Visualizing 3D Dynamical Systems on the Basis of Poincaré Maps

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Overview

- **Visualizing 3D Dynamical Systems on the Basis of Poincaré Maps –**
  Introduction on dynamical systems and flow visualization
  Improved techniques for the visualization of 2D Poincaré map
  Combining 2D Poincaré map and 3D flow visualization
  Specialities with Poincaré maps

- **Viz @ cg.tuwien.ac.at during ’96 & ’97 –** Overview
  Further work on 3D dynamical systems
  New techniques on 2D flow visualization
  Using virtual reality and the Internet
  . . .

Dynamical Systems – Introduction

$$r = \sqrt{x^2 + y^2}$$
$$a = A \cdot (r - 1/r)$$
$$\begin{aligned} \dot{x} &= a \cdot x - y \\ \dot{y} &= a \cdot y + x \\ \dot{z} &= C \cdot z \end{aligned}$$

x, y, z . . . system variables
A, C . . . system parameters

Sample system characteristics:

- **3D system (2+1), 2 parameters**
- **continuous = flow** (vs. discrete = map)
- **analytically specified** (vs. sampled set)
- **time independend** (vs. time dependend)
- **non-linear = complex** (vs. linear)
- **periodic**

Dynamical Systems & Flow Visualization

2D techniques

Hedgehog plots

Streamlines, streaklines, etc.

Spot noise, line integral convolution

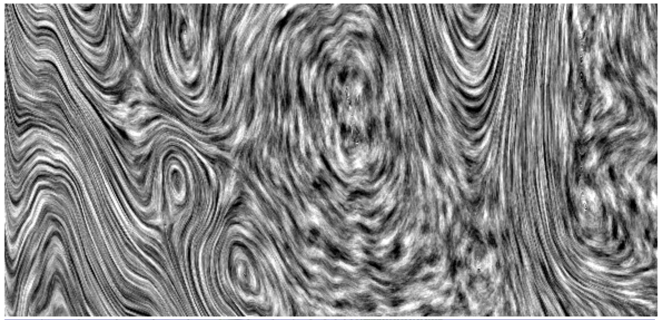
3D techniques

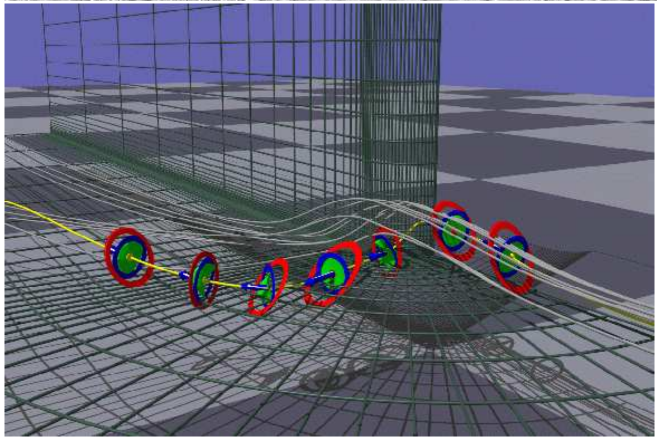
Streamlines, streamribbons, streamtubes, etc.

Streamsurfaces, particle systems

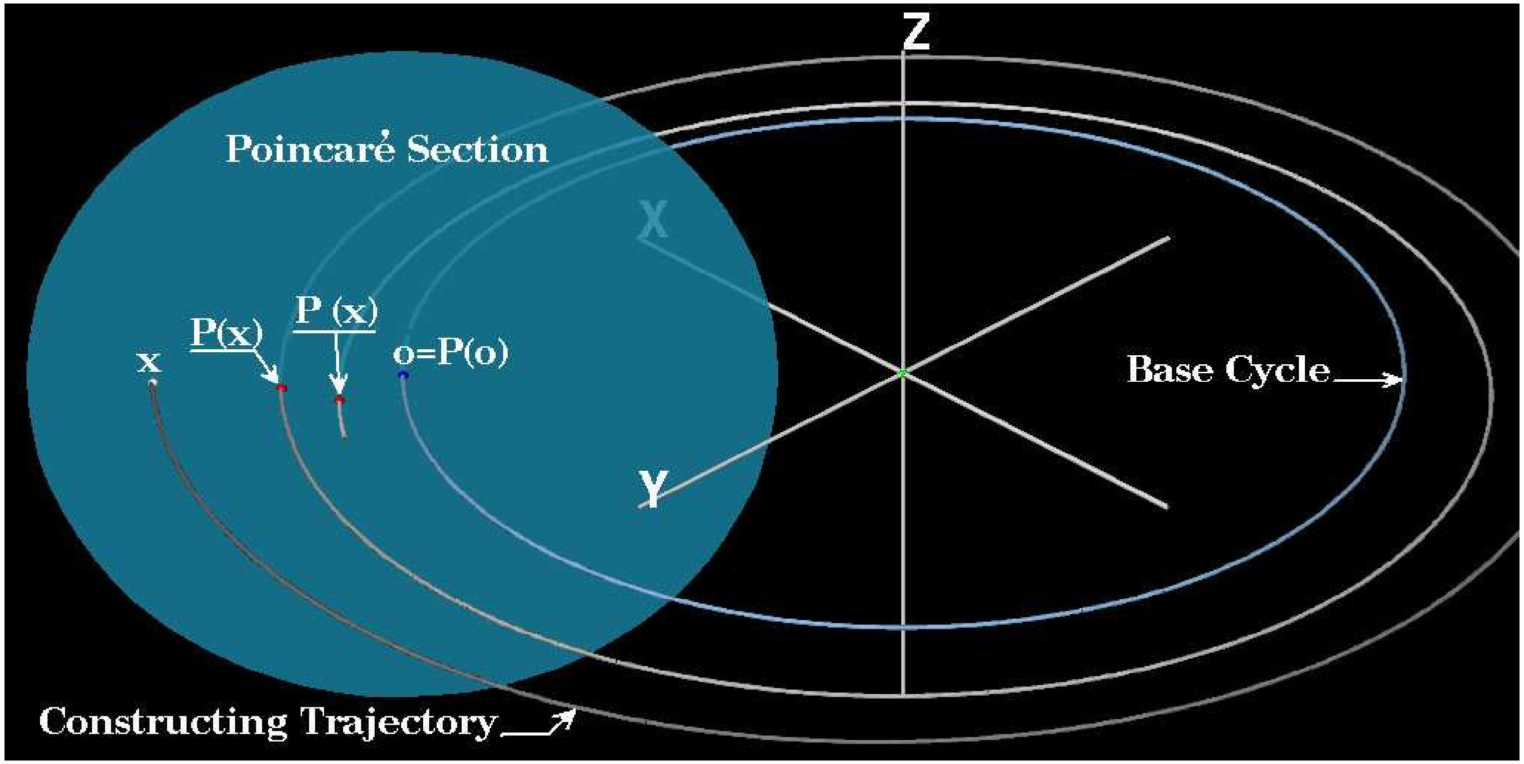
Flow volumes, virtual smoke

Topology visualization, skin-friction lines, etc.

