VU Rendering SS 2014
186.101

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VU Rendering SS 2014

Unit 07 – Participating Media

So far...

Light interaction with surfaces:

$$L_o(x,\vec{\omega}) = \underbrace{L_e(x,\vec{\omega})}_{emitted} + \underbrace{\int_{\Omega} L_i(x,\vec{\omega}')\, f_r(\vec{\omega},x,\vec{\omega}')\, \cos\theta \, d\vec{\omega}'}_{reflected\ incoming\ light}$$

So far...

Light interaction with surfaces:

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Assumes:

- Interaction directly at the surface (true for metals)
- No interaction with the volume in between (true for vacuum)

Fog

Water

Della-Stock

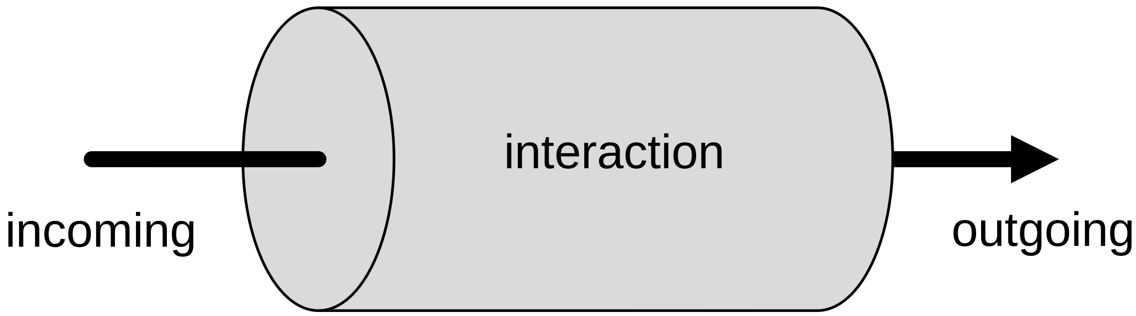
Scope

Surface approxiation not always valid → need to extend our model of light transport for materials that

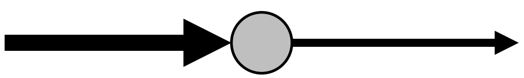
- allow perceivable light penetration and
- perceivably interact with light.

Interactions

Possible interactions:



- absorption



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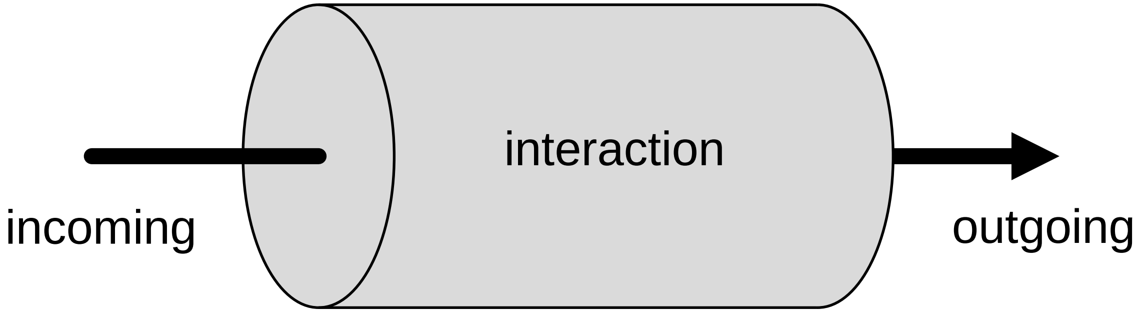
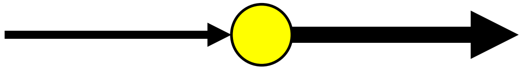


- absorption
- emission



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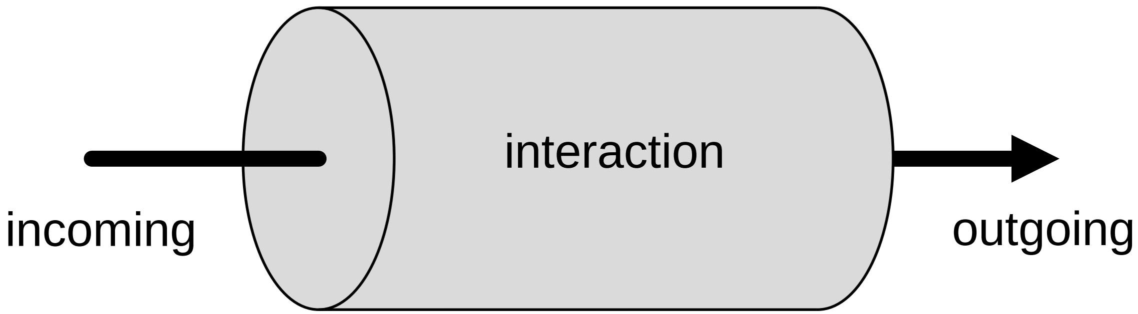
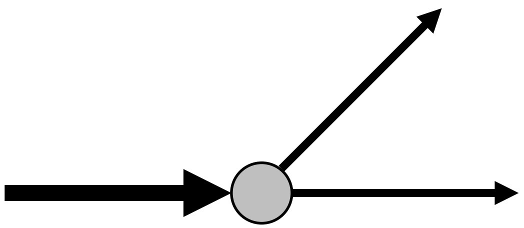


- absorption
- emission
- out-scattering



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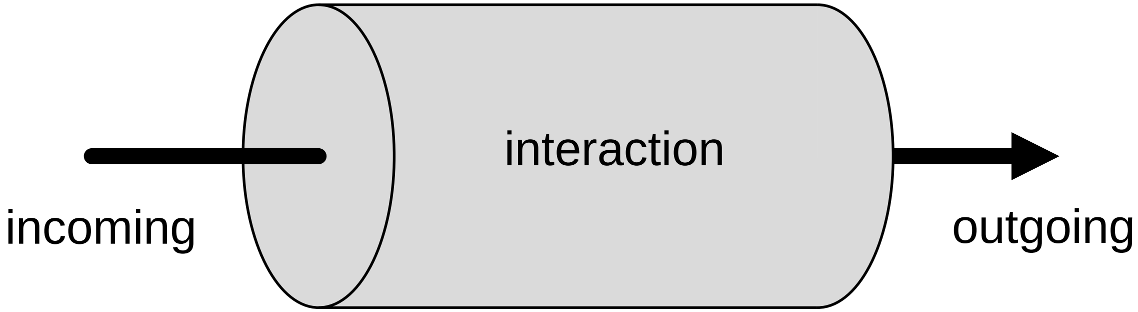
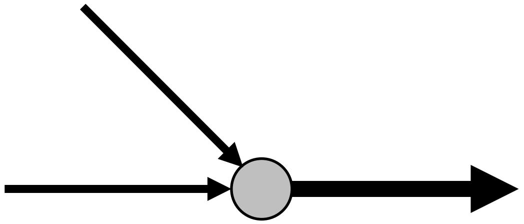
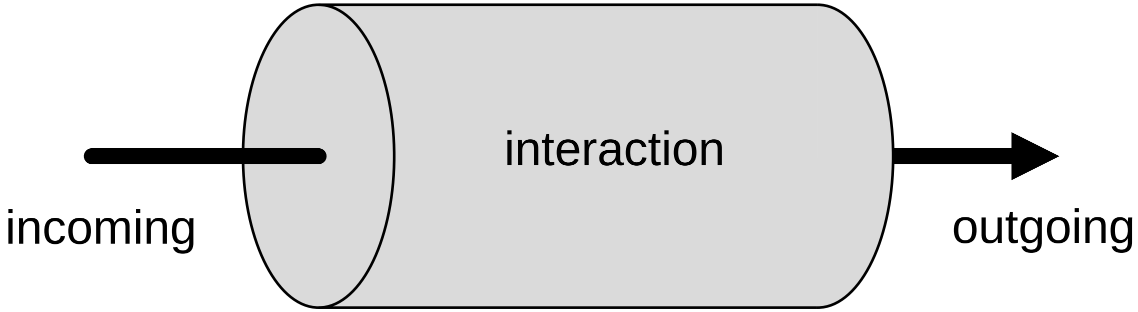




- absorption
- emission
- out-scattering
- in-scattering

Interactions

Possible interactions:



- absorption
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Interactions

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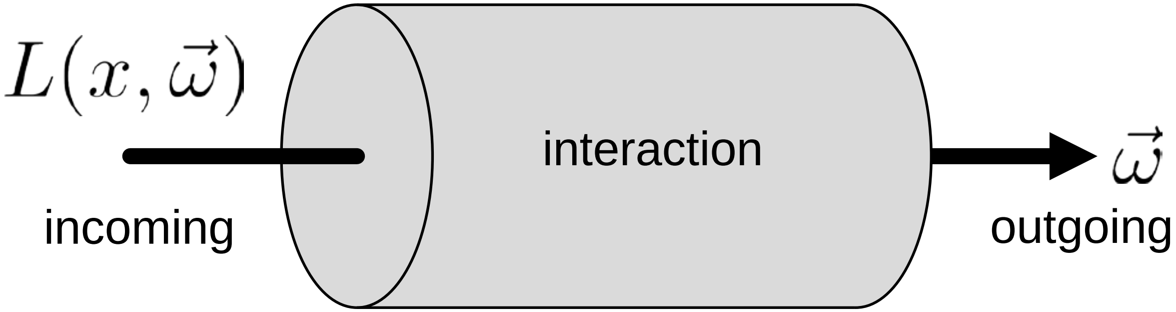

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Interactions

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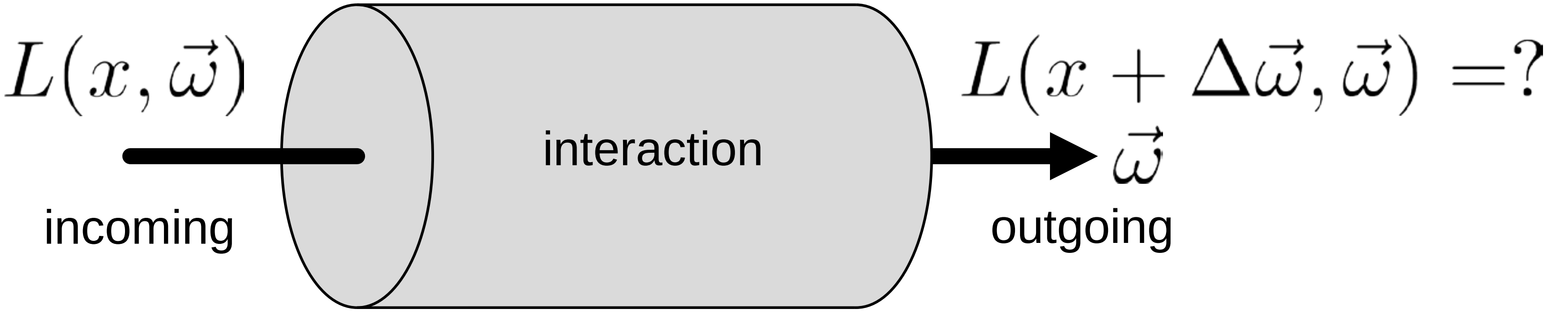

Interactions

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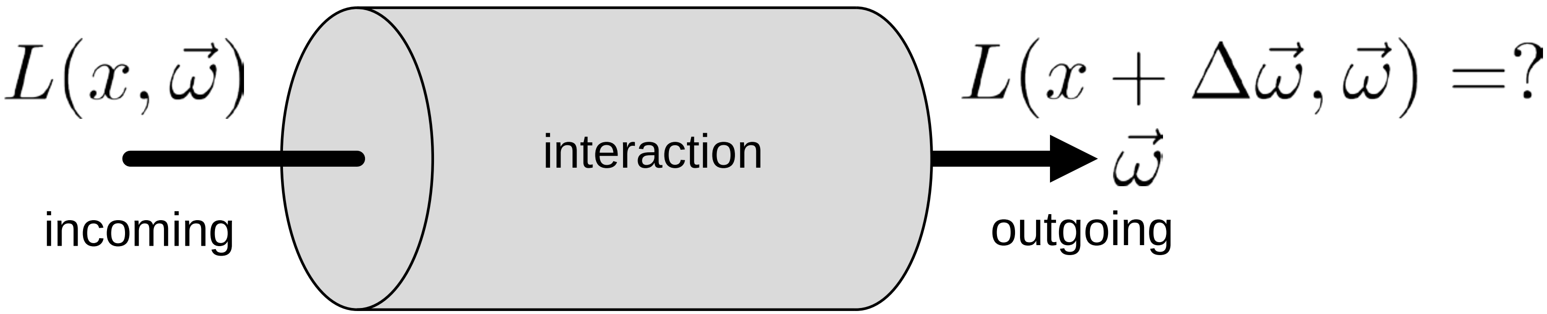
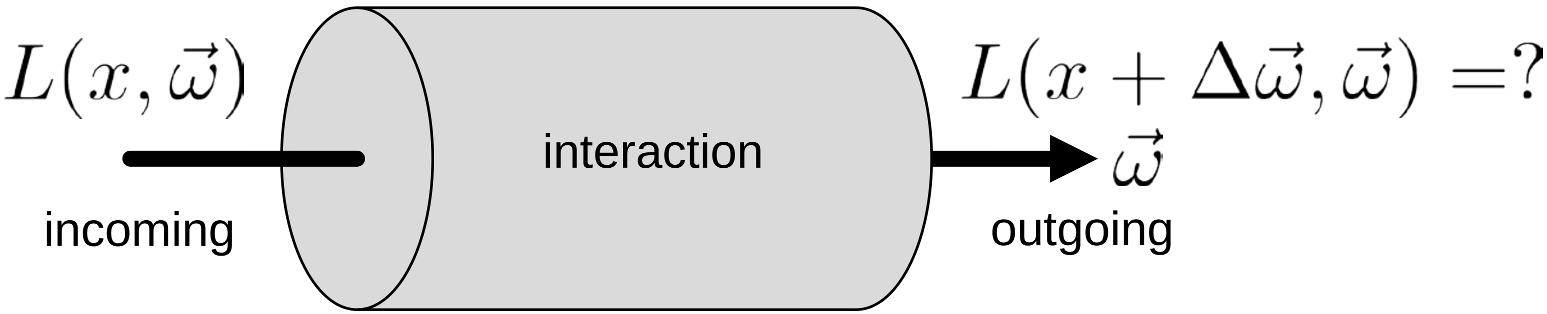


$$\vec{\omega} = (1, 0, 0) : \quad \frac{L(x + (\Delta, 0, 0), \vec{\omega}) - L(x, \vec{\omega})}{\Delta}$$

