Rendering the Effect of Labradoescence

Andrea Weidlich*
Vienna University of Technology

Alexander Wilkie†
Charles University in Prague

ABSTRACT

Labradorescence is a complex optical phenomenon that can be found in certain minerals, such as Labradorite or Spectrolite. Because of their unique colour properties, these minerals are often used as gemstones and decorative objects. Since the phenomenon is strongly orientation dependent, such minerals need a special cut to make the most of their unique type of colourful sheen, which makes it desirable to be able to predict the final appearance of a given stone prior to the cutting process. Also, the peculiar properties of the effect make a believable reproduction with an ad-hoc shader difficult even for normal, non-predictive rendering purposes.

We provide a reflectance model for labradorescence that is directly derived from the physical characteristics of such materials. Due to its inherent accuracy, it can be used for predictive rendering purposes, but also for generic rendering applications.

Index Terms: I.3.3 [Computer Graphics]: Three-Dimensional Graphics and Realism—

1 INTRODUCTION

Labradorescence, sometimes also called *Schiller*, is an iridescence phenomenon that occurs in some minerals, such as the namesake Labradorite, and Spectrolite. A typical example of the effect can be seen in Figure 1. Such minerals are normally gray, but when they are viewed under certain conditions, vivid metallic colours flash out of the stone. Commonly, these are a very intense bright blue or green, but sometimes also red, gold or violet.

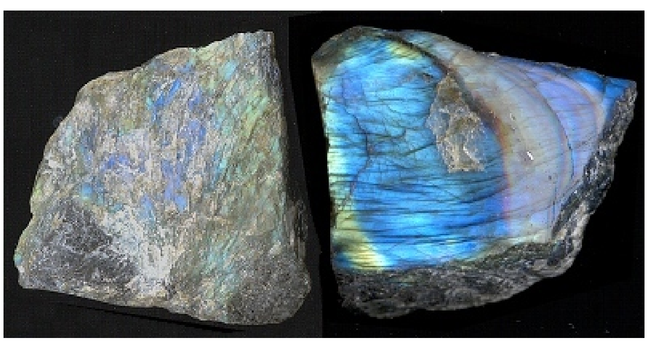

Figure 1: Photo of a Labradorite, on the left a raw specimen, and on the right a polished stone. The bright colour, which is similar to that of the wings of certain tropical butterflies, is caused by complex thin film effects, and changes when the stone is rotated. The colour effects are strongest when the stone is cut parallel to the layers. Image © 2008 by Andrew Alden, geology.about.com

The reason for this colour play is that the material is made up from repeated, microscopically thin twinned crystal lamellae, i.e. thin layers. Such twinned structures can appear when two types of crystal with similar structure inter-grow, and both end up sharing the same crystal lattice. The two different crystals form alternating, parallel layers that are approximately 50 to 100 nm thick. On each of these lamellae, light is partly reflected and partly refracted – which, together with the thinness of the layers, can cause significant interference colours. In contrast to most other materials that exhibit this effect, though, the interference colours are highly directional in this case; this is one of the main characteristics of labradorescence. A schematic illustration of this process can be seen in Figure 2.

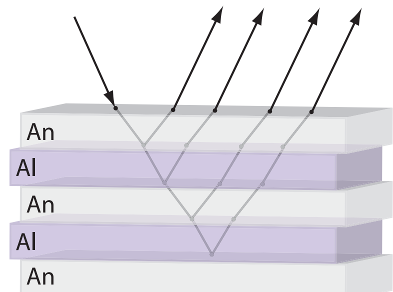

Figure 2: Schematic illustration of reflection and refraction on alternating Albite and Anorithe layers. The reflected light waves can constructively or destructively interfere with each other.

The phenomenon of labradorescence is, however, not exclusively caused by a simple thin film interference effect alone. This can be confirmed by examining the colours that are shown at varying angles. As with genuine thin films, they do vary continuously, but no alternating dark and bright bands can be seen. Nor are the first order colours of the Newtonian sequence visible; instead, the colour sequence immediately begins with a bright blue. It can be demonstrated that, apart from interference, diffraction also plays a major role in the appearance of crystals that exhibit labradorescence, and that it is responsible for the deviations from a pure interference effect behaviour.

