Flow Visualization

Part 2 (of 3)

Retrospect: Flow Visualization, Part 1

- introduction, overview
  - simulation vs. measurement vs. modelling
  - 2D vs. surfaces vs. 3D
  - steady vs time-dependent
  - direct vs. indirect FlowVis
- experimental FlowVis
  - general possibilities
  - PIV + example
- visualization of models
- flow visualization with arrows

Overview: Flow Visualization, Part 2

- numerical integration
  - Euler-integration
  - Runge-Kutta-integration
- streamlines
  - in 2D
  - particle paths
  - in 3D, sweeps
  - illuminated streamlines
- streamline placement

Integration of Streamlines

Numerical Integration

Streamlines – Theory

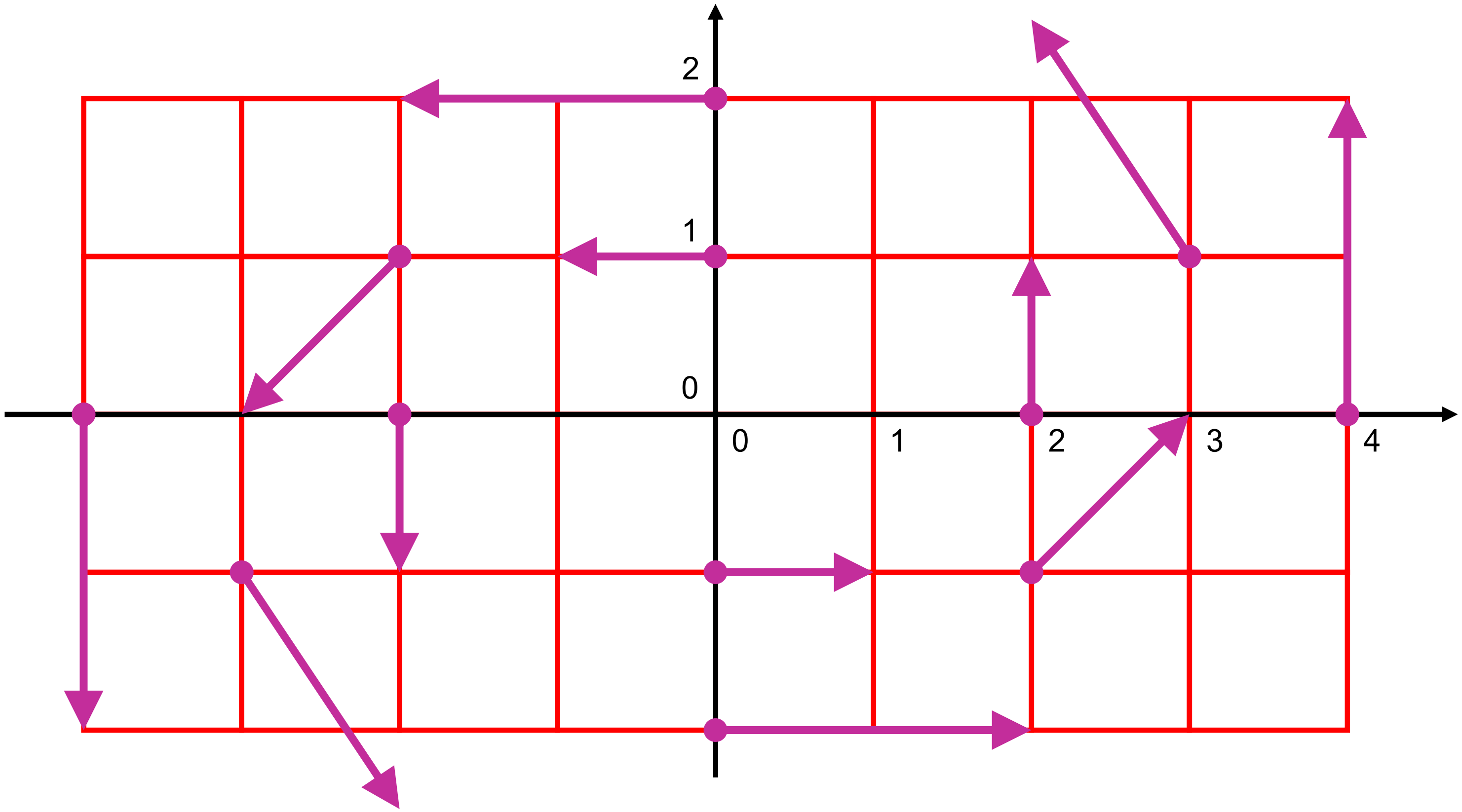
- Correlations:
  - flow data $\mathbf{v}$: derivative information
  - $d\mathbf{x}/dt = \mathbf{v}(\mathbf{x})$;
    spatial points $\mathbf{x} \in R^n$, time $t \in R$, flow vectors $\mathbf{v} \in R^n$
  - streamline $\mathbf{s}$: integration over time,
    also called trajectory, solution, curve
  - $\mathbf{s}(t) = \mathbf{s}_0 + \int_{0 \le u \le t} \mathbf{v}(\mathbf{s}(u))\, du$;
    seed point $\mathbf{s}_0$, integration variable u
  - difficulty: result $\mathbf{s}$ also in the integral $\Rightarrow$ analytical solution usually impossible!

Streamlines – Practice

- Basic approach:
  - theory: $\mathbf{s}(t) = \mathbf{s}_0 + \int_{0 \le u \le t} \mathbf{v}(\mathbf{s}(u))\,\mathrm{d}u$
  - practice: numerical integration
  - idea:
    (very) locally, the solution is (approx.) linear
  - Euler integration:
    follow the current flow vector $\mathbf{v}(\mathbf{s}_i)$ from the current streamline point $\mathbf{s}_i$ for a very small time (dt) and therefore distance
  - Euler integration: $\mathbf{s}_{i+1} = \mathbf{s}_i + \mathrm{d}t \cdot \mathbf{v}(\mathbf{s}_i)$,
    integration of small steps (dt very small)

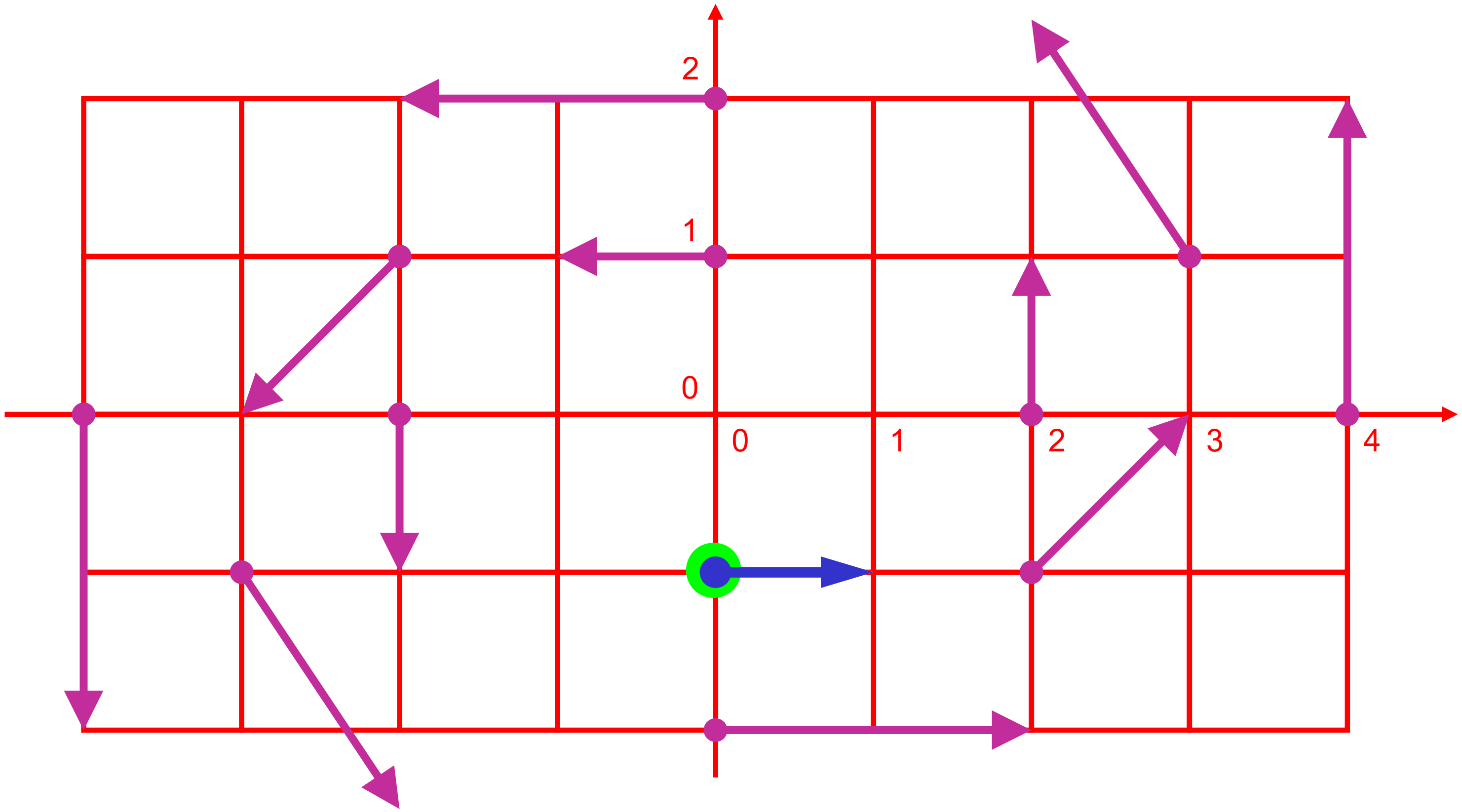
Euler Integration – Example

- 2D model data: $v_x = dx/dt = -y$
  $v_y = dy/dt = x/2$
- Sample arrows:
- True solution: ellipses!



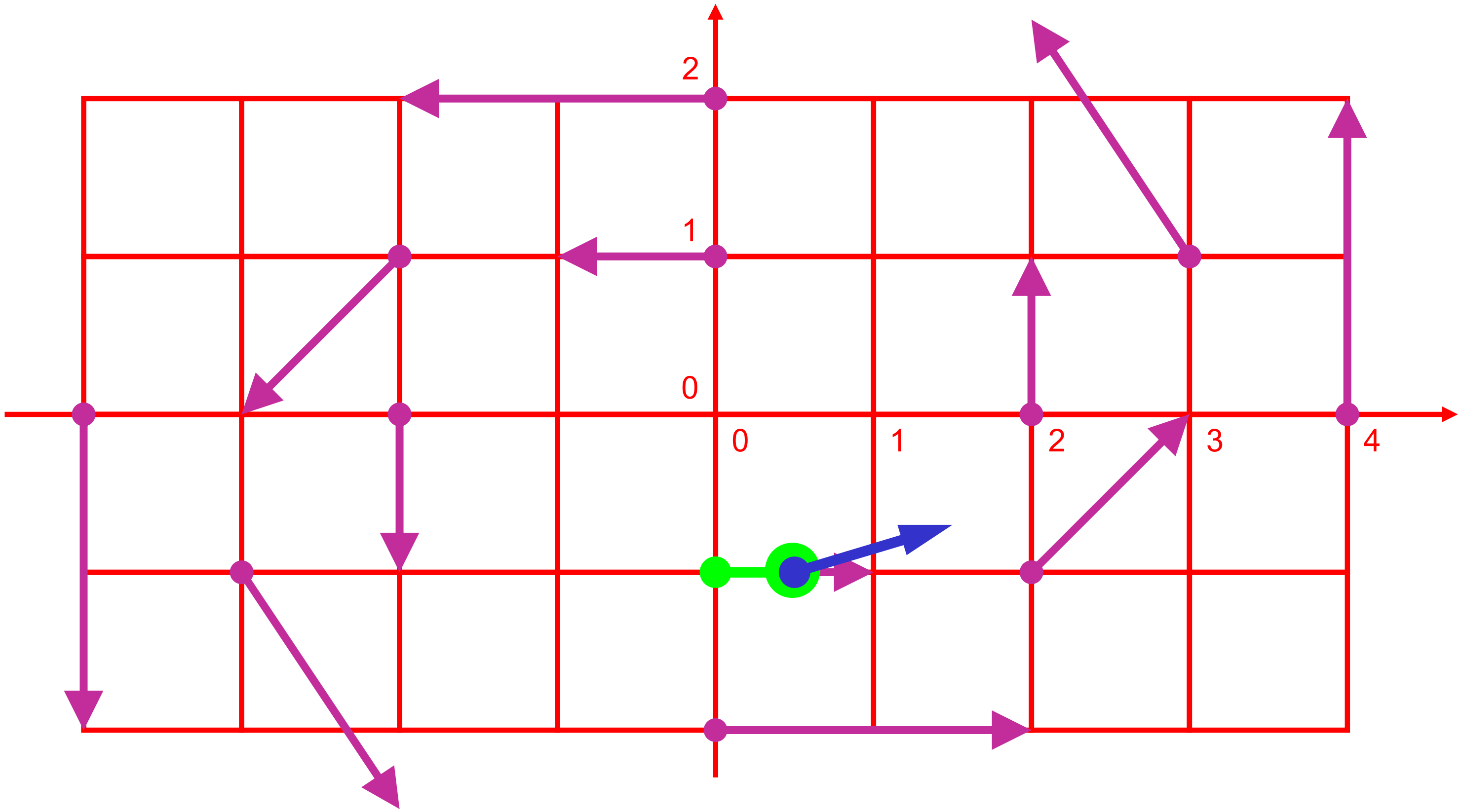
Euler Integration – Example

- Seed point $\mathbf{s}_0 = (0|-1)^T$;
  current flow vector $\mathbf{v}(\mathbf{s}_0) = (1|0)^T$;
  $dt = 1/2$



