VU Rendering SS 2013

186.101

Thomas Auzinger
Károly Zsolnai

Assignment 1

Results as email to both lecturers using the
[Rendering_SS2013_1]_0123456,Thomas Auzinger
format (with your matriculation number and name) in both the email's subject and attached archive file (e.g. .zip, .rar, .7z, ...).

Part 1 : Ray Tracing

The problems of part 1 can be found at http://cg.tuwien.ac.at/~zsolnai/rendering/02_Rendering _Zsolnai _Ray _Tracing.ppt.

Part 2: Rendering Basics

- **Problem 1**

Why are light transport simulations not based on radiant energy or radiant flux as basic measurement units? Why are irradiance/exitance/radiosity inadequate to build a simulation program upon them?
Optional question: Is there a special case where irradiance/exitance/radiosity is sufficient?

- **Problem 2**

Draw a simple test scene and run the ray tracing algorithm by hand until two bounces. The following characteristics should be contained in your drawing:

- The location of the viewer and an associated perspective camera plane.
- Reflection and refraction directions on suitable objects.
- Diffuse, specular and ambient shading.
- At least one shadow ray computation.
- What happens to V on the first bounce.

Note that for this exercise you do not have to perform the actual computations or draw the shading itself. Symbolic expressions that capture the relevant formulas (e.g. writing $L \cdot N$ near an intersection) are sufficient. The final layout should contain the essential information on the characteristics above and should be clear and understandable.

- **Problem 3**

Plot $(R \cdot V)^n$ as a function of $n \in \mathbb{R}_+$. What changes as n is increased? What is the intuitive meaning of this?
Hint: Choose fixed representative values for $R \cdot V$ to show the dependency on n.

- **Problem 4**

Write down the rendering equation. Sketch a technical drawing to demonstrate its meaning and give an intuitive explanation for every term in it.

Part 3: Caustics

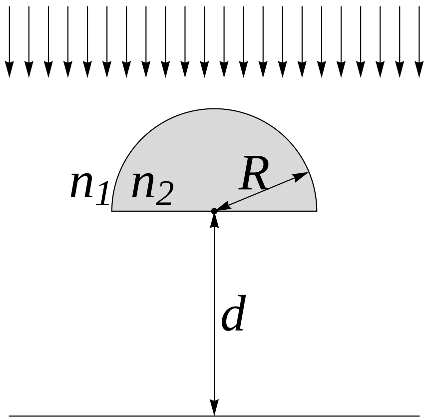
The aim of this assignment is to compute the caustic that is generated by the refraction of light by a simple geometric object. Apart from basic geometric reasoning, the knowledge of Snell's Law and the Fresnel Equations is needed.

- **Problem 1**

Sketch an air-material interface. The material can be chosen arbitrarily with an index of refraction of at least 1.2 (e.g. water, glass, diamond, ...). What is the critical angle of this interface?
How does the critical angle change as the refractive index of the material is increased? List your expectations first and give an intuitive explanation of the phenomenon.