Due to their special colour and reflection properties, crystals that exhibit labradorescence are often used as gemstones or decorative objects. Most Labradorescent minerals are not particularly rare, but a proper cut can greatly increase the value of what is an otherwise inexpensive stone. The main goal of this paper is to transport relevant physics knowledge to the realm of graphics engineering, and to enable the convincing replication of the effect in renderings. However, a secondary aim of this paper is also to explore the possibilities of performing gemstone prototyping for Labradorescent stones in a way that is helpful for improving the cut of a given rough stone.

To demonstrate our technique, we used the actual mineral Labradorite as the focus of our investigation; it is the most common, and probably also most spectacular representative of all materials that exhibit labradorescence. However, the background theory holds true for similar minerals as well, so the method we propose can be adapted to represent a variety of different layered twinning crystals, such as Spectrolite.

2 RELATED WORK

The mineral Labradorite, and the origin of its peculiar colour, have frequently been discussed in physics literature

*e-mail: weidlich@cg.tuwien.ac.at
†e-mail: wilkie@cgg.mff.cuni.cz

[17][18][16][15][25]. However, the direct application of this accumulated knowledge to computer graphics is not always easy, or even possible.

To the best of our knowledge, the effect of labradorescence itself has so far not been investigated in the computer graphics community. However, there are quite a lot of publications that deal with thin film effects [3][12][13][6][8] and diffraction [20][5][23][24]. Labradorescence is, however, a special phenomenon where interference and diffraction both play a part, and the overall result is not a simple combination of the two. Consequently, none of the models for pure interference and diffraction can be used for Labradorescent materials. What comes closer to our work are the models that deal with multiple thin films [11][10][21][22][7][19], although they are usually designed for only a few layers of film. This limits their applicability for our purposes - the appearance of Labradorescent materials is caused by large numbers of lamellae. Please note that due to the huge volume of work that deals with layered surfaces, this list is by no means complete, but only gives a brief overview of those papers that are relevant to the following work.

3 A Model of Labradorescencent Reflectance

Labradorite is a plagioclase feldspar variation and is – apart from minor inclusions – a mixture of Albite and Anorithe, with a small percentage of Orthoclase. Chemical analysis shows that Labradorite always has an Anorithe contribution of 44 to 61 percent, which in mineralogy is referred to as the ”Bøggild Range”. Within the material, the twinned Anorithe and Albite crystals are aligned in parallel lamellae of varying thickness, usually according to the *albite law*. This means that the twin plane is the (010) plane, i.e. the plane of one of the unit crystal cell faces for this mineral. These lamellae are responsible for the appearance of a stone.

To derive a useable reflectance model for labradorescence, we take a closer look at the properties of the effect, and the theoretical explanations that are available for it.

3.1 Optical Properties

When examining a mineral that exhibits labradorescence, the following optical properties can be observed:

1. The colour originates inside the stone and is mainly due to interference.

2. The interference colours only start with the second order colour sequence.

3. The effect is highly directional.

4. The coloured light is unpolarised.

An important consequence of the first point is that a useable reflectance model that captures labradorescence will perform colour and reflection computations that are specific to the effect only for rays that are refracted into the material. Although in general labradorescent materials exhibit subsurface-scattering like the materials discussed in e.g. [14], [9] or [4], this effect can be neglected for the coloured reflections since the lamellae are so thin that the light exits more or less at the same position.

The normal specular reflection off the polished stone surface obeys the standard rules of Fresnel reflectance. The remainder of this paper deals with the model that is needed to describe the behaviour of the refracted light that enters the stone.

3.2 The Colour of the Effect

One remarkable property of Labradorescent minerals is that their iridiscence colour hue can be predicted from their chemical composition alone. A good model for this effect was proposed by [1]; basically, the thickness of the lamellae is directly dependent on the ratio between the different minerals in the stone. And with increasing thickness of the lamellae, the iridescence colour shifts towards longer wavelengths. According to Bragg's law,

$$n\lambda = 2d \sin\theta \tag{1}$$

which means that the longer wavelengths λ are more strongly reflected from thicker lamella layers. d is the distance between the lamellae, θ the incoming angle and n the order of diffraction. Figure 3 shows the effect of the composition on the iridescence colour.