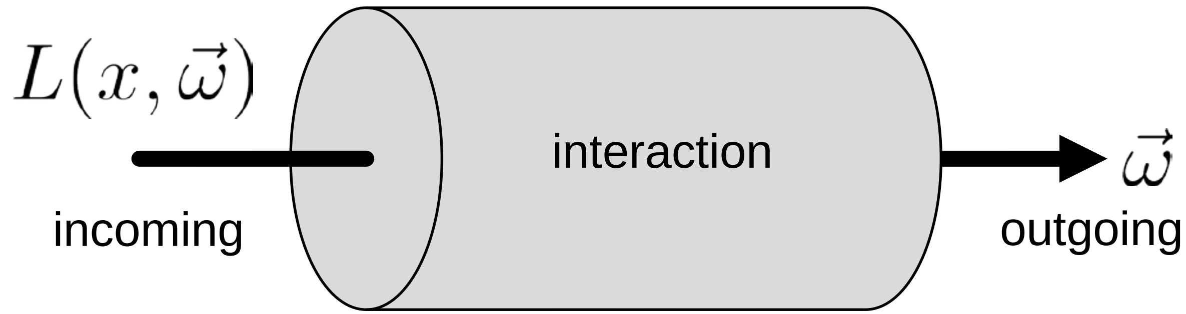
Possible interactions:



$$\vec{\omega} = (1, 0, 0) : \quad \frac{L(x + (\Delta, 0, 0), \vec{\omega}) - L(x, \vec{\omega})}{\Delta}$$

$$= \frac{dL(x, \vec{\omega})}{dx_1} \quad \text{for} \quad \Delta \to 0$$

Possible interactions:



$$\vec{\omega} = (\omega_1, \omega_2, \omega_3) = (1, 0, 0) : \frac{dL(x, \vec{\omega})}{dx_1} = ?$$

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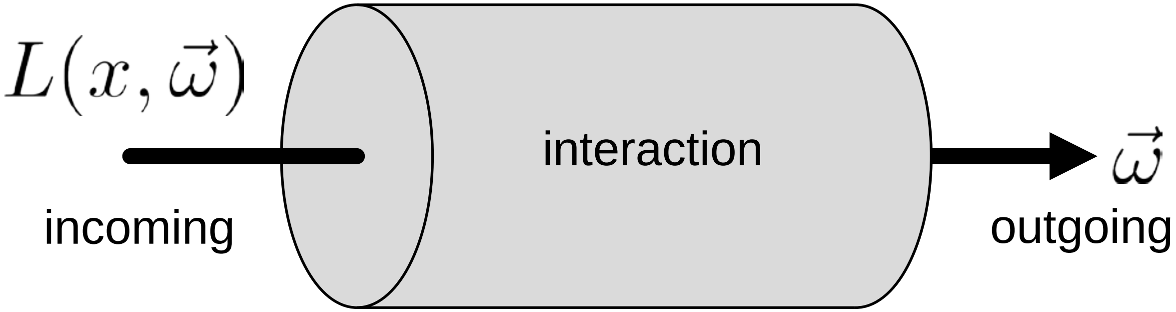

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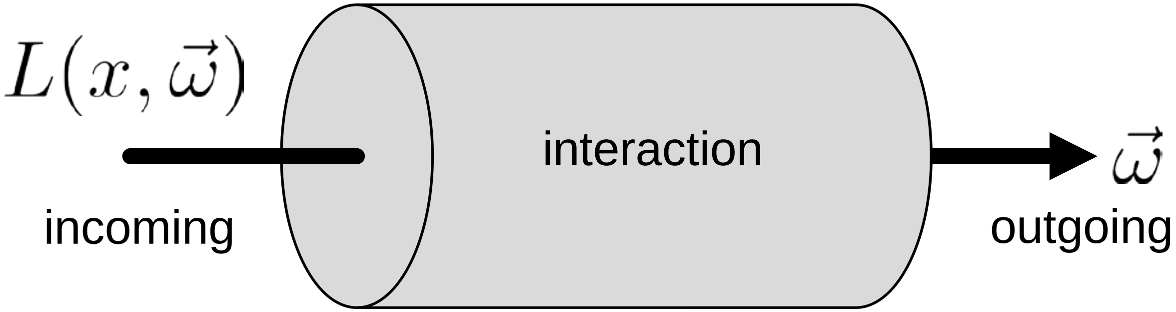
$$\vec{\omega} = (\omega_1, \omega_2, \omega_3) = (0, 1, 0) : \frac{dL(x, \vec{\omega})}{dx_2} = ?$$

Interaction Equation

Possible interactions:



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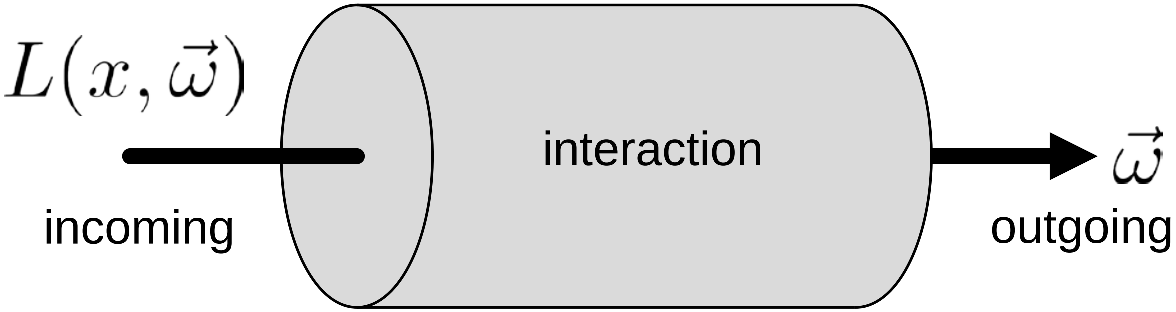
$$\vec{\omega} = (\omega_1, \omega_2, \omega_3) = (0, 0, 1) : \frac{dL(x, \vec{\omega})}{dx_3} = ?$$

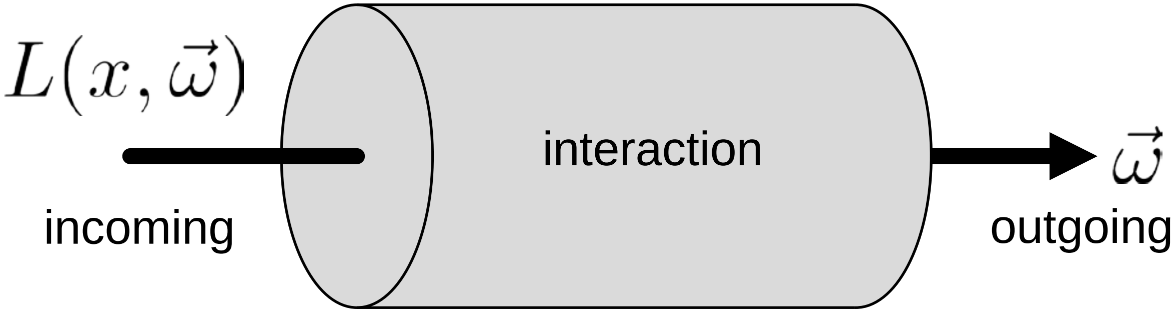
Interaction Equation

Possible interactions:



$$\left(\omega_1 \frac{dL(x, \vec{\omega})}{dx_1}, \omega_2 \frac{dL(x, \vec{\omega})}{dx_2}, \omega_3 \frac{dL(x, \vec{\omega})}{dx_3} \right) =?$$

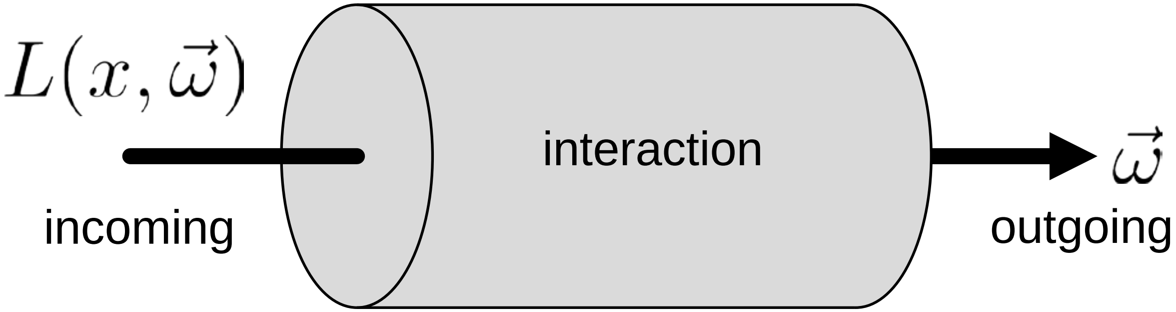
Possible interactions:



$$\left(\omega_1 \frac{d}{dx_1}, \omega_2 \frac{d}{dx_2}, \omega_3 \frac{d}{dx_3}\right) L(x, \vec{\omega}) = ?$$

Interaction Equation

Possible interactions:

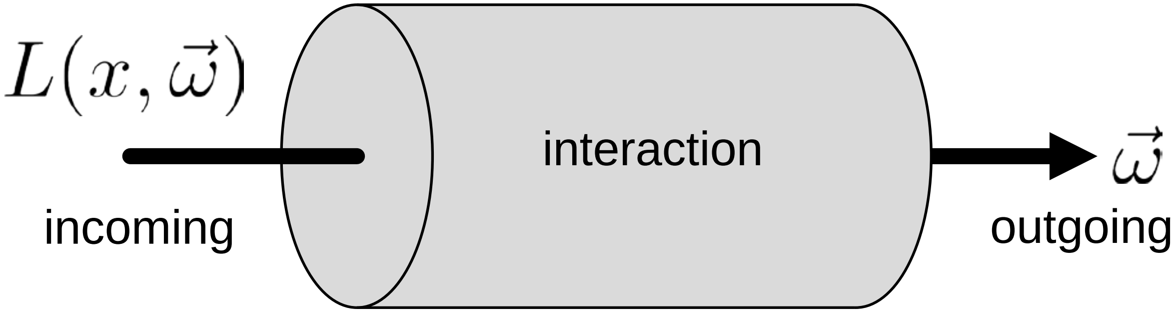


$$\left(\omega_1 \frac{d}{dx_1}, \omega_2 \frac{d}{dx_2}, \omega_3 \frac{d}{dx_3}\right) L(x, \vec{\omega}) = ?$$

$$\nabla = \left(\frac{d}{dx_1}, \frac{d}{dx_2}, \frac{d}{dx_3}\right)$$

Interaction Equation

Possible interactions:

