Energy Preserving Reconstruction of the Illumination Function for Radiosity Methods

Pavol EliÆÜ

elias@fmph.uniba.sk

[1] Institute of Computer Graphics

Technical University of Vienna
Karlspaltz 13/186-2, A-1040
Vienna, Austria

[2] Department of Computer Graphics and Image Processing
Faculty of Mathematics and Physics
Comenius University
MlynskÆdolina, 842 15, Bratislava, Slovakia

Abstract. The problem of reconstruction of illumination functions for radiosity based computations is presented, with a special attention to the issues of energy conservation principles on surfaces of the scene. Former reconstruction techniques are compared, taking into account different solution schemes (deterministic radiosity, hierarchical radiosity, stochastic radiosity). A new approach is proposed and discussed, based on the idea of preserving the total amount of received power, during the reconstruction step.

1 Introduction

Global illumination methods became a very popular solution for simulating light propagation in realistic scenes. Especially radiosity systems, based on light conservation principles, are improving their performance rapidly. In order to achieve a faster and more accurate solution, different improvements have been developed. The reconstruction of the illumination function over the selected areas of the scene belong amongst the most important ones. Instead of evaluating the lighting conditions for each point of the scene (or pixel of the screen), less expensive methods have been created, based on the smooth reconstruction of the illumination from a set of computed samples (discretize + reconstruct). Reasons for using these methods lies in the area of human perception.

Òs usually sufficient to solve the global illumination problem using the assumption of the constant illumination over the patches of the scene (this is fast and efficient) and then, in order to improve the perceptual quality of the result, the smooth reconstruction technique is used. If the reconstruction step is good enough, people canÒt find out, that the solution has been achieved, using several simplifying assumptions. However, the reconstructed solution can be used in several ways. It can be perceived by a very imperfect human visual system, but it can be often used as a reference for other methods (perhaps based on the evaluation of the illumination function for each pixel like ray-tracing), or it can be used for other engineering purposes, where the accuracy may play a crucial role. Especially due to these cases, the presented study of accuracy of reconstruction methods contributes to the general knowledge in the field of global illumination.

The presented tasks of reconstruction can be summarized as follows:

Problem: Given computed light power values for set of patches of the scene, find out Ãthe bestÓ bilinear (or higher order) reconstruction of the illumination function for a given surface.

2 Former Approaches

In order to interpolate illumination values (using linear or higher order interpolation), the illumination function has to be known at selected points. The patch vertices provide the most natural selection - the bilinear (or higher order) interpolation can be performed very efficiently, if the interpolated values are known at the vertices of the patches (this process is in computer graphics usually known as). On the other hand, as we mentioned earlier, the illumination information is not always computed for the patch vertices directly. Therefore, in order to interpolate, the values must be obtained somehow from the known information. We can basically distinguish following techniques:

1. compute the radiosity values for patch corners (vertices of polygon) by averaging the neighboring patch values (see [1] or [7]):

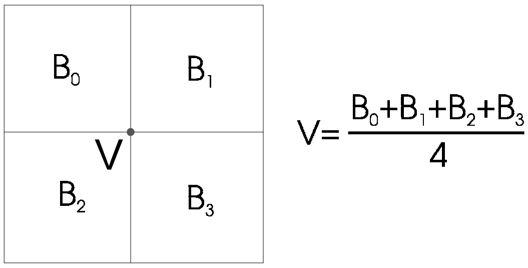


Fig. 1 evaluation of the illumination information at the patch vertices

2. compute radiosity values directly for the patch vertices. Therefore, the illumination values can be interpolated directly (see e.g. Wallace et al. in [9]).
3. construct a new triangulation (or subdivision of the surface into rectangles, etc.) using patch centers of the old triangulation as a patch vertices for a new one (see e.g. [8]). In this way the radiosity values, computed for patch centers can be interpolated using the newly constructed triangulation.

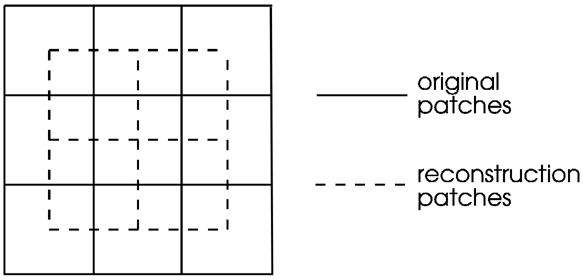


Fig. 2 Re-meshing using patch centers as new patch vertices

3 Reconstruction problems

Question: How good are these approaches?