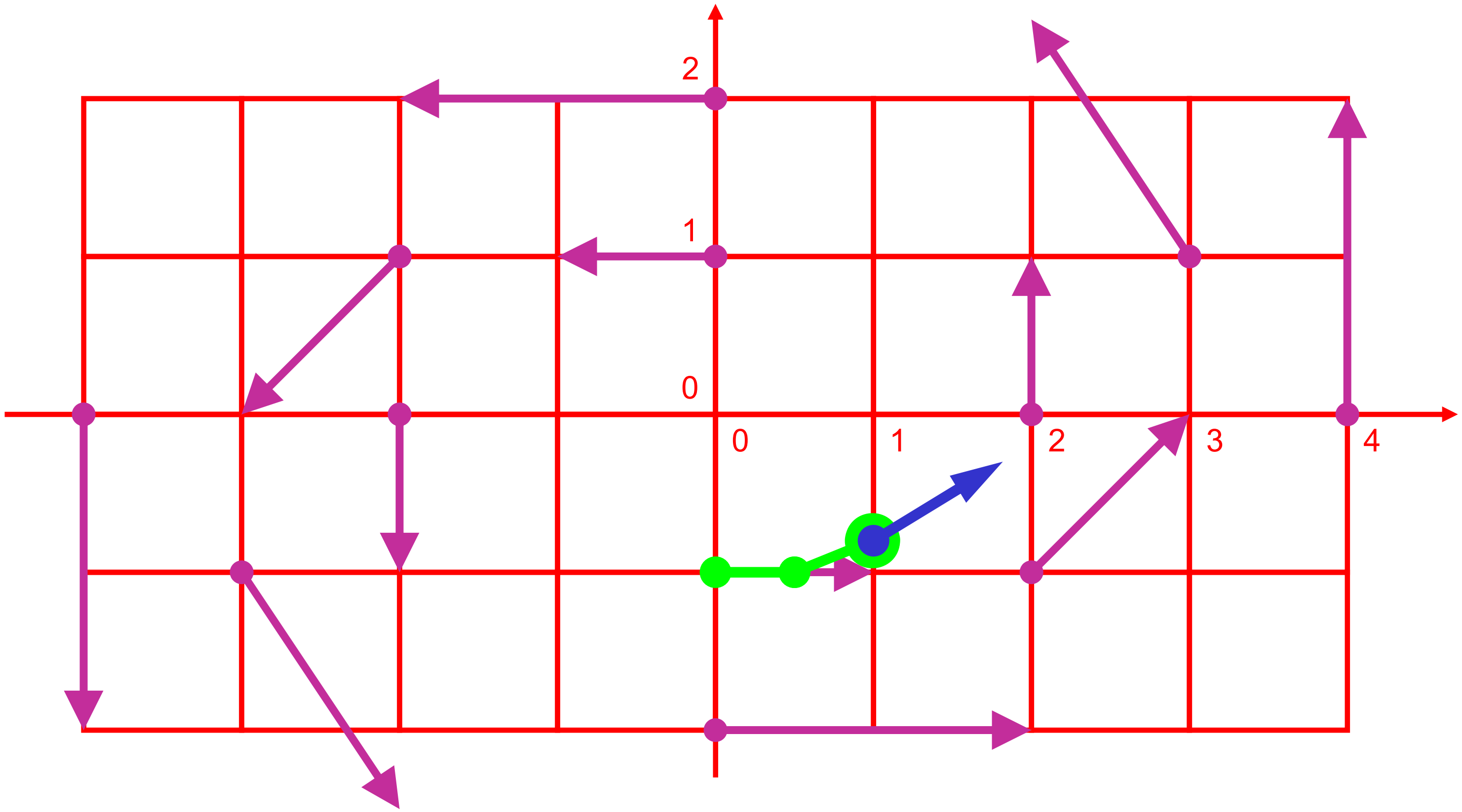
Euler Integration – Example

- New point $\mathbf{s}_1 = \mathbf{s}_0 + \mathbf{v}(\mathbf{s}_0) \cdot dt = (1/2|-1)^T$;
  current flow vector $\mathbf{v}(\mathbf{s}_1) = (1|1/4)^T$;



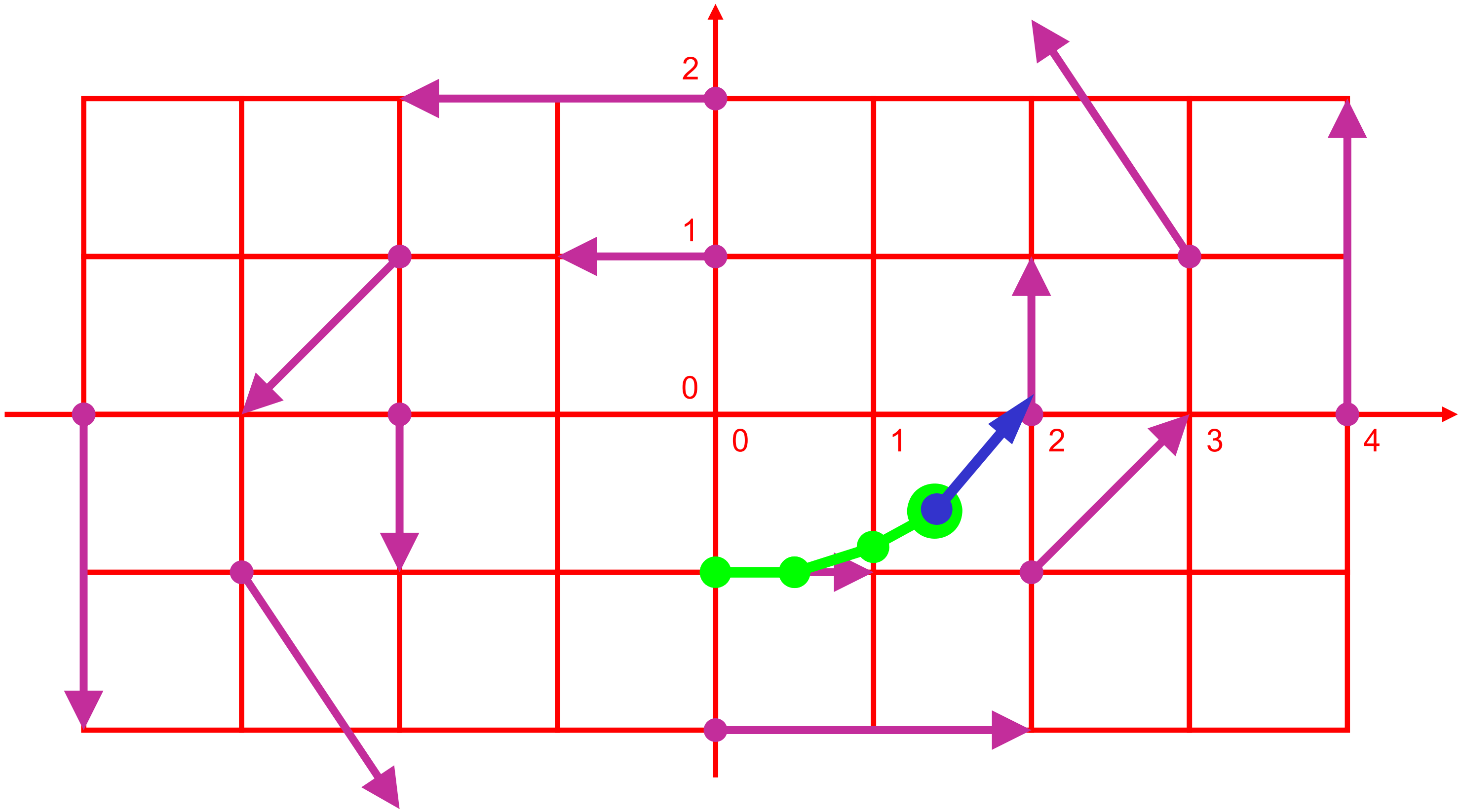
Euler Integration – Example

- New point $\mathbf{s}_2 = \mathbf{s}_1 + \mathbf{v}(\mathbf{s}_1) \cdot \mathrm{d}t = (1 \,|\, -7/8)^T$;
  current flow vector $\mathbf{v}(\mathbf{s}_2) = (7/8 \,|\, 1/2)^T$;



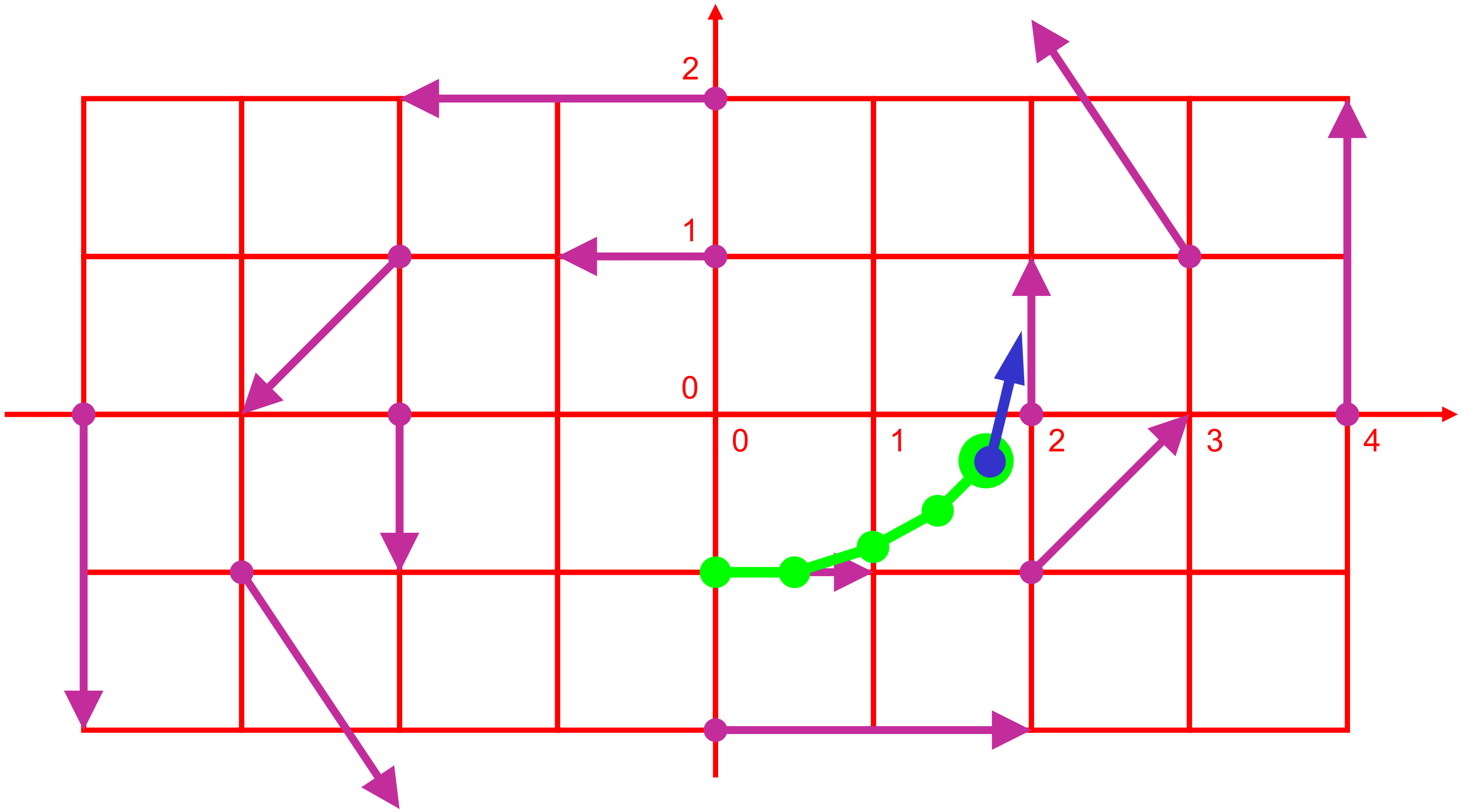
Euler Integration – Example

- $\mathbf{s}_3 = (23/16\,|\,{-5/8})^T \approx (1.44\,|\,{-0.63})^T;$
  $\mathbf{v}(\mathbf{s}_3) = (5/8\,|\,23/32)^T \approx (0.63\,|\,0.72)^T;$



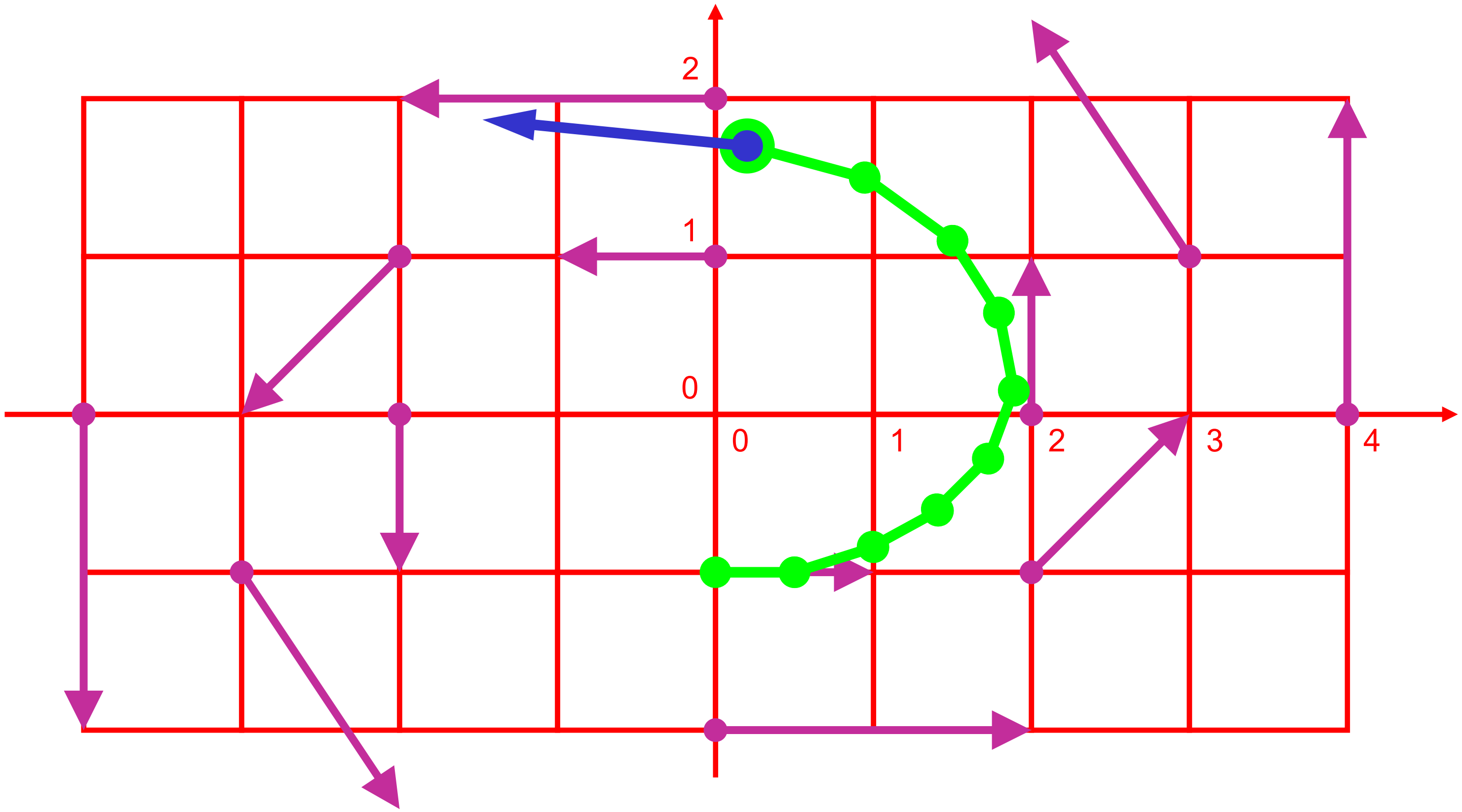
Euler Integration – Example

- $\mathbf{s}_4 = (7/4 \mid -17/64)^T \approx (1.75 \mid -0.27)^T;$
  $\mathbf{v}(\mathbf{s}_4) = (17/64 \mid 7/8)^T \approx (0.27 \mid 0.88)^T;$