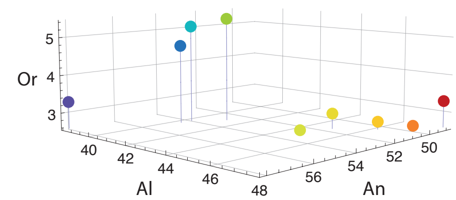

Figure 3: Relationship between iridescence colour hue, and mineral composition (in percent) for several sample concentrations found in real stones. The data is taken from [15]; Al = Albite, An = Anorithe, Or = Orthoclase.

3.2.1 Reflection Intensity and Direction

For graphics purposes, we not only need a model for the reflectance colour hue, but also for the reflection intensity and direction of the coloured reflection that occurs within the stone. It can be simulated with a statistical model which is based on three assumptions

1. The birefringence of both types of inter-growing crystal is so small that it can be ignored.

2. Multiple reflections within lamella layers are ignored.

3. The main beam is not weakened on its downward path through the layers of the crystal, because the amount of light reflected from each plane is very small.

These three assumptions may seem somewhat arbitrary at first, but they are actually not implausible, and they do make a treatment of the effect much easier. The birefringence of Albite and Anorithe is so small that the material can for all practical purposes be considered to be isotropic, and the second assumption is in accordance with the kinematical theory of diffraction. A good justification for the second and third assumption is also the fact that the differences between the indices of refraction (IOR) of the layers are very small; for interfaces with such almost negligible IOR differences, the Fresnel terms predict very little reflection. Also, both materials exhibit very weak absorption, which further reinforces the third assumption. Comparisons with measured data show that the model is indeed suitable to capture and simulate the effect [1]. Based on these assumptions, a statistical model that assumes Labradorite to consist of many very thin, parallel aligned lamellae can be formulated. The path difference between light that is reflected from two successive lamellae is given by

$$(x_i - x_{i-1}) 2 \sin\theta \tag{2}$$

if

$$x_1, x_2, x_3, \ldots, x_i, \ldots, x_{2N-1}, x_{2N} \tag{3}$$

are the positions of the reflecting planes, and θ is the angle between incident light ray and a lamella. $x_i - x_{i-1}$ is therefore the thickness

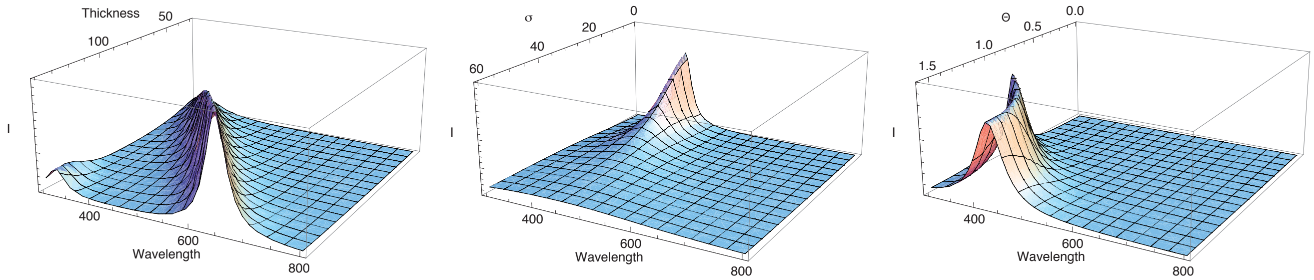

Figure 4: Spectral distribution of the reflection coefficient of different samples with decreasing mean thickness d in nm of one layer (left), decreasing variance σ of one layer (middle) and with different angle of incidence (right).

of a given layer. The total reflection amplitude V is given by

$$\begin{aligned} V = r[1- & \\ & \exp i\{p_a(x_2-x_1)\} + \\ & \exp i\{p_a(x_2-x_1)+p_b(x_3-x_2)\} - \\ & \exp i\{p_a(x_2-x_1)+p_b(x_3-x_2)+p_a(x_4-x_3)\} - \ldots - \\ & \exp i\{p_a(x_2-x_1)+\ldots+p_a(x_N-x_{N-1})\}] \end{aligned}$$