Problems:

1. Averaging the radiosity values at patch vertices works well, if all patches have the same area. Otherwise (what is usually the case when using discontinuity meshing or adaptive subdivision - or other mesh adjusting technique), the approach leads to the wrong results. That means, the total energy received in the illumination step by the surface is not preserved during the reconstruction step.

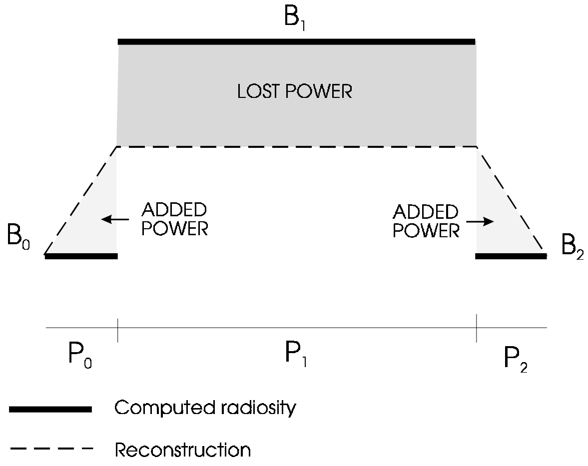


Fig. 3 energy loss due to radiosity averaging

2. Computation of the illumination information directly at the patch vertices is probably the best method, but requires the computation of point to patch formfactors (formfactor is a measure for energy transfer between points or patches in the scene, see [1] or [7]). In other words, if one can evaluate the illumination function directly at selected points, then bilinear interpolation of obtained values is the best what we can do (in the case of higher order reconstruction, higher order interpolation is used instead of bilinear interpolation, see [6]). But note, evaluating the illumination function directly at the selected points is only one of the known methods of solving the problem of the energy transfer. Several methods (especially hierarchical radiosity (see [2]) and stochastic approaches (see [3],[4]), to name the most used ones) evaluate the energy transfers in the patch to patch fashion. Therefore, this reconstruction technique canÕt be efficiently used in several situations.
3. Re-triangulation as a reconstruction method tries to get the best features of method 2, while keeping the generality of formfactor computation. The problem is, the results of the radiosity computation do not match the reconstruction technique. If patch to patch formfactors have been used, results of the radiosity simulation are power values for each patch rather than radiosity values at given points (patch centers) on each patch. The selection of patch centers as a points to represent the converted power value for each patch is the most natural one, but this method still can lead to the wrong results (see Fig. 4). Note, that patch to patch formfactors (implicit or explicit) are probably the most used ones, because they are used in the most popular radiosity systems to

date - hierarchical radiosity (see [2]) and stochastic (or Monte Carlo) radiosity (see [3] and [4]).

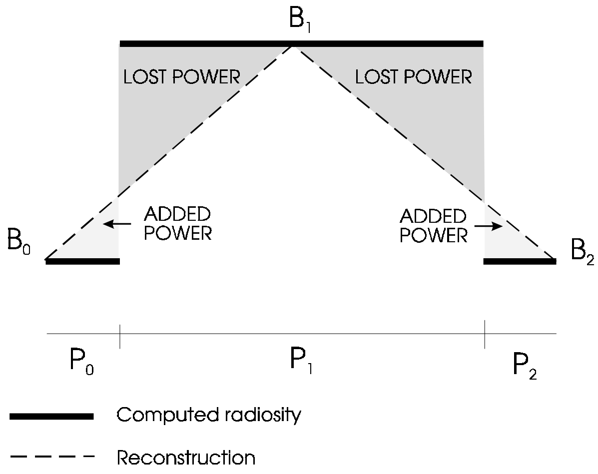
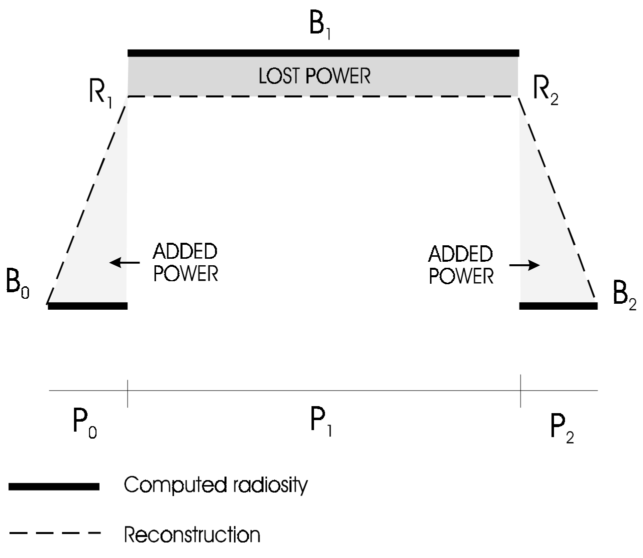