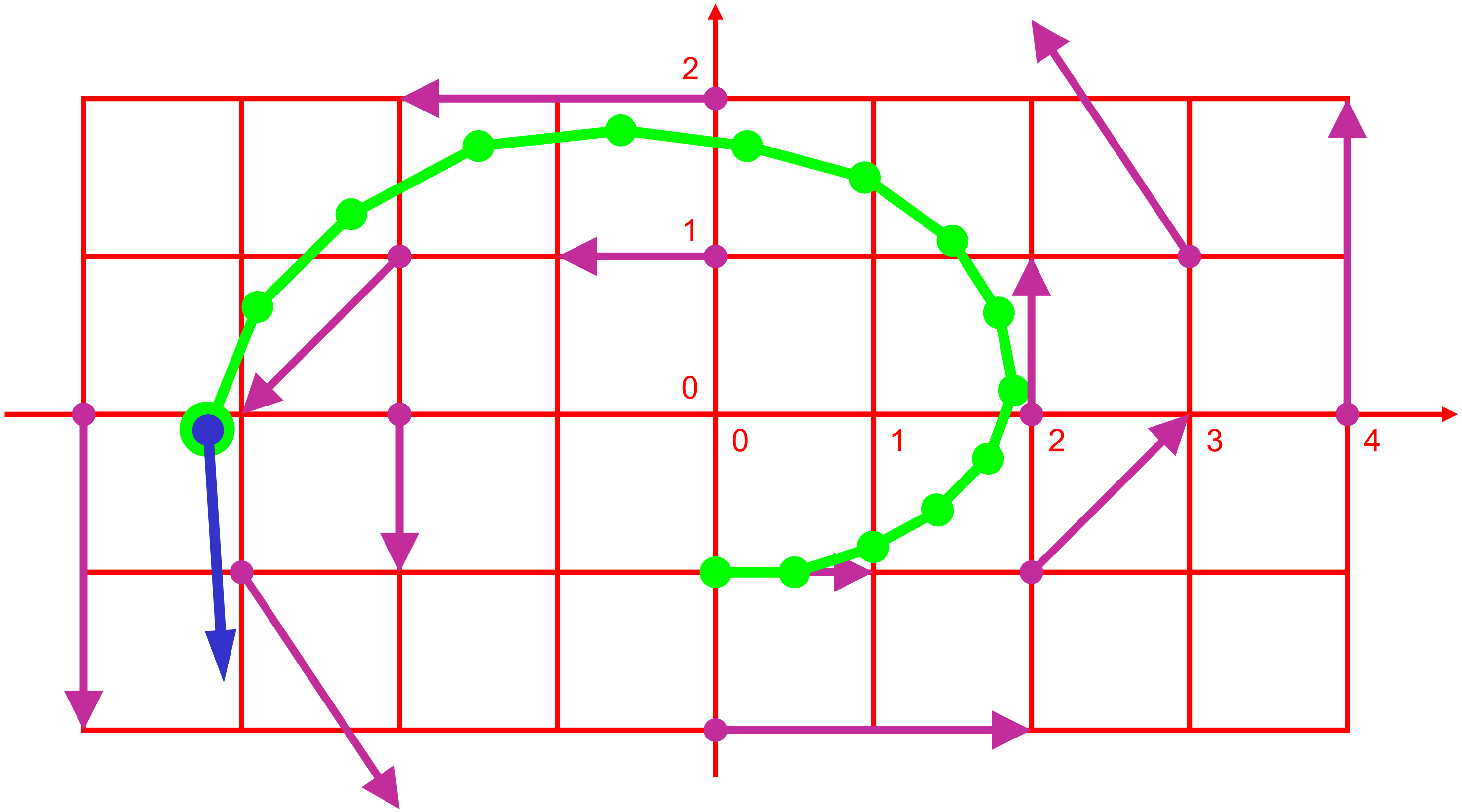
Euler Integration – Example

- $\mathbf{s}_9 \approx (0.20 \,|\, 1.69)^T;$
  $\mathbf{v}(\mathbf{s}_9) \approx (-1.69 \,|\, 0.10)^T;$



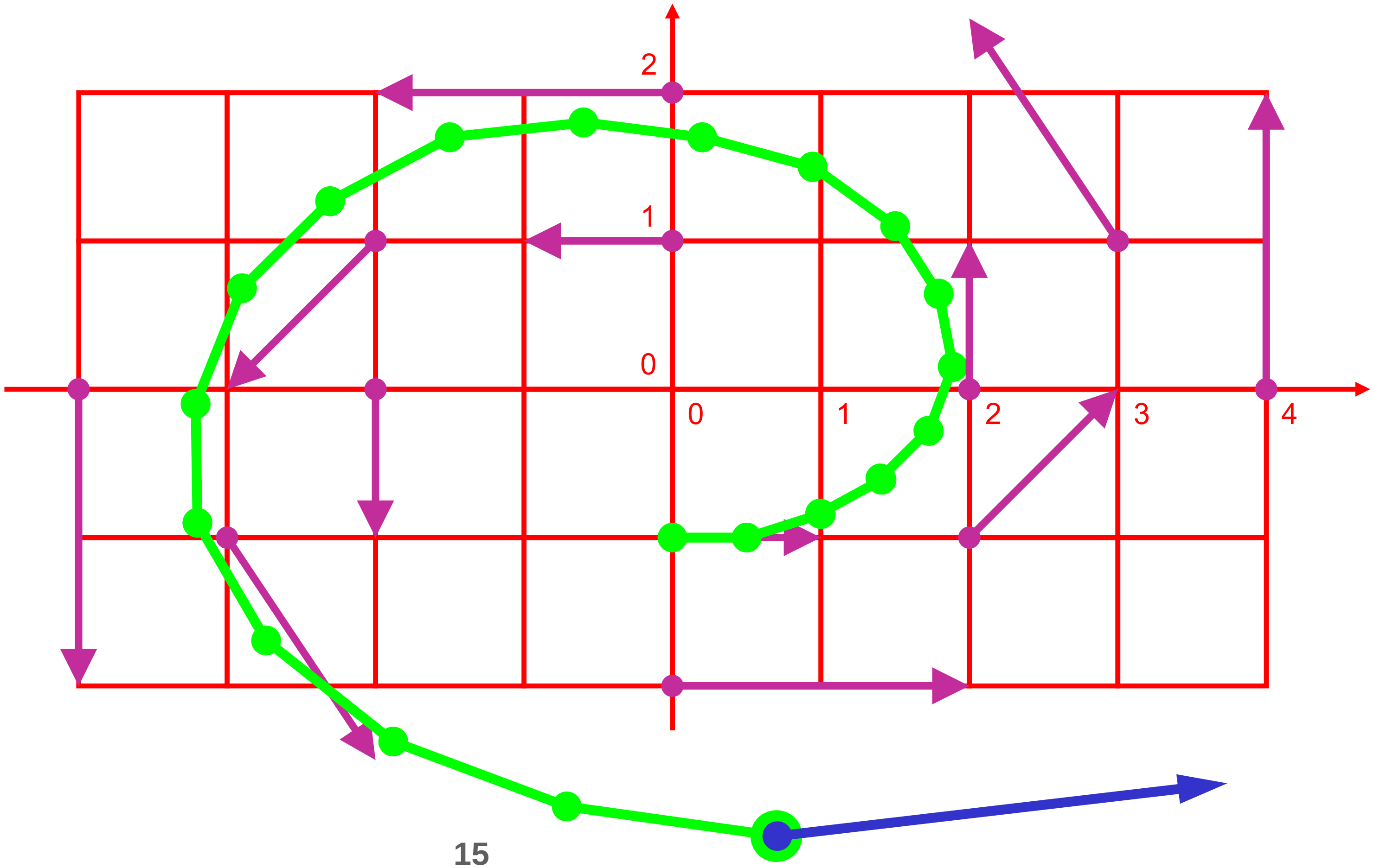
Euler Integration – Example

- $\mathbf{s}_{14} \approx (-3.22 \,|\, -0.10)^T;$
  $\mathbf{v}(\mathbf{s}_{14}) \approx (0.10 \,|\, -1.61)^T;$



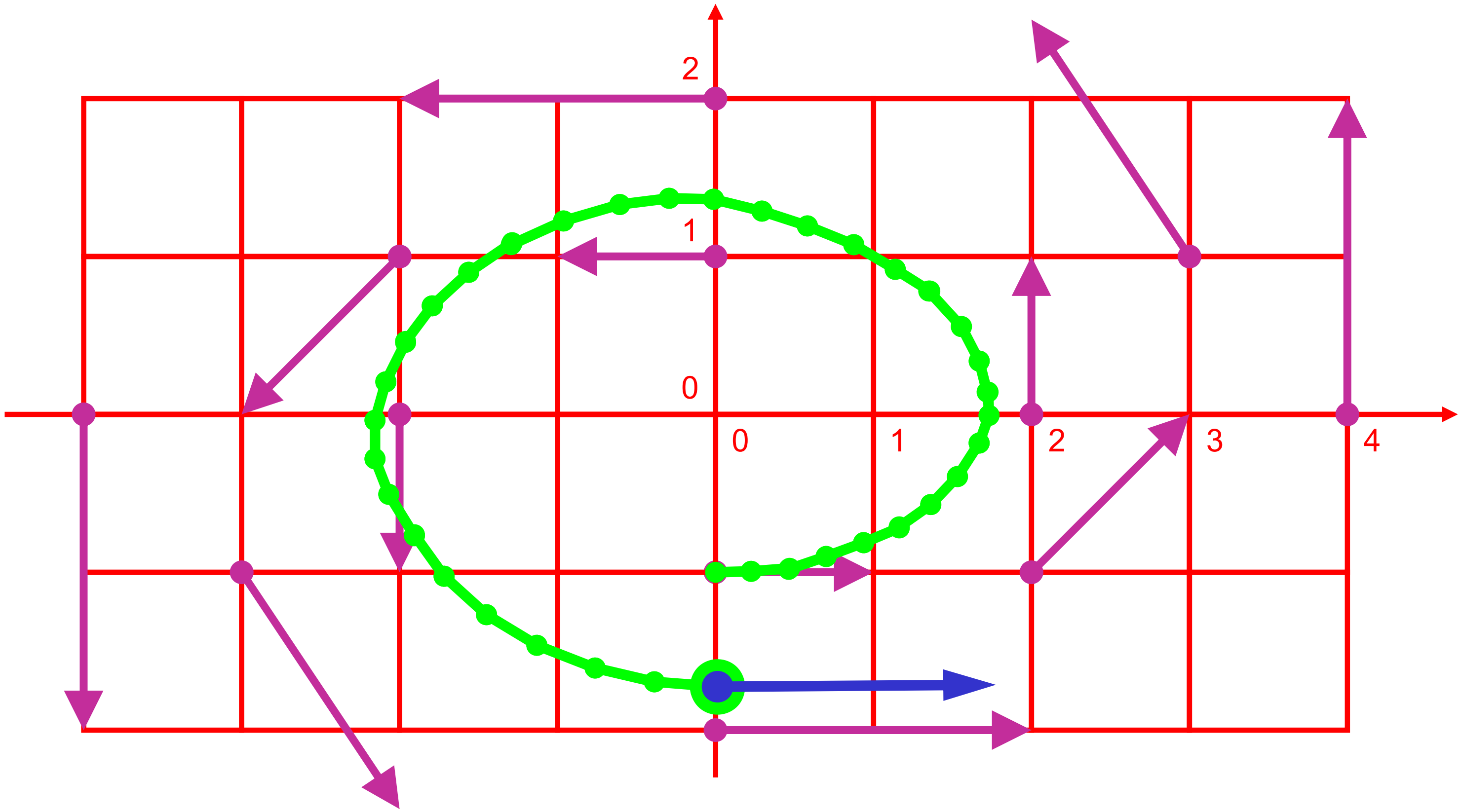
Euler Integration – Example

- $\mathbf{s}_{19} \approx (0.75 \mid -3.02)^T$; $\mathbf{v}(\mathbf{s}_{19}) \approx (3.02 \mid 0.37)^T$;
  clearly: large integration error, d*t* too large!
  19 steps



