Acquisition of 3D Data
Tomographic Techniques

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3D Data

- **Animation, film**: a sequence of images with *time* as the third coordinate:
  \[ [x, y, t] \]

- **Volume data**: a sequence of images with *space* as the third coordinate:
  \[ [x, y, z] \]

- **Volume data in general**: a set of data samples measured on a grid with certain properties
Scattered Points

- Sample position
  - $[x_i, y_j, z_k]$  

- Neighborhood relation:
  - None
Unstructured Grid

- Sample position
  - $[x_p, y_p, z_p]$  
- Neighborhood relation:
  - Explicit $i-k$, $i-l$, ...
Structured Grid

- Sample position: based on coordinates at \([i,j,k]\)
- Neighborhood relation: Implicit, neighbors from the grid
Rectilinear Grid

- **Sample position:**
  - Coordinates \([i,j,k]\)
  - Distance between \(X,Y\) and \(Z\) planes

- **Neighborhood relation:**
  - Implicit, neighbors from the grid
Regular Grid

- Sample position:
  - Coordinates \([i,j,k]\)
  - Cell size \([X,Y,Z]\)

- Neighborhood relation:
  - Implicit, neighbors from the grid
Cartesian Grid

- Sample position:
  - Coordinates \([i,j,k]\)
  - Cell dimension \(X\)

- Neighborhood relation:
  - Implicit, neighbors from the grid
Basic elements of volume data

- Sample:
  - Dimensionless point with value

- Cell:
  - 8 (4) samples in vertices of a cube

- Voxel
  - Homogeneous cube centered at sample position
  - Analogue of the 2D pixel
Acquisition of Volumetric Data

- **3D imaging techniques**
  - **Anatomically** oriented techniques:
    - Computer tomography-CT, magnetic resonance imaging-MRI, ultrasound imaging-US
  - **Physiologically** oriented techniques:
    - Positron emission tomography-PET, single-photon emission tomography-SPECT, functional MRI

- **Synthetic data**
  - Voxelization
Computed Tomography (CT)

- Also: Computer Aided Tomography (CAT, CAT scan)

- Principle:
  - Measurement of X-ray attenuation along a viewing ray (projections)

\[ S = \int \mu(l) \ dl \]

- Production of images: reconstruction from projections
A CT Tomograph
A Sample CT Scan
Measurement of Projections

- Standard setup:
  - cca 200 1D projections for one slice
Measurement of Projections

- Spatial configuration of source and detector arrays:
  - Aligned rotation
  - 180 deg
Recent Setup: Fan-Beam Spiral CT
Standard CT vs. Spiral CT

25 Slices in > 2 Minutes

25 Slices in 25 Seconds
Multi-Detector CT Scanner

State of the art:

- Detectors
  - Up to 256
  - Spacing < 1mm
- Scanning speed:
  - 1m/30s
Reconstruction from Projections

- **Theory:**
  - Johann Radon

- **First Tomograph:**
  - Godfrey Hounsfield
  - Nobel prize 1979

- **Reconstruction methods:**
  - Algebraic methods
  - Filtered backprojection
  - Fourier methods

Filtered Backprojection

- For all projections:
  - Projection filtration (high pass filter, derivative)
  - Distribution of the filtered projection to the image in the direction of the projection
CT Tomogram - an Example
Dual Energy CT

- Attenuation coefficient $\mu$ depends on the wavelength
- Scanning with two X-ray lamp voltages (energies)
- Advantage: Discrimination of tissue types
Hounsfield Unit

Normalization of measured values to a -1000 – 3095 (12 bit) scale:

<table>
<thead>
<tr>
<th>Substance</th>
<th>Value [HU]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air</td>
<td>-1000</td>
</tr>
<tr>
<td>Water</td>
<td>0</td>
</tr>
<tr>
<td>Fat</td>
<td>70 – 120</td>
</tr>
<tr>
<td>Soft tissues</td>
<td>15 – 80</td>
</tr>
<tr>
<td>Bone</td>
<td>&gt;1000</td>
</tr>
</tbody>
</table>

Tissue density is defined by its physical properties
CT Data Properties

- Axial slices 0.5-10 mm thick
- Images 256 x 256 to 512 x 512 pixels of 0.5-2 mm side
- Up to 2000 images per study
- High spatial resolution
- Consistent values (HU scale)
- X-ray irradiation
Application Fields