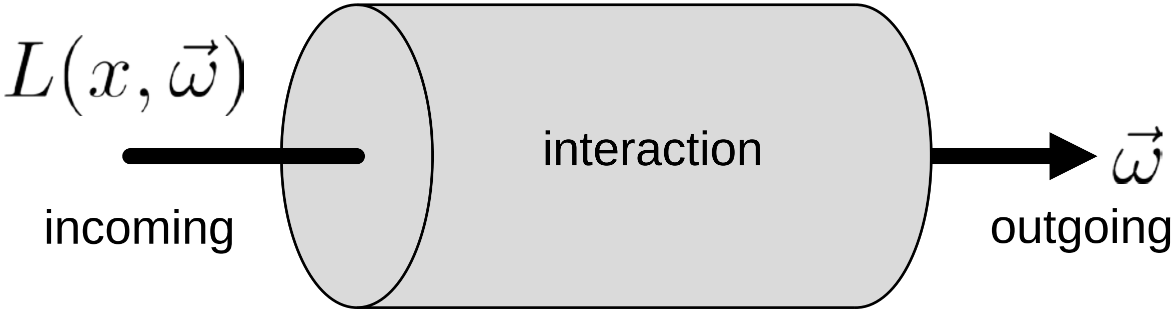


$$\left(\omega_1 \frac{d}{dx_1}, \omega_2 \frac{d}{dx_2}, \omega_3 \frac{d}{dx_3}\right) L(x, \vec{\omega}) = ?$$

$$\nabla = \left(\frac{d}{dx_1}, \frac{d}{dx_2}, \frac{d}{dx_3}\right)$$

$$\boxed{(\vec{\omega} \cdot \nabla) L(x, \vec{\omega}) = ?}$$

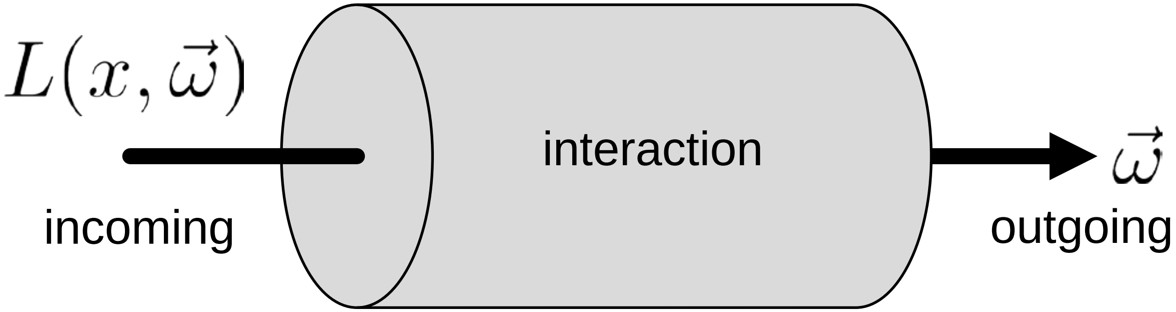
Possible interactions:



$$(\vec{\omega}\cdot\nabla)L(x,\vec{\omega}) = ?$$

Interaction Equation

Possible interactions:



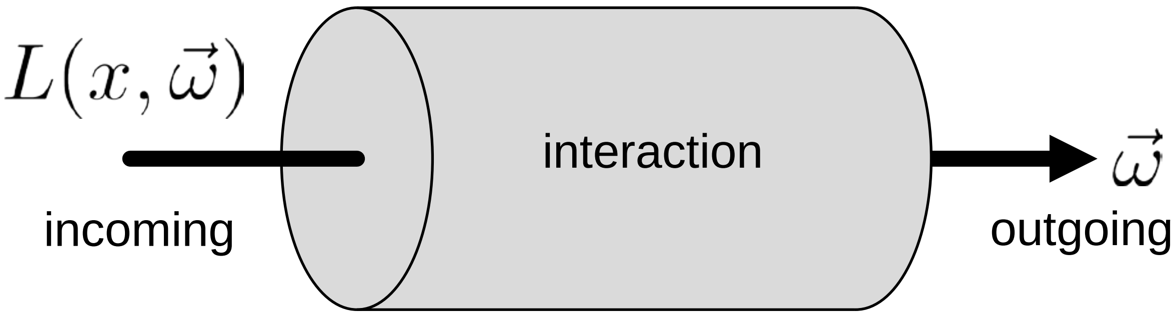
$$(\vec{\omega} \cdot \nabla) L(x, \vec{\omega}) = ?$$

- absorption

$$= -\sigma_a(x) L(x, \vec{\omega})$$

Interaction Equation

Possible interactions:

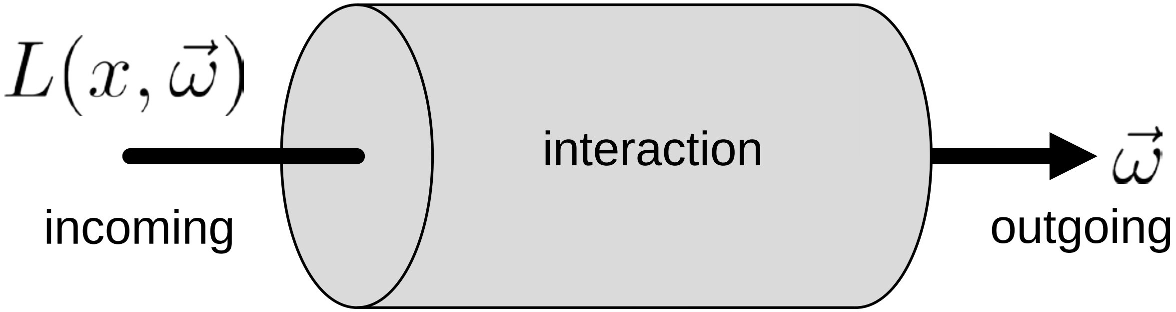


$$(\vec{\omega}\cdot\nabla)L(x,\vec{\omega}) = ?$$

- absorption $\quad = -\sigma_a(x)L(x,\vec{\omega})$
- emission $\quad = \varepsilon(x)$

Interaction Equation

Possible interactions:

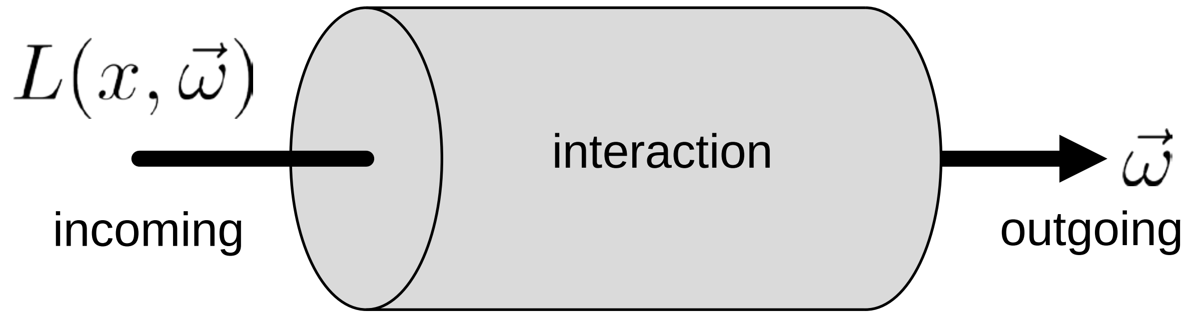


$$(\vec{\omega} \cdot \nabla) L(x, \vec{\omega}) = ?$$

- absorption $\quad = -\sigma_a(x) L(x, \vec{\omega})$
- emission $\quad = \varepsilon(x)$
- out-scattering $\quad = -\sigma_s(x) L(x, \vec{\omega})$

Interaction Equation

Possible interactions:



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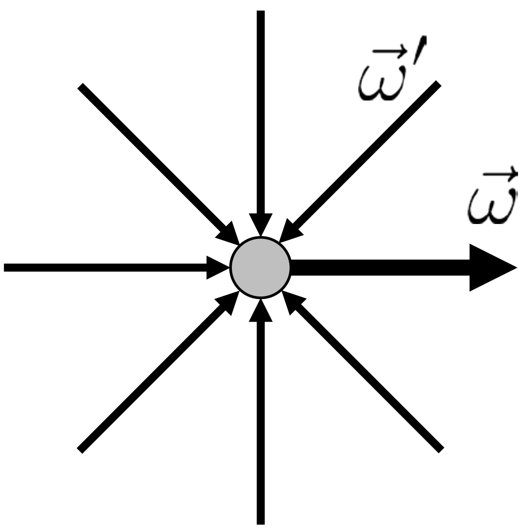
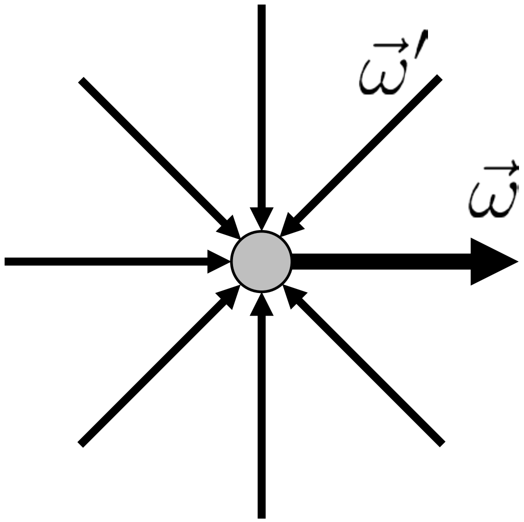


Phase Function

For incoming direction $\vec{\omega}'$ how much radiance is scattered into direction $\vec{\omega}$?



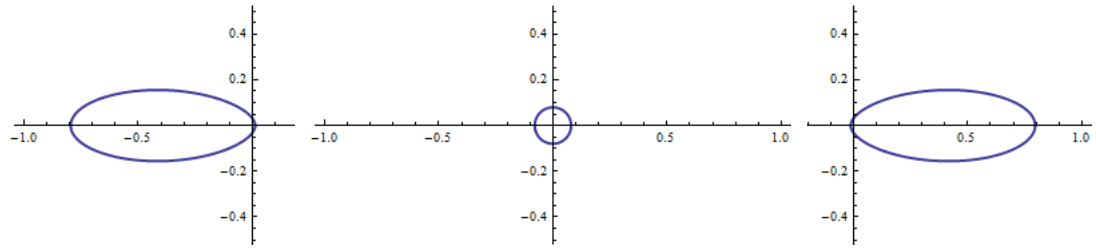
Phase function: $p(x, \vec{\omega}, \vec{\omega}')$

- Depends on the material
  - Size of particles
  - Geometry of particles
- Normalized, i.e., $\int_{4\pi} p(x, \vec{\omega}, \vec{\omega}')d\vec{\omega}' = 1$

Phase Function

- Henyey-Greenstein
  - Interstellar dust
  - Analytic
  - Anisotropy g

$$p(\theta) = \frac{1}{4\pi} \frac{1 - g^2}{\left(1 + g^2 - 2g\cos(\theta)\right)^{3/2}}$$



$g = -0.6$ $g = 0.0$ $g = 0.6$

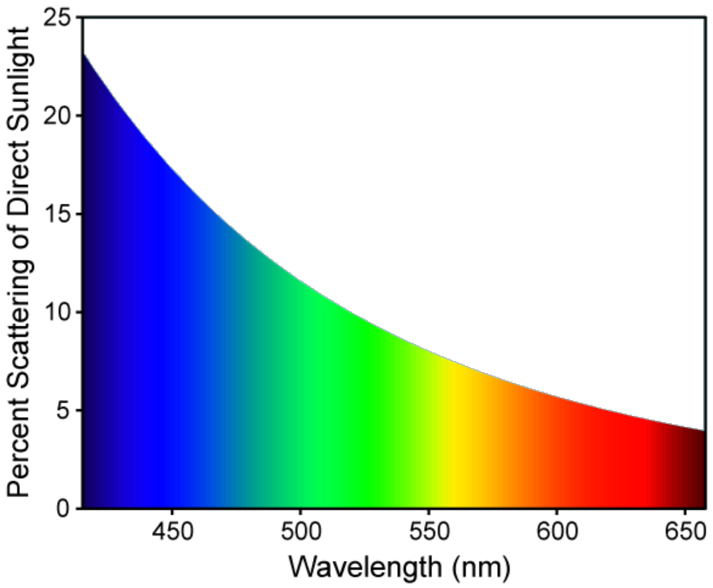
- Schlick Approxim.

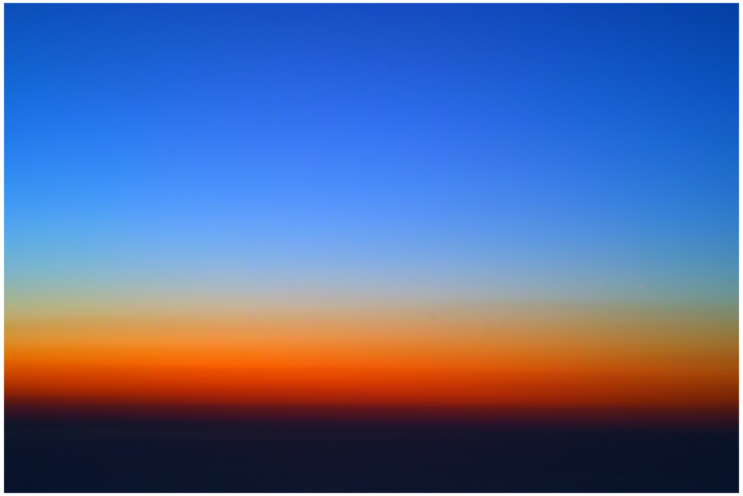
$$p(\theta) = \frac{1}{4\pi} \frac{1 - k^2}{\left(1 - k\cos(\theta)\right)^{2}}, k = 1.55g - 0.55g^3$$

- Lorenz-Mie Scattering
  - Spherically homogeneous particles
  - Full electrodynamic computation

Phase Function

- Rayleigh Scattering
  - Small particle approximation of Lorenz-Mie
  - Covers scattering by pure air
  - Depends on the light‘s wavelength