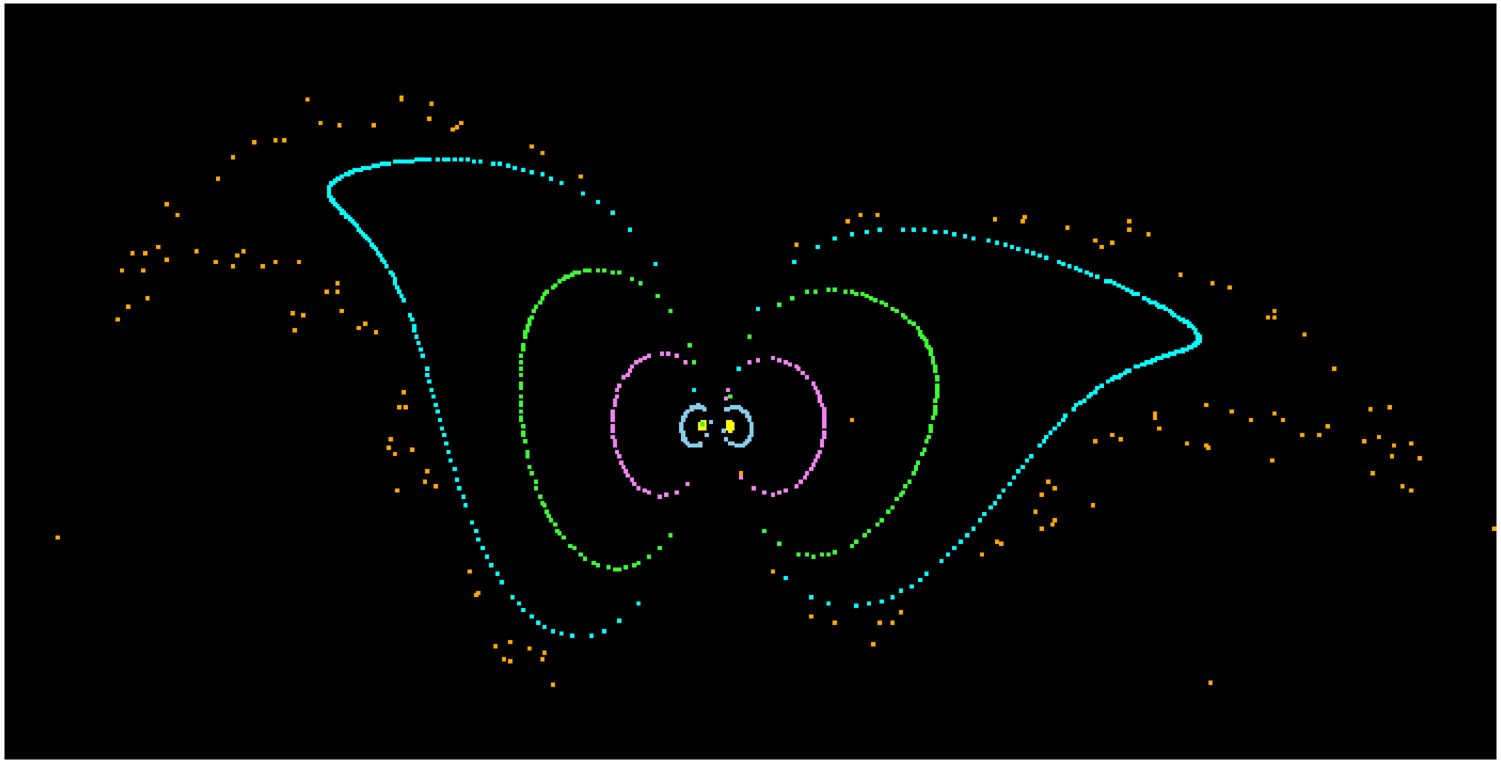




Poincaré Section & Poincaré Map



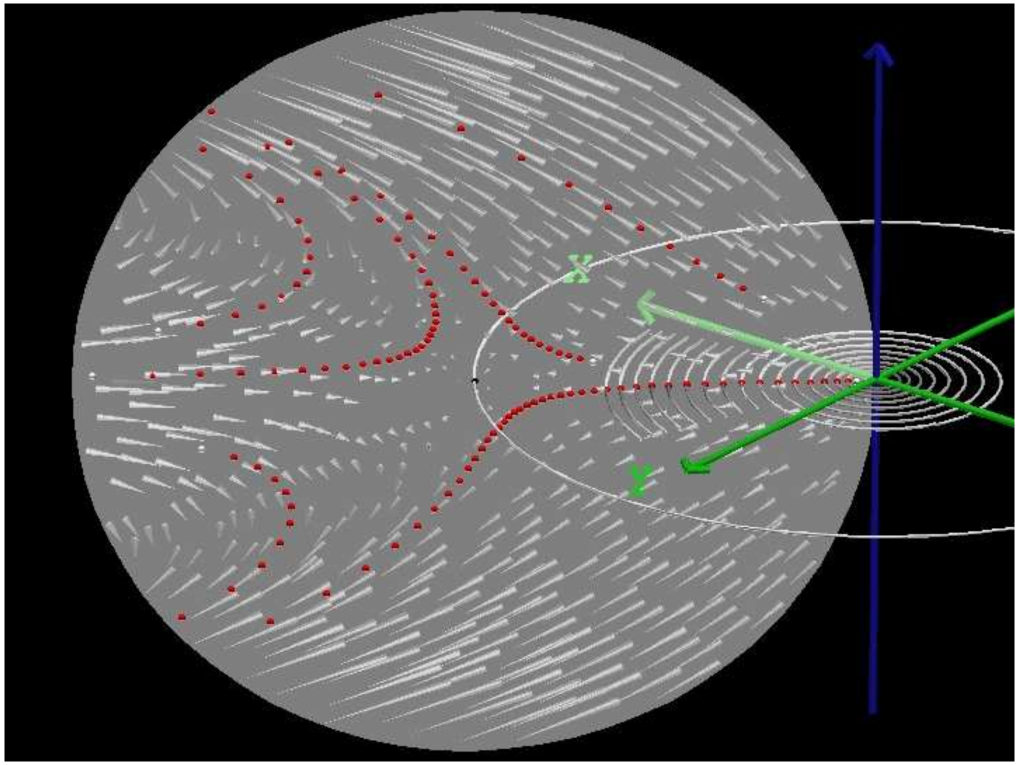
2D Poincaré Map – Traditional Example



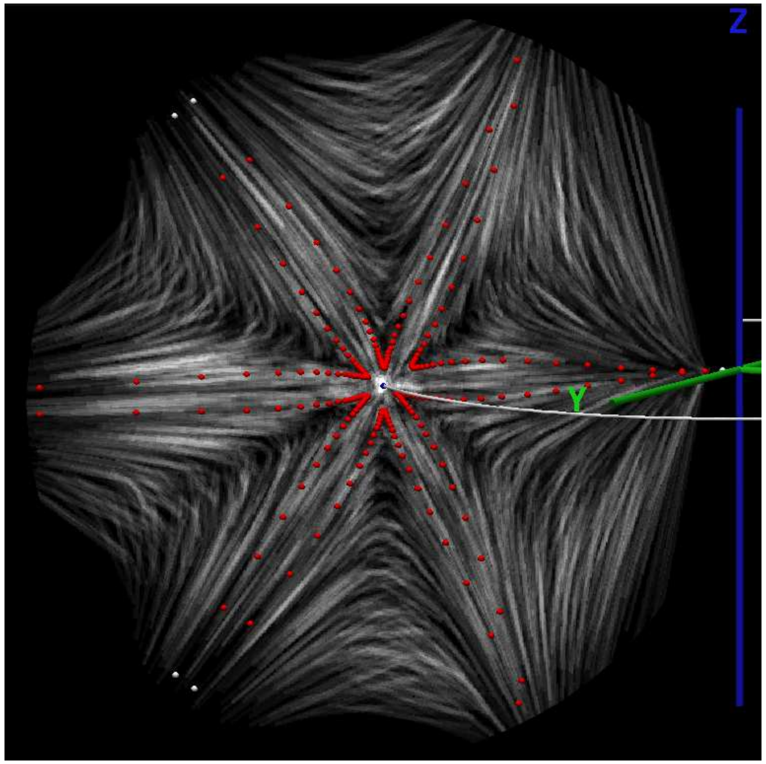
2D Poincaré Map – Advanced Vis. Techniques

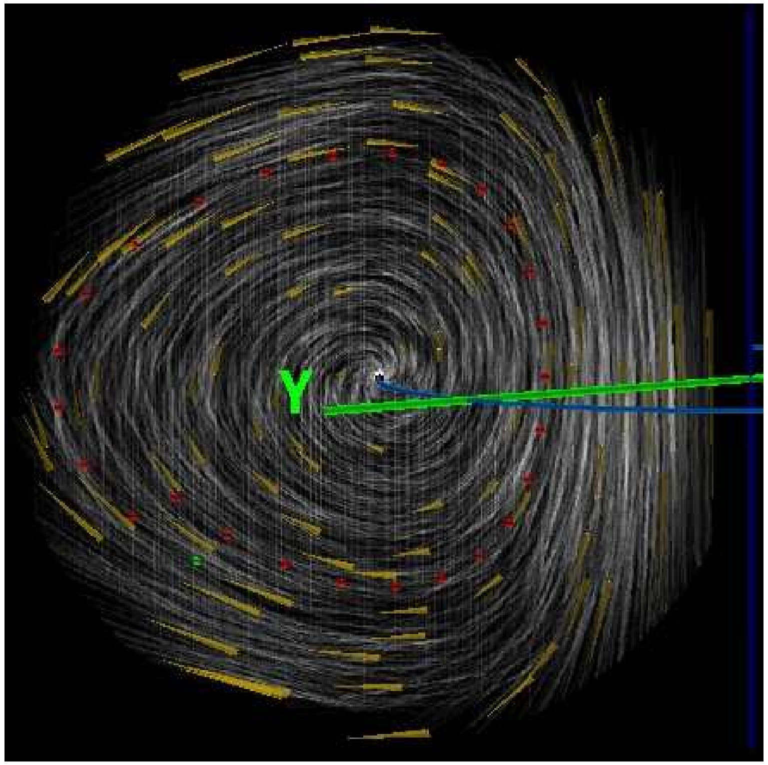
Visualizing $P(x_i)$:
- representing $P(x_i) - x_i$

Visualizing $P^i(x)$:
- with constructing trajectory
- with short streamlets
- just $P^i(x)$