where

$$p_a = \frac{2\pi}{\lambda_a}\cdot 2\sin\theta_a n_a \quad (4)$$

$$p_b = \frac{2\pi}{\lambda_b}\cdot 2\sin\theta_b n_b \quad (5)$$

and r is the amplitude reflection coefficient and $N(d_a+d_b)$ is the overall thickness of the crystal. The subscripts a and b refer to the two different types of alternating lamellae, and n_a and n_b are their indices of refraction. The actual thickness of each layer is then replaced by a set of random variables $\{\gamma_m\}$

$$(x_i - x_{i-1}) = (1+\gamma_i)d_a \quad (6)$$

with d_a as the mean thickness of the lamellae. A similar expression exists for the b-type of lamella. The random variables have a symmetric probability distribution so that the average of the variables $<\gamma_m>=0$. For each random variable γ_i an approximation of

$$<e^{iL\gamma}>=e^{-\alpha} \quad (7)$$

is possible where

$$\alpha_{a,b} = \frac{(p_{a,b}d_{a,b})^2\sigma_{a,b}^2 d_{a,b}^2}{2} \quad (8)$$

σ_a and σ_b are the variance of the lamellae thickness from the mean thickness. Since each lamella is very thin, the overall number of lamellae N is very large. Therefore only the terms of order N are retained and the intensity $I=<|V|>^2/2Nr^2$ can be written as

$$I = \quad (9)$$

$$[1-e^{-2\alpha_a-2\alpha_b}]\times \frac{1-\cos M\cos N\dfrac{e^{-\alpha_a}-e^{-\alpha_b}}{1-e^{-2\alpha_a-2\alpha_b}}+\sin M\sin N\dfrac{e^{-\alpha_a}+e^{-\alpha_b}}{1+e^{-2\alpha_a-2\alpha_b}}}{1+e^{-2\alpha_a-2\alpha_b}-2\cos(2M)e^{-\alpha_a-\alpha_b}}$$

$$M = \frac{(p_a d_a)+(p_b d_b)}{2}$$

$$N = \frac{(p_a d_a)-(p_b d_b)}{2}$$

Equation 9 yields the intensity of the reflection for every wavelength, and is only based on the the IOR n_a, n_b for the two types of lamellae, the mean layer thickness d_a, d_b and its variance σ_a, σ_b. Figure 4 shows the influence of these parameters on the spectral reflectance distribution for the refracted ray that enters the stone, i.e. the appearance of the coloured reflection within the crystal. Two interesting properties can be observed:

- The peak of the reflection intensity moves towards the longer wavelengths if the mean thickness of one of the lamellae is increased. This means that the interference colour changes from blue to red.
- If the variance is increased, the peak becomes less prominent and ultimately vanishes. In that case, the colour of the material turns gray, and no interference can be observed any more. Interestingly, only height and width of the peak change, but not the wavelength at which the maximum reflection intensity can be seen, which still follows Bragg's law.

It is important to note that with this model it is possible to predict the appearance of a given piece of Labradorite, if its composition is exactly known. As discussed later, though, it is sufficient in most cases to predict the overall appearance of a specific stone. For this task, it is much more important to find the direction in which the lamellae are oriented, because this "plane of schiller" determines how a stone must be cut. In a real stone, this plane can be easily determined by a gemmologist, since it is parallel to the (010) cleavage; and the average the thickness of the lamellae can be estimated from the colour of the stone.

3.2.2 Colour Zoning

As stated earlier, the phenomenon of Labradorite is not limited to one particular reflection colour. In fact, different *zones* of colour can usually be seen in a single stone; it is rare to find stones with completely monochromatic reflection patterns. The reason for this is that the individual lamellae do not grow regularly, but that the overall chemical composition of the mineral, and therefore also the thickness of the individual lamellae, usually changes slightly throughout the material. Figure 5 shows an example of such zoning.

