Volumetric Data Registration

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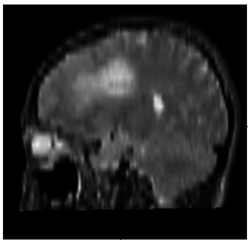
What Is It All About?

- Different modalities provide us with complementary information.
- Combination of those enhances the possibilities for interpretation.
- There is an ever-increasing need for accuracy and speed.
- Registration is NOT fusion!!!

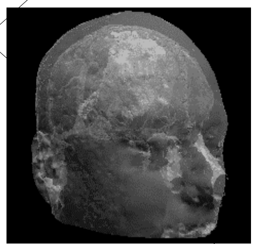
An Example: SPGR+T2

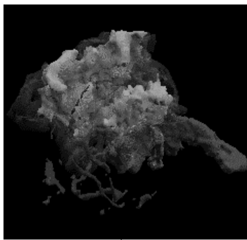
SPGR



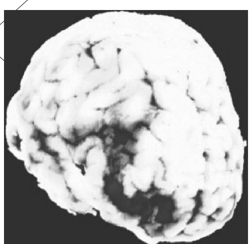
T2

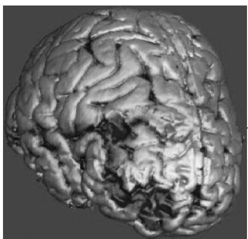
3D-Reconstructions





Another Example: MRI+SPECT





Informal Statement of the Problem

Develop a method which transforms geometrically the point samples of one data set to the point samples of another one in such a way that they fit together optimally.

Identification of the Involved Subproblems

- Matching Transformation
  - – Correlation
  - – Alignment
- (Dis)similarity Function
  - – Misregistration
  - – Cost
- Optimization Task

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Overview of Registration Methods

- Classification according to different criteria:
  - – What is to be registered?
  - – How is it to be registered?
  - – When is it to be registered?
  - – Why is it to be registered?

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What?

- Inter- vs. Intra-modality
- Inter- vs. Intra-subject
- Image-to-atlas
- 2D vs. 3D

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How?

- Rigid – affine – non-linear
- Extrinsic – intrinsic
- Points – lines – surfaces – voxels
- Interactive – semi-automatic – automatic

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When?

- Prospective
- Realtime
- Retrospective

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Why?

- Diagnostics
- Treatment planning
- Image-guided surgery
- Treatment evaluation

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Popular Methods

- Stereotactic frame
- Fiducial marks
- Principal axes
- Atlas-oriented
- Surface similarity
- Voxel-based

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Formal Statement of the Problem

- Definitions:

Let

$$\mathbf{M} = \{ f_m | \text{is localizable model feature} \}$$

and

$$\mathbf{O} = \{ f_o | \text{is localizable object feature} \}$$

be model resp. object.

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Find a *matching transformation*

$$\tau : \mathrm{R}^3 \rightarrow \mathrm{R}^3$$

specified by a *parameter vector*

$$v \in \mathrm{M} \subset \mathrm{R}^3$$

where

$$\mathrm{M} = \{ v \in \mathrm{R}^k : r_0^{\min} \leq v_0 \leq r_0^{\max}, \cdots ; r_k^{\min} \leq v_k \leq r_k^{\max} \}$$

is the *feasible region*

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such that a *cost function*

$$C : \mathrm{R}^k \rightarrow \mathrm{R}$$

takes its *optimum* in

$$v_{opt} = \arg\min \{ C(v) | v \in \mathrm{M} \}$$

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Matching Transformations

- Rigid-body transformations
  - Translations
  - Rotations
  - Scalings
- Affine transformations
  - Reflexion
  - Stretch
  - Skew
- Non-linear transformations

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Rigid-body Transformations

- Translation:

$$T_d = \begin{Bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ d_x & d_y & d_z & 1 \end{Bmatrix}$$