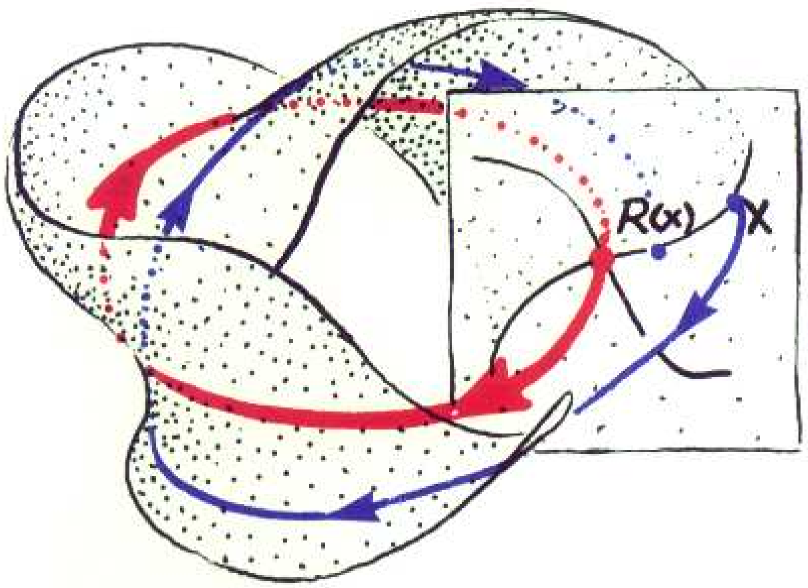


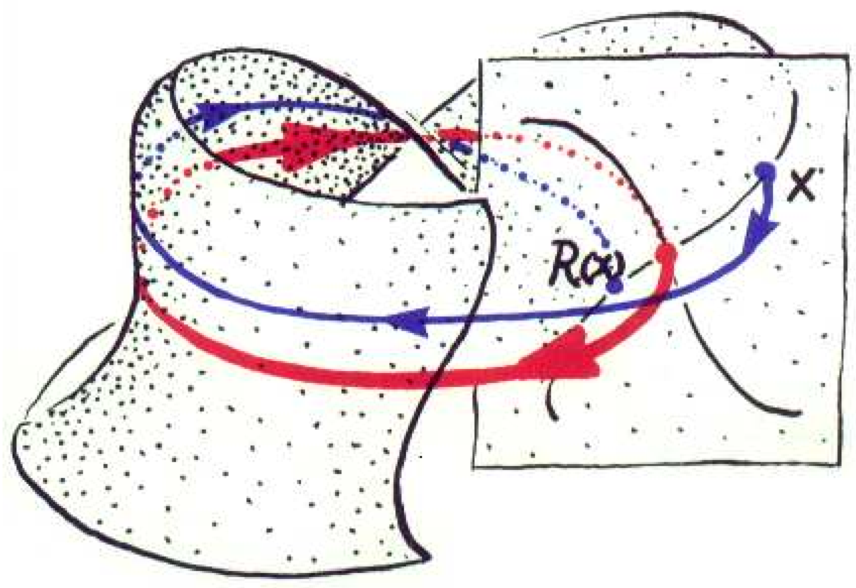
2D Poincaré Map Viz Using Spot Noise



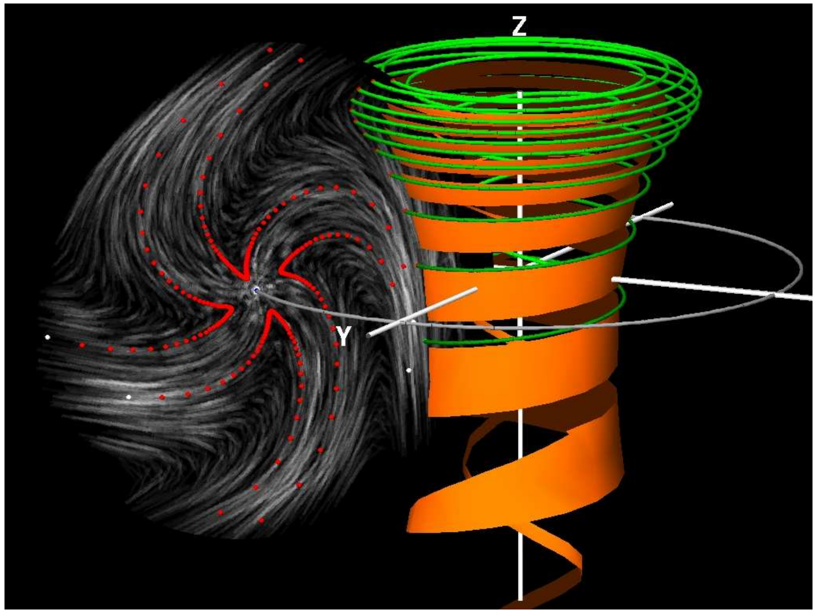


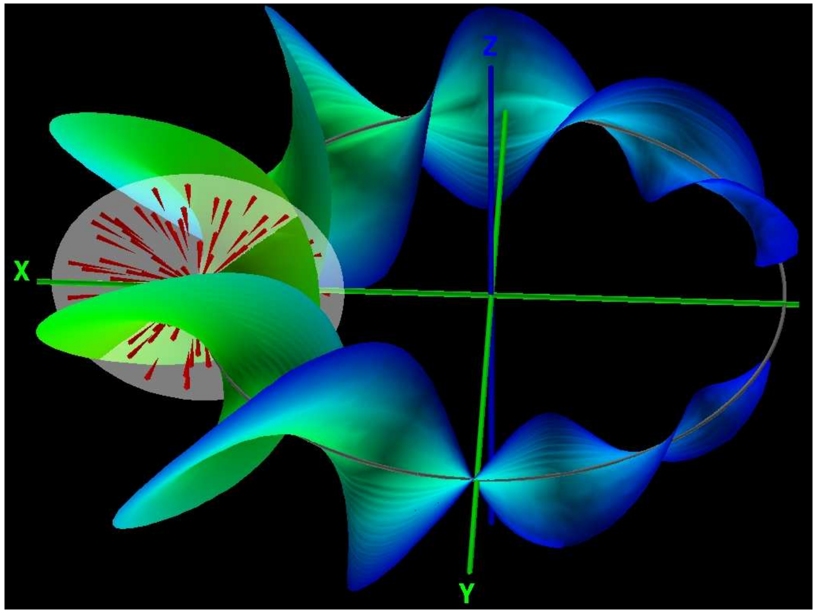
Poincaré Map + Flow Viz [Abraham/Shaw '92]



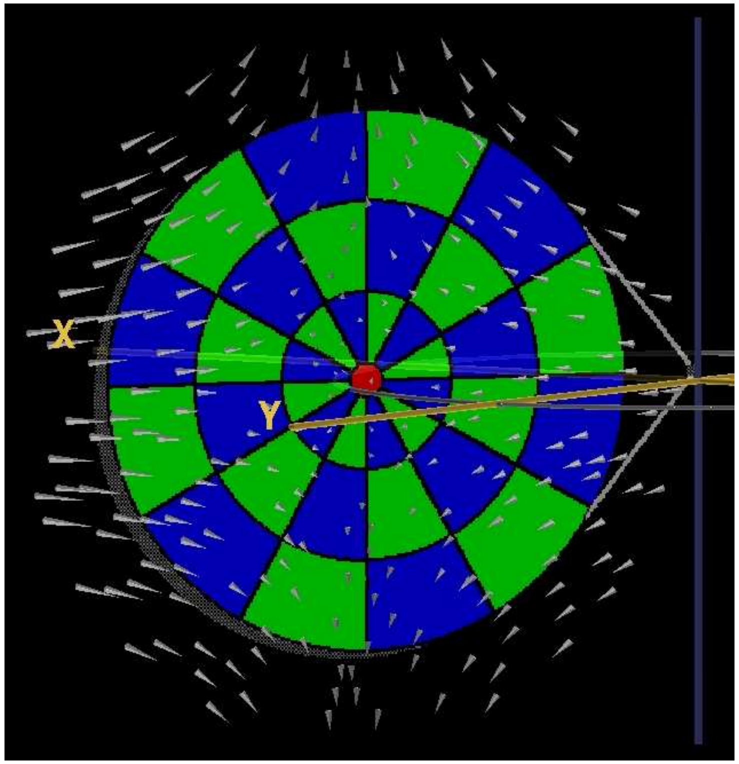


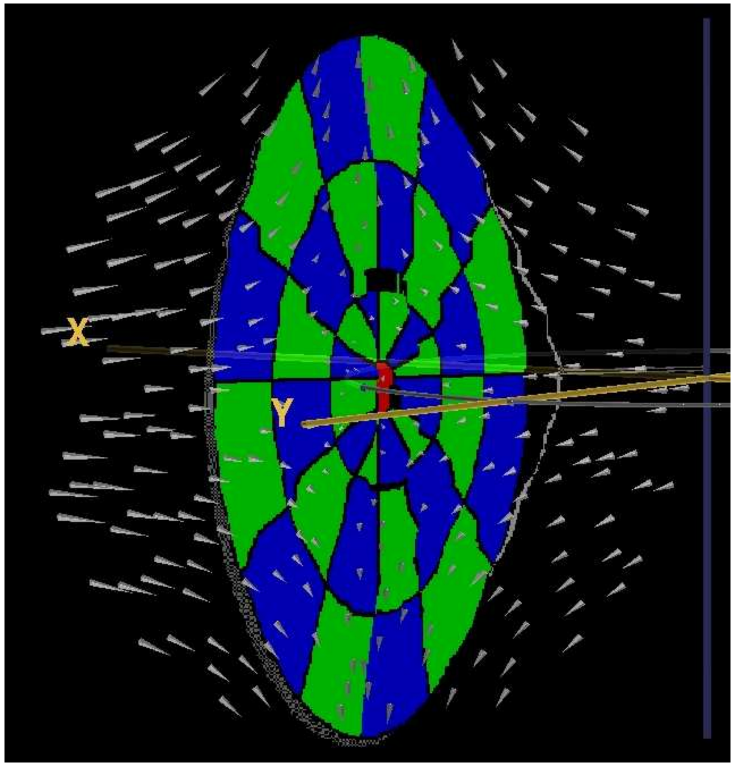
Combining Poincaré Map & Flow Visualization