Interaction Equation

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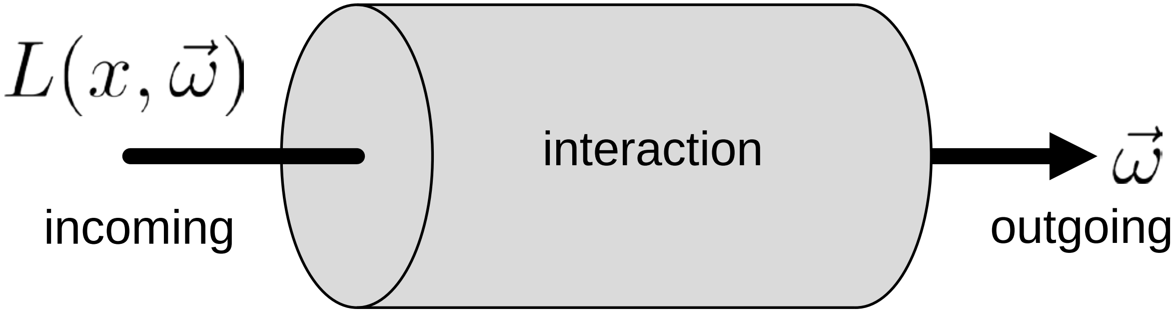
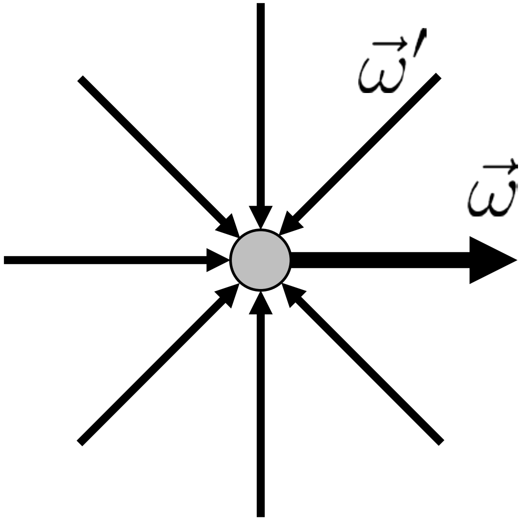


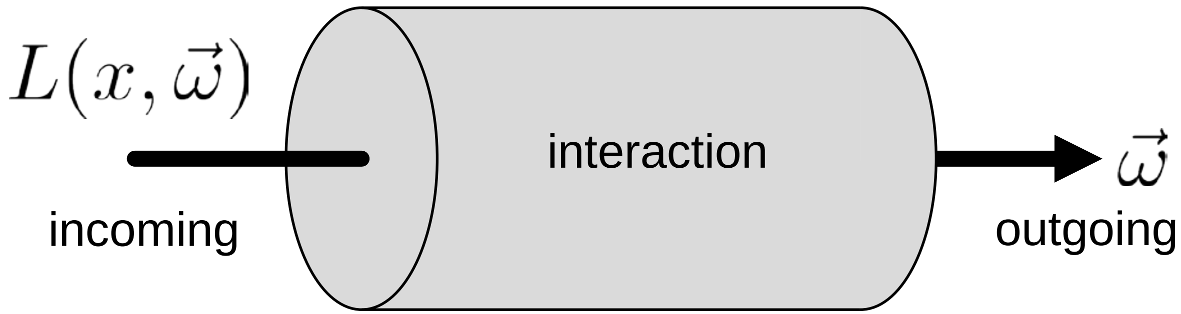
$$(\vec{\omega}\cdot\nabla)L(x,\vec{\omega}) = ?$$



Phase function: $p(x,\vec{\omega},\vec{\omega}')$

Interaction Equation

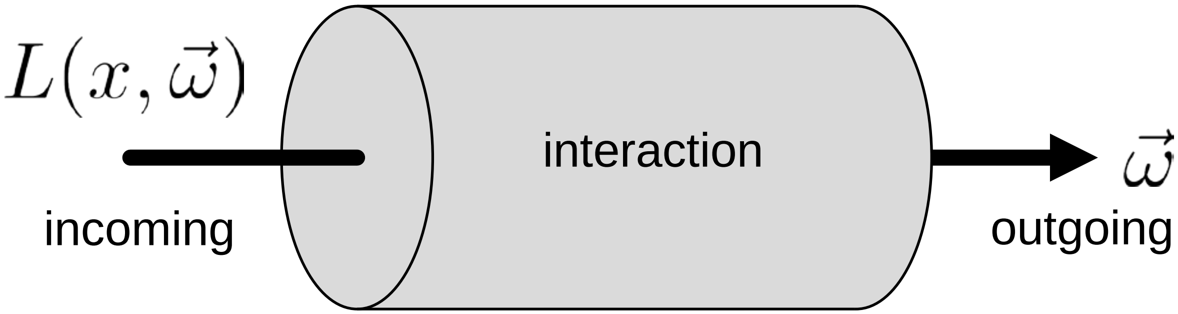
Possible interactions:



$$(\vec{\omega}\cdot\nabla)L(x,\vec{\omega}) = ?$$

- absorption $= -\sigma_a(x)L(x,\vec{\omega})$
- emission $= \varepsilon(x)$
- out-scattering $= -\sigma_s(x)L(x,\vec{\omega})$
- in-scattering $= \sigma_s(x)\int_{4\pi} p(x,\vec{\omega},\vec{\omega}')L(x,\vec{\omega}')d\vec{\omega}'$

Radiative Transfer Equation

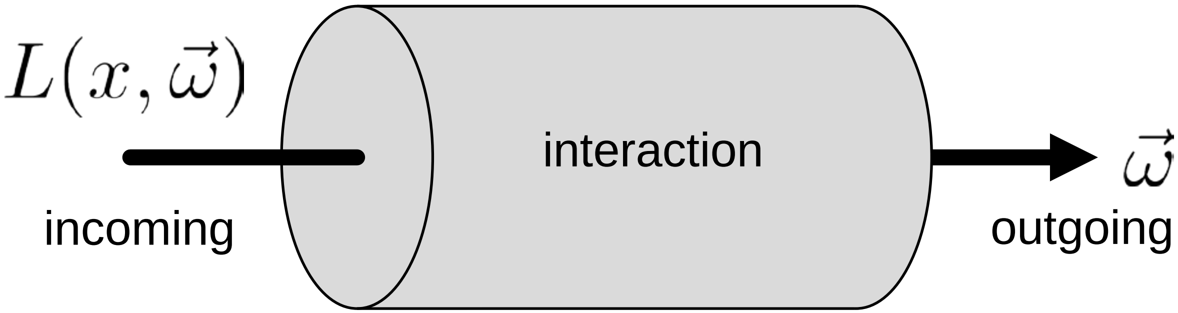


Also known as **Radiative Transport Equation**

$$(\vec{\omega} \cdot \nabla) L(x, \vec{\omega}) = - \underbrace{\sigma_a(x) L(x, \vec{\omega})}_{\text{absorption}} - \underbrace{\sigma_s(x) L(x, \vec{\omega})}_{\text{out-scattering}} + \underbrace{\varepsilon(x)}_{\text{emission}}$$

$$+ \underbrace{\sigma_s(x) \int_{4\pi} p(x, \vec{\omega}, \vec{\omega}') L(x, \vec{\omega}') d\vec{\omega}'}_{\text{in-scattering}}$$

Radiative Transfer Equation

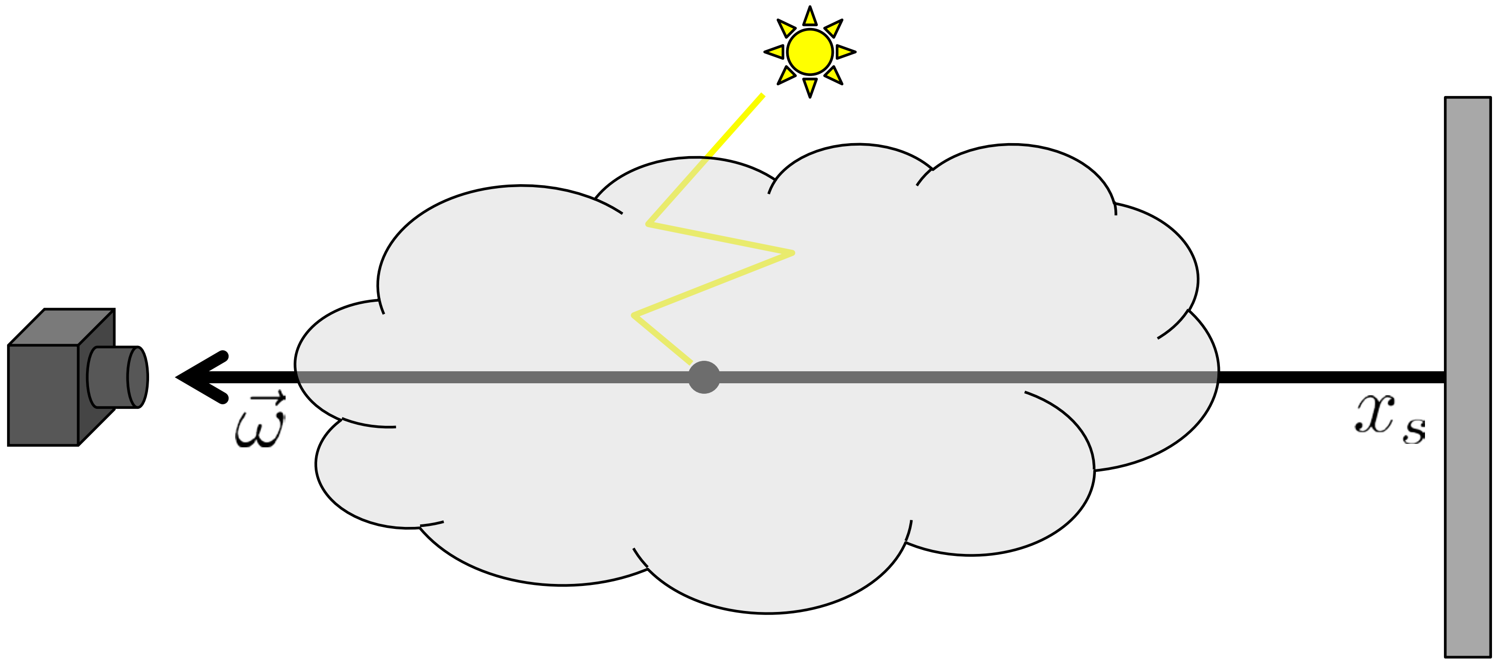


Also known as **Radiative Transport Equation**

$$(\vec{\omega} \cdot \nabla) L(x, \vec{\omega}) = -\underbrace{\sigma_t(x) L(x, \vec{\omega})}_{\text{extinction}} + \underbrace{\varepsilon(x)}_{\text{emission}}$$

$$+ \underbrace{\sigma_s(x) \int_{4\pi} p(x, \vec{\omega}, \vec{\omega}') L(x, \vec{\omega}') d\vec{\omega}'}_{\text{in-scattering}}$$

Volume Rendering Equation



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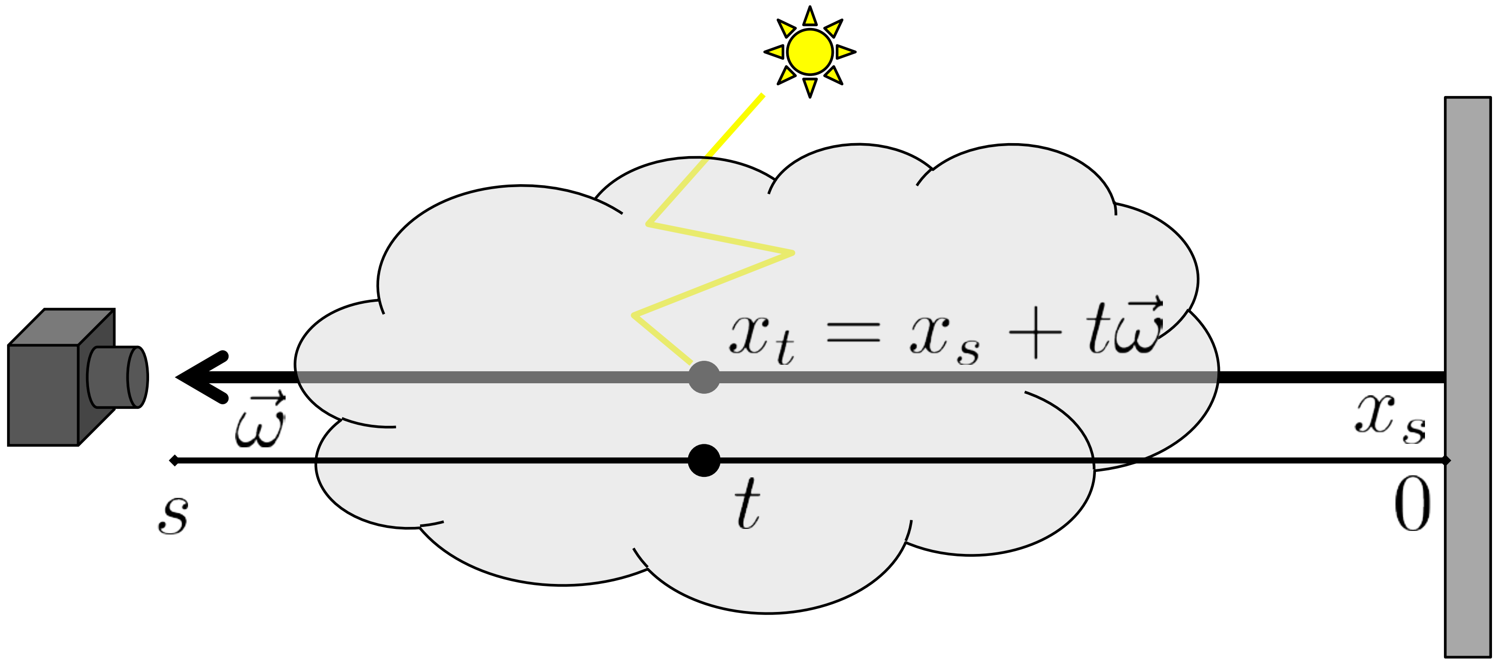