It would of course be possible to measure the composition of a real stone with an appropriate device and use these values, but it is obvious that for most applications this approach is neither necessary nor practical. It has to be noted that our proposed method would be capable of using such measured data from real stones as input, and would yield correct results – as far as the rendering step of a predictive rendering application is concerned, we provide a solution that solves the problem. The remaining difficulty in obtaining predictive images of a given stone lies not in the replication of the effect

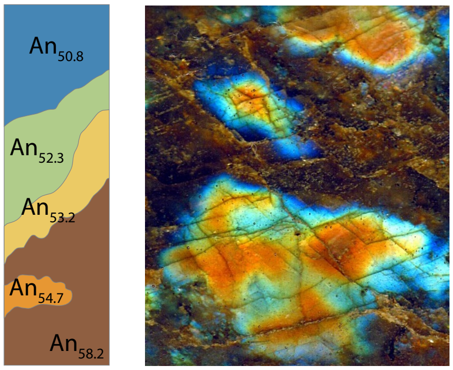

Figure 5: Colour zones in Labradorite. **Left:** sketch of the relation between colour and composition; the Figures are the Anorithe percentages for each area (after [15]). **Right:** photo of colour zones in a real Labradorite sample.

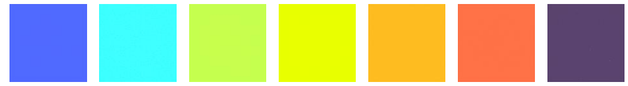

Figure 7: A series of rendered cube surfaces with increasing Anorithe content. The dominant colour shifts towards longer wavelengths with increasing Anorithe content. Please note that the images are gamut mapped. The data was taken from [15].

Figure 8: A series of rendered cube surfaces with increasing variance of lamellae thickness. σ_a increases (from left to right) from 1.0 to 10, 20, 30, 40, 50 and 80nm. The variance of the second lamellae type is fixed at 16nm. All images are again gamut mapped. Note the progressive de-saturation of the resulting colour.

as such, but solely in the fact that reliable data for the distribution of the lamellae is very hard to obtain.

For normal computer graphics purposes, a much better approach is to find an adequate procedural texture that can simulate the round and smoothly changing zones of colour. For such applications, our method has the significant advantage that it yields intrinsically realistic results, since it is a true simulation of the effect. These texture maps that one has to generate have to provide values for the mean thickness and the variance of the stone; this can be either done in a 2D or a 3D texture. This texture map is then applied to the object, and used during the reflection computations.

We found out that Perlin noise can simulate the natural structure of these zones very well, so we used such a noise function to create a texture map that contains values between 0 and 1. These values are then used to index two or more surfaces with different parameters for mean thickness and variance. Between these parameters linear interpolation is performed to create smooth colour change. A schematic illustration of this process can be seen in Figure 6.

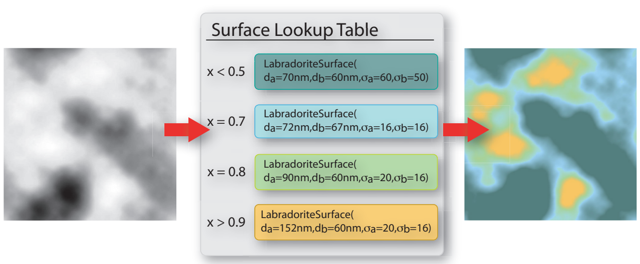

Figure 6: A lookup table is used to linearly blend surfaces with different parameters. Values below 0.5 and above 0.9 are not blended while a value of e.g. 0.6 would mean that the parameter sets of two different forms of the material are interpolated. Which is permissible, since such gradual variations also exist in nature, and are due to the kind of gradual change in chemical composition of the material that such an interpolation describes very well.

3.3 Polarisation

Labradorite is a biaxial crystal, and since birefringence gives rise to polarisation effects, this aspect should be included in the discussion of labradorescence for the sake of completeness, even if polarisation is often neglected in rendering applications.

Labradorite does exhibit polarisation effects. This can easily be seen when viewing a Labradorite sample through a linear polarisation filter. Light that is reflected directly from the top of the surface and that does not undergo refraction is linearly polarised. The part of light that enters the material and returns as coloured reflection, however, is unpolarised. Although the refracted light is split into two components that are polarised perpendicularly to each other, both rays have the same phase when they emerge from the crystal. This is not particularly surprising: the birefringence of the mineral is so weak that from a macroscopic viewpoint it can be considered to be isotropic. Also, any orientation of the oscillation would be destroyed during the diffusion of light that takes place between the lamellae.

Given these observations, capturing of these properties is not complicated at all, since the standard polarisation techniques for Fresnel reflectance apply for the directly reflected rays, and a depolarising Mueller matrix has to be used for the coloured reflection that takes place within the stone [26].