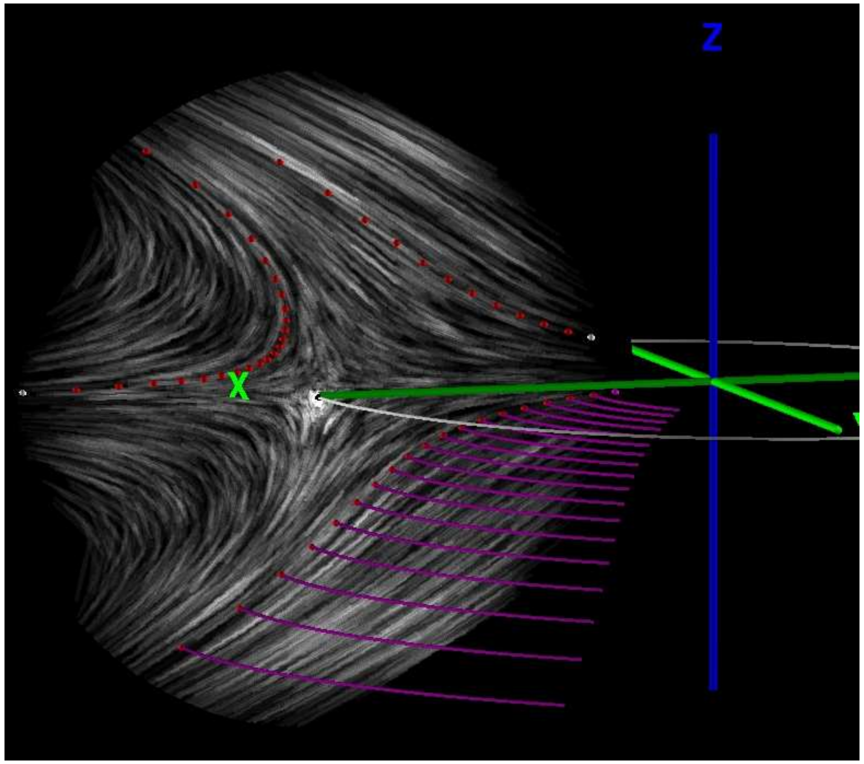


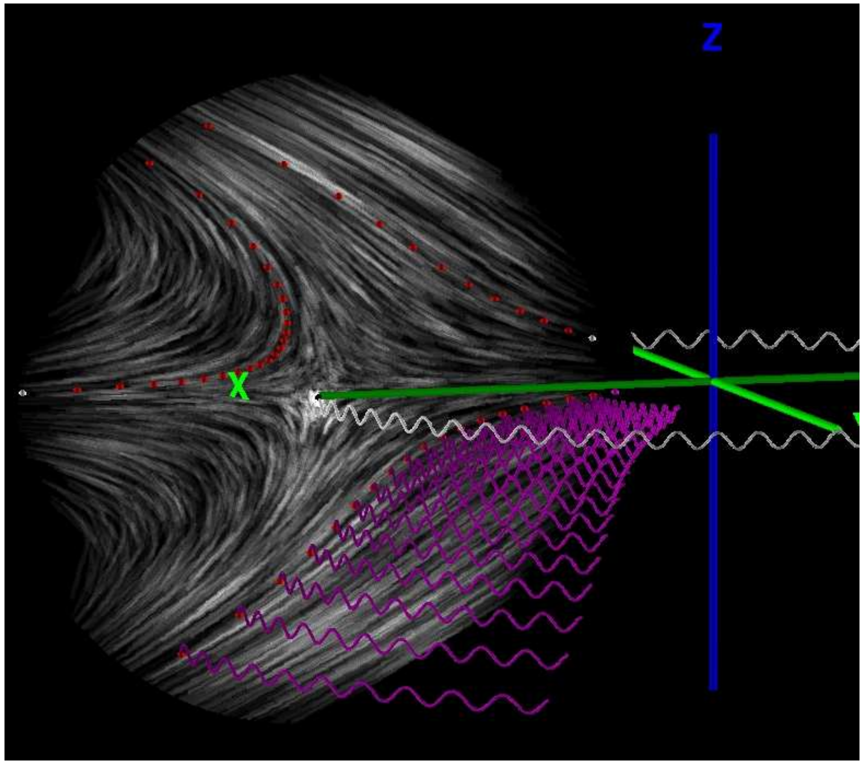
Poincaré Map Visualization using Animation