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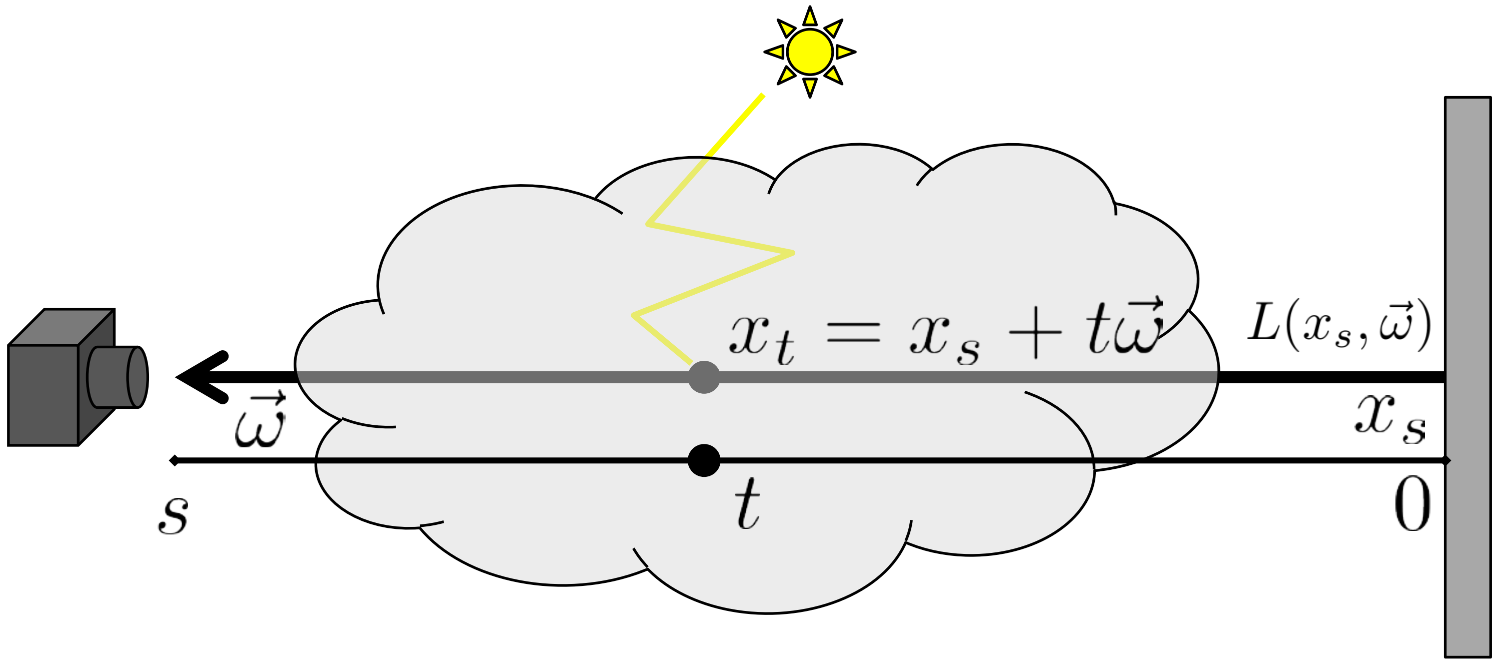

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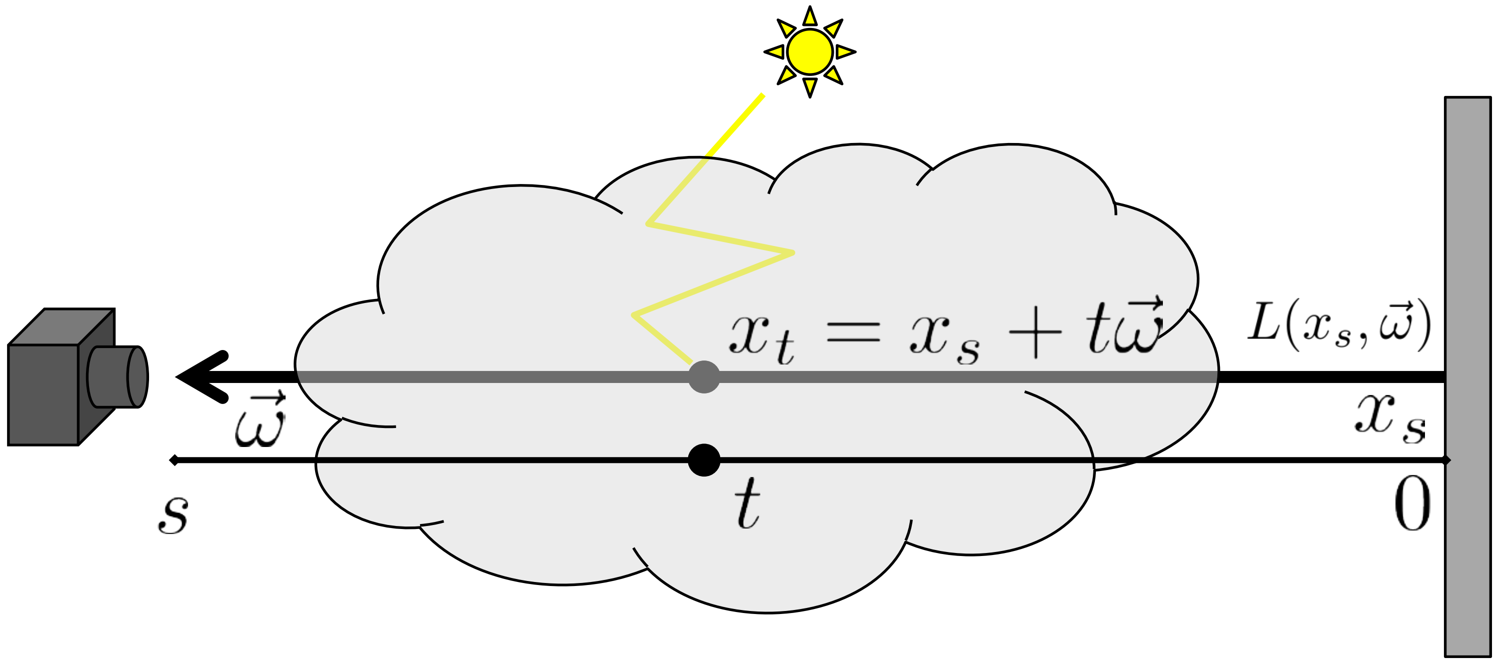
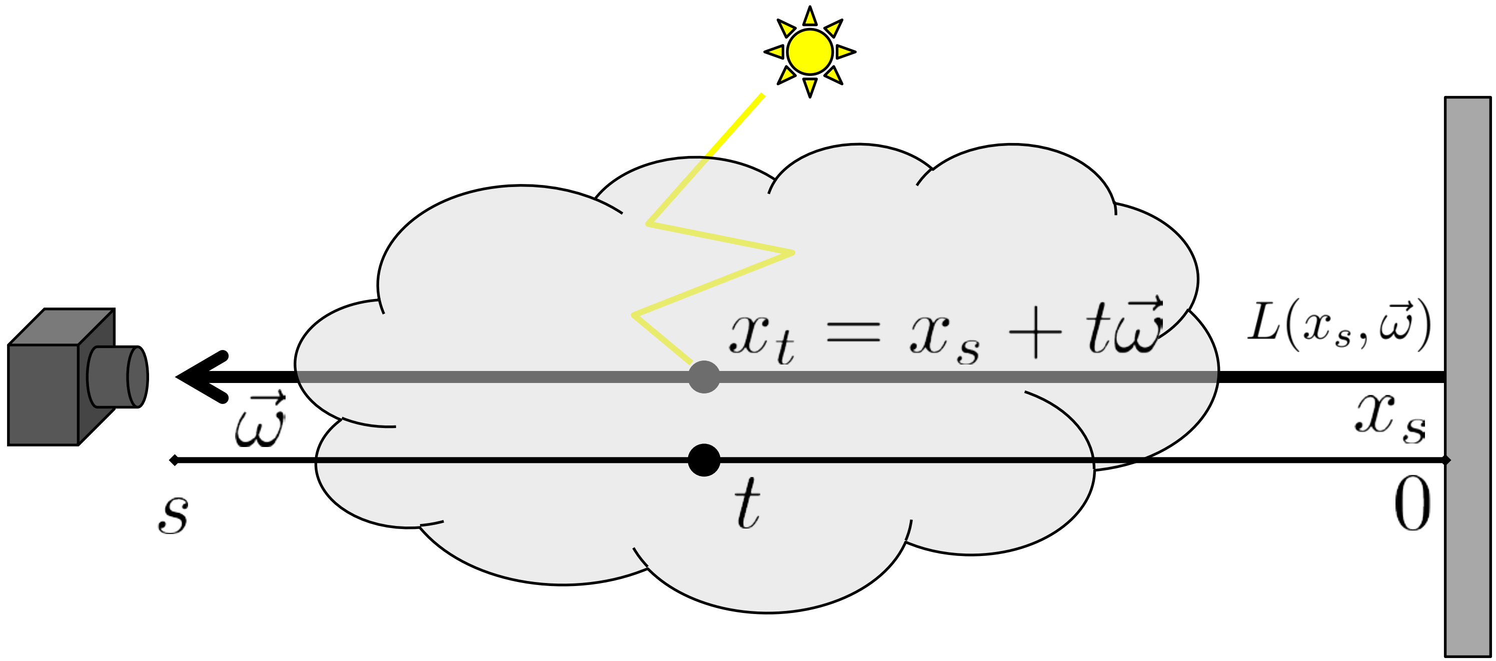


$$L(x, \vec{\omega}) = \int_0^s \underbrace{\underbrace{\qquad}_{\text{extinction}} \qquad\qquad\qquad}_{\text{in-scattering}} dt + \underbrace{\qquad}_{\text{extinction}} L(x_s, \vec{\omega})$$

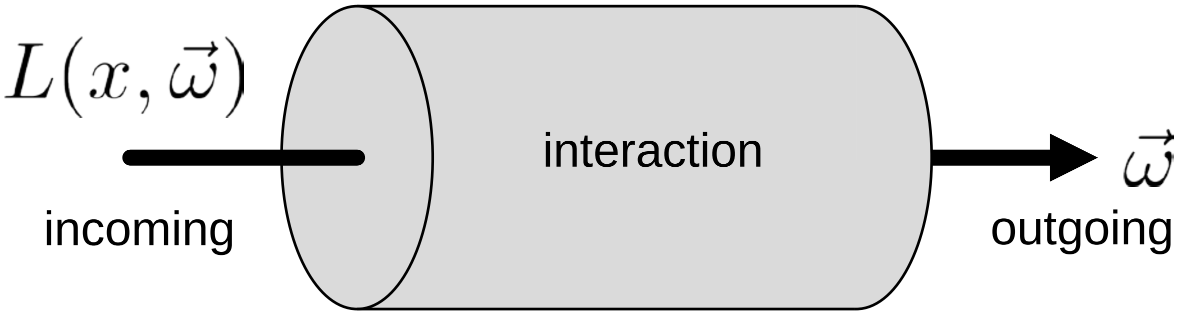
Volume Rendering Equation



$$L(x, \vec{\omega}) = \int_0^s \underbrace{T_r(x, x_t)}_{\text{extinction}} \quad dt + \underbrace{T_r(x, x_s)}_{\text{extinction}} L(x_s, \vec{\omega})$$