Fig. 4 energy loss due to re-meshing

4 Energy preserving reconstruction

LetÒs assume, that the radiosity solution to the global illumination problem has been computed. Further assume, the patch to patch formfactors (implicit or explicit) have been used during the solution process to evaluate the energy transfers. Therefore, the results are the radiosity values computed for each patch of the given scene (note, radiosity methods are based on Finite element method). In order to create C^0 (or smoother) solution, the radiosity values at the patch vertices are computed as an area weighted average of the constant radiosity values of all neighboring patches. The radiosity values at the patch vertices are than interpolated by bilinear (or higher order) interpolation scheme. Figure 5 shows a simple 1D example of a new technique.



$$R_1 = \frac{B_0 P_0 + B_1 P_1}{P_0 + P_1}$$

$$R_2 = \frac{B_1 P_1 + B_2 P_2}{P_1 + P_2}$$

Fig. 5 energy preserving reconstruction

Note, that the proposed technique differs from the standard reconstruction technique (see e.g. Cohen, Wallace in [1]) by taking the area weighted average instead of simple average of the patch values. The following proof shows, that the new technique is really energy preserving. For simplicity, the proof is given for one dimensional case. Note, that the total received power of the surface can be computed as a sum (through all patches) of radiosity multiplied by the area of a patch (we assume constant basis functions for radiosity computation here).

Notation:

- $W_{received}$ - total received power of a surface
- $W_{reconstructed}$ - total power of a surface after the reconstruction of the illumination
- B_i - radiosity of patch i
- P_i - area of a patch i
- R_i - radiosity of a vertex between patch i-1 and patch i

Proof:

From the definition we have:

$$W_{received} = \sum_i B_i P_i$$

After the reconstruction we have a piecewise linear illumination function, therefore the power can be evaluated in the following way:

$$W_{reconstructed} = \sum_i \frac{R_{i-1} + R_i}{2} P_i, \quad where \quad R_i = \frac{B_{i-1} P_{i-1} + B_i P_i}{P_{i-1} + P_i}$$

After the substitution for R_{i-1} and R_i:

$$W_{reconstructed} = \sum_i \left(\frac{B_{i-1} P_{i-1} + B_i P_i}{P_{i-1} + P_i} + \frac{B_i P_i + B_{i+1} P_{i+1}}{P_i + P_{i+1}} \right) \frac{P_i}{2}$$

$$W_{reconstructed} = \sum_i \left(B_{i-1} \frac{P_{i-1}}{P_{i-1} + P_i} + B_i \left(\frac{P_i}{P_{i-1} + P_i} + \frac{P_i}{P_i + P_{i+1}} \right) + B_{i+1} \frac{P_{i+1}}{P_i + P_{i+1}} \right) \frac{P_i}{2}$$

Rearranging the sum for B_i:

$$W_{reconstructed} = \sum_i \frac{B_i}{2} \left(\frac{P_i P_{i+1}}{P_i + P_{i+1}} + \frac{P_i P_i}{P_{i-1} + P_i} + \frac{P_i P_i}{P_i + P_{i+1}} + \frac{P_{i-1} P_i}{P_{i-1} + P_i} \right) = \sum_i B_i P_i$$

As a result:

$$W_{received} = W_{reconstructed}$$

The presented derivation shows as, that this reconstruction technique really fulfills the requirement and itÒs energy preserving. Similar derivation can be done for 2-dimensional case and for higher order reconstruction. Please note, that the reconstruction of the illumination is 2D problem - it reconstructs a function over a 2D domain of a surface in 3D space.

5 Summary and Future Trends

A new reconstruction technique has been presented based on the idea of local preservation of the received energy in radiosity algorithms. The former approaches were mainly based on the observation, that the reconstruction needs to be performed, in order to create a visually plausible resulting image. Not enough attention has been paid to the energy equilibrium features. Therefore, the visually appealing but wrong images have been produced. Fortunately, the characteristics of human visual system ÓcorrectÔ these problems. On the other hand, the new reconstruction method keeps local energy equilibrium and thus the results can be compared with results, computed using different algorithms, where the illumination function is evaluated directly, without the need of reconstruction.

Future area of research should cover the idea of using approximation of illumination function instead of interpolation at selected points, while keeping the advantage of energy preserving reconstruction. This topic seems to be especially promising in the are of stochastic radiosity methods, where the illumination information is estimated with error anyway (the error caused by a variance and the reconstruction should work as a filter). In this case, no exact values to interpolate are available, so it should be possible to take advantage of usage of smoother reconstruction, while sacrificing some of the features of the computed stochastic solution.

6 Acknowledgment

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