4 Results

Spectrolite or Labradorite gemstones are most often cut *en cabochon*, an unfaceted cut with a round surface. Sometimes more fancy cuts are applied, or they are used as the material for complex decorative objects. The shapes of real Labradorite gems are usually a trade-off between an appealing shape and maximisation of the effect, though, since the effect actually can be seen best on simple geometries. For this reason we just used cubes and spheres for the generation of most of the results shown in this paper.

4.1 Effect of the Parameters

To demonstrate the change of appearance when the individual parameters are altered, we reproduced some of the characteristics that can be seen in the plots from Figure 4 on real objects.

We first rendered a series of images that show a cube with varying Anorithe content under diffuse illumination (Figure 7); the cube is viewed from the top in close-up, so that only the reflection colour of the material can be seen. As expected, the dominant colour shifts towards longer wavelengths with increasing Anorithe content. The reason for this is that with increasing concentration, the Anorithe lamellae become thicker and the shorter wavelengths are increasingly subjected to destructive interference.

In contrast to that, if the variance of the lamellae is changed, the dominant wavelength stays the same, but changes from more or less pure spectral colour to a de-saturated version and becomes neutral when the variance between the lamellae becomes too big, i.e. the lamellae thickness is not regular enough for the effect of labradorescence to manifest itself (Figure 8). Again, this is in agreement with observations made on real stones, and our understanding of the causes of the phenomenon.

4.2 Smooth vs. Sharp Transitions

It is possible to either blend between the parameters linearly, or to generate sharp edges between the colour zones; both cases occur in nature. Labradorite often shows smooth transitions between colour zones, while Spectrolite tends to have sharp edges between them. However, no general rules can be given; the appearance of such materials differs from stone to stone, and very often both forms of transition are present.

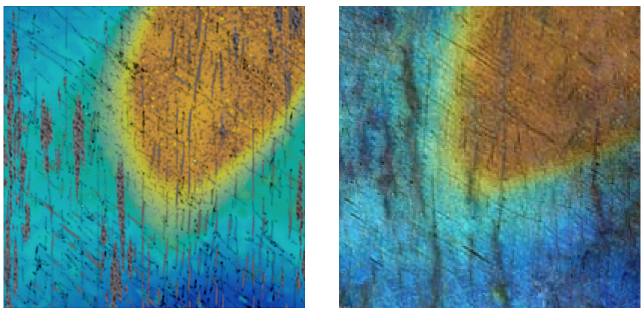

Figure 9: Smooth transitions between colour zones in Labradorite. **Left:** a synthetic image rendered with a Perlin noise function that controls lamella thickness and variance. A fracture texture was overlaid on the stone geometry for greater similarity with the cracks that are present in the real stone. **Right:** a photograph of a real Labradorite.

Both colour patterns can be simulated with the method we described in Section 3.2.2. The only differences are how smooth the noise function is, and how much we interpolate between the different parameters. Figure 9 shows an example of a smooth pattern, while Figure 10 is an example of the second form of transition. A schematic overview of the basic shader structure that was used to generate Figure 9 is shown in Figure 11. The cracks that were overlaid on the stone geometry to provide a better match with real Labradorite samples were done with a comparatively simple procedural texture that seems to be sufficient in this case. Arguably, more sophisticated approaches for crack generation like [2] could further improve this aspect of overall gemstone appearance.

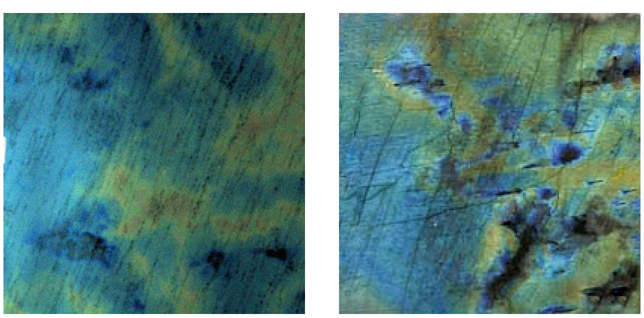

Figure 10: Sharp transitions between colour zones in Labradorite. **Left:** a synthetic image rendered with a Perlin noise function, As in Figure 9, a fracture texture was overlaid on the stone geometry. **Right:** a photograph of a real Labradorite.