in-scattering

Radiative Transfer Equation

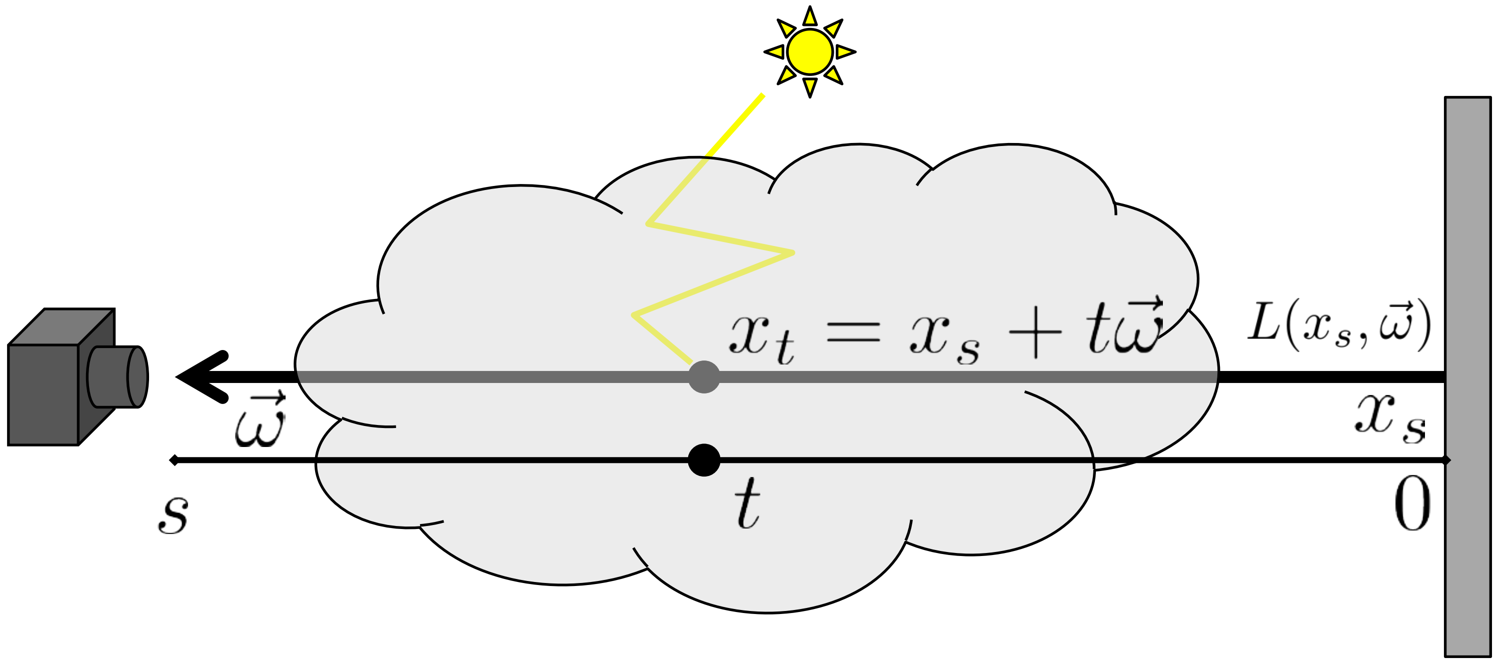


Also known as **Radiative Transport Equation**

$$(\vec{\omega} \cdot \nabla) L(x, \vec{\omega}) = -\underbrace{\sigma_t(x) L(x, \vec{\omega})}_{\text{extinction}} + \underbrace{\varepsilon(x)}_{\text{emission}}$$

$$+ \underbrace{\sigma_s(x) \int_{4\pi} p(x, \vec{\omega}, \vec{\omega}') L(x, \vec{\omega}') d\vec{\omega}'}_{\text{in-scattering}}$$

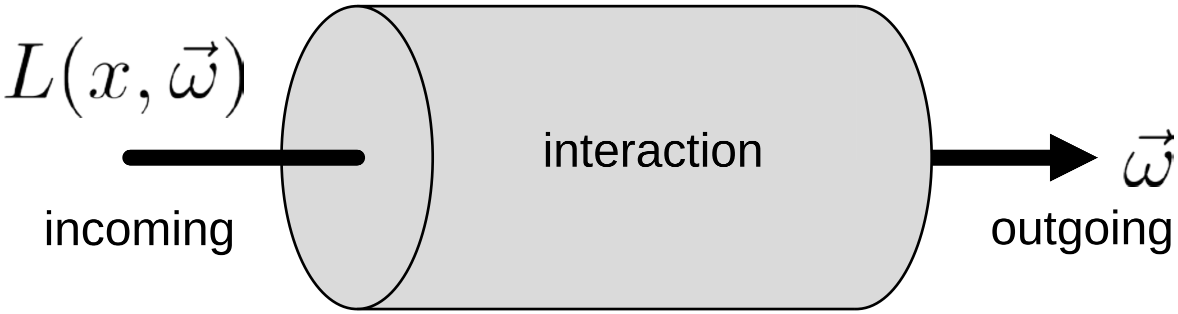
Volume Rendering Equation



$$L(x, \vec{\omega}) = \int_0^s \underbrace{T_r(x, x_t)}_{\text{extinction}} \underbrace{\qquad\qquad\qquad\qquad\qquad\qquad}_{\text{in-scattering}} dt + \underbrace{T_r(x, x_s)}_{\text{extinction}} L(x_s, \vec{\omega})$$

$$T_r(x, x_s) = \exp\left(- \int_0^s \sigma_t(x + t\omega) dt \right) \qquad \text{(solution of } y'(x) = -f(x)y(x)\text{)}$$

Radiative Transfer Equation

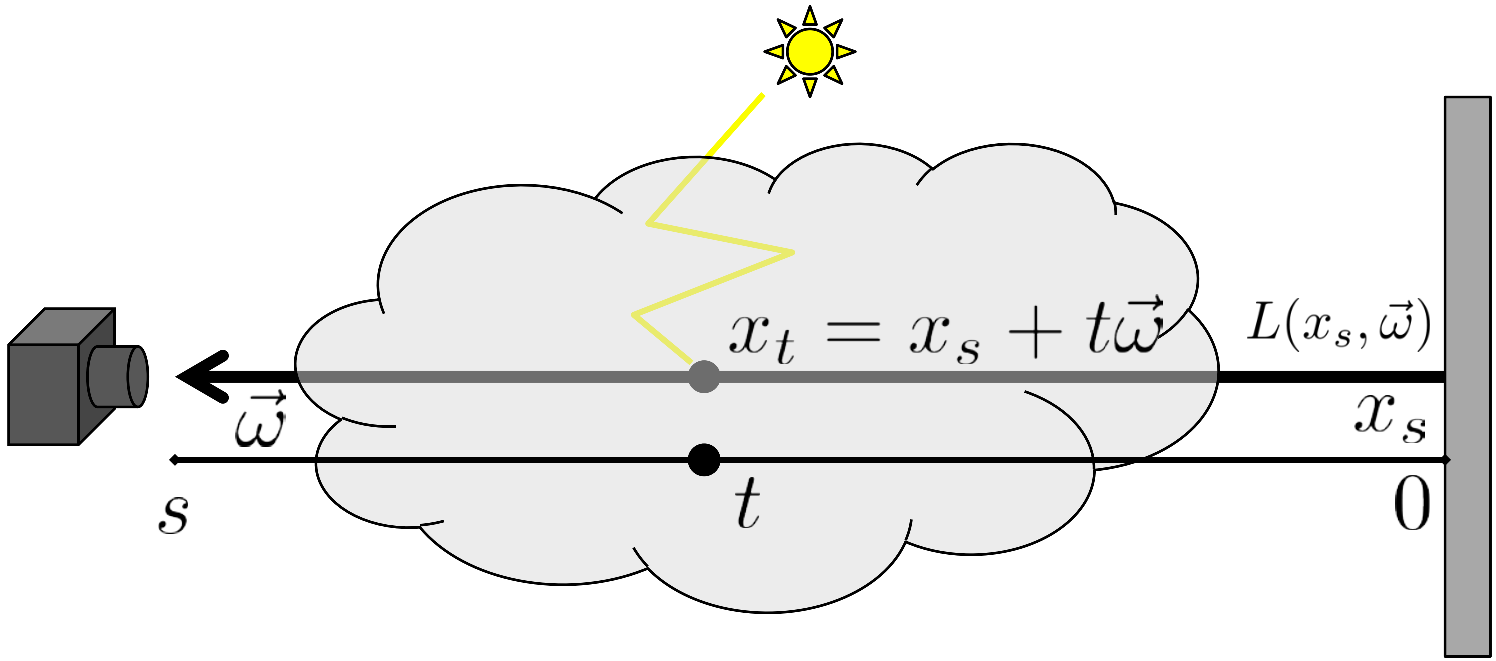


Also known as **Radiative Transport Equation**

$$(\vec{\omega}\cdot\nabla)L(x,\vec{\omega}) = -\underbrace{\sigma_t(x)L(x,\vec{\omega})}_{\text{extinction}} + \underbrace{\varepsilon(x)}_{\text{emission}}$$

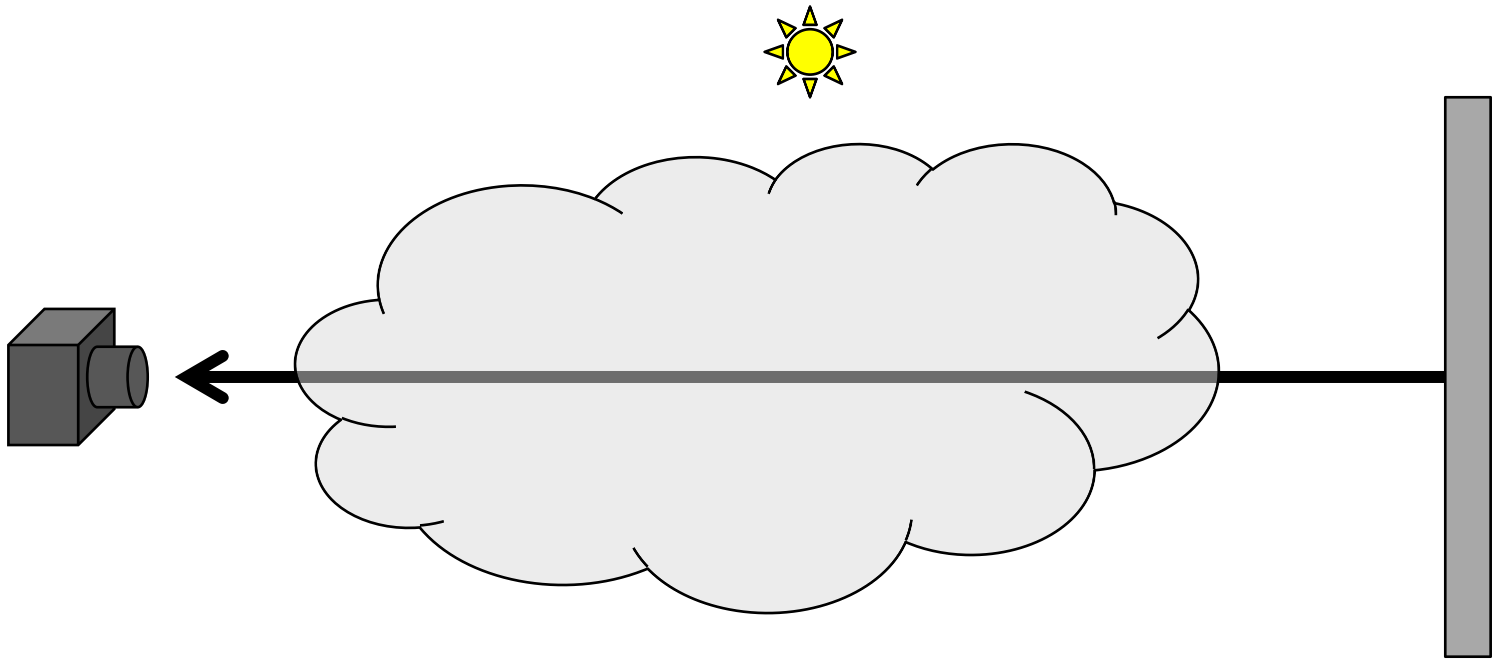
$$\underbrace{+\,\sigma_s(x)\int_{4\pi} p(x,\vec{\omega},\vec{\omega}')L(x,\vec{\omega}')d\vec{\omega}'}_{\text{in-scattering}}$$

Volume Rendering Equation



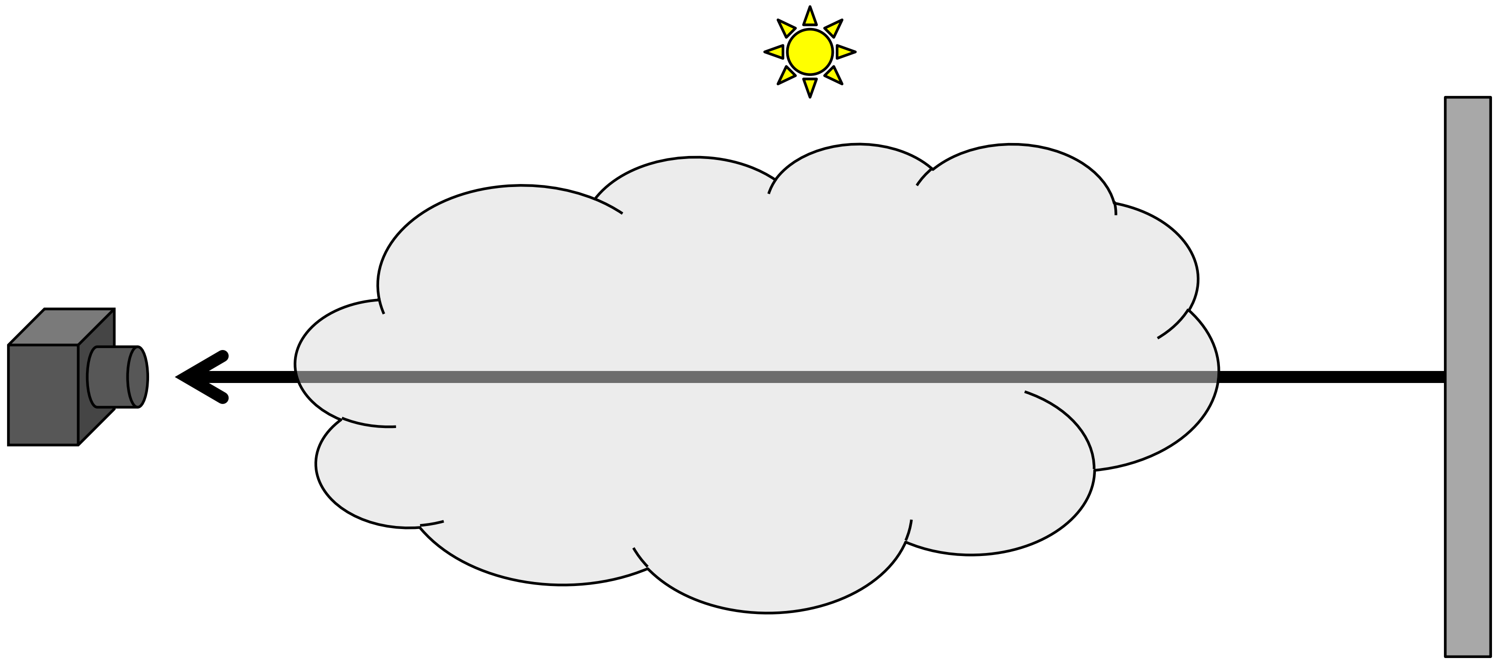
$$L(x,\vec{\omega}) = \int_0^s \underbrace{T_r(x,x_t)}_{\text{extinction}} \underbrace{\sigma_s(x_t) \int_{4\pi} p(x,\vec{\omega},\vec{\omega}') L(x_t,\vec{\omega}') d\vec{\omega}'}_{\text{in-scattering}} \, dt + \underbrace{T_r(x,x_s)}_{\text{extinction}} L(x_s,\vec{\omega})$$