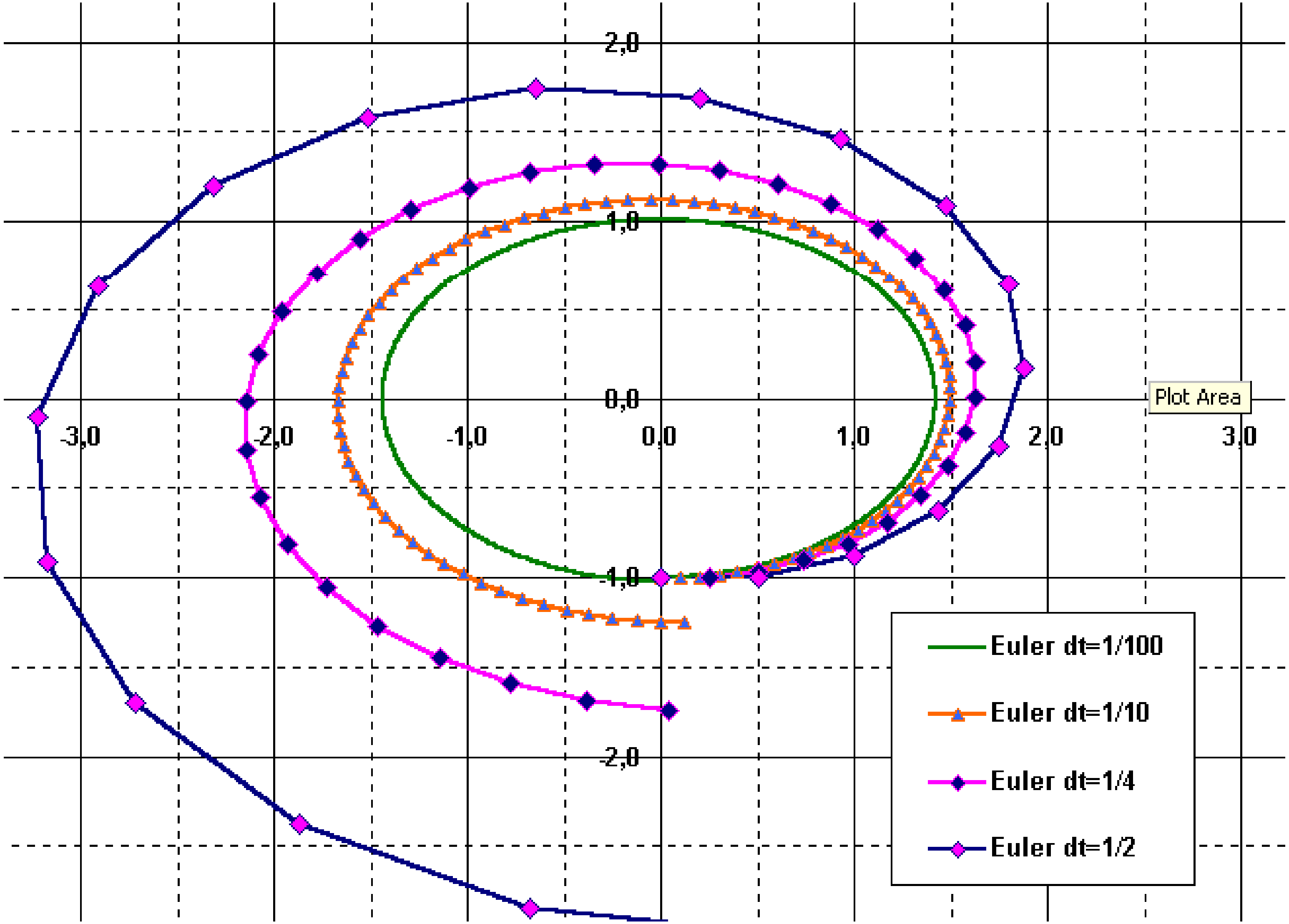
Euler Integration – Example

- d*t* smaller (1/4): more steps, more exact!
  $\mathbf{s}_{36} \approx (0.04 \,|\, -1.74)^T$; $\mathbf{v}(\mathbf{s}_{36}) \approx (1.74 \,|\, 0.02)^T$;
- 36 steps



Comparison Euler, Step Sizes

Euler
is getting
better
propor-
tionally
to d*t*



Euler Example – Error Table

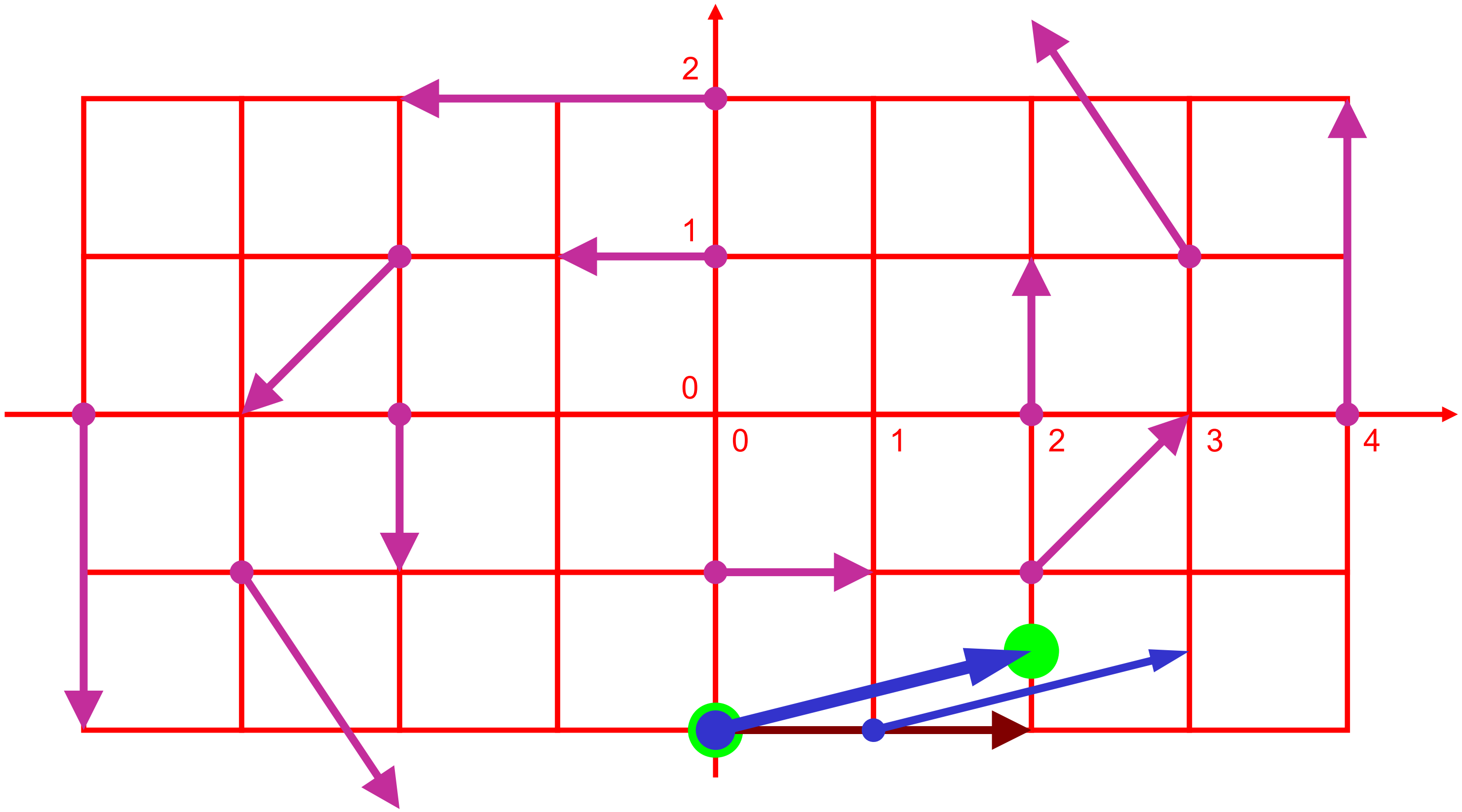
d*t*	#steps	error	
1/2	19	~200%	
1/4	36	~75%	
1/10	89	~25%	
1/100	889	~2%	
1/1000	8889	~0.2%	✓

Better than Euler Integr.: RK

- Runge-Kutta Approach:
  - theory: $\mathbf{s}(t) = \mathbf{s}_0 + \int_{0 \le u \le t} \mathbf{v}(\mathbf{s}(u))\,\mathrm{d}u$
  - Euler: $\mathbf{s}_i = \mathbf{s}_0 + \Sigma_{0 \le u < i} \mathbf{v}(\mathbf{s}_u) \cdot \mathrm{d}t$
  - Runge-Kutta integration:
    - idea: cut short the curve arc
    - RK-2 (second order RK):
      1.: do half a Euler step
      2.: evaluate flow vector there
      3.: use it in the origin
    - RK-2 (two evaluations of **v** per step):
      $\mathbf{s}_{i+1} = \mathbf{s}_i + \mathbf{v}(\mathbf{s}_i + \mathbf{v}(\mathbf{s}_i) \cdot \mathrm{d}t/2) \cdot \mathrm{d}t$

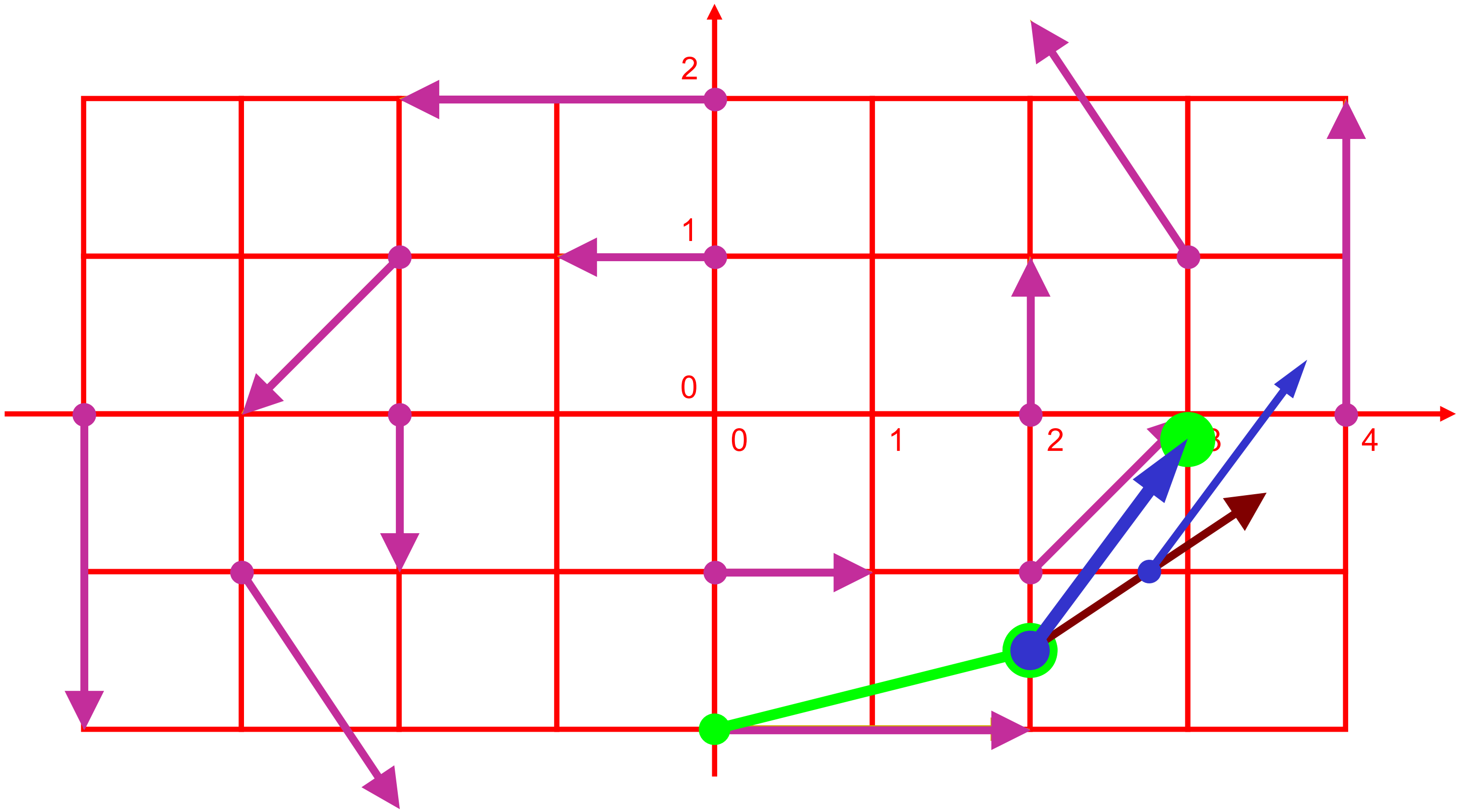
RK-2 Integration – One Step

- Seed point $\mathbf{s}_0 = (0|-2)^T$;
  current flow vector $\mathbf{v}(\mathbf{s}_0) = (2|0)^T$;
  preview vector $\mathbf{v}(\mathbf{s}_0 + \mathbf{v}(\mathbf{s}_0) \cdot dt/2) = (2|0.5)^T$;
  $dt = 1$