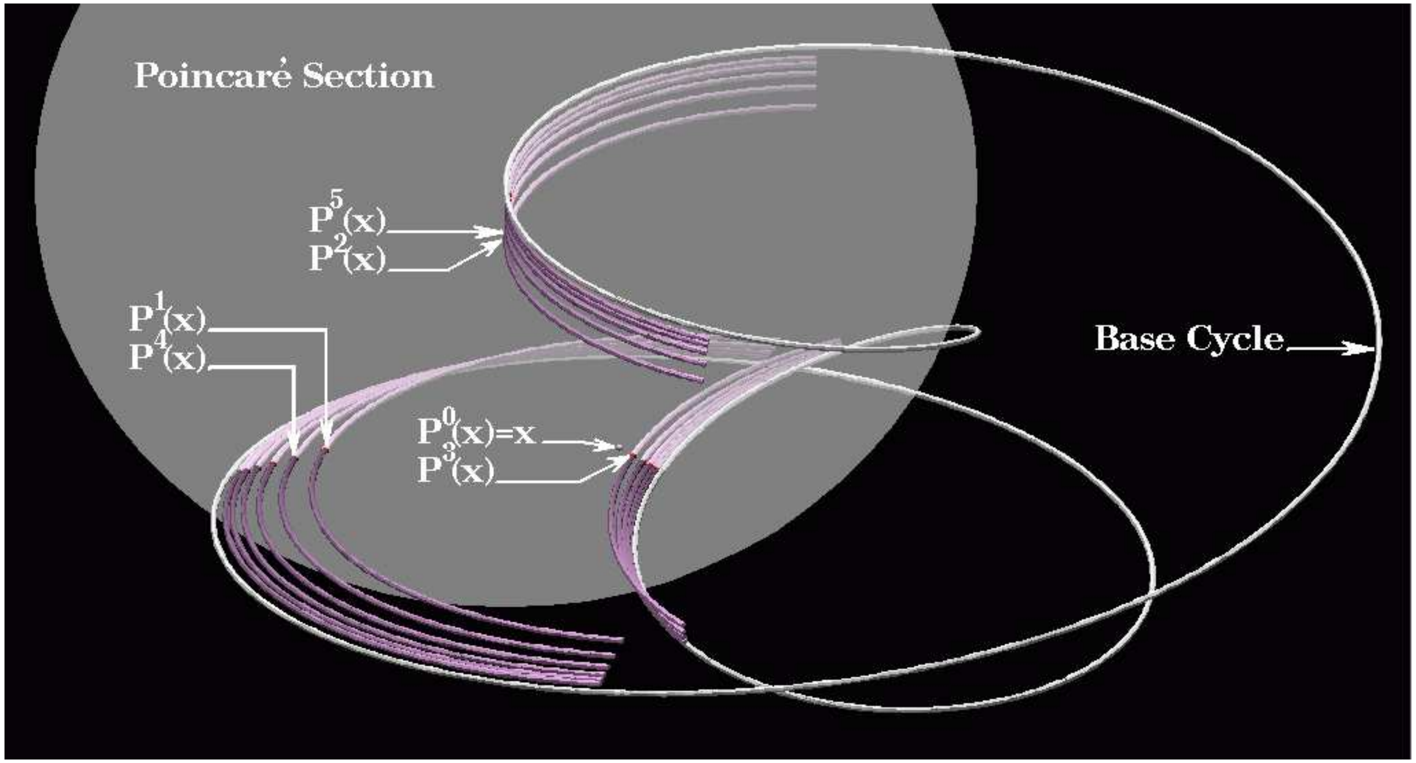


Combined Visualization – Additional Insights

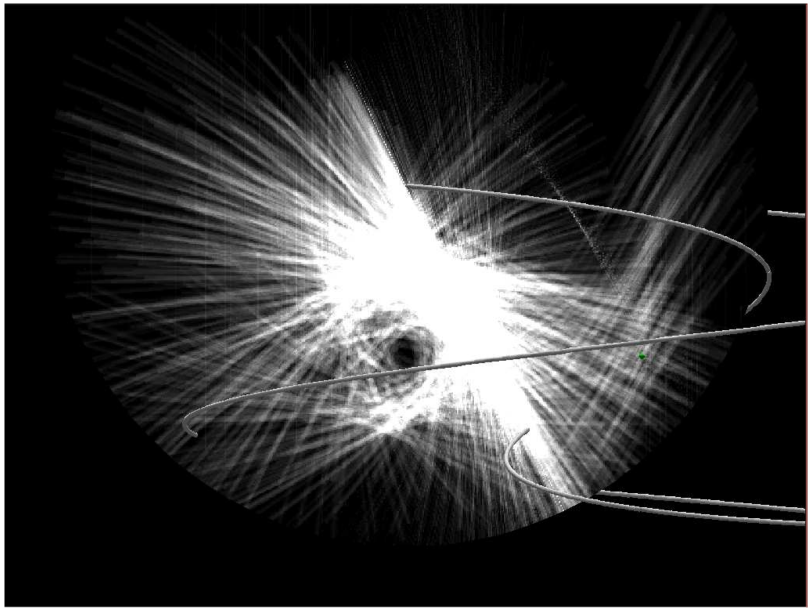


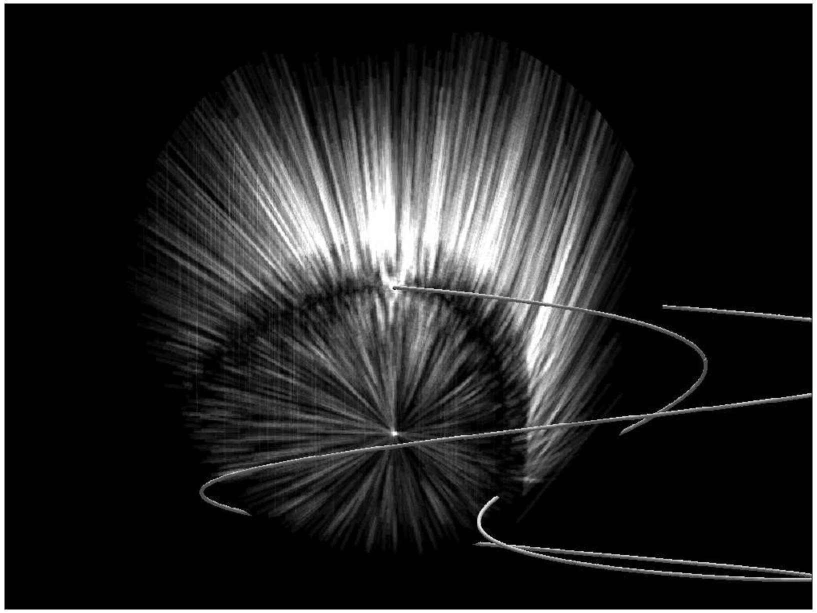


Visualizing P^q instead of P

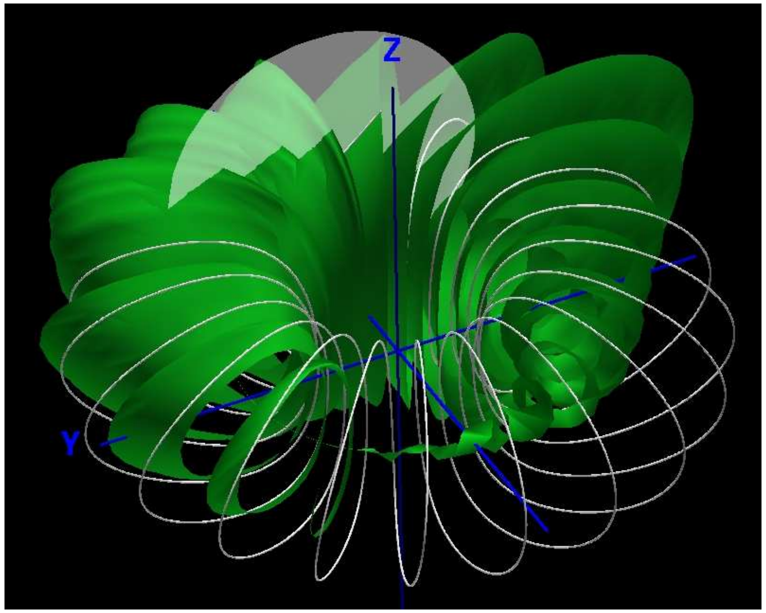


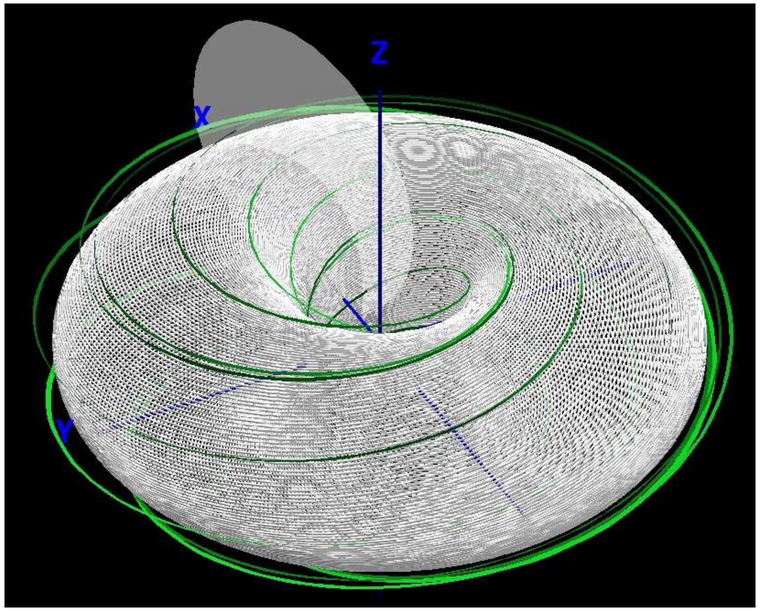
Comparing the Visualization of P vs. P^q



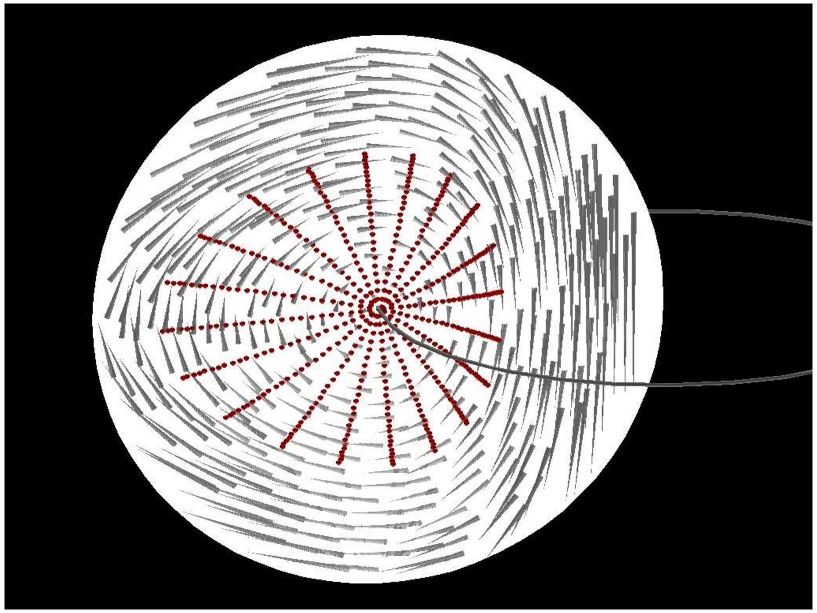


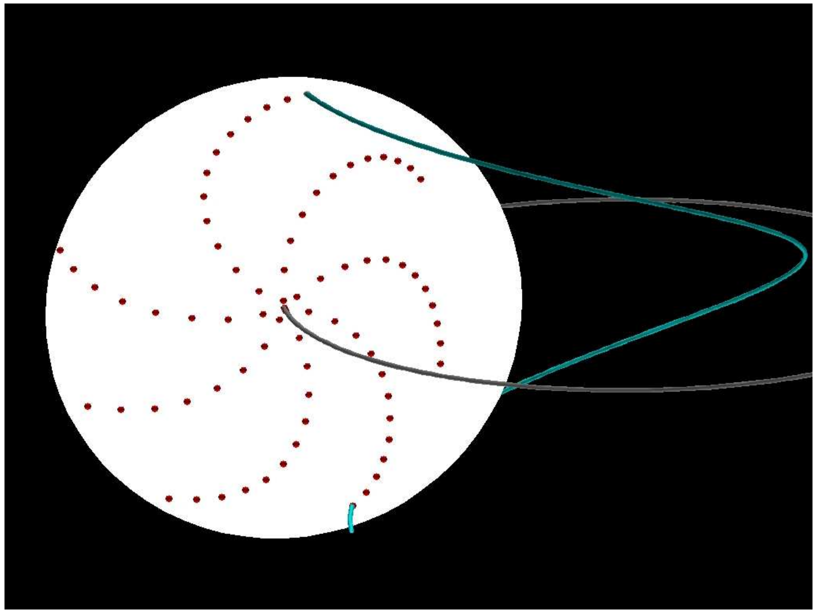
Extreme Phase Relations as Difficult Cases





