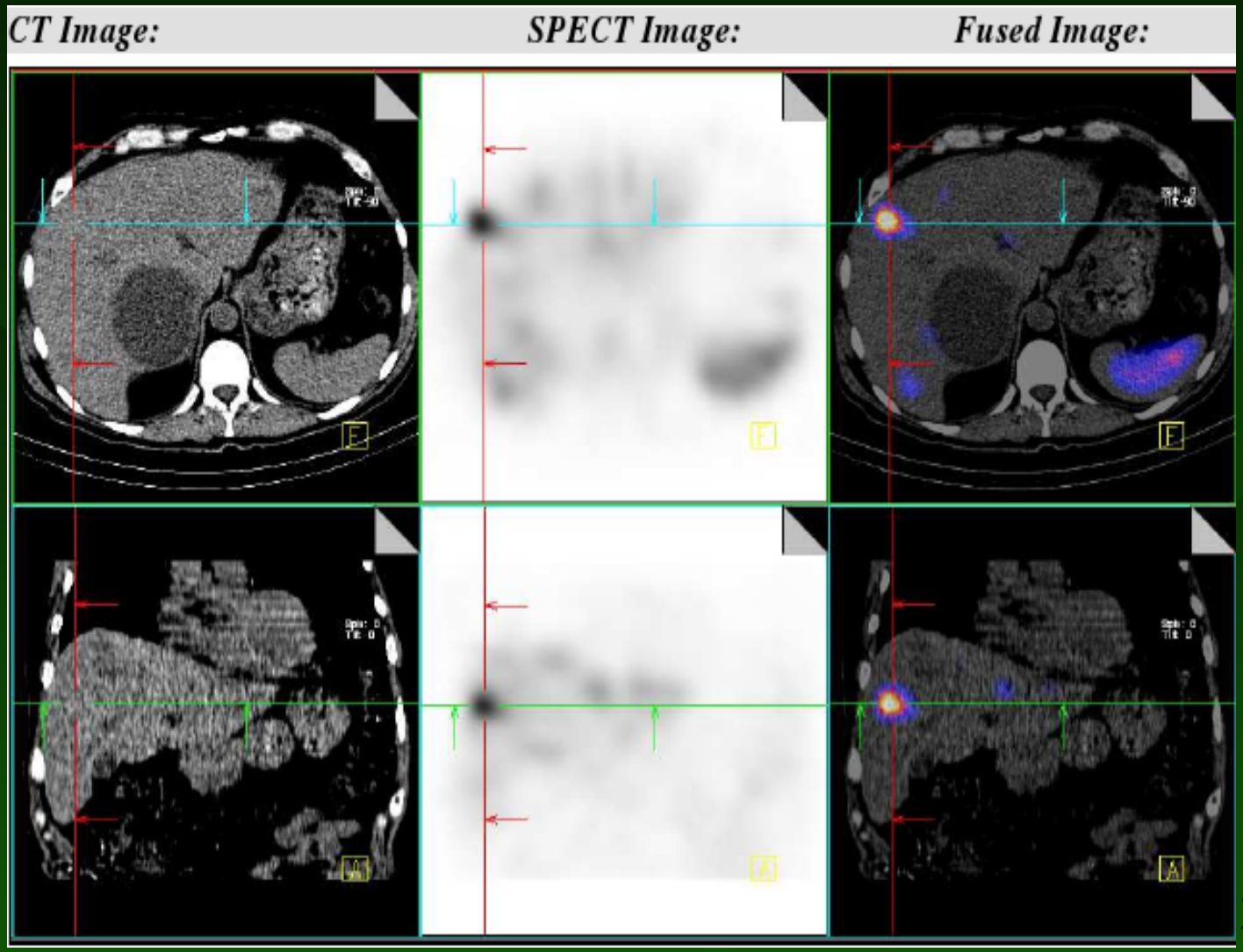
2D cost function landscape

Dual Modality Scanners

- Two imaging modalities in one device
- Registered images directly produced



Dual Modality Scanners



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- PJM van Laarhoven and EHJ Aarts (1987) Simulated Annealing: Theory and Applications. Series: Mathematics and its Applications, D. Reidel Publishing company, Dordrecht.