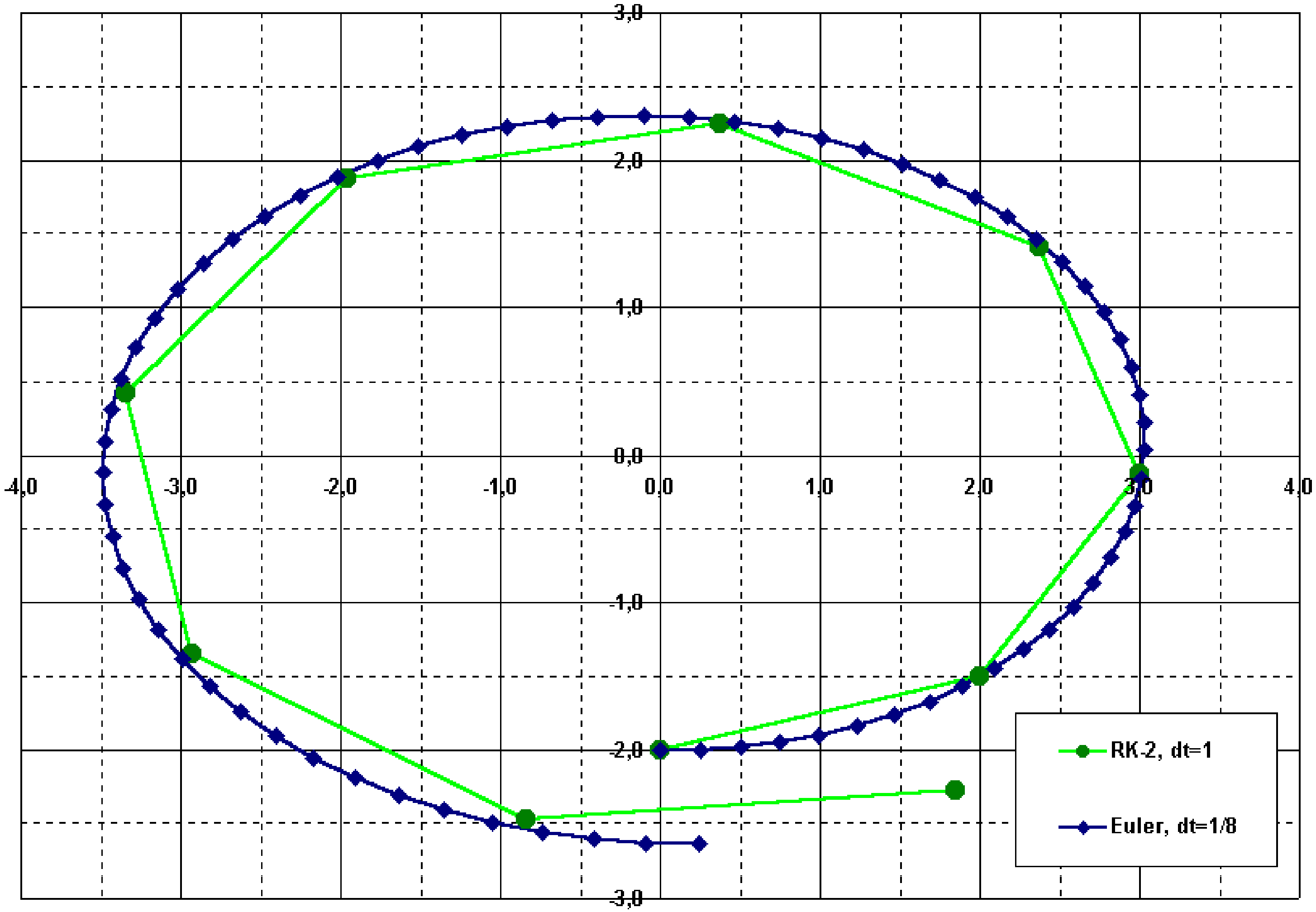
RK-2 – One more step

- Seed point $\mathbf{s}_1 = (2|-1.5)^T$;
  current flow vector $\mathbf{v}(\mathbf{s}_1) = (1.5|1)^T$;
  preview vector $\mathbf{v}(\mathbf{s}_1+\mathbf{v}(\mathbf{s}_1)\cdot dt/2) \approx (1|1.4)^T$;
  $dt = 1$



RK-2 – A Quick Round

- RK-2: even with dt=1 (9 steps) better than Euler with dt=1/8 (72 steps)

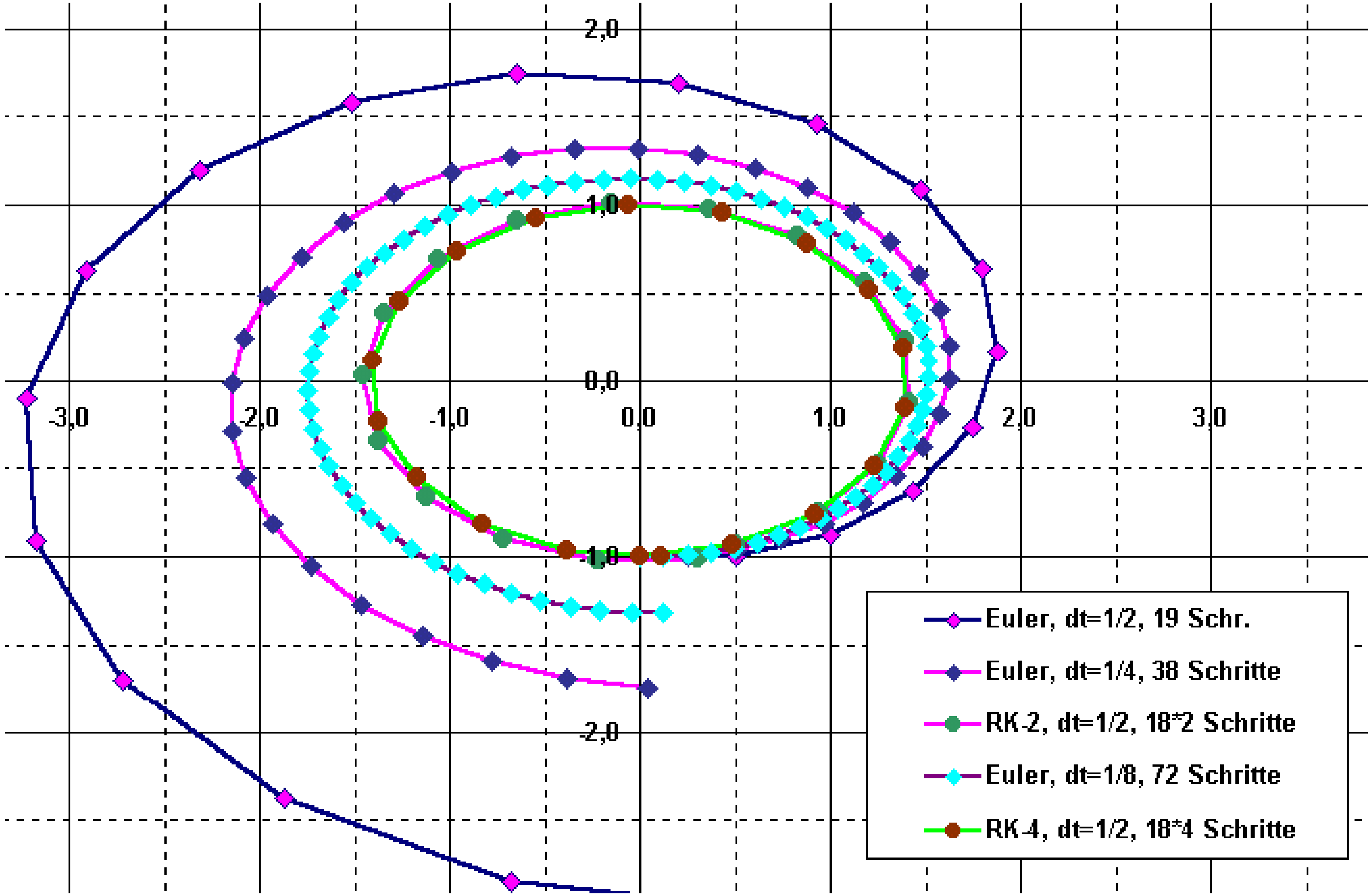


RK-4 vs. Euler, RK-2

- Even better: fourth order RK:
  - four vectors **a**, **b**, **c**, **d**
  - one step is a convex combination:
    $\mathbf{s}_{i+1} = \mathbf{s}_i + (\mathbf{a} + 2 \cdot \mathbf{b} + 2 \cdot \mathbf{c} + \mathbf{d})/6$
  - vectors:
    - $\mathbf{a} = \mathrm{d}t \cdot \mathbf{v}(\mathbf{s}_i)$ … original vector
    - $\mathbf{b} = \mathrm{d}t \cdot \mathbf{v}(\mathbf{s}_i + \mathbf{a}/2)$ … RK-2 vector
    - $\mathbf{c} = \mathrm{d}t \cdot \mathbf{v}(\mathbf{s}_i + \mathbf{b}/2)$ … use RK-2 …
    - $\mathbf{d} = \mathrm{d}t \cdot \mathbf{v}(\mathbf{s}_i + \mathbf{c})$ … and again!

Euler vs. Runge-Kutta

- RK-4: pays off only with complex flows
- Here approx. like RK-2



Integration, Conclusions

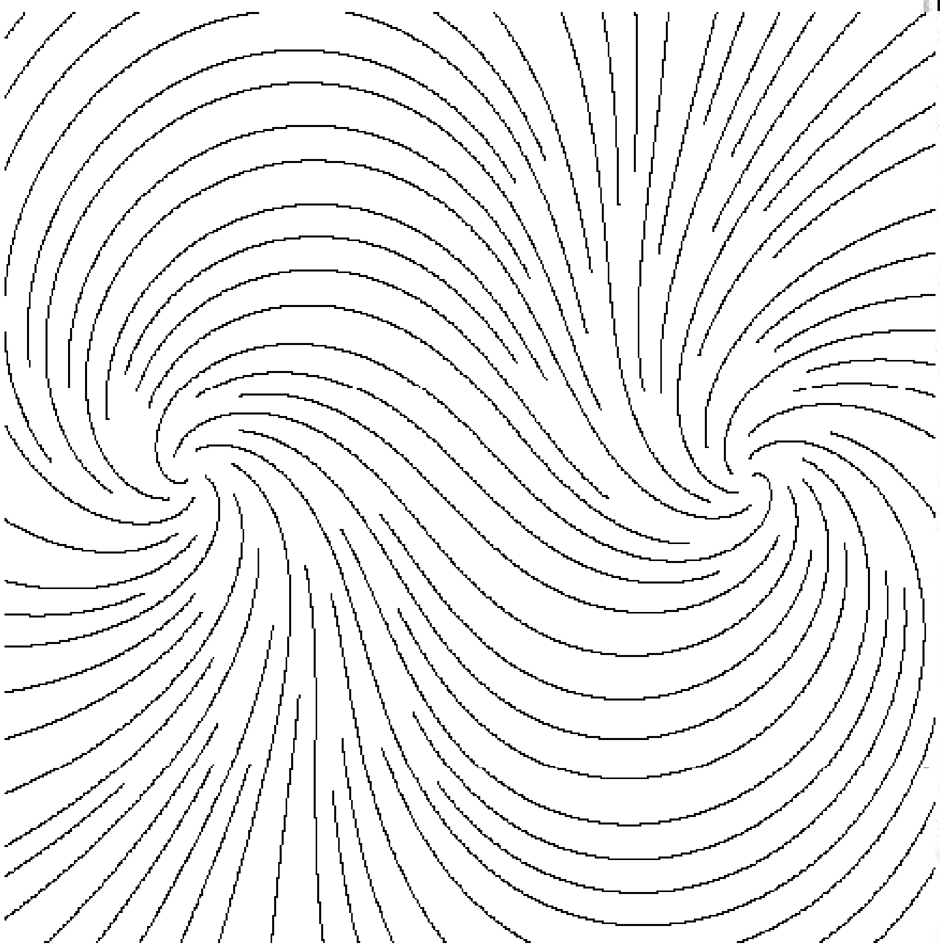
- Summary:
  - analytic determination of streamlines usually not possible
  - hence: numerical integration
  - several methods available (Euler, Runge-Kutta, etc.)
  - Euler: simple, imprecise, esp. with small dt
  - RK: more accurate in higher orders
  - furthermore: adaptive methods, implicit methods, etc.

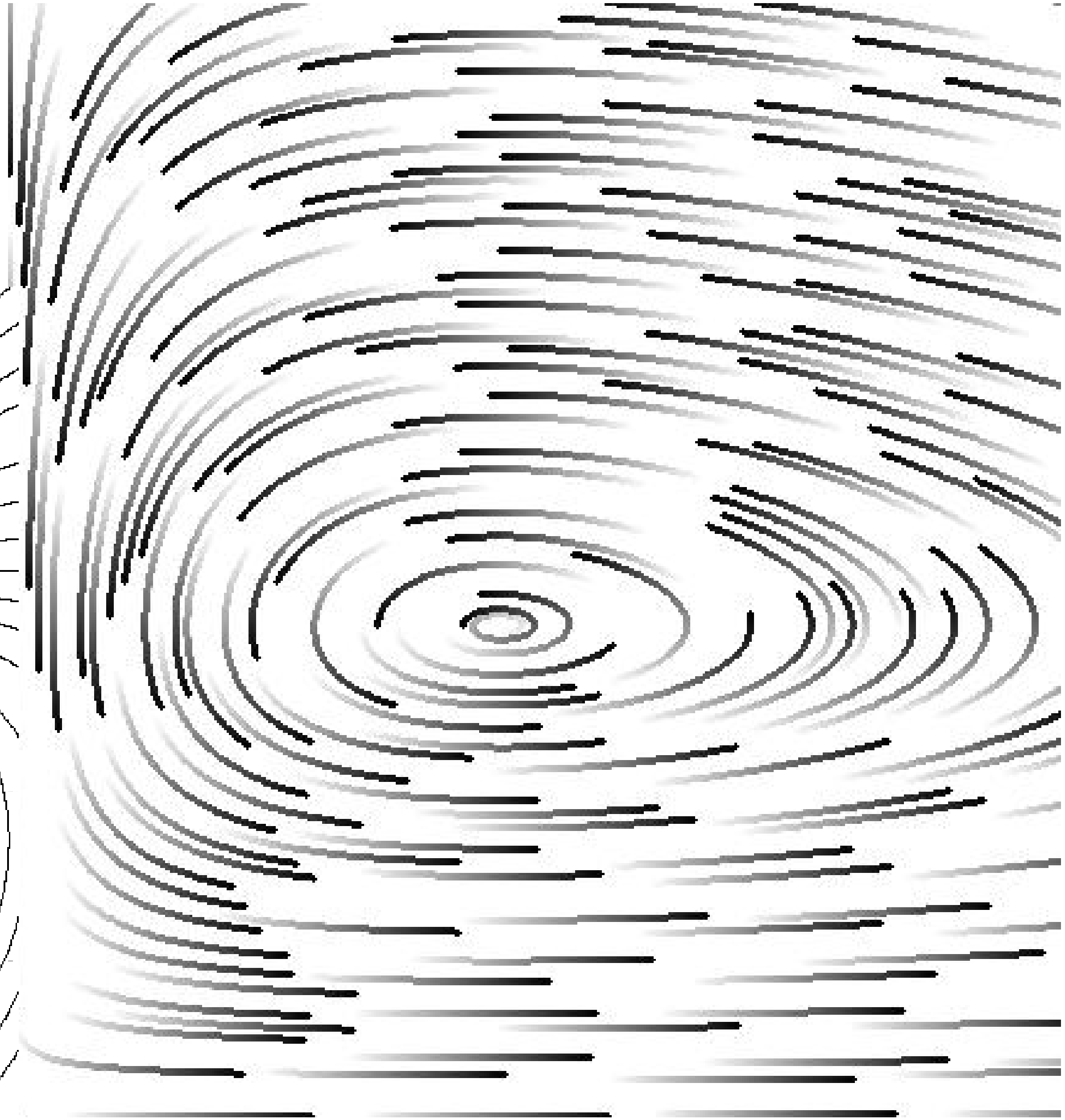
Flow Visualization with Streamlines

Streamlines,
Particle Paths, etc.