- Contrast determined by tissue density:
  - Bone imaging
  - Calcifications
  - Inflammations
  - Hematomata
  - Tumors

- Imaging with contract agent:
  - Blood vessels, vessel lumen
Overview

- Computer tomography (CT, CAT)
- Magnetic resonance imaging (MRI)
- fMRI
- SPECT
- PET
Magnetic Resonance Imaging

- **Nuclear Magnetic Resonance (NMR)**
  - **Magnetic Resonance Imaging (MRI)**

- **Physical principle**
  - Interaction of atom nuclei with an external magnetic field (resonance)
  - Requirement: Nonzero magnetic or spin moment of the atoms

- **Resonance**
  - Energy transfer between coupled systems with equal characteristic frequency
  - Example: The Tacoma Narrows bridge
<table>
<thead>
<tr>
<th>Isotope</th>
<th>Spin</th>
<th>% abundance</th>
<th>$\gamma$ MHz/T</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^1$H</td>
<td>1/2</td>
<td>99.985</td>
<td>42.575</td>
</tr>
<tr>
<td>$^2$H</td>
<td>1</td>
<td>0.015</td>
<td>6.53</td>
</tr>
<tr>
<td>$^{13}$C</td>
<td>1/2</td>
<td>1.108</td>
<td>10.71</td>
</tr>
<tr>
<td>$^{14}$N</td>
<td>1</td>
<td>99.63</td>
<td>3.078</td>
</tr>
<tr>
<td>$^{15}$N</td>
<td>1/2</td>
<td>0.37</td>
<td>4.32</td>
</tr>
<tr>
<td>$^{17}$O</td>
<td>5/2</td>
<td>0.037</td>
<td>5.77</td>
</tr>
<tr>
<td>$^{19}$F</td>
<td>1/2</td>
<td>100</td>
<td>40.08</td>
</tr>
<tr>
<td>$^{23}$Na</td>
<td>3/2</td>
<td>100</td>
<td>11.27</td>
</tr>
<tr>
<td>$^{31}$P</td>
<td>1/2</td>
<td>100</td>
<td>17.25</td>
</tr>
</tbody>
</table>
Spins in a Magnetic Field

Atoms with uncompensated magnetic moment

\[ \text{M} \neq 0 \]

\[ \text{B}_0 = 0 \]

\[ \text{B}_0 \neq 0 \]

\[ \text{M} - \text{Magnetization (material)} \]

\[ \text{B} - \text{Induction (field strength)} \]
Spin in a Magnetic Field (Zeeman Splitting, Spin ½)

$m_i = +\frac{1}{2}$

$P_{+1/2} = 0.5000049$

$m_i = -\frac{1}{2}$

$P_{-1/2} = 0.4999951$

1.5T, T=310K,

⇒ Total magnetization $M$ is parallel to $B_0$
Spin in a Magnetic Field

- $\mu$ rotates around $B_0$
- $B_0$ – external mg. field
- $\mu$ – spin moment
  - rotates around $B_0$ with Larmor frequency $f_L = \gamma B_0$
- $\gamma$ – gyromagnetic ratio ($\gamma = 42.58 \text{ MHz/T for } H$)
- Total magnetization: $M = \sum \mu_i$
Magnetic Field of a Tomograph

- Induced magnetic field:
  - Electromagnets
  - Superconductive electromagnets
  - Permanent magnets

- Field strength
  - 0.5T – 3T, (up to 15T for research)

- Earth’s mg. field: 0.3 - 0.7 \times 10^{-5}T
Principle of MR Imaging

Absorption and emission of energy by spins in external magnetic field (resonance)

- **Equilibrium**: $M \parallel B_{0}$: Uncoupled $\mu_{i}$ vectors:
  
  $$M_{z} = M, \quad M_{xy} = 0$$

- **Absorption of RF (radiofrequency) energy**: excitation ($f = f_{L}$)
  - Transition to higher energy state: $M_{z} < M$ owing to vector $\mu_{i}$ flipping
  - Coupled rotation of $\mu_{i}$: $M_{xy} \neq 0$
  - Measurable signal due to $M_{xy}$