Advanced Viz to Disambiguate Results





Viz @ cg.tuwien.ac.at – ’96 & ’97 – Overview

What we did/do

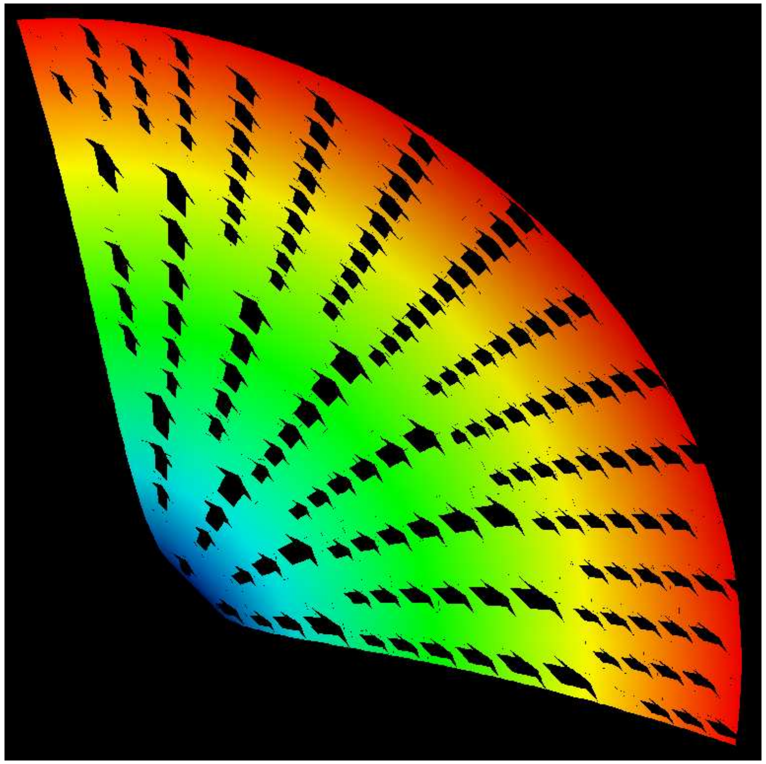
Hierarchical streamarrows, 3D extensions
OLIC, FROLIC, and virtual ink droplets
Dynamical system visualization via WWW
Visualization of high-dimensional systems
Collaborative augmented reality
Differential interferometry: modeling vs. measurements
NC-Milling: simulation and visualization

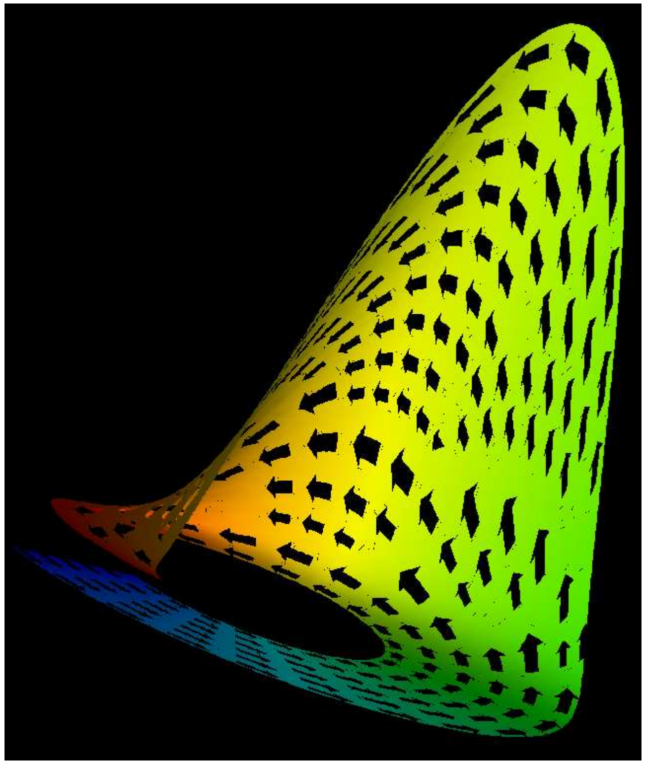
Who we are

Eduard Gröller
Helwig Löffelmann
Rainer Wegenkittl

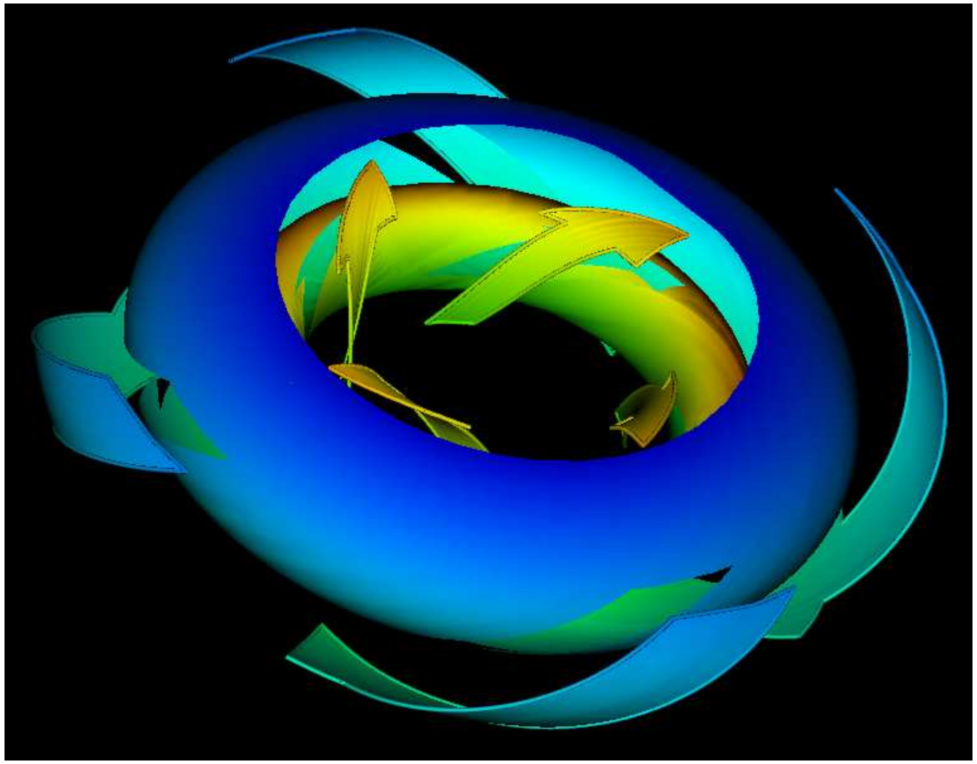
See also `http://www.cg.tuwien.ac.at/research/vis/`