Streamlines in 2D

- Adequate for overview

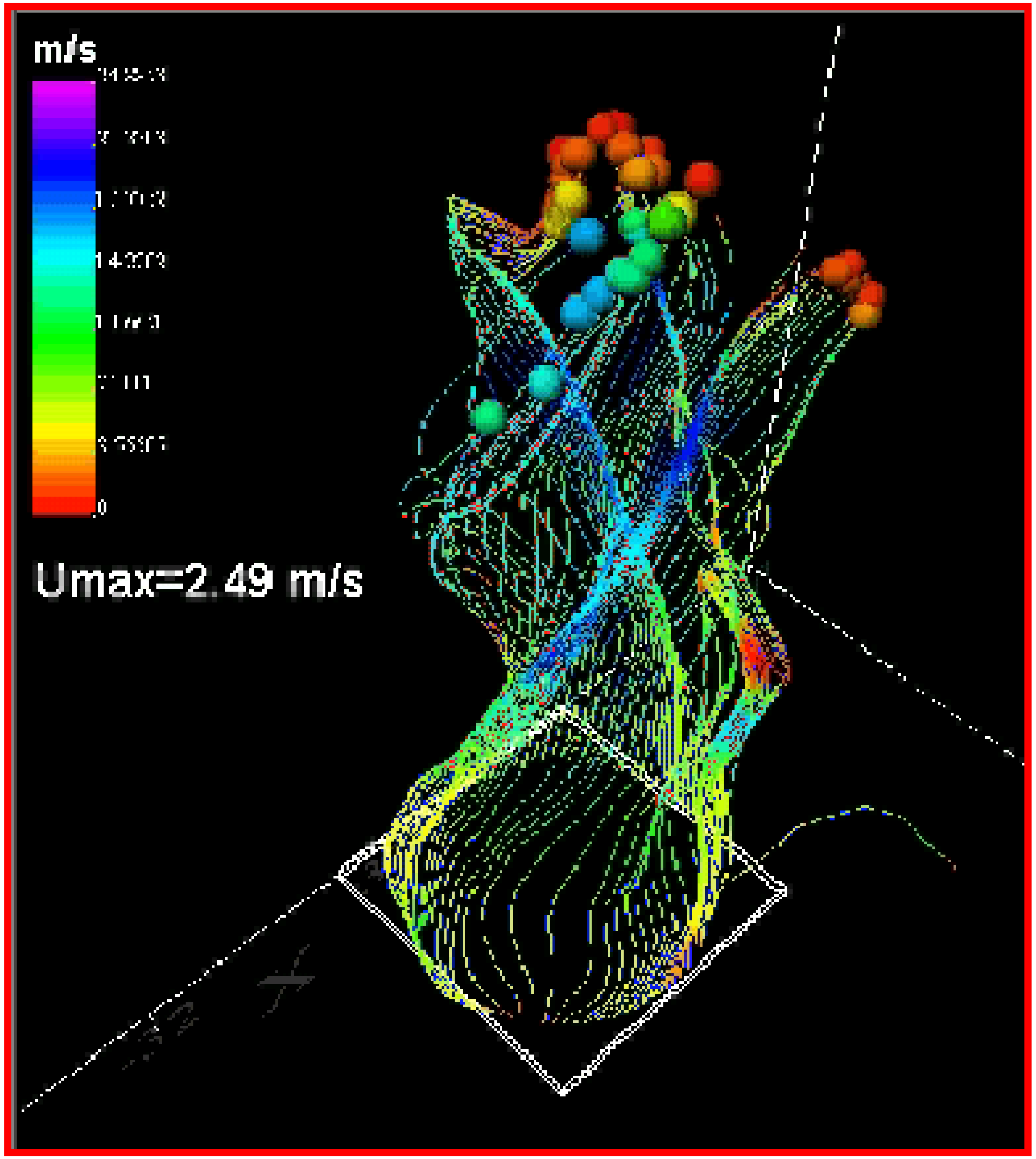




Visualization with Particles

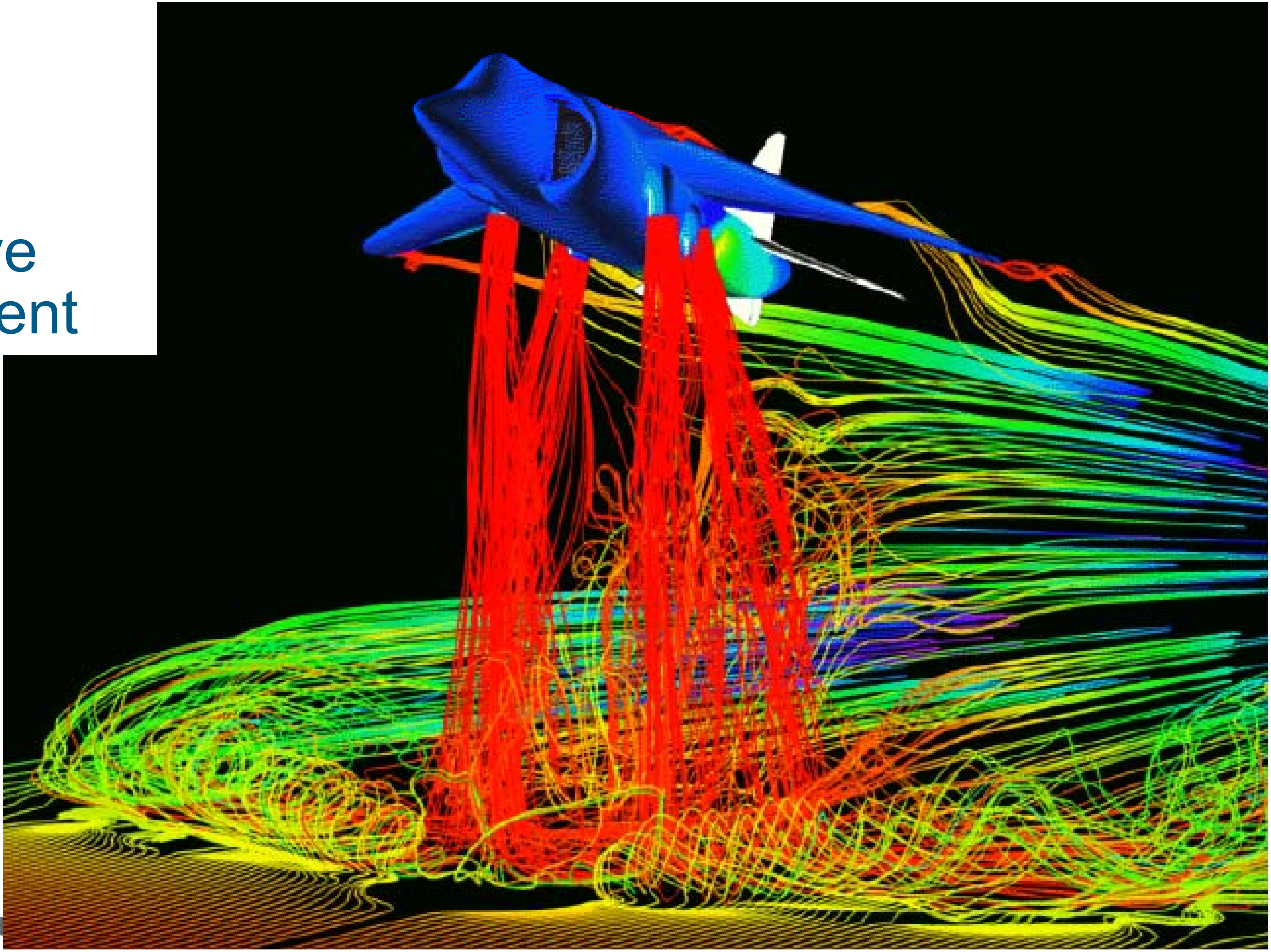
- Particle paths = streamlines (steady flows)
- Variants (time-dependent data):
  - streak lines: steadily new particles
  - path lines: long-term path of one particle

click2demo (F9!)



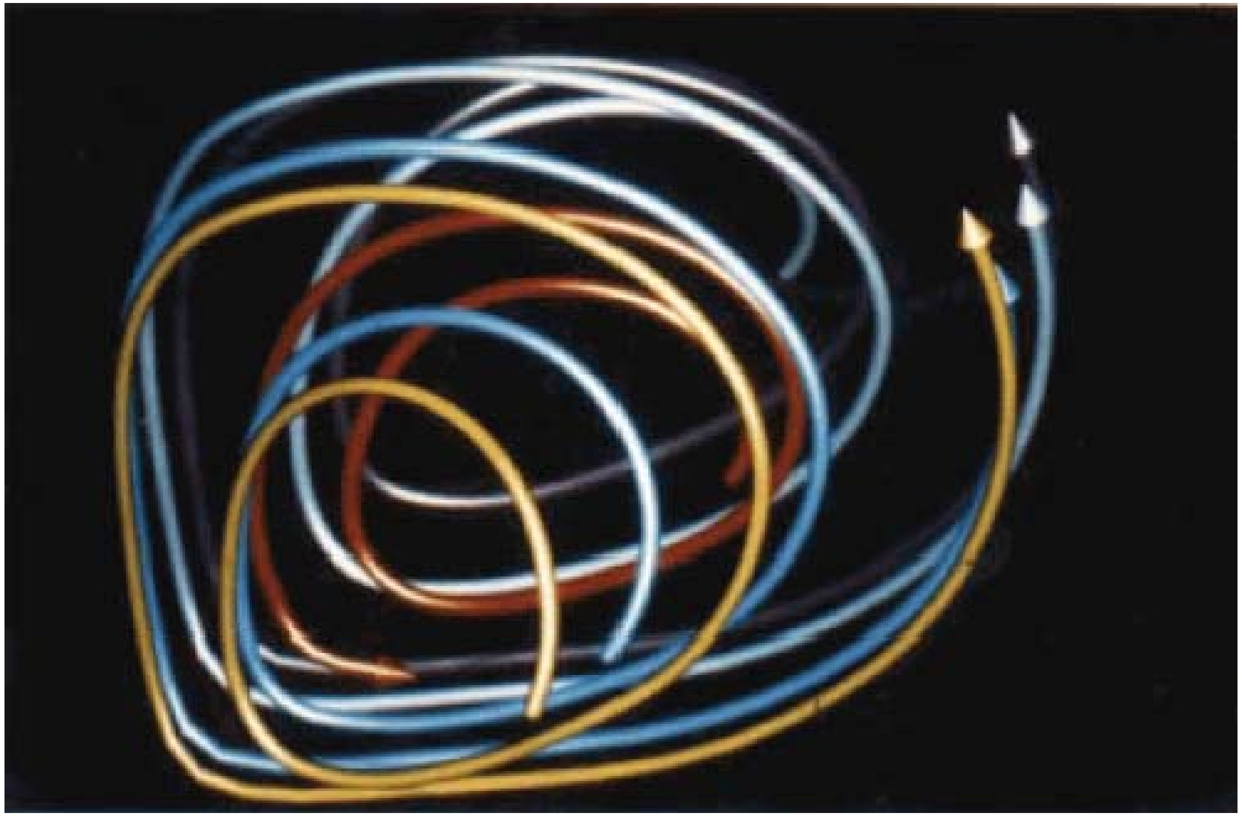
Streamlines in 3D

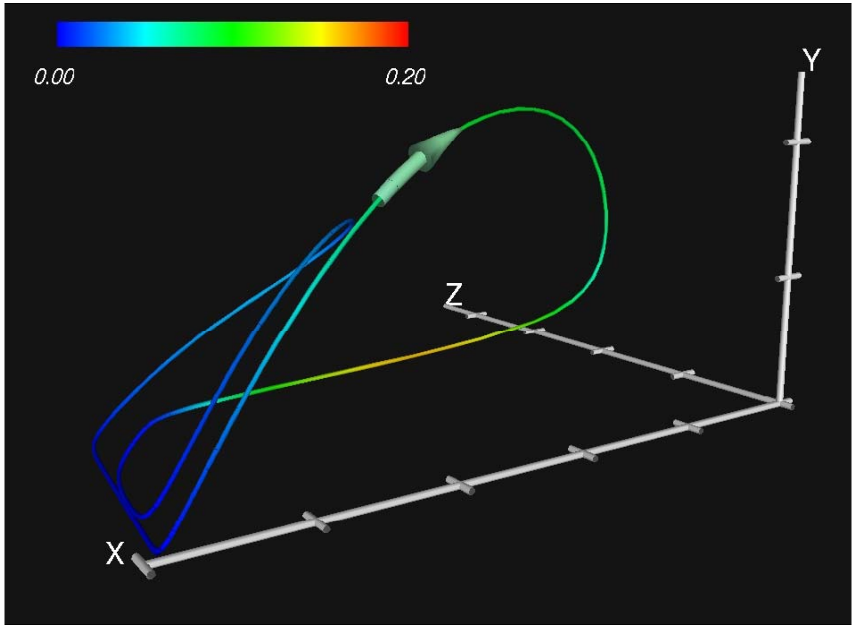
- Color coding: Speed
- Selective Placement