- **Relaxation**: return to equilibrium
Spin Excitation and Relaxation

Equilibrium

Excitation by RF impulse

Relaxation: Return back to equilibrium.
Relaxation

Return to equilibrium state after the RF pulse

- **Longitudinal relaxation** (spin-lattice, rate $T_1$):
  - Energy transfer to surrounding tissue
  - Flipping of $\mu_i$ vectors to original orientation

- **Transversal relaxation** (spin-spin, rate $T_2$):
  - Decoupling of $\mu_i$ due to field inhomogeneities and spin-spin interaction

- Measured signal during the relaxation
  - **FID** (Free Induction Decay)
Both $T_1$ and $T_2$ depend on the chemical environment of the nucleus (spin).

TE: time of measurement, echo time
Tissue Contrast

- Effect of TE (echo time, time of measurement)
Tissue Contrast

- Effect of echo time
Tissue Contrast

- Effect of echo time
Tissue Contrast

- Effect of echo time
MRI Measurement

- TR: repetition time
- TE: excitation time, moment of measurement
Different combinations of TR and TE yield different tissue contrast:

- **Long TR (>1500ms)**
  - Short TE (<25ms) PD (Proton Density) weighting (no $T_1$ and $T_2$ influence)
  - Long TE (>50ms) $T_2$ weighting

- **Short TR (<500ms)**
  - Short TE (<25ms) $T_1$ weighting
$T_1$, $T_2$ and PD Images
MRI Overview

- Physical Background
- Excitation and Relaxation
  - $T_1$ Relaxation
  - Spin Echo and $T_2$ Relaxation
- Coding of Spatial Information
  - Slice Selection
  - Read-out Gradient
  - Phase Encoding
Position encoding by means of gradient fields:

- **Z-gradient**: slice selection by changing the Larmor frequency
  \[ f_L = \gamma (B_0 + G_z \cdot z) \]
  Z-gradient is applied during the RF pulse
Slice Selection

RF pulse

\( G_z \)

\( \Delta \omega \)

frequency

\( \omega_0 \)
X, Y encoding

- Similar tricks to encode the $x$ and $y$ coordinates
  - Only one row measured in one excitation
Examination Time

- Image is measured row-by-row
- Basic formula: \( T = TR \times R \times N \) [ms]
  - \( TR \): repetition time
  - \( R \): number of image rows
  - \( N \): Number of accumulations (noise)
- Example: \( T = 2000 \times 256 \times 2 = 17 \) min
- Speed-up:
  - Low excitation angle \( \Rightarrow \) shorter TR (low energy pulse)
  - Multislice techniques
Multislice Technique

- Interleaved excitation of several (64) slices:
Imaging in Other Planes

- $B_0$ always remains in the same direction
- Choice of imaging plane depends on the order of gradients’ application
- Oblique planes: simultaneous application of 2 gradients

![Sagittal](Image)
![Coronal](Image)
![Transversal](Image)

Sagittal  Coronal  Transversal
Scanning Protocols

- **Protocol**: a sequence of pulses, gradients and signal measurements
- Protocols influence image properties and examination time
- Patented and sold by scanner vendors
- FISP, FAST, FLASH, STAGE, STERF
Properties of MR data

- Measurement in arbitrary planes
- Typically 256x256 (512x512)pixels
- No absolute scale for measured values
- Significant level of noise
- Good soft tissue contrast
- Spatial inhomogeneities - bias
- No irradiation
MR data - examples

$T_1$, PD, MRA
Overview

- Computer tomography (CT, CAT)
- Magnetic resonance imaging (MRI)
- fMRI
- SPECT
- PET
Assumption: Active brain area needs more oxygen than inactive ones

fMRI: statistical detection of areas with oxygenated blood flow

Visualization by merging with regular MRI data
fMRI Example

Right hand finger tapping at 2Hz
Overview

- Computer tomography (CT, CAT)
- Magnetic resonance imaging (MRI)
- fMRI
- Emission Tomography
  - SPECT
  - PET
Emission Tomography

ECT - Emission computed tomography

- Introduction of a radioactive agent into the patient’s body (tagged substance)
- Agent’s distribution based on metabolism
- Measurement of its spatial distribution
SPECT

Single-Photon Emission Computed Tomography

- Isotopes emitting $\gamma$-photons (Tc-99, I-125, I-131)
- Uniform distribution of photons in all directions
- Scanner - a set of detectors with collimators - gamma camera
Gamma Camera

Diagram showing the components of a gamma camera:
- Head
- Collimator
- Photomultiplier Tubes
- NaI Crystal
- Computer
- Display
Gamma Camera

radiation area and creates an image.