Figure 12 shows such patterns applied to cabochon cut geometries. Note that as with real labradorescent stones cut *en cabochon*, the coloured flash is the dominant reflection feature. However, interference colours are not only evident in the "prime position" on top of the stones – as with many real Labradorite gemstones, a very

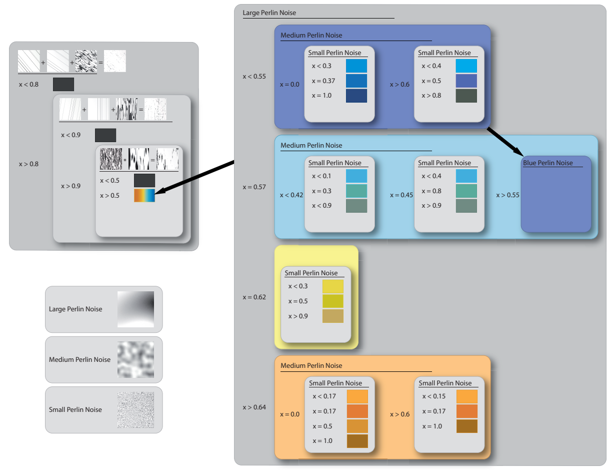

Figure 11: Schematic illustration of the shader hierarchy used for the pattern seen in Figure 9. As in Figure 6, x corresponds to the single float value that the Perlin noise function generates, and the ranges are those for which a particular set of parameters is chosen. Each of the coloured swatches represents one particular set of these numerical parameters; in the interest of clarity, the we just show the resulting dominant interference colour for each set, instead of the actual parameters like in Figure 6. Note that each individual mapping function node (i.e. each box) has its own, local value scale for x. In each node, x lies in the range $[0,1]$, and values that are passed from higher-level nodes are appropriately translated before use.

weak coloration can also be seen in other places, e.g. at half height around the circumference of the left object. This does not contradict the theory that the coloured flash only occurs on faces parallel to the (010) axis; rather, such seemingly off-axis interference colours are due to ambient light that is reflected from within the stone. As a consequence, such off-axis colours are only visible in a global illumination renderer that takes all components of light transport into account.

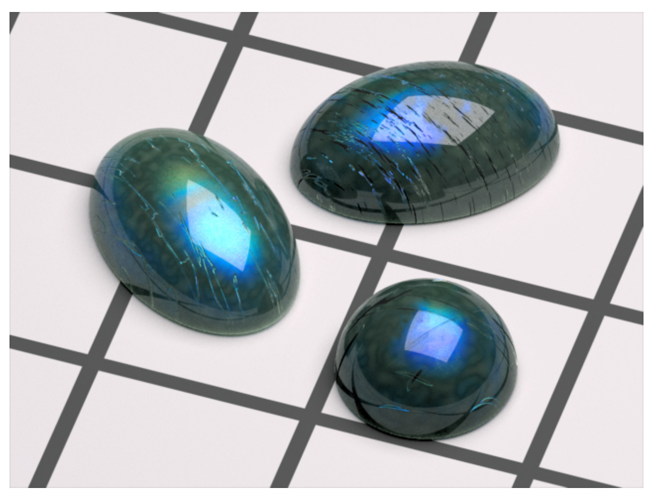

Figure 12: Textures that are roughly similar to those shown in Figures 9 and 10, applied to cabochon cut geometries.

In Figure 12, the patterns appear as less colourful than in figures 9 and 10 partly because the stone geometry is curved, and the other images are of cube faces that are viewed under optimal lighting conditions. Real labradorescent gemstones are cut to curved