Volumetric Path Tracing



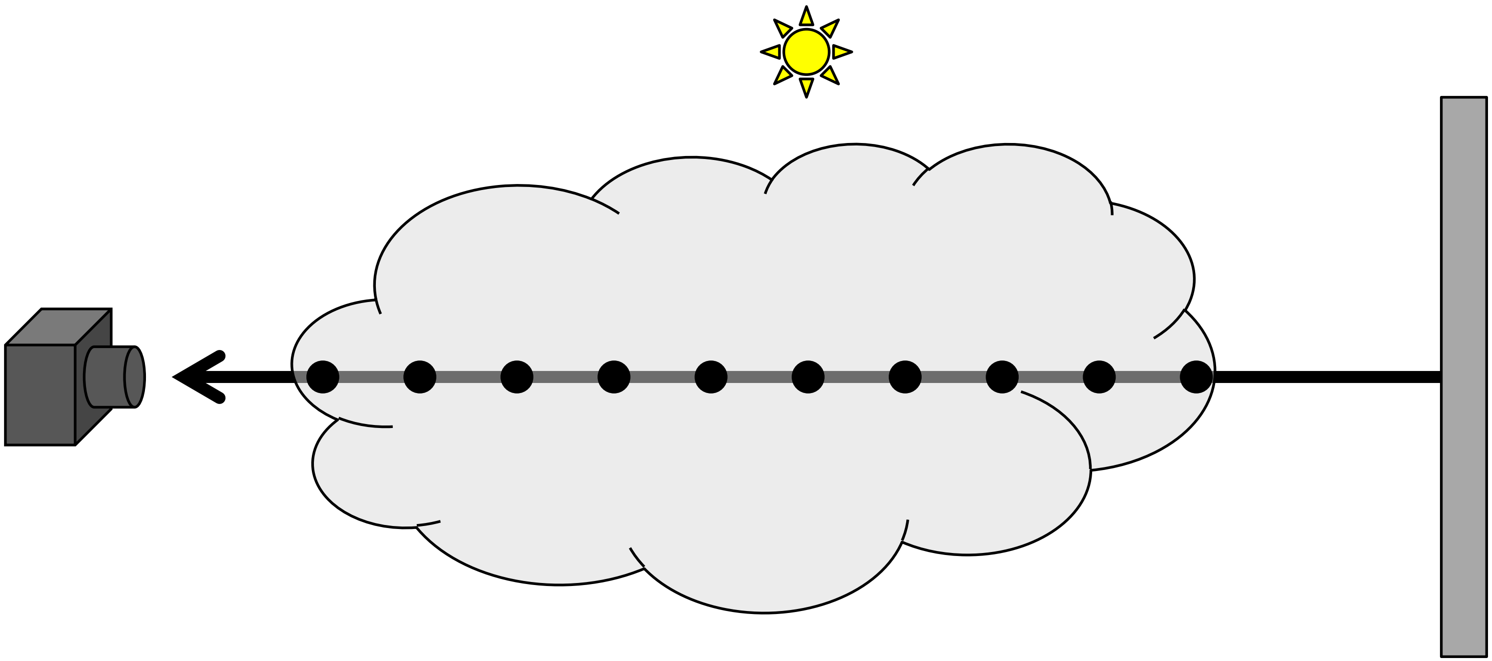
$$L(x,\vec{\omega}) = \int_0^s \underbrace{T_r(x,x_t)}_{\text{extinction}} \underbrace{\sigma_s(x_t) \int_{4\pi} p(x,\vec{\omega},\vec{\omega}')L(x_t,\vec{\omega}')d\vec{\omega}'}_{\text{in-scattering}}\, dt + \underbrace{T_r(x,x_s)}_{\text{extinction}} L(x_s,\vec{\omega})$$

Volumetric Path Tracing



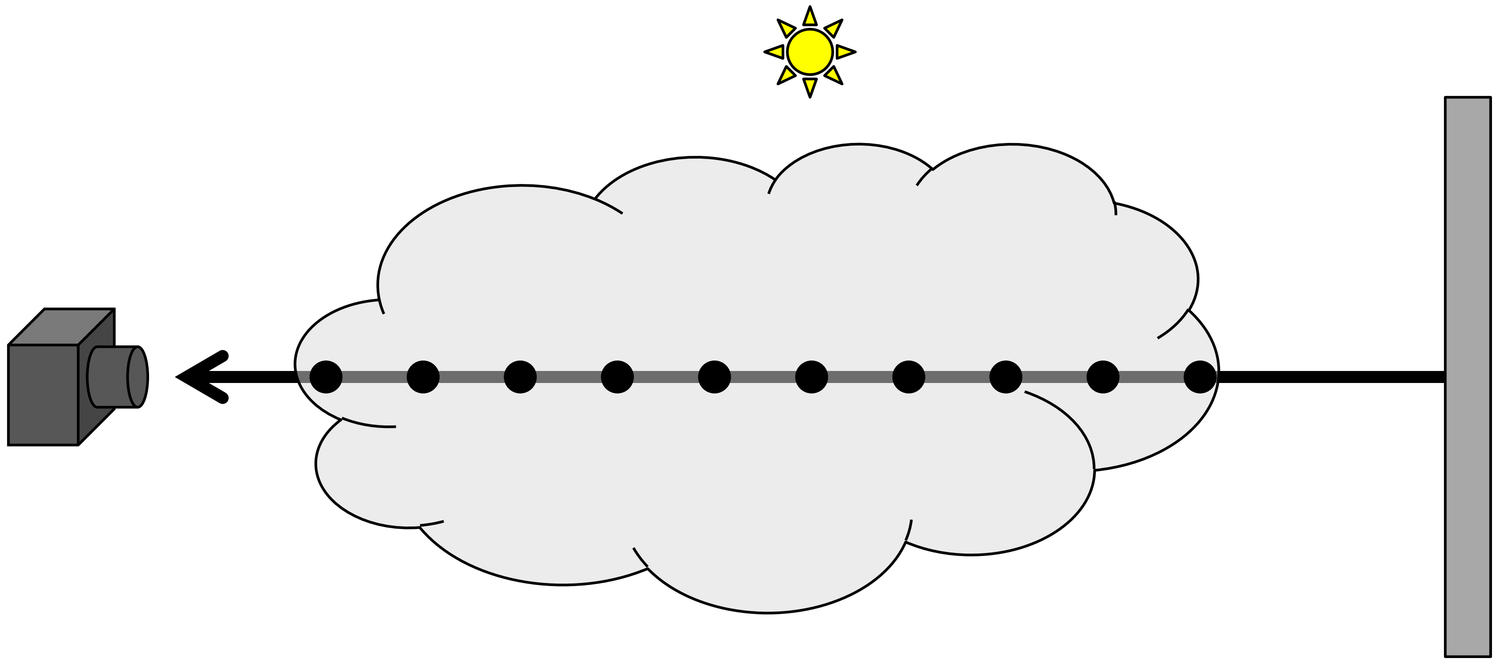
$$L(x,\vec{\omega}) = \int_0^s \underbrace{T_r(x,x_t)}_{\text{extinction}} \underbrace{\sigma_s(x_t)\int_{4\pi} p(x,\vec{\omega},\vec{\omega}')L(x_t,\vec{\omega}')d\vec{\omega}'}_{\text{in-scattering}}\, dt + \underbrace{T_r(x,x_s)}_{\text{extinction}} L(x_s,\vec{\omega})$$

Volumetric Path Tracing



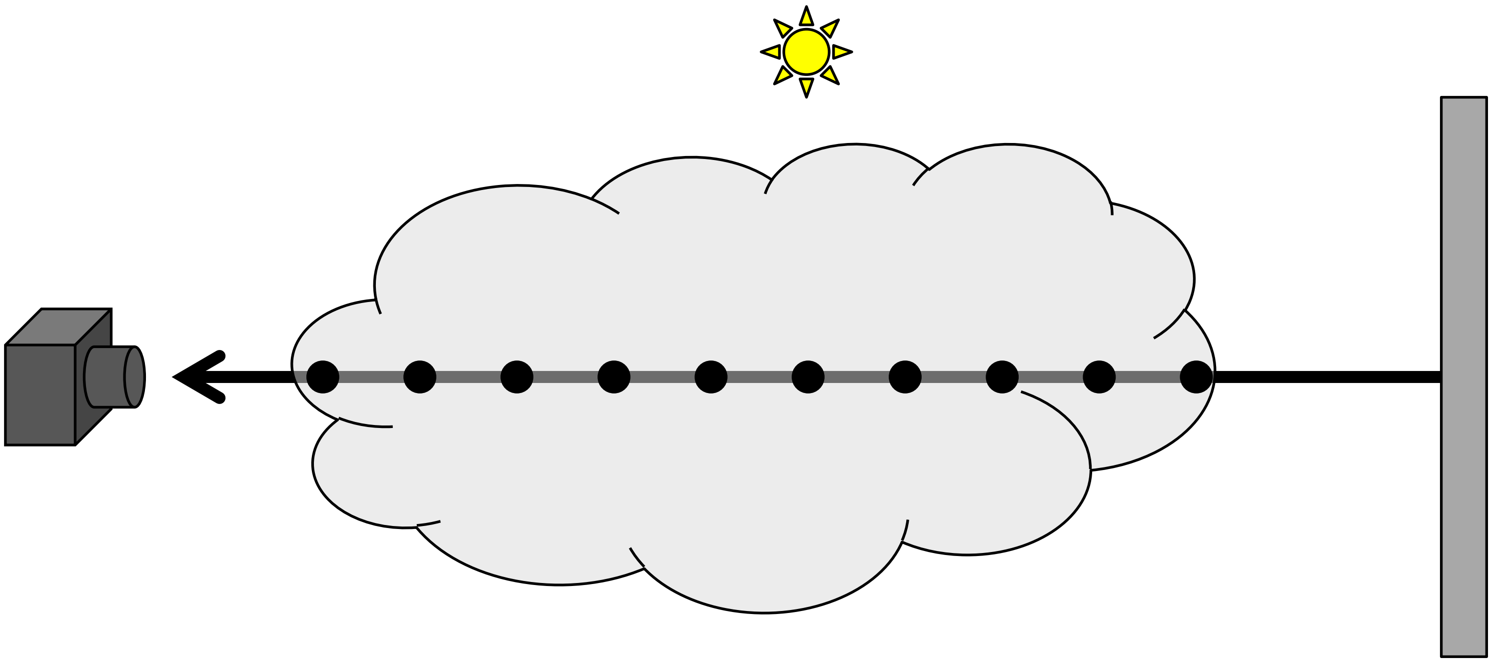
$$L(x,\vec{\omega}) = \sum_{t=0}^{S-1} \underbrace{T_r(x,x_t)}_{\text{extinction}} \underbrace{\sigma_s(x_t)\int_{4\pi} p(x,\vec{\omega},\vec{\omega}')L(x_t,\vec{\omega}')d\vec{\omega}'}_{\text{in-scattering}} \Delta_t + \underbrace{T_r(x,x_s)}_{\text{extinction}} L(x_s,\vec{\omega})$$