- Rotation:

$$R_x = \begin{Bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha & 0 \\ 0 & -\sin\alpha & \cos\alpha & 0 \\ d_x & d_y & d_z & 1 \end{Bmatrix}$$

- Scaling:

$$S = \begin{Bmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{Bmatrix}$$

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Combined Transformation

The transformation τ mediated by

$$T = T_o \circ S_o \circ T_d \circ R_x \circ R_y \circ R_z \circ S_r \circ T_r$$

transforms an object point to a model point

$$\tau(P_o) = P_m = P_o \circ T$$

Affine Transformations

- General form:

$$T = \begin{Bmatrix} a_{00} & a_{01} & a_{02} & 0 \\ a_{10} & a_{11} & a_{12} & 0 \\ a_{20} & a_{21} & a_{22} & 0 \\ a_{30} & a_{31} & a_{32} & 1 \end{Bmatrix}$$

Non-linear Transformations

- Polynomial form:

$$\begin{Bmatrix} x_m = \sum_{p=0}^{n}\sum_{q=0}^{m}\sum_{r=0}^{l} a_{pqr} x_o^p y_o^q z_o^r \\ y_m = \sum_{p=0}^{n}\sum_{q=0}^{m}\sum_{r=0}^{l} b_{pqr} x_o^p y_o^q z_o^r \\ z_m = \sum_{p=0}^{n}\sum_{q=0}^{m}\sum_{r=0}^{l} c_{pqr} x_o^p y_o^q z_o^r \end{Bmatrix}$$

Transformations Summary

- Geometrical point transformations are the tool of registration
- There are different classes of them variously suited to different registration contexts
- Goal of the registration process is to find the best suited one

Cost Functions

- Surface-based
  - Employ spatial information
- Grayvalue-based
  - Employ histogram information

Desirable Properties of Cost Functions

- Continuous on the feasable region
- Invariant to imaging conditions and object positioning
- Global minimum determines anatomical optimum

Surface-based Cost Functions

- Features are surface points
- Euclidean distance of a point P to the model M:

$$d(P) = \min_{R \in M} \|P - R\|$$

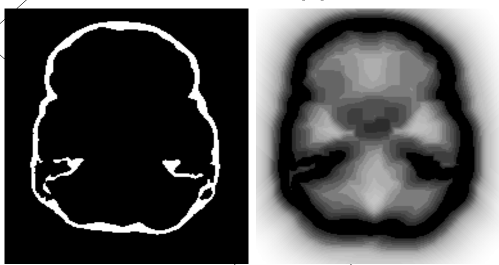
- Surface similarity measure:

$$C(v) = \sum_{i=1}^{N} d^2(\tau_v(P_i))$$

Digital Distance Transforms

- Binary image filter
- Labels voxels with their distance to objects
- Several types:
  - Euclidean
  - Chamfer
  - Chessboard
  - Cityblock

An Image with Distance Transform Applied



Grayvalue-based Similarity Functions

- Cross-correlation
  - Multiplicative
  - Subtractive
- Scatter-plot based
  - Histogram moments
  - Information entropy
  - Mutual information

Multiplicative Cross-correlation

- Defined as

$$C(v) = \frac{\sum_{i=1}^{N} g_m(\tau_v(P_i)) \cdot g_o(P_i)}{\sqrt{\sum_{i=1}^{N} g_m^2(\tau_v(P_i))} \cdot \sqrt{\sum_{i=1}^{N} g_o^2(P_i)}}$$

- where g_m, g_o are model resp. object grayvalues.