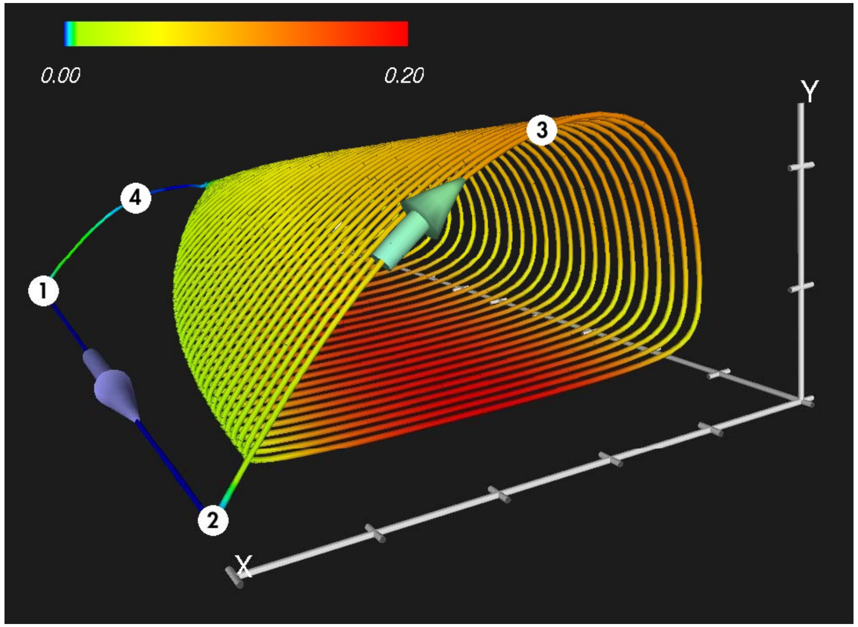


3D Streamlines with Sweeps

- Sweeps:
  better spatial 3D perception

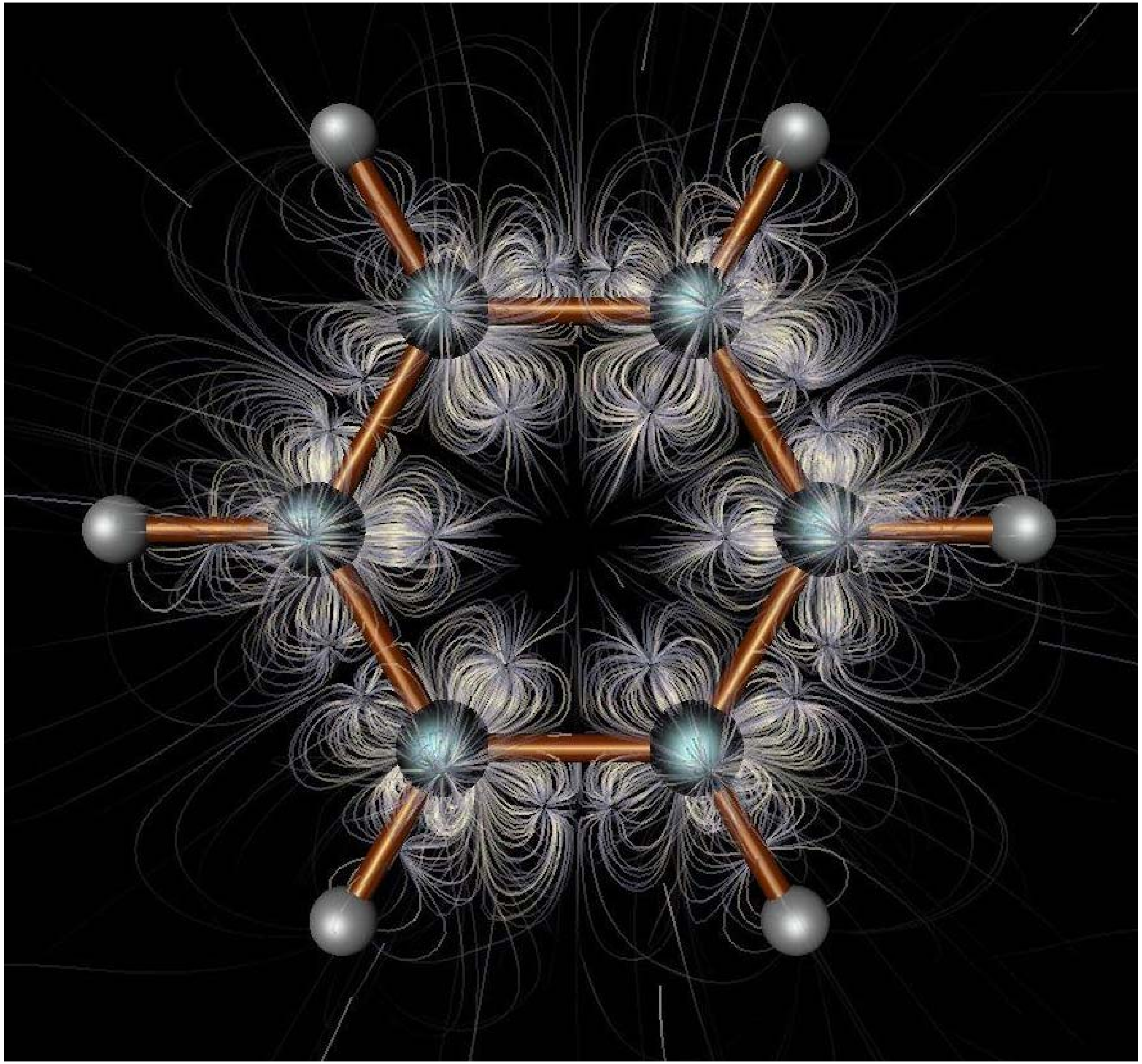






Illuminated Streamlines

- Illuminated 3D curves ⇒ better 3D perception!

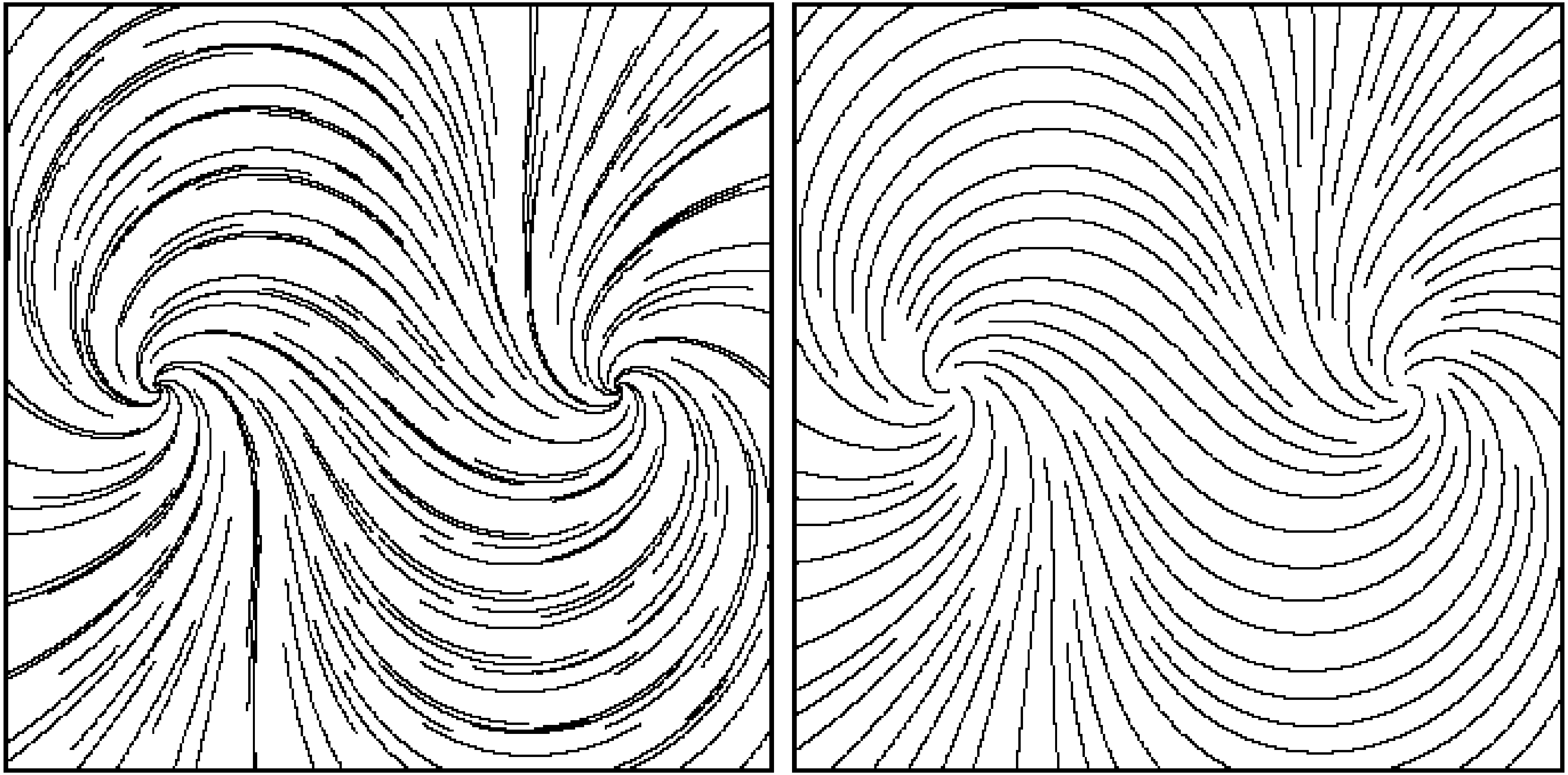


Streamline Placement

in 2D

Problem: Choice of Seed Points

- Streamline placement:
  - If regular grid used: very irregular result



Helw

Overview of Algorithm

- Idea: streamlines should not get too near to each other
- Approach:
  - choose a seed point with distance d_{sep} from an already existing streamline
  - forward- and backward-integration until distance d_{test} is reached (or …).
  - two parameters:
    - d_{sep} … start distance
    - d_{test} … minimum distance

Algorithm – Pseudocode

- Compute initial streamline, put it into a queue
- Initial streamline becomes current streamline
- WHILE not finished DO:

    TRY: get new seed point which is d_{sep} away from current streamline

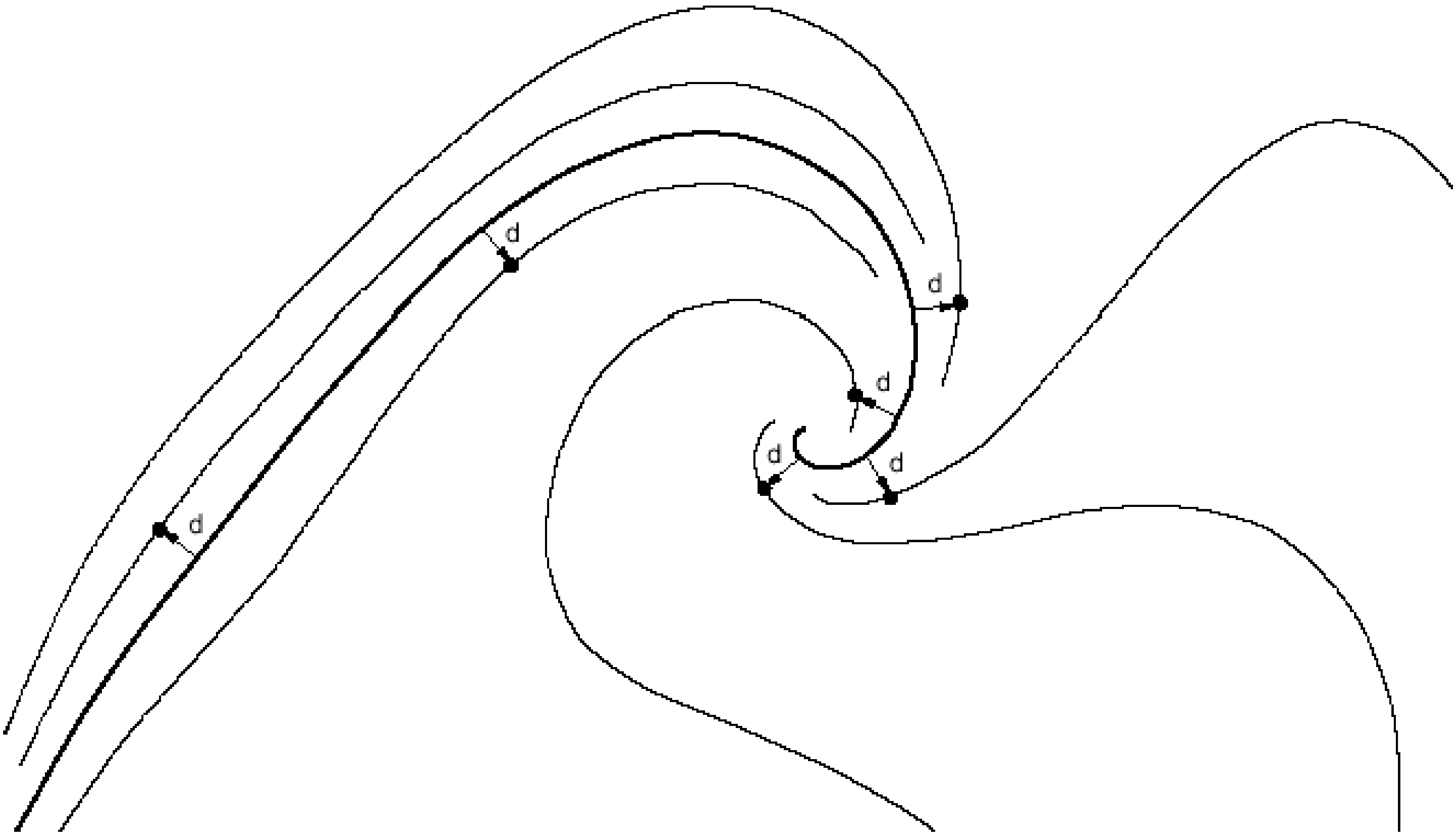
    IF successful THEN compute new streamline and put to queue

    ELSE IF no more streamline in queue
    THEN exit loop
    ELSE next streamline in queue becomes current streamline

Streamline Termination

- When to stop streamline integration:
  - when dist. to neighboring streamline ≤ d_{test}
  - when streamline leaves flow domain
  - when streamline runs into fixed point (**v**=0)
  - when streamline gets too near to itself
  - after a certain amount of maximal steps