Volumetric Path Tracing



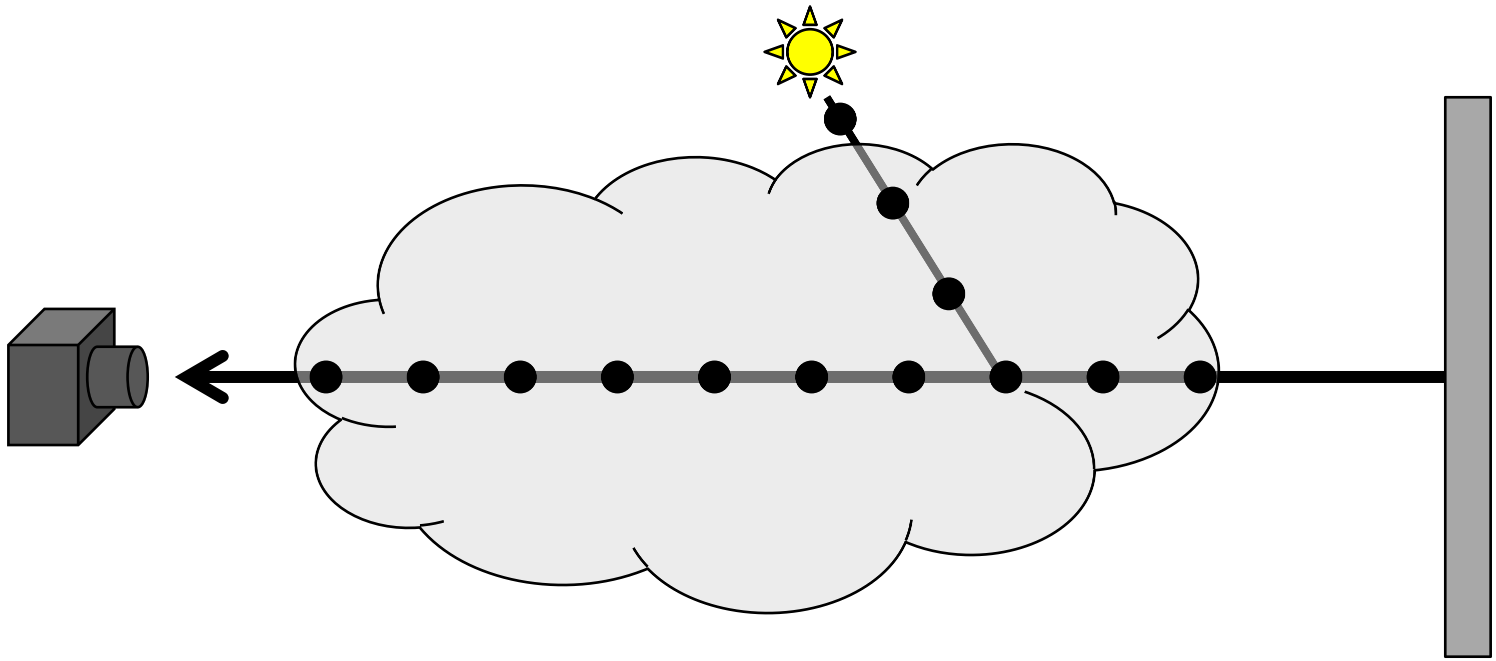
$$L(x,\vec{\omega}) = \sum_{t=0}^{S-1} \underbrace{T_r(x,x_t)}_{\text{extinction}} \underbrace{\sigma_s(x_t) \int_{4\pi} p(x,\vec{\omega},\vec{\omega}')L(x_t,\vec{\omega}')d\vec{\omega}'\,\Delta_t}_{\text{in-scattering}} + \underbrace{T_r(x,x_s)}_{\text{extinction}} L(x_s,\vec{\omega})$$

$$T_r(x_1,x_3) = T_r(x_1,x_2)T_r(x_2,x_3)$$ compute incremetally



$$L(x,\vec{\omega}) = \sum_{t=0}^{S-1} \underbrace{T_r(x,x_t)}_{\text{extinction}} \underbrace{\sigma_s(x_t) \int_{4\pi} p(x,\vec{\omega},\vec{\omega}')L(x_t,\vec{\omega}')d\vec{\omega}'}_{\text{in-scattering}} \Delta_t + \underbrace{T_r(x,x_s)}_{\text{extinction}} L(x_s,\vec{\omega})$$

Volumetric Path Tracing

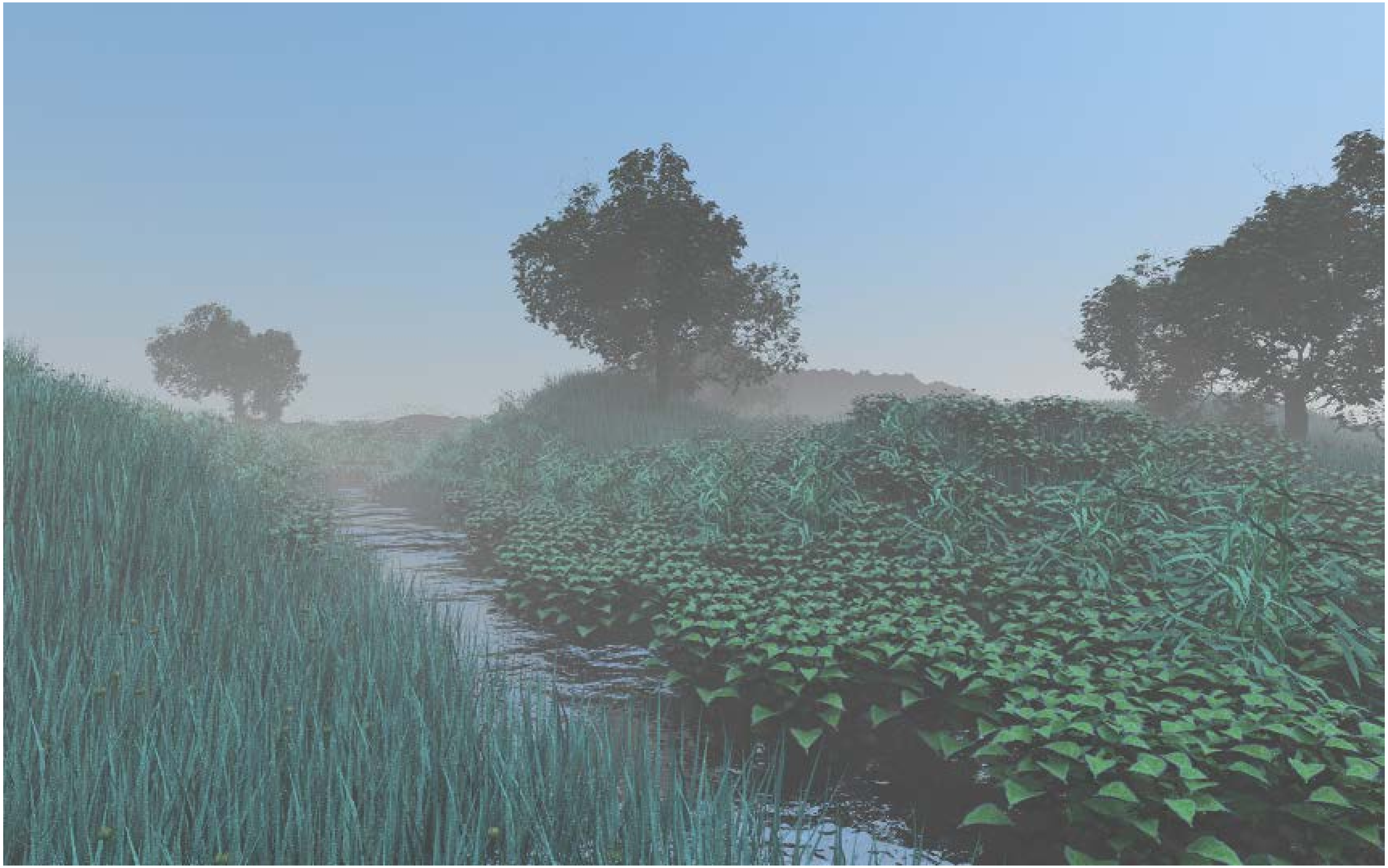


$$L(x,\vec{\omega}) = \sum_{t=0}^{S-1} \underbrace{T_r(x,x_t)}_{\text{extinction}} \underbrace{\sigma_s(x_t) \int_{4\pi} p(x,\vec{\omega},\vec{\omega}')L(x_t,\vec{\omega}')d\vec{\omega}'}_{\text{in-scattering}} \Delta_t + \underbrace{T_r(x,x_s)}_{\text{extinction}} L(x_s,\vec{\omega})$$

Single scattering: compute $T_r(x_L, x_t)$ to light source

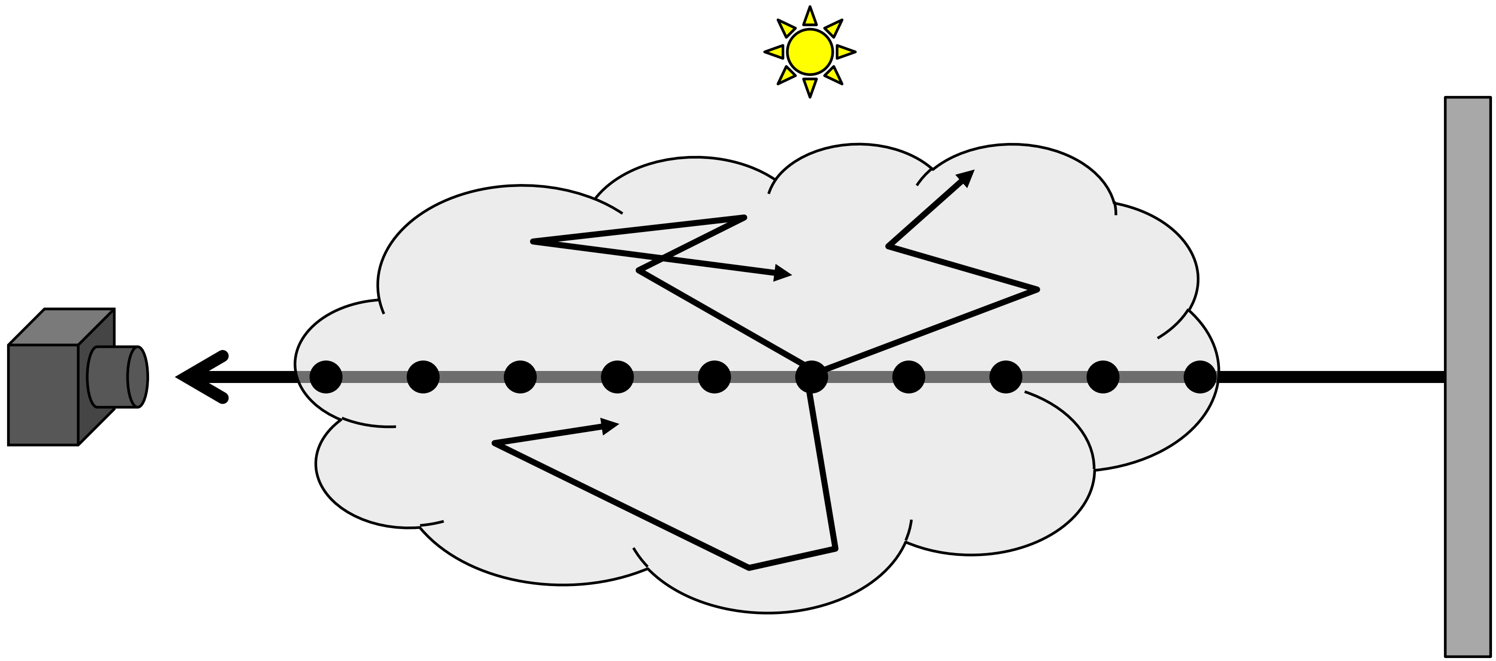
Conventional Rendering

Exponential Fog



Single Scattering

Volumetric Path Tracing



$$L(x,\vec{\omega}) = \sum_{t=0}^{S-1} \underbrace{T_r(x,x_t)}_{\text{extinction}} \underbrace{\sigma_s(x_t) \int_{4\pi} p(x,\vec{\omega},\vec{\omega}')L(x_t,\vec{\omega}')d\vec{\omega}'}_{\text{in-scattering}} \Delta_t + \underbrace{T_r(x,x_s)}_{\text{extinction}} L(x_s,\vec{\omega})$$

Multiple scattering: compute random walk

Volumetric Path Tracing

- Sample phase function $p(x, \vec{\omega}, \vec{\omega}')$

  e.g. Henyey-Greenstein $p(\theta) = \frac{1}{4\pi} \frac{1 - g^2}{(1 + g^2 - 2g\cos\theta)^{3/2}}$

  by inversion $\cos\theta = \frac{1}{2g}\left(1 + g^2 - \left(\frac{1 - g^2}{1 - g + 2g\xi}\right)^2\right)$

- For a given direction, choose a distance d to travel based on $T_r(\, , \, , \,)$
  - If d is closer than the nearest surface → scatter
  - If not, compute surface radiance

Volumetric Path Tracing

- Distance d is given by the free-flight distance

  Sample with $d = \frac{-\ln(1-\xi)}{\sigma_t}$ (homogeneous media)

Volumetric Path Tracing - Code

```
color VPT(o,ω)
      s = nearestSurfaceDist(o,ω)
      d = -ln(1 – random()) / σt
      if (d<s)
            // Media scattering
            o +=  d*ω
            return σs / σt * VPT(o, samplePhase())
      else
            // Surface scattering
            o += s*w
            (ωi, pdfi) = sampleBRDF(o,ω)
            return BRDF(o,ω,ωi) * VPT(o,ωi) / pdfi
```

Questions?