Gamma camera
SPECT data
CT-SPECT

- Combination of different techniques – supplementary information
- Registration is required
  - Not a trivial task
  - Solution: CT-SPECT scanner
CT-SPECT (3)
PET

Positron Emission Tomography

- $\beta^+$ decay (positrons)
- Annihilation - a pair of photons with energy 511keV in opposite directions
- Detection without collimators: registration of concurrent events in detector pairs
- 3D measurement, statistical reconstruction (MRF)
PET Scanner
PET data
PET Scanners

Research prototype at BNL (1961)

PET today
Combined PET/CT, MRI
Other Imaging Modalities

- Ultrasound
- Confocal microscopy
- Electrical impedance tomography
- Optical coherence tomography
- ...

![Image of imaging modality](image_url)
Overview

- Computed tomography (CT / CAT)
- Magnetic resonance imaging (MRI)
- fMRI
- SPECT
- PET
- Synthetic Data
Voxelization of Geometric Objects

- Preparation of “synthetic” data
- A CG alternative
- Simultaneous visualization of geometric and volume data
- Modeling of volumetric properties (e.g. weathering)
Surface and Volume Graphics

- **Surface Graphics**: Geometric rendering of Continuous Spatial Models (CSM)
- **Volume Graphics**: Voxelization of a CSM, manipulation, and volume rendering of a Volumetric Model (VM)
Voxelization

♦ A **process** of approximating a continuous geometric primitive in the 3D discrete space
♦ The **result** of this process

**Binary voxelization**
Voxels & occupancy $O \in \{0,1\}$ (Background, Foreground}, {Black, White})

**Non-binary voxelization** (fuzzy, filtered)
Grid points & densities $D \in \mathbb{R}$
Binary vs. nonbinary

Binary

Non binary
Non-binary Voxelization

Suppression of aliasing in 3D model by smooth density transition in the surface area

- Truncated distance field

- Surface reconstruction
  - Interpolation and thresholding
Solids in Truncated Distance Fields

Object surface

Density transition area

Surface density profile

Surface density profile

1

0.5

2\(\delta\)
**Voxelization by Direct Distance Computation**

Simple primitives (sphere, torus, polygon)

**Example: Voxelization of a triangle:**

- Bounding box
- Voxel-by-voxel update of distances to plane, edges & vertices
- Density according to the minimal distance
Voxelized Polygonal Model
Voxelization of Parametric Surfaces

\[ P(u,v) = [x(u,v), y(u,v), z(u,v)] \]

\[ [u, v] \in (u_0, u_1) \times (v_0, v_1) \]

1. Splatting – adding small voxelized balls
2. Approx uniform sampling by binary domain subdivision

Domain subdivision

Surface subdivision
Voxelization of Parametric Surfaces
Voxelization of Implicit Solids

\[
\{[x,y,z] : f(x, y, z) < 0\}
\]

• Distance estimation by linear approximation:

\[
d(x, y, z) = \frac{f(x, y, z)}{\|f'(x, y, z)\|}
\]
Voxelization of Implicit Solids

\[(y^2 + s)(x^2 + z^2) - s \quad (2x^2 + y^2 + z^2)^3 - (x^2/10 + y^2)z^3\]
# CSG Operations

<table>
<thead>
<tr>
<th>Operation</th>
<th>Density</th>
<th>Color</th>
</tr>
</thead>
</table>
| **Intersection** | $d_{A \ast B} =$  
                 | $\min(d_A,d_B)$       | $C_{A \ast B} =$  
                 | $WA(A,B)$             |
| **Union**     | $d_{A+B} =$           | $C_{A+B} =$         |
|               | $\max(d_A,d_B)$       | $WA(A,B)$            |
| **Difference**| $d_{A-B} =$           | $C_{A-B} =$         |
|               | $\max(0,d_A-d_B)$     | $C_A$ if $d_{A-B} > 0$ |
CSG Operations

\[ a * (b + c) \]