Subtractive Cross-correlation

- Absolute differences instead of products:

$$C(v) = \frac{\sum_{i=1}^{N} \left| g_m(\tau_v(P_i)) - g_o(P_i) \right|}{\sqrt{\sum_{i=1}^{N} g_m^2(\tau_v(P_i))} \cdot \sqrt{\sum_{i=1}^{N} g_o^2(P_i)}}$$

Feature-space Histogram or Scatter-plot

- Maps the set of ordered grayvalue pairs into a set of counts:

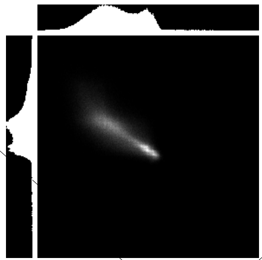
$$S: G_o \times G_m \to G_s \subset N$$

- Each value (scatter-plot pixel) represents the number of such ordered pairs:

$$S_\tau(g_o, g_m) = \left|\left\{\left(g_o(P), g_m(\tau(P))\right)\right\}\right|$$

Scatter-plot: One Object

Scatter-plot: Two Objects



Scatter-plot Histogram

- Joint probabilities:

$$P(g_o, g_m) = P(\{g_o, g_m\}) = \frac{S_\tau(g_o, g_m)}{N}$$

- 1D Scatter-plot histogram: $h(p_i)$

Scatter-plot Histogram Moments

- Third-order moments:

$$C(v) = -\sum_{i=1}^{l} h(p_i) \cdot p_i^3$$

- Normalized version:

$$C(v) = -\frac{\sum_{i=1}^{l} h(p_i) \cdot p_i^3}{\sum_{i=1}^{l} h(p_i)}$$

Scatter-plot Histogram Entropy

- Information entropy is the amount of uncertainty in a probability distribution

$$H(X) = H(p_1, \ldots, p_l) = -\sum_{i=1}^{l} p_i \cdot \log(p_i)$$

- Information entropy cost function:

$$C(v) = -\sum_{g_i \in G_o} \sum_{g_j \in G_m} P(g_i, g_j) \log\left(P(g_i, g_j)\right)$$

Mutual Information

- Generalized mutual dependence:

$$C(v) = -\sum_{g_i \in G_o} \sum_{g_j \in G_m} P(g_i, g_j) \log \frac{P(g_i, g_j)}{P(g_i)P(g_j)}$$

- Mutual information in terms of entropies:

$$C(v) = H(g_o) + H(g_m) - H(g_o, g_m)$$

Similarity Functions Summary

- The quality of registration is measured by similarity functions
- The registration process searches a function parameter space for an optimal solution
- There is a great diversity of them
- They are subject of active research

Optimization Task

- The problem is
  - Multivariate
  - Continuous
  - Non-linear
  - Constrained
- Solution: numerical algorithms

Optimization Methods

- Exhaustive search
- Gradient-based
- Simulated annealing
- Genetic algorithms

Exhaustive search

- $N \times N \times N$ model
- Possibilities for object position: $O(N^3)$
- Possible (discrete) angle triples: $O(N^3)$
- Search space for rigid-body transformations: $O(N^6)$
- For $N = 256 \Rightarrow \approx 10^{14} \Rightarrow$ ES infeasible!!!

Gradient-based Optimization

- Hill-climbing techniques
- Generally deterministic
- Susceptible to local minima
- Alternative: genetic algorithms

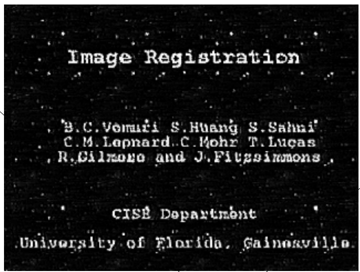
Simulated Annealing

- Probabilistic hill-climbing technique
- Based on thermodynamical processes
- Boltzmann´s distribution: $P(s) = e^{\frac{-E(s)}{k_B T}}$
- Better suited for situations with many local minima and one global optimum

Optimization Summary

- The registration process is generally formulated as an optimization task
- It can´t be solved in general by simple methods due to its complexity
- Numerical iterative approximation algorithms are used instead
- Simulated annealing and genetic algorithms are among the most promising ones

Movie demonstration of different techniques



Conclusions

- There is no ideal registration method for medical volumetric data
- Feature-based methods can hardly be automated
- Scatter-plot methods do not take into account spatial information
- A cross-metrics hybrid approach combined with anatomical knowledge may be the solution

References

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