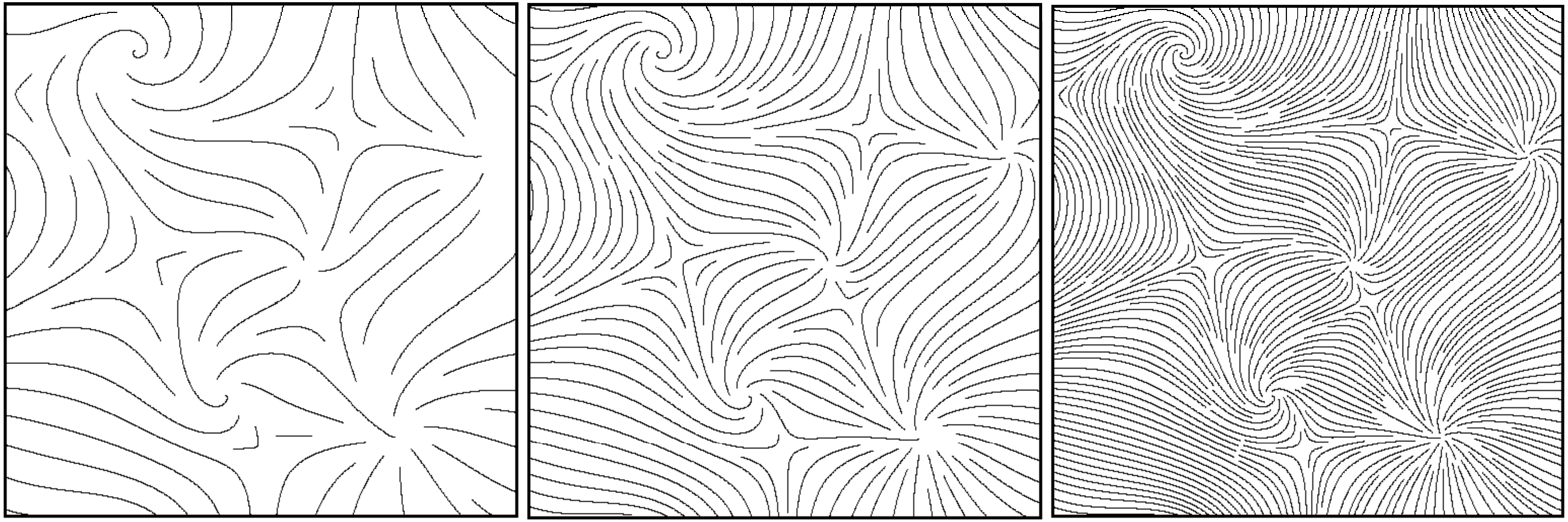
New Streamlines



Different Streamline Densities

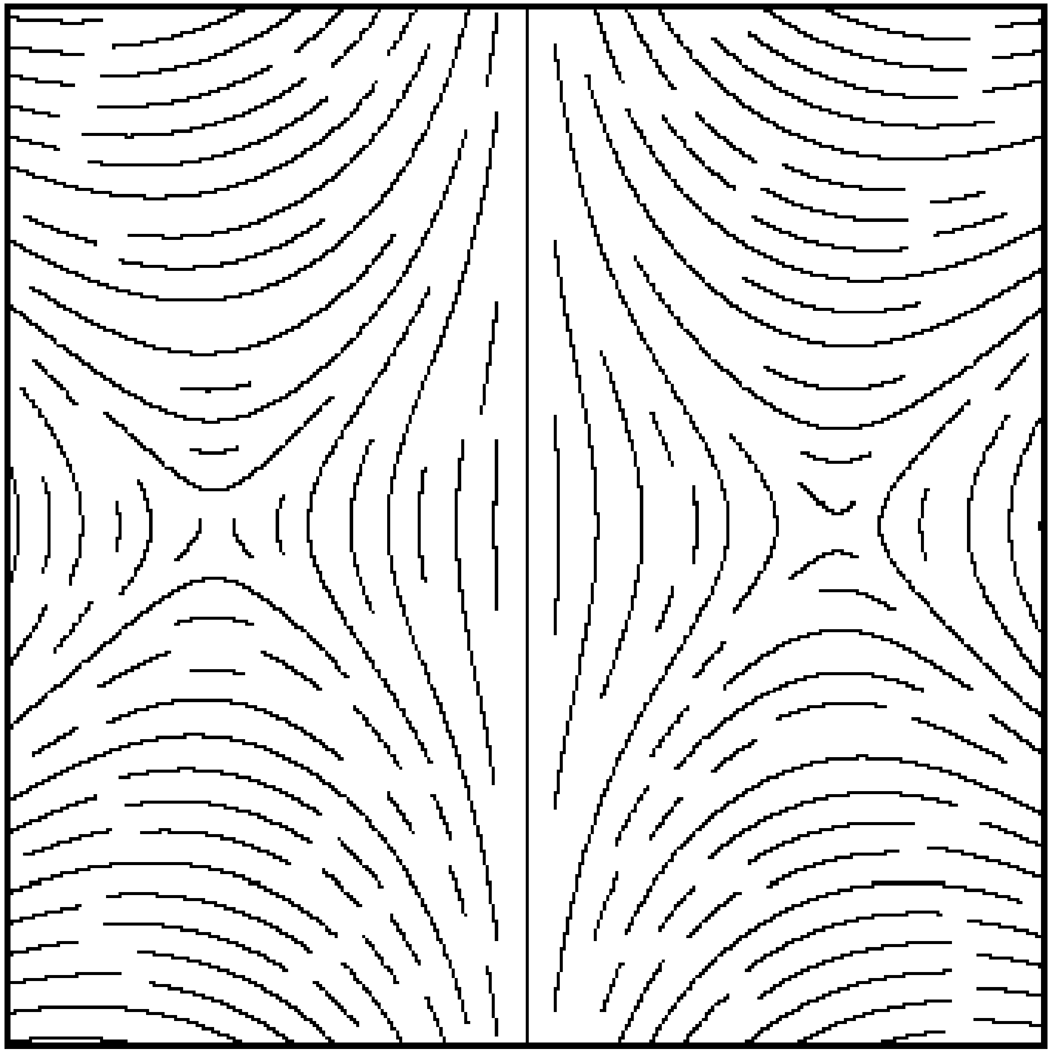
- Variations of d_{sep} in rel. to image width:

6% 3% 1.5%

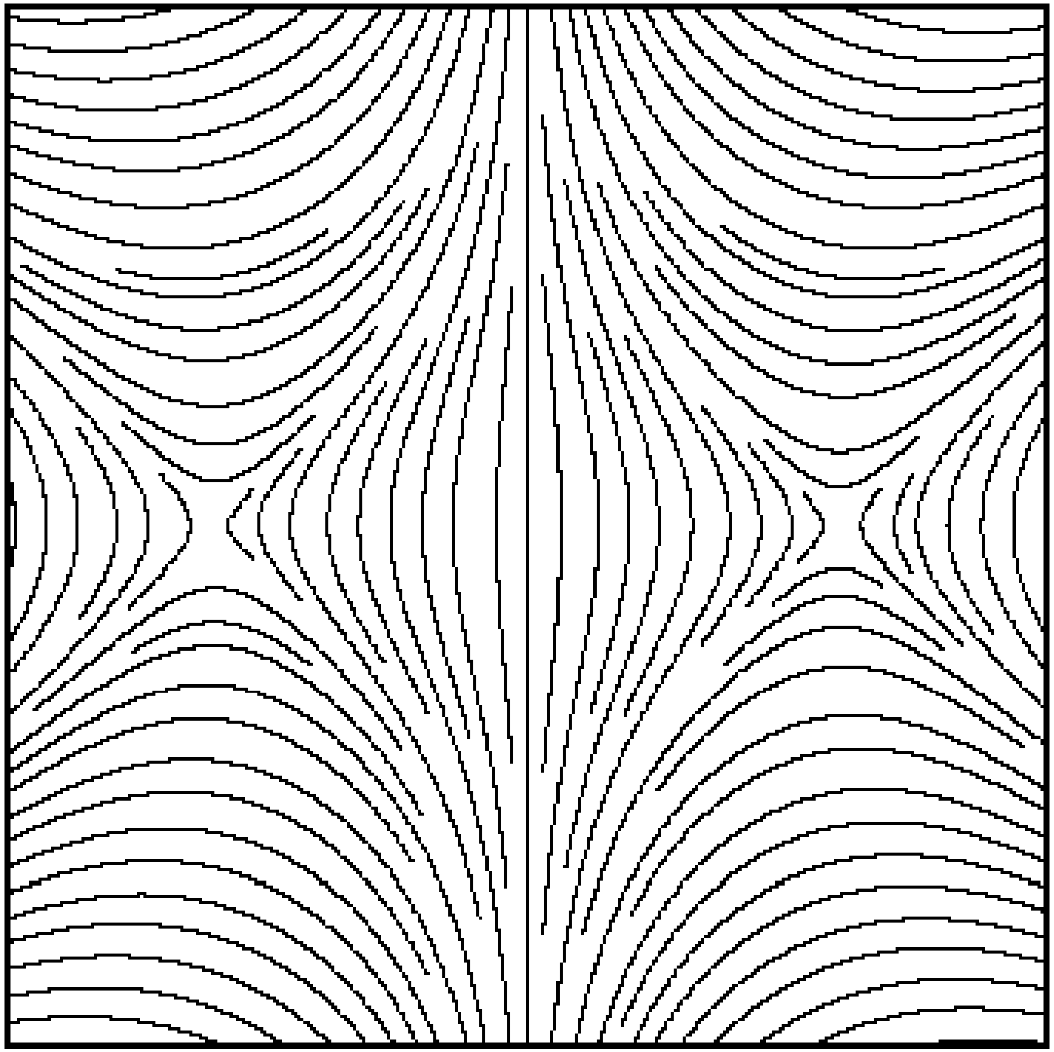


d_{sep} vs. d_{test}

$d_{test} = 0.9 \cdot d_{sep}$



$d_{test} = 0.5 \cdot d_{sep}$

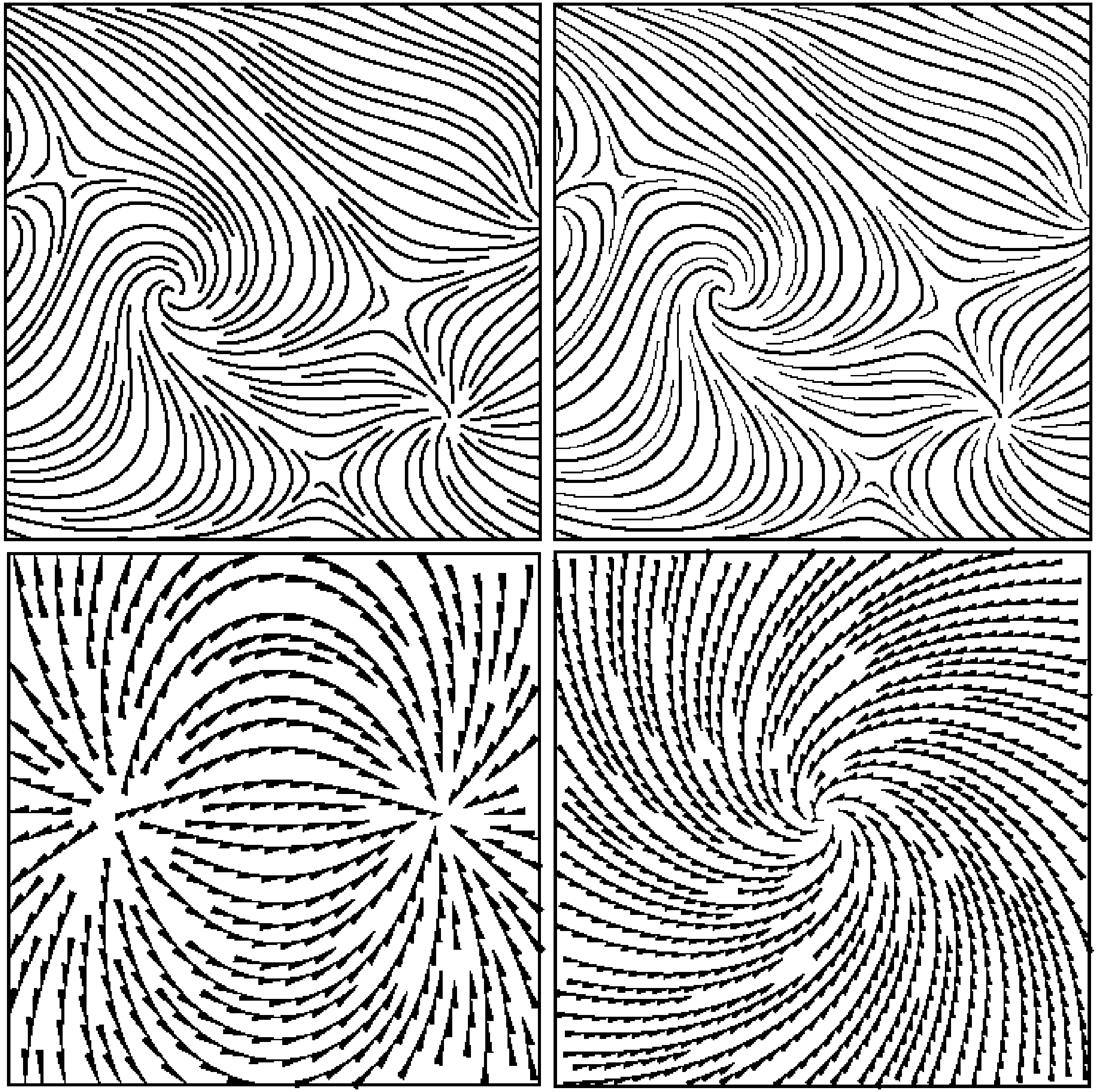


Tapering and Glyphs

- Thickness in rel. to dist.

$$\begin{cases} 1.0 & \forall d \geq d_{sep} \\ \dfrac{d - d_{test}}{d_{sep} - d_{test}} & \forall d < d_{sep} \end{cases}$$

- Directional glyphs:



Literature

- Paper (more details):
  - B. Jobard & W. Lefer: "**Creating Evenly-Spaced Streamlines of Arbitrary Density**" in *Proceedings of 8th Eurographics Workshop on Visualization in Scientific Computing*, April 1997, pp. 45-55

Acknowledgements

- For material used in this lecture:
  - Bruno Jobard
  - Malte Zöckler
  - Georg Fischel
  - Frits Post
  - Roger Crawfis
  - myself... ;–) (i.e., Helwig Hauser)
  - etc.