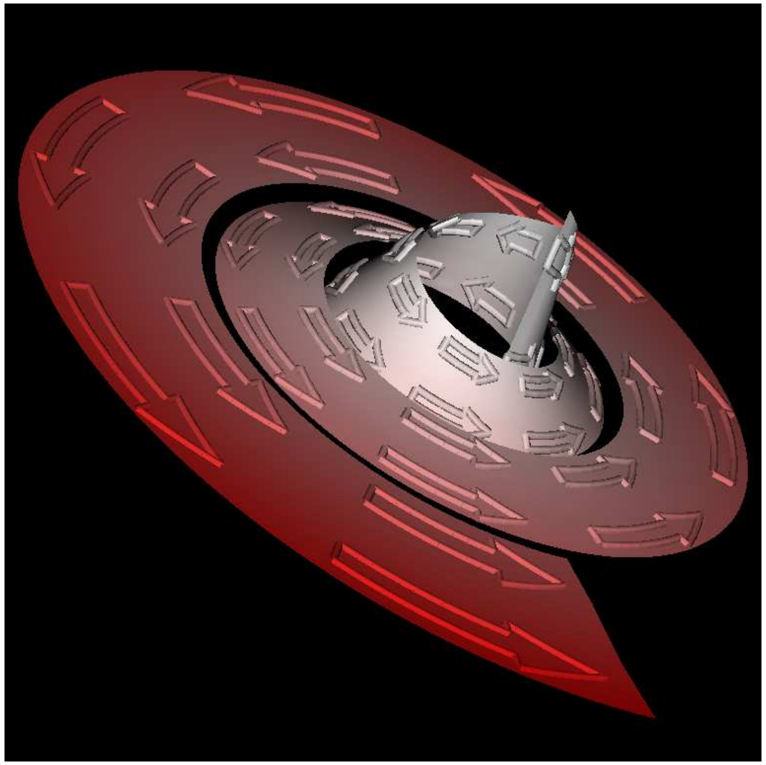
Hierarchical Streamarrows



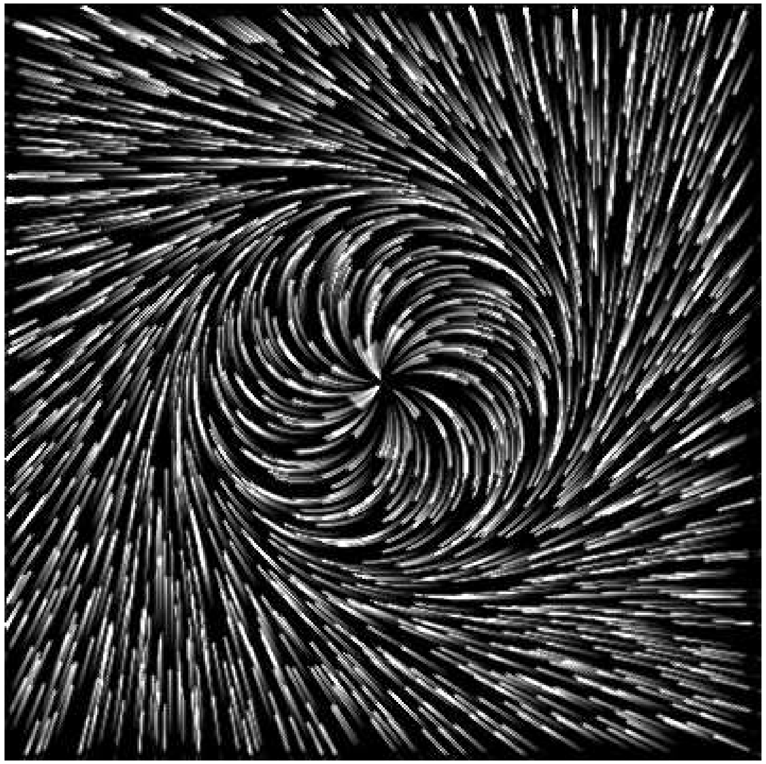


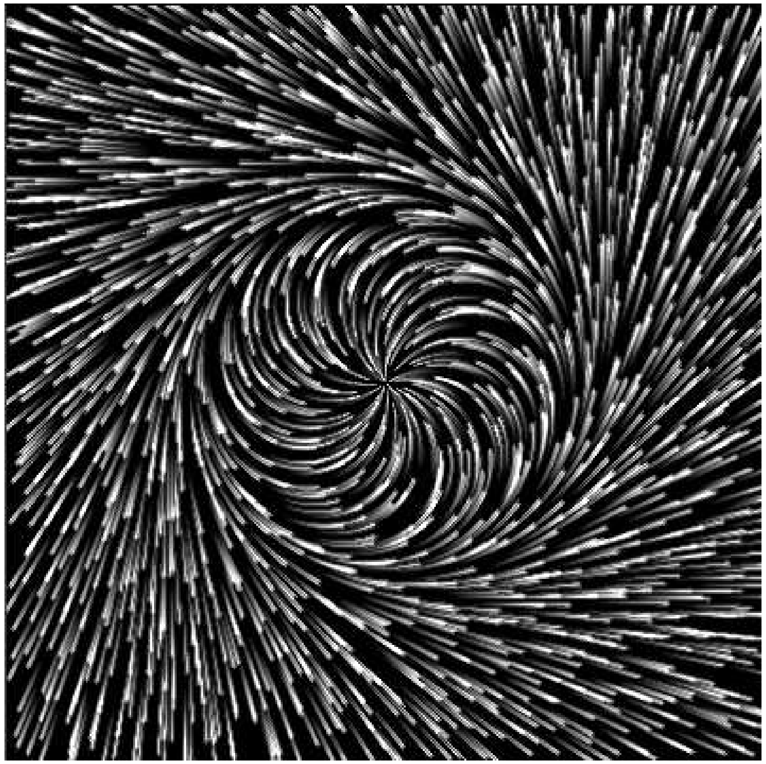
Streamarrows – 3D Extensions



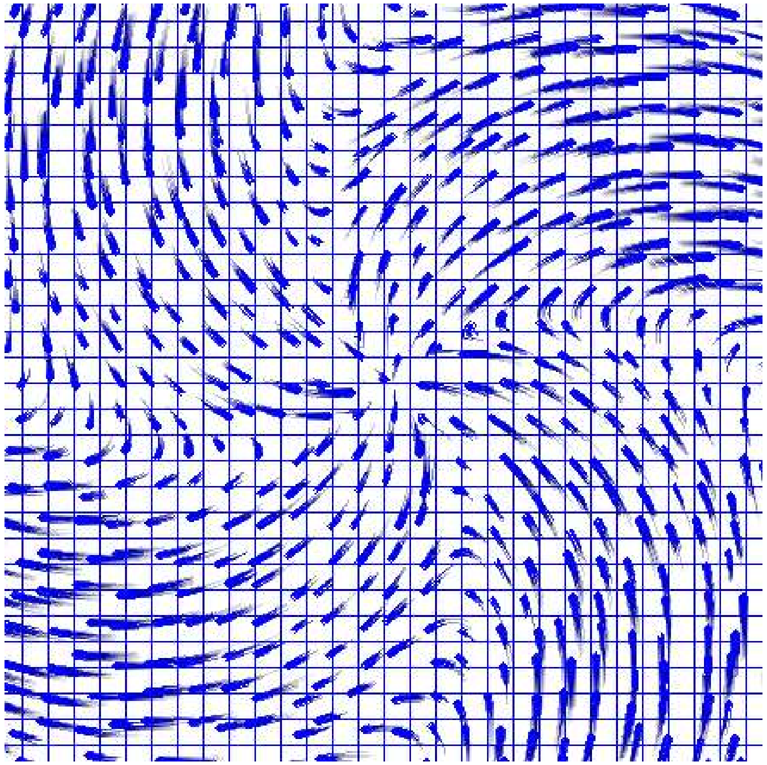


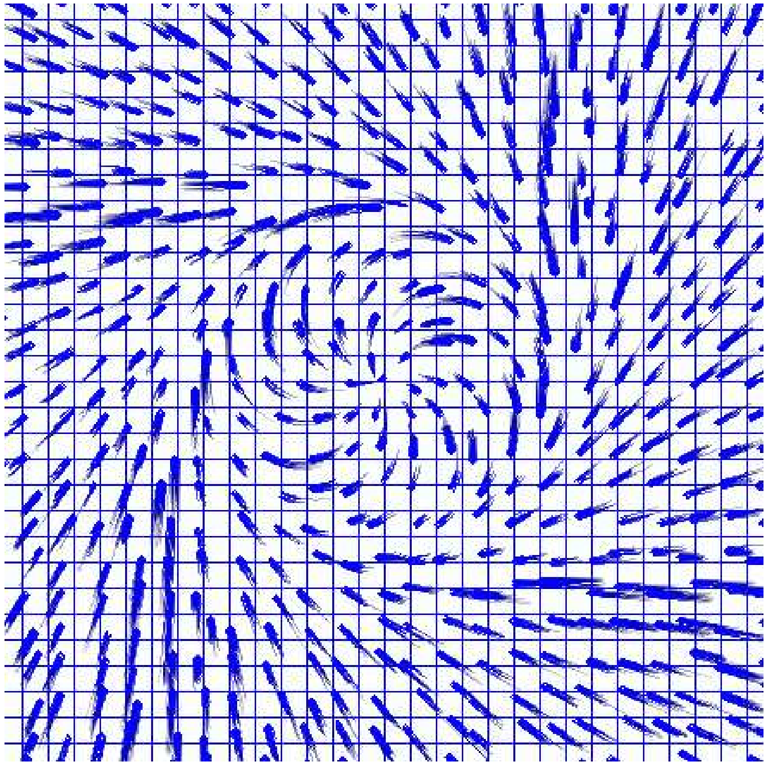
OLIC and FROLIC



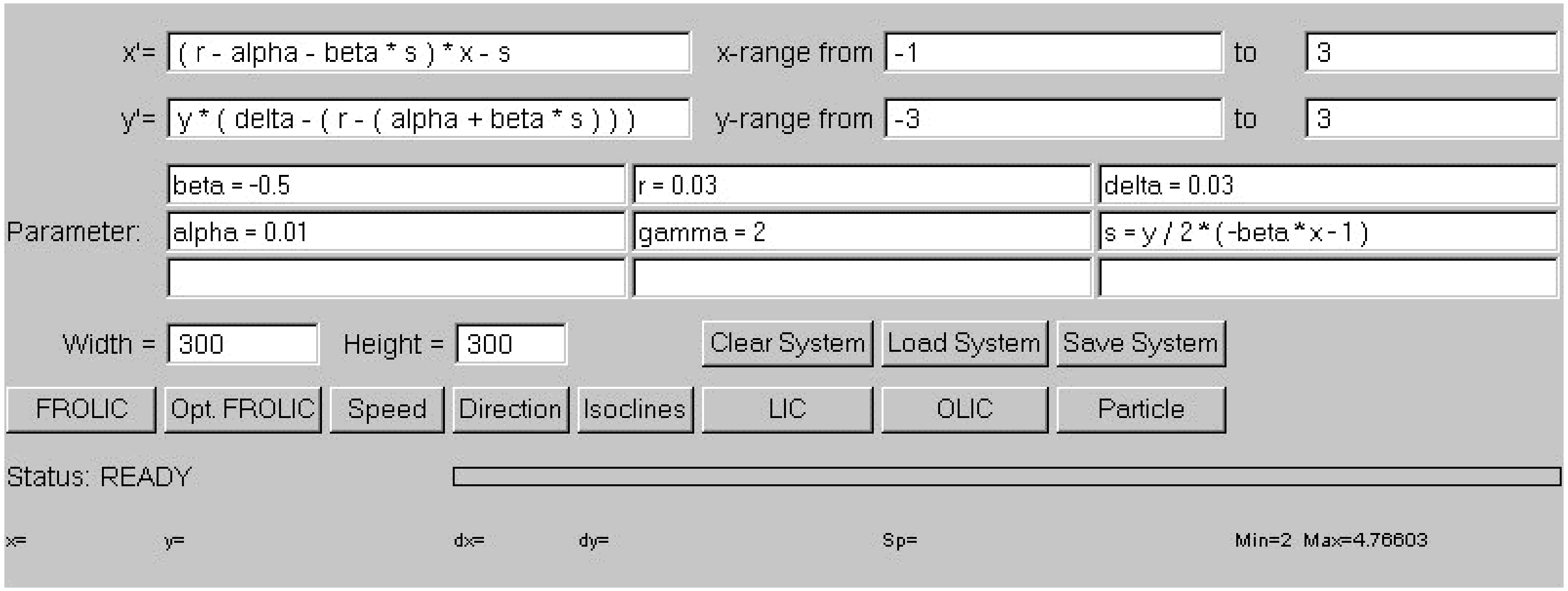


Virtual Ink Droplets



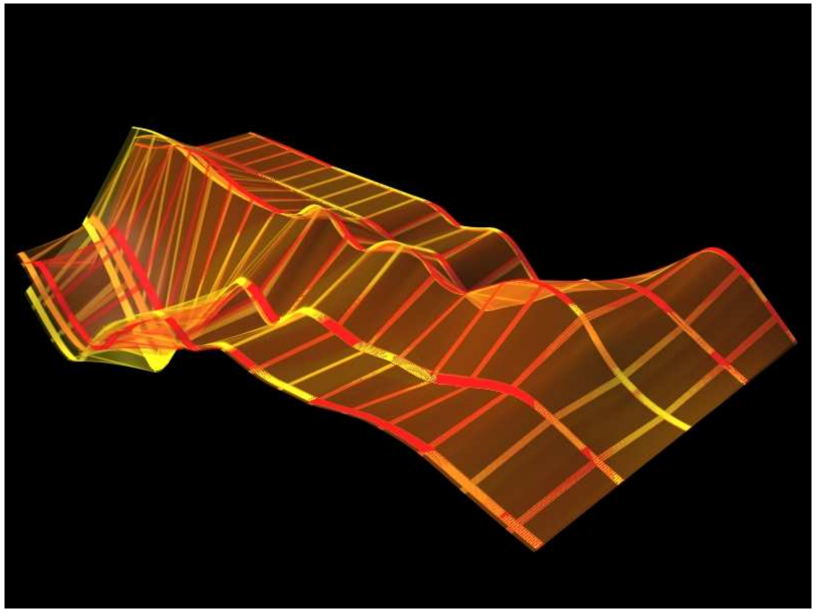