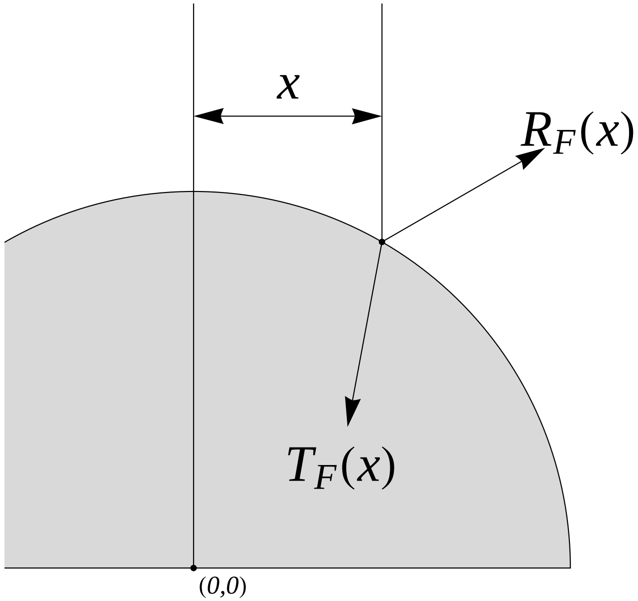
Half Sphere

In the following exercise, the refracting object is a half sphere with radius R made out of an homogeneous material with refractive index n_2 as shown below. The directional light source is placed above the object, i.e. the corresponding point source is at $(0, \infty)$. The surrounding space is assumed to have refractive index n_1. To observe the optical effects, a horizontal image plane is placed at a distance d from the object. Since this setup is rotationally symmetric around a ray through the center of the sphere, all subsequent problems are stated in 2D.



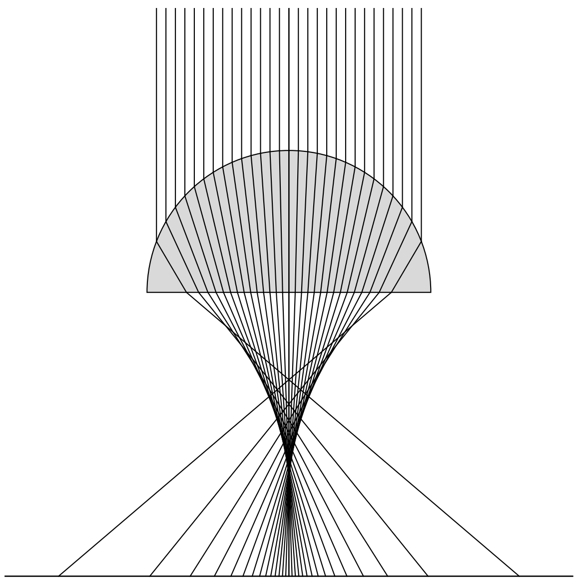
- **Problem 2**

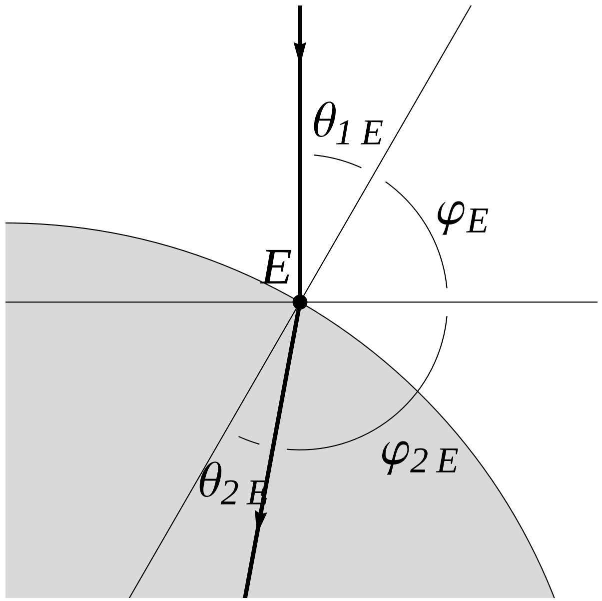
For the rays that hit the object, compute the Fresnel reflection coefficient $R_F(x)$ and the transmission coefficient $T_F(x)$ as a function of the rays' x-coordinate x as shown in the figure below.
Assume unpolarized light and plot the coefficients over x.