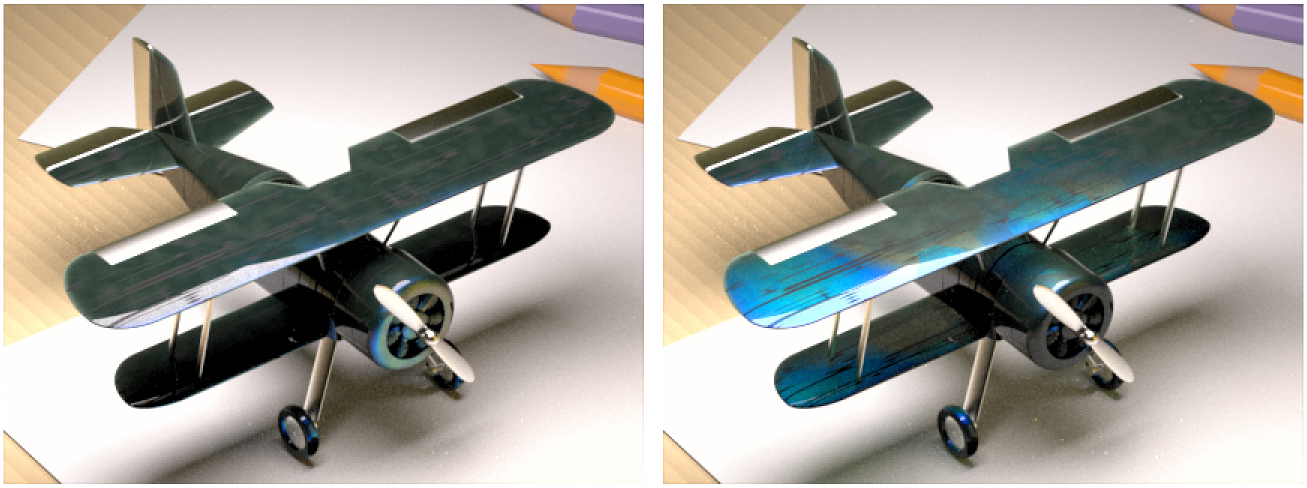

Figure 13: Rendered images of a complex decorative biplane model, some parts of which are assumed to be cut from labradorite. The only difference between the images is the lamella orientation. Given the properties of the effect, the coloured flash will only appear in one location for a given cut; no change of lighting and viewing angle can e.g. cause the large coloured area seen in the right image to appear in the left image, where the stone from which the parts are cut has been oriented differently.

shapes to provide a good trade-off between visibility of the coloured flash, and its intensity. The visibility of the flash is poor for a cube geometry (or indeed any faceted shape) – it can only be seen from a few combinations of viewpoint and lighting direction for planar facets, and only occurs on two faces of the cube; curved shapes are much better in this regard. On the other hand, the intensity of the flash is optimal for planar faces cut along the (010) axis, and curved geometries can only yield a smaller, less intense flash. Still, their more benign viewing characteristics make them the geometry of choice for such materials.

4.3 Orientation of the Lamellae

It is not difficult to properly cut a labradorescent stone *en cabochon*, since the overall orientation of the lamellae can be determined comparatively easily, and the choice of how to cut the stone (and in particular, how to orient the cut with respect to the lamella orientation) is straightforward in this case.

However, for more complex gemstone shapes the question of how one should orient the cut in order to maximise the effect quickly becomes difficult. In this scenario one can potentially benefit from a model that can predict the appearance of such a material. It should be noted that for such a simulation to deliver helpful results, it is not necessary to know the exact distribution of the colour zones within the material, since the goal is just to determine which faces of the object in question will look best when aligned with the lamellae.

Figure 13 shows a sample scenario for this kind of imagery: two rendered images of the same decorative object, a biplane model, some parts of which are assumed to be made of Labradorite. The general properties of the material are the same, but the stone from which the biplane parts are cut is rotated differently in the two images. As can be seen, the overall appearance is fairly different, and a designer could conceivably use such renderings to determine the best lamella orientation for a given, complex shape that he wants to produce from Labradorite, or a similar material. It should be noted that while the images in Figure 13 are not predictive in the narrow sense of the term (mainly, as discussed in Section 3.2.2, because exact information on the structure of the uncut stone is so hard to come by), they are still a reliable forecast of where the coloured flash will appear on the object. The exact texture of the cut stone will vary, but this main property will not.

5 Conclusion

In this paper we presented a model for rendering the effect of labradorescence solely based on the same few material parameters that govern its appearance in reality. Although we only demonstrated our approach for the namesake mineral Labradorite, is is not limited to this substance, because no assumptions were made that are only true for Labradorite. Due to its relative simplicity our approach is easily integrated in any existing renderer, and could also be implemented in a real-time environment.

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