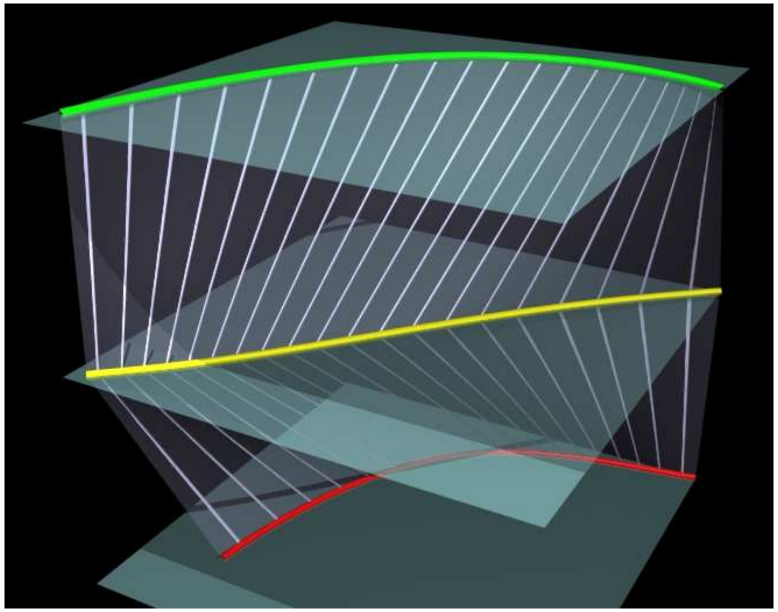
Visualizing Dynamical Systems via WWW



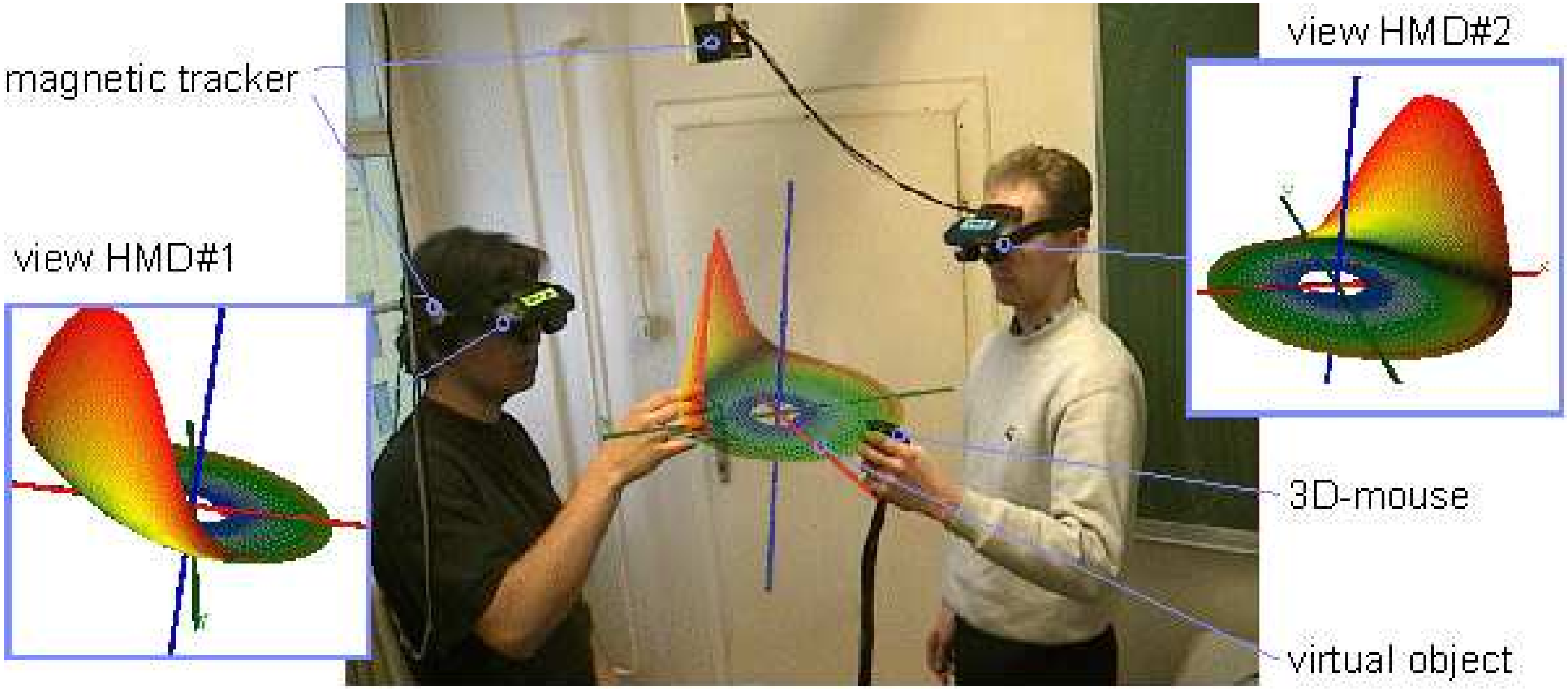
URL `http://www.cg.tuwien.ac.at/research/vis/dynsys/frolic/`

Visualizing Dynamical Systems in nD, n>3

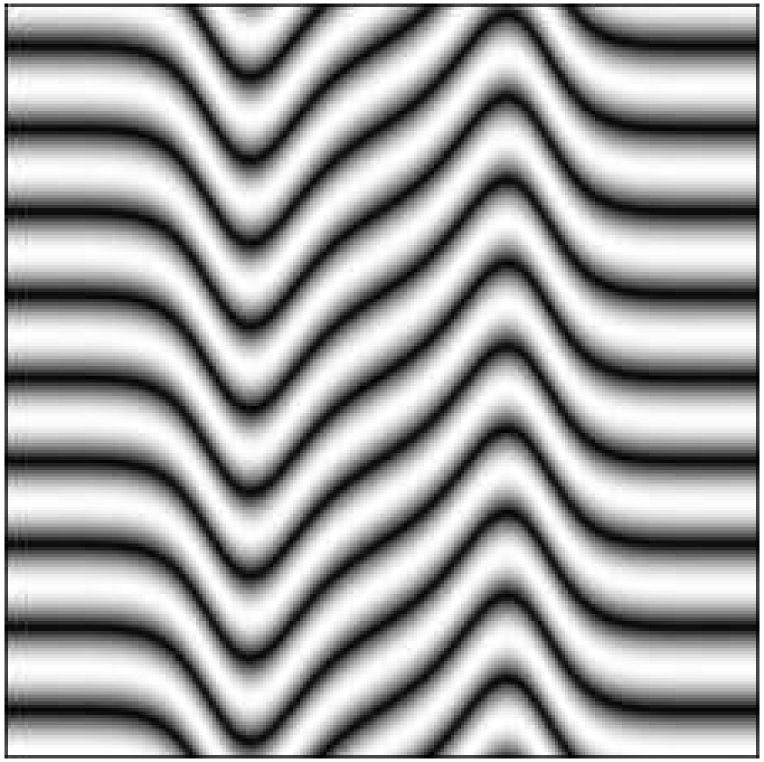




Collaborative Augmented Reality



Differential Interferometry and Visualization



NC-Milling – Simulation and Visualization