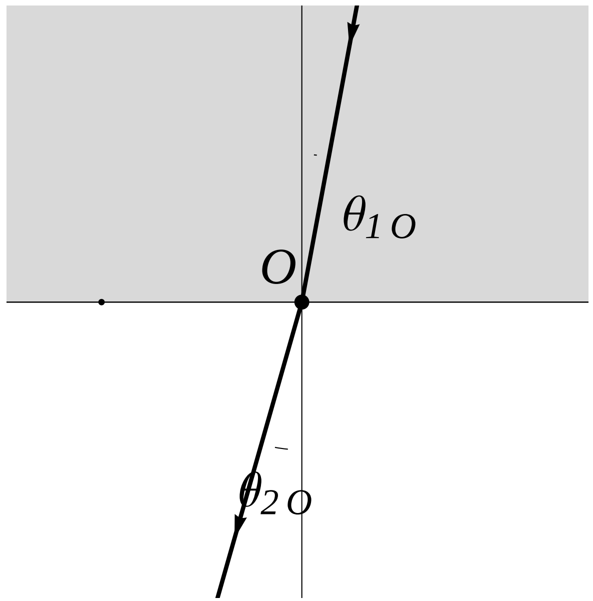
- **Problem 3**

Compute the path of the rays when traveling trough the object and their displacement F on the image plane. As a first step, compute the path for 3 distinct rays given by their initial x-coordinate $x = 0, \frac{1}{2}, \frac{1}{\sqrt{2}}$ where we assume the surrounding medium to be air ($n_1 = 1$) and the half sphere to be made out of glass ($n_2 = 1.5$). As an additional **optional** exercise, compute the displacement $F(x)$ as a function of the rays initial x-coordinate x. The figures below should serves as both a sketch of the geometric relations and as a suggestion on how to name the intermediate variables.
Note that we only look at the special case of one refraction into the object and one refraction out of the object; multiple light bounces do not have to be considered.

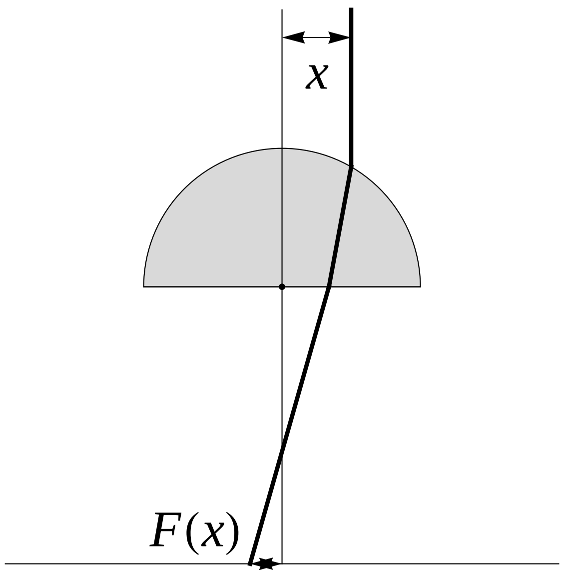




$\theta_{1\,E}$
φ_E
E
$\theta_{2\,E}$
$\varphi_{2\,E}$



$\theta_{1\,O}$
O
$\theta_{2\,O}$



x
$F(x)$

Hints:

- This problem is symmetric; only positive values of x have to considered, i.e. $F(-x) = -F(x)$.
- Not all rays hit the object.
- If the surrounding is optically denser than the object ($n_1 > n_2$), then not all rays are refracted due to total internal reflection. In this case it is also possible that refracted rays intersect the curved upper part of the object a second time; this case can be ignored.
- If the object is optically denser than the surrounding ($n_2 > n_1$), then not all refracted rays leave the object after the second intersection with the boundary due to total internal reflection.
- Due to the nature of Snell's Law, the computations make heavy use of trigonometric functions.
- The use of Wolfram Alpha or software for symbolic computations can prove useful in simplifying various terms throughout the computation.
- The final result of the optional exercise is
$F(x) = x - \sqrt{R^2 - x^2}\ \tan\left(\sin^{-1}\left(\frac{x}{R}\right) - \sin^{-1}\left(\frac{n_1 x}{n_2 R}\right)\right) - d\tan\left(\sin^{-1}\left(\frac{n_2}{n_1}\sin\left(\sin^{-1}\left(\frac{x}{R}\right) - \sin^{-1}\left(\frac{n_1 x}{n_2 R}\right)\right)\right)\right)$, where $|x| \le \frac{n_1 R}{\sqrt{-2 n_1 \sqrt{n_2^2 - n_1^2} + n_1^2 + n_2^2}}$ for $n_1 < n_2$.

■ Problem 4

The following figures show the behavior of $F(x)$ over x for different constellations of refractive indices. The red line depicts the case of no intermediate object and serves as a reference. Interpret what you can observe.